Title: Measure the cosmic expansion history of the Universe using GW sources

Speakers: Jonathan Gair

Collection: Gravitational Waves Beyond the Boxes II

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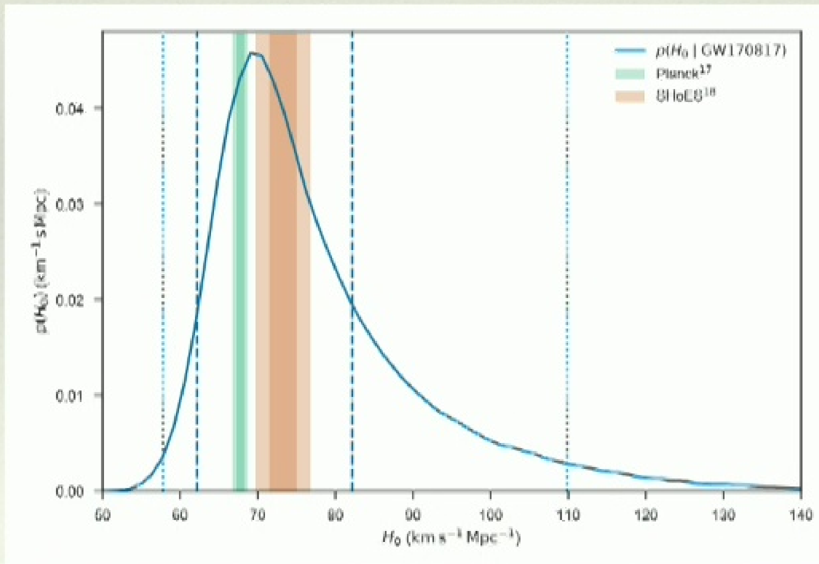
Measuring the cosmic expansion history using GW sources

Jonathan Gair, Albert Einstein Institute Potsdam

Gravitational Waves Beyond the Boxes II, Perimeter Institute, April 4th 2022



Talk outline

- Overview of cosmology with gravitational wave sources (counterparts, galaxy catalogue, mass function).
- Current results
  - Counterparts: GW170817, GW190521
  - Statistical/mass function: GW170817, GW170814, GWTC-1, GWTC-3
- Prospects for the future: 2G and 3G detectors, plus LISA.
- Sources of systematics in GW constraints on cosmology.

Cosmological models

- Standard cosmological model starts with homogeneous and isotropic line element

$$ds^2 = c^2 d\tau^2 = dt^2 - a^2(t) d\Sigma^2, \qquad d\Sigma^2 = \frac{dr^2}{1 - kr^2} + r^2 \left(d\theta^2 + \sin^2 \theta d\phi^2 \right)$$

- and stress-energy tensor of perfect fluid

$$T_{\mu\nu} = (\rho + p) u_\mu u_\nu + p g_{\mu\nu}$$

- Einstein's equations then yield the (Friedmann) equations

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} - \frac{\Lambda}{3} = \frac{8\pi}{3}\rho$$

$$2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} - \Lambda = -\frac{8\pi}{3}p$$

- The expansion rate $H = \dot{a}/a$ is called *the Hubble parameter* and its value today, *the Hubble constant*, is denoted H_0.

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The Hubble Constant

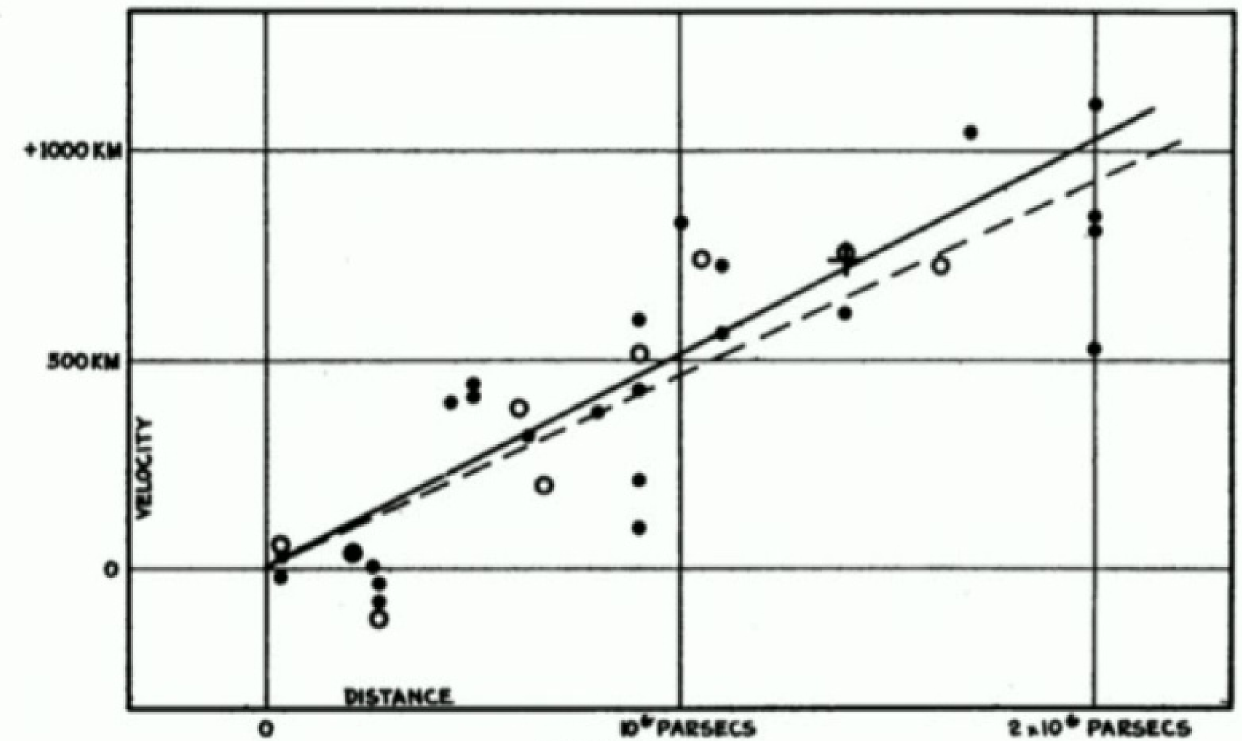
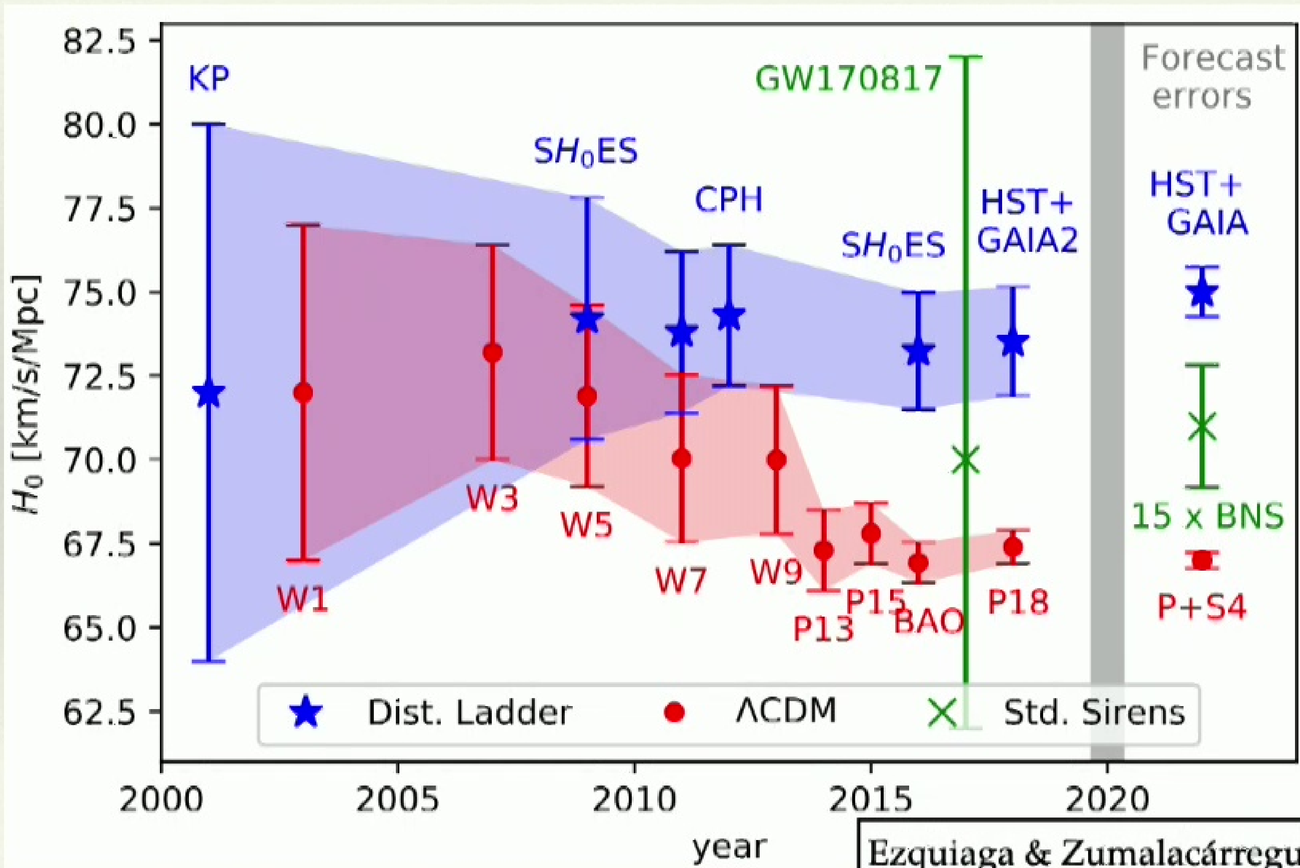


FIGURE 1

Velocity-Distance Relation among Extra-Galactic Nebulae.

Current H_0 Tension



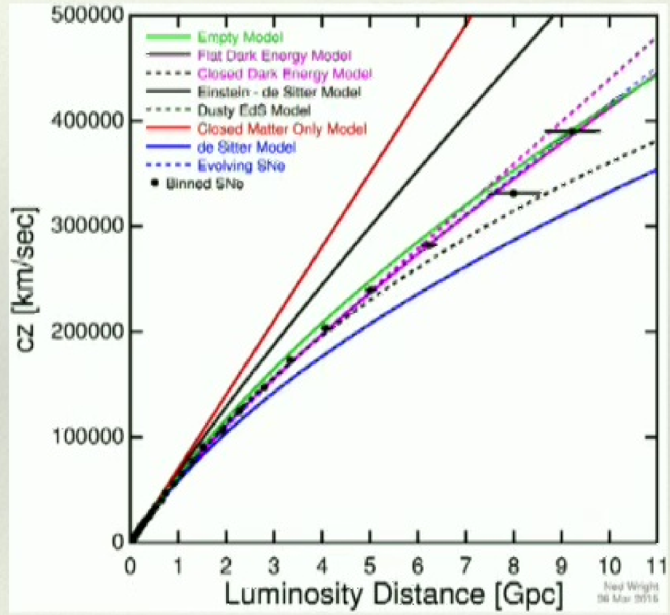
Ezquiaga & Zumalacárregui 2018

Standard Sirens

- Basic idea: gravitational wave strain scales as

$$h_{+,\times} \sim f_{+,\times}(\cos\iota)\frac{\mathcal{M}}{\mathcal{D}} \sim f_{+,\times}(\cos\iota)\frac{(1+z)M}{D_L(z)}$$

- Phase evolution determines intrinsic parameters (e.g., mass) to high accuracy. Amplitude then determines distance (Schutz 1986).
- If redshift can be obtained, we get a point on the D_L-z relationship
- Problem is to obtain redshift.



Plot from Betoule et al. (2004)

Obtaining redshift information

- Obtain a redshift either using EM information or assumptions about the source population
  - **Counterpart observations**: direct measurement of z from EM observation of the same source.
  - **Galaxy catalogues ("statistical method")**: use galaxy catalogues to identify possible host redshifts.
  - **Cross-correlation method**: see next talk.
  - **Mass-function assumptions**: assume knowledge of source mass, obtain redshift from measurement of $M(1+z)$.
  - **Tidal effects in BNS**: obtain source mass from tidal coupling parameter and universal relations (Messenger & Read 2011, Chatterjee+ 2021)

$$\bar{\lambda}(M) = \sum_{k=0}^{\infty} \frac{\bar{\lambda}_0^{(k)}}{k!} \left(1 - \frac{M}{m_0}\right)^k$$

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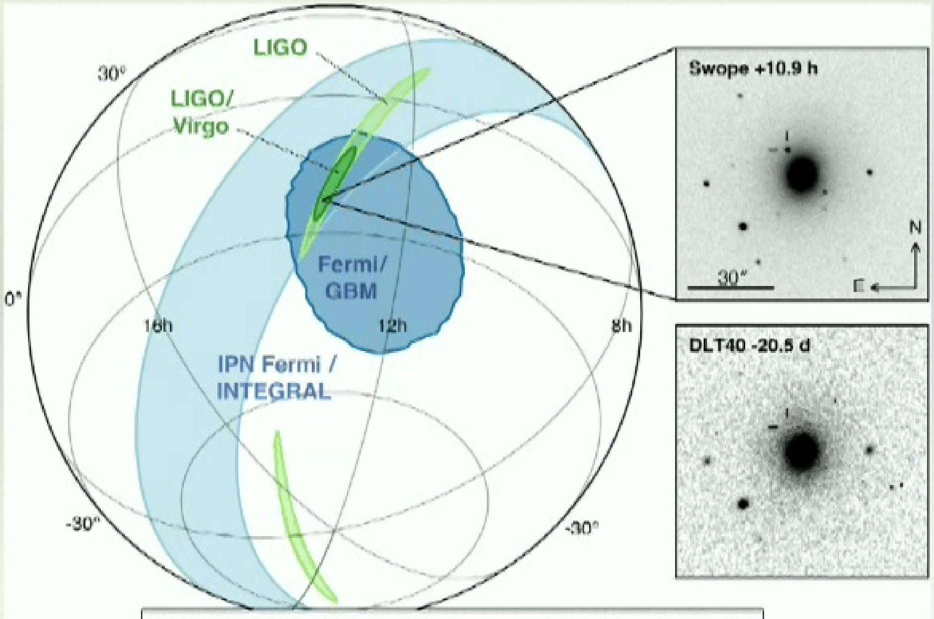
Counterpart measurements

- If the source redshift, z, can be obtained electromagnetically then the observed luminosity distance posterior, $p(d_L)$, can be converted to a posterior on H_0, $p(H_0) = (cz/H_0^2)\, p(cz/H_0)$ (low-z illustration).
- BNS are most likely CBC source with counterparts, e.g., kilonova for GW170817.
- The optical counterpart identified the host galaxy as NGC 4993, a galaxy in the constellation of Hydra at sky location ra=13h09m48s, dec = -23°22′53″.



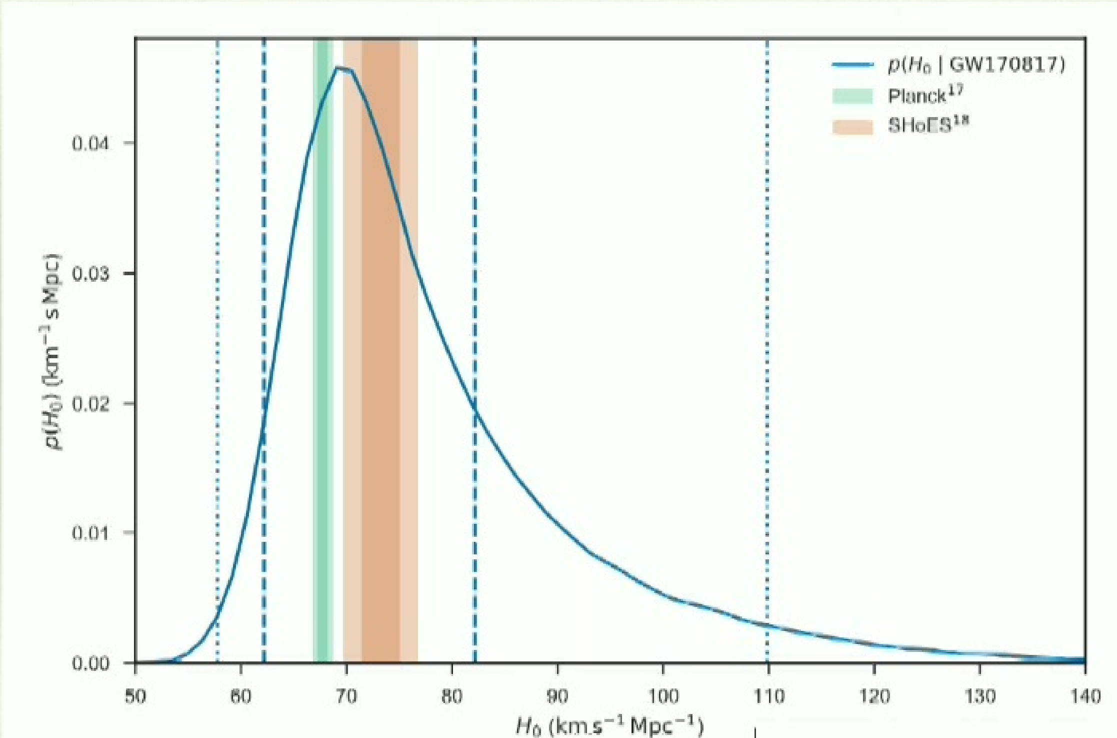
LVC+, *Astrophys. J. Lett.* **848** L12 (2017)

Hubble constant measurement with counterparts

- Redshift of NGC 4993, $v_{\rm rec} = 3327 \pm 72\,\mathrm{km\,s^{-1}}$, and GW distance of GW170817, $d = 43.8^{+2.9}_{-6.9}\,\mathrm{Mpc}$, gives $H_0 = v_{rec}/d = 76.0$ km s^{-1} Mpc^{-1}.
- There are two potential complications:
  - 1) selection effects in gravitational wave and electromagnetic measurements. Our detectors have finite sensitivity so the sample of GW events with counterparts is incomplete. At the time of GW170817 we were GW-selection dominated and so the cosmology-dependence of this selection **was negligible.**
  - 2) NGC 4993 is sufficiently close that it has a significant peculiar velocity. This can be corrected using measurements of the peculiar velocity field. For GW170817, estimated $v_p = 310 \pm 150\mathrm{km\,s^{-1}}$.

Hubble constant measurement

- Final result is $H_0 = 70.0^{+12.0}_{-8.0}\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$



LVC+, *Nature Lett.* **551** 85 (2017)

Statistical H_0 measurements

- General form of likelihood is

$$p(H_0|\mathbf{d}^{GW}, \mathbf{d}^{EM}, \mathrm{det}) \propto \frac{p_h(H_0)}{\mathcal{N}_s(H_0)} \int p(\mathbf{d}^{GW}, \mathbf{d}^{EM}|z_t, H_0, \vec{\lambda}) p_z(z_t) \mathrm{d}z_t \mathrm{d}\vec{\lambda}$$

- If do not have EM data, prior on $p_z(z_t)$ important. Inference requires combining multiple observations, using

$$p(H_0|\left\{\mathbf{d}^{GW}, \mathrm{det}\right\}) \propto \frac{p_h(H_0)}{(\mathcal{N}_s(H_0))^{N_{\mathrm{obs}}}} \prod_{i=1}^{N_{\mathrm{obs}}} \left[\int p_z(z_t) p(\vec{\lambda}) p_{GW}(\mathbf{d}_i^{GW}|cz_t/H_0, z_t, \vec{\lambda}) \mathrm{d}\vec{\lambda} \mathrm{d}z_t\right]$$

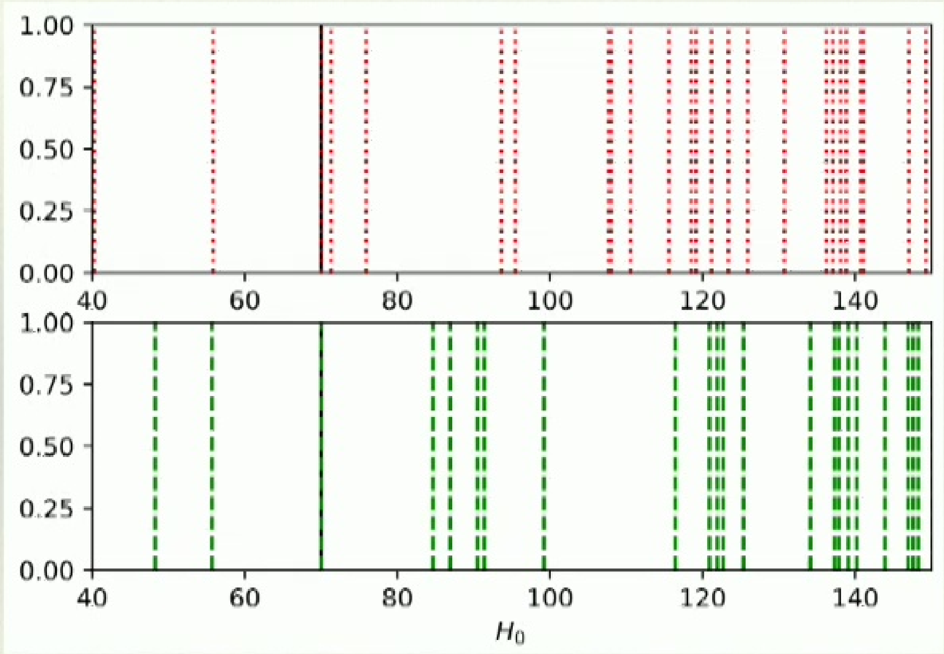
- Can construct $p_z(z_t)$ from a galaxy catalogue, but need to handle incompleteness of catalogue.

$$p_z(z) = p_{\mathrm{cat}} \sum_i \delta(z - z_i) + \frac{1}{V_c(z_{\mathrm{max}})}(1 - f(z|H_0)) \frac{\mathrm{d}V_c}{\mathrm{d}z}$$

$$f(z|H_0) = \int_{M_{\mathrm{th}}(z, H_0, m_{\mathrm{th}})}^{\infty} p(M|I) \mathrm{d}M$$

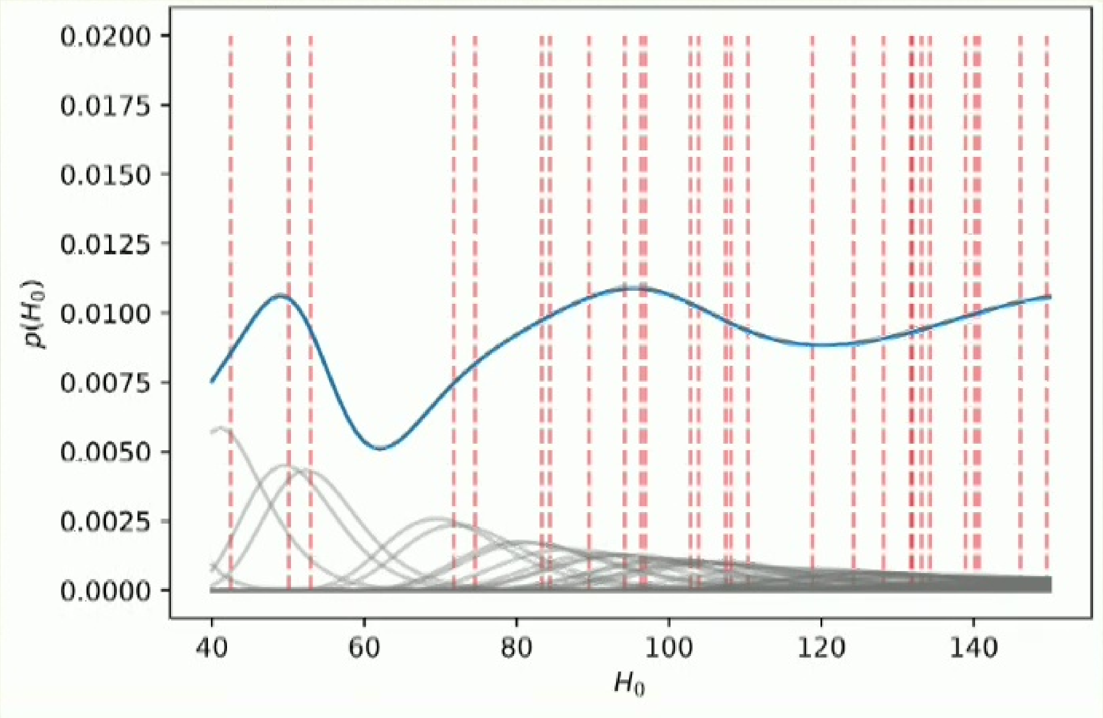
How does the statistical method work?

- Assuming $p(z) \sim z^2$, $p(d_L) \sim d_L^2$ for any cosmology -> no information.
- **But** the Universe is clumpy - galaxies have discrete redshifts.
- In the absence of measurement errors, one event gives a set of possible $H_0 = cz_i/d_L$ values. A second observation gives another set. Exactly one value will agree.



How does the statistical method work?

- Measurement errors smear things out, so individual events are not informative. But, there will be a feature at the true H0, which will accumulate as events are combined.



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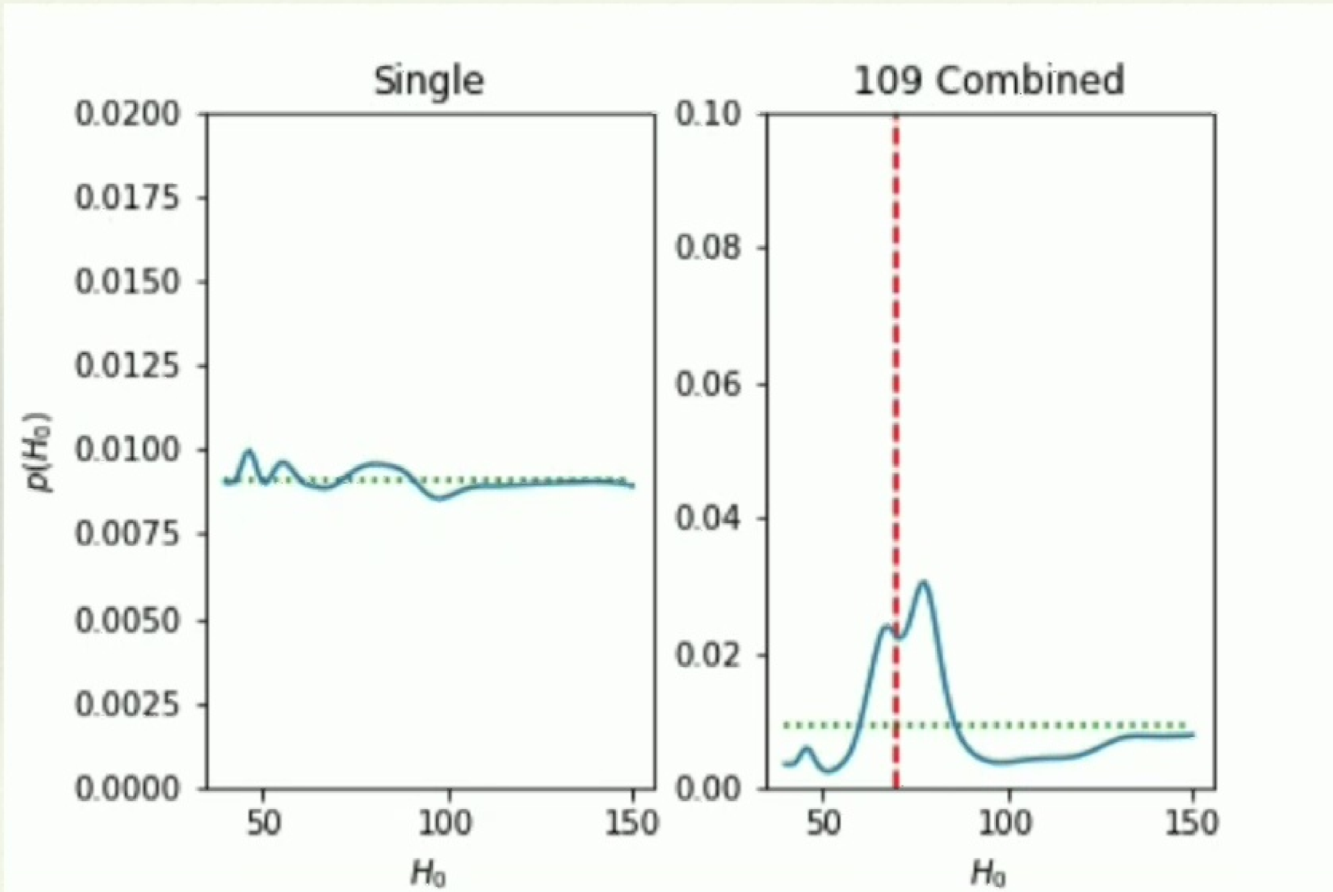

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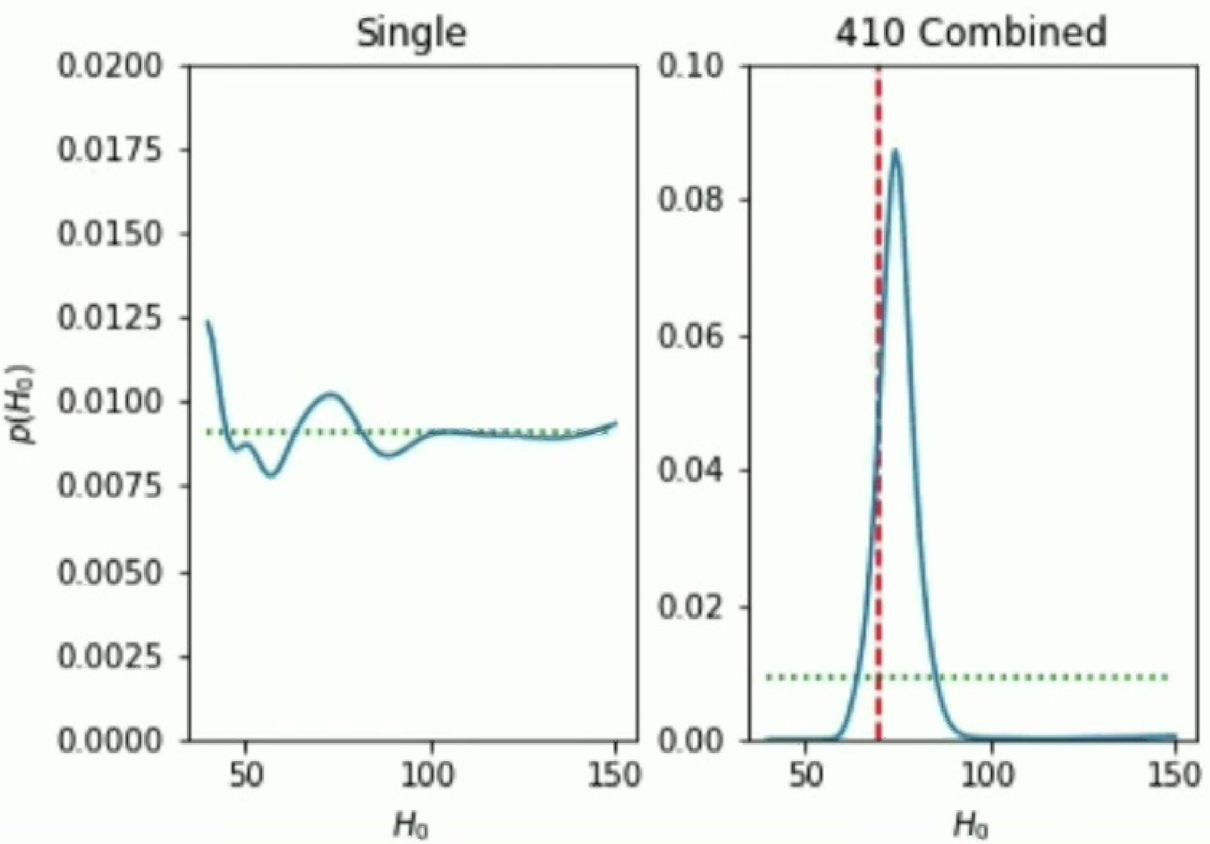

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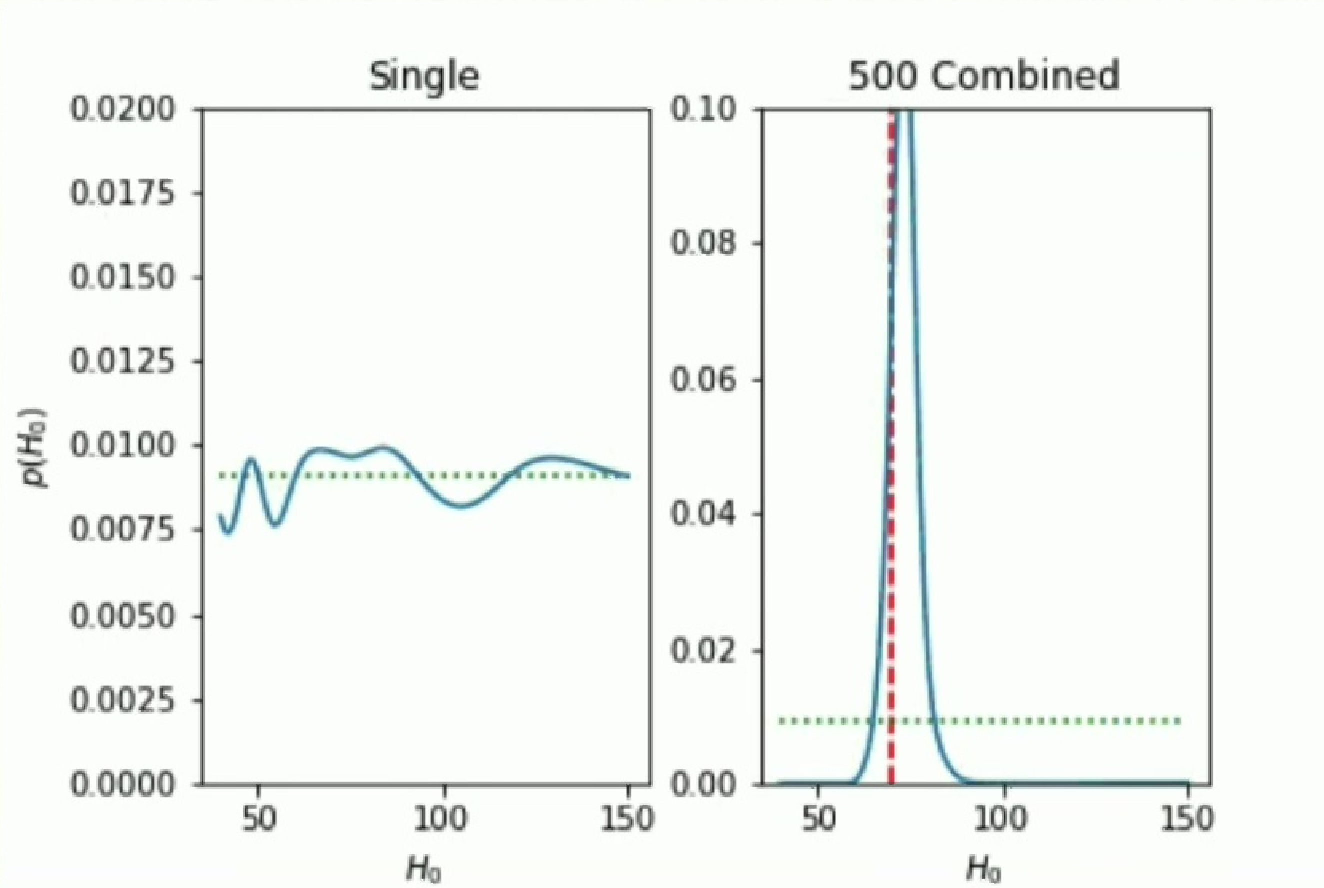
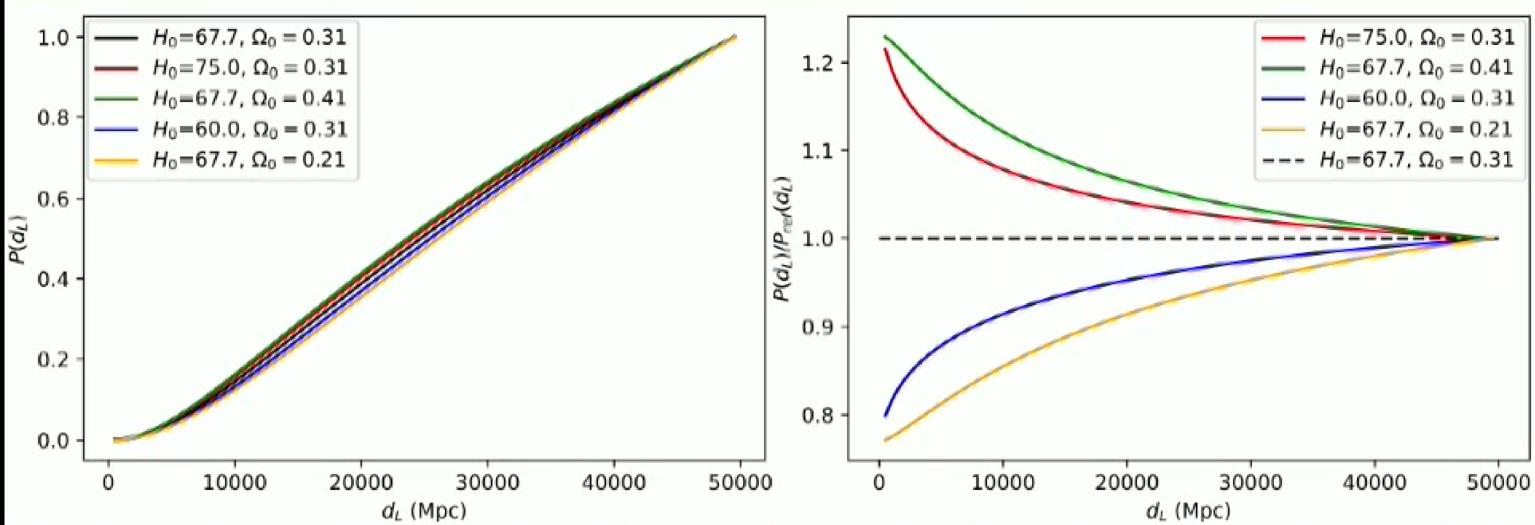


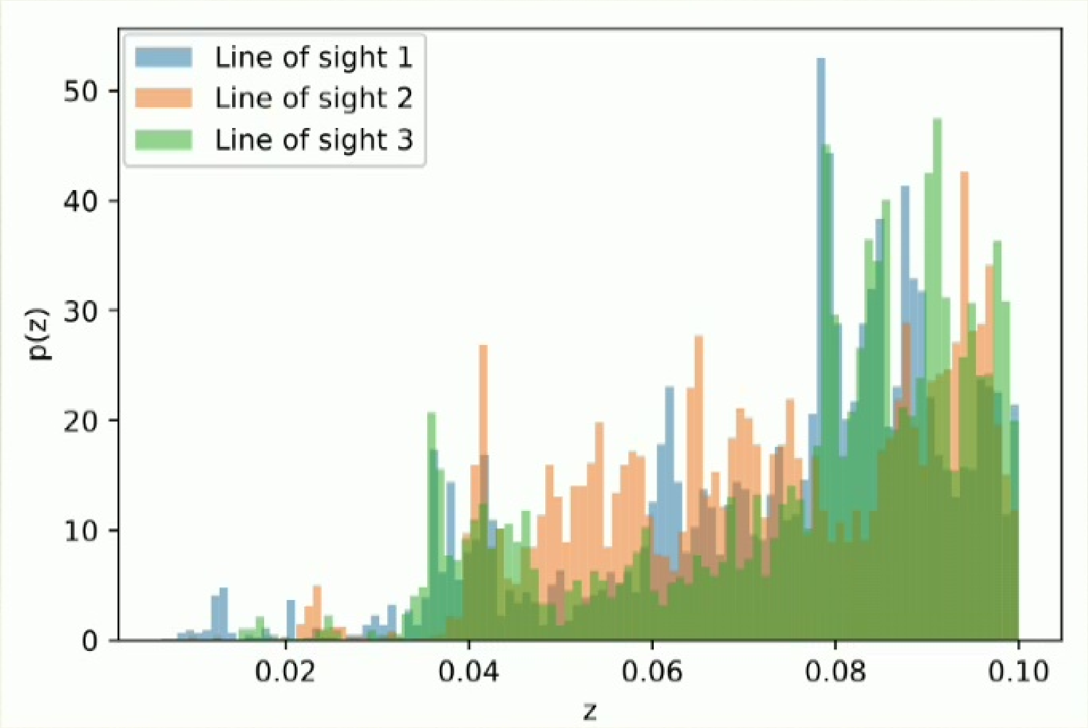
How does the statistical method work?

- Various other effects help to accelerate convergence.
- At higher redshift, deviations of cosmology from a simple power-law become important.
- Finite range in d_L then provides a handle on cosmological parameters.



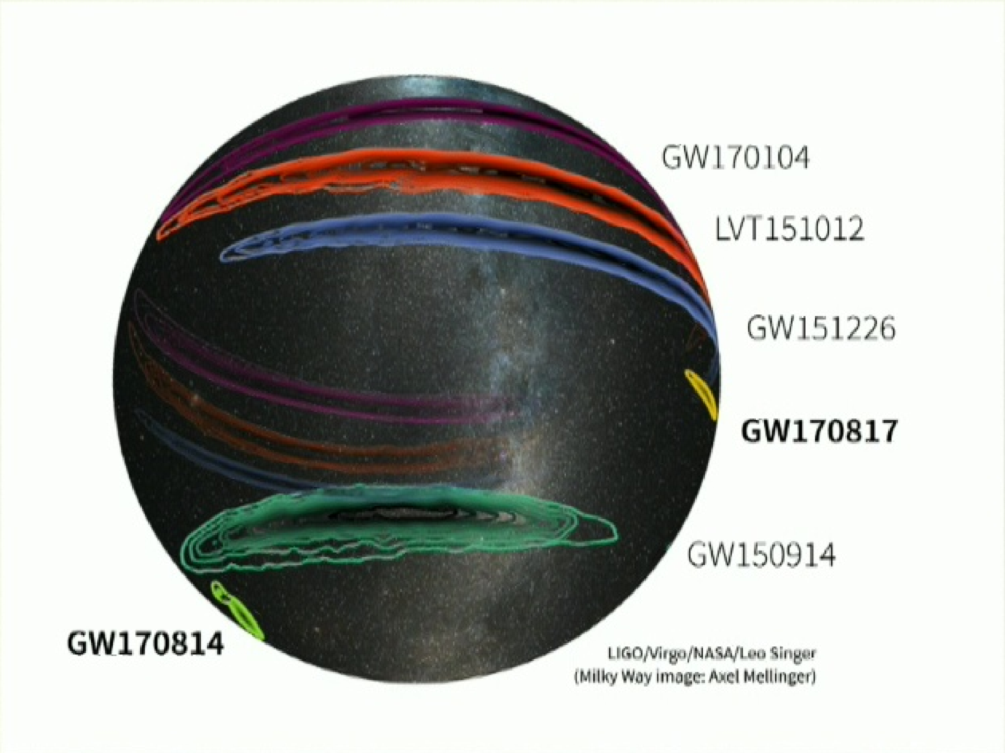
How does the statistical method work?

- Galaxies tend to cluster which makes features in redshift distribution more distinct.



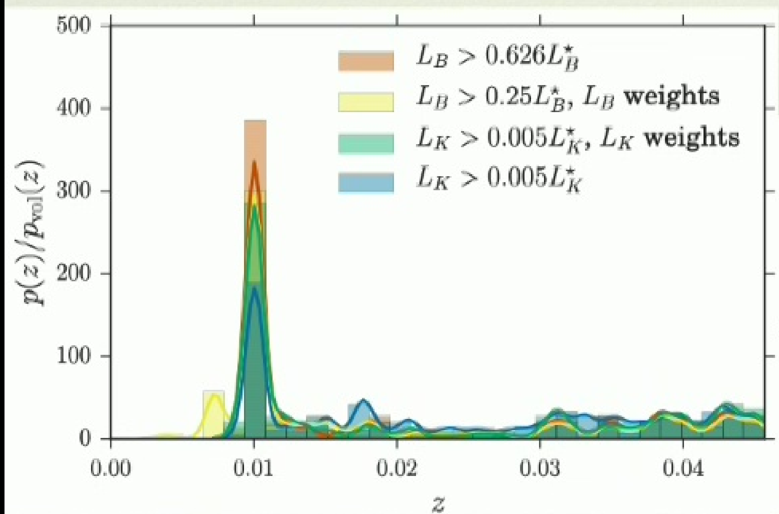
Statistical H_0 measurements

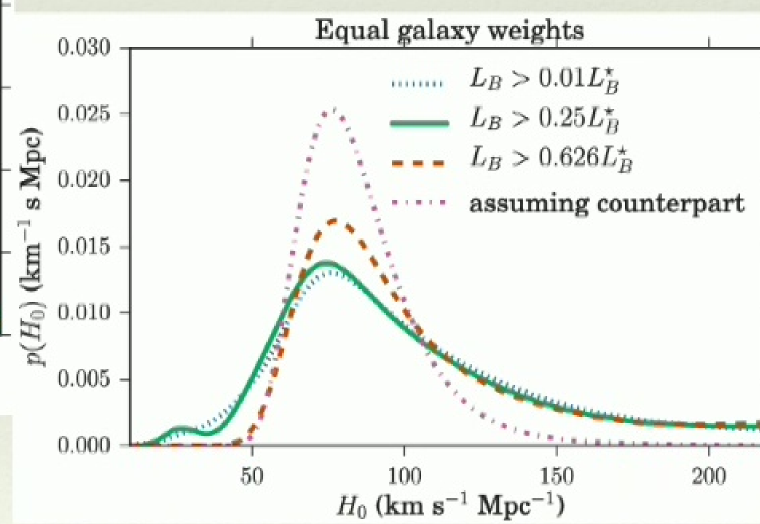
- Need small uncertainties for maximum power - events after Adv. Virgo came online.



First statistical measurements: GW170817

- Carried out proof of principle of this measurement using GW170817. More informative than average since very close, but statistical measurement weaker than counterpart measurement.

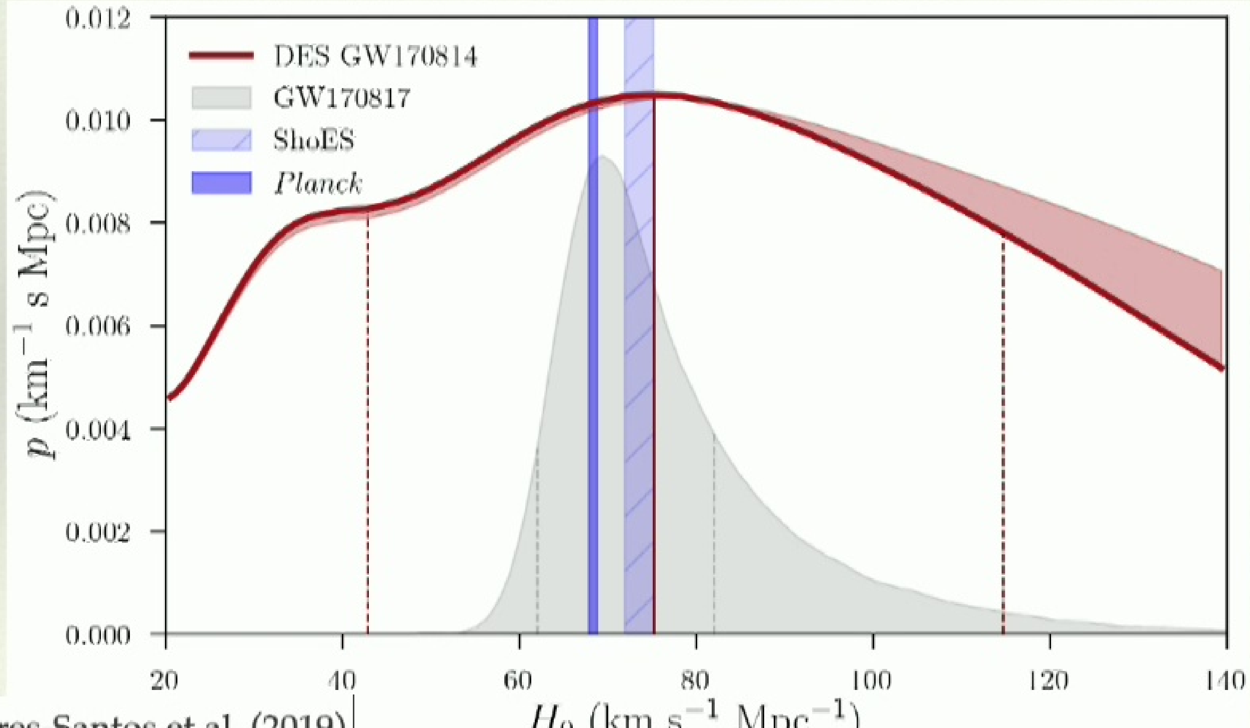




Fishbach, …, **JG**+ (2018)

Statistical measurements: GW170814

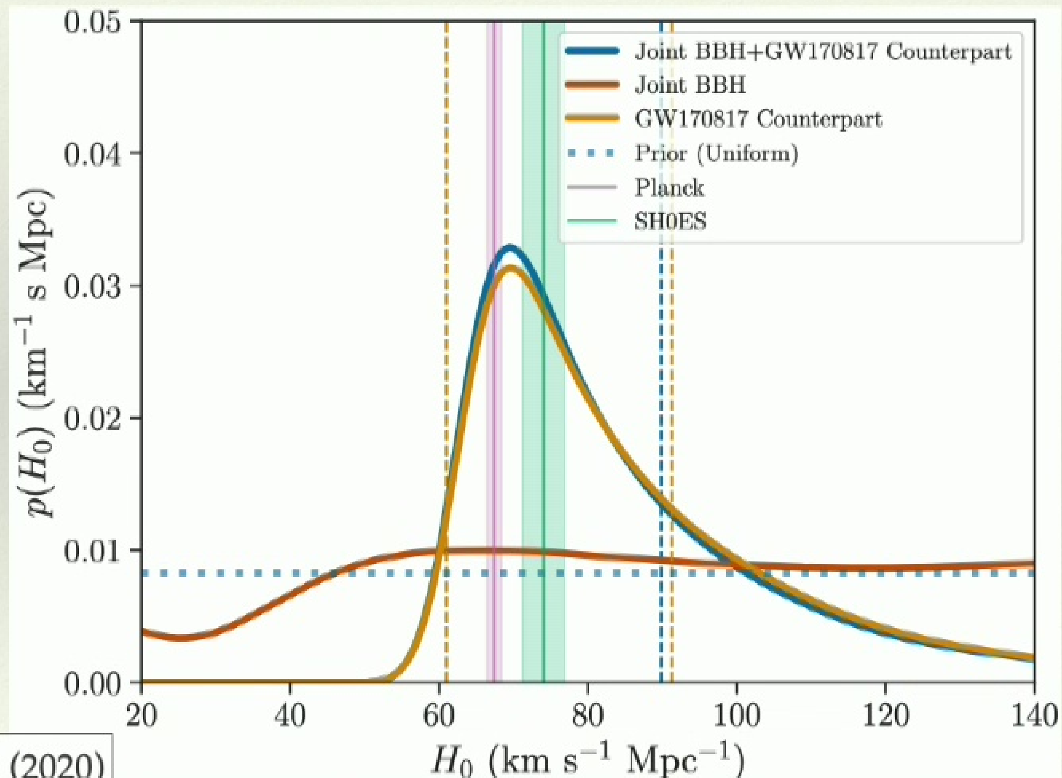
- Also applied to GW170814 by DES collaboration + LVC. First measurement using a dark siren.



Soares-Santos et al. (2019)

Statistical measurements: O1+O2

- Result from O1+O2 was ~10% tighter than from GW170817 alone.



Abbott et al. (2020)

H_0 measurements: mass function

- Another approach: assume mass function has sharp features. For example, BNS component masses have narrow distribution (Taylor+ 2011)

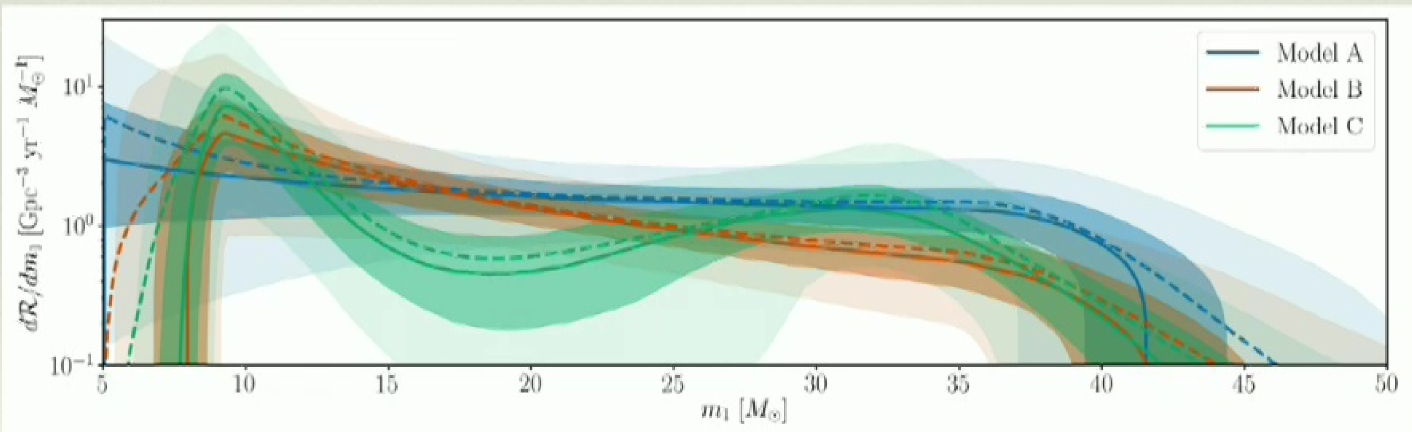
$$m_i \sim N(\mu, \sigma^2)$$

- Then precisely measured $M_z = M(1+z)$ determines z.
- Advantage - no completeness problem (posterior dies away to large H_0 values), no need for collection of expensive galaxy catalogues.
- Disadvantage: relies on the NS mass function being narrow enough, and on having a reliable model for it.
- There are large uncertainties at low redshift

$$M_z = m(1+z) \Rightarrow \Delta z \approx \frac{\Delta(1+z)}{1+z} = \frac{\Delta M_z}{M_z} + \frac{\Delta m}{m} = \frac{\Delta M_z}{M_z} + \frac{\sigma}{\mu}$$

H_0 measurements: mass gap

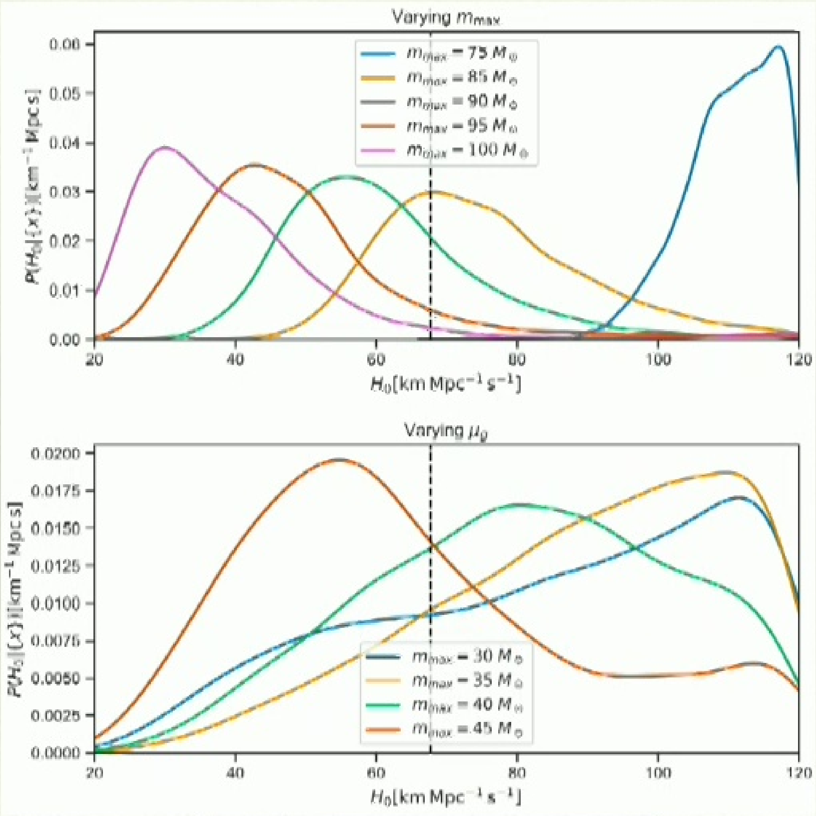
- Although the BNS mass function may not be narrow enough, the existence of the *pair instability mass gap* provides a feature in the BBH mass distribution that can be used in a similar way (Farr+ 2019).



LVC *BBH properties from GWTC-1* (2019)

Mass function measurements: GWTC-1

- With a fixed power law+Gaussian population model, the measurement of H_0 using GWTC-2 is informative.
- However, it is sensitive to what model is assumed.
- Need to simultaneously fit the population and cosmology to get robust constraints.

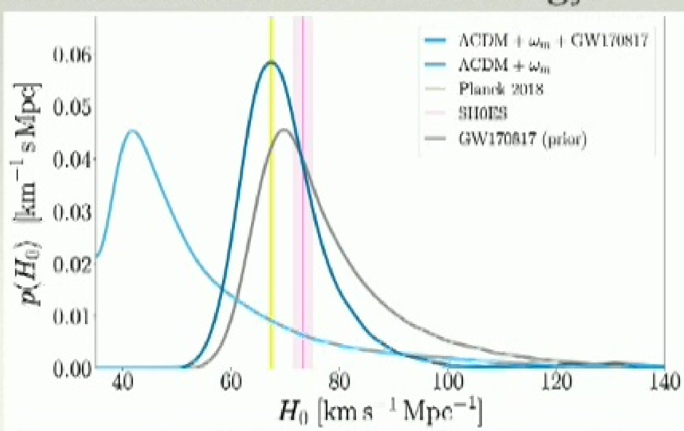


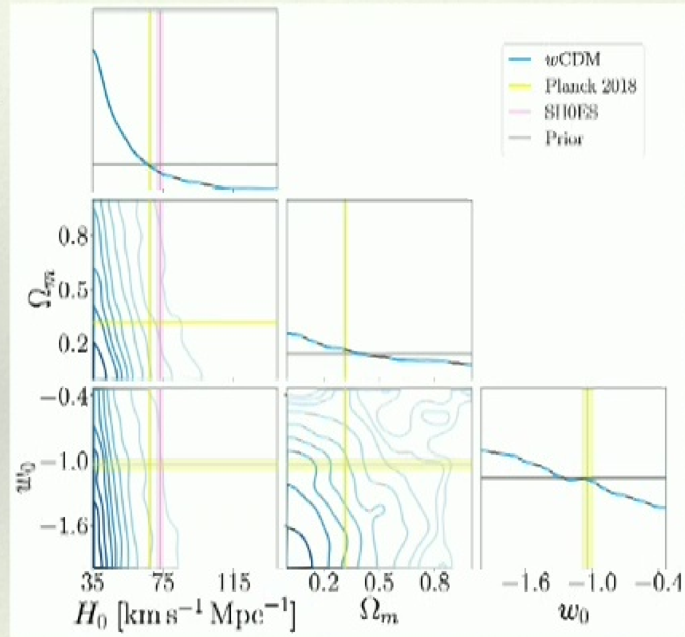
Mastrogiovanni et al. (2021)

Current results

Counterpart (?) measurements: GW190521

- During O3 a high-mass BBH event, GW190521, was observed at $d_L \sim 6$ Gpc.
- The Zwicky Transient Facility reported a possible association with an electromagnetic transient, *ZTF19abanrhr*.
- Assuming this association is real, this event can be used for cosmology.





Chen+ (2021)

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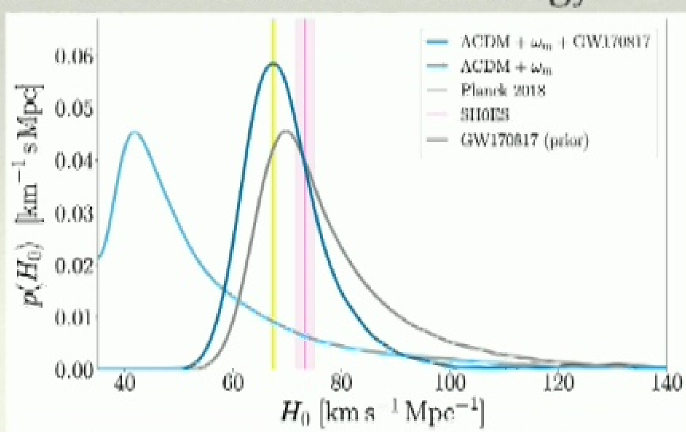
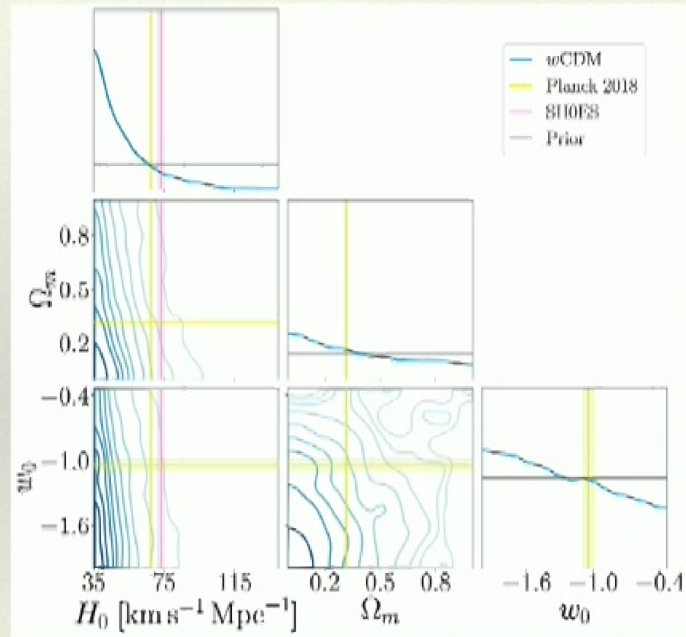
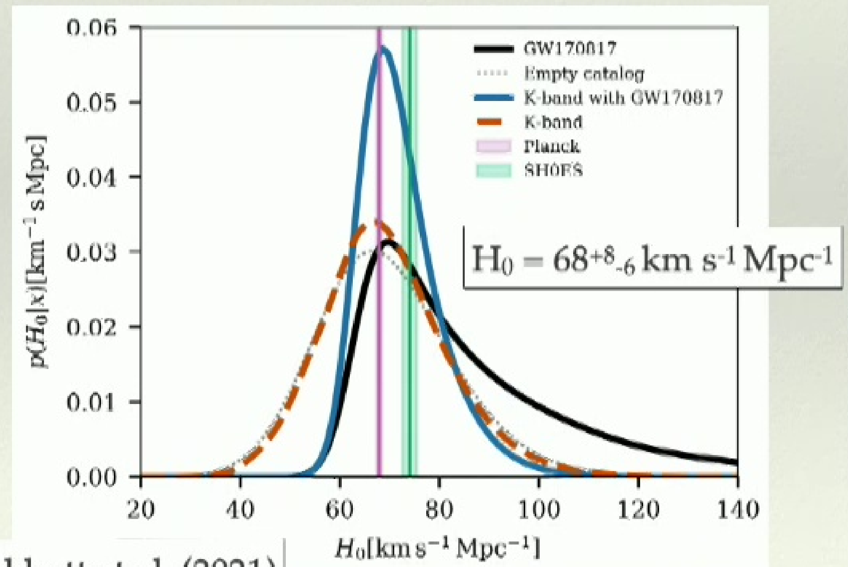




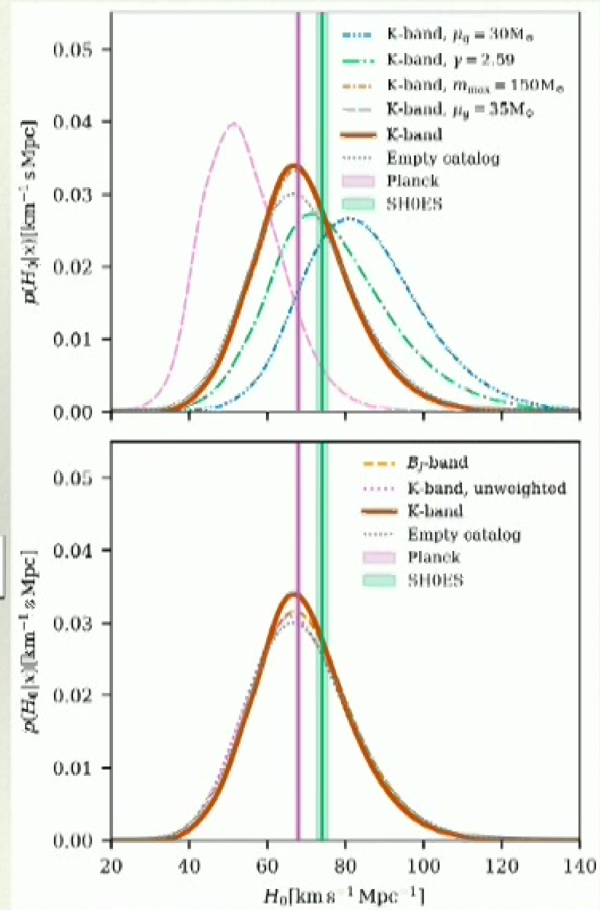
Chen+ (2021)

Statistical measurements: O3

- In O3, LVK published an updated result based on two assumptions. The first used galaxy catalogues, with a fixed population model. This was a ~40% improvement over GWTC-1.

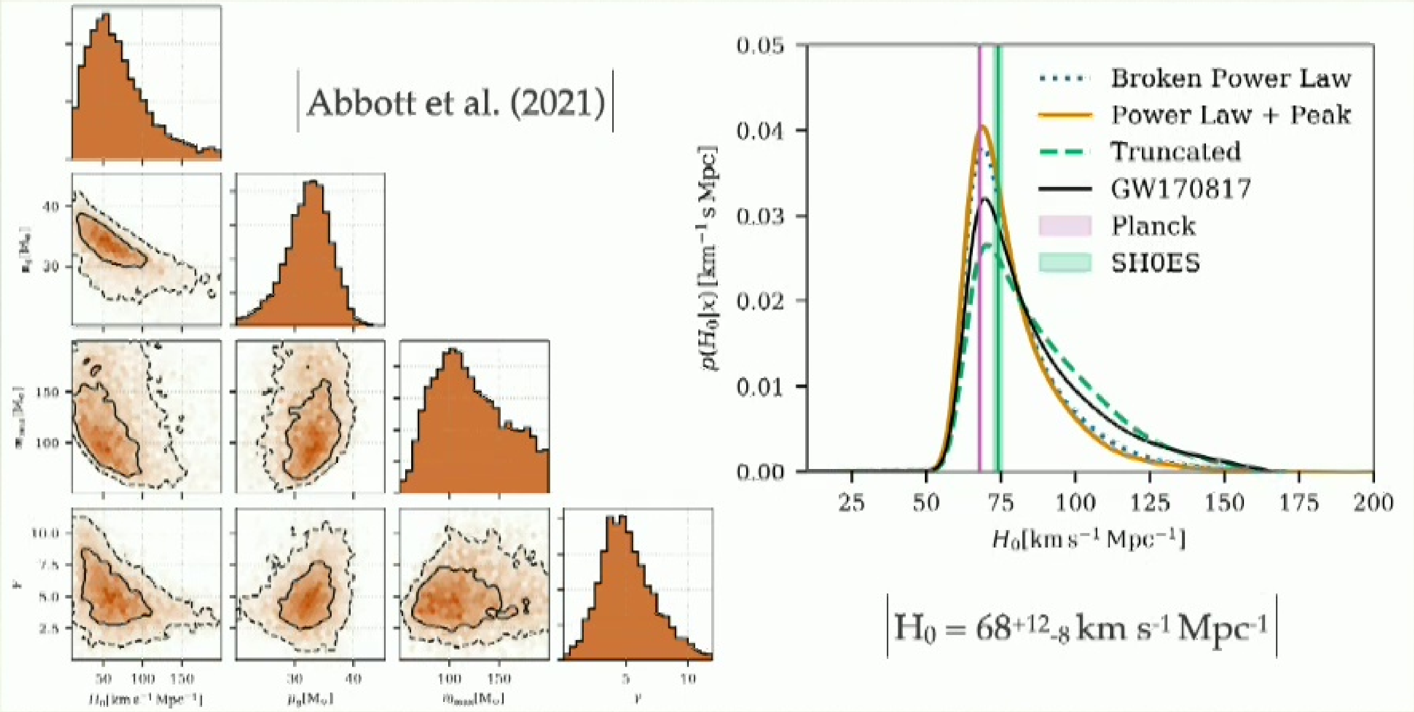


Abbott et al. (2021)



Mass function measurements: O3

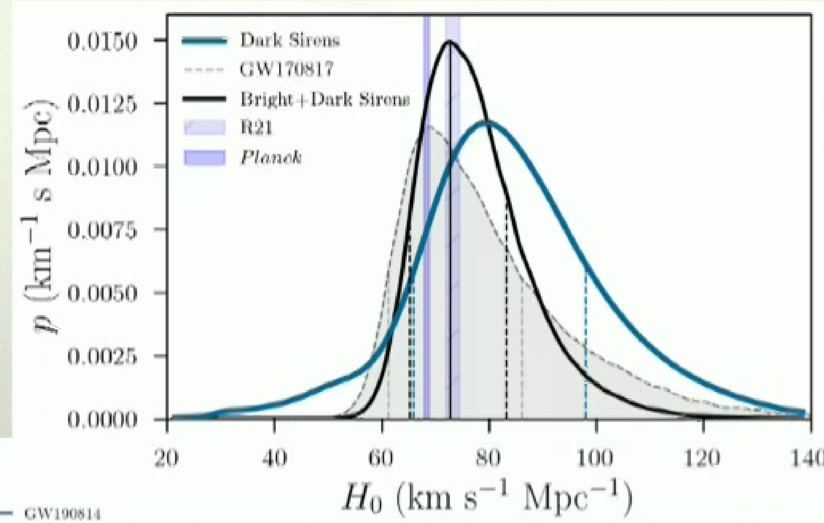
- The second LVK O3 analysis used an empty catalogue and simultaneously fitted a power law + Gaussian population model.

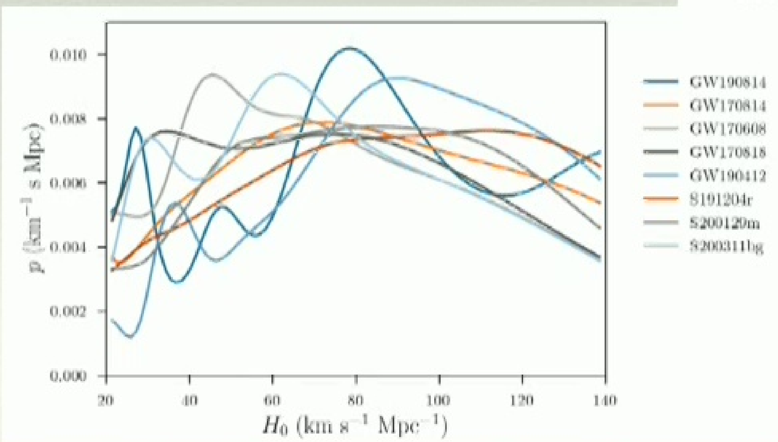


$H_0 = 68^{+12}_{-8}$ km s^{-1} Mpc^{-1}

Other statistical measurements in O3

- Using DES observations for several of the best-localised BBHs, Palmese et al. obtained a comparable result using only 8 of the O3 events.





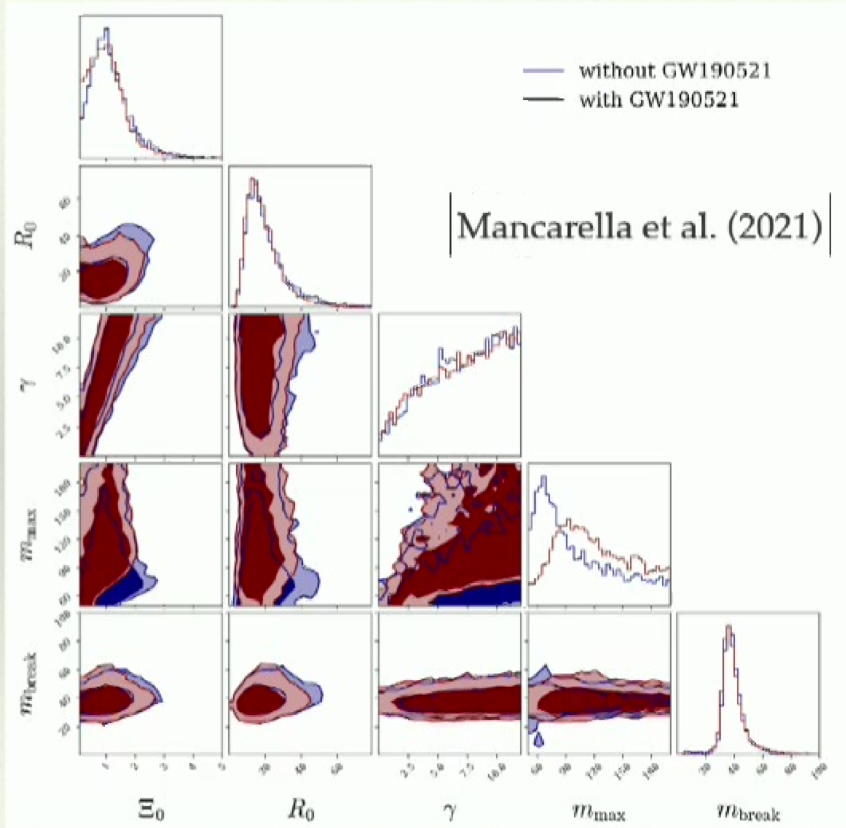
$H_0 = 72.77^{+11}_{-7.55}$ km s^{-1} Mpc^{-1}

Palmese et al. (2021)

Other statistical measurements in O3

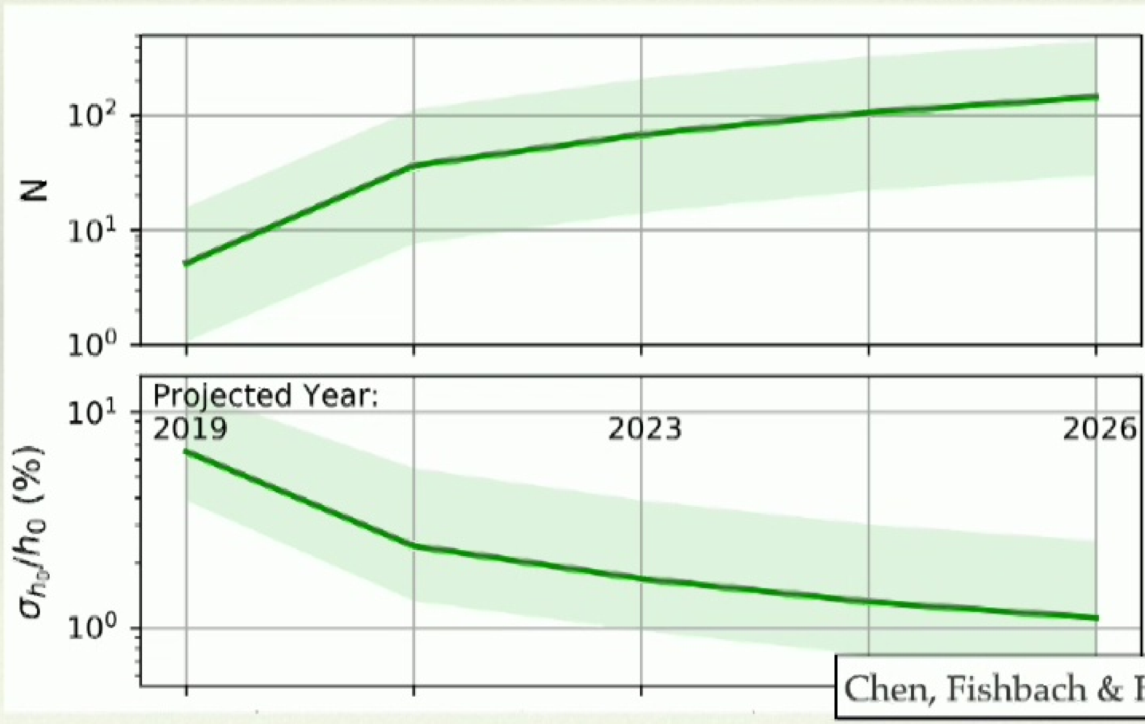
- Mancarella et al. combined the GWTC-3 observations with Planck cosmological measurements to place constraints on modified GW propagation.

$$\frac{d_L^{\rm gw}(z)}{d_L^{\rm em}(z)} \equiv \Xi(z) = \Xi_0 + \frac{1-\Xi_0}{(1+z)^n}$$



Future H_0 measurements: counterparts

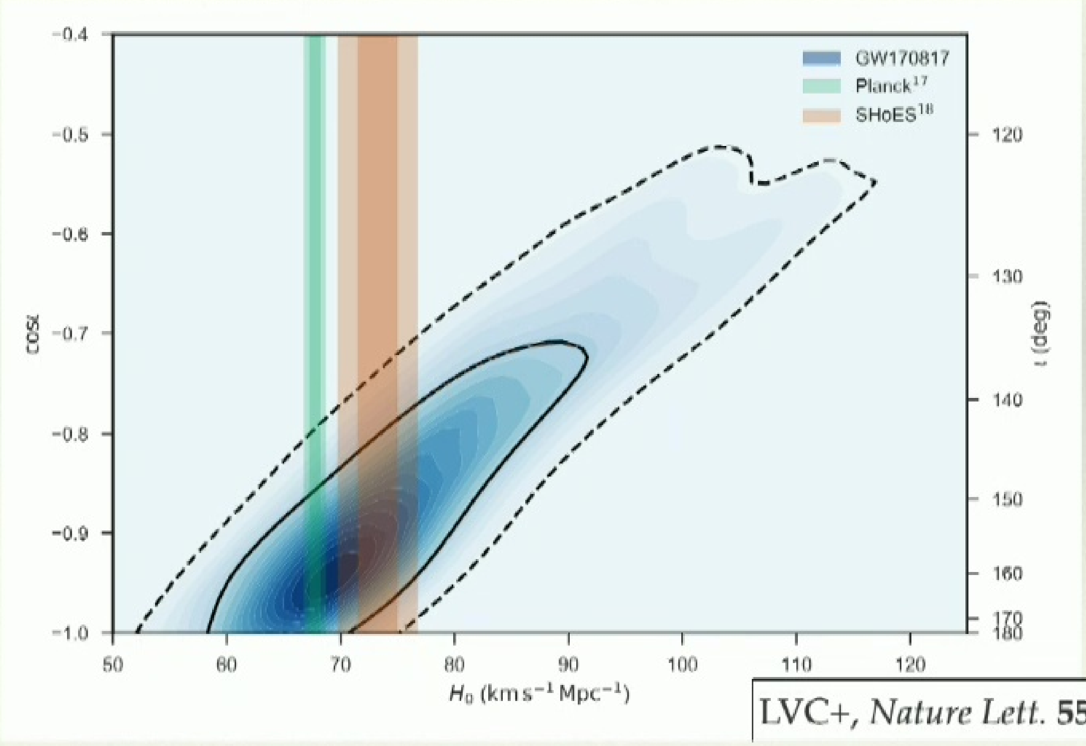
- 2% measurement achievable with ~60 events, 1% with ~240 events. Addition of KAGRA and LIGO India improves precision per event by ~15%.



Chen, Fishbach & Holz (2018)

H_0-inclination degeneracy

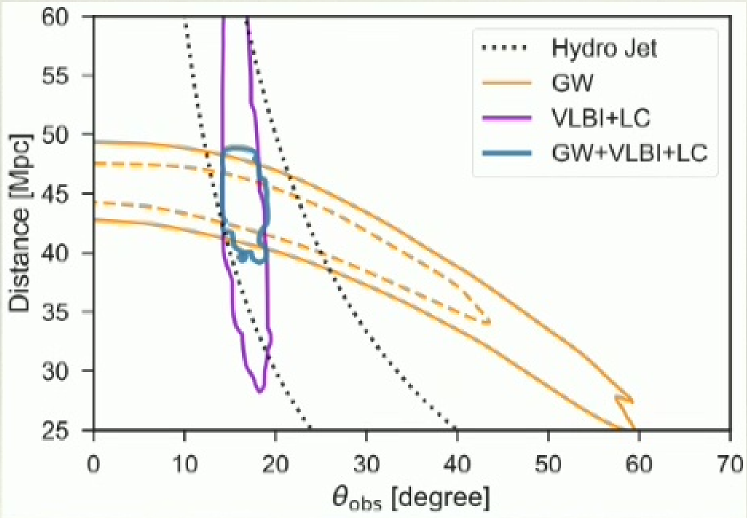
- Uncertainty in measurement largely driven by degeneracy between distance and inclination of the source.



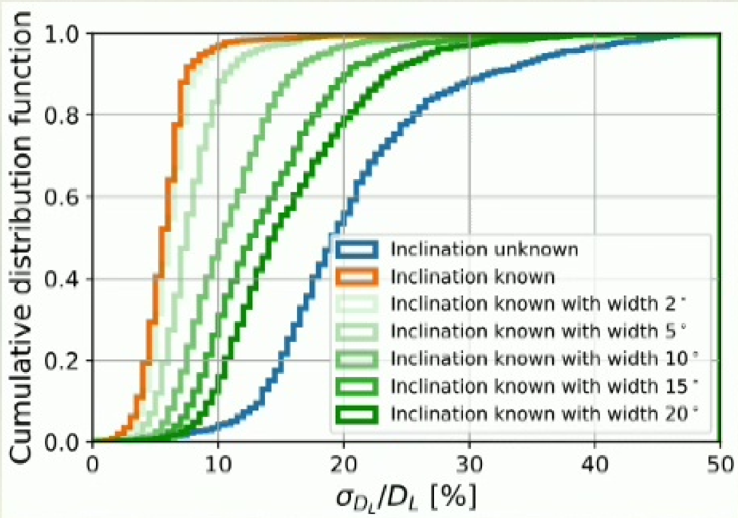
LVC+, *Nature Lett.* **551** 85 (2017)

H_0-inclination degeneracy

- EM observations can break d_L-i degeneracy, proving 2-3 times better precision on individual events -> 1% measurement with ~50 events.



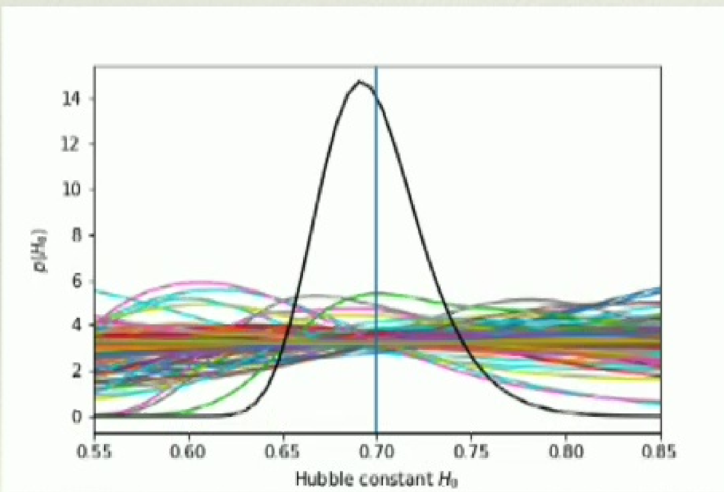
Hotokezaka+ (2019)

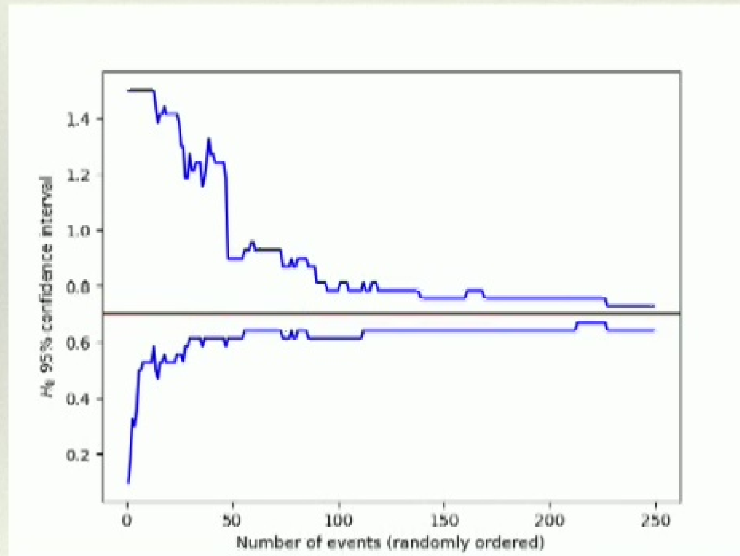


Chen, Vitale & Narayan (2019)

Future H_0 measurements: statistical

- Assuming complete catalogues and nearby events, could reach a precision of ~3km s^{-1} Mpc^{-1} with 250 events. This would be achieved with ~15 counterpart events.

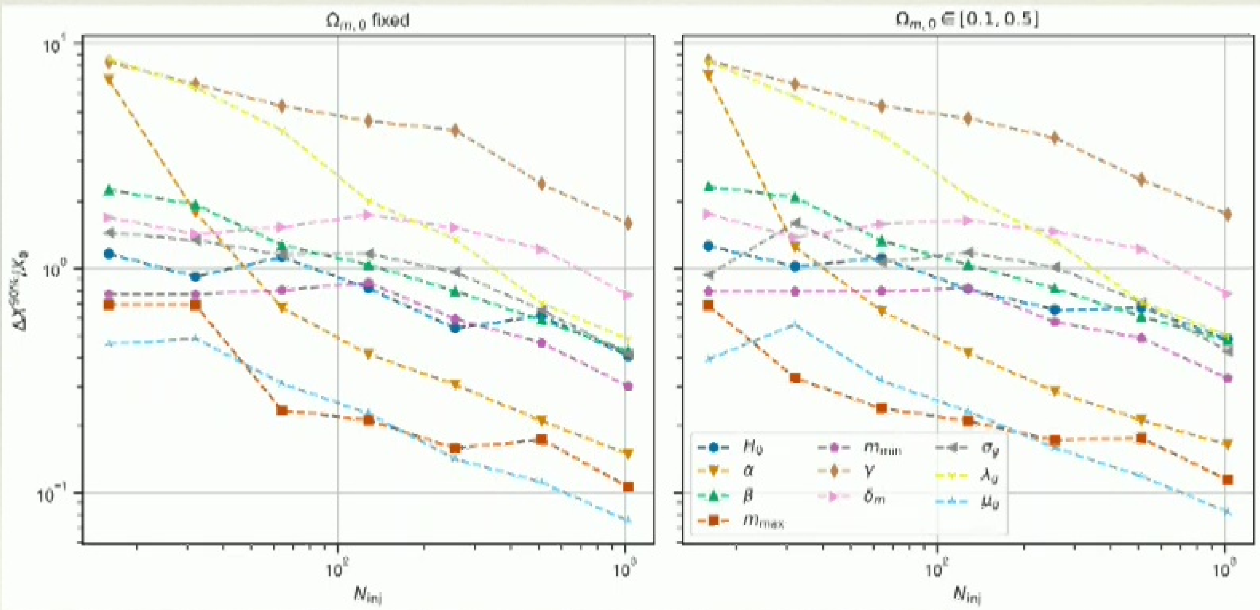




Gray, …, **JG**, et al. (2020)

Future measurements: mass gap

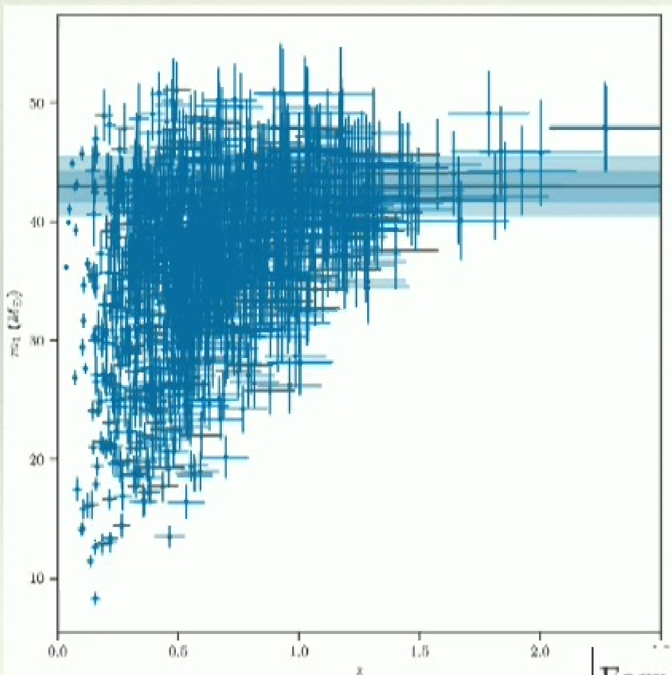
- Without catalogues, and assuming the BBH mass function is a power-law + Gaussian, to achieve the same precision (a few percent) would require several 1000 observations of BBHs at design sensitivity.

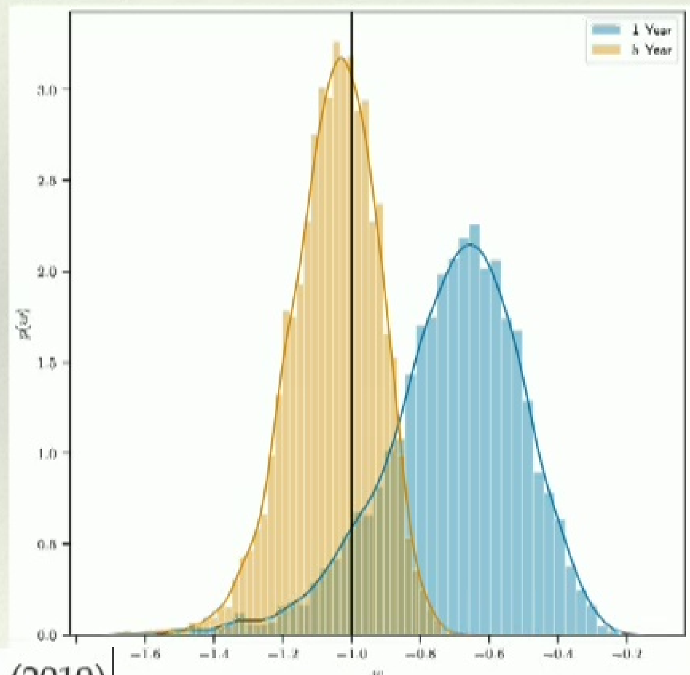


Mastrogiovanni et al. (2021)

Future measurements: mass gap

- But, dark sirens observed at higher redshift are better at placing constraints on other cosmological parameters. 3G detectors could measure the dark energy EoS to 10% in 10 years of observations.

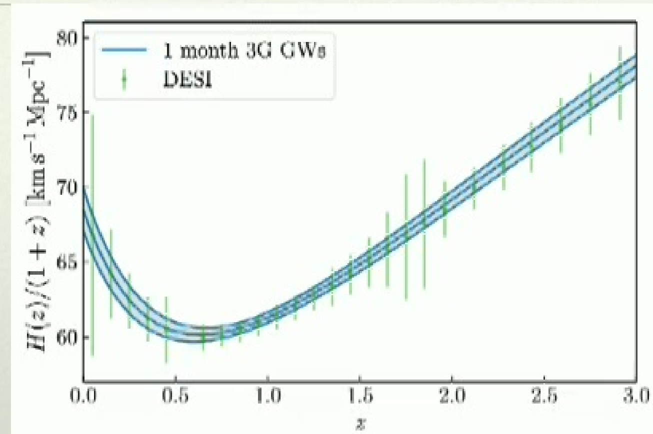


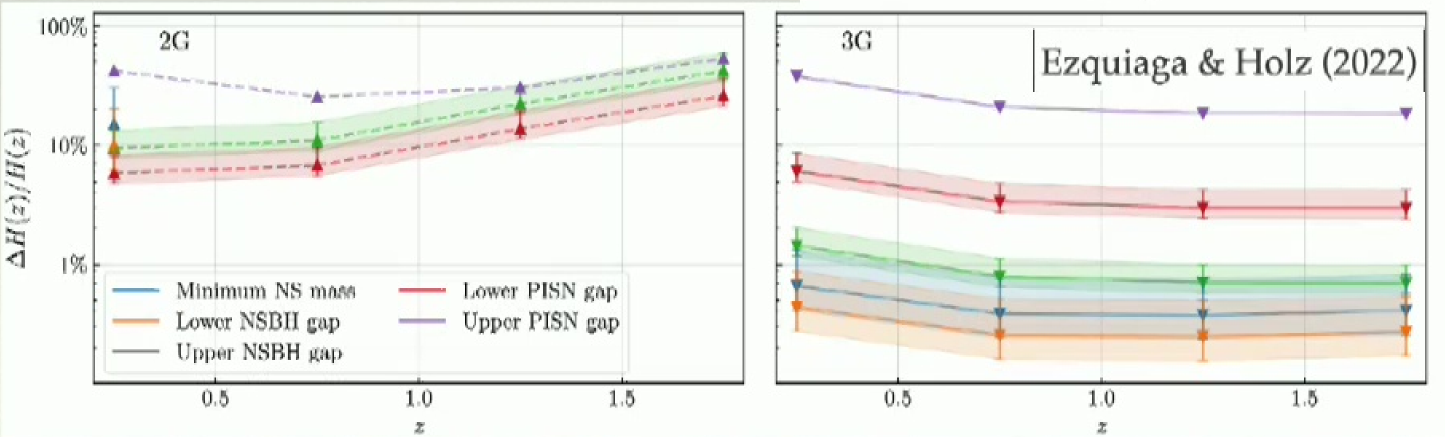


Farr et al. (2019)

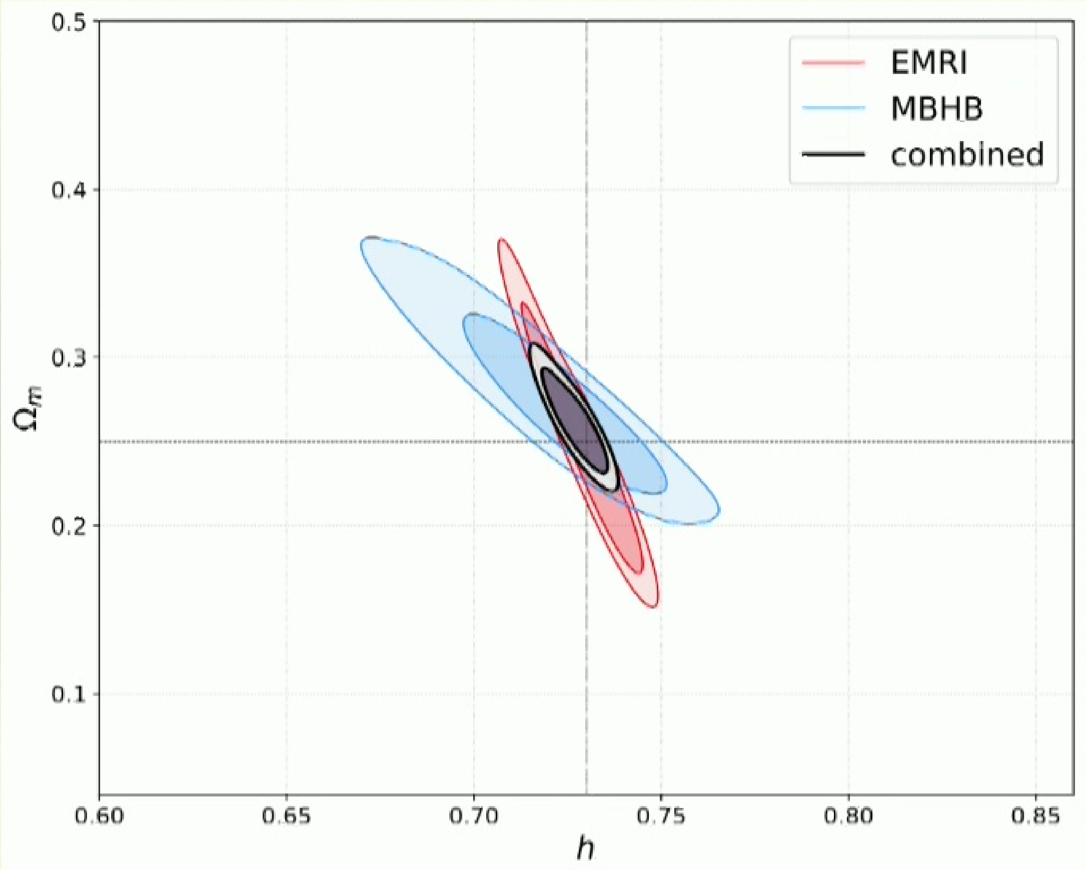
Future measurements: mass gap

- The *upper mass gap* dominates constraints from 2G detectors, but information from the *lower mass gap* between NS and BH, could dominate for 3G.
- Expect constraints to be competitive with contemporary EM probes.





Future H_0 measurements: LISA



- LISA will offer comparable sensitivity with fewer events, and probe a wide range of redshifts.

Laghi+ (in prep.)

Prospects for the future: which method will "win"?

Method	Pros	Cons
Counterpart	Clean measurement. Systematics well understood.	Event rate highly uncertain.
Galaxy catalogue	Can use all sirens - high event rate. Do not rely on intrinsic properties of population.	Lots of systematics need to be controlled. Limited by survey completeness.
Mass function	No external data needed.	Relies on intrinsic population having sharp mass features, and correctly modelling them. Need to make assumptions about population evolution.

General statistical framework

- Considering all EM and GW observations together, general framework can be written

$$
\begin{aligned}
&p(\vec{\theta}|\{\mathbf{d}_{\rm hist}^{EM,{\rm obs}}\},\{\mathbf{d}^{GW,{\rm obs}}\},\{\mathbf{d}^{EM,{\rm obs}}\}) \\
&\quad\propto p(\vec{\theta})\sum_{N_{\rm gal}}\int\left\{\prod_{i=1}^{N_{\rm obs}}\left[\int\left(\sum_{g_i=1}^{N_{\rm gal}}p(\mathbf{d}_i^{\rm GW,obs}|\vec{\Lambda}_i,g_i,\vec{\theta})p(\mathbf{d}_i^{\rm EM,obs}|\vec{\Lambda}_i,g_i,\vec{\theta})p(\vec{\Lambda}_i,g_i|\{z\},\{\vec{\Omega}\},\{\vec{\lambda}\},\vec{\theta})\right){\rm d}\vec{\Lambda}_i\right]\right. \\
&\qquad\times N^{N_{\rm obs}}\exp\left[-p_{\rm det}\left(\vec{\theta}|\{z_i\},\{\vec{\Omega}_i\},\{\vec{\lambda}_i\}\right)N\right]\times\binom{N_{\rm gal}}{N_{\rm obs}^{\rm hist}}p(N_{\rm gal}|\vec{\theta}) \\
&\qquad\left.\times\prod_{l=1}^{N_{\rm obs}^{\rm hist}}p(\mathbf{d}_l^{EM,{\rm obs}}|z_l,\vec{\lambda}_l,\vec{\Omega}_l,\vec{\theta})\prod_{m=N_{\rm obs}^{\rm hist}+1}^{N_{\rm gal}}\left\{(1-p_{\rm hist}^{\rm det}(z_m,\vec{\lambda}_m,\vec{\Omega}_m,\vec{\theta}))\right\}p(\{z\},\{\vec{\Omega}\},\{\vec{\lambda}\}|\vec{\theta})\right\} \\
&\qquad\qquad p(N|\vec{\theta})\,{\rm d}N{\rm d}\{z\}{\rm d}\{\vec{\Omega}\}{\rm d}\{\vec{\lambda}\}.
\end{aligned}
$$

Prior on galaxy locations/properties

General statistical framework

- Considering all EM and GW observations together, general framework can be written

$$
\begin{aligned}
&p(\vec{\theta}|\{\mathbf{d}^{EM,\mathrm{obs}}_{\mathrm{hist}}\},\{\mathbf{d}^{GW,\mathrm{obs}}\},\{\mathbf{d}^{EM,\mathrm{obs}}\}) \\
&\quad\propto p(\vec{\theta})\sum_{N_{\mathrm{gal}}}\int\left\{\prod_{i=1}^{N_{\mathrm{obs}}}\left[\int\left(\sum_{g_i=1}^{N_{\mathrm{gal}}} p(\mathbf{d}_i^{\mathrm{GW,obs}}|\vec{\Lambda}_i,g_i,\vec{\theta})\,p(\mathbf{d}_i^{\mathrm{EM,obs}}|\vec{\Lambda}_i,g_i,\vec{\theta})\,p(\vec{\Lambda}_i,g_i|\{z\},\{\vec{\Omega}\},\{\vec{\lambda}\},\vec{\theta})\right)\mathrm{d}\vec{\Lambda}_i\right]\right. \\
&\qquad\times N^{N_{\mathrm{obs}}}\exp\left[-p_{\mathrm{det}}\left(\vec{\theta}|\{z_i\},\{\vec{\Omega}_i\},\{\vec{\lambda}_i\}\right)N\right]\times\binom{N_{\mathrm{gal}}}{N^{\mathrm{hist}}_{\mathrm{obs}}}p(N_{\mathrm{gal}}|\vec{\theta}) \\
&\qquad\left.\times\prod_{l=1}^{N^{\mathrm{hist}}_{\mathrm{obs}}}p(\mathbf{d}_l^{EM,\mathrm{obs}}|z_l,\vec{\lambda}_l,\vec{\Omega}_l,\vec{\theta})\prod_{m=N^{\mathrm{hist}}_{\mathrm{obs}}+1}^{N_{\mathrm{gal}}}\left\{1-p^{\mathrm{det}}_{\mathrm{hist}}(z_m,\vec{\lambda}_m,\vec{\Omega}_m,\vec{\theta})\right\}p(\{z\},\{\vec{\Omega}\},\{\vec{\lambda}\}|\vec{\theta})\right\} \\
&\qquad p(N|\vec{\theta})\,\mathrm{d}N\mathrm{d}\{z\}\mathrm{d}\{\vec{\Omega}\}\mathrm{d}\{\vec{\lambda}\}.
\end{aligned}
$$

GW observations

Historical observations

Catalogue incompleteness

Prior on galaxy locations/properties

General statistical framework

- Considering all EM and GW observations together, general framework can be written

Counterpart observations

GW observations

GW source population

$$p(\vec{\theta}|\{\mathbf{d}_{\rm hist}^{EM,\rm obs}\},\{\mathbf{d}^{GW,\rm obs}\},\{\mathbf{d}^{EM,\rm obs}\})$$

$$\propto p(\vec{\theta})\sum_{N_{\rm gal}}\int\left\{\prod_{i=1}^{N_{\rm obs}}\left[\int\left(\sum_{g_i=1}^{N_{\rm gal}} p(\mathbf{d}_i^{\rm GW,obs}|\vec{\Lambda}_i,g_i,\vec{\theta})\,p(\mathbf{d}_i^{\rm EM,obs}|\vec{\Lambda}_i,g_i,\vec{\theta})\,p(\vec{\Lambda}_i,g_i|\{z\},\{\vec{\Omega}\},\{\vec{\lambda}\},\vec{\theta})\right)\mathrm{d}\vec{\Lambda}_i\right]\right.$$

$$\times N^{N_{\rm obs}}\exp\left[-p_{\rm det}\left(\vec{\theta}|\{z_i\},\{\vec{\Omega}_i\},\{\vec{\lambda}_i\}\right)N\right]\times\binom{N_{\rm gal}}{N_{\rm obs}^{\rm hist}}p(N_{\rm gal}|\vec{\theta})$$

$$\left.\times\prod_{l=1}^{N_{\rm obs}^{\rm hist}}p(\mathbf{d}_l^{EM,\rm obs}|z_l,\vec{\lambda}_l,\vec{\Omega}_l,\vec{\theta})\prod_{m=N_{\rm obs}^{\rm hist}+1}^{N_{\rm gal}}\left\{(1-p_{\rm hist}^{\rm det}(z_m,\vec{\lambda}_m,\vec{\Omega}_m,\vec{\theta}))\right\}p(\{z\},\{\vec{\Omega}\},\{\vec{\lambda}\}|\vec{\theta})\right\}$$

$$p(N|\vec{\theta})\,\mathrm{d}N\mathrm{d}\{z\}\mathrm{d}\{\vec{\Omega}\}\mathrm{d}\{\vec{\lambda}\}.$$

Historical observations

Catalogue incompleteness

Prior on galaxy locations/properties

General statistical framework

- Systematic errors can potentially enter all of these terms.

$$
\begin{aligned}
&p(\vec{\theta}|\{\mathbf{d}^{EM,\mathrm{obs}}_{\mathrm{hist}}\},\{\mathbf{d}^{GW,\mathrm{obs}}\},\{\mathbf{d}^{EM,\mathrm{obs}}\}) \\
&\quad\propto p(\vec{\theta})\sum_{N_{\mathrm{gal}}}\int\Bigg\{\prod_{i=1}^{N_{\mathrm{obs}}}\Bigg[\int\Bigg(\sum_{g_i=1}^{N_{\mathrm{gal}}} p(\mathbf{d}_i^{\mathrm{GW,obs}}|\vec{\Lambda}_i,g_i,\vec{\theta})\,p(\mathbf{d}_i^{\mathrm{EM,obs}}|\vec{\Lambda}_i,g_i,\vec{\theta})\,p(\vec{\Lambda}_i,g_i|\{z\},\{\vec{\Omega}\},\{\vec{\lambda}\},\vec{\theta})\Bigg)\,\mathrm{d}\vec{\Lambda}_i\Bigg] \\
&\quad\times N^{N_{\mathrm{obs}}}\exp\left[-p_{\mathrm{det}}\left(\vec{\theta}|\{z_i\},\{\vec{\Omega}_i\},\{\vec{\lambda}_i\}\right)N\right]\times\binom{N_{\mathrm{gal}}}{N^{\mathrm{hist}}_{\mathrm{obs}}}\,p(N_{\mathrm{gal}}|\vec{\theta}) \\
&\quad\times\prod_{l=1}^{N^{\mathrm{hist}}_{\mathrm{obs}}} p(\mathbf{d}_l^{EM,\mathrm{obs}}|z_l,\vec{\lambda}_l,\vec{\Omega}_l,\vec{\theta})\prod_{m=N^{\mathrm{hist}}_{\mathrm{obs}}+1}^{N_{\mathrm{gal}}}\left\{(1-p^{\mathrm{det}}_{\mathrm{hist}}(z_m,\vec{\lambda}_m,\vec{\Omega}_m,\vec{\theta}))\right\}p(\{z\},\{\vec{\Omega}\},\{\vec{\lambda}\}|\vec{\theta})\Bigg\} \\
&\quad\quad p(N|\vec{\theta})\,\mathrm{d}N\mathrm{d}\{z\}\mathrm{d}\{\vec{\Omega}\}\mathrm{d}\{\vec{\lambda}\}.
\end{aligned}
$$

GW selection effects

- Can mitigate these uncertainties in the analysis, but must be aware of them, and mitigation will reduce precision of cosmological measurements.

General statistical framework

- Systematic errors can potentially enter all of these terms.

Calibration and model uncertainties

$$
\begin{aligned}
&p(\vec{\theta}|\{\mathbf{d}^{EM,\mathrm{obs}}_{\mathrm{hist}}\},\{\mathbf{d}^{GW,\mathrm{obs}}\},\{\mathbf{d}^{EM,\mathrm{obs}}\}) \\
&\quad \propto p(\vec{\theta}) \sum_{N_{\mathrm{gal}}} \int \Bigg\{ \prod_{i=1}^{N_{\mathrm{obs}}} \left[\int \left(\sum_{g_i=1}^{N_{\mathrm{gal}}} p(\mathbf{d}_i^{\mathrm{GW,obs}}|\vec{\Lambda}_i, g_i, \vec{\theta})\, p(\mathbf{d}_i^{\mathrm{EM,obs}}|\vec{\Lambda}_i, g_i, \vec{\theta})\, p(\vec{\Lambda}_i, g_i|\{z\},\{\vec{\Omega}\},\{\vec{\lambda}\},\vec{\theta}) \right) \mathrm{d}\vec{\Lambda}_i \right] \\
&\quad \times N^{N_{\mathrm{obs}}} \exp\left[-p_{\mathrm{det}}\left(\vec{\theta}|\{z_i\},\{\vec{\Omega}_i\},\{\vec{\lambda}_i\}\right) N\right] \times \binom{N_{\mathrm{gal}}}{N_{\mathrm{obs}}^{\mathrm{hist}}}\, p(N_{\mathrm{gal}}|\vec{\theta}) \\
&\quad \times \prod_{l=1}^{N_{\mathrm{obs}}^{\mathrm{hist}}} p(\mathbf{d}_l^{EM,\mathrm{obs}}|z_l, \vec{\lambda}_l, \vec{\Omega}_l, \vec{\theta}) \prod_{m=N_{\mathrm{obs}}^{\mathrm{hist}}+1}^{N_{\mathrm{gal}}} \left\{(1 - p_{\mathrm{hist}}^{\mathrm{det}}(z_m, \vec{\lambda}_m, \vec{\Omega}_m, \vec{\theta}))\right\} p(\{z\},\{\vec{\Omega}\},\{\vec{\lambda}\}|\vec{\theta}) \Bigg\} \\
&\quad p(N|\vec{\theta})\, \mathrm{d}N \mathrm{d}\{z\} \mathrm{d}\{\vec{\Omega}\} \mathrm{d}\{\vec{\lambda}\}.
\end{aligned}
$$

- Can mitigate these uncertainties in the analysis, but must be aware of them, and mitigation will reduce precision of cosmological measurements.

General statistical framework

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\begin{aligned}
&p(\vec{\theta}|\{\mathbf{d}_{\rm hist}^{EM,\rm obs}\},\{\mathbf{d}^{GW,\rm obs}\},\{\mathbf{d}^{EM,\rm obs}\}) \\
&\quad\propto p(\vec{\theta})\sum_{N_{\rm gal}}\int\left\{\prod_{i=1}^{N_{\rm obs}}\left[\int\left(\sum_{g_i=1}^{N_{\rm gal}} p(\mathbf{d}_i^{\rm GW,obs}|\vec{\Lambda}_i,g_i,\vec{\theta})p(\mathbf{d}_i^{\rm EM,obs}|\vec{\Lambda}_i,g_i,\vec{\theta})p(\vec{\Lambda}_i,g_i|\{z\},\{\vec{\Omega}\},\{\vec{\lambda}\},\vec{\theta})\right)\mathrm{d}\vec{\Lambda}_i\right]\right. \\
&\quad\quad\times N^{N_{\rm obs}}\exp\left[-p_{\rm det}\left(\vec{\theta}|\{z_i\},\{\vec{\Omega}_i\},\{\vec{\lambda}_i\}\right)N\right]\times\binom{N_{\rm gal}}{N_{\rm obs}^{\rm hist}}p(N_{\rm gal}|\vec{\theta}) \\
&\quad\quad\left.\times\prod_{l=1}^{N_{\rm obs}^{\rm hist}} p(\mathbf{d}_l^{EM,\rm obs}|z_l,\vec{\lambda}_l,\vec{\Omega}_l,\vec{\theta})\prod_{m=N_{\rm obs}^{\rm hist}+1}^{N_{\rm gal}}\left\{(1-p_{\rm hist}^{\rm det}(z_m,\vec{\lambda}_m,\vec{\Omega}_m,\vec{\theta}))\right\}p(\{z\},\{\vec{\Omega}\},\{\vec{\lambda}\}|\vec{\theta})\right\} \\
&\quad\quad p(N|\vec{\theta})\,\mathrm{d}N\mathrm{d}\{z\}\mathrm{d}\{\vec{\Omega}\}\mathrm{d}\{\vec{\lambda}\}.
\end{aligned}
$$

Galaxy population

- Can mitigate these uncertainties in the analysis, but must be aware of them, and mitigation will reduce precision of cosmological measurements.

General statistical framework

- Systematic errors can potentially enter all of these terms.

$$
\begin{aligned}
&p(\vec{\theta}|\{\mathbf{d}^{EM,\mathrm{obs}}_{\mathrm{hist}}\},\{\mathbf{d}^{GW,\mathrm{obs}}\},\{\mathbf{d}^{EM,\mathrm{obs}}\}) \\
&\quad\propto p(\vec{\theta})\sum_{N_{\mathrm{gal}}}\int\left\{\prod_{i=1}^{N_{\mathrm{obs}}}\left[\int\left(\sum_{g_i=1}^{N_{\mathrm{gal}}} p(\mathbf{d}_i^{\mathrm{GW,obs}}|\vec{\Lambda}_i,g_i,\vec{\theta})\,p(\mathbf{d}_i^{\mathrm{EM,obs}}|\vec{\Lambda}_i,g_i,\vec{\theta})\,p(\vec{\Lambda}_i,g_i|\{z\},\{\vec{\Omega}\},\{\vec{\lambda}\},\vec{\theta})\right)\mathrm{d}\vec{\Lambda}_i\right]\right. \\
&\qquad\times N^{N_{\mathrm{obs}}}\exp\left[-p_{\mathrm{det}}\left(\vec{\theta}|\{z_i\},\{\vec{\Omega}_i\},\{\vec{\lambda}_i\}\right)N\right]\times\binom{N_{\mathrm{gal}}}{N^{\mathrm{hist}}_{\mathrm{obs}}}\,p(N_{\mathrm{gal}}|\vec{\theta}) \\
&\qquad\left.\times\prod_{l=1}^{N^{\mathrm{hist}}_{\mathrm{obs}}} p(\mathbf{d}_l^{EM,\mathrm{obs}}|z_l,\vec{\lambda}_l,\vec{\Omega}_l,\vec{\theta})\prod_{m=N^{\mathrm{hist}}_{\mathrm{obs}}+1}^{N_{\mathrm{gal}}}\left\{\left(1-p^{\mathrm{det}}_{\mathrm{hist}}(z_m,\vec{\lambda}_m,\vec{\Omega}_m,\vec{\theta})\right)\right\}p(\{z\},\{\vec{\Omega}\},\{\vec{\lambda}\}|\vec{\theta})\right\} \\
&\qquad p(N|\vec{\theta})\,\mathrm{d}N\mathrm{d}\{z\}\mathrm{d}\{\vec{\Omega}\}\mathrm{d}\{\vec{\lambda}\}.
\end{aligned}
$$

Catalogue incompleteness

- Can mitigate these uncertainties in the analysis, but must be aware of them, and mitigation will reduce precision of cosmological measurements.

Summary

- Gravitational waves are beginning to be used as standard sirens to probe the expansion of the Universe
  - First measurement of H_0 made possible by GW170817, since host galaxy redshift determined. Measured $H_0=70^{+12}_{-8}$ km s^{-1} Mpc^{-1}.
  - Can obtain additional constraints from dark sirens using galaxy catalogues to provide potential host redshifts, or by exploiting characteristic features in the mass spectrum. These methods have provided a few tens of percent improvement in the constraints.
  - In the future, ~60-130 counterpart events would allow us to reach 2% precision, enough to resolve the current tension. After ~250-500 events we could obtain a 1% measurement.
  - With the thousands of BBH systems expected to be observed by 2G and 3G detectors, we should be able to place competitive constraints on $H(z)$ at the level of a few percent.
- Systematics will affect, and potentially limit, all these measurements. But, these can be mitigated and are different to those from EM observations and will provide a complementary probe with comparable sensitivity.