Title: Quantum Gravity

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Collection: Quantum Gravity (2021-2022)

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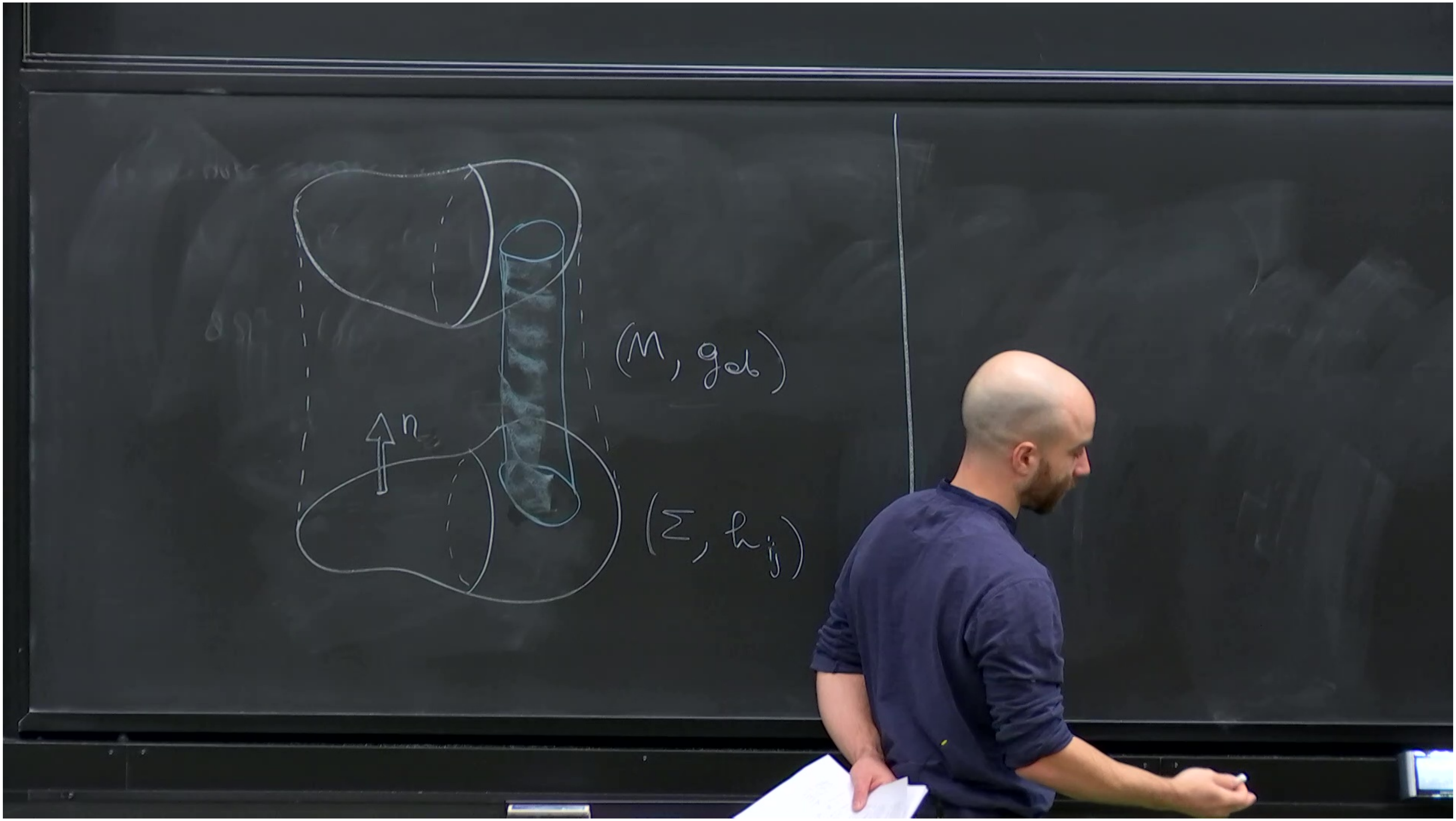
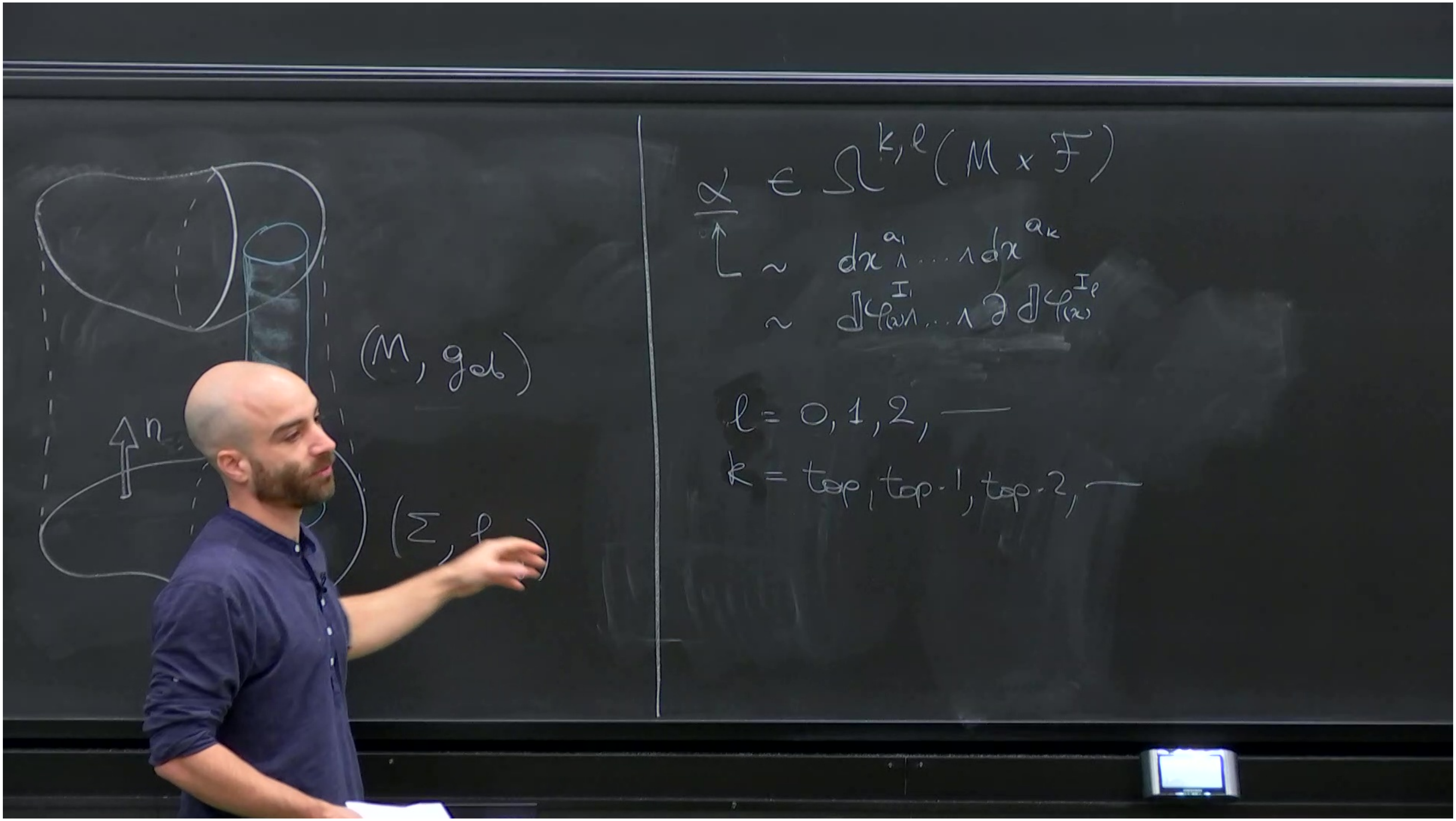
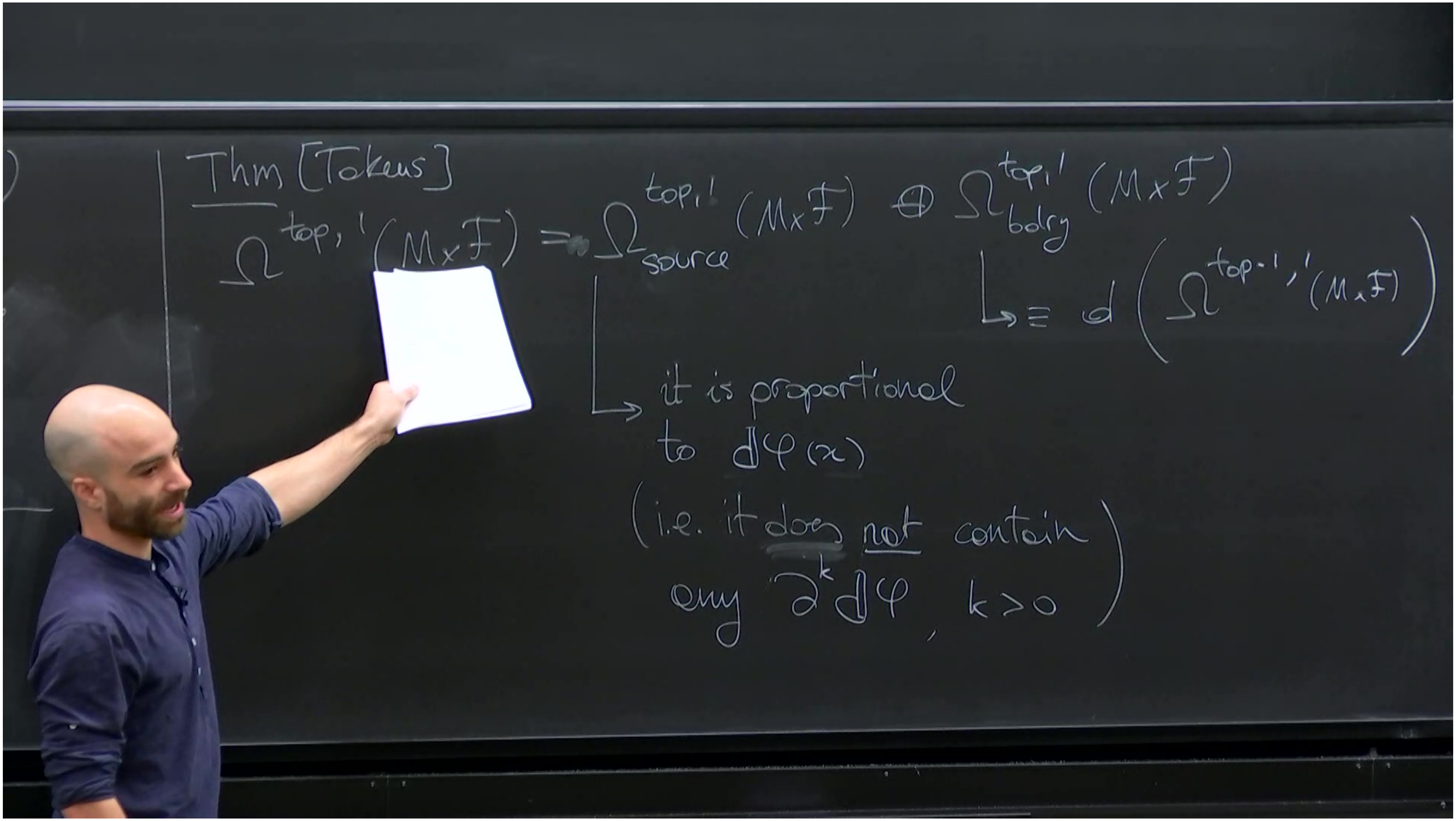
Abstract: Topics will include (but are not limited to): Canonical formulation of constrained systems, The Dirac program, First order formalism of gravity, Loop Quantum Gravity, Spinfoam models, Research at PI and other approaches to quantum gravity.

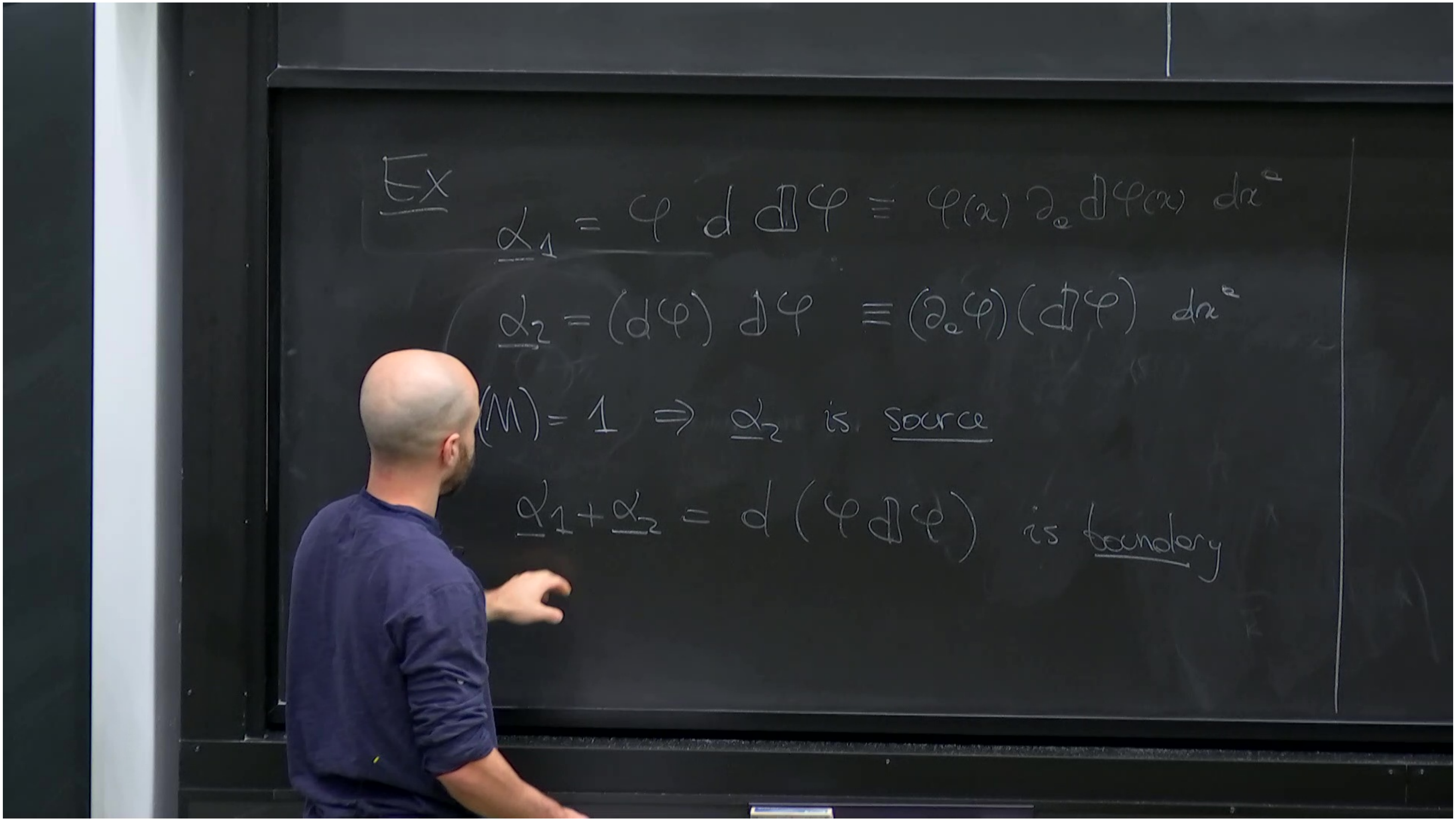


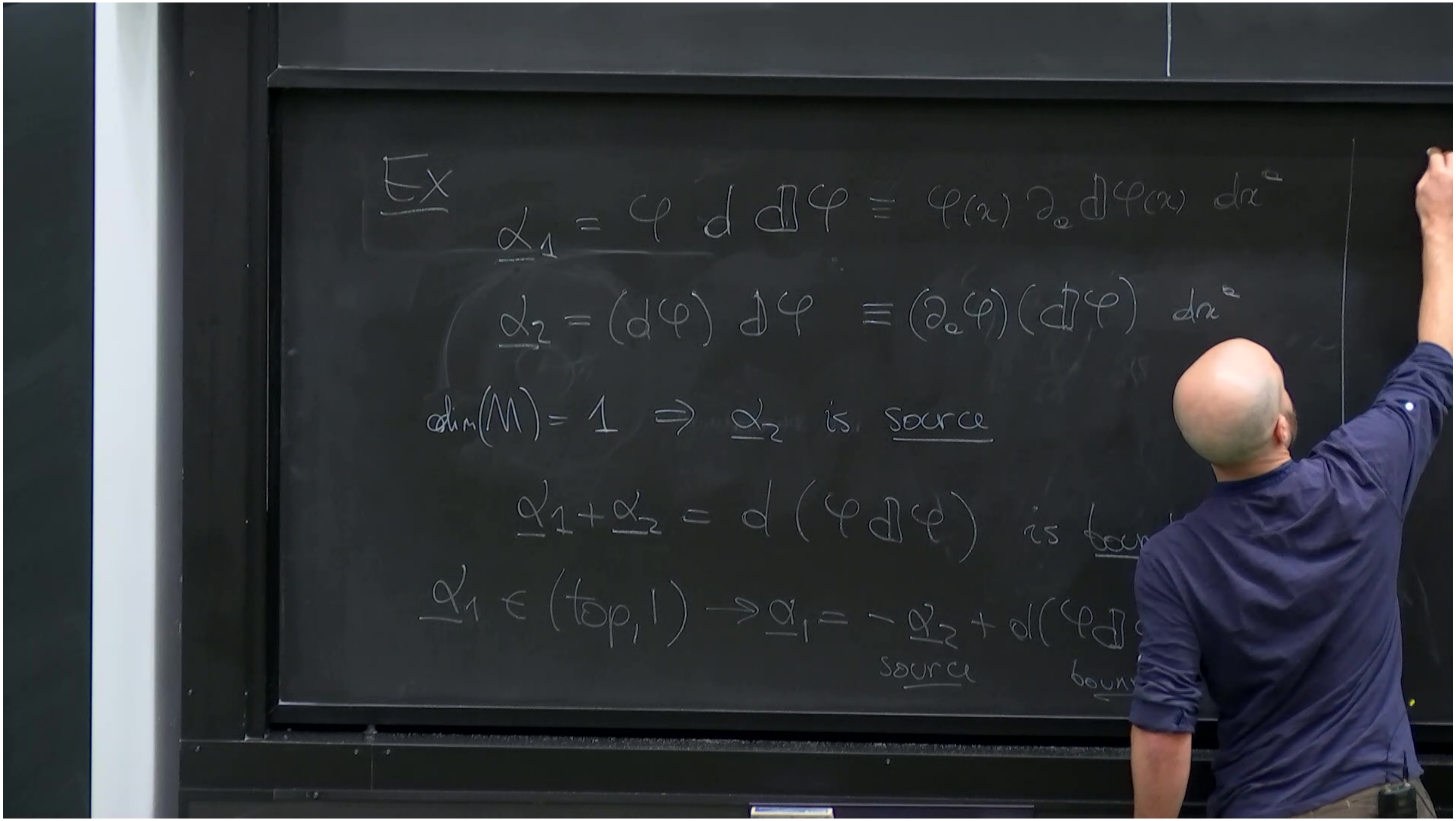
(M, g_{ab})
(Σ, h_{ij})
n



(M, gab)
α ∈ Ω^{k,ℓ}(M × F)
~ dx^{a1} ∧ … ∧ dx^{ak}
ℓ = 0, 1, 2, —
k = top, top-1, top-2, —


Thm [Takens]
$\Omega^{top,1}(M\times F) = \Omega^{top,1}_{source}(M\times F) \oplus \Omega^{top,1}_{bdry}(M\times F)$
$= d\left(\Omega^{top-1,1}(M\times F)\right)$
it is proportional to $d\varphi(x)$
(i.e. it does not contain any $\partial^k d\varphi$, $k>0$)


Ex
$\alpha_1 = \varphi \, d \, \mathbb{d}\varphi \equiv \varphi(x) \, \partial_a \mathbb{d}\varphi(x) \, dx^a$
$\alpha_2 = (d\varphi) \, \mathbb{d}\varphi \equiv (\partial_a \varphi)(\mathbb{d}\varphi) \, dx^a$
$(M) = 1 \Rightarrow \alpha_2$ is source
$\alpha_1 + \alpha_2 = d(\varphi \, \mathbb{d}\varphi)$ is boundary


Ex
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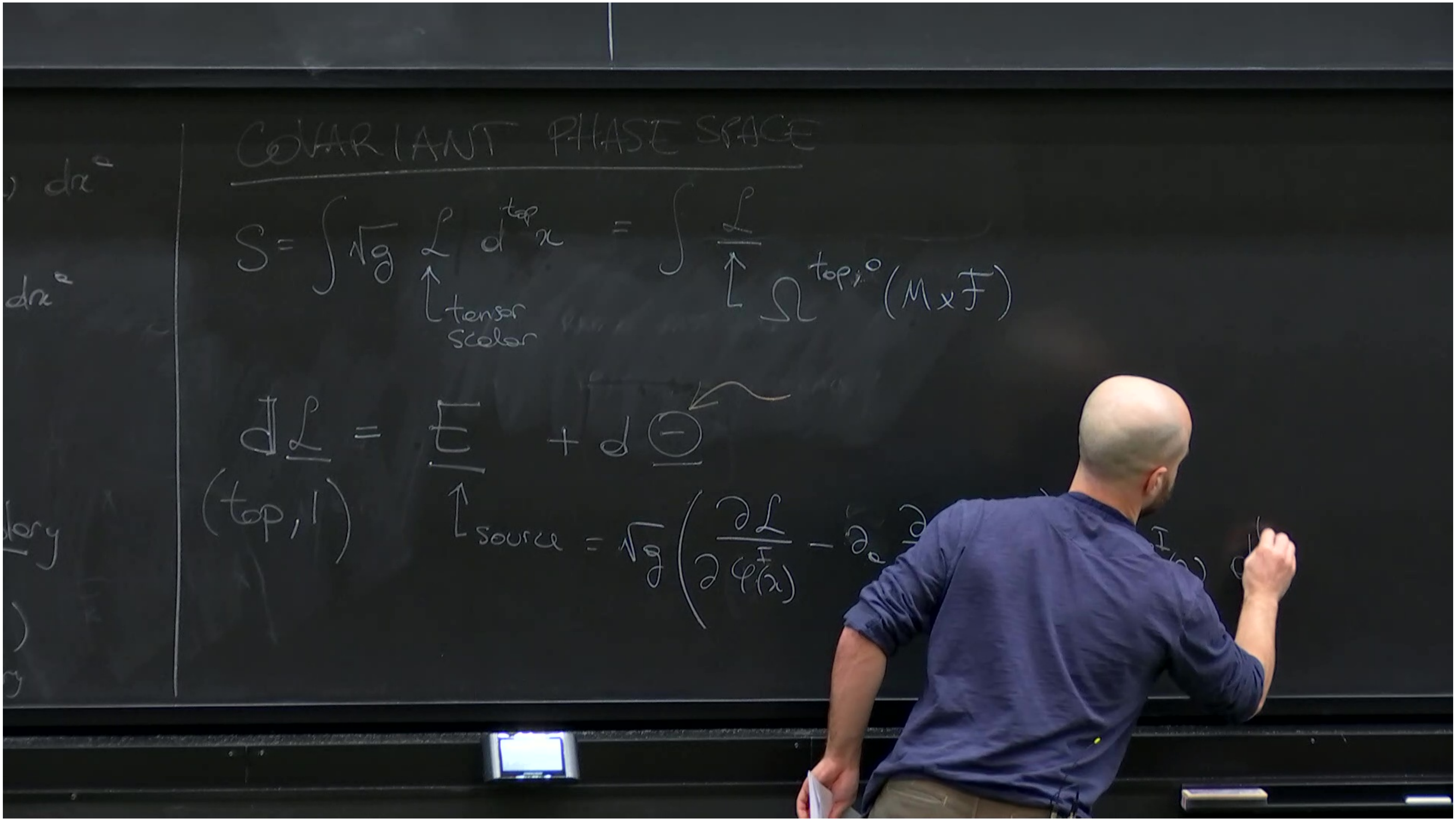
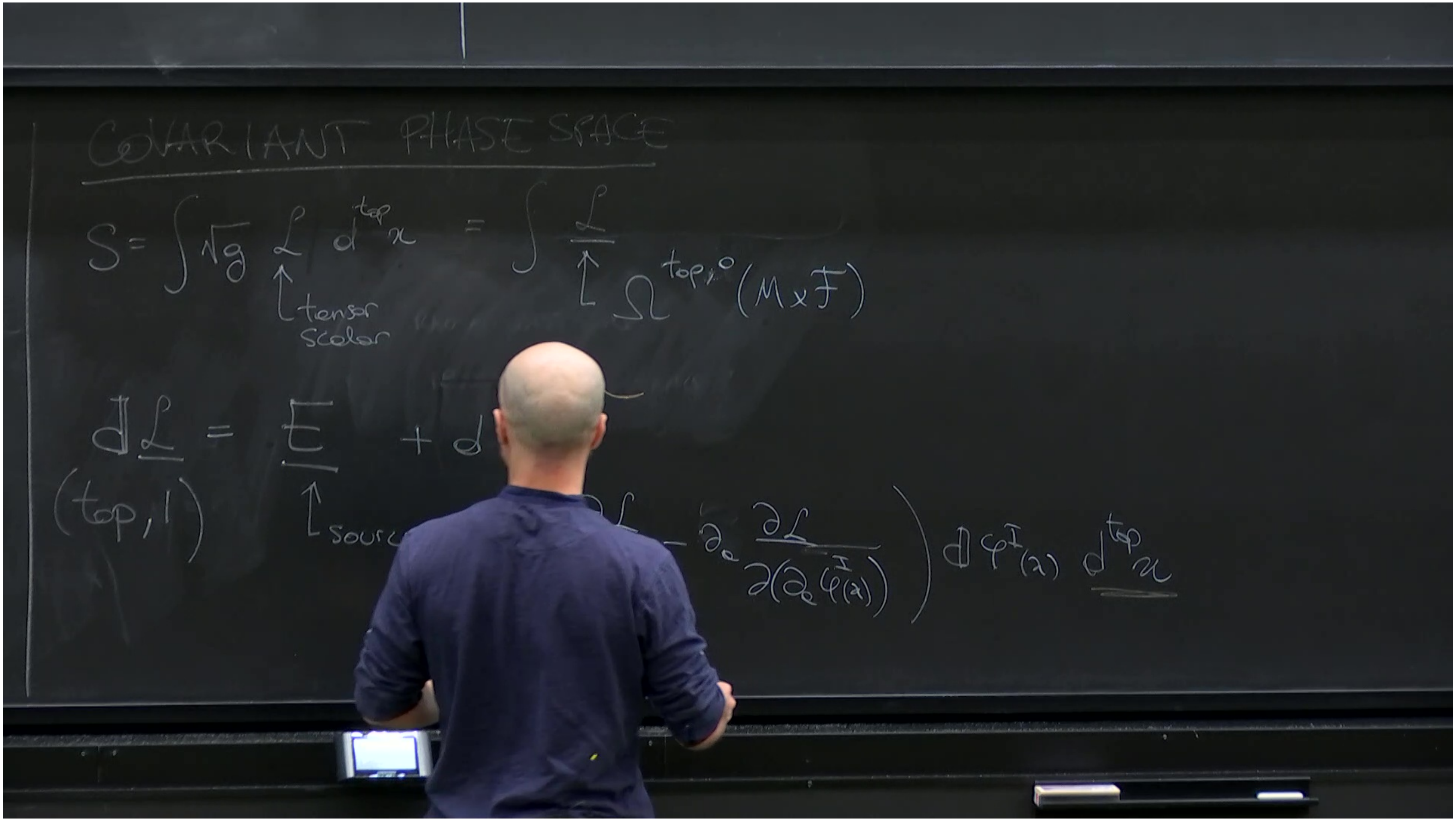
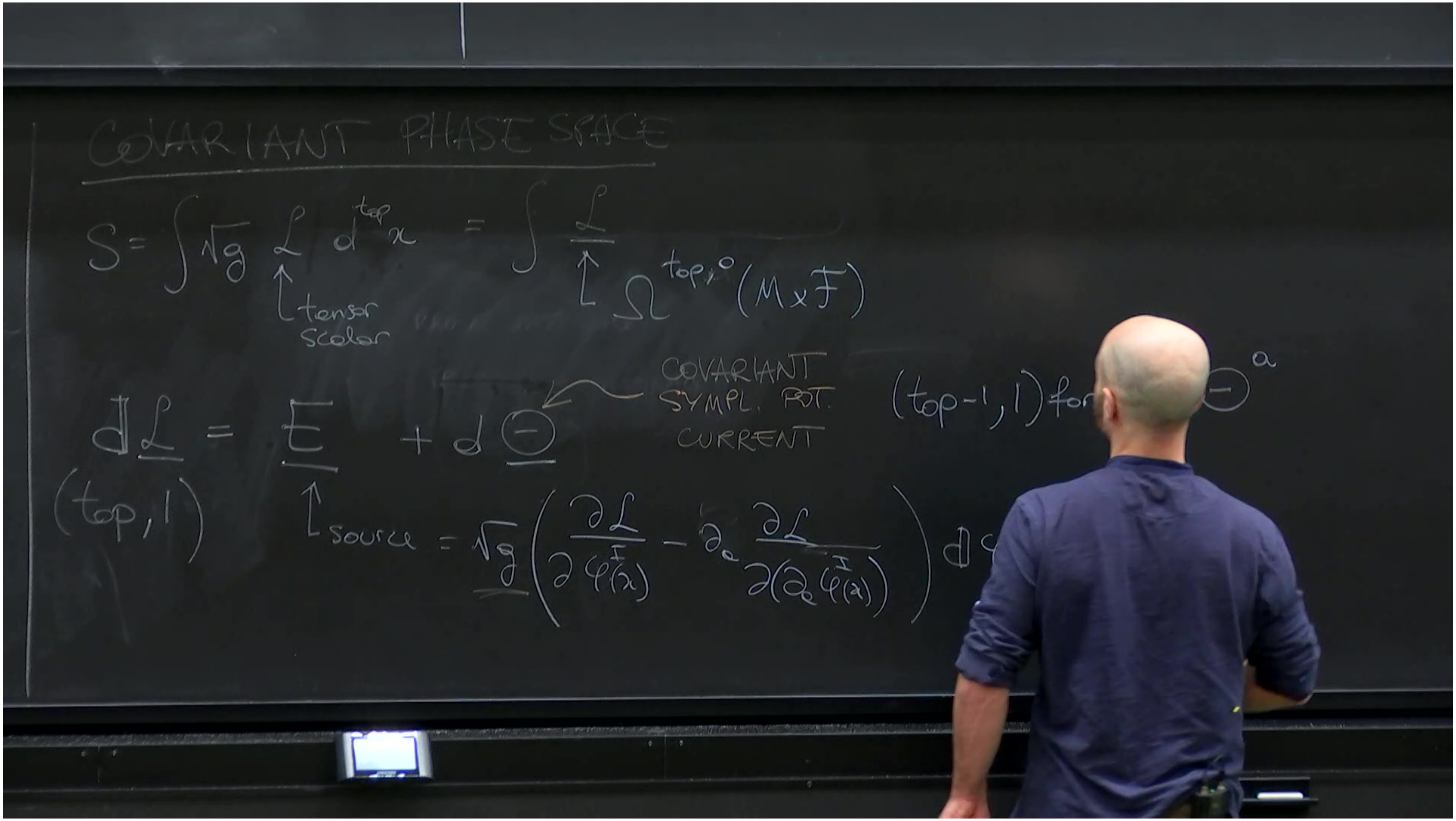
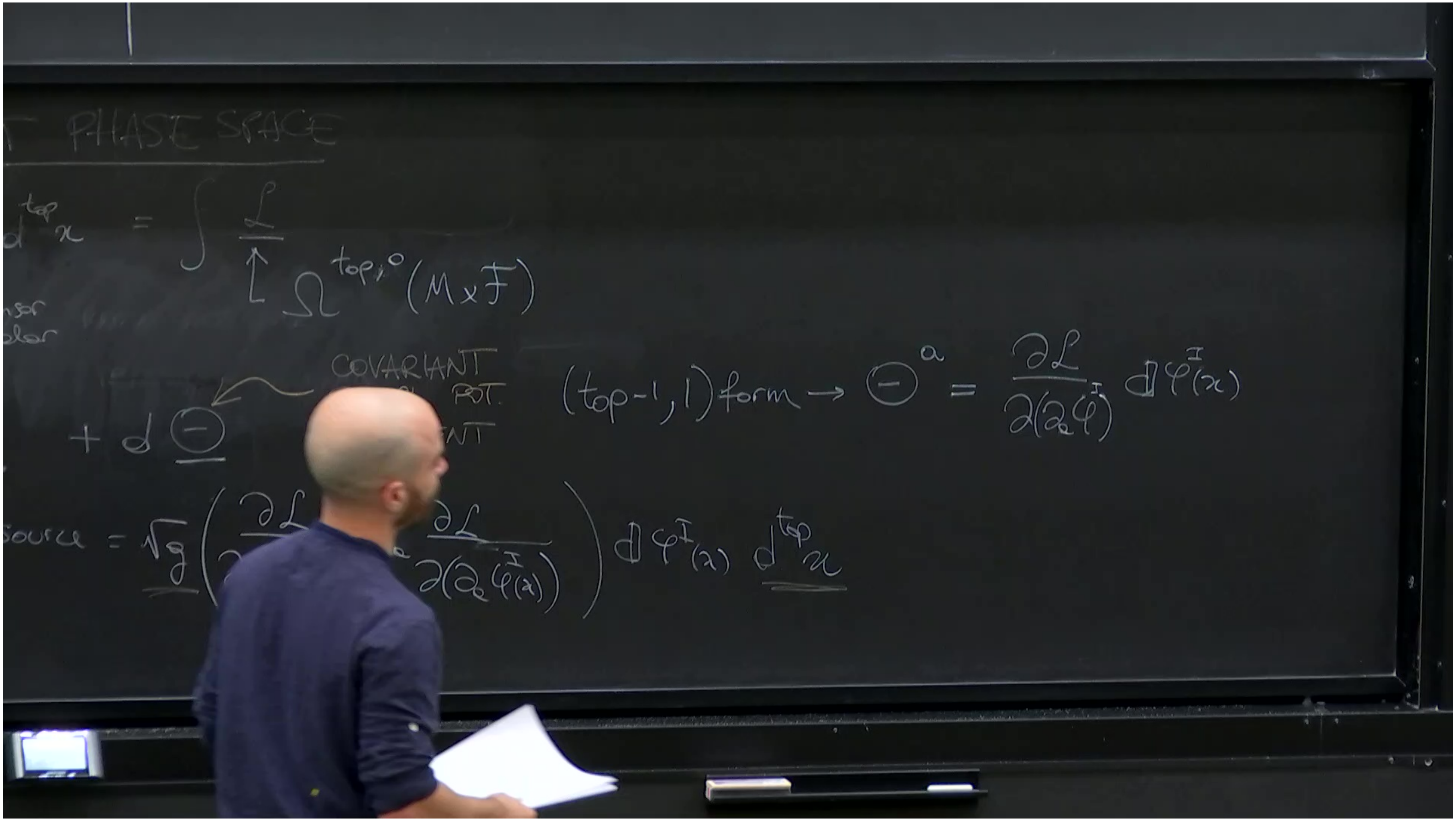
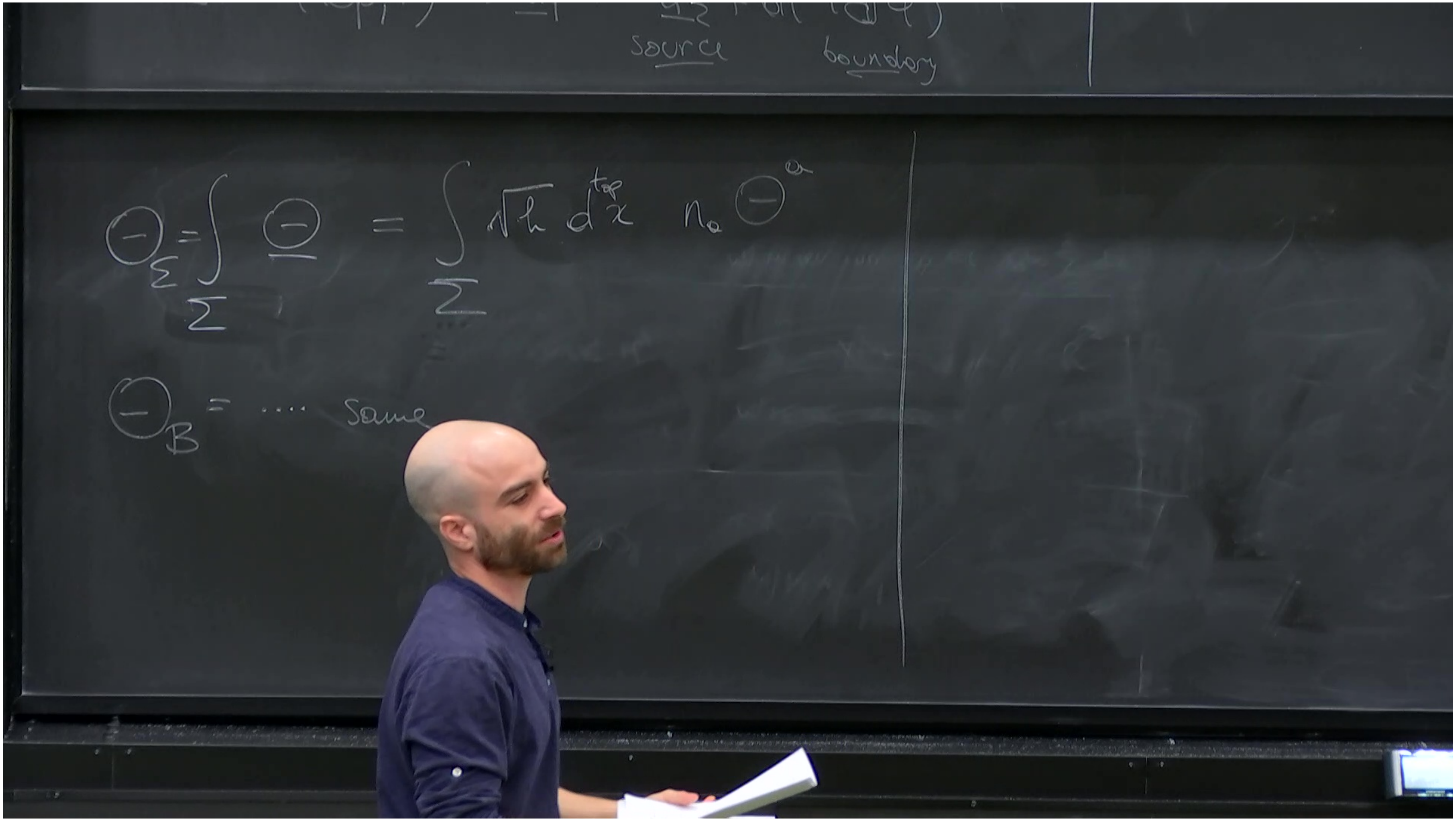
$\alpha_2 = (d\varphi)\, \mathbb{d}\varphi \equiv (\partial_a \varphi)(\mathbb{d}\varphi)\, dx^a$
$\dim(M) = 1 \Rightarrow \alpha_2$ is source
$\alpha_1 + \alpha_2 = d(\varphi \mathbb{d}\varphi)$ is boun
$\alpha_1 \in (\mathrm{top}, 1) \to \alpha_1 = -\alpha_2 + d(\varphi\mathbb{d}$
source
boun

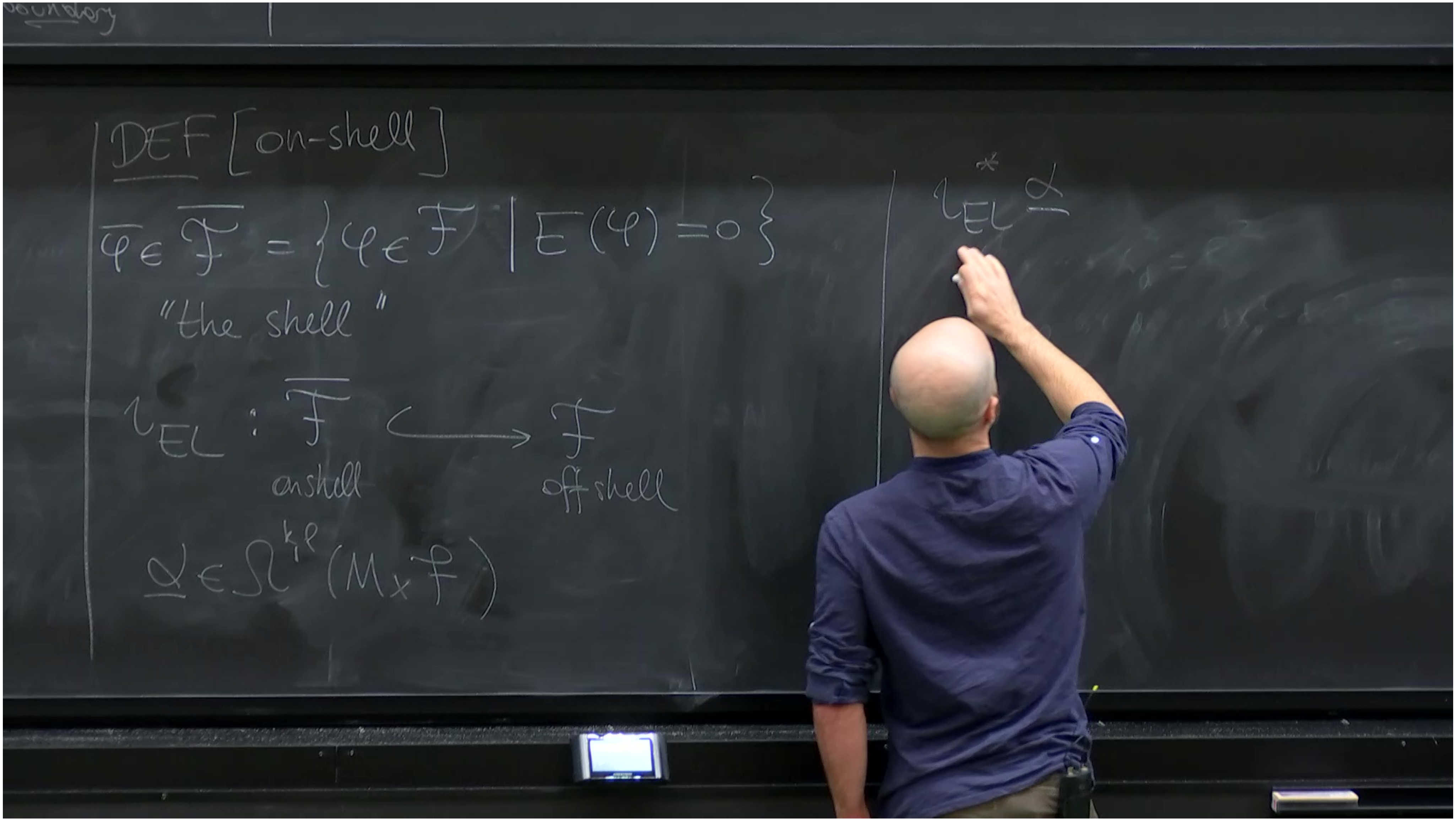

COVARIANT PHASE SPACE
S = ∫√g ℒ d^top x = ∫ ℒ
tensor
scalar
Ω^top,0 (M × F)
dℒ = E + dΘ
(top, 1)
source = √g (∂ℒ / ∂φ^I(x) − ∂_a ∂


COVARIANT PHASE SPACE
$S = \int \sqrt{g}\, \mathcal{L}\, d^{top}x = \int \underline{\mathcal{L}}$
tensor scalar
$\Omega^{top,0}(M \times \mathcal{F})$
$\mathbb{d}\underline{\mathcal{L}} = \underline{E} + d$
(top,1)
sour
$- \partial_a \frac{\partial \mathcal{L}}{\partial(\partial_a \varphi^I(x))} \Big) \mathbb{d}\varphi^I(x)\, \underline{d^{top}x}$


COVARIANT PHASE SPACE
$S = \int \sqrt{g}\, \mathcal{L}\, d^{top}x = \int \underline{\mathcal{L}}$
tensor scalar
$\Omega^{top,0}(M \times \mathcal{F})$
COVARIANT SYMPL. POT. CURRENT
$(top-1,1)$ form Θ^a
$\mathbb{d}\underline{\mathcal{L}} = \underline{E} + d\,\underline{\Theta}$
$(top,1)$
source $= \underline{\sqrt{g}}\left(\frac{\partial \mathcal{L}}{\partial \varphi^I(x)} - \partial_a \frac{\partial \mathcal{L}}{\partial(\partial_a \varphi^I(x))}\right) \mathbb{d}\varphi$


PHASE SPACE
COVARIANT
POT.
$(top-1, 1)$ form $\rightarrow \Theta^a = \frac{\partial \mathcal{L}}{\partial(\partial_a \varphi^I)} \, d\varphi^I(x)$
$\Omega^{top,0}(M \times F)$


source
boundary
same


DEF [on-shell]
$\bar{\varphi} \in \bar{\mathcal{F}} = \{\varphi \in \mathcal{F} \mid E(\varphi) = 0\}$
"the shell"
$\iota_{EL} : \bar{\mathcal{F}} \hookrightarrow \mathcal{F}$
onshell
offshell
$\alpha \in \Omega^{k,p}(M \times \mathcal{F})$
$\iota_{EL}^{*} \alpha$

$\varphi) = 0 \}$

ell

$\underline{d} = (d\varphi) \mathbb{d}\varphi$

$\iota_{EL}^{*} \underline{\alpha}$

not only $\varphi = \overline{\varphi}$ satisfies the eom

but also $\mathbb{d}\varphi$ —ıı— the linearized eqs.

$\underline{\text{Ex}}$:
$$\begin{cases} \Box\overline{\varphi} + m\overline{\varphi} + \frac{\lambda}{3}\overline{\varphi}^3 = 0 & (EL) \\ \Box \mathbb{d}\varphi + m\,\mathbb{d}\varphi + \lambda\overline{\varphi}^2 \mathbb{d}\varphi = 0 \end{cases}$$

$) = 0 \}$

$\iota^*_{EL}\,\alpha$

not only $\varphi = \bar{\varphi}$ satisfies the eom

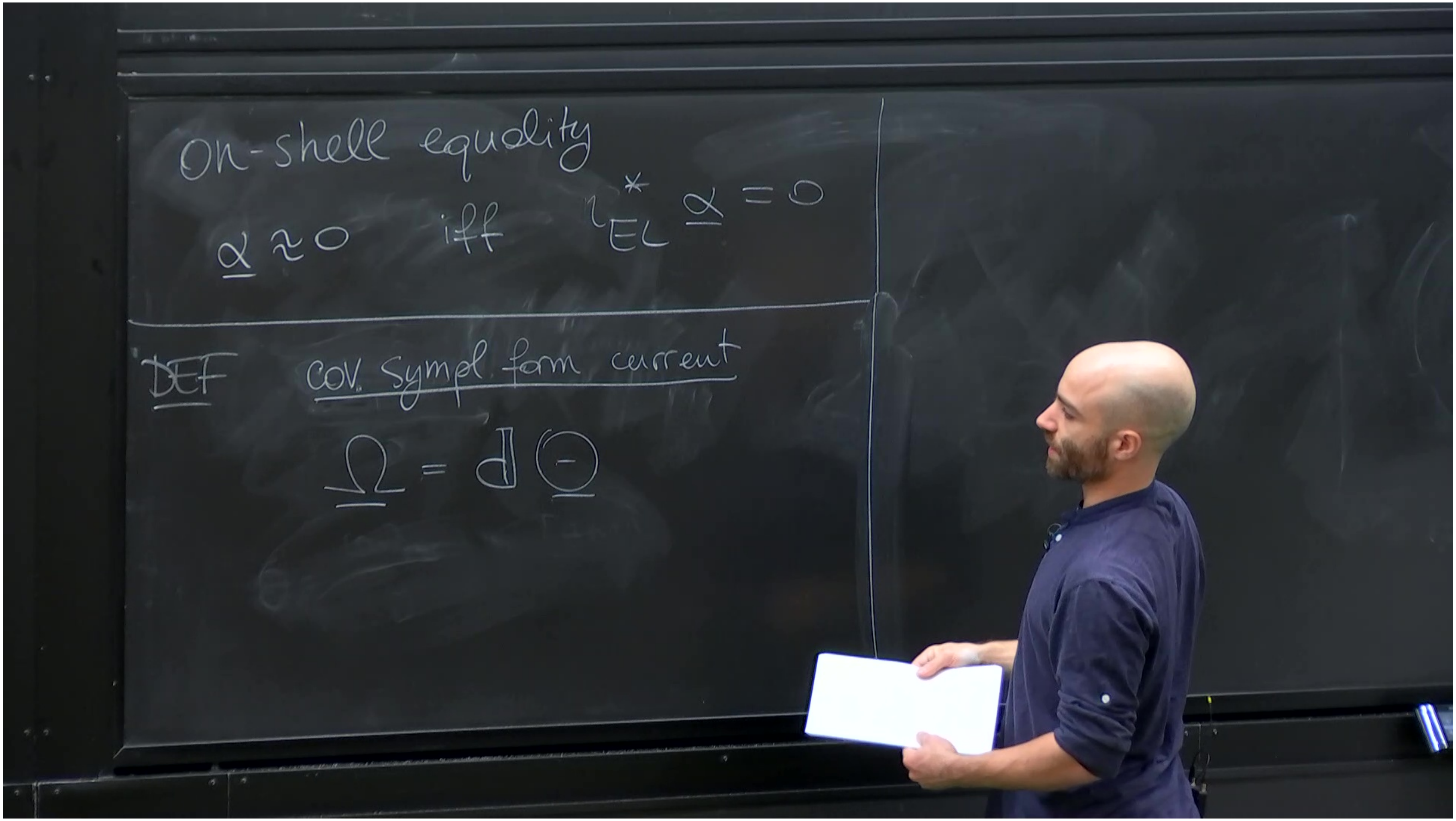
but also $\mathbb{d}\varphi$ — " — the linearized eqns.

ll

$= (\mathbb{d}\varphi)\mathbb{d}\varphi$

Ex:

$$\begin{cases} \Box\bar{\varphi} + m\bar{\varphi} + \frac{\lambda}{3}\bar{\varphi}^3 = 0 & (EL) \\ \Box\,\mathbb{d}\bar{\varphi} + m\,\mathbb{d}\bar{\varphi} + \lambda\bar{\varphi}^2\,\mathbb{d}\bar{\varphi} = 0 \end{cases}$$

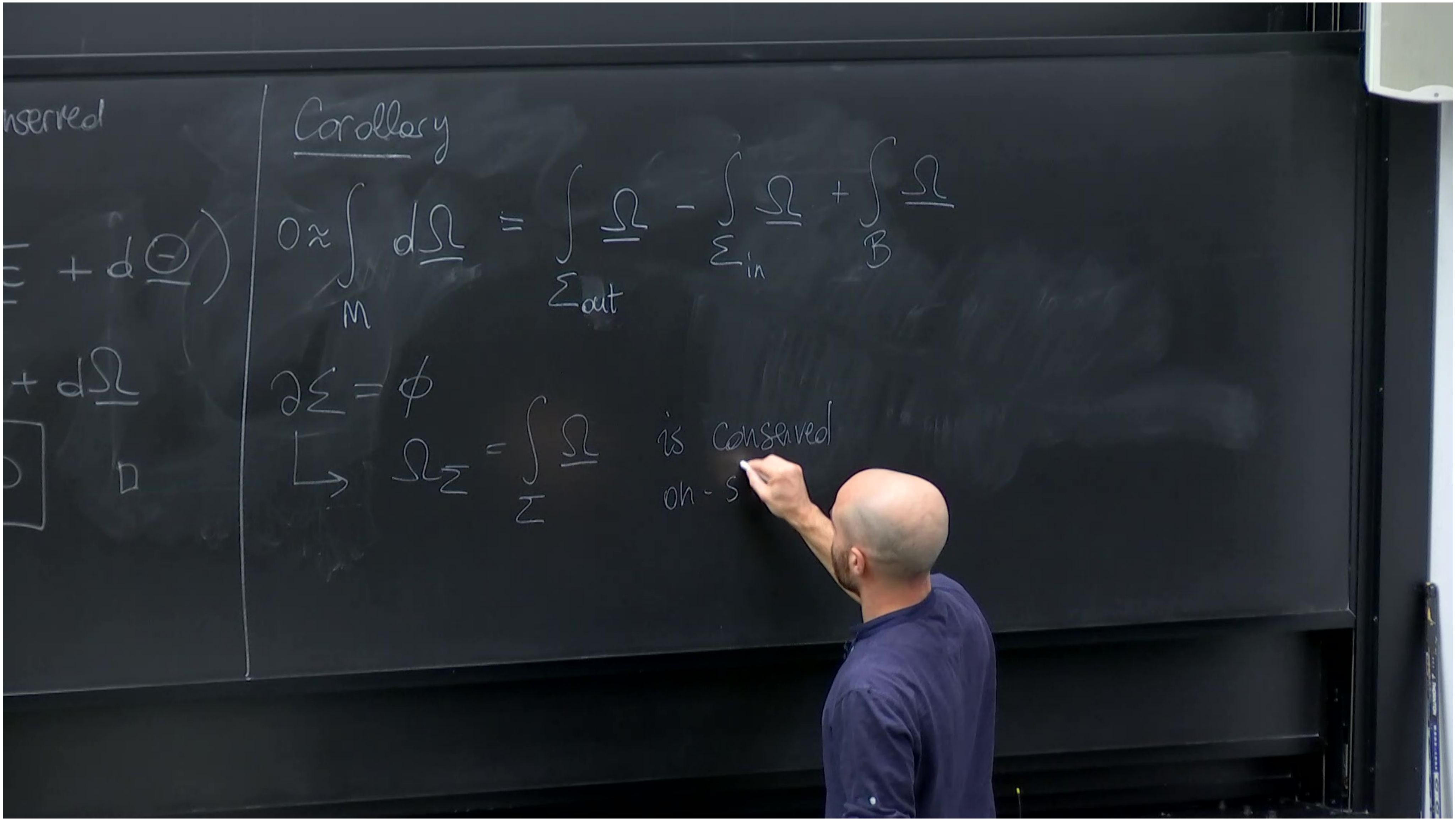

On-shell equality
$\underline{\alpha} \approx 0$ iff $\iota^*_{EL}\, \underline{\alpha} = 0$
DEF cov. sympl form current
$\underline{\Omega} = d\, \underline{\Theta}$

THM $\underline{\Omega}$ is conserved on-shell

Pf $\mathbb{d}\left(\mathbb{d}\underline{L} = \underline{E} + d\underline{\Theta}\right)$

$$0 = \mathbb{d}E + d\underline{\Omega}$$

$$\boxed{d\underline{\Omega} \approx 0} \qquad \square$$


Corollary
$0 \approx \int_M d\underline{\Omega} = \int_{\Sigma_{out}} \underline{\Omega} - \int_{\Sigma_{in}} \underline{\Omega} + \int_B \underline{\Omega}$
$\partial\Sigma = \phi$
$\hookrightarrow \Omega_\Sigma = \int_\Sigma \underline{\Omega}$ is conserved
on-s

...nserved

$$\ldots + d\underline{\Theta}\Big)$$

$$+ d\underline{\Omega}$$

$\square$

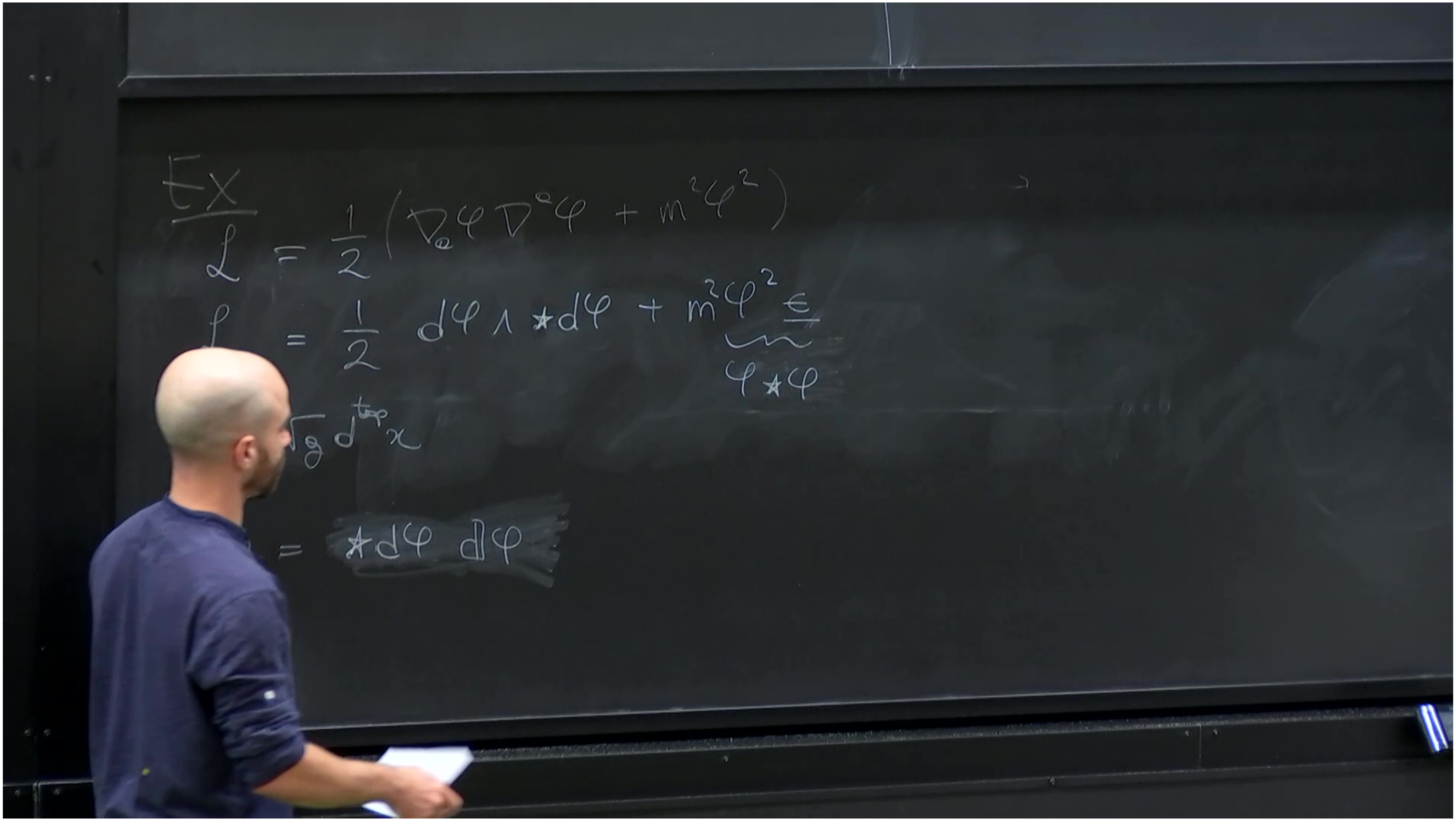
Corollary

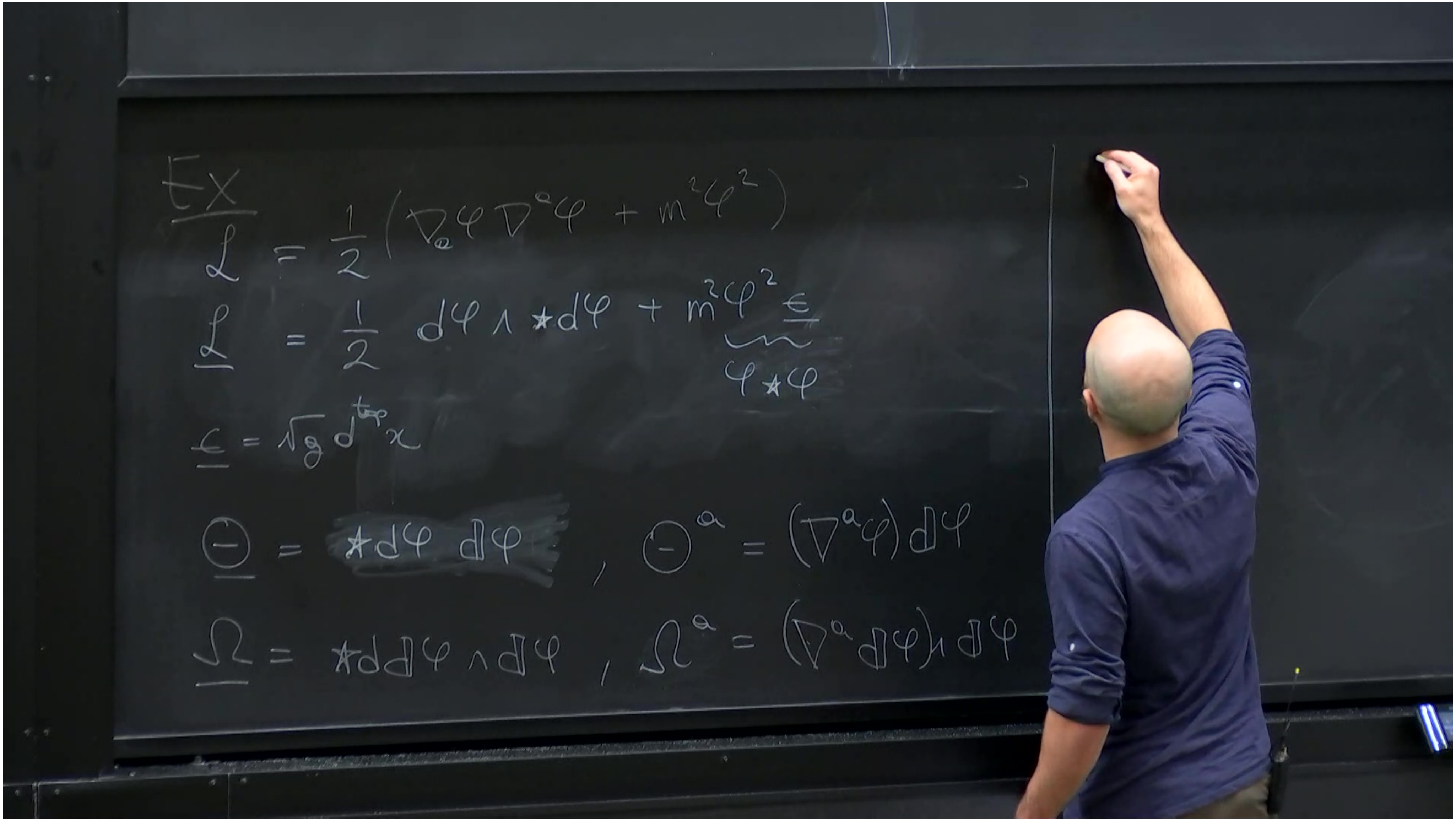
$$0 \approx \int_M d\underline{\Omega} = \int_{\Sigma_{out}} \underline{\Omega} - \int_{\Sigma_{in}} \underline{\Omega} + \int_B \underline{\Omega}$$

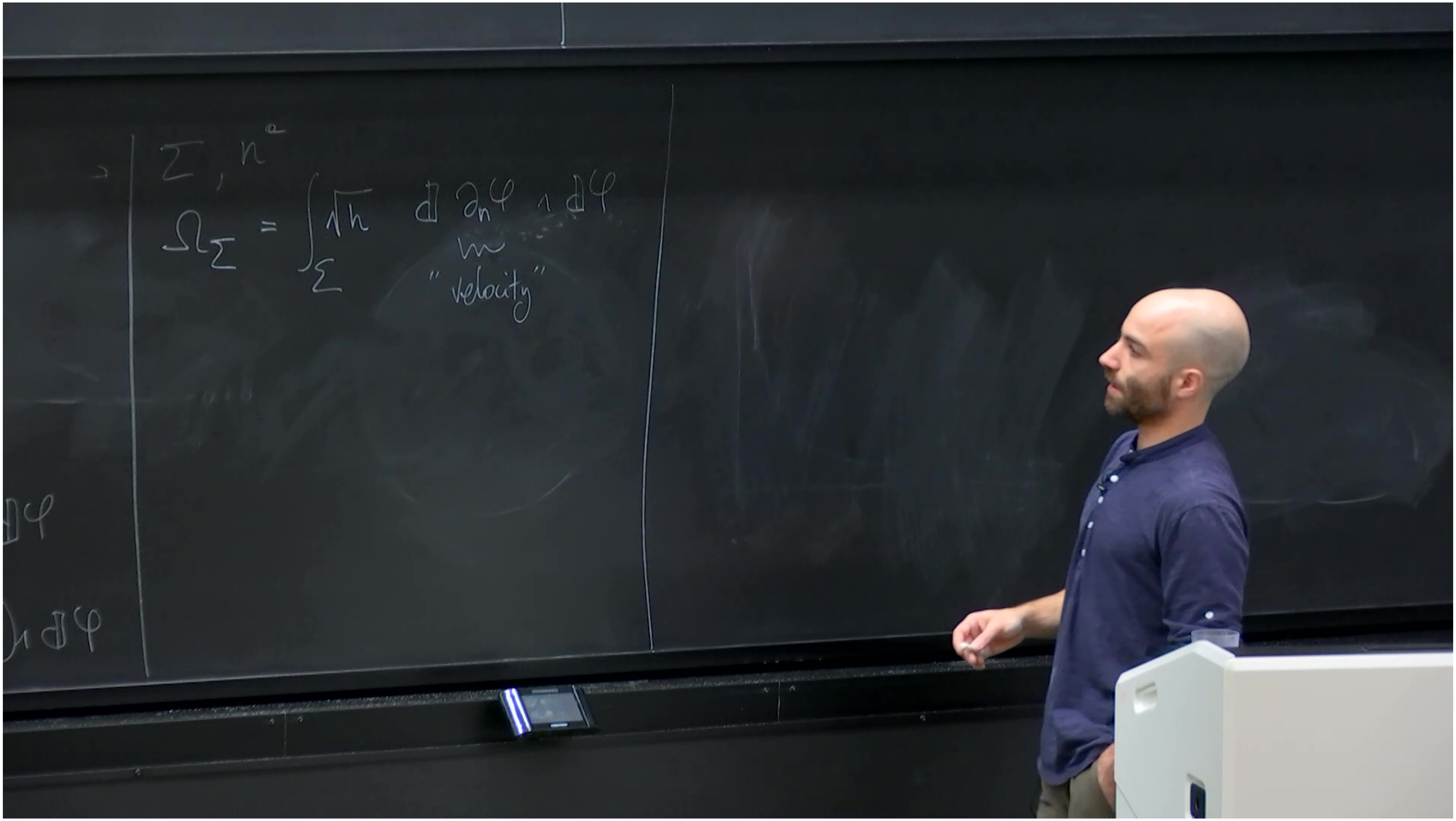
$\partial\Sigma = \phi$

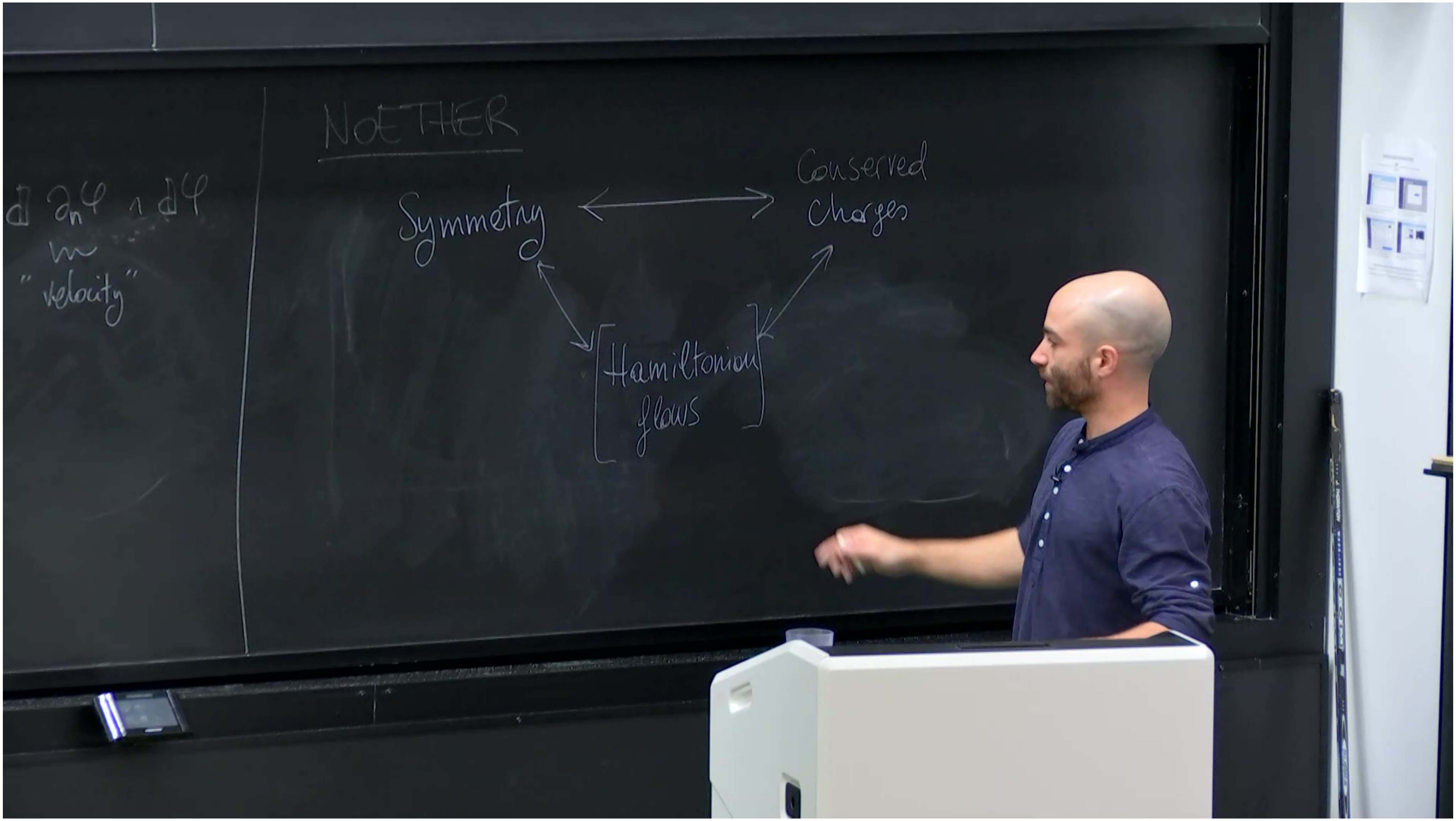
$\hookrightarrow \Omega_\Sigma = \int_\Sigma \underline{\Omega}$ is conserved on-shell

$\Longrightarrow$ we expect that $(\mathcal{F}, \Omega_\Sigma) =$ "phase space"


Ex
$\mathcal{L} = \frac{1}{2}(\nabla_a\varphi\nabla^a\varphi + m^2\varphi^2)$
$\ell = \frac{1}{2}\, d\varphi \wedge \star d\varphi + m^2\varphi^2\epsilon$
$\varphi\star\varphi$
$\sqrt{g}\, d^{top}x$
$= \star d\varphi\ d\varphi$


Ex
$\mathcal{L} = \frac{1}{2}(\nabla_a\varphi\nabla^a\varphi + m^2\varphi^2)$
$\underline{\mathcal{L}} = \frac{1}{2}\, d\varphi \wedge \star d\varphi + m^2\varphi^2\underline{\epsilon}$
$\varphi \star \varphi$
$\underline{\epsilon} = \sqrt{g}\, d^{top}x$
$\underline{\Theta} = \star d\varphi\, \mathbb{d}\varphi \; , \; \Theta^a = (\nabla^a\varphi)\mathbb{d}\varphi$
$\underline{\Omega} = \star d\mathbb{d}\varphi \wedge \mathbb{d}\varphi \; , \; \Omega^a = (\nabla^a\mathbb{d}\varphi)\wedge\mathbb{d}\varphi$


Σ, n^a
$\Omega_\Sigma = \int_\Sigma \sqrt{h}\; \mathbb{d}\, \partial_n \varphi \wedge \mathbb{d}\varphi$
"velocity"



NOETHER
Symmetry
Conserved Charges
Hamiltonian flows
"velocity"