Title: Quantum Gravity

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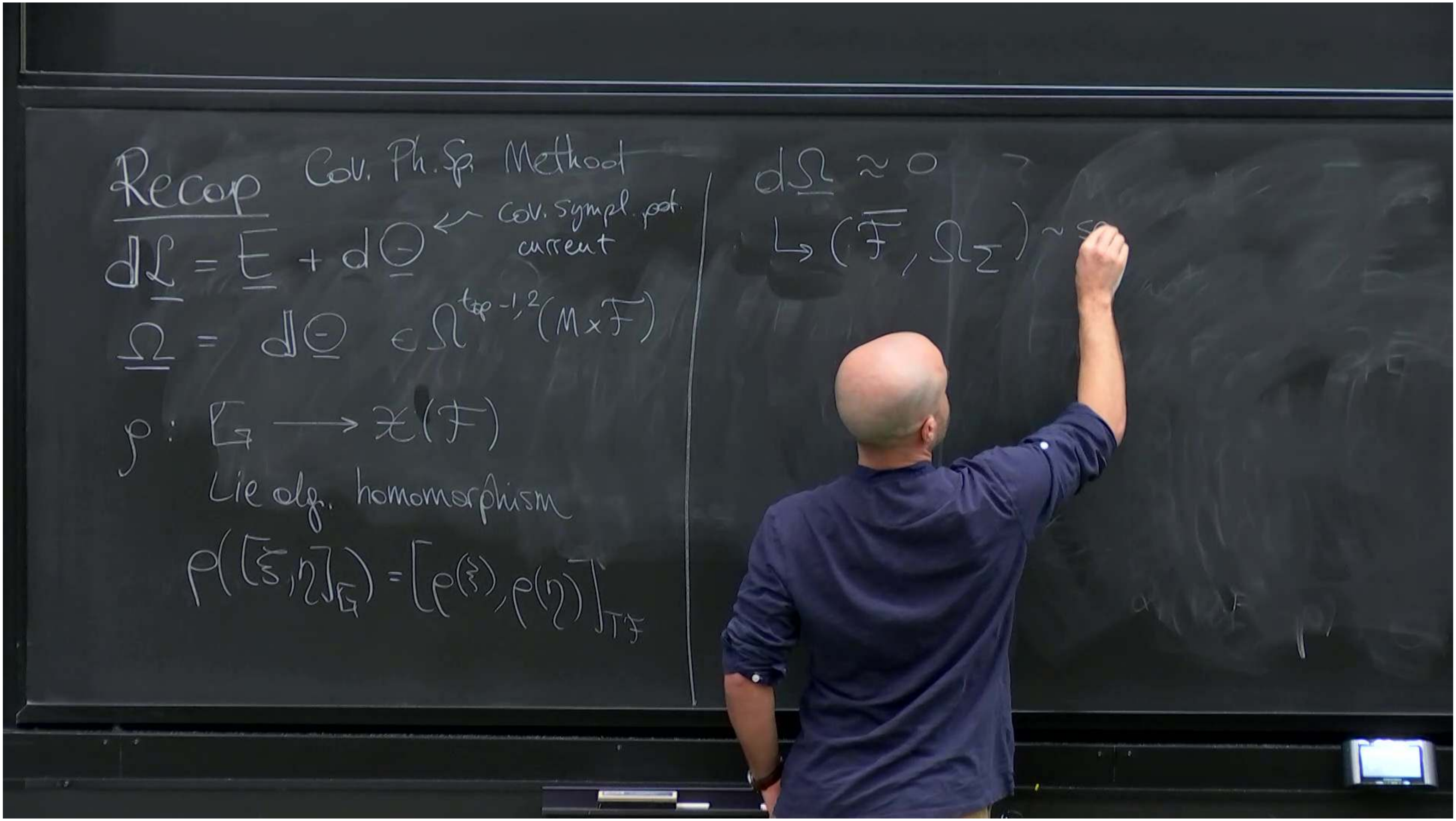
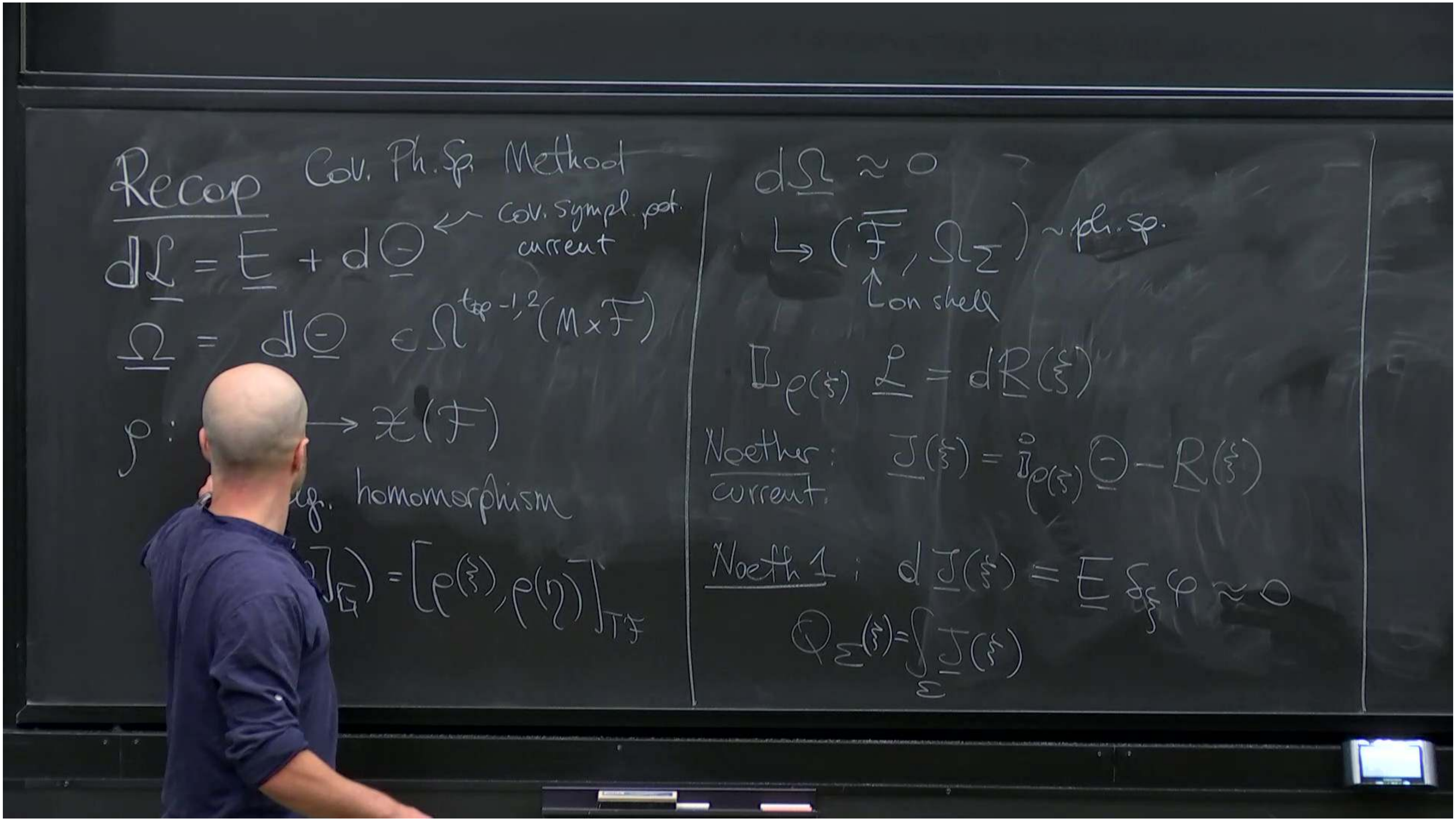
Collection: Quantum Gravity (2021-2022)

Date: April 18, 2022 - 10:15 AM

URL: https://pirsa.org/22040060

Abstract: Topics will include (but are not limited to): Canonical formulation of constrained systems, The Dirac program, First order formalism of gravity, Loop Quantum Gravity, Spinfoam models, Research at PI and other approaches to quantum gravity.

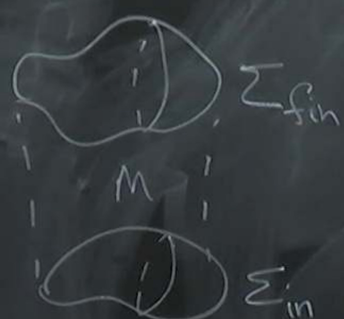

Recap Cov. Ph. Sp. Method
$d\underline{L} = \underline{E} + d\underline{\Theta}$ ← cov. sympl. pot. current
$\underline{\Omega} = d\underline{\Theta} \in \Omega^{top-1,2}(M \times \mathcal{F})$
$\rho : \mathfrak{G} \longrightarrow \mathfrak{X}(\mathcal{F})$
Lie alg. homomorphism
$\rho([\xi,\eta]_{\mathfrak{G}}) = [\rho(\xi), \rho(\eta)]_{T\mathcal{F}}$
$d\underline{\Omega} \approx 0$
$\hookrightarrow (\mathcal{F}, \Omega_\Sigma)$



Recap Cov. Ph. Sp. Method
$d\!\!\!d \underline{\mathcal{L}} = \underline{E} + d\underline{\Theta}$
cov. sympl. pot. current
$\underline{\Omega} = d\!\!\!d \underline{\Theta} \in \Omega^{top-1,2}(M \times \mathcal{F})$
$\rho : \ \longrightarrow \mathfrak{X}(\mathcal{F})$
homomorphism
$]_{\mathfrak{G}}) = [\rho(\xi), \rho(\eta)]_{T\mathcal{F}}$
$d\underline{\Omega} \approx 0$
$\hookrightarrow (\overline{\mathcal{F}}, \Omega_\Sigma) \sim$ ph. sp.
on shell
$\mathbb{L}_{\rho(\xi)} \underline{\mathcal{L}} = d\underline{R}(\xi)$
Noether current: $\underline{J}(\xi) = \mathbb{i}_{\rho(\xi)} \underline{\Theta} - \underline{R}(\xi)$
Noeth 1: $d\underline{J}(\xi) = \underline{E}\, \delta_\xi \varphi \approx 0$
$Q_\Sigma(\xi) = \int_\Sigma \underline{J}(\xi)$

Local sym (Noether 2)

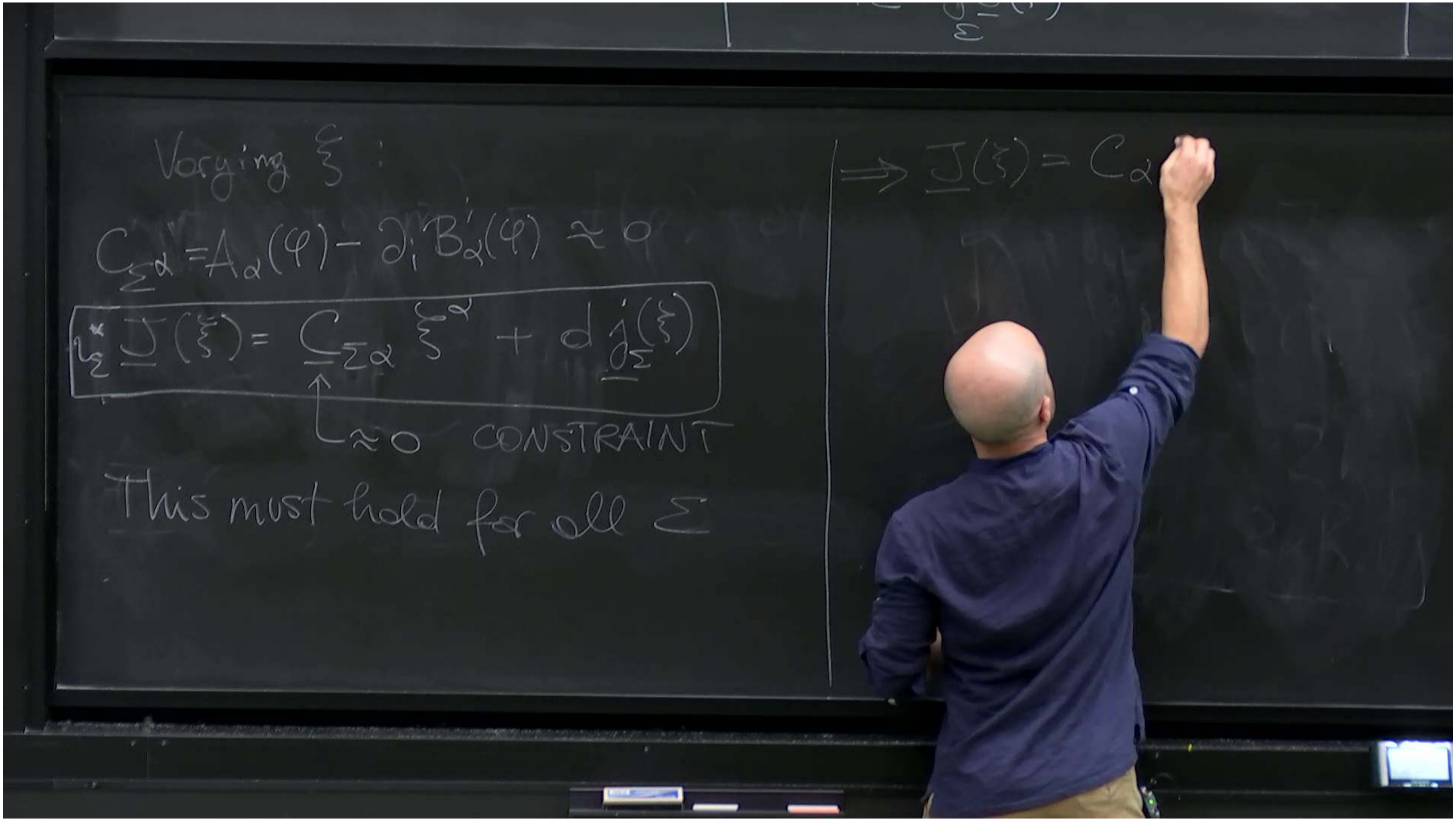
$\xi = \xi(x)$

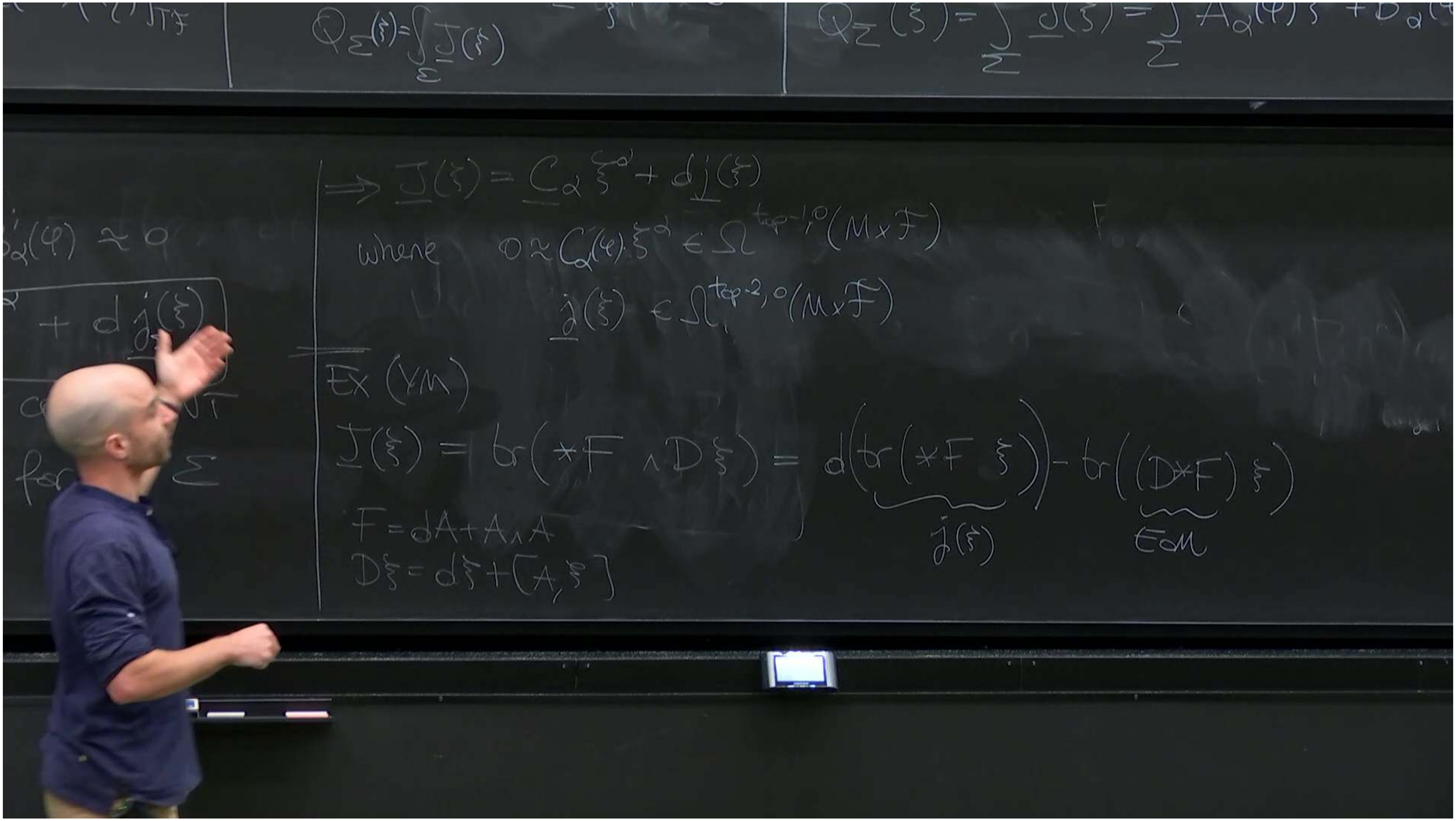


$Q_\Sigma(\xi) \approx 0$

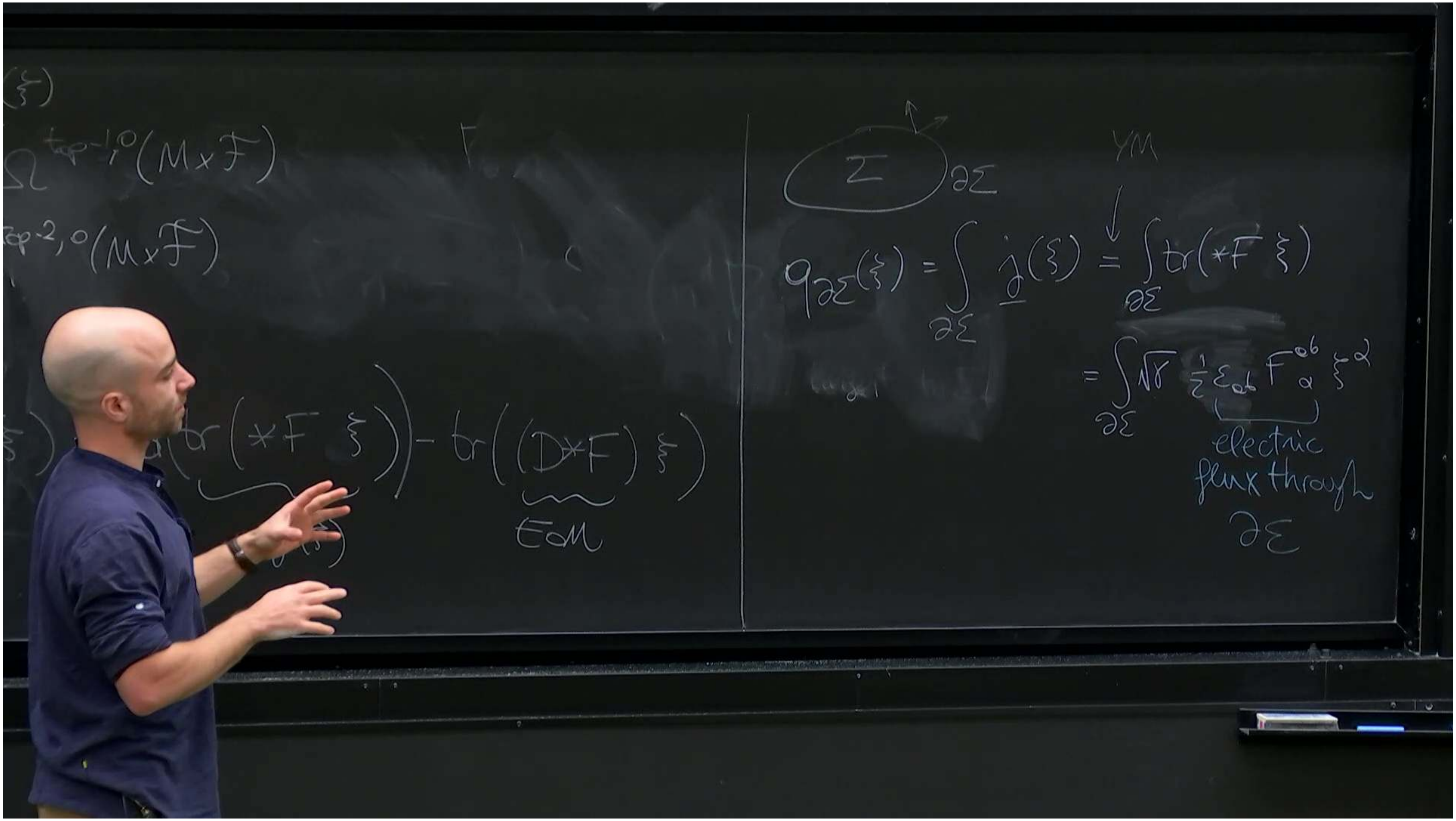
$$Q_\Sigma(\xi) \approx 0 \quad \forall \xi$$

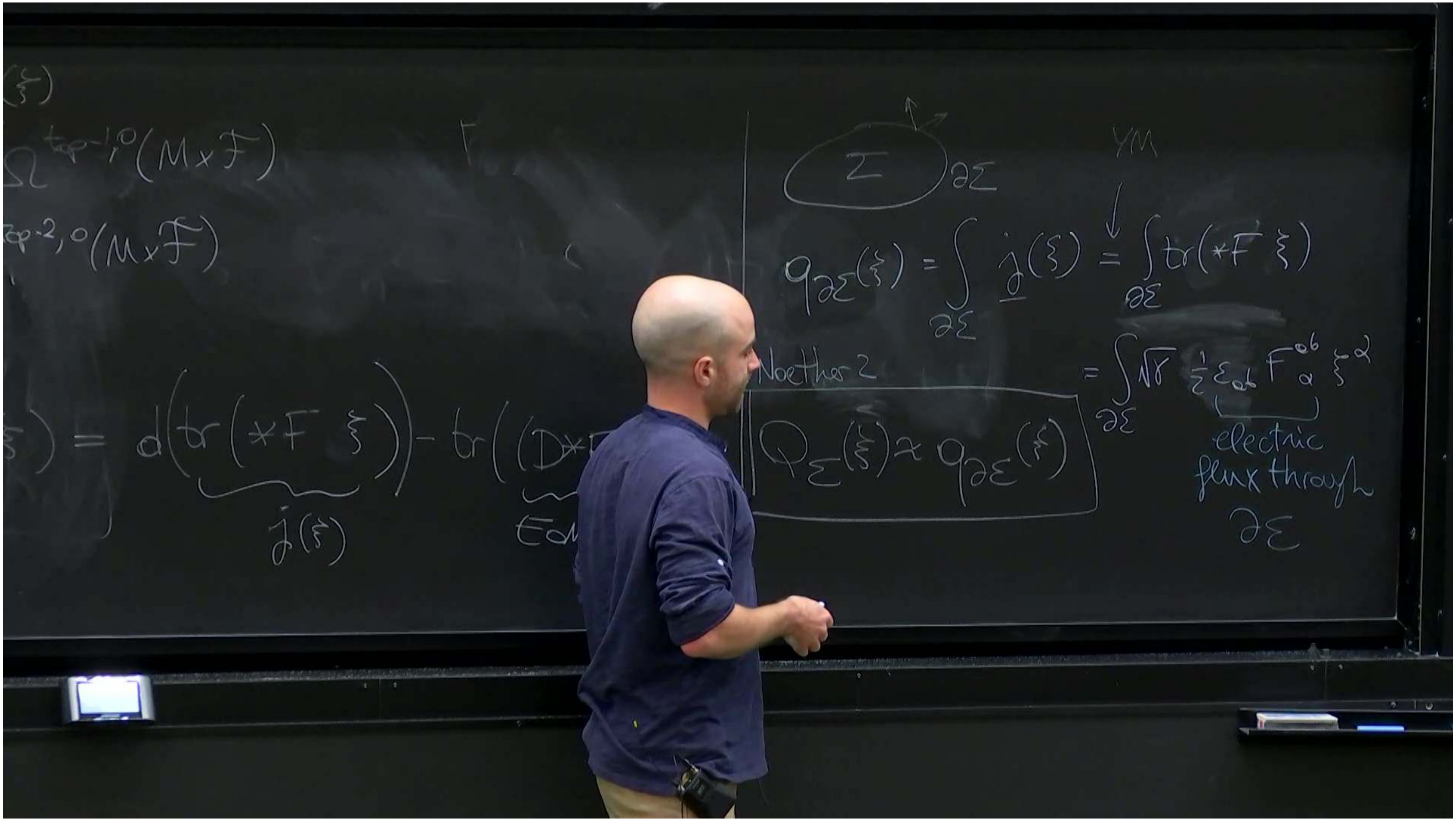
$$Q_\Sigma(\xi) = \int_\Sigma \underline{J}(\xi) = \int_\Sigma A_\alpha(\varphi)\,\xi^\alpha + B^i_\alpha(\varphi)\,\partial_i \xi^\alpha$$

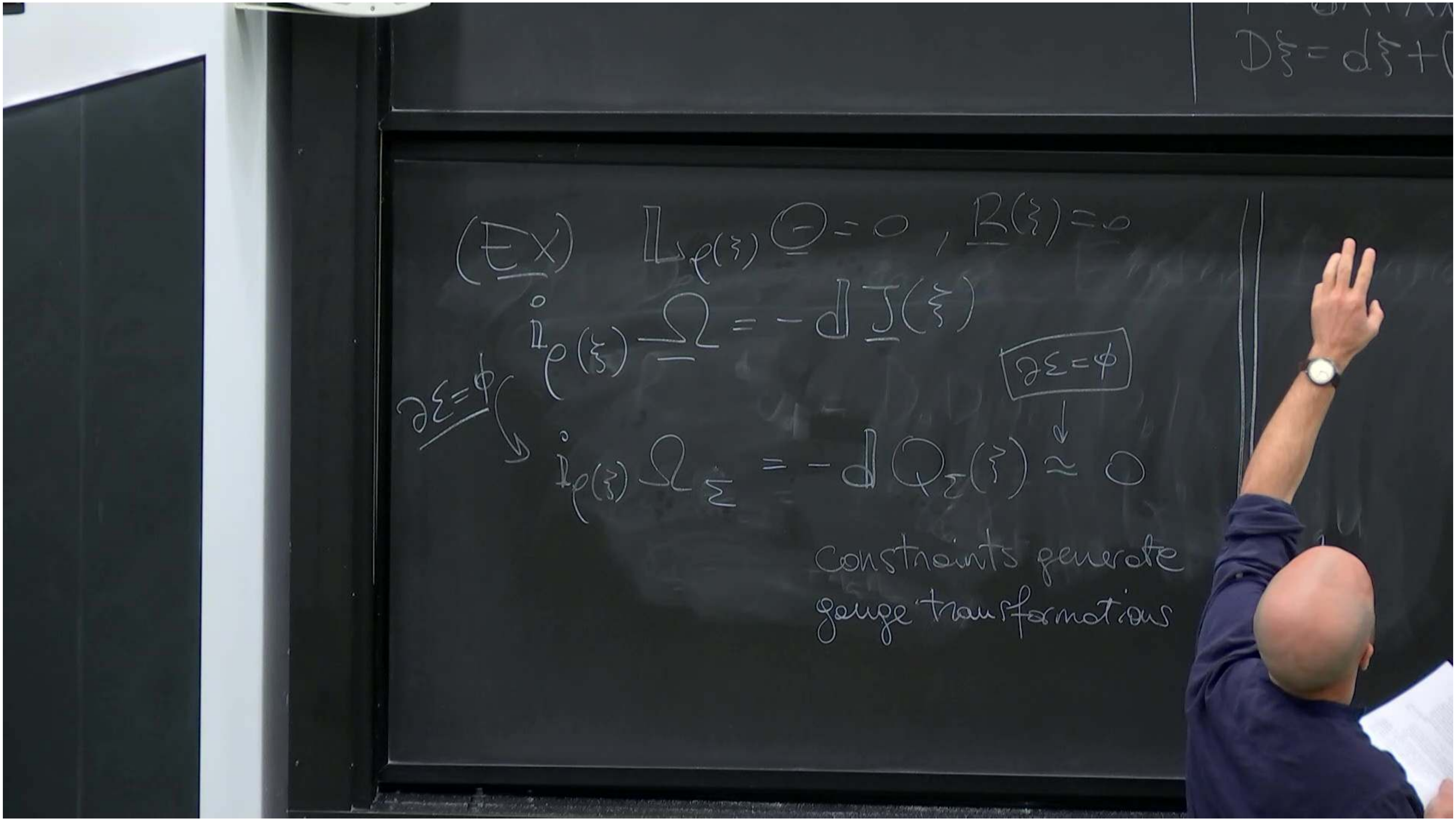

Varying ξ :
C_Σα = A_α(φ) − ∂_i B^i_α(φ) ≈ 0
ι*_Σ J(ξ) = C_Σα ξ^α + d j_Σ(ξ)
≈ 0 CONSTRAINT
This must hold for all Σ
⇒ J(ξ) = C_α



$Q_\Sigma(\xi) = \int_\Sigma \underline{J}(\xi)$
$Q_\Sigma(\xi) = \int_\Sigma \underline{J}(\xi) = \int_\Sigma A_\alpha(\varphi)\,\xi + B_\alpha$
$\Rightarrow \underline{J}(\xi) = \underline{C}_\alpha \xi^\alpha + d\underline{j}(\xi)$
where $0 \approx C_\alpha(\varphi)\cdot\xi^\alpha \in \Omega^{top-1,0}(M \times \mathcal{F})$
$\underline{j}(\xi) \in \Omega^{top-2,0}(M \times \mathcal{F})$
Ex (YM)
$\underline{J}(\xi) = tr(*F \wedge D\xi) = d(\underbrace{tr(*F\,\xi)}_{j(\xi)}) - tr((\underbrace{D*F}_{EoM})\,\xi)$
$F = dA + A \wedge A$
$D\xi = d\xi + [A, \xi]$


$\Omega^{top-1,0}(M\times\mathcal{F})$
$\Omega^{top-2,0}(M\times\mathcal{F})$
$\mathrm{tr}(*F\ \xi)) - \mathrm{tr}((D*F)\ \xi)$
EoM
Σ
∂Σ
YM
$q_{\partial\Sigma}(\xi) = \int_{\partial\Sigma} j(\xi) = \int_{\partial\Sigma} \mathrm{tr}(*F\ \xi)$
$= \int_{\partial\Sigma} \sqrt{\gamma}\ \frac{1}{2}\epsilon_{ab} F^{ab}{}_{\alpha}\ \xi^{\alpha}$
electric flux through ∂Σ


$\Omega^{top-1,0}(M \times \mathcal{F})$
$\Omega^{top-2,0}(M \times \mathcal{F})$
$= d(\underbrace{tr(*F\ \xi)}_{j(\xi)}) - tr((D*F$
Σ
$\partial\Sigma$
YM
$q_{\partial\Sigma}(\xi) = \int_{\partial\Sigma} \underline{j}(\xi) \doteq \int_{\partial\Sigma} tr(*F\ \xi)$
$= \int_{\partial\Sigma} \sqrt{\gamma}\ \frac{1}{2}\varepsilon_{ab} F^{ab}{}_{\alpha}\ \xi^{\alpha}$
Noether 2
$Q_\Sigma(\xi) \approx q_{\partial\Sigma}(\xi)$
electric
flux through
$\partial\Sigma$


$D\xi = d\xi + ($
(Ex) $\mathbb{L}_{\rho(\xi)} \underline{\Theta} = 0$, $\underline{B}(\xi) = 0$
$\mathring{\mathbb{I}}_{\rho(\xi)} \underline{\Omega} = -d \underline{J}(\xi)$
$\boxed{\partial\Sigma = \phi}$
$\partial\Sigma = \phi$
$\mathring{\mathbb{I}}_{\rho(\xi)} \Omega_{\Sigma} = -d Q_{\Sigma}(\xi) \approx 0$
constraints generate
gauge transformations

$D\xi = d\xi + [A, \xi]$

GENERAL RELATIVITY

$$S = \int_M \underline{L}$$

$$\underline{L} = \underline{\epsilon}\left(\frac{1}{2} g^{ab} R_{ab} - \Lambda\right)$$

$\underline{\epsilon} = \sqrt{g}\, d^{top}x$

$$\mathcal{F} = \{ g_{ab}(x) \}$$

$$\mathbb{d}\varphi^I \equiv \mathbb{d} g_{ab}$$

$$\mathbb{d} g^{ab} := g^{aa'} g^{bb'}\, \mathbb{d} g_{a'b'} \,, \quad \mathbb{d} g := g^{ab}\, \mathbb{d} g_{ab}$$

$\mathbb{d}\underline{L}$

...VITY

$$\mathcal{L} = \epsilon\left(\frac{1}{2} g^{ab} R_{ab} - \Lambda\right)$$

$$\epsilon \leftarrow \sqrt{g}\, d^{top}x$$

$$, \quad \delta g = g^{ab}\, \delta g_{ab}$$

$$\delta \mathcal{L} = E_{ab}\, \delta g^{ab} + d\Theta$$

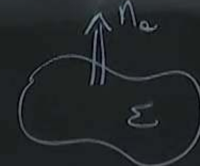
$$E_{ab}\, \delta g^{ab} = -\frac{1}{2}\epsilon\left(G_{ab} + \Lambda g_{ab}\right)\delta g^{ab}$$

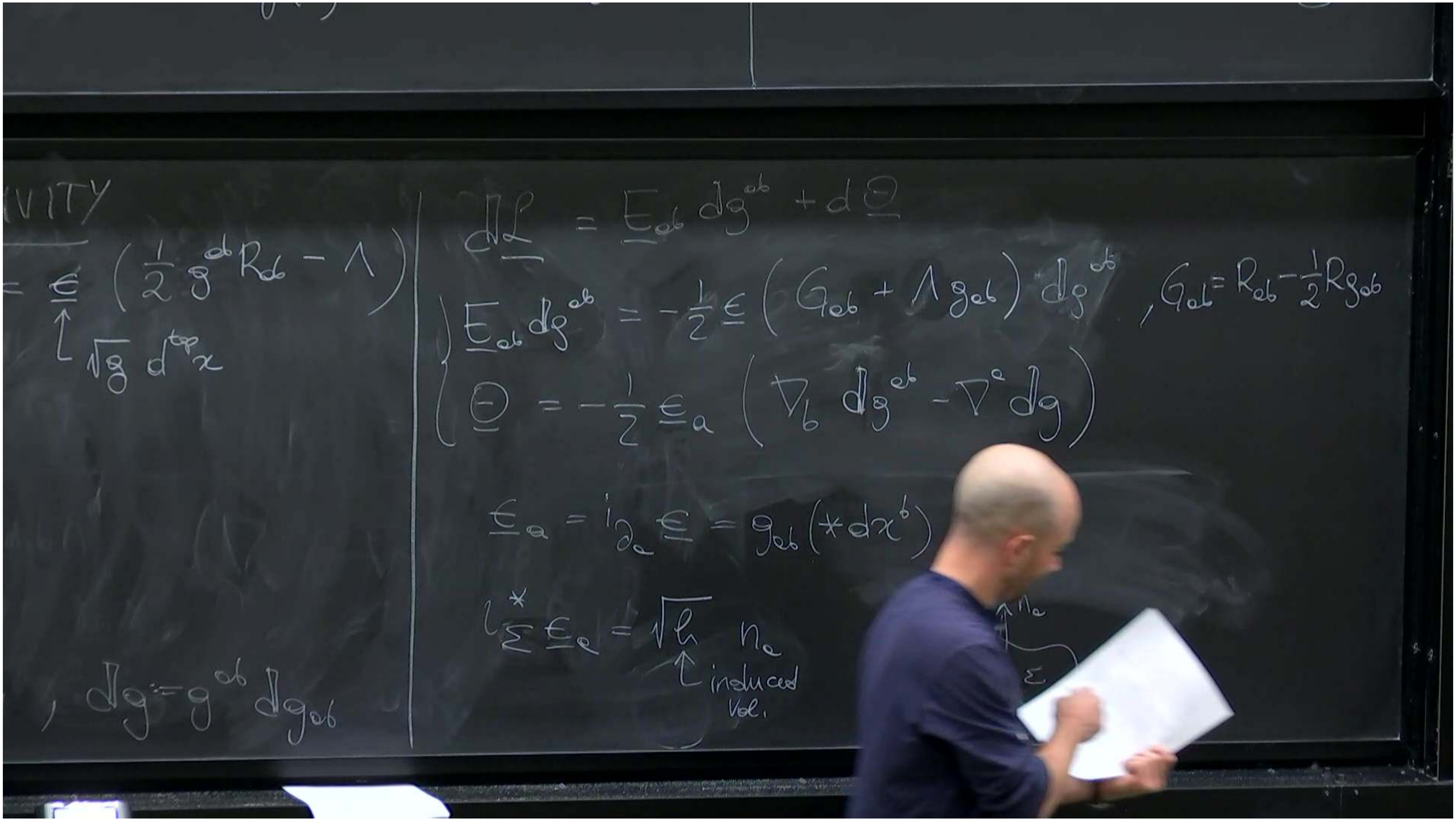
$$\Theta = -\frac{1}{2}\epsilon_a\left(\nabla_b\, \delta g^{ab} - \nabla^a \delta g\right)$$

$$\epsilon_a = i_{\partial_a}\epsilon = g_{ab}\left(* dx^b\right)$$

$$\iota_\Sigma^* \epsilon_a = \sqrt{h}\; n_a$$

$\sqrt{h}$: induced vol.





VITY
$= \underline{\epsilon} \left(\frac{1}{2} g^{ab} R_{ab} - \Lambda \right)$
$\sqrt{g}\, d^{top}x$
$, \; \mathbb{d}g := g^{ab} \mathbb{d}g_{ab}$
$\mathbb{d}\underline{L} = \underline{E}_{ab}\, \mathbb{d}g^{ab} + d\underline{\Theta}$
$\underline{E}_{ab}\, \mathbb{d}g^{ab} = -\frac{1}{2} \underline{\epsilon} \left(G_{ab} + \Lambda g_{ab} \right) \mathbb{d}g^{ab}$
$, \; G_{ab} = R_{ab} - \frac{1}{2} R g_{ab}$
$\underline{\Theta} = -\frac{1}{2} \underline{\epsilon}_a \left(\nabla_b \mathbb{d}g^{ab} - \nabla^a \mathbb{d}g \right)$
$\underline{\epsilon}_a = i_{\partial_a} \underline{\epsilon} = g_{ab} (\star dx^b)$
$\iota^*_\Sigma \underline{\epsilon}_a = \sqrt{h}\, n_a$
induced vol.
n_a
Σ

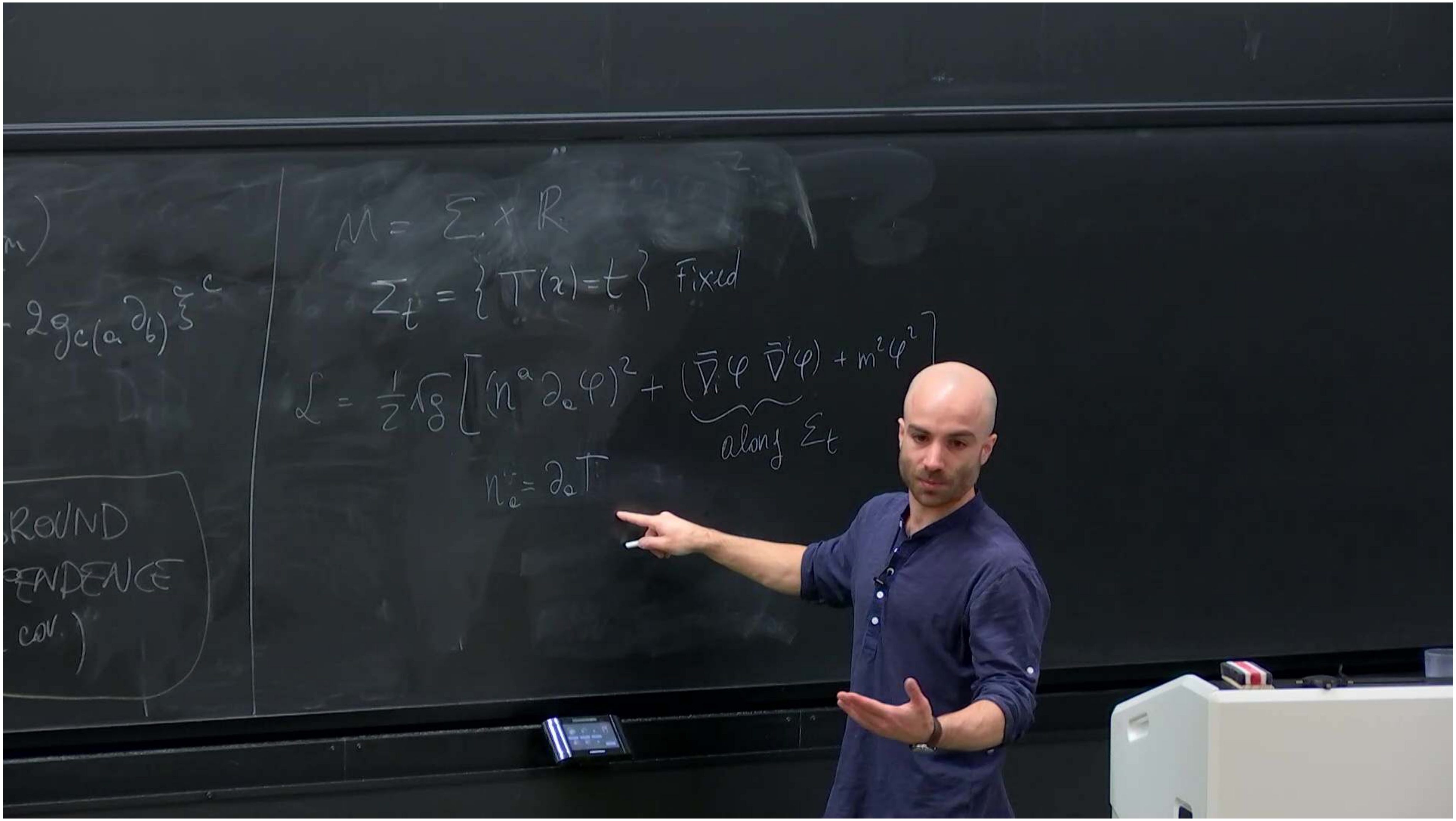
Diffeomorphisms

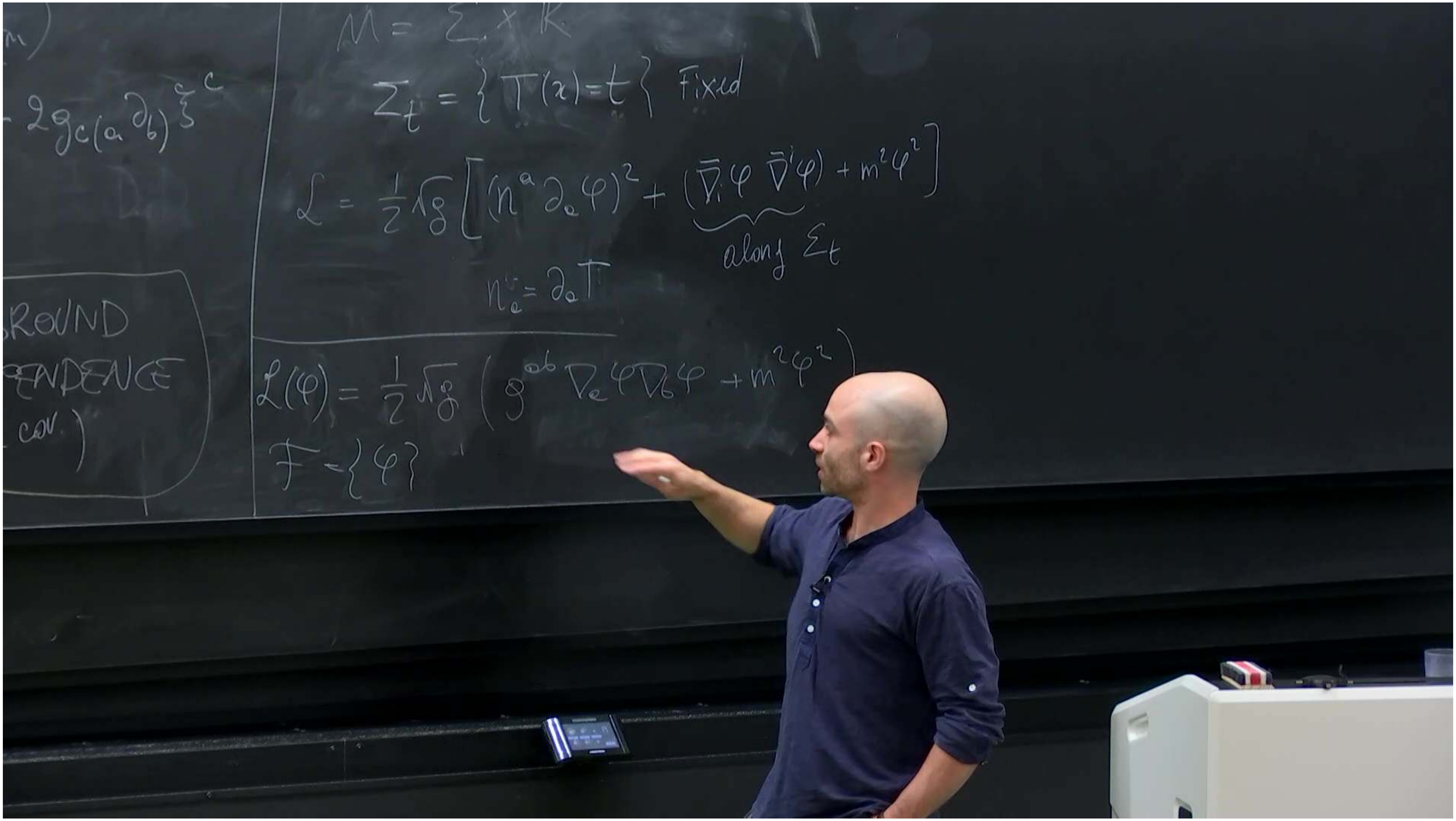
$$G = \mathrm{diff}(M) \simeq (\mathfrak{X}(M), [\cdot,\cdot]_{TM})$$

$$\delta_\xi g_{ab} = L_\xi g_{ab} = \xi^c \partial_c g_{ab} - 2 g_{c(a} \partial_{b)} \xi^c$$

$$= 2 \nabla_{(a} \xi_{b)}$$

$$\boxed{\mathbb{L}_{\rho(\xi)} \underline{\mathcal{L}} = L_\xi \underline{\mathcal{L}} \quad \text{BACKGROUND INDEPENDENCE (general cov.)}}$$


M = Σ × R
Σt = { T(x) = t } Fixed
along Σt
ROUND
ENDENCE
cov.)


$M = \Sigma \times \mathbb{R}$
$\Sigma_t = \{T(x) = t\}$ Fixed
$\mathcal{L} = \frac{1}{2}\sqrt{g}\left[(n^a \partial_a \varphi)^2 + (\underbrace{\bar{\nabla}_i \varphi \bar{\nabla}^i \varphi}_{\text{along } \Sigma_t}) + m^2\varphi^2\right]$
$n_a = \partial_a T$
$\mathcal{L}(\varphi) = \frac{1}{2}\sqrt{g}\left(g^{ab}\nabla_a\varphi\nabla_b\varphi + m^2\varphi^2\right)$
$F = \{\varphi\}$
$2g_{c(a}\partial_{b)}\xi^c$
ROUND
ENDENCE
cov.

Since $\underline{L} \in \Omega^{top,0}(M \times F)$

$$L_\xi \underline{L} = \cancel{i_\xi d \underline{L}} + d i_\xi \underline{L}$$

$$\Rightarrow d\underline{R}(\xi) = \mathbb{L}_{\rho(\xi)} \underline{L} = d i_\xi \underline{L}$$

bckgrnd ind.

$$\Rightarrow \boxed{\underline{R}(\xi) = i_\xi \underline{L}}$$

$$= \left(\frac{1}{2} R - \Lambda\right)$$

Since $\underline{L} \in \Omega^{top,0}(M \times \mathcal{F})$

$$L_\xi \underline{L} = \cancel{i_\xi d \underline{L}} + d i_\xi \underline{L}$$

$$\Rightarrow \quad d\underline{R}(\xi) = \mathbb{L}_{\rho(\xi)} \underline{L} \underset{\text{bckgrnd ind.}}{=} d i_\xi \underline{L}$$

$$\Rightarrow \quad \boxed{\underline{R}(\xi) = i_\xi \underline{L}}$$

$$= \left(\tfrac{1}{2} R - \Lambda\right) \underbrace{i_\xi \underline{\epsilon}}_{\xi^a \underline{\epsilon}_a}$$