Title: Quantum Gravity

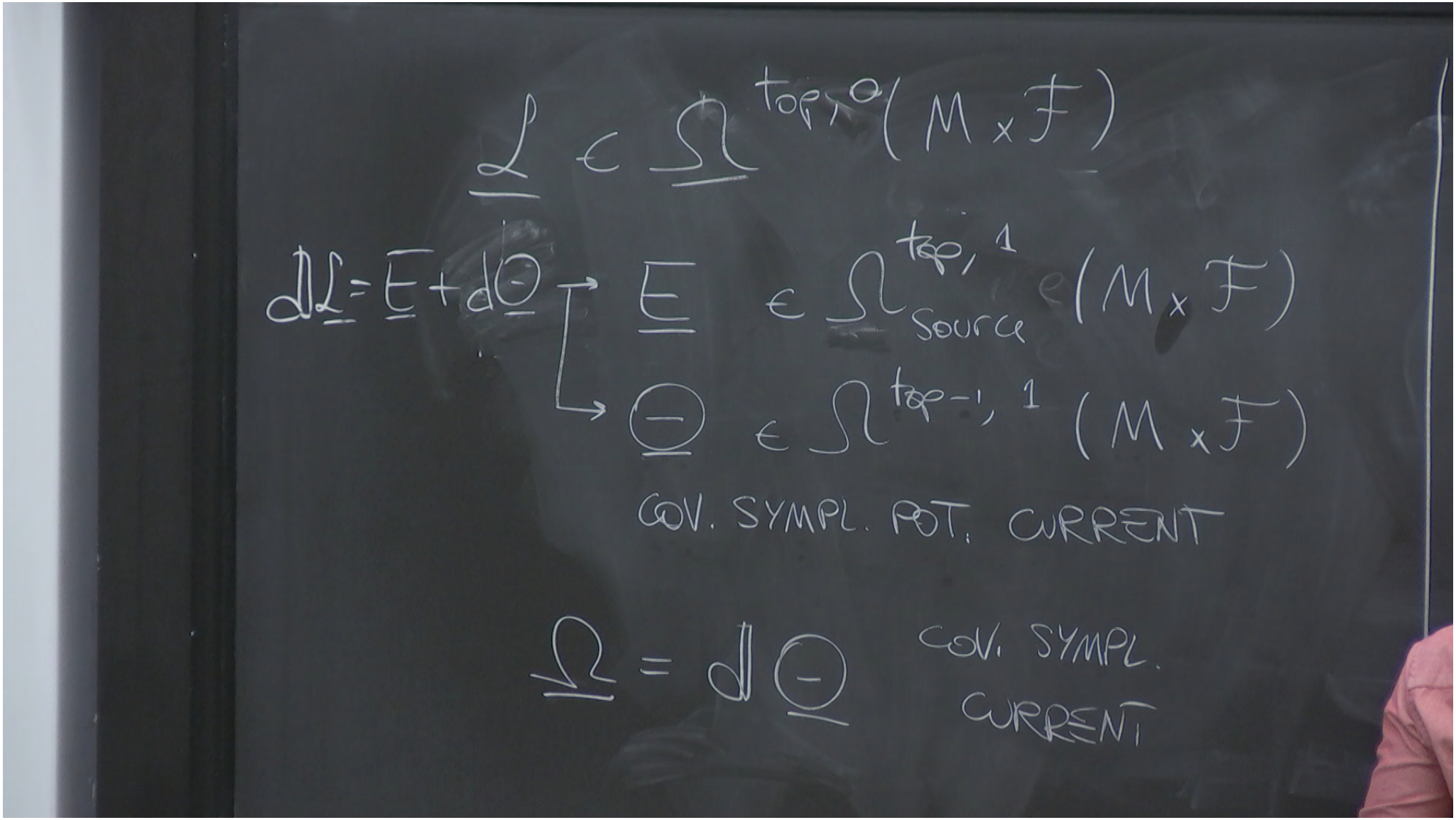
Speakers: Aldo Riello

Collection: Quantum Gravity (2021-2022)

Date: April 14, 2022 - 10:15 AM

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Abstract: Topics will include (but are not limited to): Canonical formulation of constrained systems, The Dirac program, First order formalism of gravity, Loop Quantum Gravity, Spinfoam models, Research at PI and other approaches to quantum gravity.


$\underline{\mathcal{L}} \in \underline{\Omega}^{top,0}(M \times \mathcal{F})$
$d\underline{\mathcal{L}} = \underline{E} + d\underline{\Theta}$
$\underline{E} \in \underline{\Omega}^{top,1}_{source}(M \times \mathcal{F})$
$\underline{\Theta} \in \underline{\Omega}^{top-1,1}(M \times \mathcal{F})$
COV. SYMPL. POT. CURRENT
$\underline{\Omega} = d\underline{\Theta}$
COV. SYMPL. CURRENT

THM

$d\Omega \approx 0$ on-shell

$(\mathcal{F}, \Omega_\Sigma) \sim$ ph. sp.

$\Sigma \hookrightarrow M$

$\Omega_\Sigma = \int_\Sigma \Omega \in \Omega^2(\mathcal{F})$

$\Omega^{top,1}_{Source}(M \times \mathcal{F})$

$\Omega^{top-1,1}(M \times \mathcal{F})$

SYMPL. POT. CURRENT

COV. SYMPL. CURRENT

THM

$d\underline{\Omega} \approx 0$ on-shell

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$\Sigma \hookrightarrow M$

$\Omega_\Sigma = \int_\Sigma \underline{\Omega} \in \Omega^2(\mathcal{F})$

LAGRANGIA SYMS

$\mathfrak{G}$ Lie alg.

$\rho : \mathfrak{G} \longrightarrow \mathfrak{X}^1(F)$ action

$$\rho([\xi,\eta]) = [\rho(\xi),\rho(\eta)]$$

DEF Lagr. Sym

$$\mathbb{L}_{\rho(\xi)} \underline{L} = d\underline{R}(\xi)$$

$\underline{R}(\xi) \in \Omega^{top-1,0}(M \times F)$ linear in ξ

F) action

$(\xi), \rho(\eta)]$

$= d\underline{R}(\xi)$

$\Omega^{\text{top}-1,0}(M \times \mathcal{F})$

THM (Noether 1)

def: $\underline{J}(\xi) := i_{\rho(\xi)} \underline{\Theta} - \underline{R}(\xi)$

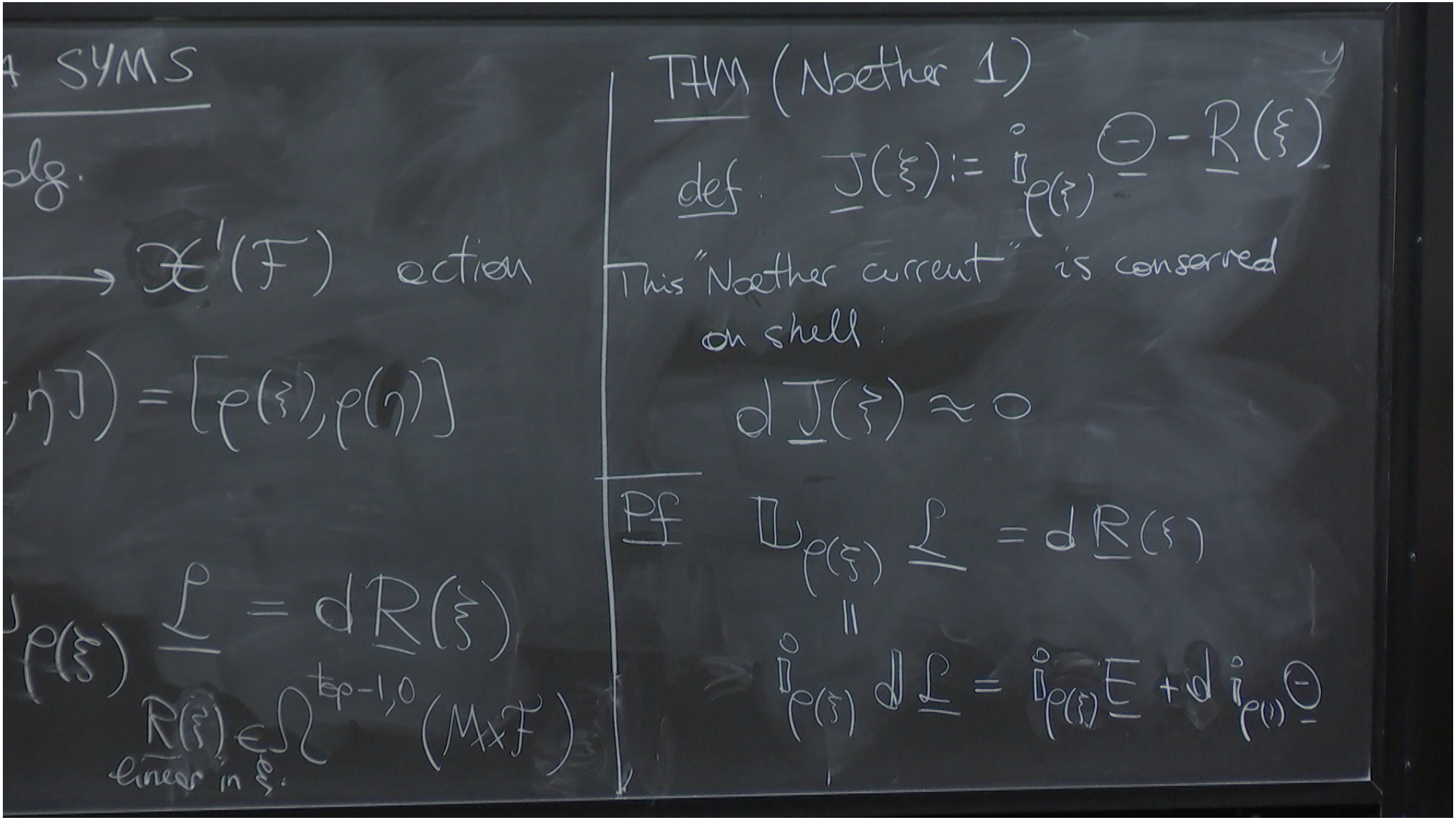
This "Noether current" is conserved on shell:

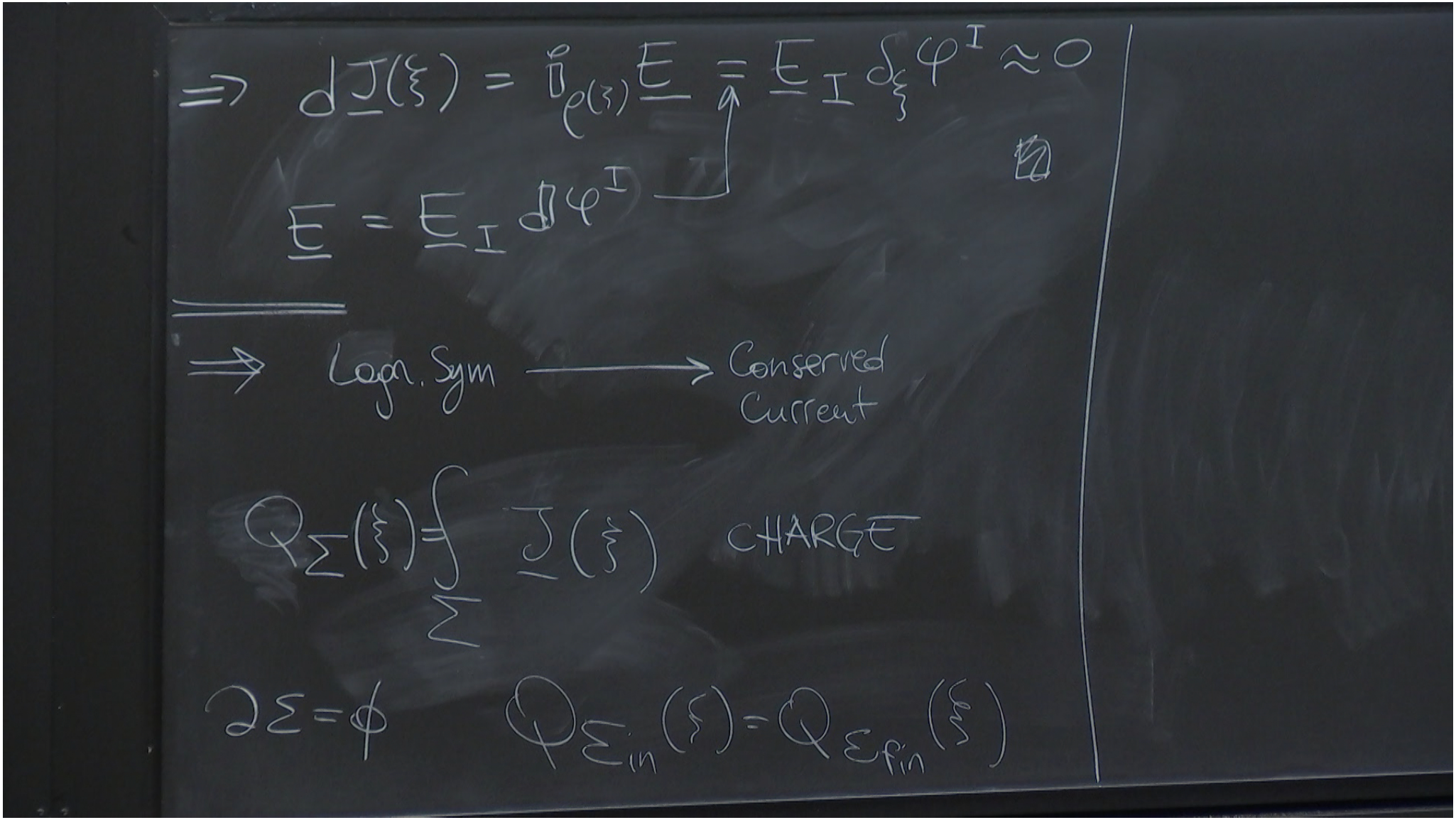
$$d\underline{J}(\xi) \approx 0$$

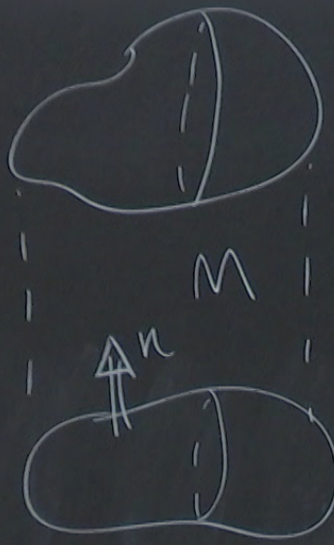
Pf $\quad \mathbb{L}_{\rho(\xi)} \underline{\mathcal{L}} = d\underline{R}(\xi)$

$$\| \quad \text{(Cartan)}$$

$$i_{\rho(\xi)} d\underline{\mathcal{L}} =$$


4 SYMS
dg.
$\longrightarrow \mathfrak{X}^1(\mathcal{F})$ action
$,\eta]) = [\rho(\xi),\rho(\eta)]$
$\rho(\xi)\ \underline{\mathcal{L}} = d\underline{R}(\xi)$
$\underline{R}(\xi) \in \Omega^{top-1,0}(M\times\mathcal{F})$
linear in ξ.
THM (Noether 1)
def: $\underline{J}(\xi) := i_{\rho(\xi)}\underline{\Theta} - \underline{R}(\xi)$
This "Noether current" is conserved on shell:
$d\underline{J}(\xi) \approx 0$
Pf $L_{\rho(\xi)}\underline{\mathcal{L}} = d\underline{R}(\xi)$
$i_{\rho(\xi)} d\underline{\mathcal{L}} = i_{\rho(\xi)}\underline{E} + d\, i_{\rho(\xi)}\underline{\Theta}$


$\Rightarrow d\underline{J}(\xi) = i_{\rho(\xi)}\underline{E} = \underline{E}_I \delta_\xi \varphi^I \approx 0$
$\underline{E} = \underline{E}_I d\varphi^I$
$\Rightarrow$ Log. Sym $\longrightarrow$ Conserved Current
$Q_\Sigma(\xi) = \int_\Sigma \underline{J}(\xi)$ CHARGE
$\partial\Sigma = \phi$ $Q_{\Sigma_{in}}(\xi) = Q_{\Sigma_{fin}}(\xi)$



Example

$$\mathcal{L} = \left(\sum_\alpha \frac{m_\alpha}{2} \|\dot{q}_\alpha\|^2 - \sum_{\alpha<\beta} V(\|q_\alpha - q_\beta\|) \right) dt$$

$M = \mathbb{R}$ (time), $\dim(M) = 1$

$$\mathcal{F} = \{ \vec{q}_\alpha(t) \}$$

$$\Theta = \sum_\alpha m_\alpha \dot{q}_\alpha^i \, dq_\alpha^i(t) \in \Omega^{0,1}(M \times \mathcal{F})$$

Example

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$$\Theta = \sum_\alpha m_\alpha \dot{q}_\alpha^i \, dq_\alpha^i(t) \in \Omega^{0,1}(M \times \mathcal{F})$$

$$P(\xi) = \sum_\alpha \epsilon_{ij}{}^k \xi^i q_\alpha^j(t) \frac{\delta}{\delta q_\alpha^k(t)}$$

Rot. $\xi \in so(3) \cong \mathbb{R}^3$

$$\sum_{\alpha<\beta} V(\|q_\alpha - q_\beta\|)\Big)\, dt$$

$M = \mathbb{R}$ (time)

$\dim(M) = 1$

$$d\, q_\alpha^k(t) \in \Omega^{0,1}(M \times \mathcal{F})$$

$$\xi^i q_\alpha^j(t) \Big) \frac{\delta}{\delta q_\alpha^k(t)}$$

$$\cong \mathbb{R}^3$$

$$J(\xi) = i_{\rho(\xi)} \Theta \in \Omega^{1,0}(M \times \mathcal{F})$$

$$= \sum_\alpha m_\alpha \dot{q}_\alpha^k \epsilon_{ij}{}^k \xi^i q_\alpha^j(t)$$

$$= \Big[\sum_\alpha (\vec{q}_\alpha \times \vec{p}_\alpha) \Big] \cdot \vec{\xi}$$

$$\partial_t J(\xi) \approx 0$$

$M = \mathbb{R}$ (time)

$M \times F$)

$$= \sum_\alpha m_\alpha \dot{q}_\alpha^k \epsilon_{ij}^k \xi^i q_\alpha^j (t)$$

$$= \left[\sum_\alpha (\vec{q}_\alpha \times \vec{p}_\alpha) \right] \cdot \vec{\xi}$$

$\partial_t J(\xi) \approx 0$

$Q_\Sigma = \{t, J(\xi)\}$ = ang. mom.

LEMMA (Best Case Scenario)

if (1) $\mathbb{L}_{e(\xi)} \underline{\mathcal{L}} = 0$ $\quad (\mathrm{d}\underline{R}(\xi) \equiv 0)$

(2) $\mathbb{L}_{e(\xi)} \underline{\Theta} = 0$

$$\Longrightarrow \boxed{\mathrm{i}_{e(\xi)} \underline{\Omega} = -\mathrm{d}\underline{J}(\xi)}$$

cov. HAM Flow Eq.

Pf Cartan's formula on (2)

THM (general case)

$$i_{\rho(\xi)}\underline{\Omega} = -\,d\underline{J}(\xi) + \underline{r}(\xi) + d\,\underline{s}(\xi)$$

$\underline{r}(\xi)$: bulk obstr (top-1, 0)

$\underline{s}(\xi)$: corner obstr. (top-2, 0)

where:

- $\mathbb{L}_{\rho(\xi)}\underline{E} = d\underline{r}(\xi)$
- $\mathbb{L}_{\rho(\xi)}\Theta - d\underline{R}(\xi) - \underline{r}(\xi) = d\,\underline{s}(\xi)$

$-d\underline{\Sigma}(\xi)$

corner
dostr.

(top-2, 0)

COROLLARY

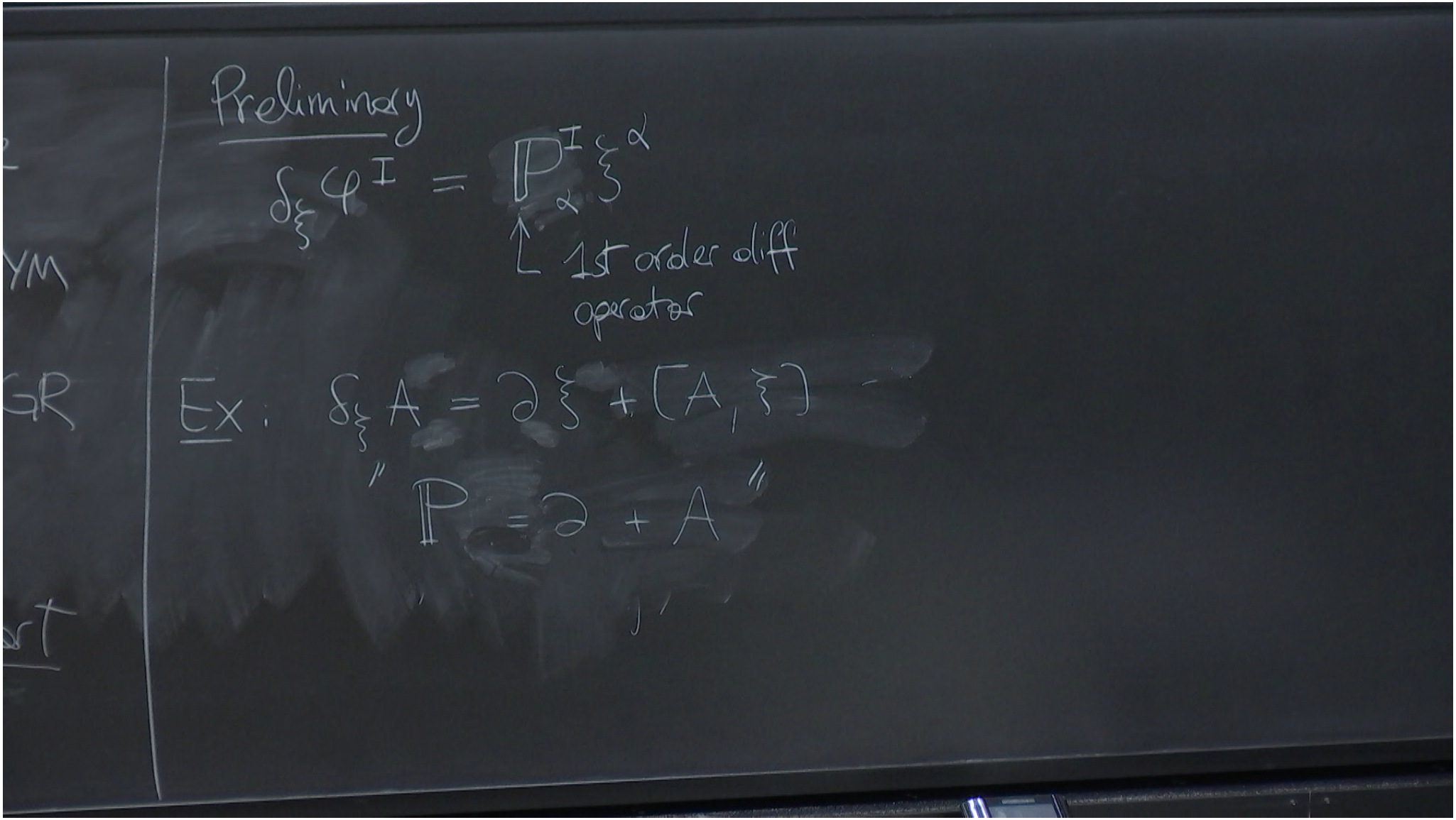
1) Off-shell if $\underline{r}(\xi) = 0$ &

either (i) $\partial\Sigma = \phi$, or

(ii) $d\underline{\Sigma}(\xi) = 0$

$\Rightarrow i_{\delta(\xi)} \Omega_\Sigma = -\delta Q_\Sigma(\xi)$

2) On-shell $(=) \rightsquigarrow (\approx)$


Preliminary
$\delta_\xi \varphi^I = P^I_\alpha \xi^\alpha$
1st order diff operator
Ex: $\delta_\xi A = \partial \xi + [A, \xi]$
"$P = \partial + A$"

$$\int_M E_I \delta_\xi \varphi^I = \int_M E_I P^I_\alpha \xi^\alpha \overset{\text{ibp}}{=} \int \left({}^{\dagger}P^I_\alpha E_I \right) \xi^\alpha$$

ξ of cpt support $\otimes$ $\Rightarrow$ no bdy terms

THM (Noether 2)

$${}^{\dagger}P^I_\alpha E_I \equiv 0$$

Pf $\int_M d\underline{J}(\xi) = \int_M E_I \delta_\xi \varphi^I = \int_M$

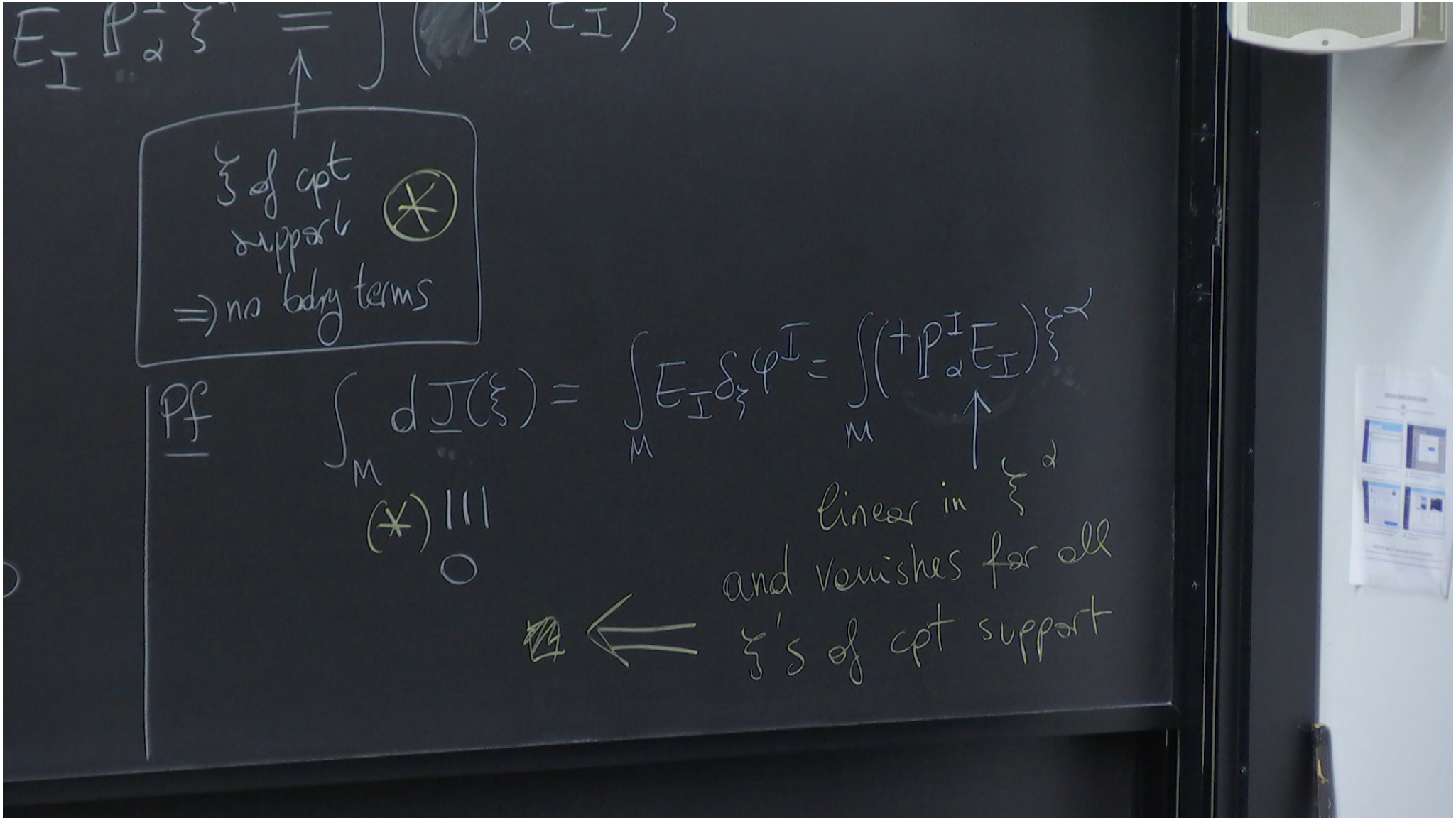
$(*)$ $\Rightarrow$ 0

line

and veri

ξ's of

$\boxtimes \Leftarrow$


$E_I P^I_\alpha \xi^\alpha = \int (P_\alpha L_I) \xi$
ξ of cpt support
$\Rightarrow$ no bdry terms
Pf
$\int_M d\underline{J}(\xi) = \int_M E_I \delta_\xi \varphi^I = \int_M (\pm P^I_\alpha E_I) \xi^\alpha$
$(*)$!!!
0
linear in ξ^α
and vanishes for all ξ's of cpt support
$\Leftarrow$

$\Rightarrow$ there are "not enough" eom !

$\varphi^I \quad \#I \rightsquigarrow E_I$

$\xi^\alpha \quad \#\alpha \rightsquigarrow B_\alpha \equiv P^I_{(\alpha)} E_I \equiv 0$

$(\#I - \#\alpha)$ indep eom

$\Rightarrow$ Evol. of the fields is UNDERDETERMINED

$\Rightarrow$ Need to identify config. related by a local sym

eom !

E_I

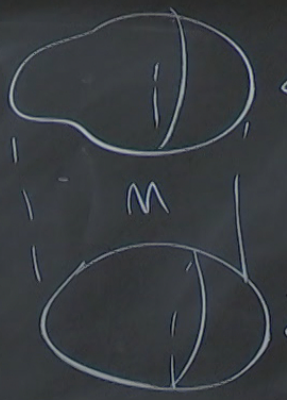
$= P^I_{(\alpha)} E_I = 0$

eom

UNDERDETERMINED

nfig. related by e local

LEMMA (Constraints)



$\partial\Sigma = \phi$

$Q_\Sigma(\xi) = \int_\Sigma J(\xi) \approx$ "const"

Noether ↑ (indep of Σ)

If ξ is a loc. sym

I can choose $\xi|_{\Sigma_{out}} \equiv 0$

$$\Rightarrow Q_\Sigma(\xi) \approx 0$$

$$\Rightarrow i_\Sigma^* J(\xi) = \underbrace{C_\Sigma(\xi)}_{\text{VANISHES ON SHELL}} + d j_\Sigma(\xi)$$

"const"
(indep of Σ)

& $C_\Sigma(\xi) = \xi^\alpha C_{\Sigma\alpha}$ ← CONSTRAINT

↑ no derivative