Title: Quantum Information and holography

Speakers: Aaron Szasz, Beni Yoshida

Collection: Quantum Information and holography (2021/2022)

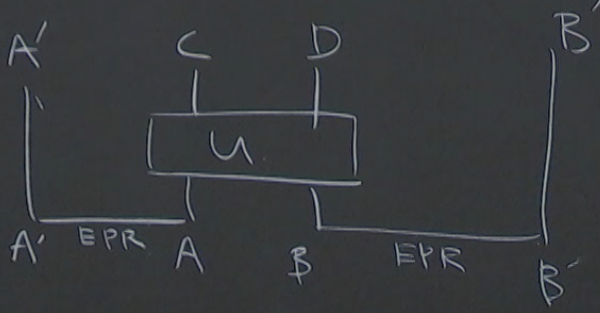
Date: April 20, 2022 - 9:00 AM

URL: https://pirsa.org/22040051

Abstract: Topics will include (but are not limited to):
- Quantum error correction in quantum gravity and condensed matter
- Quantum information scrambling and black hole information
- Physics of random tensor networks and random unitary circuits

Information recovery from scrambling

$|\Psi\rangle \equiv$



$I(A', B'D) \approx \max$

$\Updownarrow$

Recoverable

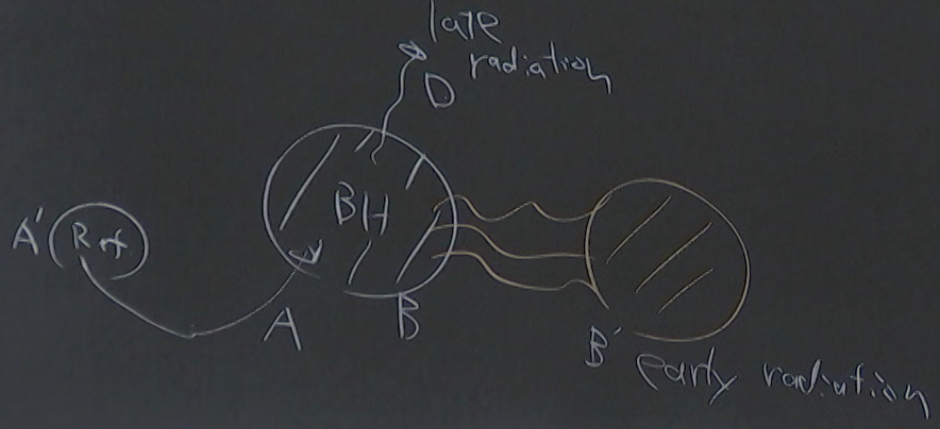
A : diary

A' : ref

B : BH

B' : early rad.

D : late rad.



O_A, O_D : unitary on A, D

$\langle O_A(0)\, O_D(t)\, O_A^\dagger(0)\, O_D^\dagger(t) \rangle$ $\int dO_A$: average over all unitary O_A.

U : fixed

$\geq \min\left(\frac{1}{d_A^2}, \frac{1}{d_D^2}\right)$ (?)

$d_A = 2^{n_A}$

$(OTOC)_{ave}$

Averaged OTOCs

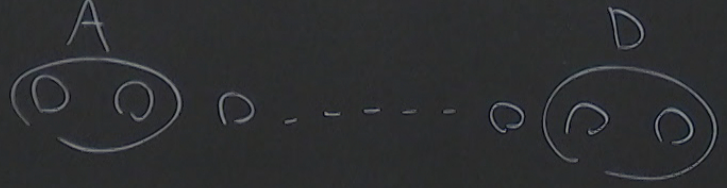
$\langle O_A(0) O_D(t) O_A^\dagger(0) O_D^\dagger(t) \rangle$ O_A, O_D: unitary on A, D

$\langle \text{OTOC} \rangle_{\text{ave}} = \iint dO_A\, dO_D \, \langle O_A(0) O_D(t) O_A^\dagger(0) O_D^\dagger(t) \rangle$ $\int dO_A$: ave

U : fixed

Scrambly $\Rightarrow$ small $\langle \text{OTOC} \rangle_{\text{ave}}$

$\geq \min\left(\frac{1}{d_A^2}, \frac{1}{d_D^2} \right)$ (?)



$d_A = 2^n$

$\mathrm{Tr}\left(\frac{1}{d} \sim \right)$

$O_D(t) = U O_D U^\dagger$

$(t)\rangle$ $\quad O_A, O_D$: unitary on A, D

$\int dO_D \langle O_A(0)\, O_D(t)\, O_A^\dagger(0)\, O_D^\dagger(t) \rangle$ $\quad \int dO_A$: average over all unitary O_A.

$\mathcal{U}$: fixed

all $(O\Gamma(\alpha)_{ave}$

$\geq \min\left(\frac{1}{d_A^2}, \frac{1}{d_D^2}\right)$ (?)

D

$\mathrm{Tr}\left(\frac{1}{d} \sim\right)$

$d_A = 2^{n_A}$

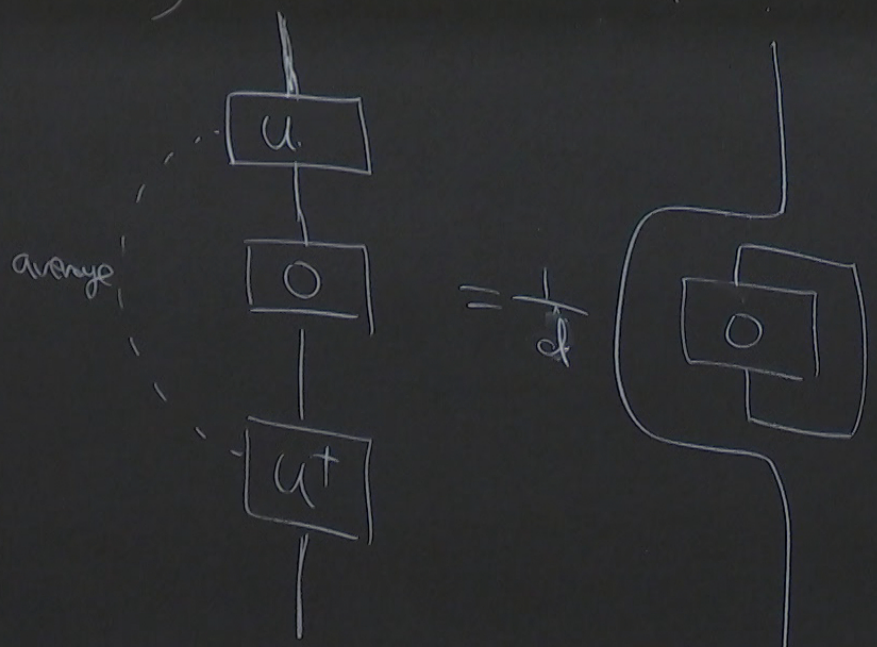
i) $\int_{\text{Haar}} dU_A \; U_A O U_A^\dagger = \int_{\text{Pauli}} dP_A \; P_A O P_A^\dagger$

$\frac{1}{4^{n_A}} \sum_{P \neq \text{Pauli}}$

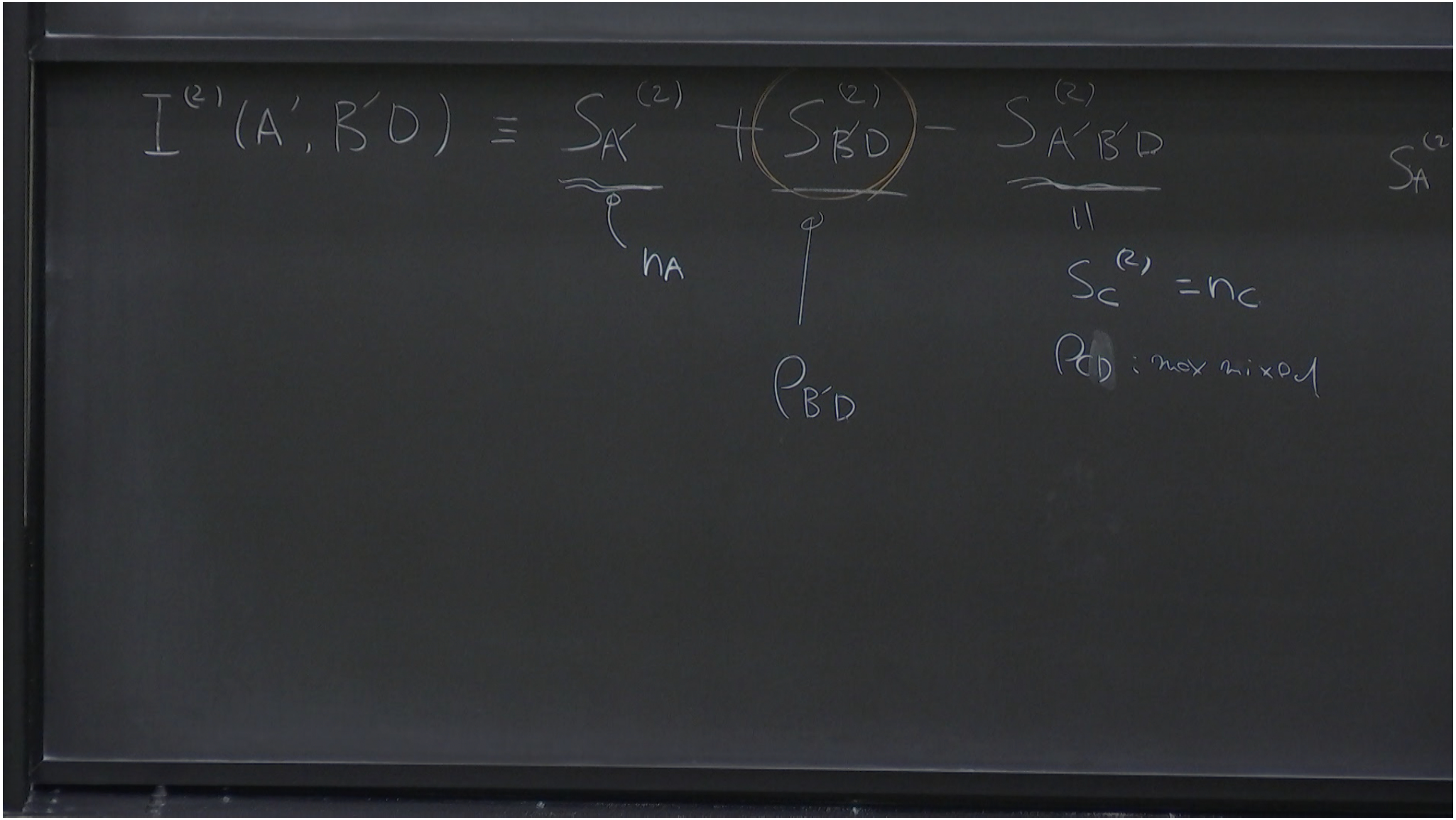
$$\langle O(t) O \rangle_{\text{ave}} = \frac{1}{(d_A^2) d_B^2} \sum_{P_A, P_B \in \text{Pauli}} \langle P_A(0) P_B(t) P_A^\dagger(0) P_B^\dagger(t) \rangle$$

4^{n_A} : # of Pauli on A

ii) $\int du\ UOU^\dagger = \frac{I}{d}\,\mathrm{Tr}(O)$ (depolarizing)



$= \frac{1}{d}$


$I^{(2)}(A', B'D) \equiv S_{A'}^{(2)} + S_{B'D}^{(2)} - S_{A'B'D}^{(2)}$
$S_A^{(2)}$
n_A
$\rho_{B'D}$
$S_C^{(2)} = n_C$
ρ_{CD} : max mixed

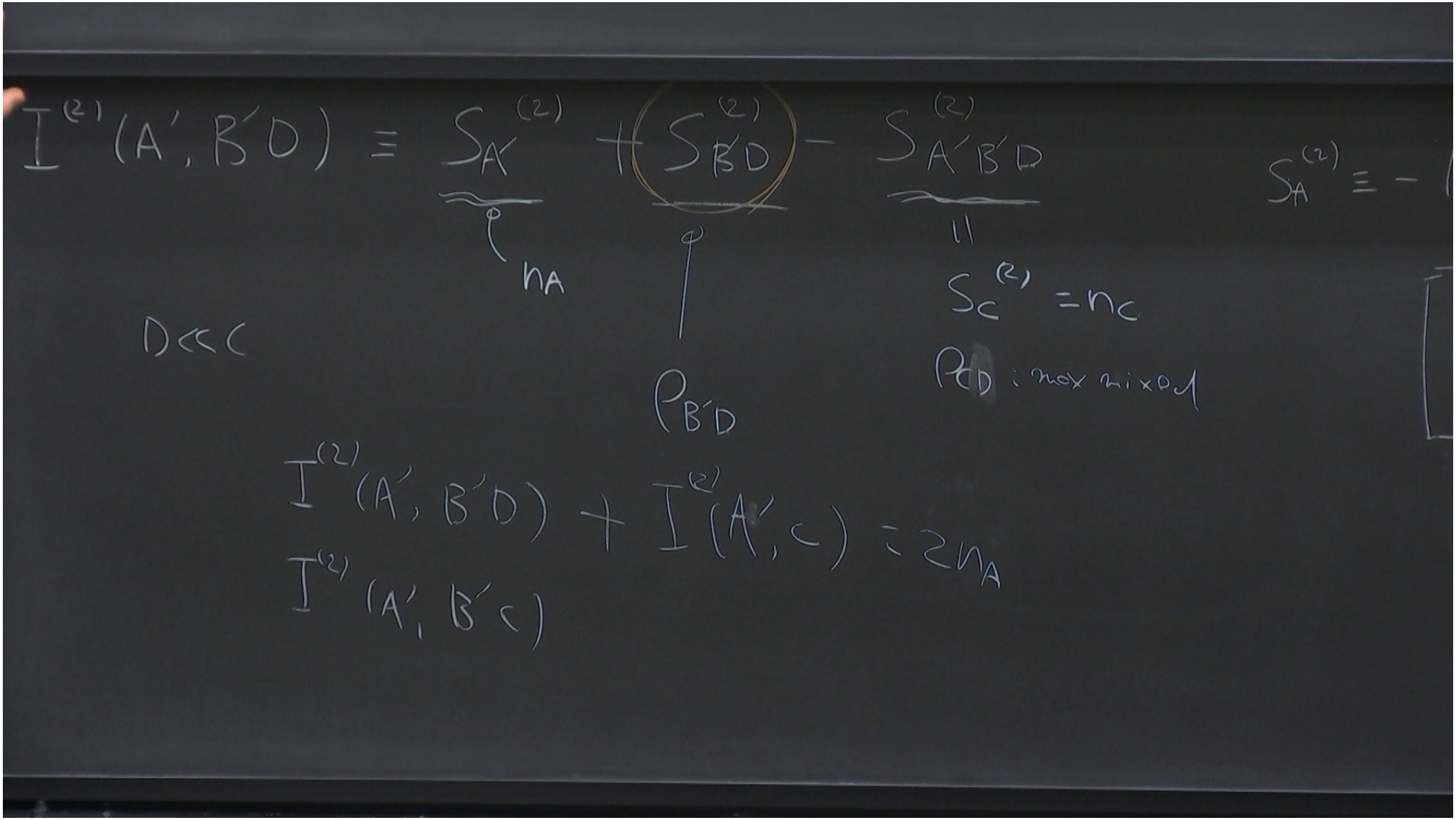
$$S_A^{(2)} \equiv -\log\left[\mathrm{Tr}(\rho_A^2)\right]$$

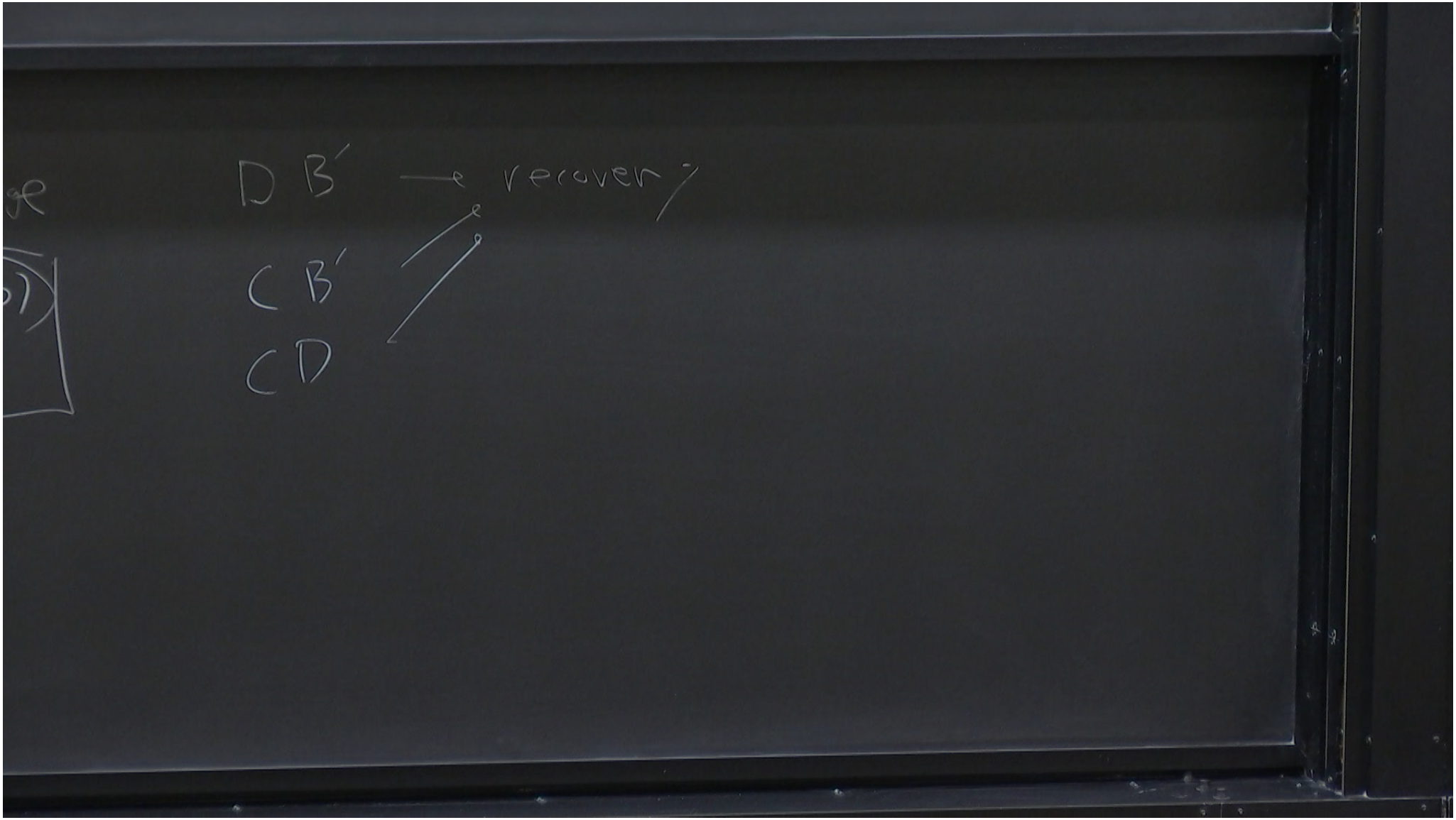
large

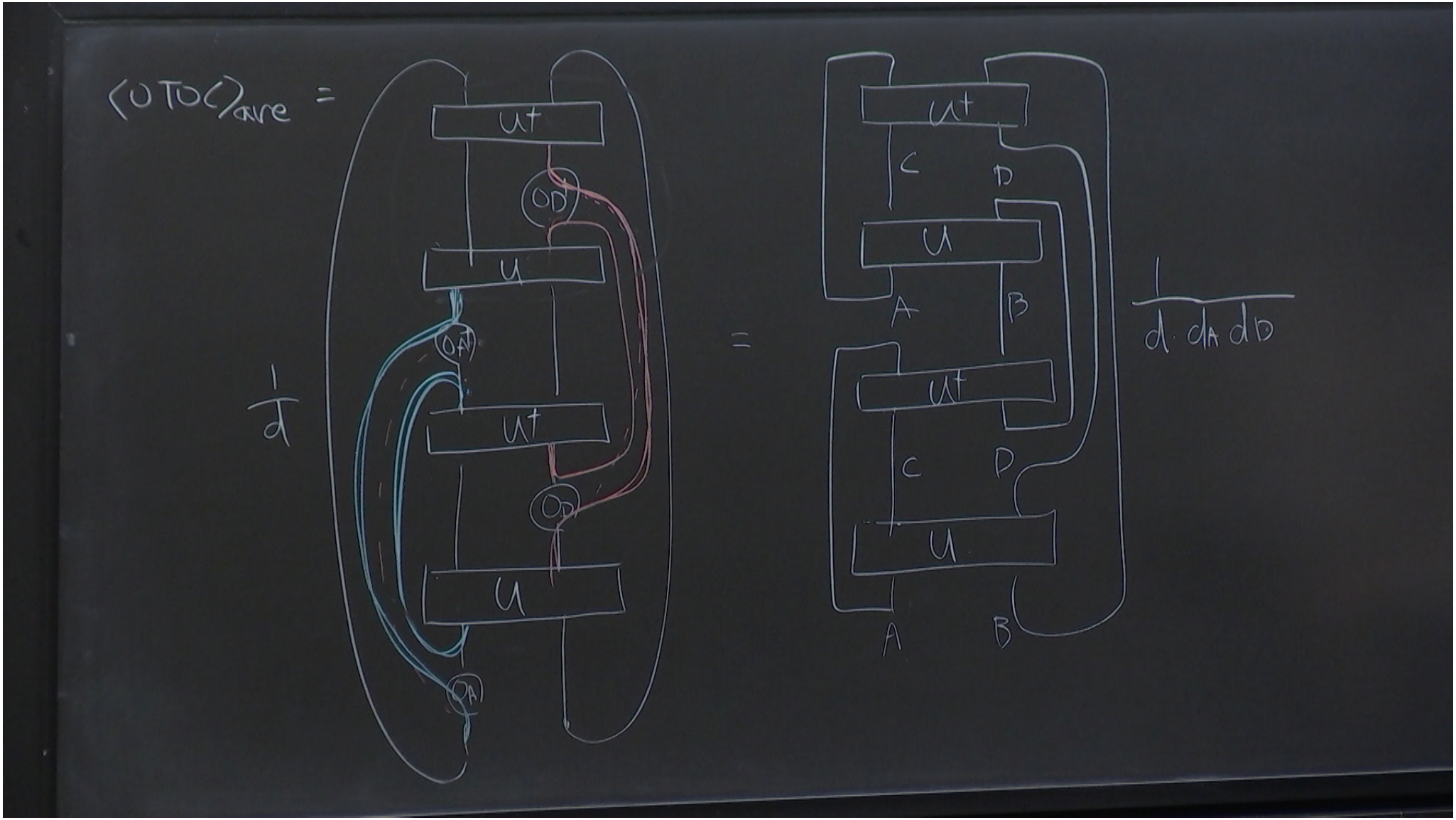
$$\langle OTOC \rangle_{ave} = 2^{-I^{(2)}(A', B'D)}$$

small

scrambly $\Rightarrow$ recoverability.

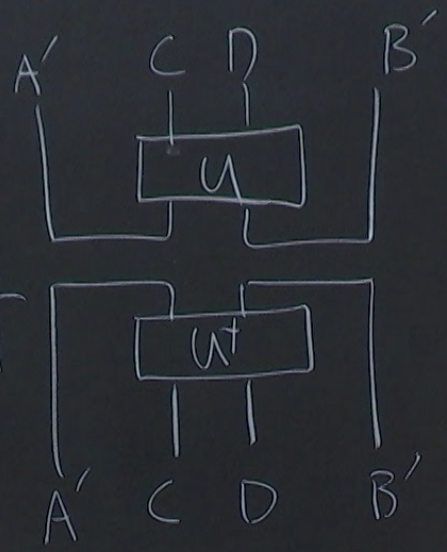

$I^{(2)}(A', B'D) \equiv S_{A'}^{(2)} + S_{B'D}^{(2)} - S_{A'B'D}^{(2)}$
$S_A^{(2)} \equiv -$
n_A
$D \ll C$
$S_C^{(2)} = n_C$
ρ_{CD} : max mixed
$\rho_{B'D}$
$I^{(2)}(A', B'D) + I^{(2)}(A', C) = 2n_A$
$I^{(2)}(A', B'C)$


DB'
recovery
CB'
CD

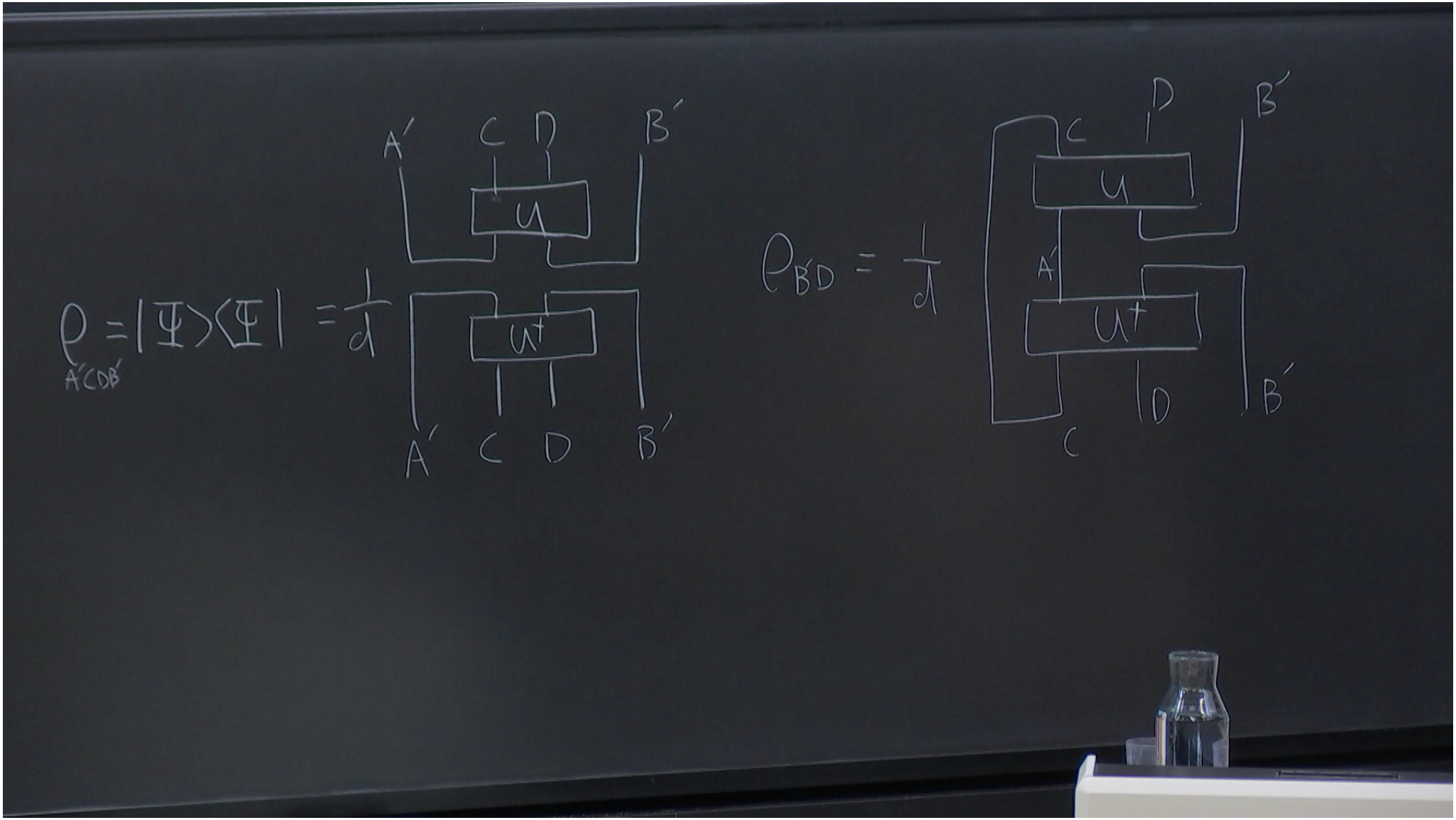

(UTOC)are =
U†
OD
U
OA
U†
OD
U
OA
=
U†
C
D
U
A
B
U†
C
D
U
A
B
1/d
1/d · dA dD

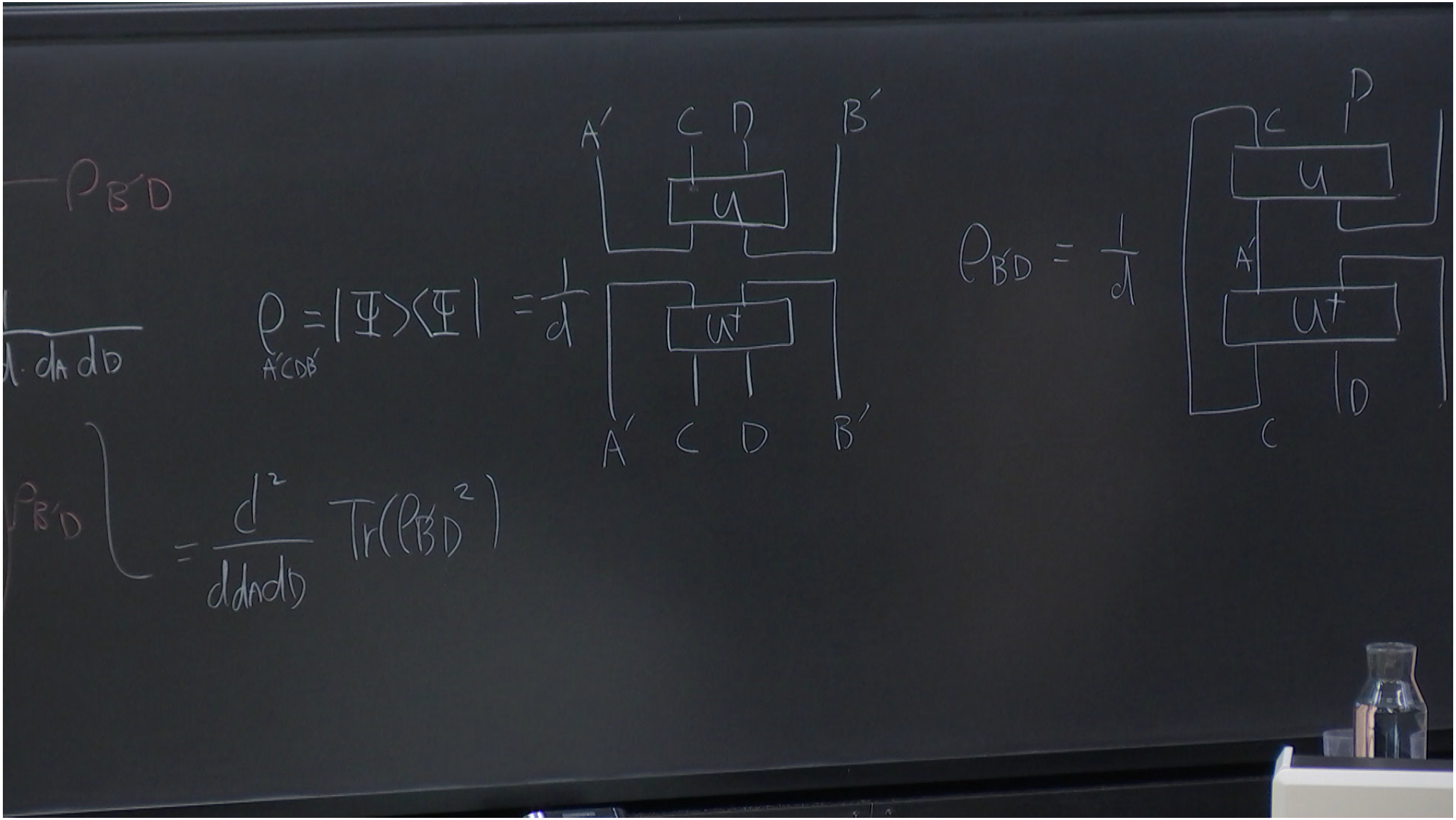
$$\frac{1}{d \cdot d_A d_D}$$

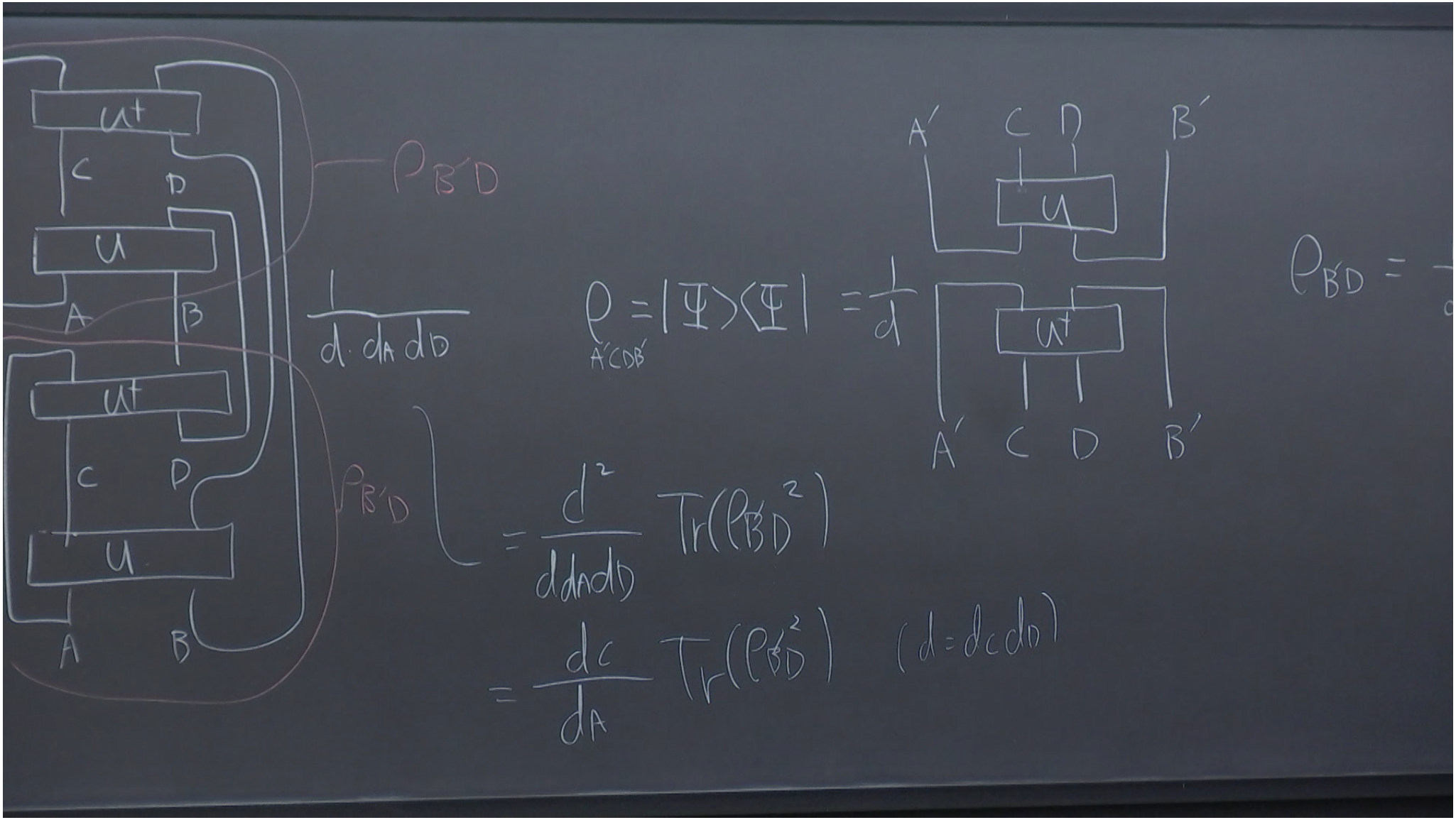
$$\rho_{A'CDB'} = |\Psi\rangle\langle\Psi| = \frac{1}{d}$$



$$\rho_{B'D} =$$


$\rho_{A'CDB'} = |\Psi\rangle\langle\Psi| = \frac{1}{d}$
A' C D B'
U
U†
A' C D B'
$\rho_{BD} = \frac{1}{d}$
C D B'
U
A'
U†
C D B'

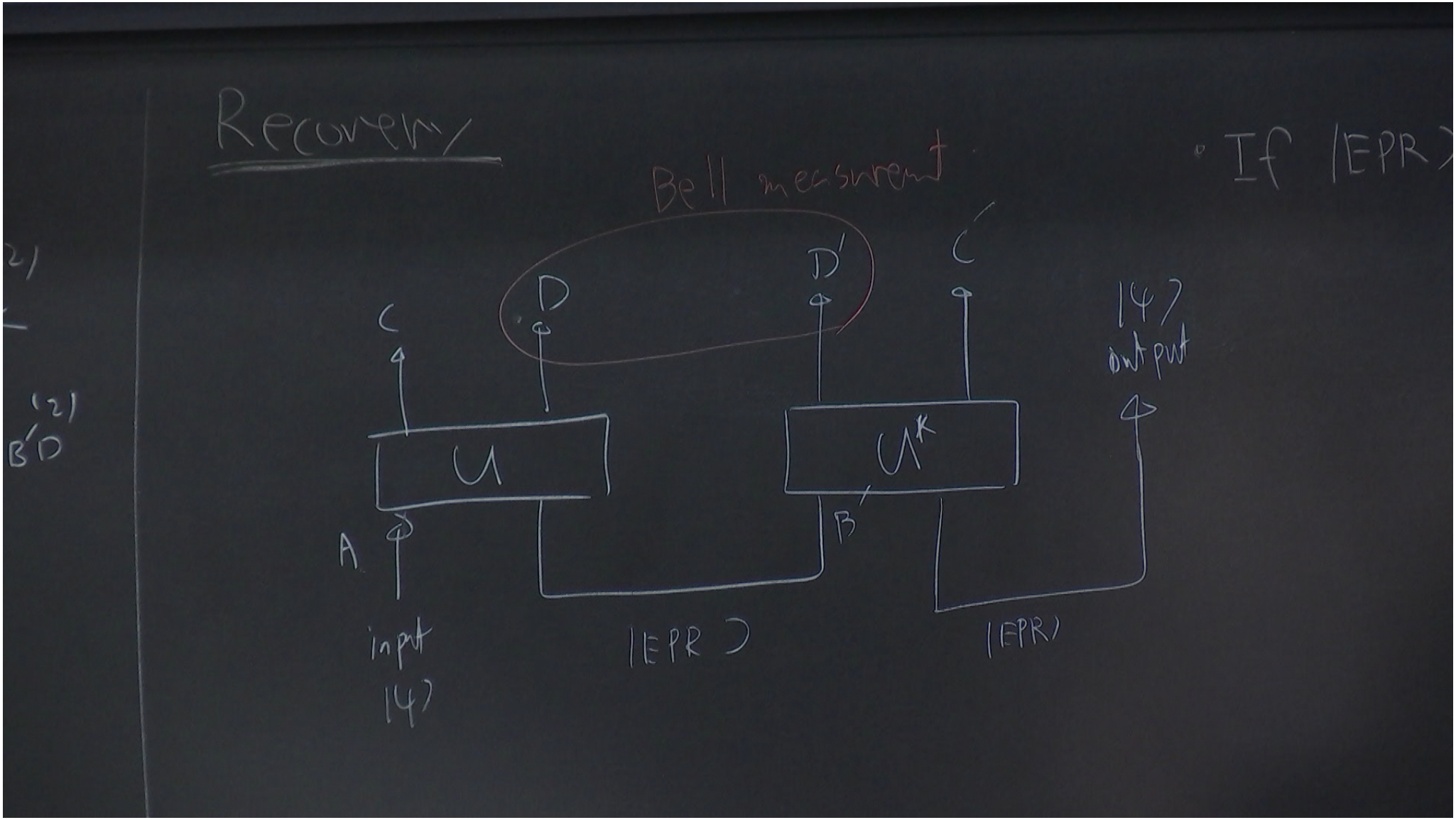

$\rho_{B'D}$
$\rho_{A'CDB'} = |\Psi\rangle\langle\Psi| = \frac{1}{d}$
A' C D B'
U
U^t
A' C D B'
$\rho_{B'D} = \frac{1}{d}$
C D
U
A'
U^t
C D
$\rho_{B'D}$ = $\frac{d^2}{d\,d_A\,d_D}$ $Tr(\rho_{B'D}^2)$

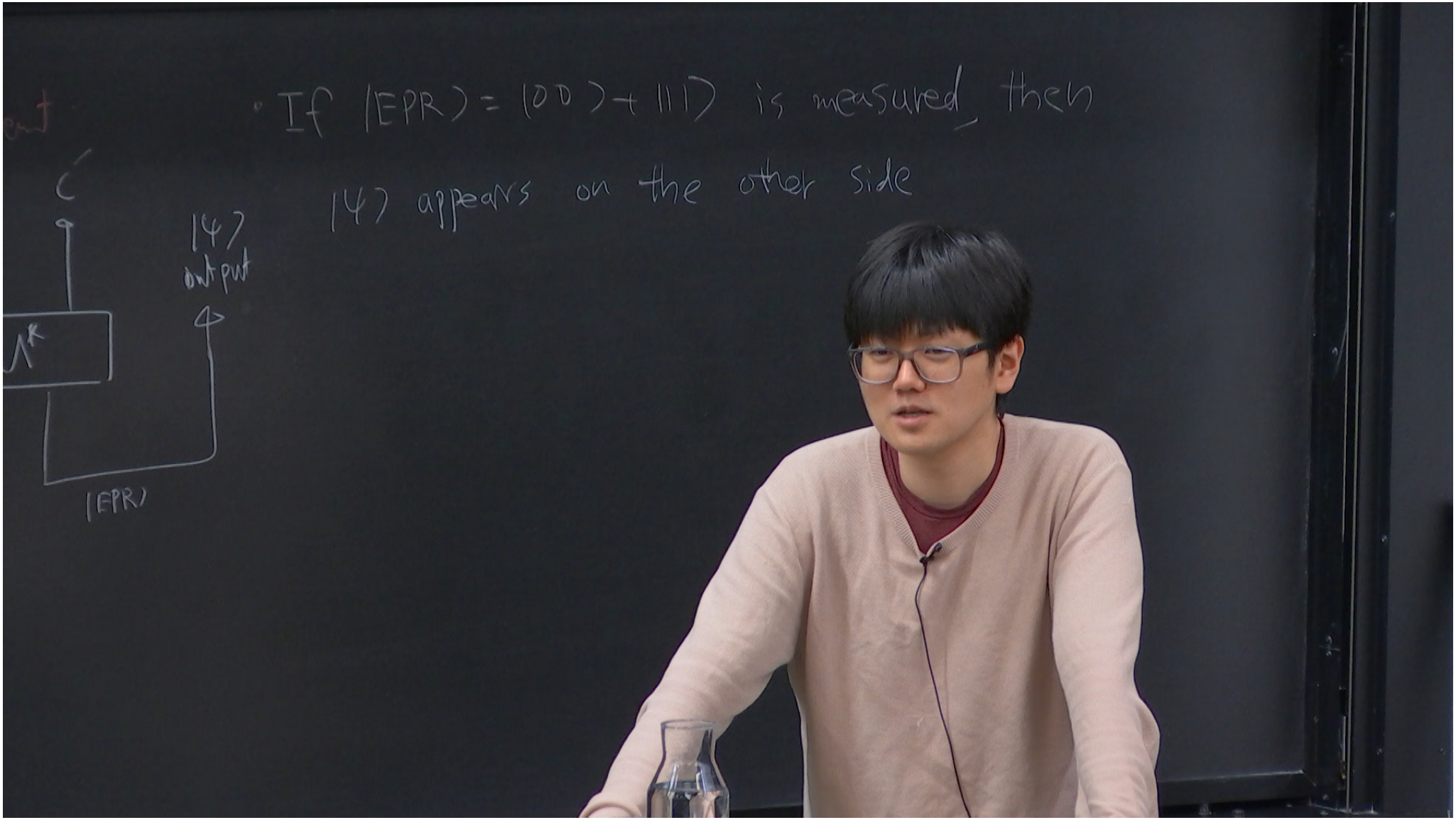

U†
C
D
U
A
B
U†
C
D
U
A
B
$\rho_{B'D}$
$\frac{1}{d \cdot d_A d_D}$
$\rho_{A'CDB'} = |\Psi\rangle\langle\Psi| = \frac{1}{d}$
A' C D B'
U
U†
A' C D B'
$\rho_{B'D}$
$= \frac{d^2}{d d_A d_D} Tr(\rho_{B'D}^2)$
$= \frac{d_C}{d_A} Tr(\rho_{B'D}^2)$ $(d = d_C d_D)$

$$(OTOC)_{ave} = \frac{dc}{dA} \mathrm{Tr}(P R' D^{\dagger})$$

$$= \frac{2^{S_C^{(2)}}}{2^{S_A^{(2)}}} 2^{-S_{BD}^{(2)}} = 2^{-S_A^{(2)} - S_{BD}^{(2)} + \underbrace{S_C^{(2)}}_{\downarrow S_{AB'D}^{(2)}}}$$

$$= 2^{-I^{(2)}(A, BD)}$$


Recovery
Bell measurement
If |EPR>
C
D
D'
C'
U
U*
A
B'
|ψ>
output
input
|ψ>
|EPR>
|EPR>


If |EPR⟩ = |00⟩ + |11⟩ is measured, then
|4⟩ appears on the other side
|4⟩
output
|EPR⟩