Title: Quantum Information and holography

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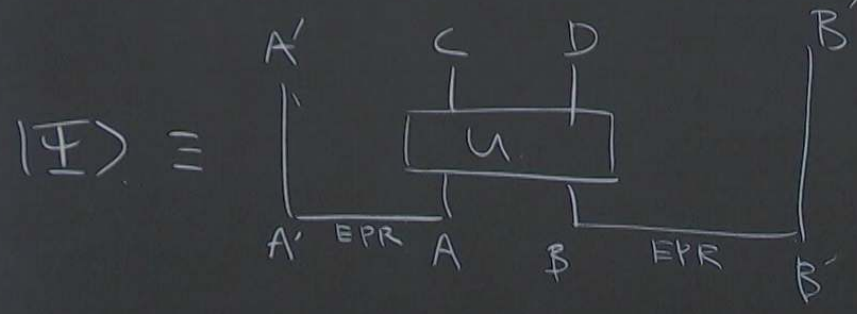
Abstract: Topics will include (but are not limited to):
- Quantum error correction in quantum gravity and condensed matter
- Quantum information scrambling and black hole information
- Physics of random tensor networks and random unitary circuits

Information recovery from scrambling



$|\Psi\rangle \equiv$

$I(A', B'D) \approx \max$

$\Updownarrow$

Recoverable

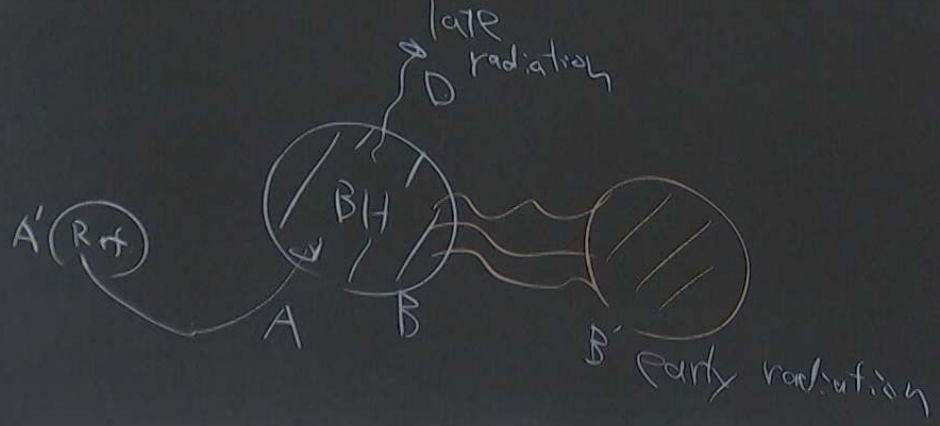
A : diary

A' : ref

B : BH

B' : early rad.

D : late rad



O_A, O_D : unitary on A, D

$\langle O_A(0)\, O_D(t)\, O_A^\dagger(0)\, O_D^\dagger(t) \rangle$ $\int dO_A$: average over all unitary O_A.

U : fixed

$\langle O(\alpha) \rangle_{ave}$

$\geq \min\left(\frac{1}{d_A^2}, \frac{1}{d_O^2} \right)$ (?)

$d_A = 2^{n_A}$

Averaged OTOCs

$\langle O_A(0) O_D(t) O_A^\dagger(0) O_D^\dagger(t) \rangle$?

O_A, O_D : unitary on A, D

$$\langle \mathrm{OTOC} \rangle_{\mathrm{ave}} \equiv \iint dO_A\, dO_D \, \langle O_A(0) O_D(t) O_A^\dagger(0) O_D^\dagger(t) \rangle$$

$\int dO_A$: ave

U : fixed

$\geq \min\left(\frac{1}{d_A^2}, \frac{1}{d_D^2} \right)$ (?)

$d_A = 2^n$

Scrambly $\Rightarrow$ small $\langle \mathrm{OTOC} \rangle_{\mathrm{ave}}$

$\mathrm{Tr}\left(\frac{1}{d} \sim \right)$

$O_D(t) = U O_D U^\dagger$

$(t)\rangle$ O_A, O_D: unitary on A, D

$\int dO_D \langle O_A(0)\, O_D(t)\, O_A^\dagger(0)\, O_D^\dagger(t)\rangle$ $\int dO_A$: average over all unitary O_A.

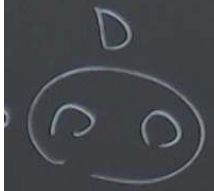
U : fixed

all $(OT(\alpha))_{ave}$

$\geq \min\left(\frac{1}{d_A^2}, \frac{1}{d_D^2}\right)$ (?)

$d_A = 2^{n_A}$

D



$Tr\left(\frac{1}{d} \sim\right)$

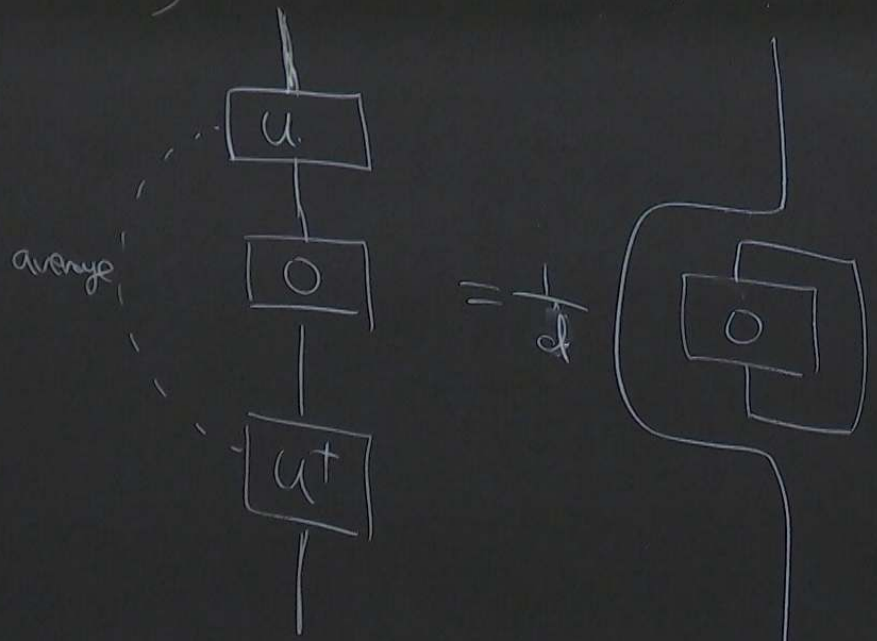
i) $\int_{\text{Haar}} dU_A \; U_A O U_A^\dagger = \int_{\text{Pauli}} dP_A \; P_A O P_A^\dagger$

$= \frac{1}{4^{n_A}} \sum_{P \neq \text{Pauli}}$

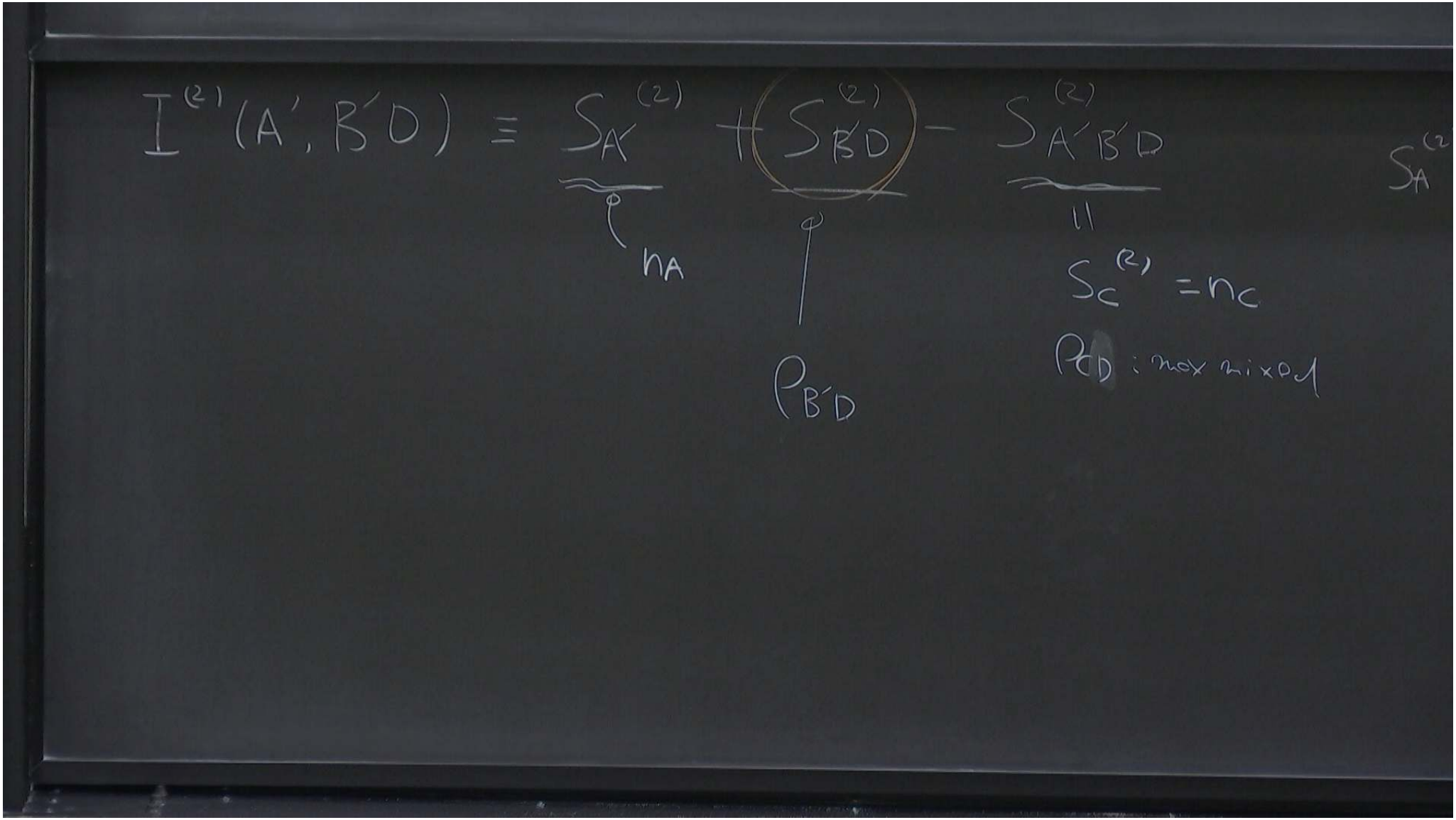
$\langle OTOC \rangle_{ave} = \frac{1}{(dA^2) d0^2} \sum_{P_A, P_0 \in \text{Pauli}} \langle P_A(0) P_0(t) P_A^\dagger(0) P_0^\dagger(t) \rangle$

4^{n_A} : # of Pauli on A

ii) $\int du\ UOU^\dagger = \frac{I}{d}\,\mathrm{Tr}(O)$ (depolarizing)



$= \frac{I}{d}$


$I^{(2)}(A', B'D) \equiv S_{A'}^{(2)} + S_{B'D}^{(2)} - S_{A'B'D}^{(2)}$
$S_{A'}^{(2)}$
n_A
$\rho_{B'D}$
$S_C^{(2)} = n_C$
ρ_{CD} : max mixed

$$S_A^{(2)} \equiv -\log\left[\mathrm{Tr}(\rho_A^2)\right]$$

large

$$\langle OTOC \rangle_{ave} = 2^{-I^{(2)}(A', B' D)}$$

small

scrambly $\Rightarrow$ recoverability.

$$I^{(2)}(A', B'D) \equiv S_{A'}^{(2)} + S_{B'D}^{(2)} - S_{A'B'D}^{(2)}$$

$S_{A'}^{(2)}$ ≡ −

$S_{A'} = n_A$

$D \ll C$

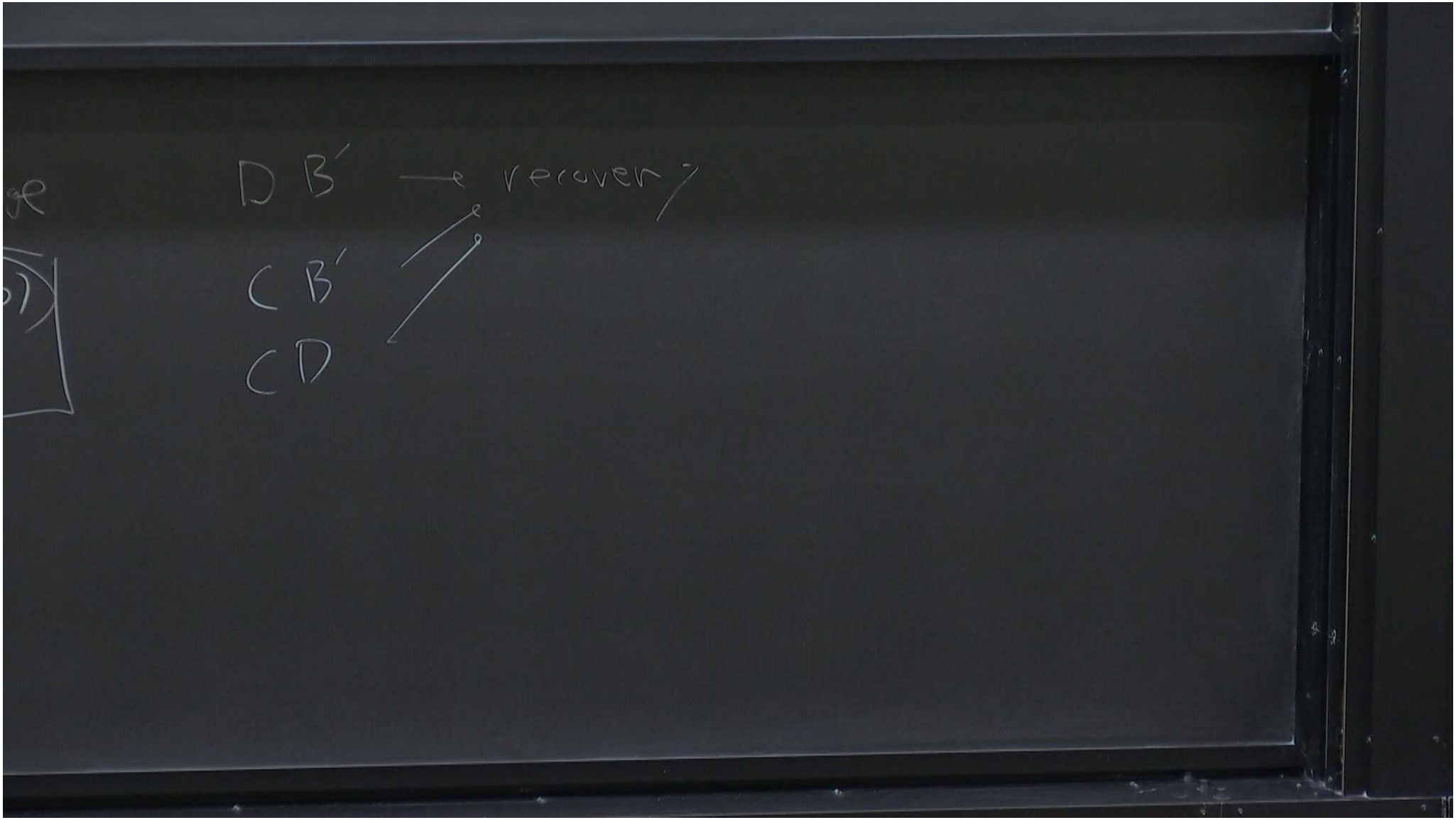
$S_{B'D}$

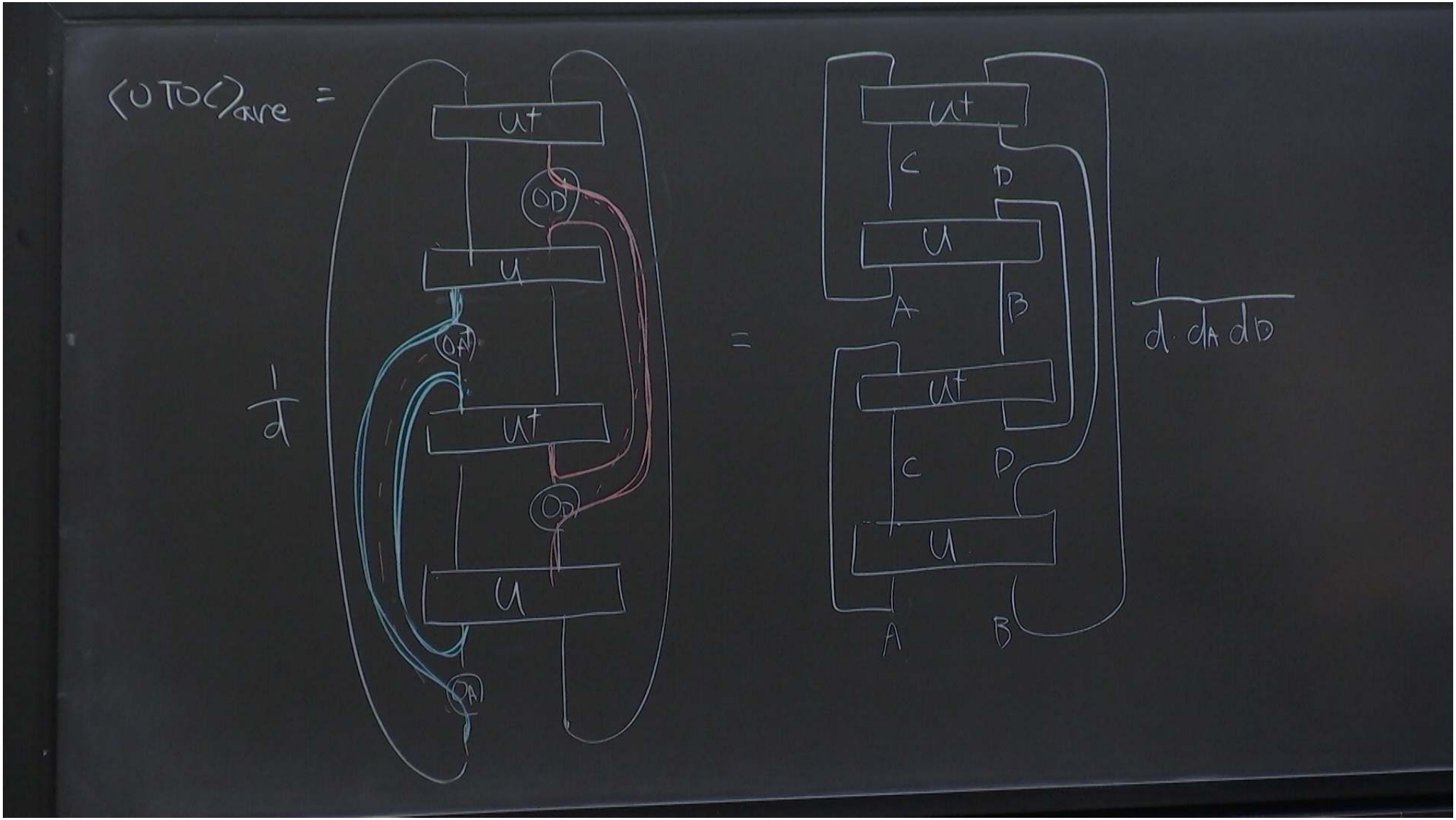
$S_{A'B'D} = S_C^{(2)} = n_C$

ρ_{CD} : max mixed

$$I^{(2)}(A', B'D) + I^{(2)}(A', C) = 2n_A$$

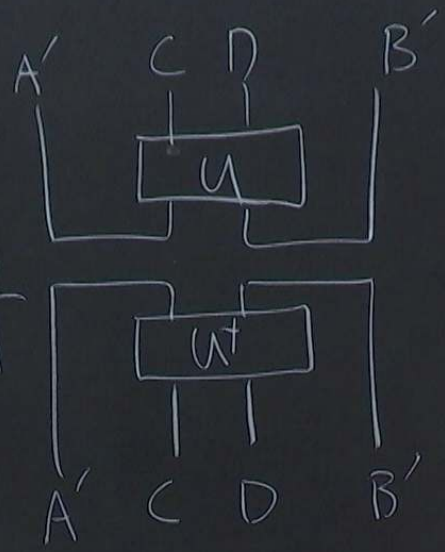
$I^{(2)}(A', B'C)$


DB'
recovery
CB'
CD

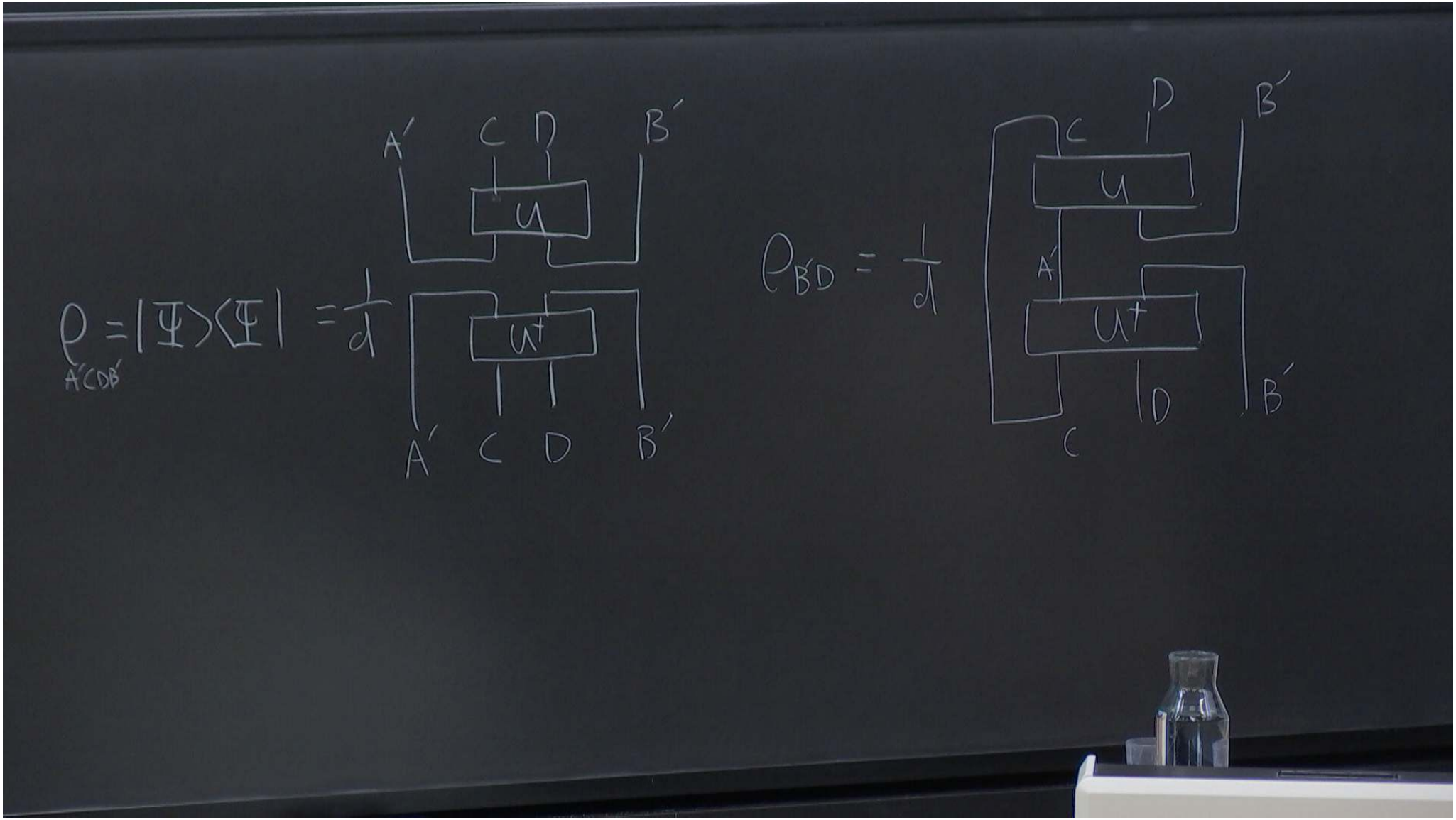

(OTOC)are =
1/d
Ut
OD
U
OA
Ut
OD
U
OA
=
Ut
C
D
U
A
B
Ut
C
D
U
A
B
1/d · dA dB

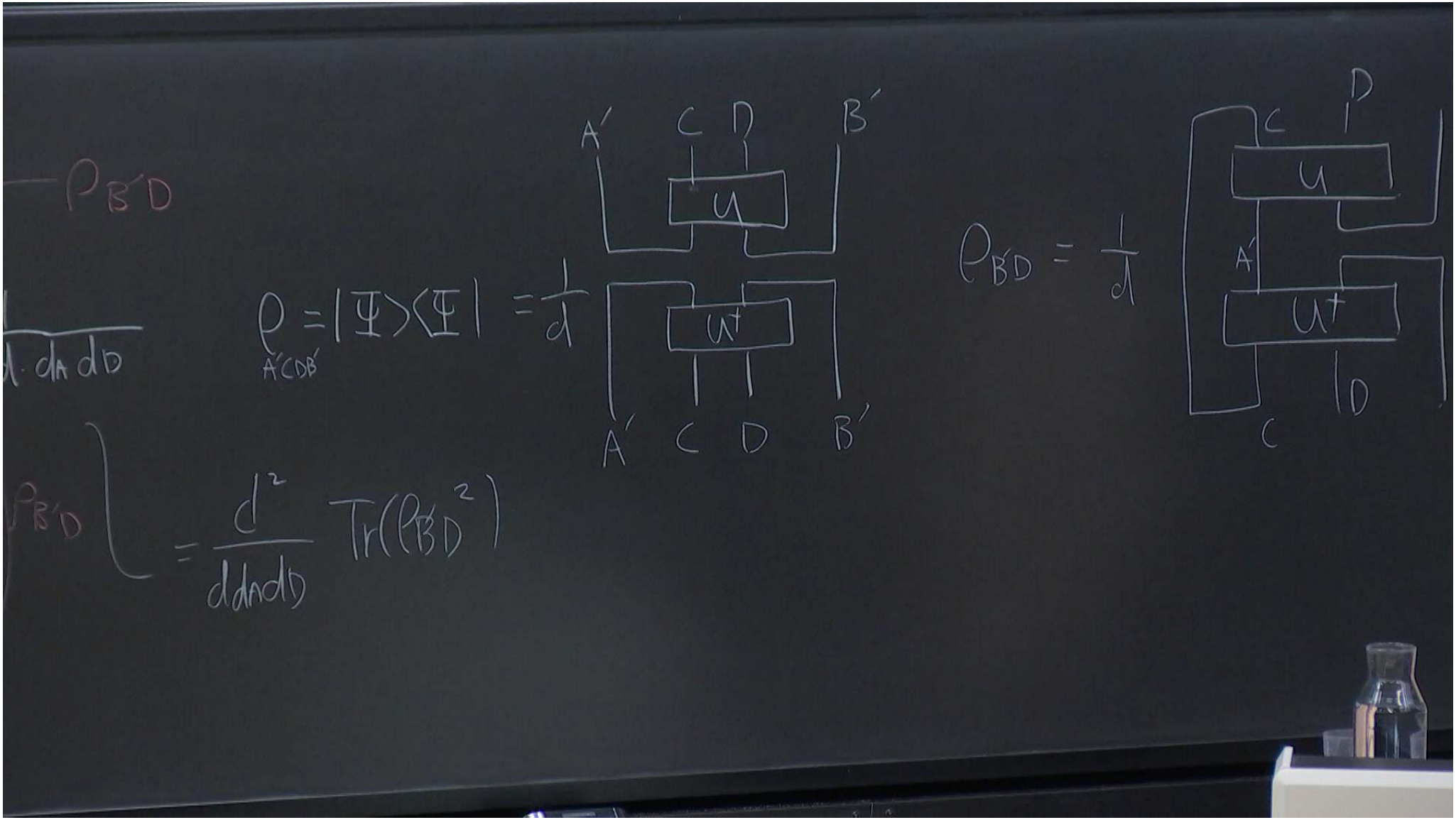
$$\frac{1}{d \cdot d_A d_D}$$

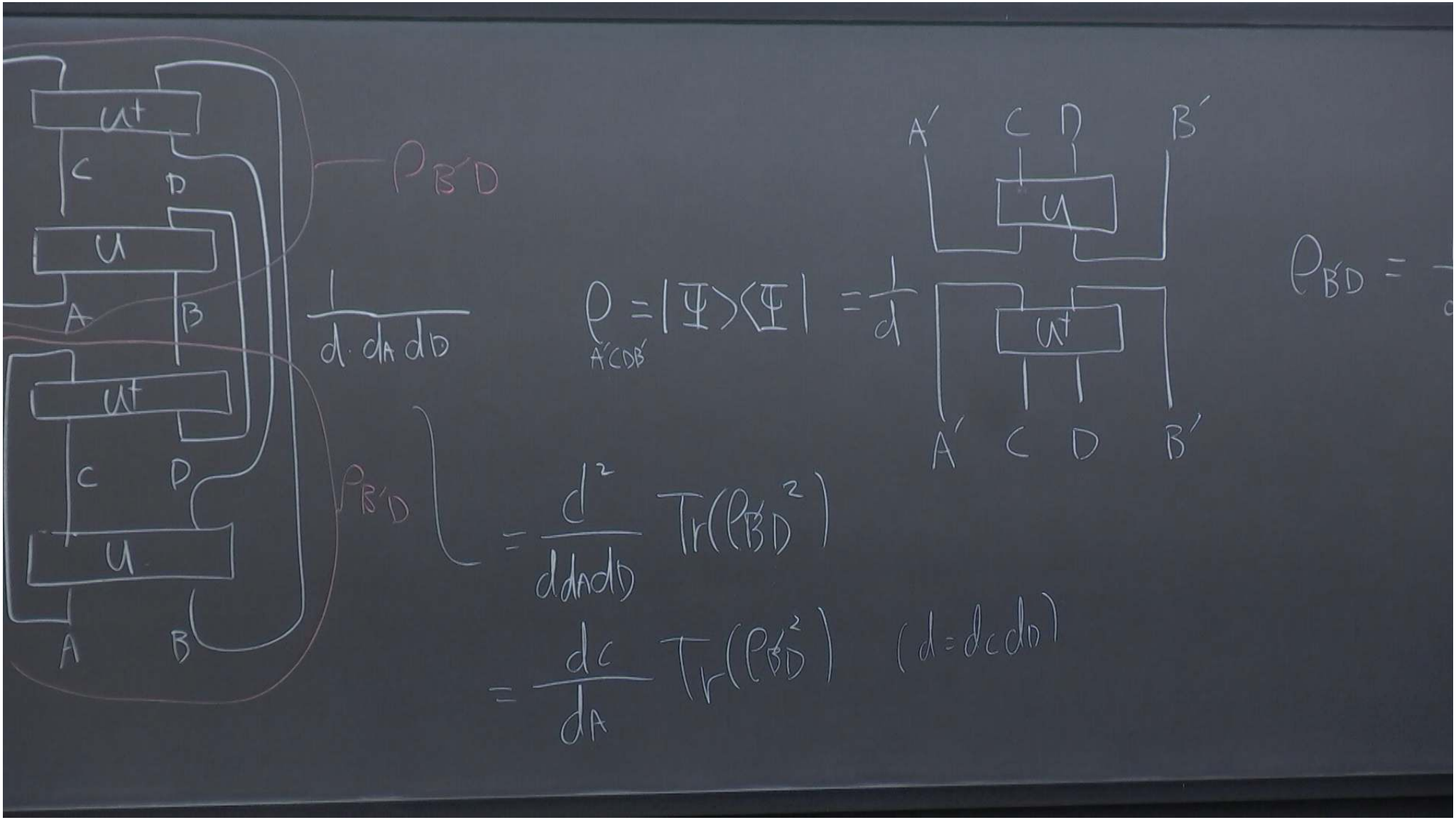
$$\rho_{A'CDB'} = |\Psi\rangle\langle\Psi| = \frac{1}{d}$$



$$\rho_{BD} =$$


$\rho_{A'CDB'} = |\Psi\rangle\langle\Psi| = \frac{1}{d}$
U
U^t
A' C D B'
$\rho_{BD} = \frac{1}{d}$
C D B'
A'
C D B'

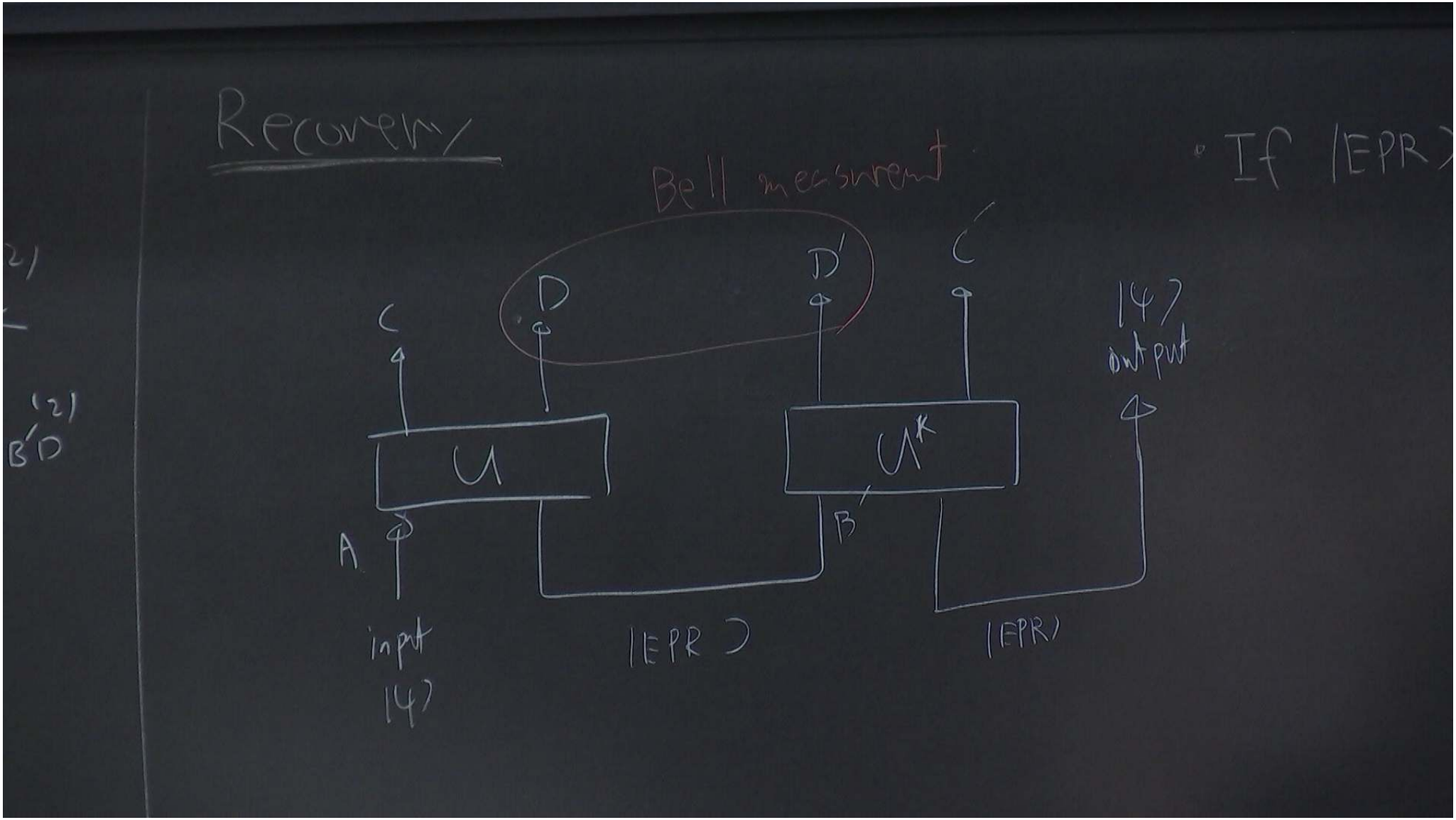

$\rho_{B'D}$
$\rho = |\Psi\rangle\langle\Psi| = \frac{1}{d}$
$\rho_{A'CDB'}$
A' C D B'
U
U^t
A' C D B'
$\rho_{B'D} = \frac{1}{d}$
C D
U
A'
U^t
C D
$\rho_{B'D}$ = $\frac{d^2}{d\,d_{A'}\,d_D}$ $\mathrm{Tr}(\rho_{B'D}^2)$

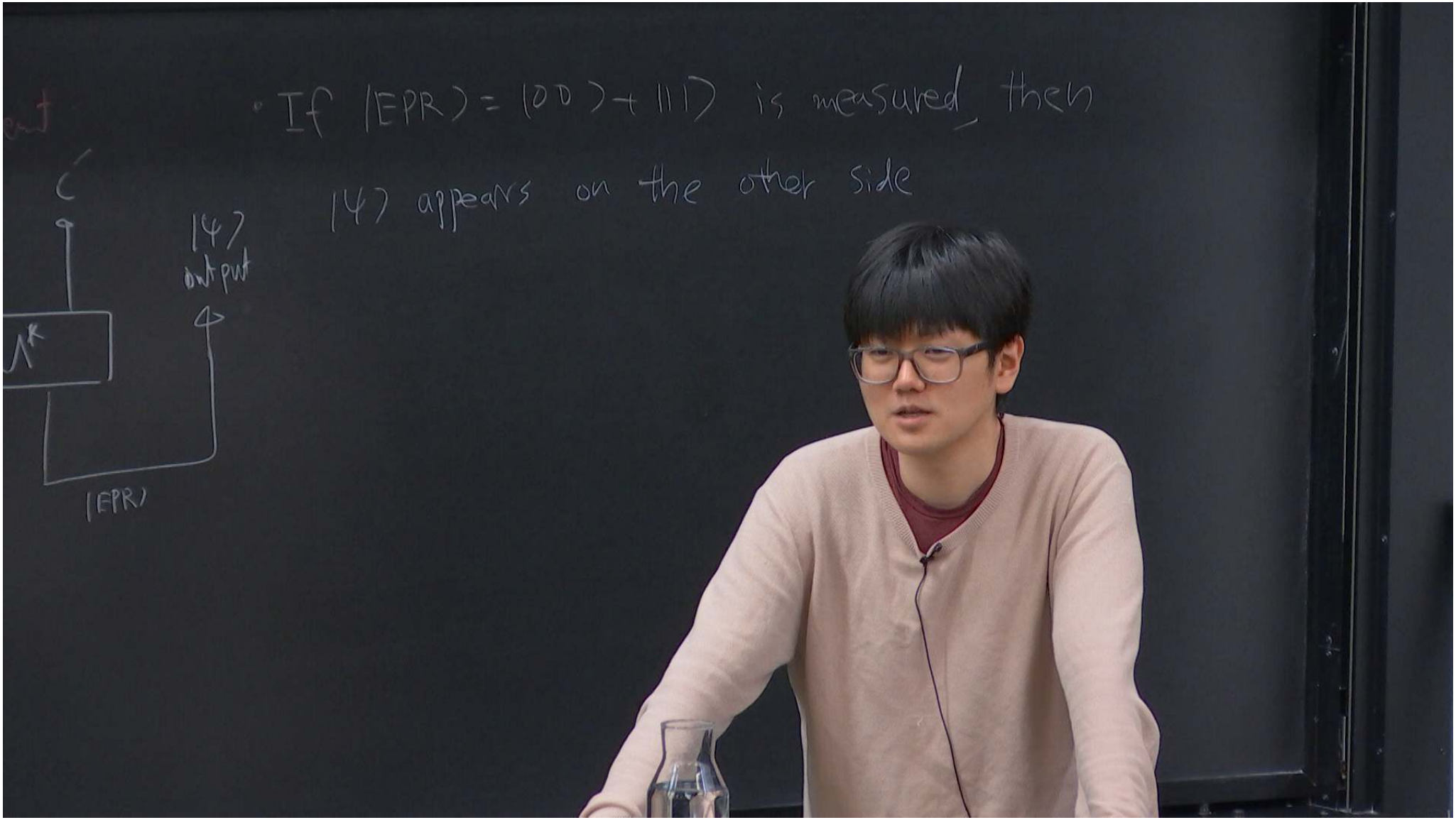

U†
C
D
U
A
B
U†
C
D
U
A
B
$\rho_{B'D}$
$\frac{1}{d \cdot d_A d_D}$
$\rho_{A'CDB'} = |\Psi\rangle\langle\Psi| = \frac{1}{d}$
A' C D B'
U
U†
A' C D B'
$\rho_{B'D}$
$= \frac{d^2}{d d_A d_D} Tr(\rho_{B'D}^2)$
$= \frac{d_C}{d_A} Tr(\rho_{B'D}^2)$ $(d = d_C d_D)$

$$(OTOC)_{ave} = \frac{dc}{dA} \mathrm{Tr}(P R' D^{\dagger})$$

$$= \frac{2^{S_C^{(2)}}}{2^{S_A^{(2)}}} 2^{-S_{BD}^{(2)}} = 2^{-S_A^{(2)} - S_{BD}^{(2)} + S_C^{(2)}}$$

$$= 2^{-I^{(2)}(A, BD)}$$


Recovery
Bell measurement
If |EPR>
C
D
D'
C'
U
U*
A
B'
input
|ψ>
|EPR>
|EPR>
|ψ>
output


If |EPR⟩ = |00⟩ + |11⟩ is measured then
|4⟩ appears on the other side
|4⟩
output
|EPR⟩