Title: Nonlinear Bosonization of Fermi Surfaces: The Method of Coadjoint Orbits

Speakers: Dam Thanh Son

Series: Quantum Matter

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Abstract: We develop a new method for bosonizing the Fermi surface. In this method, a system with a Fermi surface is described as a coadjoint orbit of the group of canonical transformations. The method naturally parametrizes the Fermi surface by a bosonic field that depends on the spacetime coordinates and the position on the Fermi surface. The Wess-Zumino-Witten term in the effective action, governing the adiabatic phase acquired when the Fermi surface changes its shape, is completely fixed by the Kirillov-Kostant-Souriau symplectic form on the coadjoint orbit. We show that the resulting local effective field theory captures both linear and nonlinear effects in Landau's Fermi liquid theory. Possible extensions of the theory are described. Reference: arXiv:2203.05004.

Zoom Link: https://pitp.zoom.us/j/95453440138?pwd=b0Fmbi9HTU9nYTA3Y2F6dWlha294UT09

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Nonlinear bosonization of Fermi liquids: the methods of *Coadjoint Orbits*

Dam Thanh Son (University of Chicago)
Perimeter Institute seminar
April 5, 2022

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Plan

- Landau's Fermi liquid theory
- LFLT as dynamics of shapes
- Coadjoint orbit
- Group of canonical transformations and its coadjoint orbits
- Reformulation of LFLT

Reference:
Luca Delacrétaz, Yi-Hsien Du, Umang Mehta, DTS
arXiv:2203.05004

SOVIET PHYSICS JETP VOLUME 3, NUMBER 6 JANUARY, 1957

The Theory of a Fermi Liquid

L. D. LANDAU
Institute for Physical Problems, Academy of Sciences, USSR

(Submitted to JETP editor March 7, 1956)
J. Exptl. Theoret. Phys. (U.S.S.R.) **30**, 1058-1064 (June, 1956)

A theory of the Fermi liquid is constructed, based on the representation of the perturbation theory as a functional of the distribution function. The effective mass of the excitation is found, along with the compressibility and the magnetic susceptibility of the Fermi liquid. Expressions are obtained for the momentum and energy flow.

If we consider a Fermi gas at temperatures which are low in comparison with the temperature of degeneration, and introduce some weak interaction between the atoms of this gas, then, as is known, the collision probability for a given atom, which is found in the diffuse Fermi zone, is proportional not only to the intensity of the interaction, but also to the square of the temperature. This shows that for a given intensity of interaction, the "indeterminacy of the momenta", associated with the finite path length, is also small for low temperatures, not only in comparison with the size of the momentum itself, but also in comparison with the width of the Fermi zone, proportional to the first power of the temperature.

- Sometimes it Fermi liquid is described as a "free Fermi gas" of Landau's quasiparticles
- This is not completely true: interaction between Landau's quasiparticles follows a specific pattern
- It is instructive to first describe the structure of Landau's fermi liquid theory as originally sketched

Free fermions: Liouville's equation

- Phase space distribution function $f(t, \mathbf{x}, \mathbf{p})$
- Liouville's equation

$$\frac{\partial f}{\partial t} + \mathbf{v_p} \cdot \frac{\partial f}{\partial \mathbf{x}} + (\mathbf{E} + \mathbf{v_p} \times \mathbf{B}) \cdot \frac{\partial f}{\partial \mathbf{p}} = 0$$

- Moreover, if the initial condition has a Fermi surface ($f = 1$ inside, $f = 0$ outside), then this is preserved under time evolution
- Dynamics of shape

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$\hbar = 1$

$\omega \ll \epsilon_F$

$q \ll k_F$

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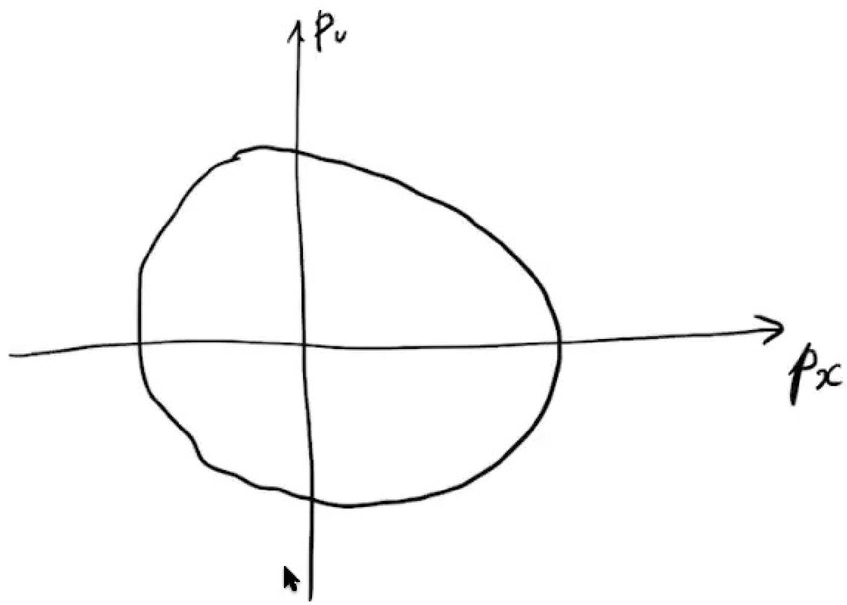
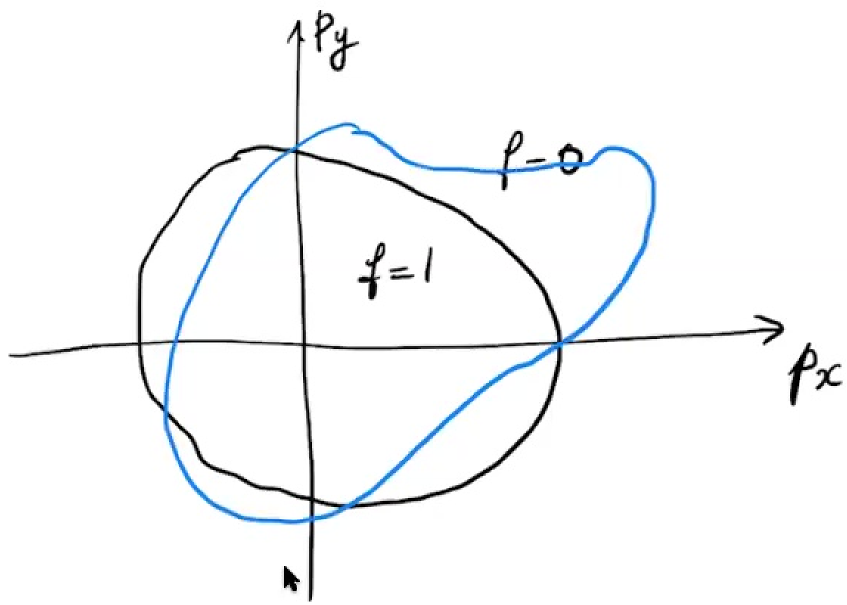
Dynamic of shapes

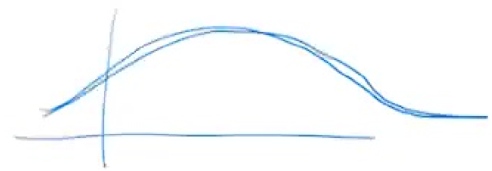


Dynamic of shapes



at given x

Free fermions: Liouville's equation



- Phase space distribution function $f(t, \mathbf{x}, \mathbf{p})$
- Liouville's equation

$\hbar = 1$

$$\frac{\partial f}{\partial t} + \mathbf{v}_{\mathbf{p}} \cdot \frac{\partial f}{\partial \mathbf{x}} + (\mathbf{E} + \mathbf{v}_{\mathbf{p}} \times \mathbf{B}) \cdot \frac{\partial f}{\partial \mathbf{p}} = 0$$

$\omega \ll \epsilon_F$

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- Moreover, if the initial condition has a Fermi surface ($f = 1$ inside, $f = 0$ outside), then this is preserved under time evolution
- Dynamics of shape

Correlation functions

- Linear response: $f = f_0 + \delta f$

$$\frac{\partial\, \delta f}{\partial t} + \mathbf{v}_{\mathbf{p}} \cdot \frac{\partial\, \delta f}{\partial \mathbf{x}} + \mathbf{E} \cdot \frac{\partial f_0}{\partial \mathbf{p}} = 0$$

from which one finds $\delta\rho(\omega, \mathbf{q}) = \Pi_{00}(\omega, \mathbf{q}) A_0(\omega, \mathbf{q})$

$$\Pi_{00}(\omega, \mathbf{q}) = -\int \frac{d\mathbf{p}}{(2\pi)^d} \frac{\mathbf{v}_{\mathbf{p}} \cdot \mathbf{q}}{\omega - \mathbf{v}_{\mathbf{p}} \cdot \mathbf{q} + i\varepsilon} \delta(\epsilon_{\mathbf{p}} - \mu)$$

- Nonlinear response can also be obtained

regime ov validity: $\omega, v_F q \ll \epsilon_F$

Landau's Fermi liquid theory

- Landau's Fermi liquid theory: modification of free fermions
- "Landau parameters" $F(\mathbf{p}, \mathbf{p}')$

$$\epsilon_{\mathbf{p}}(\mathbf{x}) = \epsilon^0_{\mathbf{p}} + \int_{\mathbf{p}'} F(\mathbf{p}, \mathbf{p}')\delta f(\mathbf{x}, \mathbf{p}')$$

- Kinetic equation

$$\frac{\partial f}{\partial t} + \mathbf{v}_{\mathbf{p}} \cdot \frac{\partial f}{\partial \mathbf{x}} + \left(\mathbf{E} + \mathbf{v}_{\mathbf{p}} \times \mathbf{B} - \frac{\partial \epsilon_{\mathbf{p}}(\mathbf{x})}{\partial \mathbf{x}} \right) \cdot \frac{\partial f}{\partial \mathbf{p}} = 0$$

LFLT as a field theory?

- LFLT is not a *field theory*
- This disallows a large toolbox of Effective Field Theory, as well as the insights from the Wilsonian viewpoint
- Related problem: Non-Fermi liquids
  example: (2+1)D fermions at finite density coupled to a gauge field

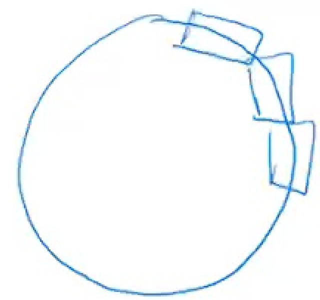
Previous work

- LFLT as an EFT of fermions: Benfatto-Gallavotti, Polchinski, Shankar (early 1990s)

$$\int dt\, d^3\mathbf{p} \left\{ i\psi^\dagger_\sigma(\mathbf{p})\partial_t\psi_\sigma(\mathbf{p}) - (\varepsilon(\mathbf{p}) - \varepsilon_F)\psi^\dagger_\sigma(\mathbf{p})\psi_\sigma(\mathbf{p}) \right\}$$

$$+ \int dt\, d^2\mathbf{k}_1\, d\mathbf{l}_1\, d^2\mathbf{k}_2\, d\mathbf{l}_2\, d^2\mathbf{k}_3\, d\mathbf{l}_3\, d^2\mathbf{k}_4\, d\mathbf{l}_4\, V(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4)$$

$$\psi^\dagger_\sigma(\mathbf{p}_1)\psi_\sigma(\mathbf{p}_3)\psi^\dagger_{\sigma'}(\mathbf{p}_2)\psi_{\sigma'}(\mathbf{p}_4)\delta^3(\mathbf{p}_1 + \mathbf{p}_2 - \mathbf{p}_3 - \mathbf{p}_4).$$



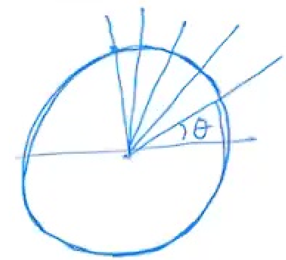
- sometimes Fermi surface is divided into patches
- scaling properties of Landau and BCS couplings

Bosonization

- In 1+1 dimensions, fermion is dual to boson

$$\rho_L = \frac{1}{2\pi}\partial_x\varphi, \quad j_L = -\frac{1}{2\pi}\partial_t\varphi$$

- Higher-dimensional bosonization: 1 chiral boson at each point on FS $\quad \varphi = \varphi(x,\theta)$



$$S = \int d\theta \left(\dot{\phi}\,\vec{n}_\theta \cdot \vec{\nabla}\varphi - (\vec{n}_\theta \cdot \vec{\nabla}\varphi)^2 \right)$$

- Not clear how to write down interaction terms
- External field: mixing of modes with different θ

Haldane; Castro Neto, Fradkin; Houghton, Kwon, Marston…

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- We want to formulate LFLT as a field theory with an action $S[\phi]$

so that $\frac{\delta S}{\delta \phi} = 0$ gives Landau's kinetic equation

- Method of coadjoint orbits

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Adjoint representation

- Consider Lie group $\mathscr{G}$, Lie algebra $\mathfrak{g}$
  - $F \in \mathfrak{g},\ e^F \in \mathscr{G}$
- Adjoint representation of $\mathfrak{g}$ and $\mathscr{G}$: realized on $\mathfrak{g}$

$$G \in \mathfrak{g} : F \to \mathrm{ad}_G F = \{G, F\}$$

$$U \in \mathscr{G} : F \to \mathrm{Ad}_U F = UFU^{-1}$$

Coadjoint representation

- Dual space $\mathfrak{g}^*$
  - $f \in \mathfrak{g}^*, F \in \mathfrak{g}$ then scalar product $\langle f, F \rangle \in \mathbb{R}$
- Coadjoint representation:

$$\langle \mathrm{Ad}_U^* f, \mathrm{Ad}_U F \rangle = \langle f, F \rangle$$

Coadjoint orbits

- Take a $f_0 \in \mathfrak{g}^*$
- *Coadjoint orbit* $\mathcal{O}_{f_0}$ is the set of all points in $\mathfrak{g}^*$ obtained from f_0 by coadjoint group action

$$\mathcal{O}_{f_0} = \{f \,|\, \exists U \in \mathcal{G} : f = \mathrm{Ad}^*_U f_0\}$$

- Coadjoint orbit is left coset $\mathcal{G}/\mathcal{H}$:
  $UV \sim U, \;\; f_0 = \mathrm{Ad}^*_V f_0$

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$\mathscr{H} f_0 = f_0$

Example: SU(2) group

- su(2) algebra: $F = \vec{F} \cdot \vec{\tau} = F_1 \tau^1 + F_2 \tau^2 + F_3 \tau^3$
- dual space: $f = (f_1, f_2, f_3)$
- scalar product: $\langle f, F \rangle = \vec{f} \cdot \vec{F}$
- Coadjoint action: SO(3) rotation of $\vec{f}$
- Coadjoint orbit: sphere, $|\vec{f}|^2 = |\vec{f}_0|^2$

Kirillov-Kostant-Souriau symplectic form

- An important theorem in the theory of coadjoint orbit: existence of a symplectic form (Kirillov-Kostant-Souriau)
- defined this way:

$$\omega_f(\delta f_1, \delta f_2) = \langle f, [F_1, F_2] \rangle \qquad \begin{aligned} \delta f_1 &= \mathrm{ad}^*_{F_1} f \\ \delta f_2 &= \mathrm{ad}^*_{F_2} f \end{aligned}$$

- ω does not depends on the choice of F_1 and F_2 and is closed

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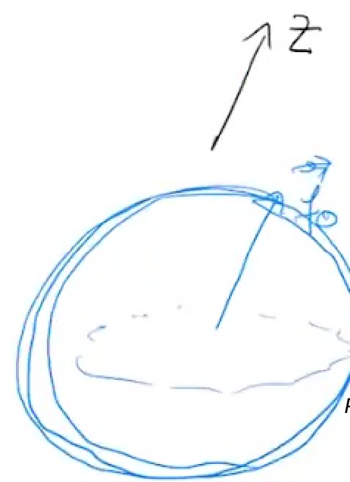

Canonical transformations

$\mathcal{G}$

- Fermi liquids will be identified with coadjoint orbits of the group of *canonical transformations*

- Infinitesimal canonical transformations: $F(\mathbf{x}, \mathbf{p})$

$$\mathbf{x} \to \mathbf{x}' = \mathbf{x} - \epsilon \nabla_{\mathbf{p}} F$$
$$\mathbf{p} \to \mathbf{p}' = \mathbf{p} + \epsilon \nabla_{\mathbf{x}} F$$

preserves Poisson brackets

- Lie algebra with commutator

$$\{F, G\} = \nabla_{\mathbf{x}} F \cdot \nabla_{\mathbf{p}} F - \nabla_{\mathbf{p}} F \cdot \nabla_{\mathbf{x}} F$$

- Lie groups: finite canonical transformations e^F

Dual space

- Dual space: also space of all functions $f(\mathbf{x}, \mathbf{p})$
- Scalar product: $\langle f, F \rangle = \int \frac{d\mathbf{x}\, d\mathbf{p}}{(2\pi)^d} f(\mathbf{x}, \mathbf{p}) F(\mathbf{x}, \mathbf{p})$
- Physical interpretation:
  - F is an observable $F = \frac{p^2}{2m}$
  - f is a state (distribution function)
  - $\langle f, F \rangle$ is the average of observable F in state f
    - quantum mechanical equivalent: $\mathrm{Tr}\,(\hat{f}\hat{F})$

Coadjoint orbit

- As the reference state f_0 : ground state
  $$f_0(\mathbf{p}) = \Theta(p_F - |\,\mathbf{p}\,|)$$
- Coadjoint orbits: states with sharp Fermi surface, with the same total number of particles
- Evolution of the system occurs on the coadjoint orbit
- Our task is to parameterize the coadjoint orbit and write an action

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Parametrizing the coadjoint orbit

- We work in perturbation theory: $f = f_0 + \delta f$
- Parametrize group element $U = \exp(-\phi)$ $\phi = \phi(x, |$

$$f = \mathrm{Ad}^*_{\exp(-\phi)} f_0 \approx f_0 - \{\phi, f_0\}$$
$$= \Theta(p_F - |\mathbf{p}|) + \delta(|\mathbf{p}| - p_F)\mathbf{n}_\theta \cdot \nabla_{\mathbf{x}} \phi$$

- Stabilizer subgroup of f_0: $V = \exp \alpha, \quad \mathbf{n}_\theta \cdot \nabla_{\mathbf{x}} \alpha \,|_{|\mathbf{p}|=p_F} = 0$
- $\phi \sim \phi - \alpha + \frac{1}{2}\{\phi, \alpha\} + \cdots$
  can be used to fix $\phi = \phi(\mathbf{x}, \theta)$

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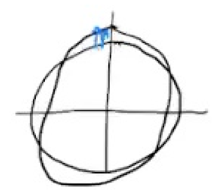

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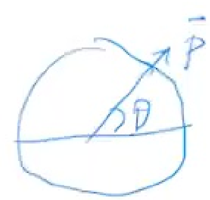
$\mathcal{G}/\mathcal{H}$

$\mathcal{H} f_0 = f_0$

$\phi = \phi(x, \hat{p})$



The action

- $S = \int dt \, \langle f_0, U^{-1} \partial_t U \rangle - \int dt \, \mathscr{H}[f]$

- The first (WZW) term can be written as

$$\int dt \int_0^1 ds \; \omega(\partial_t f, \partial_s f)$$

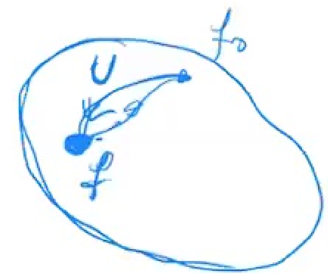
where ω is the Kirillov-Kostant-Souriau two-form $d\omega = 0$

- The Hamiltonian for free fermions is

$$\langle f, \epsilon(\mathbf{p}) \rangle = \langle f_0, U^{-1} \epsilon(\mathbf{p}) U \rangle$$

- Can be fully gauged

The action



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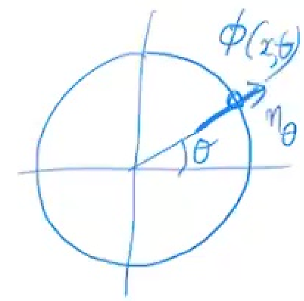
- The Hamiltonian for free fermions is

$$\langle f, \epsilon(\mathbf{p}) \rangle = \langle f_0, U^{-1} \epsilon(\mathbf{p}) U \rangle \quad = \int dx \, dp \; \varepsilon(p) f(p, x)$$

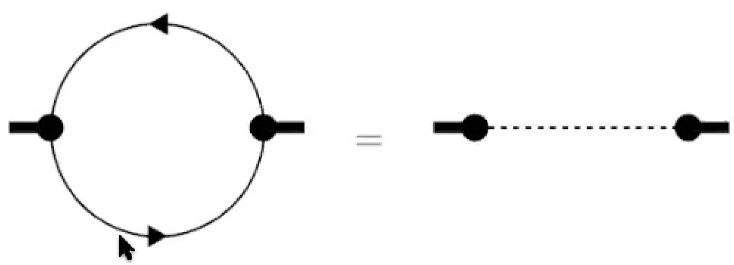
- Can be fully gauged

Quadratic action

- $$S = -\frac{p_F}{8\pi^2}\int dt\, d\mathbf{x}\, d\theta\, \mathbf{n}_\theta \cdot \nabla\phi(\dot{\phi} + \mathbf{n}_\theta \cdot \nabla\phi)$$



- density operator $\rho = -\frac{p_F}{2\pi}\int d\theta\, \mathbf{n}_\theta \cdot \nabla\phi$



$$\langle\phi\phi'\rangle(\omega, \mathbf{q}) = i\frac{(2\pi)^d}{p_F^{d-1}}\frac{\delta^{d-1}(\theta-\theta')}{\mathbf{n}_\theta\cdot\mathbf{q}(\omega - v_F\mathbf{n}_\theta\cdot\mathbf{q})}$$

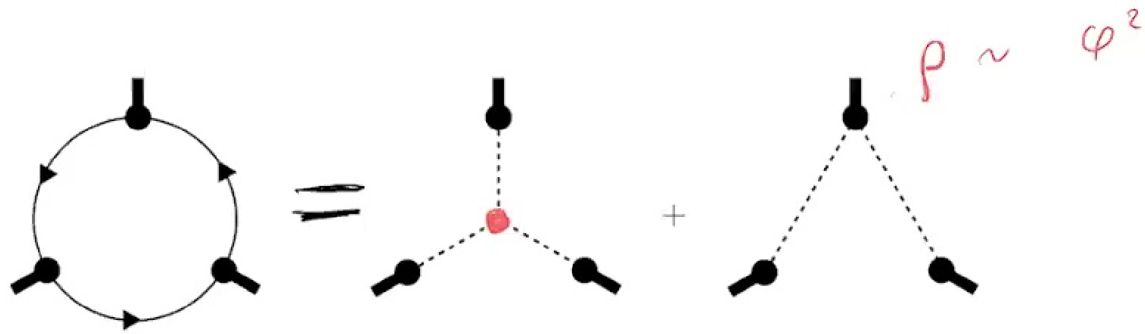
$$\langle\rho\rho\rangle(\omega, q) = i\frac{p_F}{2\pi}\frac{1}{v_F}\left(1 - \frac{|\omega|}{\sqrt{\omega^2 - v_F^2 q^2}}\right)$$

Nonlinear response

$$S_{\mathrm{WZW}} = \int dt\, \langle f_0, U^{-1}\partial_t U\rangle = \int dt\, \langle f_0, \frac{1}{2}\{\dot\phi, \phi\} - \frac{1}{3!}\{\{\dot\phi, \phi\}, \phi\} + \cdots \rangle$$

$$S_H = -\int dt\, \langle f_0, U^{-1}\epsilon U\rangle = -\int dt\, \langle f_0, \frac{1}{2}\{\phi, \{\phi, \epsilon\}\} + \frac{1}{3!}\{\phi, \{\phi, \{\phi, \epsilon\}\}\} + \cdots \rangle ,$$

$$\begin{aligned} S &= S_{\mathrm{WZW}} + S_H , \\ S_{\mathrm{WZW}} &= -p_F^{d-1}\int_{t,\mathbf{x},\theta} \frac{1}{2}\dot\phi(\mathbf{n}_\theta \cdot \nabla\phi) + \frac{1}{3!}\frac{1}{p_F}(\mathbf{n}_\theta\cdot\nabla\phi)\left(\mathbf{s}^i_\theta\cdot\nabla\phi\,\partial_{\theta^i}\dot\phi - \mathbf{s}^i_\theta\cdot\nabla\dot\phi\,\partial_{\theta^i}\phi\right) + \cdots , \\ S_H &= -p_F^{d-1}\int_{t,\mathbf{x},\theta} \frac{1}{2}\epsilon'(\mathbf{n}_\theta\cdot\nabla\phi)^2 + \frac{1}{3!}\frac{1}{p_F}\left(\frac{d-1}{2}\epsilon' + \epsilon'' p_F\right)(\mathbf{n}_\theta\cdot\nabla\phi)^3 + \cdots . \end{aligned} \tag{107}$$



Cancellation between 2 fermion-loop diagrams
when $\omega \sim q$, individual diagrams $\sim 1/q$, sum $\sim q^0$

In boson language the q^0 behavior is automatic

Exact matching: highly nontrivial to check

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Nonlinear response

$$S_{\text{WZW}} = \int dt\, \langle f_0, U^{-1}\partial_t U\rangle = \int dt\, \langle f_0, \frac{1}{2}\{\dot{\phi}, \phi\} - \frac{1}{3!}\{\{\dot{\phi}, \phi\}, \phi\} + \cdots \rangle$$

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$$S = S_{\text{WZW}} + S_H\,, \tag{107}$$

$$S_{\text{WZW}} = -p_F^{d-1}\int_{t,\mathbf{x},\theta} \frac{1}{2}\dot{\phi}(\mathbf{n}_\theta \cdot \nabla\phi) + \frac{1}{3!}\frac{1}{p_F}(\mathbf{n}_\theta \cdot \nabla\phi)\left(\mathbf{s}^i_\theta \cdot \nabla\phi\partial_{\theta^i}\dot{\phi} - \mathbf{s}^i_\theta \cdot \nabla\dot{\phi}\partial_{\theta^i}\phi\right) + \cdots\,,$$

$$S_H = -p_F^{d-1}\int_{t,\mathbf{x},\theta} \frac{1}{2}\epsilon'(\mathbf{n}_\theta \cdot \nabla\phi)^2 + \frac{1}{3!}\frac{1}{p_F}\left(\frac{d-1}{2}\epsilon' + \epsilon'' p_F\right)(\mathbf{n}_\theta \cdot \nabla\phi)^3 + \cdots\,.$$

Nonlinear response

rigid

$$S_{\mathrm{WZW}} = \int dt\, \langle f_0, U^{-1}\partial_t U\rangle = \int dt\, \langle f_0, \frac{1}{2}\{\dot\phi, \phi\} - \frac{1}{3!}\{\{\dot\phi,\phi\},\phi\} + \cdots \rangle$$

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$S[f]$ $\quad e^{\phi} \to e^{\phi} e^{\alpha}$ $\quad e^{\alpha} f_0$

$\mathcal{H}$

$$S = S_{\mathrm{WZW}} + S_H\,, \tag{107}$$

$$S_{\mathrm{WZW}} = -p_F^{d-1}\int_{t,\mathbf{x},\theta} \frac{1}{2}\dot\phi(\mathbf{n}_\theta\cdot\nabla\phi) + \frac{1}{3!}\frac{1}{p_F}(\mathbf{n}_\theta\cdot\nabla\phi)\left(\mathbf{s}^i_\theta\cdot\nabla\phi\partial_{\theta^i}\dot\phi - \mathbf{s}^i_\theta\cdot\nabla\dot\phi\partial_{\theta^i}\phi\right) + \cdots\,,$$

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Nonlinear response

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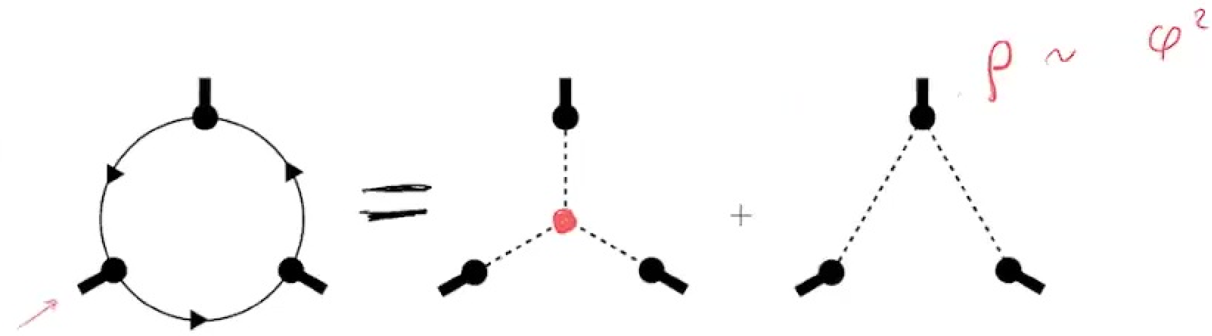
$$S = S_{\rm WZW} + S_H\,, \tag{107}$$

$$S_{\rm WZW} = -p_F^{d-1}\int_{t,\mathbf{x},\theta} \frac{1}{2}\dot\phi(\mathbf{n}_\theta\cdot\nabla\phi) + \frac{1}{3!}\frac{1}{p_F}(\mathbf{n}_\theta\cdot\nabla\phi)\left(\mathbf{s}^i_\theta\cdot\nabla\phi\,\partial_{\theta^i}\dot\phi - \mathbf{s}^i_\theta\cdot\nabla\dot\phi\,\partial_{\theta^i}\phi\right) + \cdots\,,$$

$$S_H = -p_F^{d-1}\int_{t,\mathbf{x},\theta} \frac{1}{2}\epsilon'(\mathbf{n}_\theta\cdot\nabla\phi)^2 + \frac{1}{3!}\frac{1}{p_F}\left(\frac{d-1}{2}\epsilon' + \epsilon'' p_F\right)(\mathbf{n}_\theta\cdot\nabla\phi)^3 + \cdots\,.$$



Cancellation between 2 fermion-loop diagrams
when $\omega \sim q$, individual diagrams $\sim 1/q$, sum $\sim q^0$

In boson language the q^0 behavior is automatic

Exact matching: highly nontrivial to check

9:41 AM Tue Jan 9

PI2022

Back to canonical transformations

- Fermion bilinears

$$O(x,y) = \frac{1}{2}\left(\psi(x)\psi^\dagger(y) - \psi^\dagger(y)\psi(x)\right)$$

form a close algebra

- In the long-wavelength limit: algebra of canonical transformation

Extensions

- Spinful Fermi surface: extend the algebra to include

$$\Rightarrow O^a(x,y) = \psi^\dagger(x)\frac{1}{2}\sigma^a\psi(y),\ a = 0,1,2,3$$

- BCS theory: extend the algebra to include

$$O_2(x,y) = \psi^\dagger(x)\psi^\dagger(y),\ O_{-2}(x,y) = \psi(x)\psi(y)$$

Conclusion

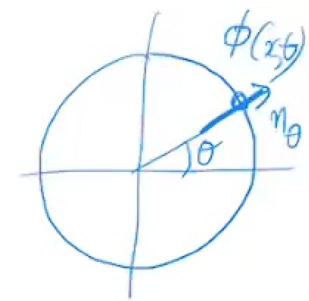
- The method of coadjoint orbits provides a natural way to write down a bosonic effective field theory of a Fermi liquid
- Reproduces linear and non-linear Fermi liquid response
- A good approach to study non-Fermi liquids?

$$f_0 = |\psi_0\rangle$$

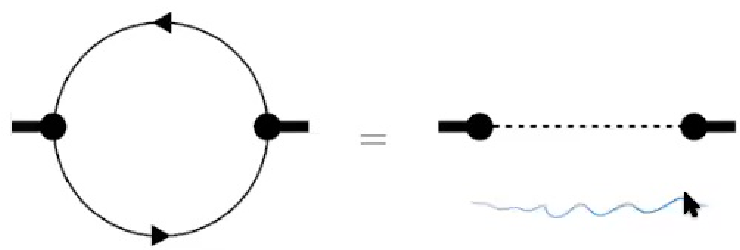
$$U \in \mathcal{G} \qquad U \sim \exp\left[i \int F(x,y)\, \psi(x)\, \psi^\dagger(y) \right]$$

Quadratic action

- $S = -\frac{p_F}{8\pi^2}\int dt\, d\mathbf{x}\, d\theta\, \mathbf{n}_\theta \cdot \nabla\phi(\dot{\phi} + \mathbf{n}_\theta \cdot \nabla\phi)$



- density operator $\rho = -\frac{p_F}{2\pi}\int d\theta\, \mathbf{n}_\theta \cdot \nabla\phi$



$$\langle \phi\phi' \rangle(\omega, \mathbf{q}) = i\frac{(2\pi)^d}{p_F^{d-1}} \frac{\delta^{d-1}(\theta - \theta')}{\mathbf{n}_\theta \cdot \mathbf{q}(\omega - v_F \mathbf{n}_\theta \cdot \mathbf{q})}$$

$$\langle \rho\rho \rangle(\omega, q) = i\frac{p_F}{2\pi}\frac{1}{v_F}\left(1 - \frac{|\omega|}{\sqrt{\omega^2 - v_F^2 q^2}}\right)$$

$f_0 = |\psi_0\rangle$

$U \in \mathcal{G} \qquad U \sim \exp\left[i \int F(x,y)\, \psi(x)\, \psi^{+}(y)\right]$

Berry curvature: $\{x, p\} = 1$

$\{x_i, x_j\} = \Omega_{ij}$

$\{x_i, p_j\} = \delta_{ij} + \ldots$

Couple theory to gauge field. $A_x(\vec{x}, \vec{p})$

$A_p(\vec{x}, \vec{p})$

No anomaly!

$$f_0 = |\psi_0\rangle$$

$$U \in \mathcal{G} \qquad U \sim \exp\left[i \int F(x,y)\,\psi(x)\,\psi^\dagger(y)\right]$$

Berry curvature: $\{x, p\} = 1$

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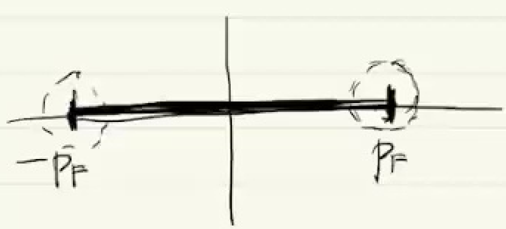

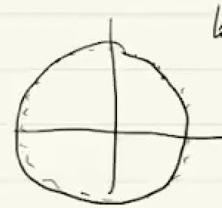

Back to canonical transformations

- Fermion bilinears

$$\underline{O(x,y)} = \frac{1}{2}\left(\psi(x)\psi^\dagger(y) - \psi^\dagger(y)\psi(x)\right)$$

$\{\psi\ \psi^\dagger\} = \delta$

form a close algebra

- In the long-wavelength limit: algebra of canonical transformations