Title: Matter Effects in Waveform Models

Speakers: Geraint Pratten

Collection: Gravitational Waves Beyond the Boxes II

Date: April 05, 2022 - 9:00 AM

URL: https://pirsa.org/22040029

UNIVERSITY OF BIRMINGHAM

LIGO VIRGO

# Matter Effects in Waveform Models

Geraint Pratten

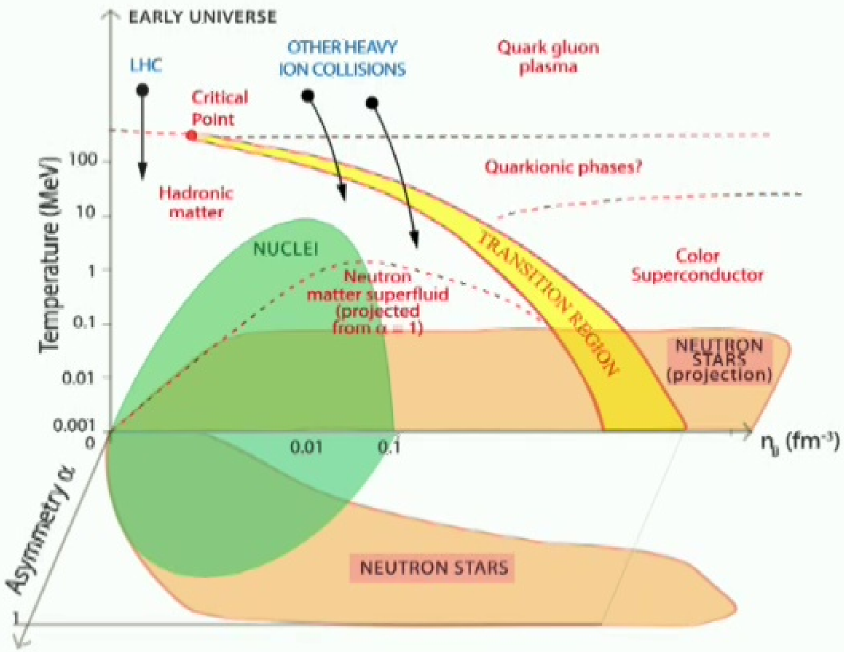
Gravitational Waves Beyond the Boxes II

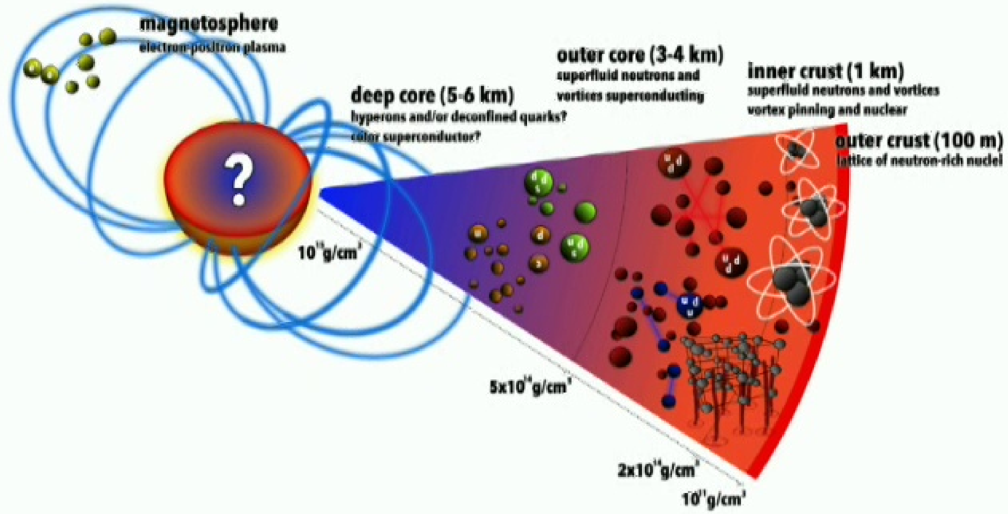
Perimeter, Canada, 05th April 2022

# Introduction (a 9am refresher)

UNIVERSITY OF BIRMINGHAM

- Gravitational-wave observations distinctive avenue for probing behaviour of ultra-dense matter in extreme environments
- Goal: study behaviour of nuclear matter across **wide range** of **temperatures**, **densities**, and **asymmetries**
- Probe QCD inaccessible to collider experiments!




Watts+14

2

Watts+14

## Introduction (a 9am refresher)

UNIVERSITY OF BIRMINGHAM

- Strong synergy with many avenues of exploration including terrestrial, astrophysical, and theoretical
  - PREx-II: measurement of neutron skin thickness
  - Heavy Ion Collisions
  - Pulsar timing (e.g. NANOGrav): mass of millisecond pulsars → important information on upper mass limit
  - Gravitational-wave observations: masses, spins, tidal deformability, post-merger, …
  - X-ray observations (NICER, XMM-Newton): measurement of mass and radius of massive pulsars
  - **QCD** d.o.f? **Phase transitions**? Chiral EFT? Quark deconfinement? **Strange** behaviour in NS core?

    (e.g. hyperons: 1+ s and u/d)

- Focus on **gravitational-waves** but the **combined** insight from all the above will be key!

## Introduction (a 9am refresher)

UNIVERSITY OF BIRMINGHAM

- Tidal signatures in GW signal arise from response of matter to spacetime curvature sourced by companion

- Static, spherically symmetric NS in static external tidal field [e.g. Thorne 98 or Hinderer 07]

tidally induced quadrupole: $Q_{ij} = -\lambda \mathcal{E}_{ij}$ ~ definition of tidal deformability

$$\frac{1-g_{tt}}{2} = -\frac{m}{r} - \frac{3}{2}\frac{Q_{ij}}{r^3}\left(n^i n^j - \frac{1}{3}\delta^{ij}\right) + \mathcal{O}(r^{-4})$$ ~ potential higher order multipole moments

~ perturbing tidal field $$+\frac{1}{2}\frac{\mathcal{E}_{ij}}{r^2}n^i n^j + \mathcal{O}(r^3)$$ ~ potential external higher order tidal fields

- **Change** in **multipole structure** of NS's exterior spacetime due to **external tidal field** imprinted in GWs

- Calculation of tidal response needs metric in asymptotic regime + equation of state

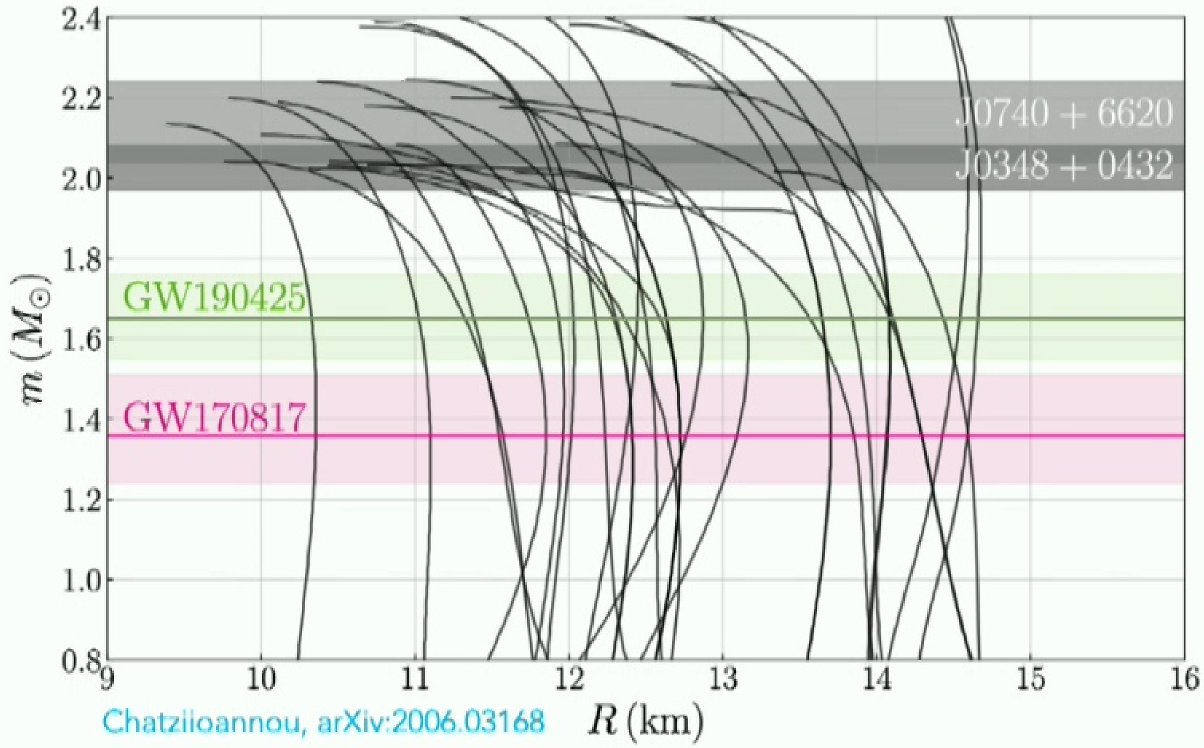
$$(P, \rho) \mapsto (m, r)$$

4

## Introduction (a 9am refresher)

UNIVERSITY OF BIRMINGHAM

- Equation of state of ~ pure neutron matter highly uncertain at densities, pressures and asymmetries in NS's
- New goal: forensically reconstruct EoS from observations + theory

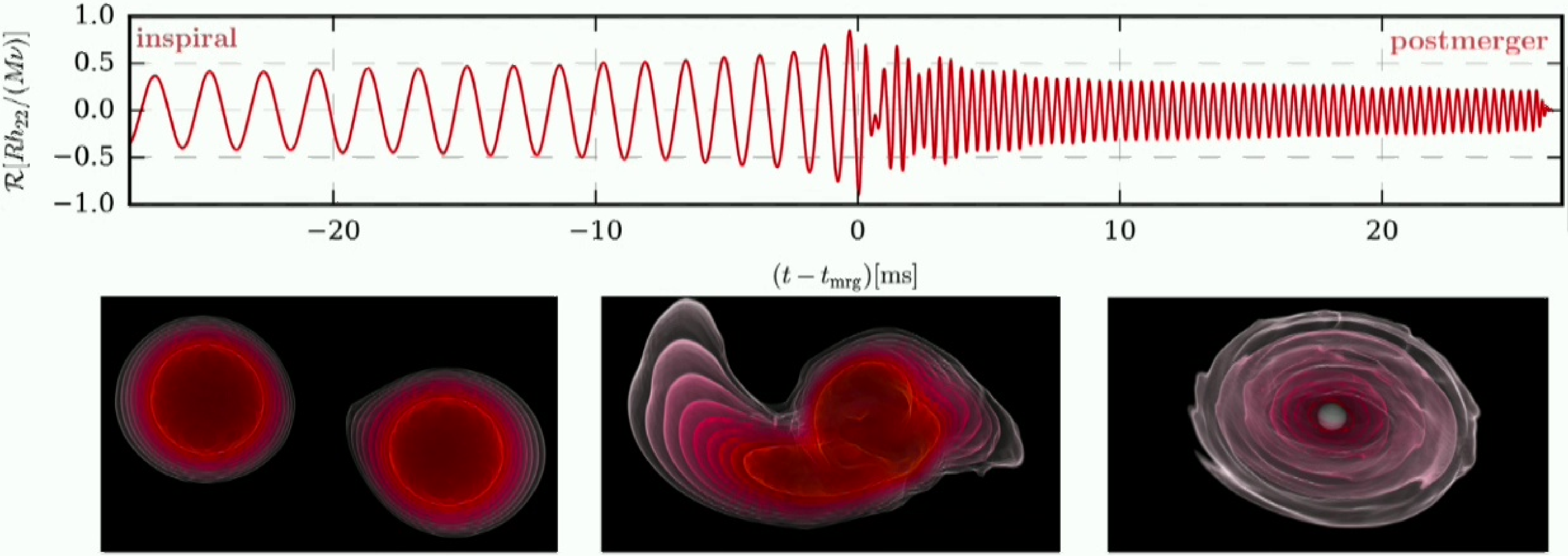



Chatziioannou, arXiv:2006.03168

5

# Anatomy of an inspiral…

UNIVERSITY OF BIRMINGHAM




Dietrich+20

6

# BNS Waveform Models

UNIVERSITY OF BIRMINGHAM

- **P**ost **N**ewtonian **[PN]**
  - Stationary phase approximation + closed-form expressions in **frequency** domain → **fast**
  - Analytical expressions breakdown as $v \sim c$
- **E**ffective **O**ne **B**ody **[EOB]**
  - Semi-analytical + **resummed** PN + calibration against NR → natural framework
  - Solve computationally expensive ODEs in time-domain → **slow**
- **Phenom**enological **[Phenom]**
  - PN baseline + terms calibrated to EOB/NR → closed form frequency domain expressions
  - ~ accuracy of EOB but speed of PN [best of both worlds]

Not going to talk about EFT, NR, ADM Hamiltonian, PM, ...

# Post Newtonian

# Post-Newtonian Models

UNIVERSITY OF BIRMINGHAM

- Orbital dynamics approximated by PN expansion $\epsilon \sim GM/rc^2 \sim v^2/c^2 \ll \mathcal{O}(1)$
- Finite-size corrections included via tidal expansion $\alpha = R/r$
- Tidal deformability ~ coupling constant in effective action for dynamics
- Multipoles arise in response to external tidal field → excites NS oscillation modes
- **Basic picture:** $\ell = 2$ fundamental mode of star treated as forced damped harmonic oscillator driven by tidal field of external companion [e.g. Flanagan + Hinderer]

$$S = S_{\rm pp} + \int dt \sum_{\ell \geq 2} \left[ -\frac{1}{\ell!} Q_L \mathcal{E}_L + \mathcal{L}^{\rm int} \right]$$

$$S_{\rm pp} = \int dt \left[ \frac{\mu}{2} v^2 + \frac{\mu M}{r} \right]$$

Point-particle terms ~ orbital motion

$$\mathcal{L}^{\rm int} = \frac{1}{2\ell! \lambda_\ell \omega_{0\ell}^2} \left[ \dot{Q}_L \dot{Q}^L - \omega_{0\ell}^2 Q_L Q^L \right]$$

Internal dynamics of multipoles ~ response to external tidal field

# Post-Newtonian Models

UNIVERSITY OF BIRMINGHAM

- **Adiabatic limit** is when NS internal time scale fast compared to variation in tidal field

$$\tau_{\rm NS} \sim \omega_{0\ell}^{-1} \sim \sqrt{R_{\rm NS}^3/m_{\rm NS}} \qquad \tau_{\rm orb} \sim \omega_{\rm orb}^{-1} \sim \sqrt{r^3/M}$$

Depends on internal structure [EoS]

$$\lambda_\ell = \frac{2}{(2\ell-1)!!} k_\ell R^{2\ell+1}$$

- Most of inspiral in adiabatic regime as $2\,\omega_{\rm orb} \ll \omega_{0\ell}$

- In this limit quadrupole ~ instantaneously tracks external tidal field $Q_L^{\rm ad.} = -\lambda_L \mathcal{E}_L$

- Kinetic terms in Lagrangian drop out $\dot{Q}_L = 0$

- Internal Lagrangian reduces to ~ elastic potential energy of deformation

$$\mathcal{L}_{\rm ad.}^{\rm int} = -\frac{Q_L Q^L}{2\ell!\lambda_\ell}$$

$$S_{\rm ad.} = S_{\rm pp} + \int dt \left[\frac{\lambda_\ell}{2\ell!}\mathcal{E}_L\mathcal{E}^L\right]$$

## Post-Newtonian Models

UNIVERSITY OF BIRMINGHAM

- Derive GW phase by imposing **flux balance** + **s**tationary **p**hase **a**pproximation (SPA)

$$\Psi(f) = 2\pi f t_{\rm ref} - \phi_{\rm ref} + 2\int_{v_f}^{v_{\rm ref}} \left(v_f^3 - v^3\right) \frac{E'(v)}{\mathcal{F}(v)}\, dv \qquad v = (M\omega_{\rm orb})^{1/3}$$

- Leading order tidal contribution at 5PN [Flanagan & Hinderer]

$$\delta\Psi = -\frac{9}{16}\frac{v^5}{\mu M^4}\left[\left(11\frac{m_2}{m_1} + \frac{M}{m_1}\right)\lambda_1 + 1 \leftrightarrow 2\right]$$

- But the oscillation modes **evolve** towards a **resonance** as $\omega_{\rm orb} \sim \omega_{0\ell}$

- Resonant f-mode frequency $\omega_{0\ell} \sim \sqrt{\frac{m_{\rm NS}}{R_{\rm NS}^3}}$ — NS radius

- Tidal driving frequency $2\omega_{\rm orb} \sim 2\sqrt{\frac{M}{r^3}}$ — Orbital separation

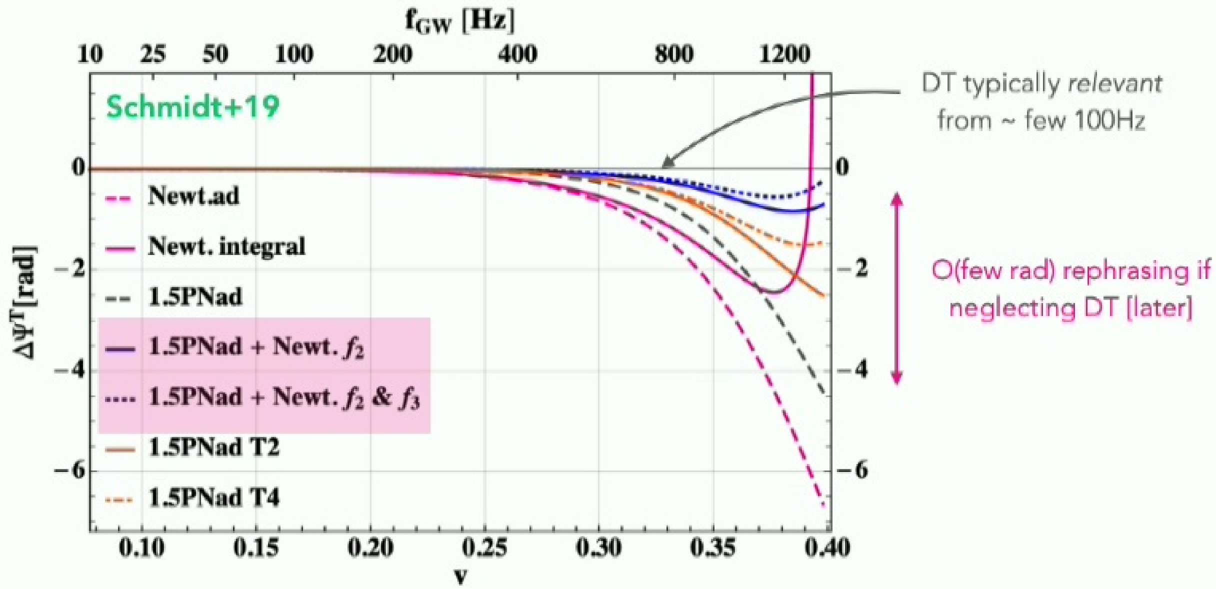
Dai+, Kokkotas+, Hinderer+, Steinhoff+, Schmidt+, Ferrari+, Maselli+, Landry+, Ho+, Ma+, Andersson+, ...

## Post-Newtonian Models

UNIVERSITY OF BIRMINGHAM

- Analytical form of f-mode excitations via regularisation [**Schmidt+19**]

$$\Psi^{\rm T}_{\rm fm} = \underbrace{-\frac{(10\sqrt{3}\pi - 27 - 30\log(2))}{96\eta}\frac{\Lambda_{2A}X_A^6}{(\Omega_{2A})^2}(155 - 147X_A)\,v^{11}}_{\text{Newt. } f_2\text{-mode}} \underbrace{-\frac{1875(5 - 6\log(2))}{16}\frac{\Lambda_{3A}X_A^7}{(\Omega_{3A})^2}v^{15}}_{\text{Newt. } f_3\text{-mode}}$$



# Post-Newtonian Models

- What is the state-of-the-art?
- Lagrangian for relativistic binaries complete up to NNLO [e.g. Henry+20]
- Einstein-Hilbert action + Effective matter action

$$S_m = \sum_{A=1}^{N} \int d\tau_A \, L_A \qquad L_A = -m_A c^2 + \frac{\mu_A^{(2)}}{4} G_{\mu\nu}^A G_A^{\mu\nu} + \frac{\sigma_A^{(2)}}{6c^2} H_{\mu\nu}^A H_A^{\mu\nu} + \frac{\mu_A^{(3)}}{12} G_{\mu\nu\alpha}^A G_A^{\mu\nu\alpha}$$

Tidal polarizabilities

$$G\mu_A^{(2)} = \left(\frac{G m_A}{c^2}\right)^2 \Lambda_A^{(2)} = \frac{2}{3} k_A^{(2)} R_A^5$$ Mass quadrupole $$G_{\mu\nu}^A = -c^2 R_{\mu\rho\nu\sigma}^A u_A^\rho u_A^\sigma$$

$$G\sigma_A^{(2)} = \frac{1}{48} j_A^{(2)} R_A^5$$ Current quadrupole $$H_{\mu\nu} = 2c^3 R_{(\mu\rho\nu)\sigma}^A u_A^\rho u_A^\sigma$$

$$G\mu_A^{(3)} = \frac{2}{15} k_A^{(3)} R_A^7$$ Mass octupole $$G_{\mu\nu\alpha} = -c^2 \nabla_{(\lambda}^{\perp} R_{\mu\rho\nu)\sigma}^A u_A^\rho u_A^\sigma$$

Rotational tidal love numbers, see also: Pani+, Landry+, Vines+...

## Post-Newtonian Models

- Up to **NNLO** including tails the tidal phase is **complete**

| $\varphi_{\rm tidal}$ | Mass Quadrupole | Current Quadrupole | Mass Octupole |
|---|---|---|---|
| 5PN (L) | [6,7,18,44,45] ✓ | ✗ | ✗ |
| 6PN (NL) | [18,44,46] ✓ | [46,47] ✓ | ✗ |
| 6.5PN (tail) | [18,46] ✓ | ✗ | ✗ |
| 7PN (NNL) | ✓ | ✓ | [46,48] ✓ |
| 7.5PN (tail) | ✓ | ✓ | ✓ |

Henry+ PRD 102, 044033 (2020)

$$\delta\psi_{\rm tidal} = -\Lambda_{\ell=2,A} c_N^A x^{5/2} \left(1 + c_1^A x + c_{3/2}^A x^{3/2} + c_2^A x^2 + c_{5/2}^A x^{5/2}\right) + A \leftrightarrow B$$

## Post-Newtonian Models

UNIVERSITY OF BIRMINGHAM

- What about **spins**?
- Multipole moments that couple to tidal field generated by NS spin → EOS signature
- Rotating BHs in GR have simple multipolar structure due to no-hair theorem

$$M_\ell^{\rm BH} + i S_\ell^{\rm BH} = m (i\chi m)^\ell$$

- For finite-size objects the quadrupole + higher order moments depend on EOS

$$M_{\ell=2}^{\rm NS} \sim -\kappa \chi^2 m^3$$

Matter dependent coefficient, the rotational Love number
Introduces EoS dependence!

- Leads to a spin-induced quadrupole-monopole at 2PN [Poisson 97]

$$\Psi_{\rm QM}(v) = -\frac{30}{128\eta}\sigma_{\rm QM} v^{-1} \qquad \sigma_{\rm QM} = -\frac{5}{2} M_{\ell=2,A}^{\rm NS} X_A^2 \left[3\left(\chi_A \cdot \hat{L}\right) - 1\right] + A \leftrightarrow B$$

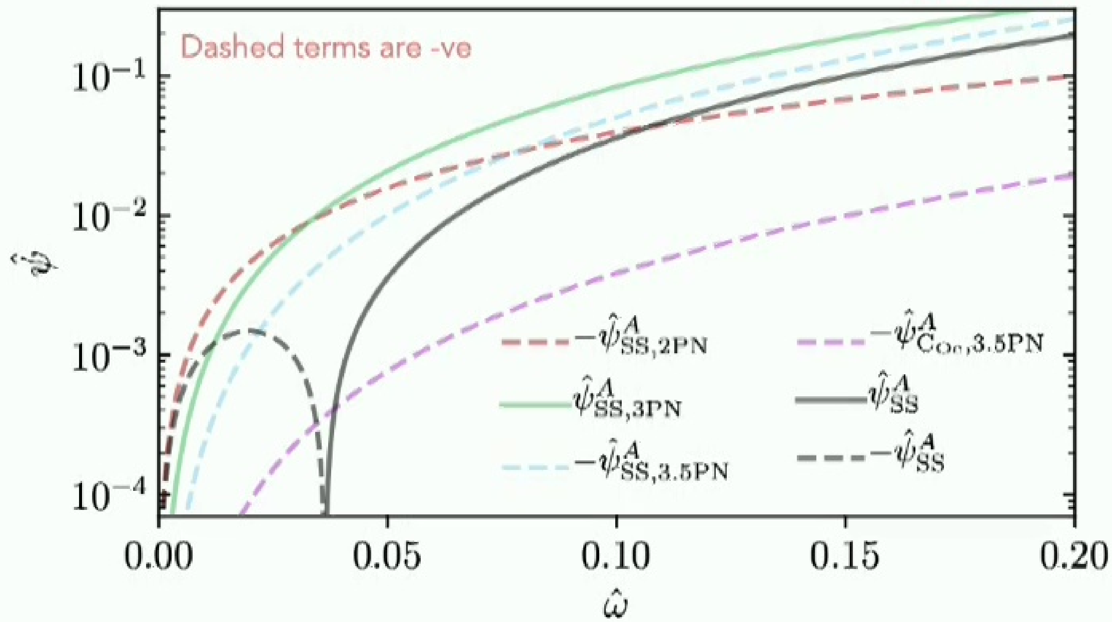
# Post-Newtonian Models

- EOS-dependent quadrupole-monopole terms known up to 3.5PN [e.g. Marsat+15, Bohé+15, Nagar+19]

$$\Psi^{\rm QM}_{\rm SS} = \Psi^{\rm QM,LO}_{\rm SS} + \Psi^{\rm QM,NLO}_{\rm SS} + \Psi^{\rm QM,tail}_{\rm SS}$$

2PN term Poisson (97) — 3PN term — 3.5PN term

Dietrich+19




Octupolar term ~ order of magnitude smaller $S^A_3 = -C^A_{\rm Oc}\chi^3_A M^4_A$

## Post-Newtonian Models

UNIVERSITY OF BIRMINGHAM

- In reality **rich spectra** of normal modes beyond just the fundamental mode are excited…
- **Interface** (i-) modes? Interaction of fluid core and crust → crust meltdown ~ $\mathcal{O}(10^2\,\mathrm{Hz})$ [Pan+20]
- **Inertial** modes [Lockitch+99; Poisson+20] ~ perturbation to NS velocity field?
  - Frequencies ~ NS rotation angular frequency
  - **r-modes** are a subset of inertial modes [Papaloizou+78, Flanagan+07]
  - Fastest known pulsar in binary ~ 60Hz? [Andrews+19]
- Nonlinear fluid instabilities? [Venumadhav+14, Weinberg+16, Essick+16]
  - Non-resonant coupling between equilibrium tide p-mode and gravity supported g-mode?
- **g**-modes fully excited but **phase shifts** $\mathcal{O}(10^{-3})$ or **small**er? [Yu+16]

## NR Tidal Models

## Post-Newtonian Models

UNIVERSITY OF BIRMINGHAM

- In reality **rich spectra** of normal modes beyond just the fundamental mode are excited…
- **Interface** (i-) modes? Interaction of fluid core and crust → crust meltdown ~ $\mathcal{O}(10^2\,\mathrm{Hz})$ [Pan+20]
- **Inertial** modes [Lockitch+99; Poisson+20] ~ perturbation to NS velocity field?
  - Frequencies ~ NS rotation angular frequency
  - **r-modes** are a subset of inertial modes [Papaloizou+78, Flanagan+07]
  - Fastest known pulsar in binary ~ 60Hz? [Andrews+19]
- Nonlinear fluid instabilities? [Venumadhav+14, Weinberg+16, Essick+16]
  - Non-resonant coupling between equilibrium tide p-mode and gravity supported g-mode?
- **g**-modes fully excited but **phase shifts** $\mathcal{O}(10^{-3})$ or **small**er? [Yu+16]

# NRTidal Models

- NRTidal [Dietrich+] is a closed form approximation with terms calibrated to NR [see also Kawaguchi+]

- Baseline is just analytical terms up to 7.5PN

$$\phi_T = -\kappa_A c_N^A x^{5/2} \left(1 + c_1^A x + c_{3/2}^A x^{3/2} + c_2^A x^2 + c_{5/2}^A x^{5/2}\right) + [A \leftrightarrow B]$$

- Introduce an *effective* tidal coupling constant

$$\kappa_{\rm eff}^T = \frac{2}{13}\left[\left(1 + 12\frac{X_B}{X_A}\right)\right]\left(\frac{X_A}{C_A}\right)^5 k_2^A + A \leftrightarrow B$$

$$\tilde{\Lambda} = \frac{16}{3}\kappa_{\rm eff}^T$$

- Effective tidal phase

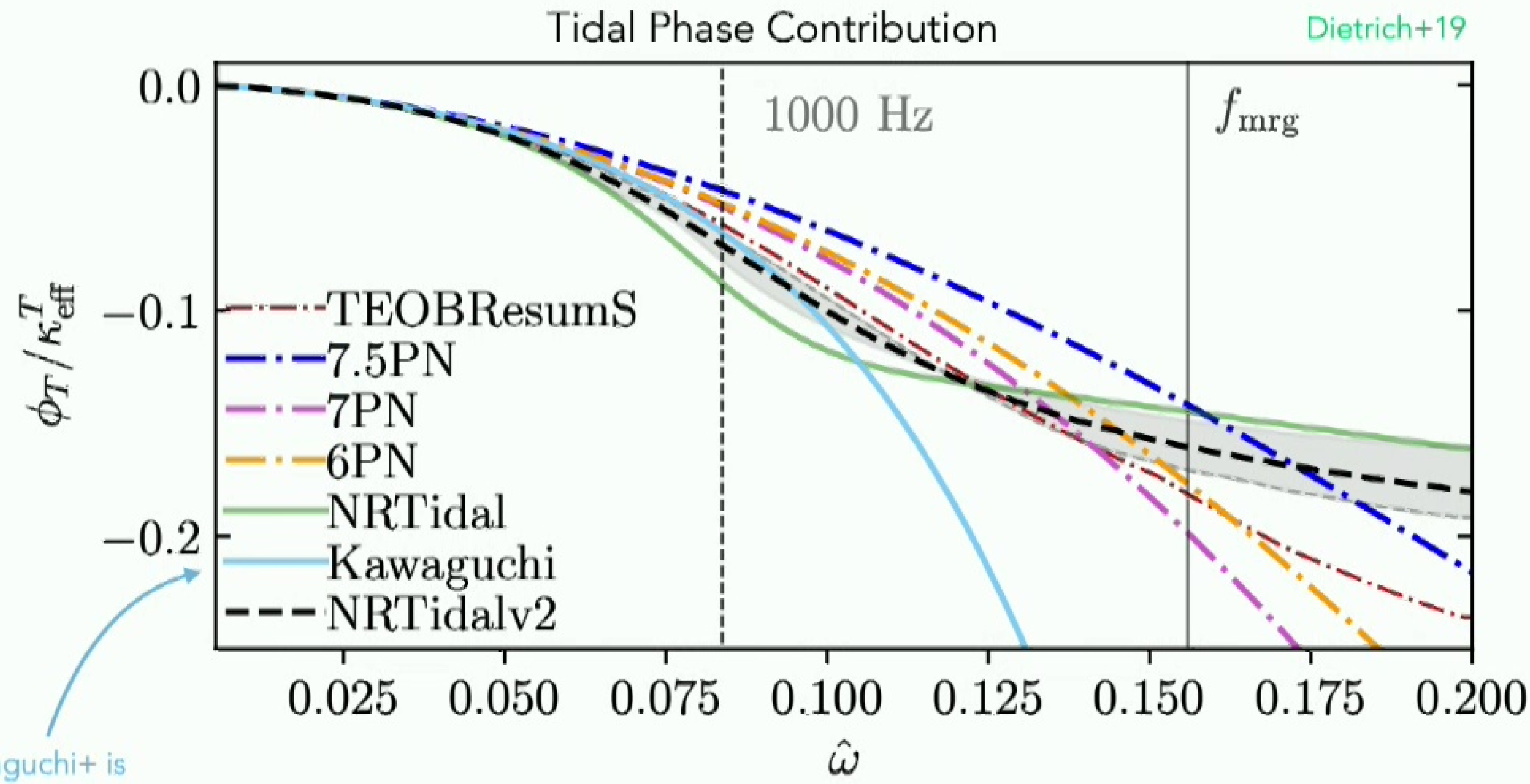
Constrained by analytical PN

$$\phi_T(x) = -\kappa_{\rm eff}^T \frac{13}{8\nu} x^{5/2} P_{\rm NRT}(x) = -\kappa_{\rm eff}^T \frac{13}{8\nu} x^{5/2} \left[\frac{1 + n_1 x + n_{3/2} x^{3/2} + n_2 x^2 + n_{5/2} x^{5/2} + n_3 x^3}{1 + d_1 x + d_{3/2} x^{3/2} + d_2 x^2}\right]$$

Free coefficients fit to NR

- Why? NR contains all [strongly caveated] physics including non-perturbative

# NRTidal Models



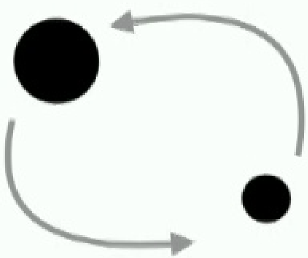

Kawaguchi+ is alternative NR calibrated model [up to 1000 Hz]

## Effective One Body

## Effective One Body

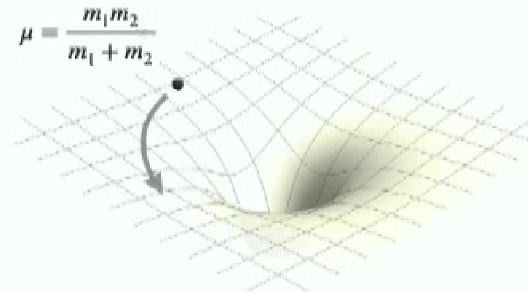
UNIVERSITY OF BIRMINGHAM

- In EOB we replace two-body dynamics $(m_1, m_2)$ with dynamics of particle of mass $\mu$ in effective metric $g_{\mu\nu}^{\text{eff}}(u)$



EOB Mapping

$$H_{\text{real}}^{\text{EOB}} = M\sqrt{1 + 2\nu\left(\frac{H_{\text{eff}}^{\nu}}{\mu} - 1\right)}$$




- EOB equations of motion

$$\dot{\mathbf{r}} = (A/B)^{1/2}\,\partial_{\mathbf{p_{r_*}}}\hat{H}_{\text{EOB}} \qquad \dot{p}_{r_*} = -(A/B)^{1/2}\,\partial_r \hat{H}_{\text{EOB}}$$

$$\dot{\varphi} = \partial_{p_\varphi}\hat{H}_{\text{EOB}} \qquad \dot{p_\varphi} = \hat{\mathcal{F}}_\varphi$$

- EOB waveform

$$h_{\ell m}^{\text{EOB}} = \theta\left(t_m - t\right) h_{\ell m}^{\text{insplunge}}\left(t\right) + \theta\left(t - t_m\right) h_{\ell m}^{\text{ringdown}}\left(t\right)$$

## Effective One Body

UNIVERSITY OF BIRMINGHAM

- Effective Hamiltonian can be written in terms of metric potentials

$$\hat{H}_{\text{eff}} = \sqrt{p_{r_*}^2 + A(r)\left(1 + \frac{p_\varphi^2}{r^2} + 2\nu(4-3\nu)\frac{p_{r_*}^4}{r^2}\right)}$$

- **Adiabatic** tidal effects incorporated through **tidal** contributions to **potentials**

$$A = A_0 + A_T^{(+)}$$

$$A_T^{(+)} = -\sum_{\ell=2}^{4}\left[\kappa_A^{(\ell)} u^{2\ell+2}\hat{A}_A^{(\ell+)} + (A \leftrightarrow B)\right] \qquad \hat{A}_A^{(\ell+)}(u) = 1 + \alpha_1^{(\ell)} u + \alpha_2^{(\ell)} u^2 \quad \text{[PN NNLO]}$$

- Also need additive **tidal** contribution to **dissipative** sector [e.g. Damour+12]

$$h_{\ell m} = h_{\ell m}^0 + h_{\ell m}^T$$

# Effective One Body

UNIVERSITY OF BIRMINGHAM

- There are two main variants of EOB in the literature
  - Different Hamiltonians, gauge choices, gyro-gravitomagnetic couplings, calibration, …
- SEOB [Buonanno, Hinderer, Steinhoff, … ]
  - Includes both **adiabatic** and **dynamical** tides [Hinderer+16, Steinhoff+16, Steinhoff+21]
  - Octupole terms [Bini+12]
  - Leading order 2PN spin induced quadrupole
- TEOBResumS [Damour, Nagar, Bernuzzi, …]
  - **Adiabatic** tides informed by **GSF** limit [Nagar+, Ackay+] + recently **dynamical** tides [Steinhoff+]
  - $\ell = 2,3,4$ **gravitoelectric** and $\ell = 2$ **gravitomagnetic** tidal terms
  - Spin induced quadrupole terms up to 3.5PN

## Effective One Body

UNIVERSITY OF BIRMINGHAM

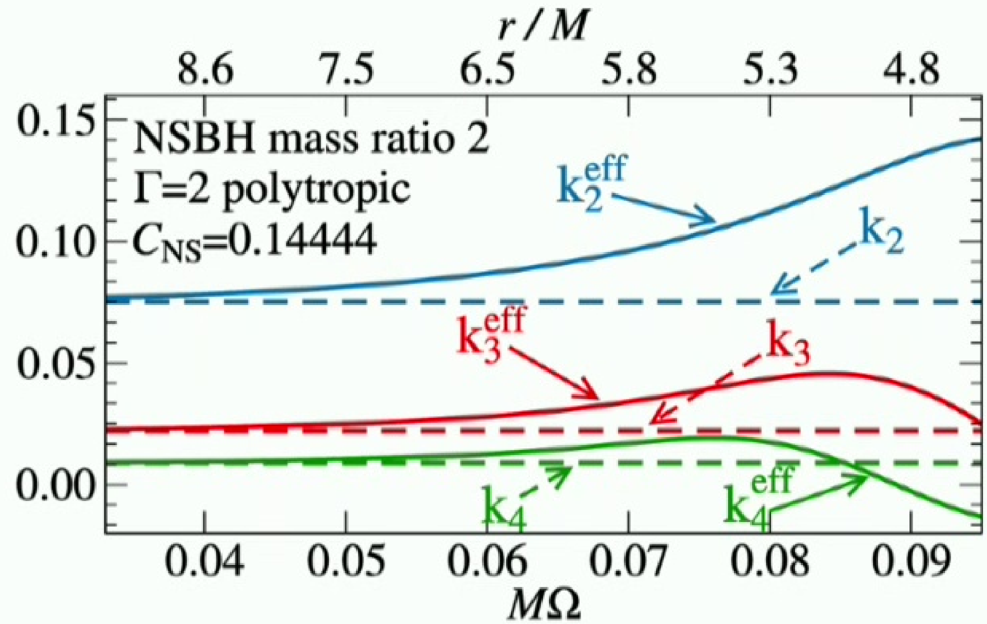
- Dynamical tides incorporated through ~ redressing of tidal Love number $k_\ell \to k_\ell \, \hat{k}_\ell^{\rm dyn}$
- Leads to **enhancement** of tidal love number [Hinderer+16, Steinhoff+16; also Ma+20]

$$\hat{k}_\ell^{\rm dyn} = a_\ell + \frac{b_\ell}{2}\left[\frac{Q^{\rm DT}_{m=\ell}}{Q^{\rm AT}_{m=\ell}} + \frac{Q^{\rm DT}_{m=-\ell}}{Q^{\rm AT}_{m=-\ell}}\right]$$

$$\frac{Q^{\rm DT}_m}{Q^{\rm AT}_m} \approx \frac{\omega_f^2}{\omega_f^2 - (m\Omega)^2} + \frac{\omega_f^2}{2(m\Omega)^2 \epsilon_f \Omega'_f(\phi - \phi_f)} \pm \frac{i\omega_f^2}{(m\Omega)^2\sqrt{\epsilon_f}} e^{\pm i\Omega'_f \epsilon_f(\phi-\phi_f)^2} \int_{-\infty}^{\sqrt{\epsilon_f}(\phi-\phi_f)} e^{\mp i\Omega'_f s^2} \mathrm{d}s$$

Equilibrium solution that increases correction long before resonance

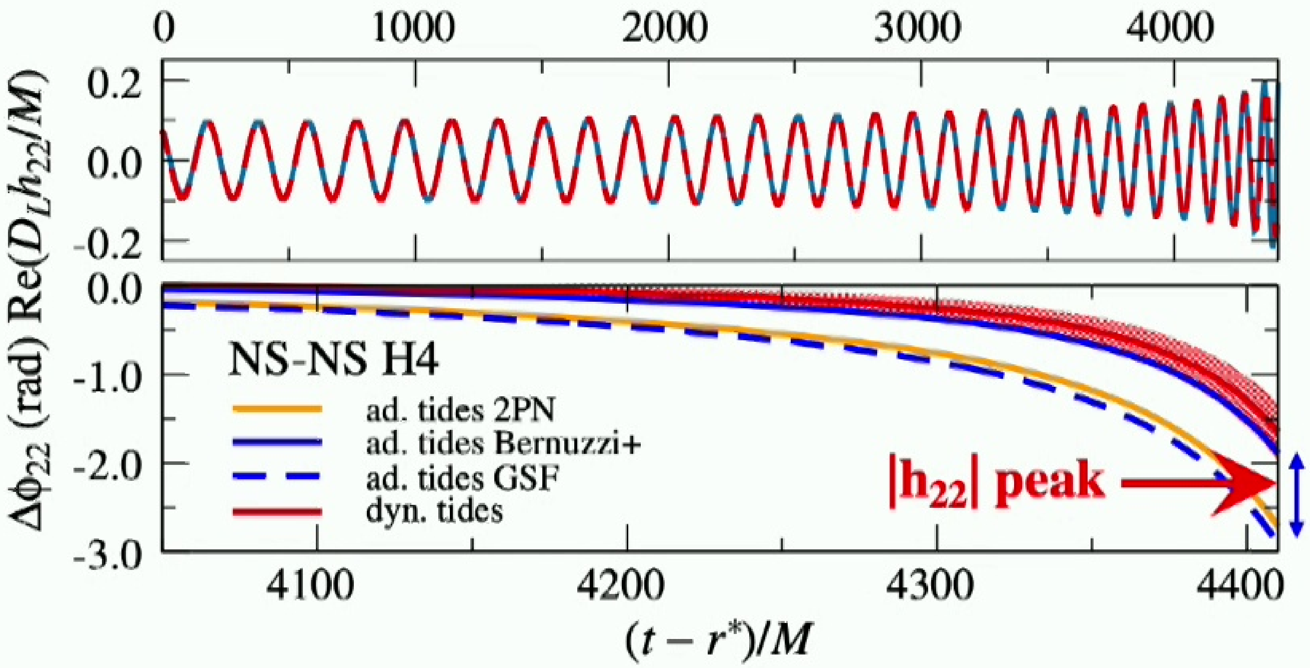
Fresnel integral capture near resonance dynamics $|t - t_r| \ll \sqrt{\pi/\dot{\Omega}_r}$



## Effective One Body

UNIVERSITY OF BIRMINGHAM

- Dynamical tides incorporated through ~ redressing of tidal Love number $k_\ell \to k_\ell \, \hat{k}_\ell^{\mathrm{dyn}}$
- Leads to **enhancement** of tidal love number [Hinderer+16, Steinhoff+16]



# Effective One Body

UNIVERSITY OF BIRMINGHAM

- Spin also leads to an induced shift of the modes ~ Coriolis type term [e.g. Steinhoff+21]

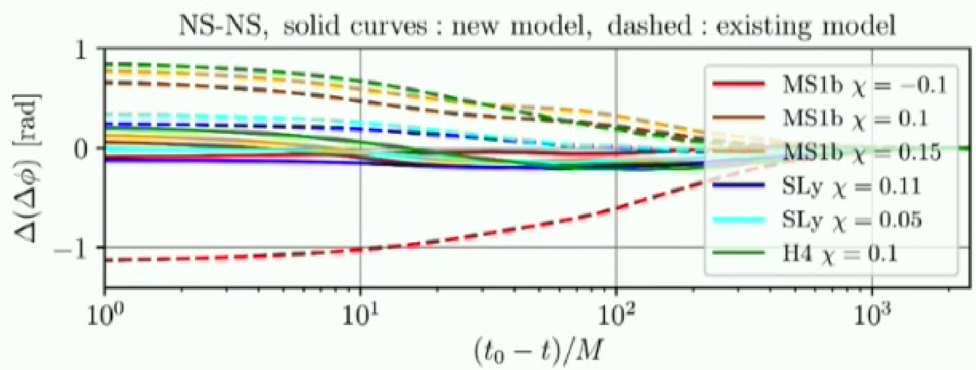
~ quasi-universal spin-tidal coupling

$$\Delta\omega_0 = \omega_f - \omega_0 \approx |m|\frac{3}{4I}\,\mathbf{S}_1 = \frac{3}{2}\Omega$$

rotation frequency of body

- Propagates into effective Love number

$$\hat{k}_\ell^{\rm eff} = a_\ell + b_\ell\left(1 - \frac{(\Delta\omega_{0\ell})^2}{\omega_{0\ell}^2}\right)\left\{\frac{\omega_{0\ell}^2}{\omega_{0\ell}^2 - (\ell\omega_{\rm orb} - \Delta\omega_{0\ell})^2} + \left[\frac{\omega_{f,\ell}^2}{2\sqrt{\epsilon_\ell}\,\hat{t}\,|\tilde{\Omega}'|\left(1+\frac{\Delta\omega_{0\ell}}{\omega_{0\ell}}\right)(\ell\omega_{\rm orb})^2}\right]_{\omega_{f,\ell}=\omega_{0\ell}+\Delta\omega_{0\ell}} + \left[\frac{\omega_{f,\ell}^2}{\sqrt{\epsilon_\ell}(\ell\omega_{\rm orb})^2\left(1+\frac{\Delta\omega_{0\ell}}{\omega_{0\ell}}\right)}\mathcal{Q}_{\ell\ell}(\hat{t})\right]_{\omega_{f,\ell}=\omega_{0\ell}+\Delta\omega_{0\ell}}\right\}$$




29

## Brief Summary

UNIVERSITY OF BIRMINGHAM

| | TaylorF2 | TEOBResumS | SEOBNRv4T | NRTidal |
|---|---|---|---|---|
| Adiabatic Tides | 2.5PN* in phasing | e.g. 2PN in A potential + GSF | e.g. 2PN in A potential | Incomplete 2.5PN in phasing [DNV] |
| Dynamic Tides | Yes | Yes* | Yes* | —* |
| SIQM | NNLO | NNLO | LO | NNLO (PN) |
| Notes | Complete NNLO adiabatic tides derived in Henry+21 | *Recent | *Inc. spin dependent corrections [Steinhoff+21] | *Dynamical tides partially absorbed in NR calibration |

DNV = Damour, Nagar & Villain, arXiv:1203.4352

Impact of Tidal Information on Gravitational-Wave Observations?

## Bayesian Inference

UNIVERSITY OF BIRMINGHAM

- Bayesian inference allows us to extract information on the binary parameters

Likelihood Priors

Posterior Probability $$p(\theta \mid d) = \frac{\mathcal{L}(d \mid \theta)\pi(\theta)}{\mathcal{Z}}$$

Evidence

Compare theoretical waveform against the data

$$\mathcal{L}(d \mid \theta) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{1}{2}\frac{|d - h(\theta)|^2}{\sigma^2}\right)$$

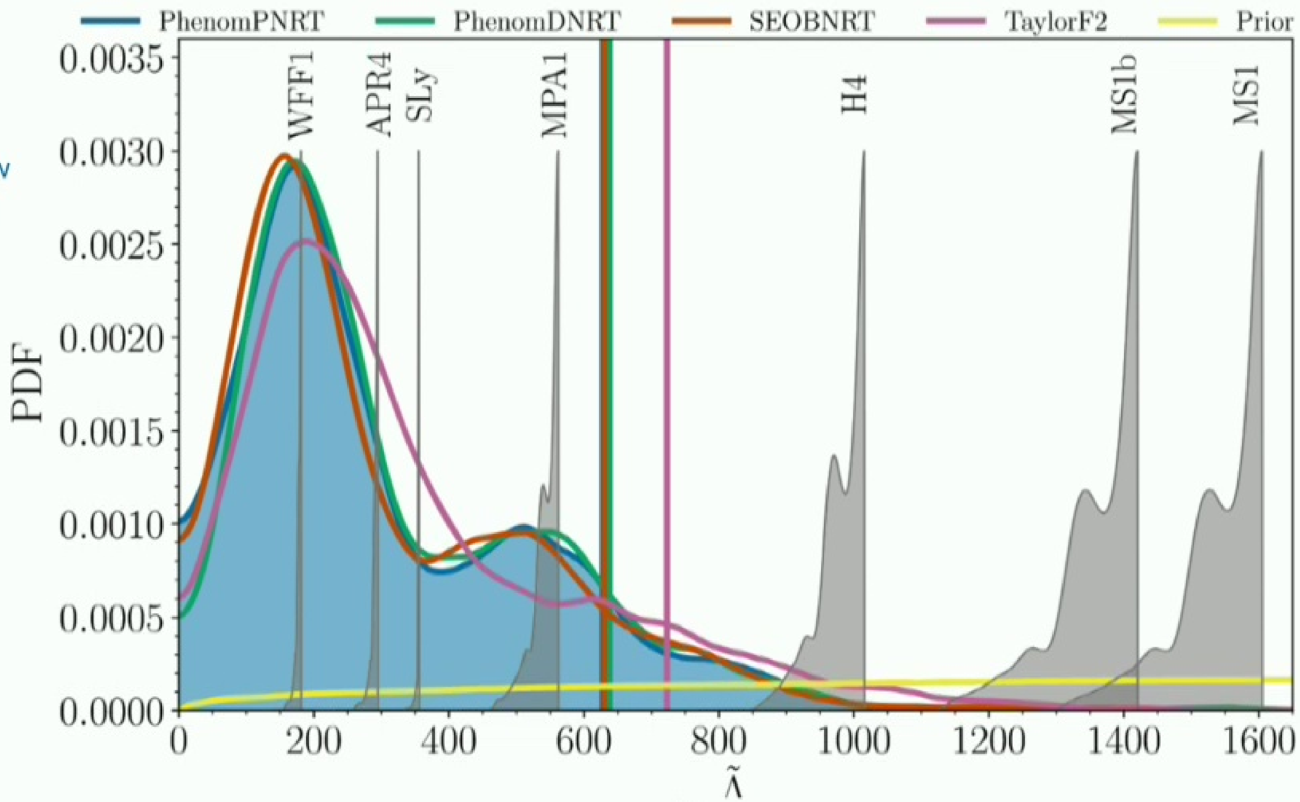
## Bayesian Inference: GW170817

UNIVERSITY OF BIRMINGHAM

- GW170817 provides excellent constraint on tidal deformability but also shows subtle **systematics**

3 BBH baselines using NRTidal show broad agreement

TaylorF2 favours larger tidal deformabilities?




33

## Model Systematics?

UNIVERSITY OF BIRMINGHAM

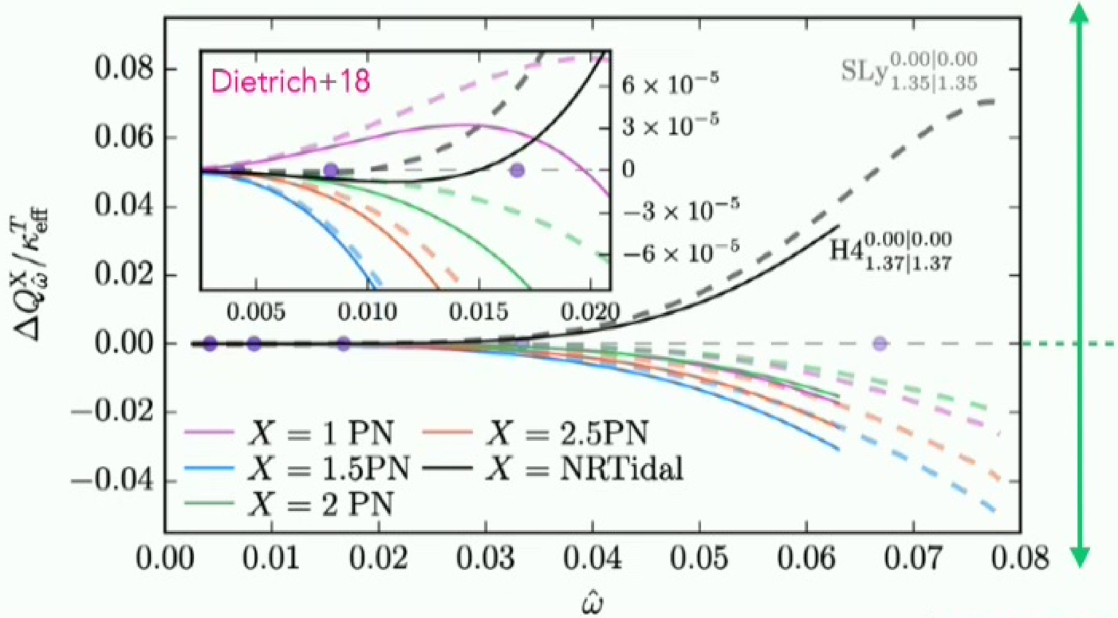
- Can we understand what is going on?
- Introduce gauge invariant expression

$$Q_\omega = \frac{\omega^2}{\dot{\omega}} = \frac{d\phi(t)}{d\ln\omega}$$

- Interested in *differences* $\Delta Q_\omega = Q_\omega^A - Q_\omega^B$
  - For fixed $\omega$ if $\Delta Q_\omega > 0$ then $\dot{\omega}^B > \dot{\omega}^A$
  - Tidal effects in model B more attractive than A
  - Bonus $\epsilon_{\text{adiabatic}} = Q_\omega^{-1}$ ~ validity of SPA

$\Delta Q_\omega^X = Q_\omega^{\text{TEOBResumS}} - Q_\omega^Y$

**More** attractive than TEOB




**Less** attractive than TEOB

TaylorF2 more **negative** than TEOBResumS and NRTidal, i.e. **less attractive** and hence need **larger tidal deformability** to produce same impact…

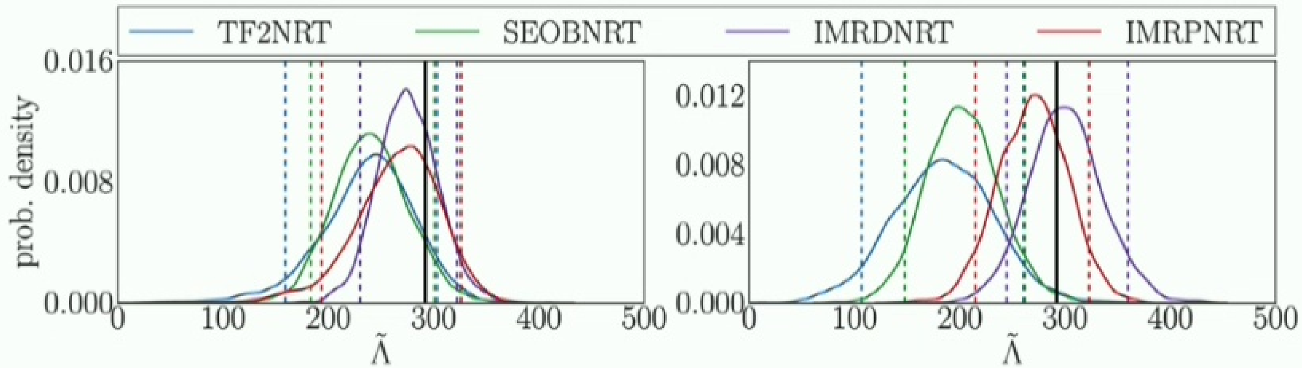
34

# Model Systematics?

UNIVERSITY OF BIRMINGHAM

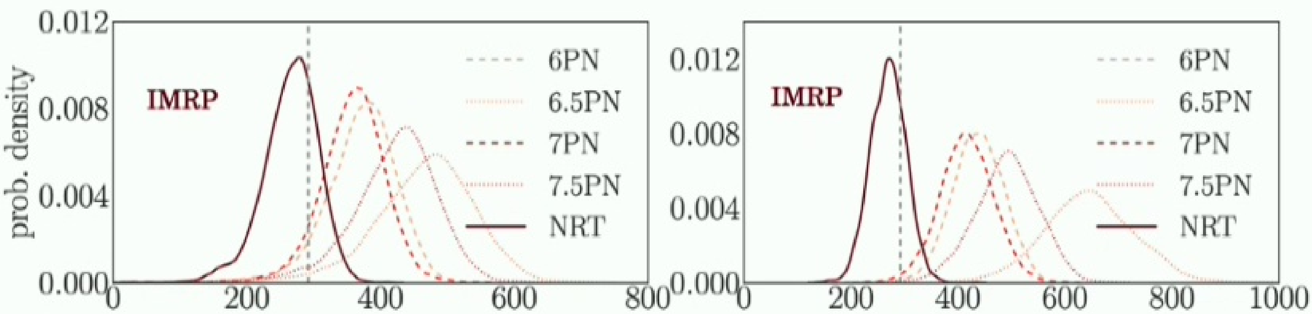
- Both **point-particle** and **tidal** sectors matter

Fix tidal sector and vary PP




Systematics from PP couple to tidal inference

Fix PP sector and vary tidal



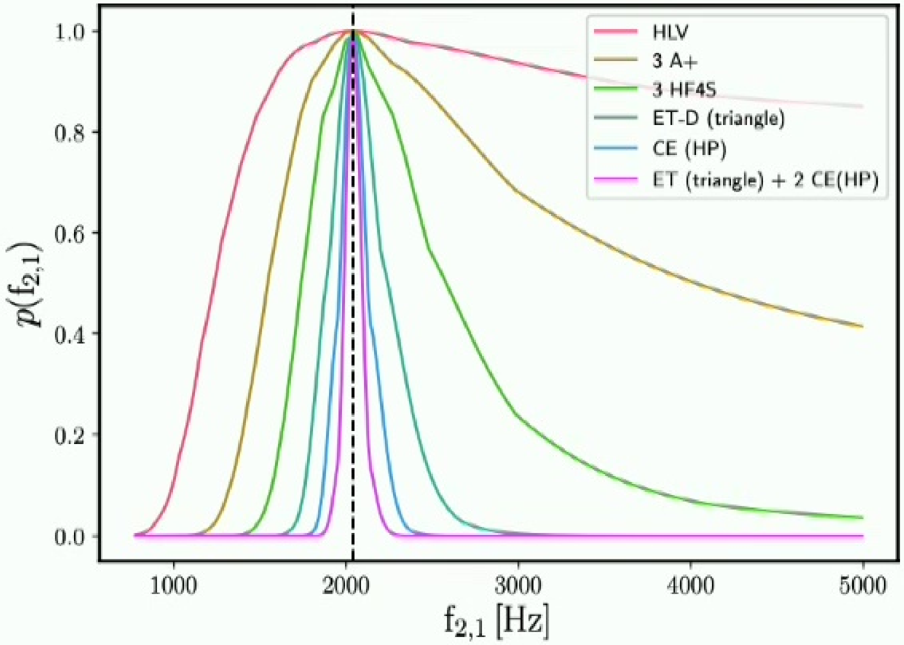

Insight into TaylorF2 holds: less attractive so larger tidal deformabilities required

# Can We Measure Dynamical Tides?

UNIVERSITY OF BIRMINGHAM

- What about **dynamical** tides?
- Consider optimally oriented GW170817-like binary
- In high SNR limit invoke linear signal approximation
- Tentatively suggests that we *could* measure the f-mode frequency to ~ $\mathcal{O}(10^1 - 10^2\,\mathrm{Hz})$
- Reality check
  - Need to include all missing physics: spin-tidal couplings, higher-order PN effects, precession, …
  - Need to understand correlations/degeneracies between parameters

Pratten, Schmidt+ arXiv:1905.00817




36

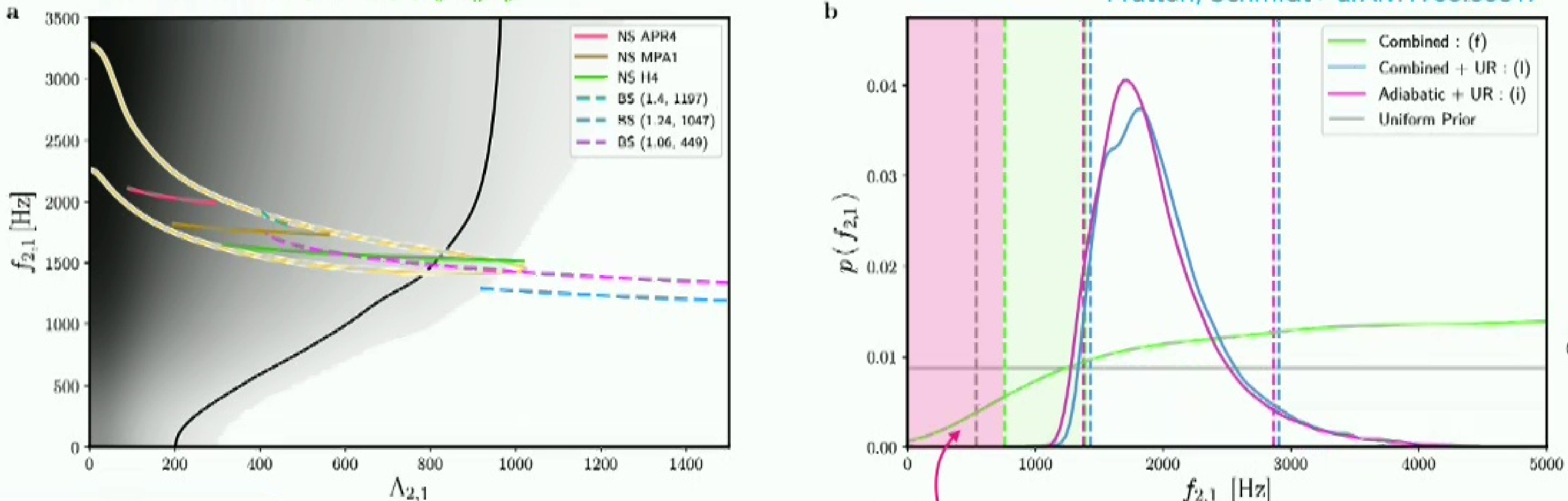
# Bayesian Inference: GW170817 + Dynamical Tides

UNIVERSITY OF BIRMINGHAM

- Analyse public strain data with TaylorF2 + dynamical tides **[Schmidt+19]**
- Place lower bound on the f-mode frequency *without* assuming universal relations $f_{2,1} \geq 1.39\,\mathrm{kHz}$
- Consistent with predictions using Universal Relations

NS EoS [solid]
Massive boson star [dashed]: $(m_b/m_n, \lambda_b)$

Pratten, Schmidt+ arXiv:1905.00817




$$\propto \frac{\Lambda_{2,A}}{f_{2,A}^2}$$

Disfavours hyper-excited dynamical tides

$m_b$ : boson mass
$\lambda_b$ : strength of self-interaction

## Impact of Dynamical Tides on Future Inference?

UNIVERSITY OF BIRMINGHAM

- Shift focus from measurability to *impact* of dynamical tides on our inference of NS EoS

- First consider a systematic series → vary the chirp mass but fix the mass ratio to be ~ 0.85
- Fix the SNR to be ~ 50 (500) for a canonical O5 (3G) detector network
- Reminder: point-particle + adiabatic + dynamical contributions

$$\varphi(f) = \varphi_{\rm pp}(f) + \varphi_{\rm ad.}(f) + \varphi_{\rm dyn.}(f)$$

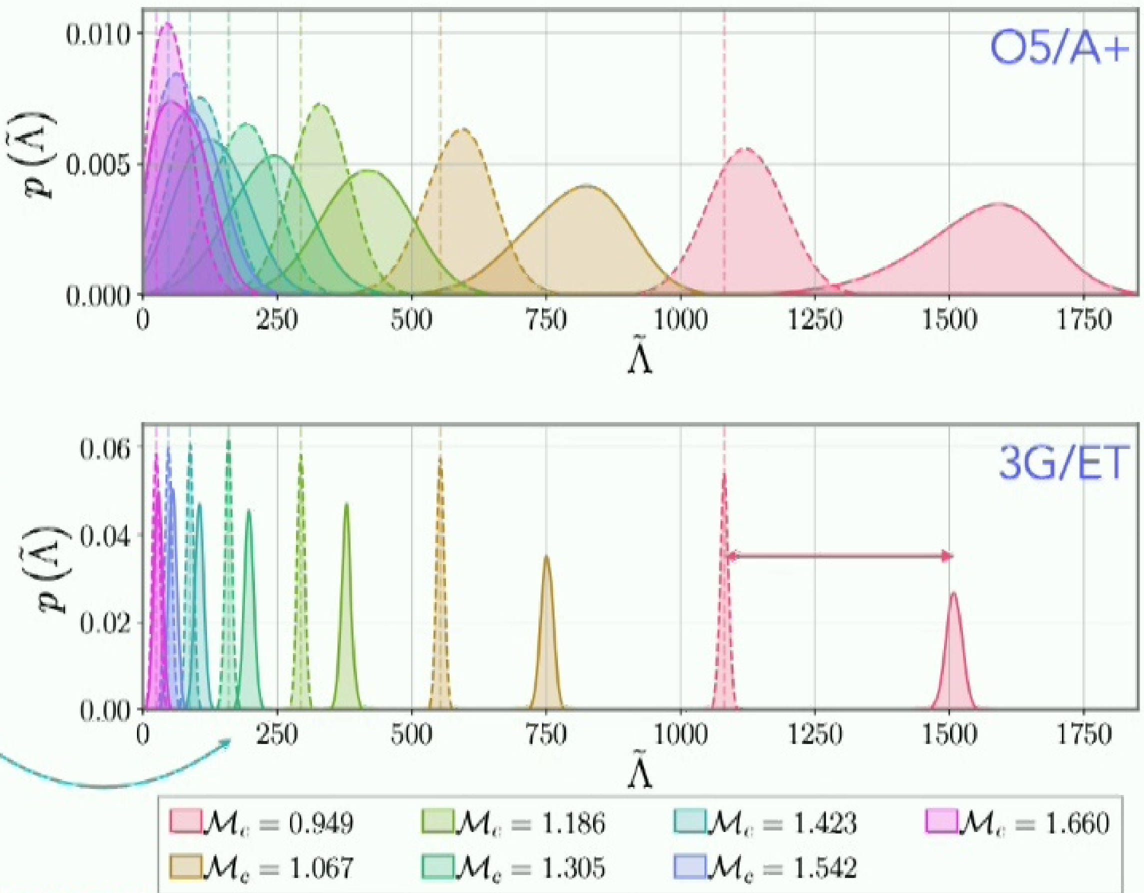
- Synthetic injection with all physics and recover with adiabatic or full dynamical model

- What would we expect *a priori*?
  - Adiabatic and dynamical tides more excited in lighter binaries and f-mode frequency is smaller
  - As dephasing scales as $\delta\varphi_{\rm dyn}(f) \propto \Lambda_{2A} / \Omega^2_{2A}$ the dynamical tides *should* have increasingly large contribution relative to adiabatic term as we decrease the chirp mass…

# Impact of Dynamical Tides on Future Inference?

UNIVERSITY OF BIRMINGHAM

Lighter NS = more excited tidal effects




- Dashed (adiabatic + dynamical)
- Solid (adiabatic)

Heavier binaries have a smaller but non-negligible bias ($m_i \sim 1.5 M_\odot$)

Neglecting dynamical tides has introduced a bias ~ O(400) on $\tilde{\Lambda}$ for the lightest binaries ($m_i \sim 1.1 M_\odot$)

Pratten, Schmidt+ arXiv:2109.07566

## Impact of Dynamical Tides on Future Inference?

UNIVERSITY OF BIRMINGHAM

- Will not observe a systematic series in real life…
- Consider impact of dynamical tides on semi-realistic **population**?
- Gaussian $m_i \sim N(\mu_i, \sigma_i)$ where $\mu_i = 1.33 M_\odot$ and $\sigma_i = 0.09 M_\odot$ [e.g. Özel+19]
  - Broadly consistent with low-mass peak in **Galactic** population [as per yesterday - caveats]
  - Large **uncertainty** in **NS mass** distribution, are NS masses lighter in BNSs? [Broekgaarden+21]
- Assume EM bright ~ small inclinations [e.g. Rezzolla+12]
- Use current BNS merger rate and 2yr Obs. In O5/A+ network (2025+) ~ 30 BNS mergers within 250 Mpc
- ~ 40% target of opportunity for Vera Rubin Observatory → **12 EM bright BNS**

## Impact of Dynamical Tides on Future Inference?

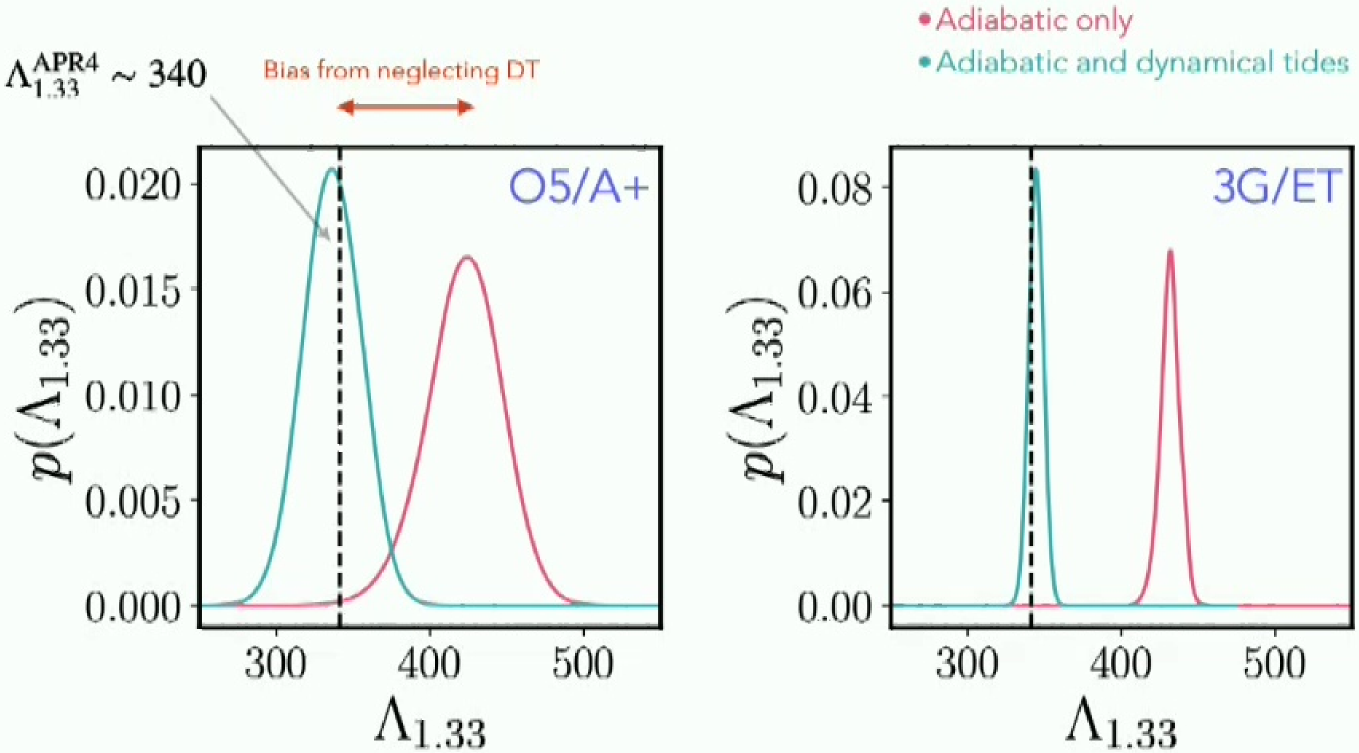
UNIVERSITY OF BIRMINGHAM

- How should we **combine** information from a population?
  - Full **hierarchical** inference? Recent work towards this, e.g. Golomb+21
  - **Non-parametric** or **parametric** model for EoS-sensitive observables? E.g. Del Pozzo+13, Agathos+16, …
- Taylor expand $\Lambda(m)$ around $(m - m_c)/M_\odot$ to linear order about reference mass $m_c = 1.33M_\odot$ [Del Pozzo+13]
  - Difficult to impose ***a priori*** information on the functional form of $\Lambda(m)$
  - Higher order coefficients in expansion are poorly constrained
- Calculate extrapolated tidal deformability $$\Lambda_{1.33} \simeq (1.33 - m_2)\frac{\Lambda_2 - \Lambda_1}{m_2 - m_1} + \Lambda_2$$
- Use population to estimate joint likelihood $$\mathcal{L}(\Lambda_{1.33}) \sim \prod_{n=1}^{N} p(\Lambda_{1.33}|d_n, H_i)\, p(\Lambda_{1.33}|H_i)^{-1}$$

## Impact of Dynamical Tides on Future Inference?

UNIVERSITY OF BIRMINGHAM

- Tidal information ~ weaker per event but systematics **compounded** in population
- Even in the O5/A+ network (2025+) systematic biases problematic and **catastrophic** by 3G/ET/CE




Pratten, Schmidt+ arXiv:2109.07566

# Impact of Dynamical Tides on Future Inference?

UNIVERSITY OF BIRMINGHAM

- **Goal:** infer EoS parameters $E = \{p_1, \Gamma_1, \Gamma_2, \Gamma_3\}$ from the population → follow **Lackey+14**
- Piecewise polytrope: $p(\rho) = K_i \rho^{\Gamma_i}$
  - $\Gamma_i$ adiabatic indices and $K_i$ enforces continuity of pressure at boundaries
- Easier to impose meaningful *a priori* constraints on the functional form of EoS
  - EoS to be thermodynamically stable: $dp/d\epsilon \geq 0$
  - Impose causality constraints: $v_s = \sqrt{dp/d\epsilon} < c$
  - Maximum NS mass compatible with heaviest pulsar (PSR J0740+6620) requires: $M_{\max} \gtrsim 2.14 M_{\odot}$
  - Discussion yesterday: beware correlations in phenomenological models [Legred+22]

## Impact of Dynamical Tides on Future Inference?

UNIVERSITY OF BIRMINGHAM

- Define pseudo-likelihood marginalised over all extrinsic and nuisance parameters

$$\mathcal{L}(d_i, \theta_{\mathrm{in},i}, \mathcal{H}, \mathcal{I}) = \int d\theta_{\mathrm{ex},i}\, p(\theta_{\mathrm{ex},i}|\mathcal{H},\mathcal{I})\, p(d_i|\theta_i, \mathcal{H}, \mathcal{I})$$

- Hypothesis $H$ denotes adiabatic or dynamical model
- $I$ denotes *a priori* information on EoS or binary parameters
- Marginalised posterior for the EoS parameters using joint-likelihood for all events marginalised over masses

$$p(\mathcal{E}|d_N,\mathcal{H},\mathcal{I}) = \frac{1}{p(d_N|\mathcal{H},\mathcal{I})}\int d\mathcal{M}\, dq\, p(\mathcal{E}|\mathcal{H},\mathcal{I}),$$
$$\times \prod_{i=1}^{N} p(\mathcal{M}_i, q_i|\mathcal{E},\mathcal{H},\mathcal{I})\mathcal{L}(d_i,\theta_{\mathrm{in},i},\mathcal{H},\mathcal{I})\Big|_{\tilde{\Lambda}_i=\tilde{\Lambda}(\mathcal{M}_i,\eta_i,\mathrm{EOS})},$$

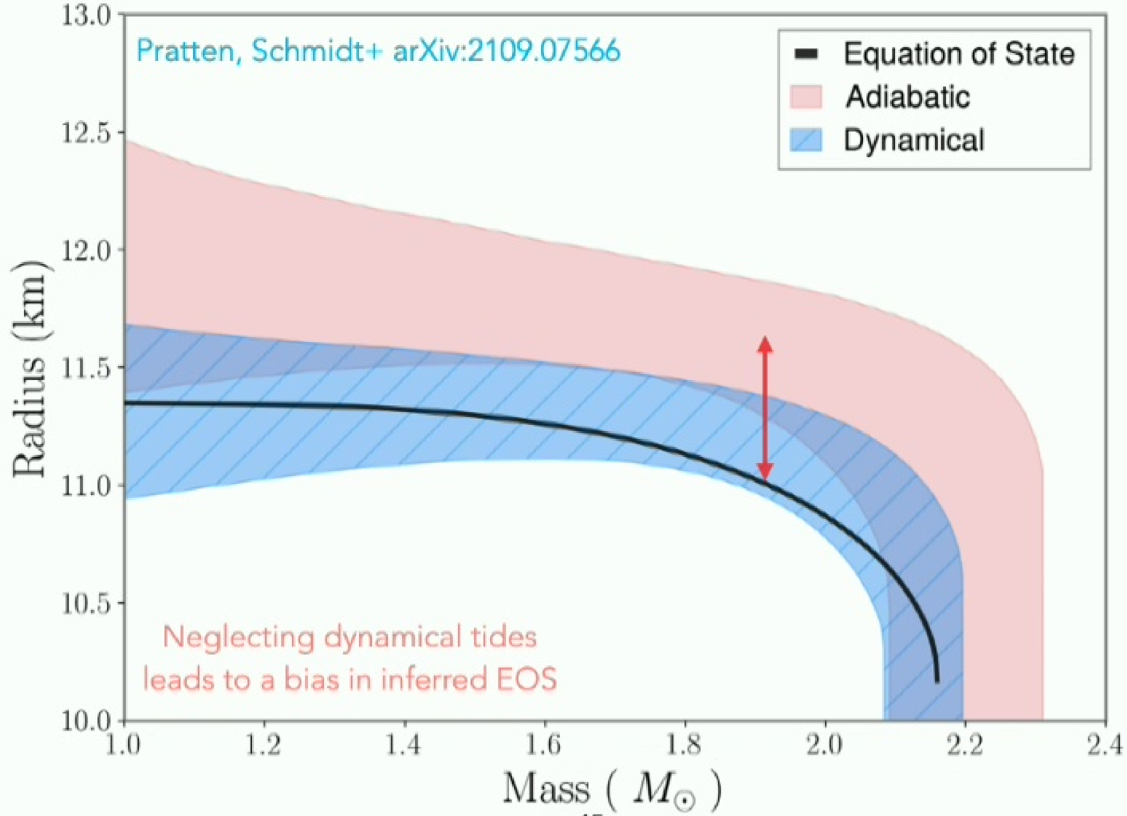
~ conditional prior ensuring masses supported by the EoS

See Lackey & Wade, arXiv:1410.8866

# Impact of Dynamical Tides on Future Inference?

UNIVERSITY OF BIRMINGHAM

- From posteriors for $E = \{p_1, \Gamma_1, \Gamma_2, \Gamma_3\}$ generate mass-radius curve
- Key message: the **adiabatic** results induce a bias in the inferred radius ~ $\mathcal{O}(0.5\mathrm{km})$




45

# Conclusion

UNIVERSITY OF BIRMINGHAM

- Inspiral of BNS is a playground full of rich spectral features and physical effects
- Significant advances on numerous fronts:
  - Modelling of **point particle** baseline
  - Higher order **adiabatic** tidal effects + extension to **dynamical** tides
  - **Spin** effects such as spin-tidal couplings or spin shifts
- Have not talked about: **post-merger**, microphysics, full spectrum of tides, magnetic fields, Universal relations, …
- **Systematics** will be **important** in future BNS observations
  - Sensitive to higher-order tidal effects e.g. dynamical tides

Thank You!