Title: Quantum Gravity

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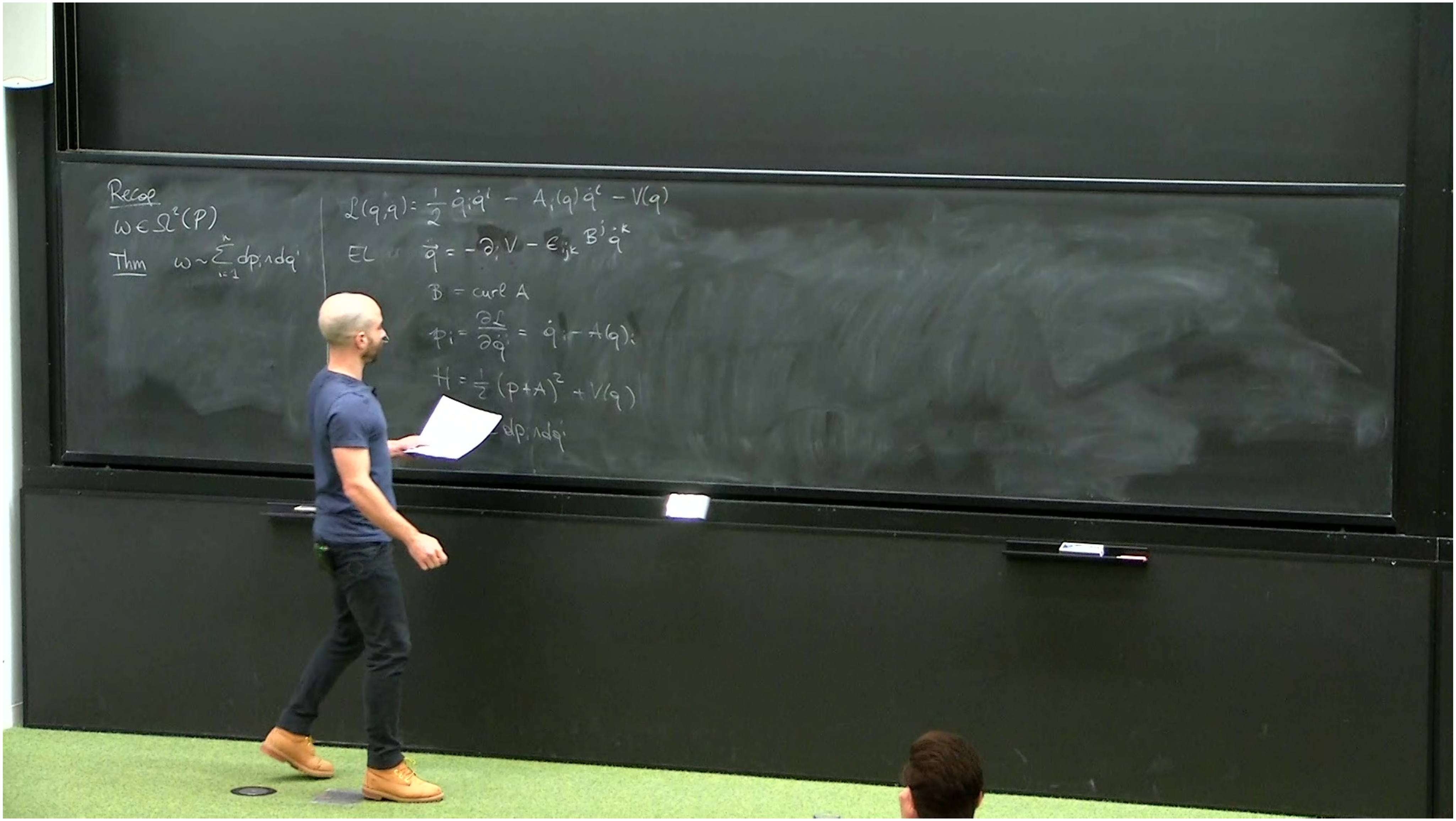
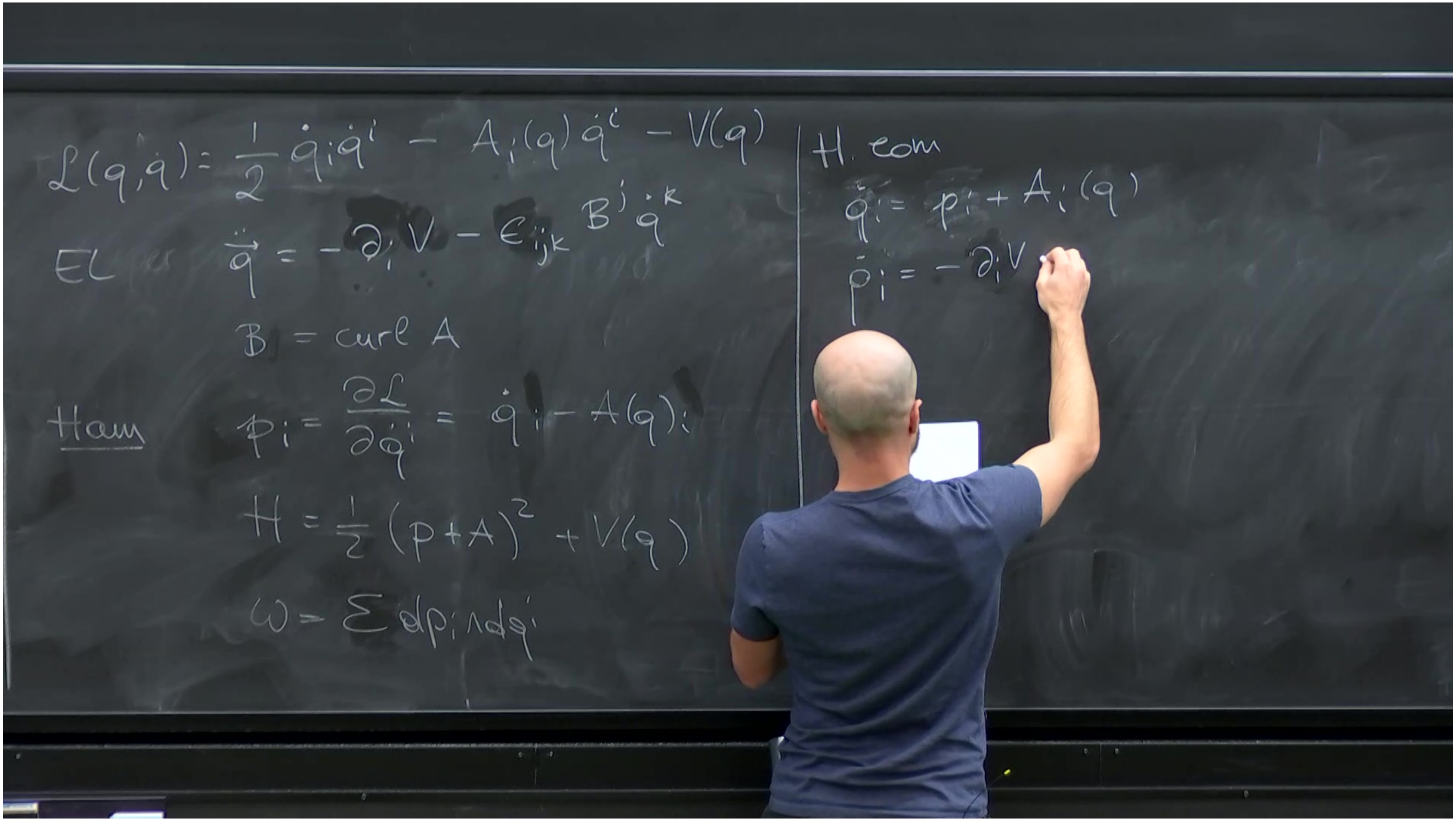
Collection: Quantum Gravity (2021-2022)

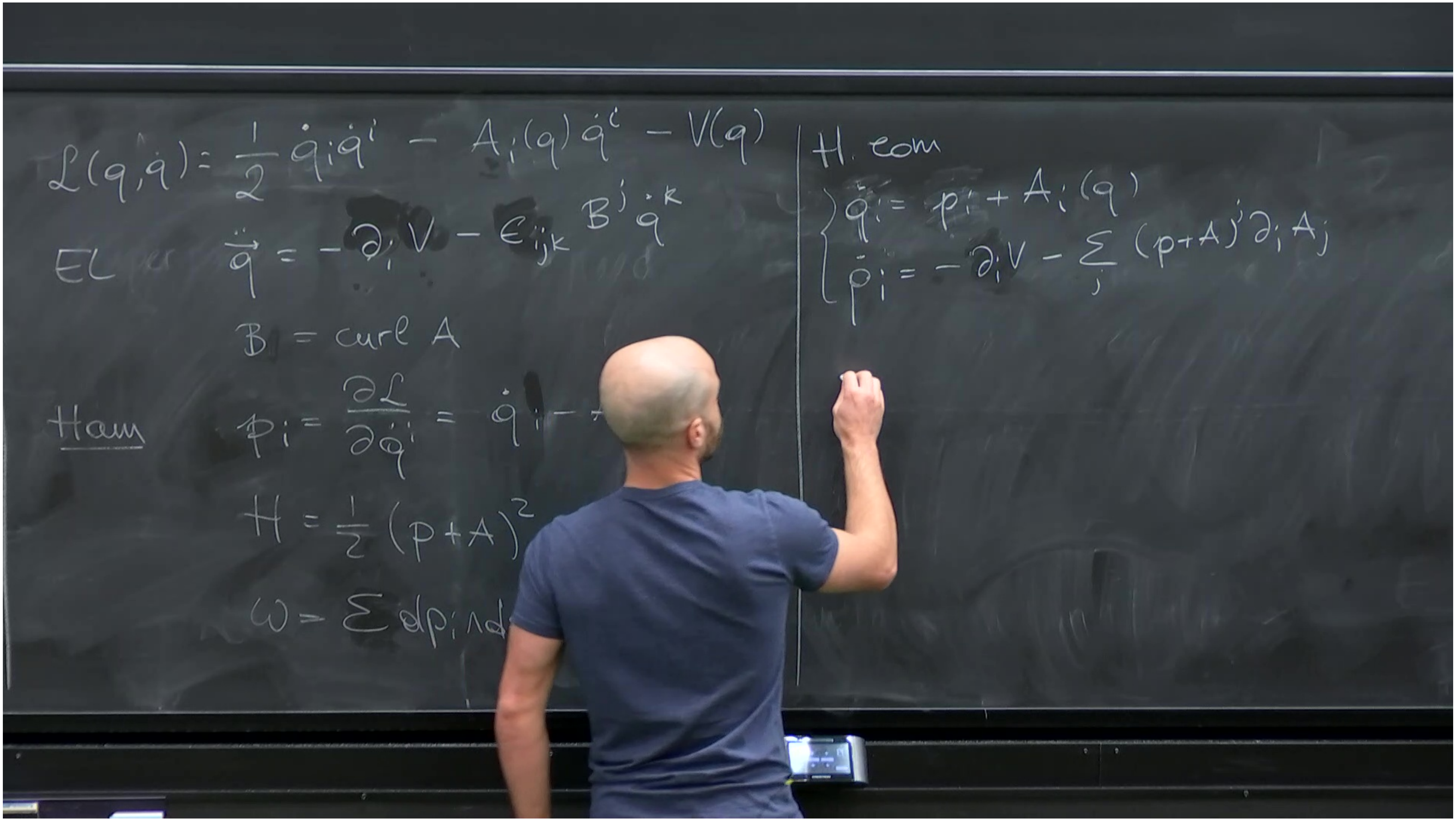
Date: April 07, 2022 - 10:15 AM

URL: https://pirsa.org/22040057

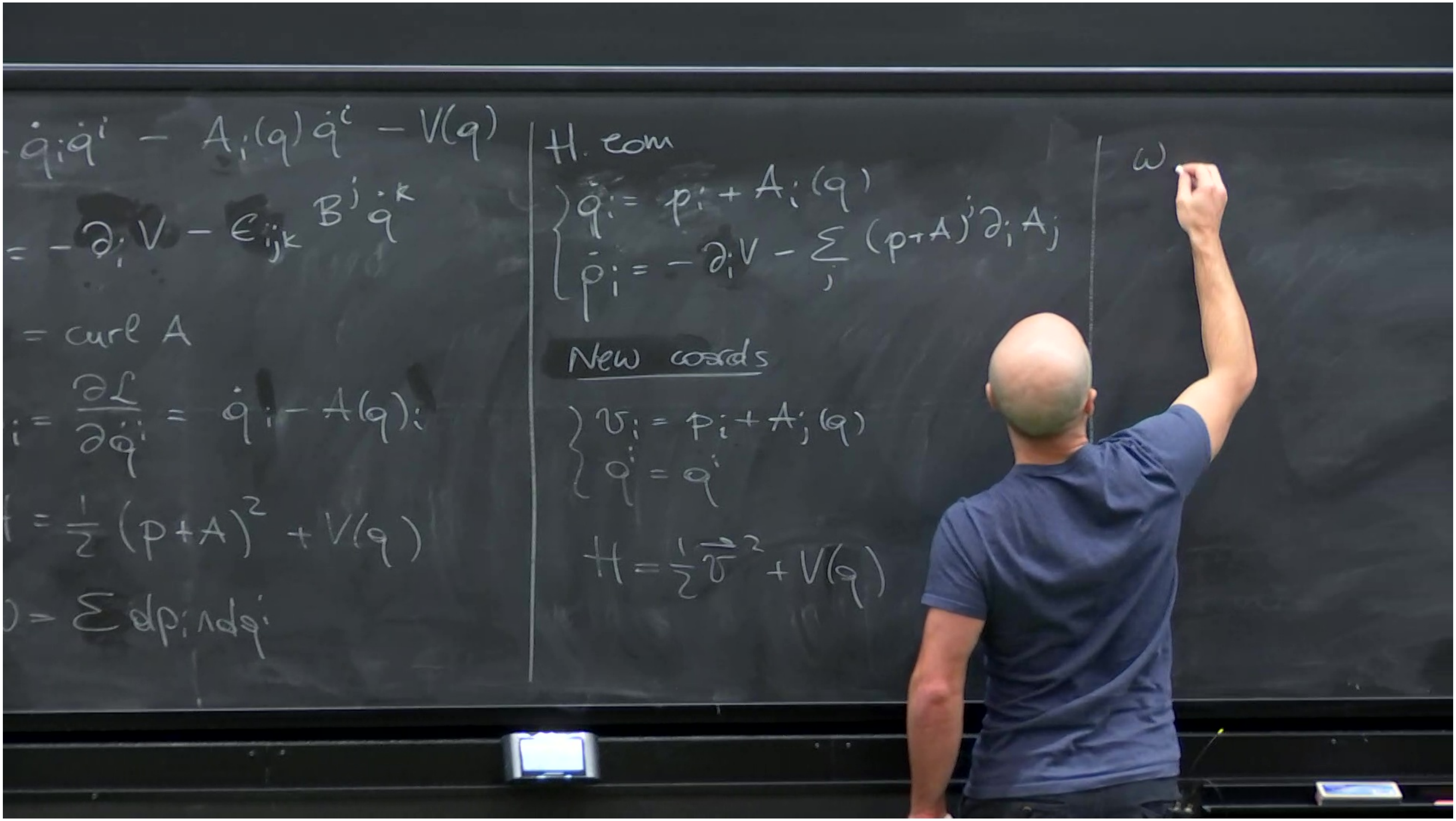
Abstract: Topics will include (but are not limited to): Canonical formulation of constrained systems, The Dirac program, First order formalism of gravity, Loop Quantum Gravity, Spinfoam models, Research at PI and other approaches to quantum gravity.


Recap
$\omega \in \Omega^2(P)$
Thm $\omega \sim \sum_{i=1}^{n} dp_i \wedge dq^i$
$\mathcal{L}(q,\dot q) = \frac{1}{2}\dot q_i \dot q^i - A_i(q)\dot q^i - V(q)$
EL $\ddot{\vec q} = -\partial_i V - \epsilon_{ijk} B^j \dot q^k$
$B = \text{curl } A$
$p_i = \frac{\partial \mathcal{L}}{\partial \dot q^i} = \dot q_i - A(q)_i$
$H = \frac{1}{2}(p+A)^2 + V(q)$
$dp_i \wedge dq^i$


$\mathcal{L}(q,\dot q) = \frac{1}{2}\dot q_i \dot q^i - A_i(q)\dot q^i - V(q)$
EL
$\ddot{\vec q} = -\partial_i V - \epsilon_{ijk} B^j \dot q^k$
$B = \mathrm{curl}\, A$
Ham
$p_i = \frac{\partial \mathcal{L}}{\partial \dot q^i} = \dot q_i - A(q)_i$
$H = \frac{1}{2}(p+A)^2 + V(q)$
$\omega = \Sigma\, dp_i \wedge dq^i$
H. eom
$\dot q_i = p_i + A_i(q)$
$\dot p_i = -\partial_i V$



$\mathcal{L}(q,\dot q) = \frac{1}{2}\dot q_i \dot q^i - A_i(q)\dot q^i - V(q)$
EL
$\ddot{\vec q} = -\partial_i V - \epsilon_{ijk} B^j \dot q^k$
$B = \mathrm{curl}\, A$
Ham
$p_i = \frac{\partial \mathcal{L}}{\partial \dot q^i} = \dot q^i - A$
$H = \frac{1}{2}(p+A)^2$
$\omega = \Sigma\, dp_i \wedge d$
H. eom
$\dot q_i = p_i + A_i(q)$
$\dot p_i = -\partial_i V - \sum_j (p+A)^j \partial_i A_j$



$\dot{q}_i \dot{q}^i - A_i(q)\dot{q}^i - V(q)$
$= -\partial_i V - \epsilon_{ijk} B^j \dot{q}^k$
$= \text{curl } A$
$_i = \frac{\partial \mathcal{L}}{\partial \dot{q}^i} = \dot{q}_i - A(q)_i$
$H = \frac{1}{2}(p+A)^2 + V(q)$
$\omega = \sum dp_i \wedge dq^i$
H. eom
$\dot{q}_i = p_i + A_i(q)$
$\dot{p}_i = -\partial_i V - \sum_j (p+A)^j \partial_i A_j$
New coords
$v_i = p_i + A_i(q)$
$q^i = q^i$
$H = \frac{1}{2}\vec{v}^2 + V(q)$
$\omega$

H. eom

$$\begin{cases} \dot{q}^i = p^i + A_i(q) \\ \dot{p}_i = -\partial_i V - \sum_j (p+A)^j \partial_i A_j \end{cases}$$

**New coords**

$$\begin{cases} v_i = p_i + A_i(q) \\ q^i = q^i \end{cases}$$

$$H = \frac{1}{2}\vec{v}^{\,2} + V(q)$$

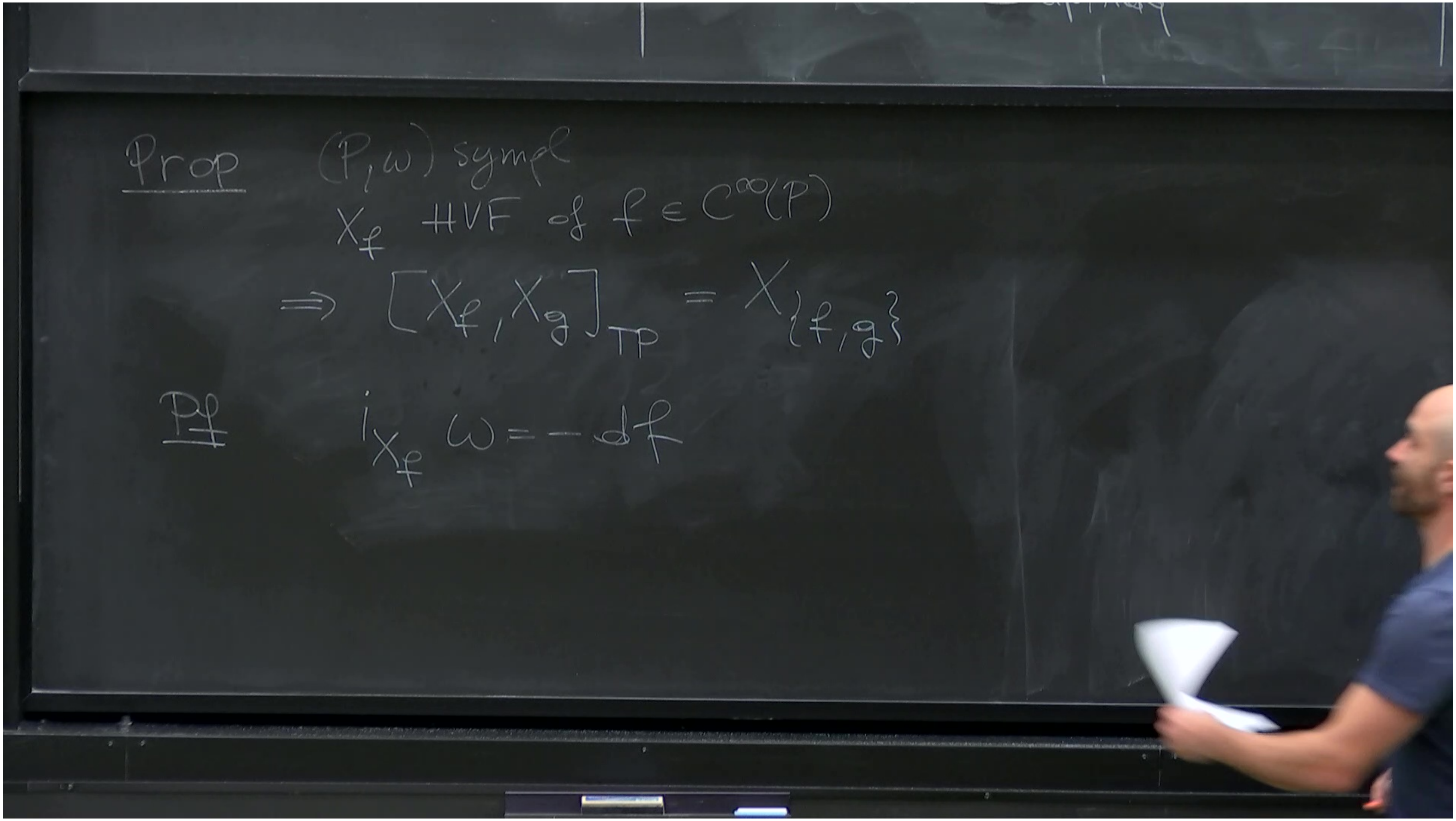
$$\omega = \sum dv_i \wedge dq^i + dA$$

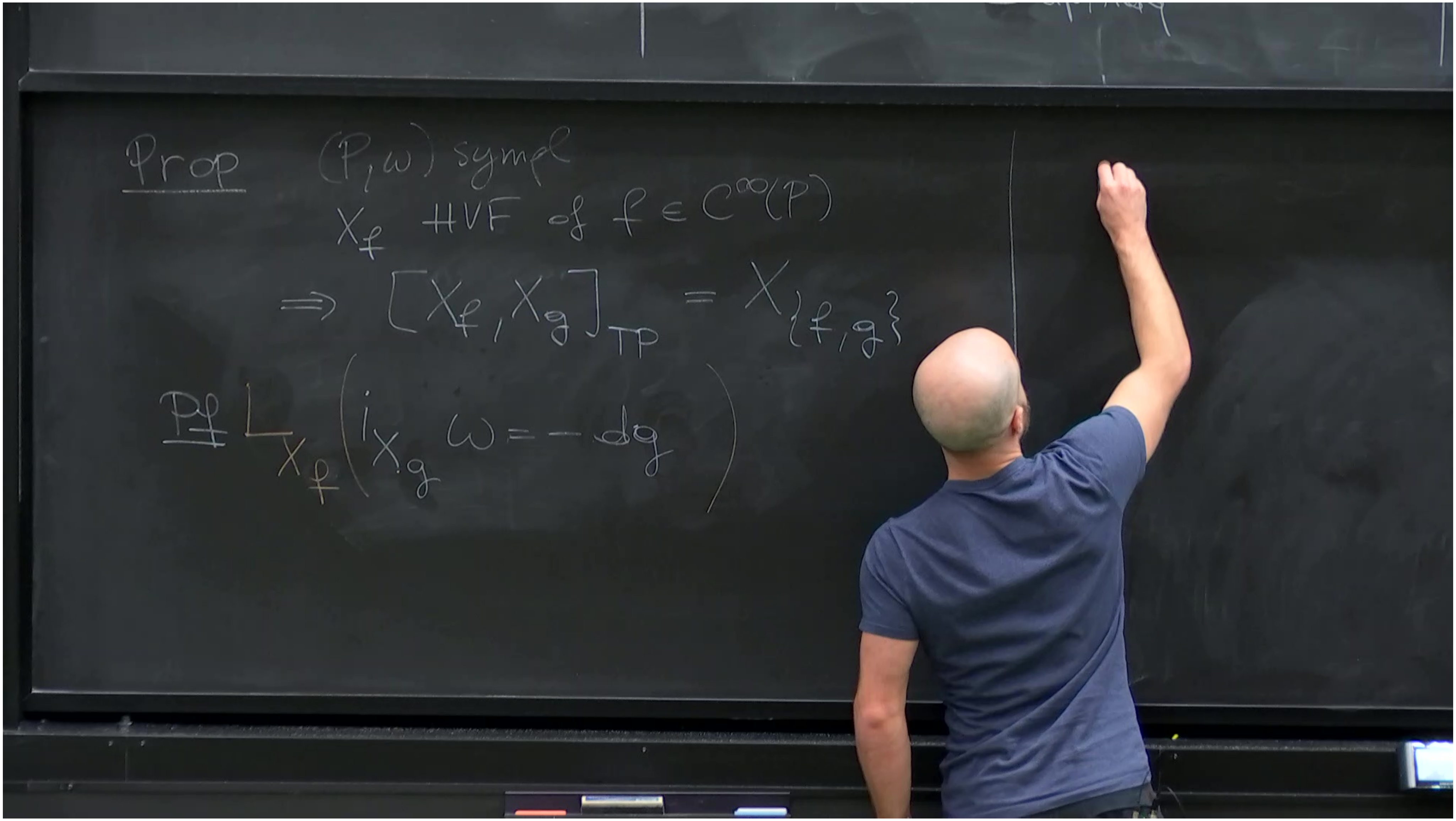
$$dA = \frac{1}{2} B_i \epsilon^i{}_{jk}\, dq^j \wedge dq^k$$

$$A = A_i\, dq^i$$

$$X_H = \sum_i v^i \frac{\partial}{\partial q^i} - \left(\partial_i V - \epsilon_{ijk} B^j \dot{q}^k\right) \frac{\partial}{\partial v^i}$$

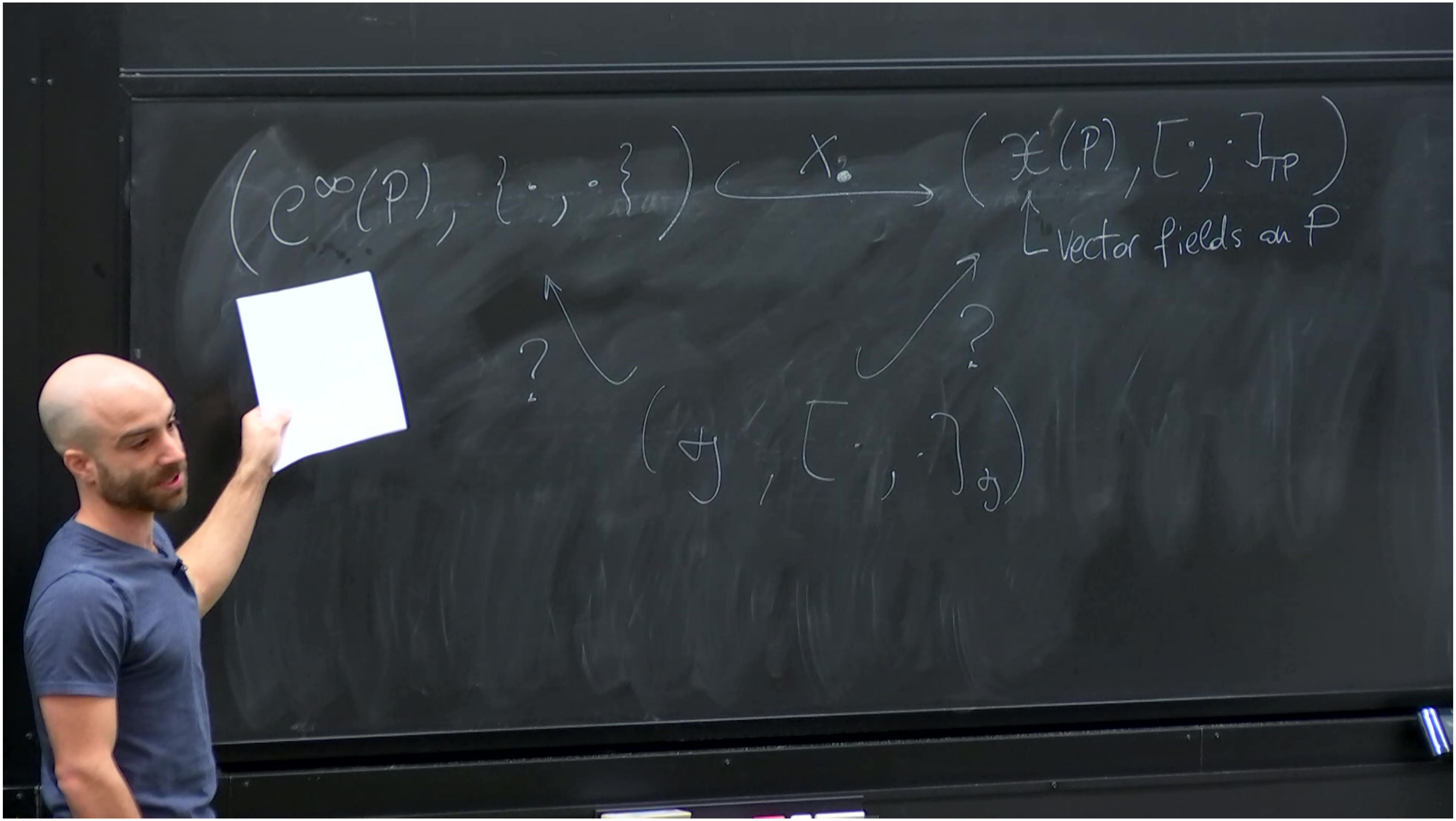
$$i_{X_H}\omega = -dH$$

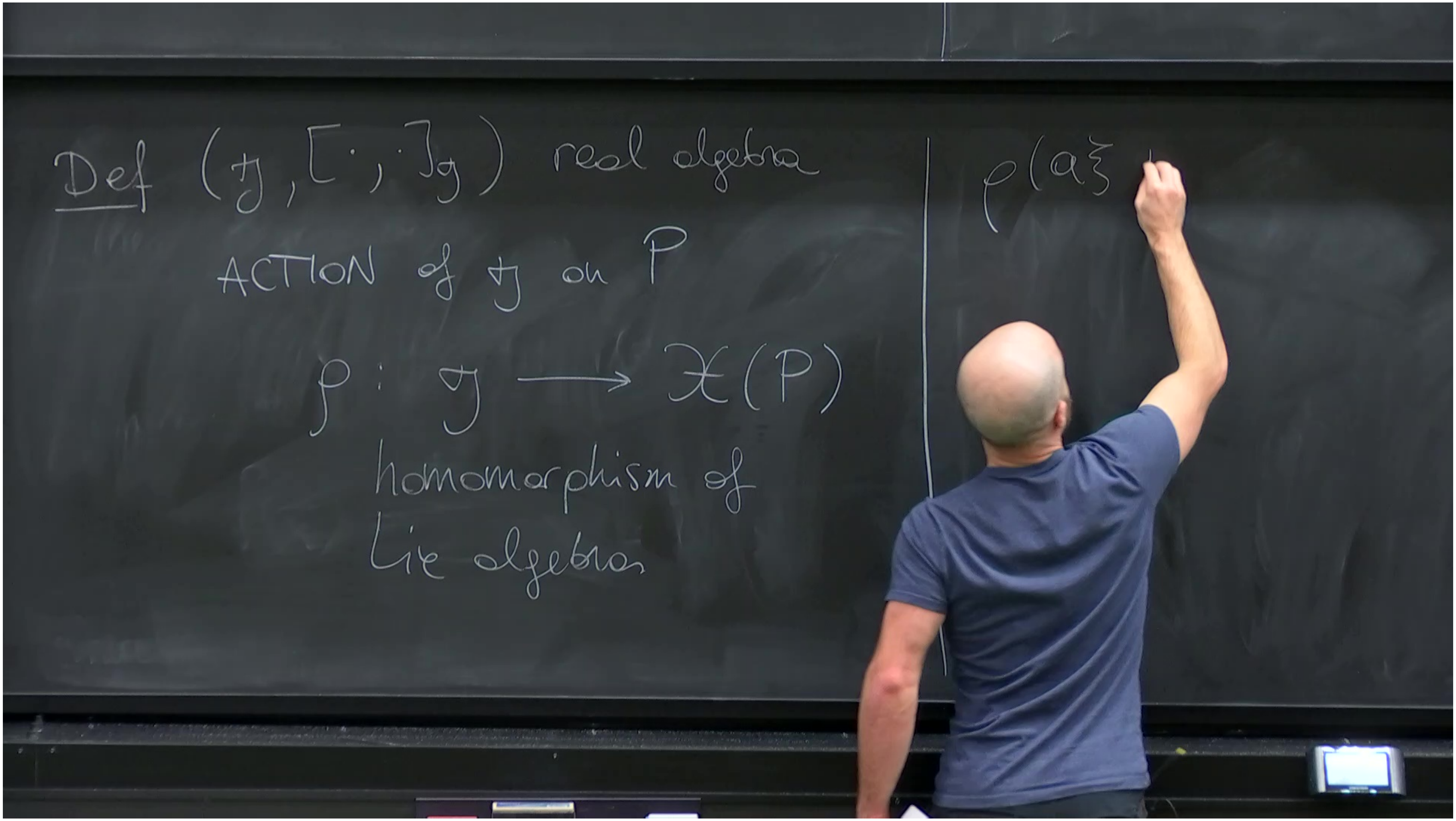

Prop $(P,\omega)$ sympl
$X_f$ HVF of $f \in C^\infty(P)$
$\Rightarrow [X_f, X_g]_{TP} = X_{\{f,g\}}$
Pf $i_{X_f}\omega = -df$

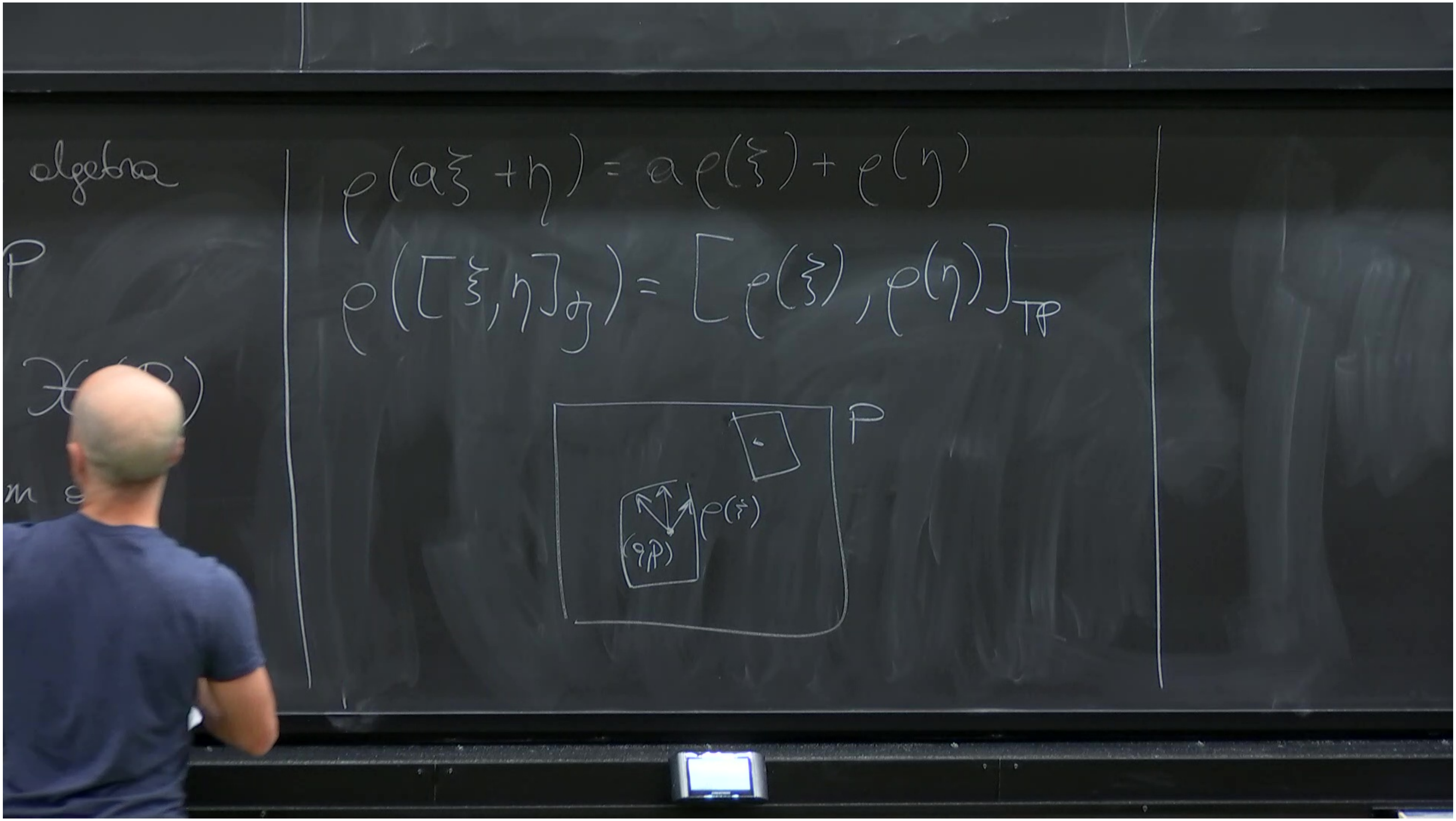

Prop $(P,\omega)$ sympl
$X_f$ HVF of $f \in C^\infty(P)$
$\Rightarrow [X_f, X_g]_{TP} = X_{\{f,g\}}$
Pf $L_{X_f}\left(i_{X_g}\omega = -dg\right)$

$$i_{L_{X_f} X_g}\,\omega + i_{X_g} L_{X_f}\,\omega = -d L_{X_f} g$$

$$i_{[X_f, X_g]}\,\omega + \cancel{i_{X_g} d(\overbrace{i_{X_f}\omega}^{-df})} = -d\, X_f(g) = -d\{f, g\}$$


$(C^\infty(P), \{\cdot,\cdot\})$
$X_\cdot$
$(\mathfrak{X}(P), [\cdot,\cdot]_{TP})$
vector fields on P
?
?
$(\mathfrak{g}, [\cdot,\cdot]_{\mathfrak{g}})$


Def $(\mathfrak{g}, [\cdot,\cdot]_{\mathfrak{g}})$ real algebra
ACTION of $\mathfrak{g}$ on $P$
$\rho : \mathfrak{g} \longrightarrow \mathfrak{X}(P)$
homomorphism of
Lie algebras
$\rho(a\xi$


algebra
P
$\rho(a\xi + \eta) = a\rho(\xi) + \rho(\eta)$
$\rho([\xi,\eta]_{\mathfrak{g}}) = [\rho(\xi), \rho(\eta)]_{TP}$
P
$\rho(\xi)$
$(q,p)$

$\rho(\eta)$

$), \rho(\eta)]_{TP}$

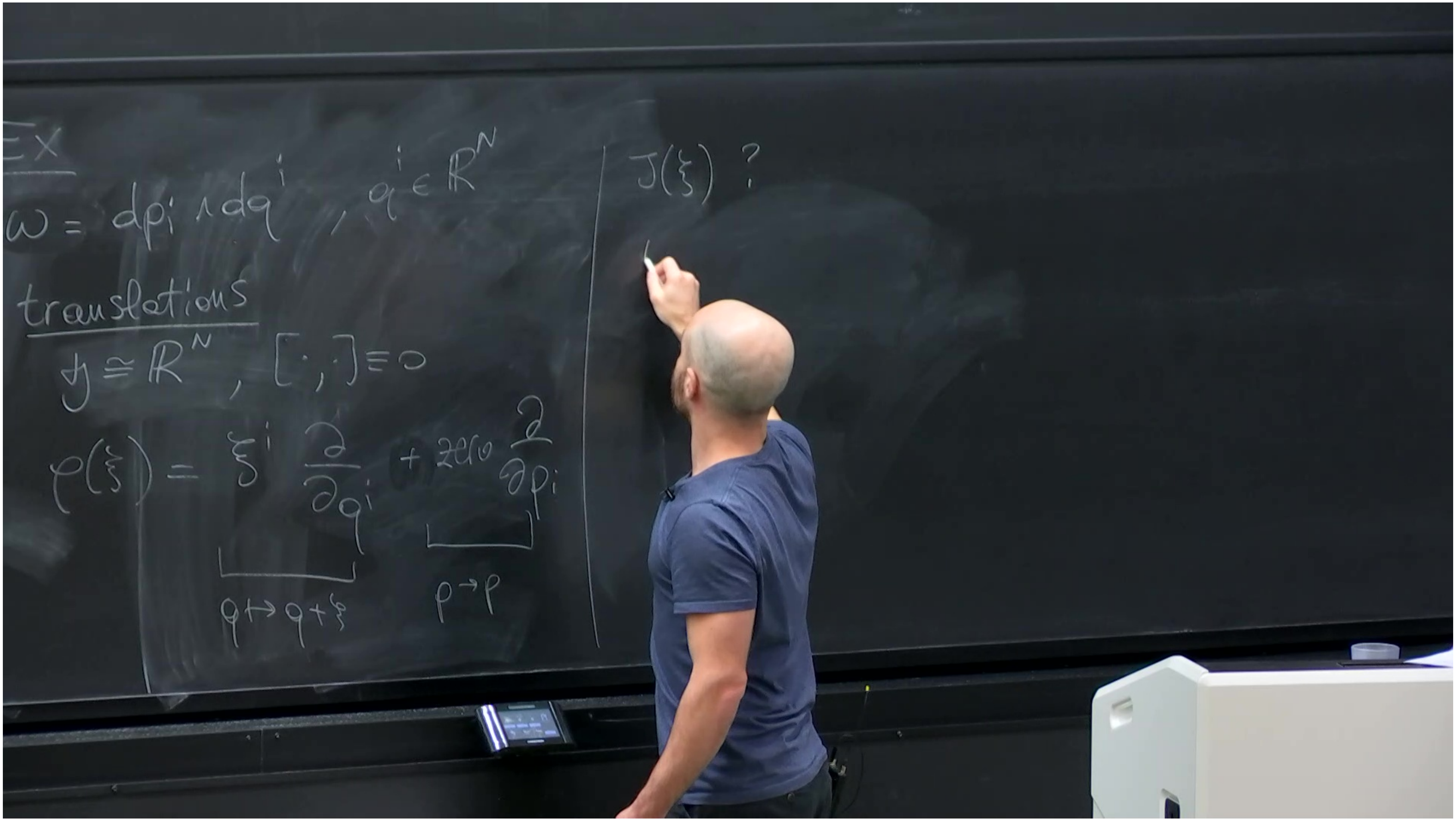
$P$

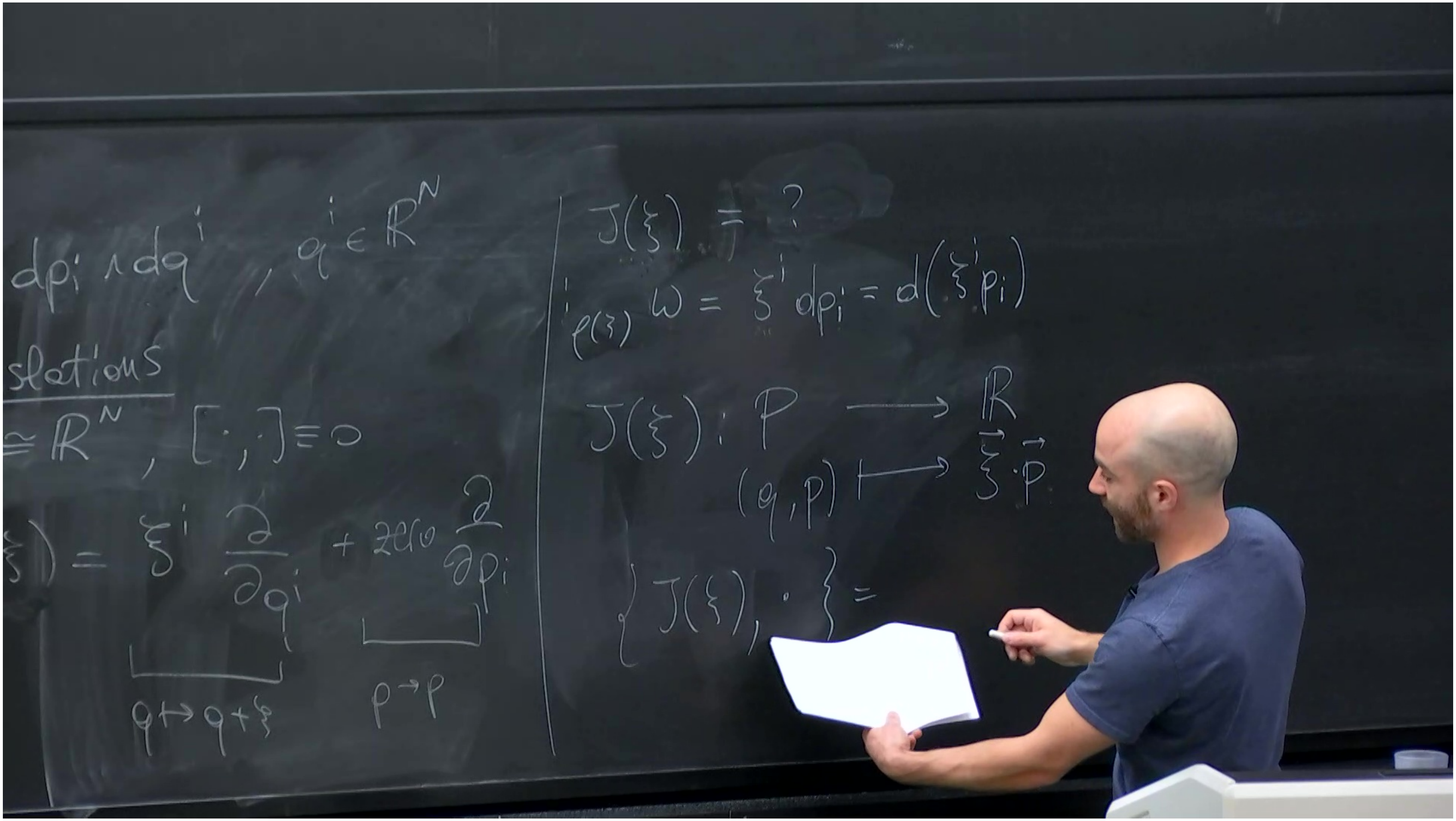
<u>Def</u> $\mathfrak{g}$ Ham. sym. (kinematical)

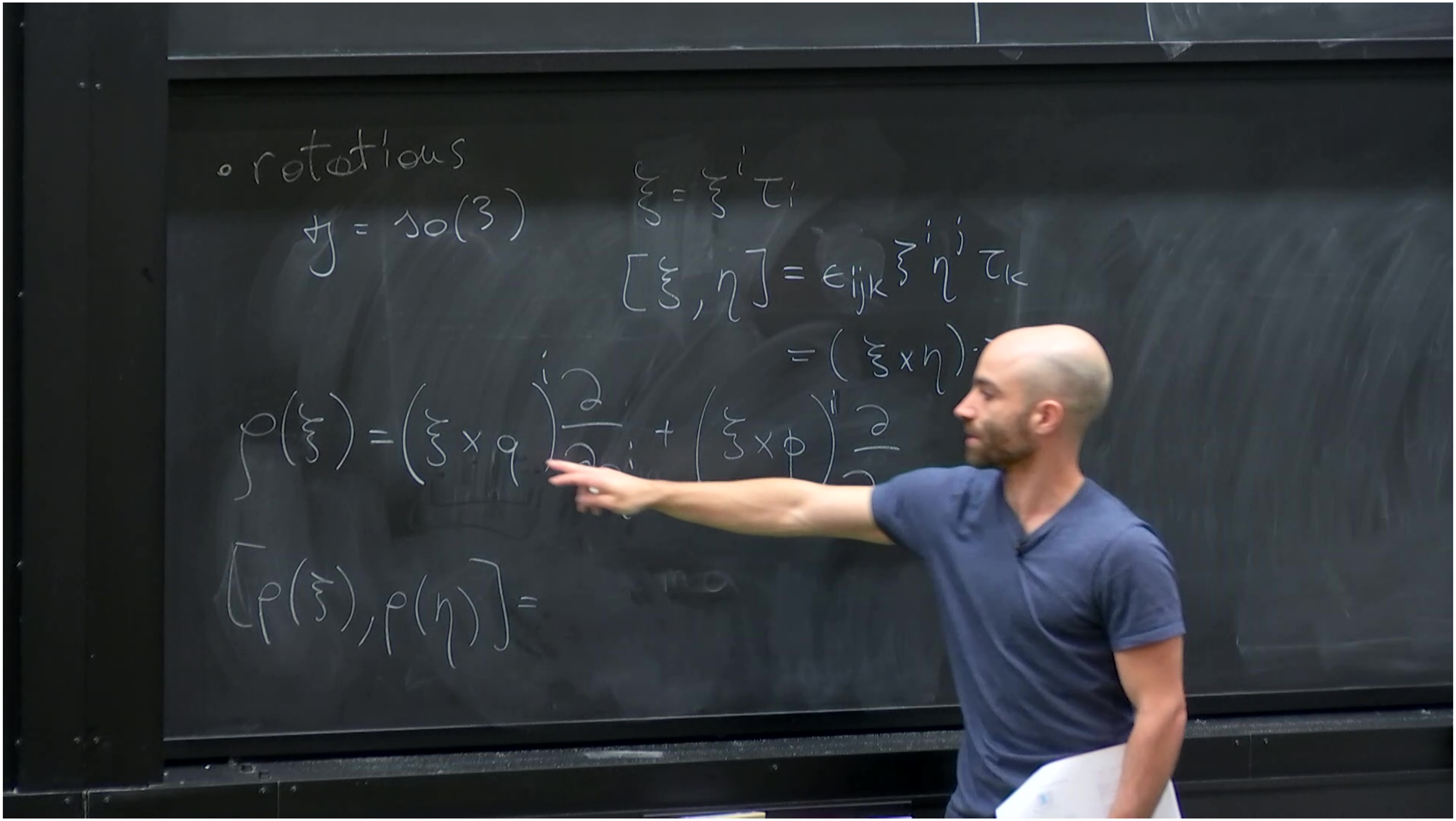
iff $\exists\, J^*: \mathfrak{g} \longrightarrow C^\infty(P)$ :

$$i_{\rho(\xi)}\,\omega = - d J^*(\xi)$$

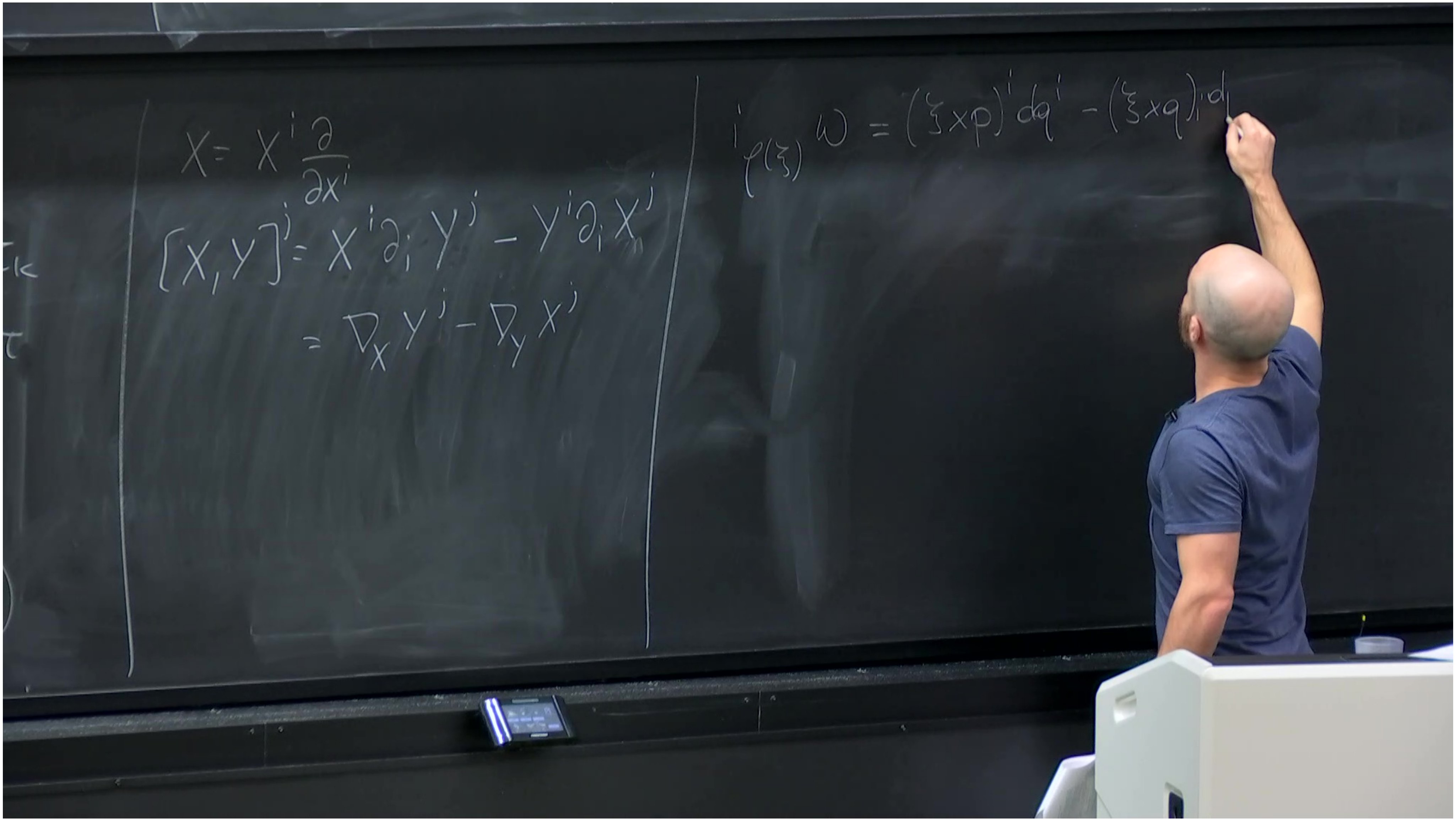
$J^*$ called "comomentum map"

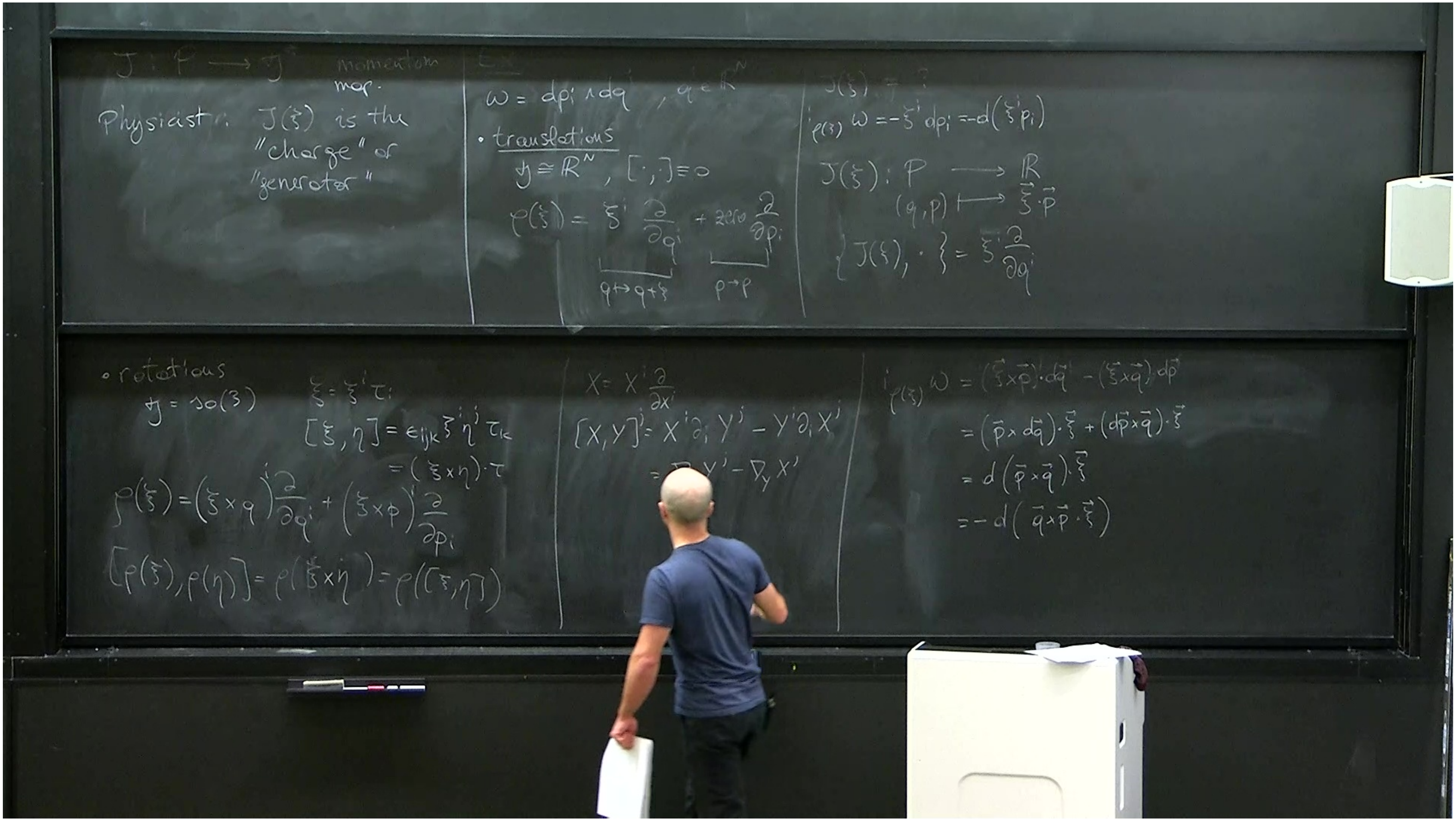

Ex
ω = dpᵢ ∧ dqⁱ , qⁱ ∈ ℝᴺ
translations
g ≅ ℝᴺ , [·,·] = 0
ρ(ξ) = ξⁱ ∂/∂qⁱ + zero ∂/∂pᵢ
q ↦ q+ξ
p → p
J(ξ) ?

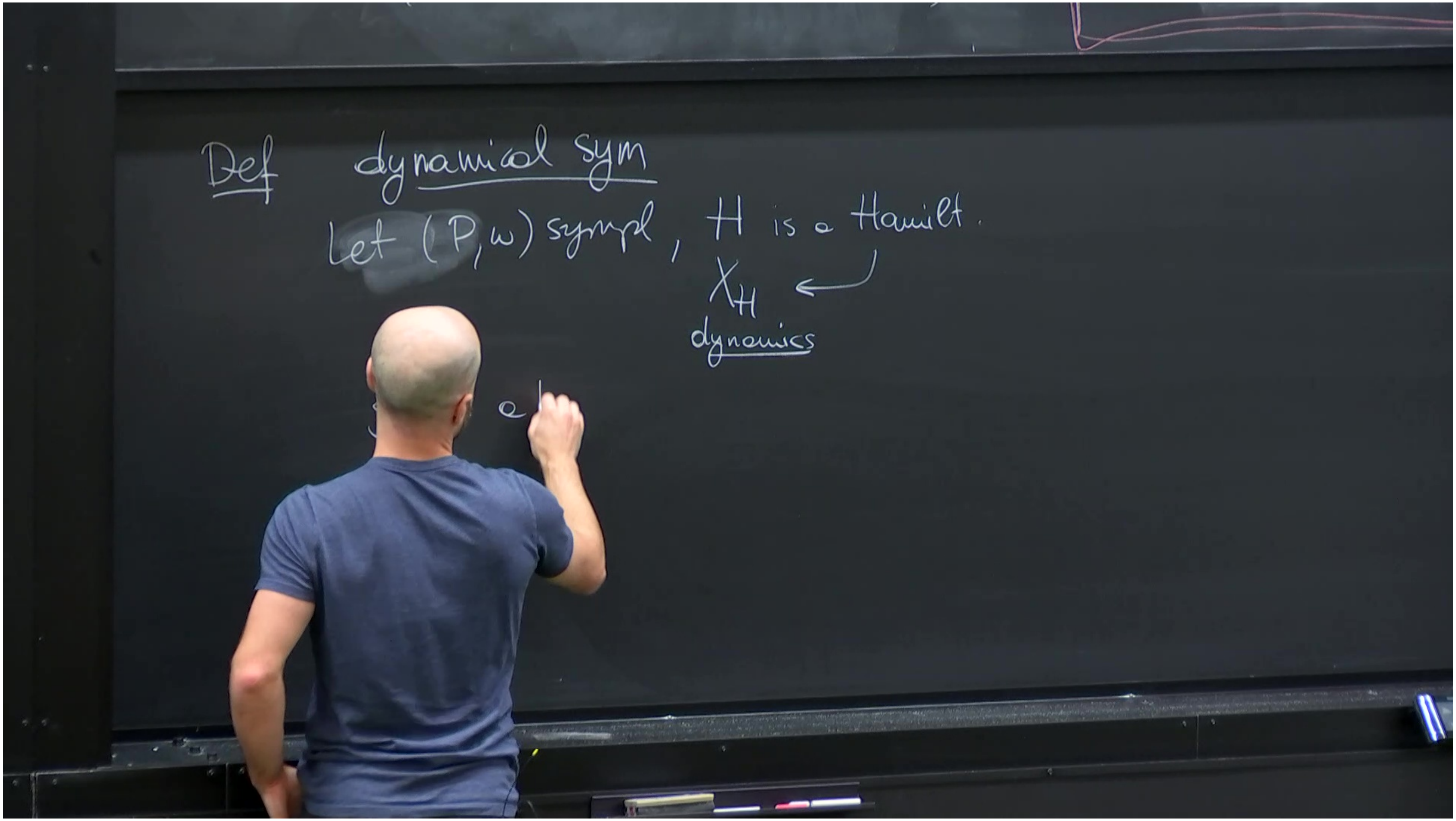

dpᵢ ∧ dqⁱ , qⁱ ∈ ℝᴺ
slations
≅ ℝᴺ , [·,·] ≡ 0
ξ) = ξⁱ ∂/∂qⁱ + zero ∂/∂pᵢ
q ↦ q + ξ
p → p
J(ξ) = ?
ι_{ρ(ξ)} ω = ξⁱ dpᵢ = d(ξⁱ pᵢ)
J(ξ) : P ⟶ ℝ
(q, p) ⟼ ξ⃗ · p⃗
{ J(ξ), · } =

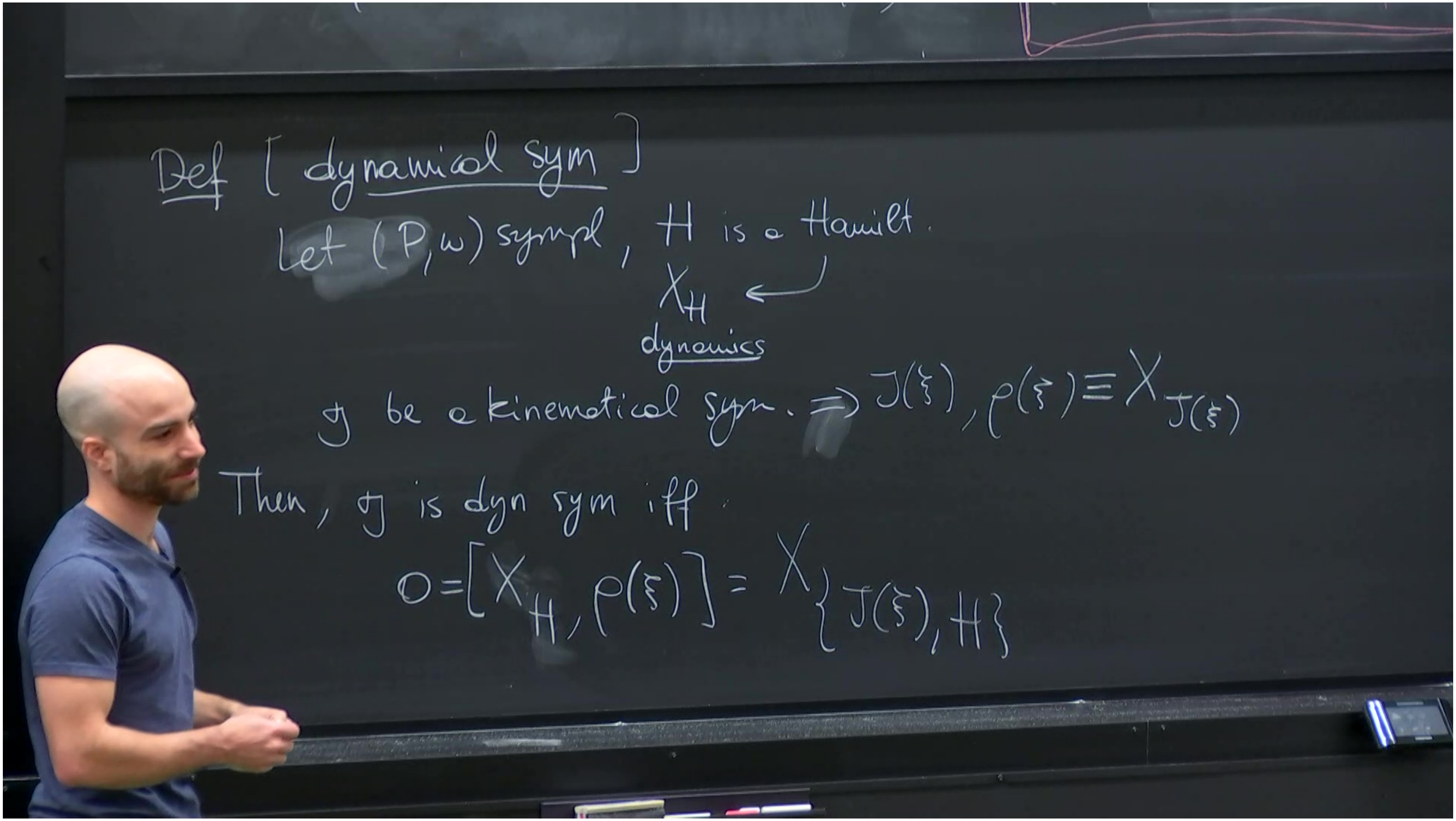


• rotations
$\mathfrak{g} = so(3)$
$\xi = \xi^i \tau_i$
$[\xi, \eta] = \epsilon_{ijk} \xi^i \eta^j \tau_k$
$= (\xi \times \eta)^i \tau$
$\rho(\xi) = (\xi \times q)^i \frac{\partial}{\partial q^i} + (\xi \times p)^i \frac{\partial}{\partial}$
$[\rho(\xi), \rho(\eta)] =$


$X = X^i \frac{\partial}{\partial x^i}$
$[X,Y]^j = X^i \partial_i Y^j - Y^i \partial_i X^j$
$= \nabla_X Y^j - \nabla_Y X^j$
$i_{\rho(\xi)}\,\omega = (\xi \times p)^i dq^i - (\xi \times q)_i d$


J : P ⟶ g* momentum map.
Physicist: J(ξ) is the "charge" or "generator"
Ex
ω = dp_i ∧ dq^i , q ∈ R^N
• translations
g ≅ R^N , [·,·] ≡ 0
ρ(ξ) = ξ^i ∂/∂q^i + zero ∂/∂p_i
q ↦ q+ξ   p → p
J(ξ) = ?
i_ρ(ξ) ω = −ξ^i dp_i = −d(ξ^i p_i)
J(ξ) : P ⟶ R
(q,p) ⟼ ξ·p
{J(ξ), · } = ξ^i ∂/∂q^i
• rotations
g = so(3)   ξ = ξ^i τ_i
[ξ, η] = ε_ijk ξ^i η^j τ_k
= (ξ × η)·τ
ρ(ξ) = (ξ × q)^i ∂/∂q^i + (ξ × p)^i ∂/∂p_i
[ρ(ξ), ρ(η)] = ρ(ξ × η) = ρ([ξ,η])
X = X^i ∂/∂x^i
[X,Y]^j = X^i ∂_i Y^j − Y^i ∂_i X^j
= ∇_X Y^j − ∇_Y X^j
i_ρ(ξ) ω = (ξ × p)·dq − (ξ × q)·dp
= (p × dq)·ξ + (dp × q)·ξ
= d(p × q)·ξ
= −d(q × p · ξ)


Def dynamical sym
Let (P, ω) sympl, H is a Hamilt.
X_H
dynamics


Def [ dynamical sym ]
Let $(P, \omega)$ sympl, $H$ is a Hamilt.
$X_H$
dynamics
$\mathfrak{g}$ be a kinematical sym. $\Rightarrow J(\xi), \rho(\xi) \equiv X_{J(\xi)}$
Then, $\mathfrak{g}$ is dyn sym iff
$0 = [X_H, \rho(\xi)] = X_{\{J(\xi), H\}}$

Def [ dynamical sym ]

Let $(P, \omega)$ sympl, $H$ is a Hamilt.

$X_H \leftarrow$

dynamics

$\mathfrak{g}$ be a kinematical sym. $\Rightarrow J(\xi)$, $\rho(\xi) \equiv X_{J(\xi)}$

Then, $\mathfrak{g}$ is dyn sym iff

$$0 = \underbrace{\{J(\xi), H\}}_{\frac{d}{dt} J(\xi)} = L_{X_H} J(\xi) = L_{\rho(\xi)} H$$