

Title: AdS/CFT

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$SO(6)$ quantum spin chain

$O_A = \frac{1}{r}(\phi^{m_1} \dots \phi^{m_j}) \longleftrightarrow |m_1 \dots m_j\rangle$

$\{\Delta\} = \text{Eigenergies}$

$H = \frac{\lambda}{\pi^2} \sum_{n=1}^J \left[\text{diagram of two sites } n, n+1 \text{ with interaction} \right] = H_{nn+1}$

$= 2P_{nn+1} - K_{nn+1} - 2I_{nn+1} + O(\lambda^2)$

From all other interactions $\leftarrow \text{Energy} = 0$

Local nearest neighbor interaction

Planarity!

$\Delta(\lambda)$ for

For physical χ
 $\#A - \#B + \#C =$

n_{21}

n_{12} oscillators
 [of whatever type]

$H | (A^+) (B^+) (C^+) \dots, (A^+) (B^+) (C^+) \dots \rangle$

n in total

$= \sum_{n_2, n_{21}} C(n, n_2, n_{21}) | \text{re-shuffled oscillators in the 2 sites} \rangle$

$\uparrow \Gamma\left(\frac{n_{12} + n_{21}}{2}\right)_{\infty} /_{\infty} (-1)^{\infty}$

$O_A = \frac{1}{r}(\phi^{m_1} \dots \phi^{m_J}) \longleftrightarrow |m_1 \dots m_J\rangle \leftarrow SO(6) \text{ quantum spin chain}$

$\{\Delta\} = \text{Eigenenergies} \left[H = \frac{\lambda}{\pi^2} \sum_{n=1}^J \left[\text{diagram of two sites } n \text{ and } n+1 \text{ with a cross-hatch} \right] = H_{nn+1} = 2P_{nn+1} - K_{nn+1} - 2I_{nn+1} \right] + O(\lambda^2)$

$\leftarrow \text{from all other interactions} \leftarrow \text{Energy} = 0 \text{ for } t \in \mathbb{Z}$

$\leftarrow \text{Local nearest neighbor interaction spin chain Hamiltonian}$

$\leftarrow \text{from } [\phi_n, \phi_m]^2$

$\leftarrow \text{10D origin}$

$\leftarrow \text{Integrable!}$

$\leftarrow \text{Planarity!}$

$\Delta(\lambda) \text{ for single}$

$H | (A^+)_{\text{vac}} (B^+)_{\text{vac}} (C^+)_{\text{vac}} \rangle, (A^+)_{\text{vac}} (B^+)_{\text{vac}} (C^+)_{\text{vac}} \rangle$

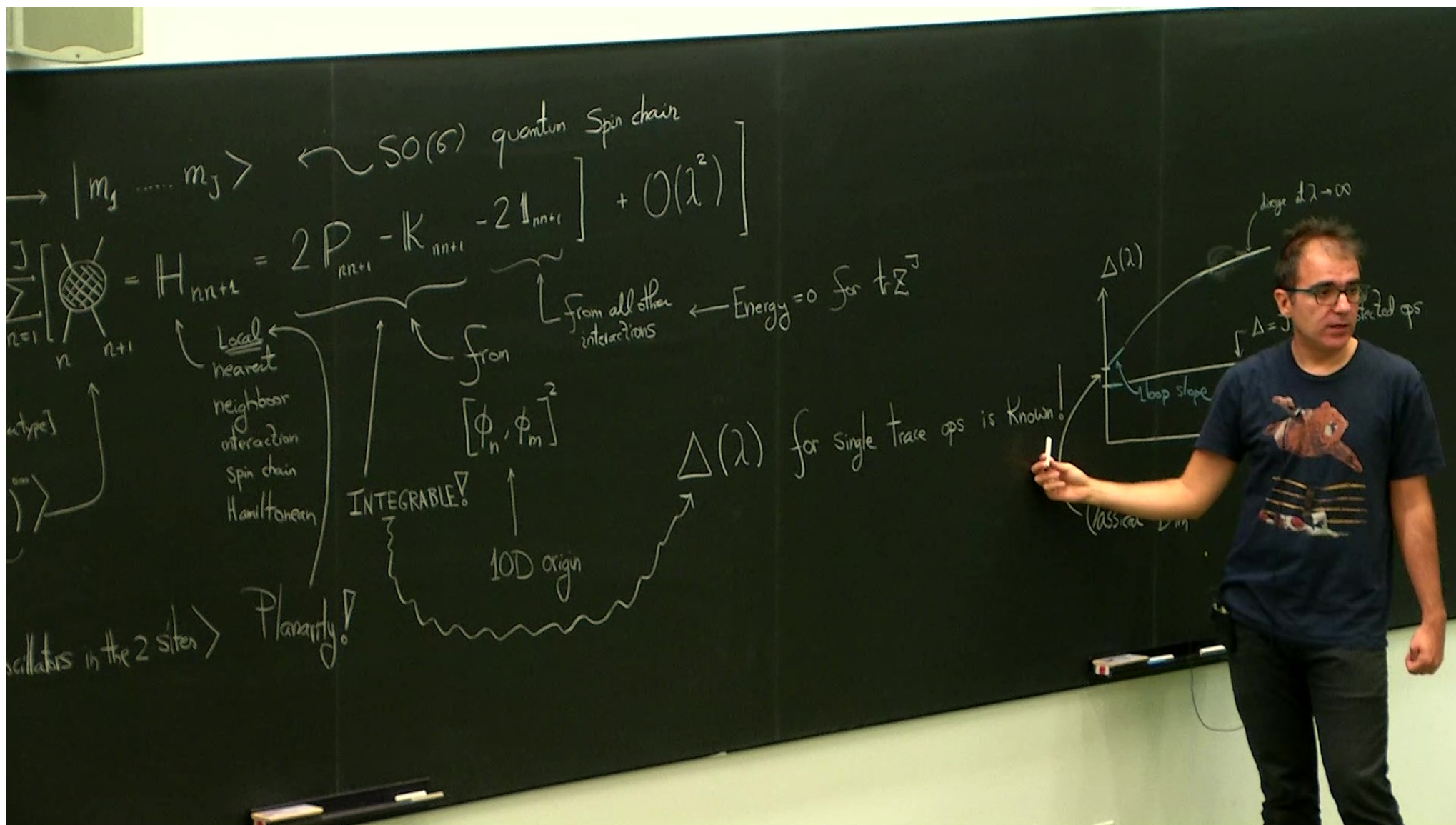
$n \text{ in total}$

$\sum_{n_{21}, n_{22}} C(n, n_{21}, n_{22}) | \text{re-shuffled oscillators in the 2 sites} \rangle$

$\leftarrow \left(\frac{n_{21} + n_{22}}{2} \right)_{\text{vac}} /_{\text{vac}} (-1)^{\text{vac}}$

n_{21} oscillators [of whatever type]

n_{22} oscillators [of whatever type]



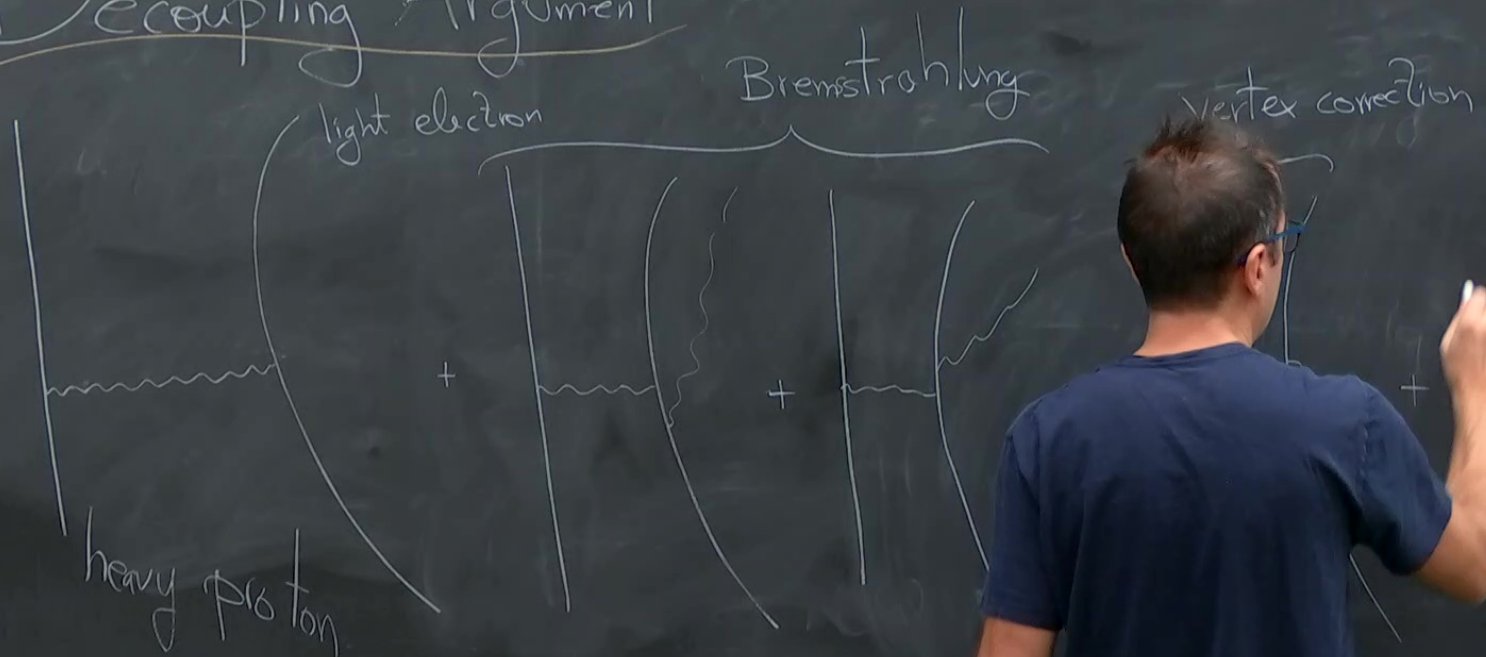
(6) quantum spin chain
 $K_{nn+1} - 2J_{nn+1} + O(\lambda^2) = \lambda \text{ (diagram)} + \lambda^2 \text{ (diagram)} + \dots \sim \text{long-range } H \text{ (but after local as } \lambda \text{ increases)} \equiv H_1$

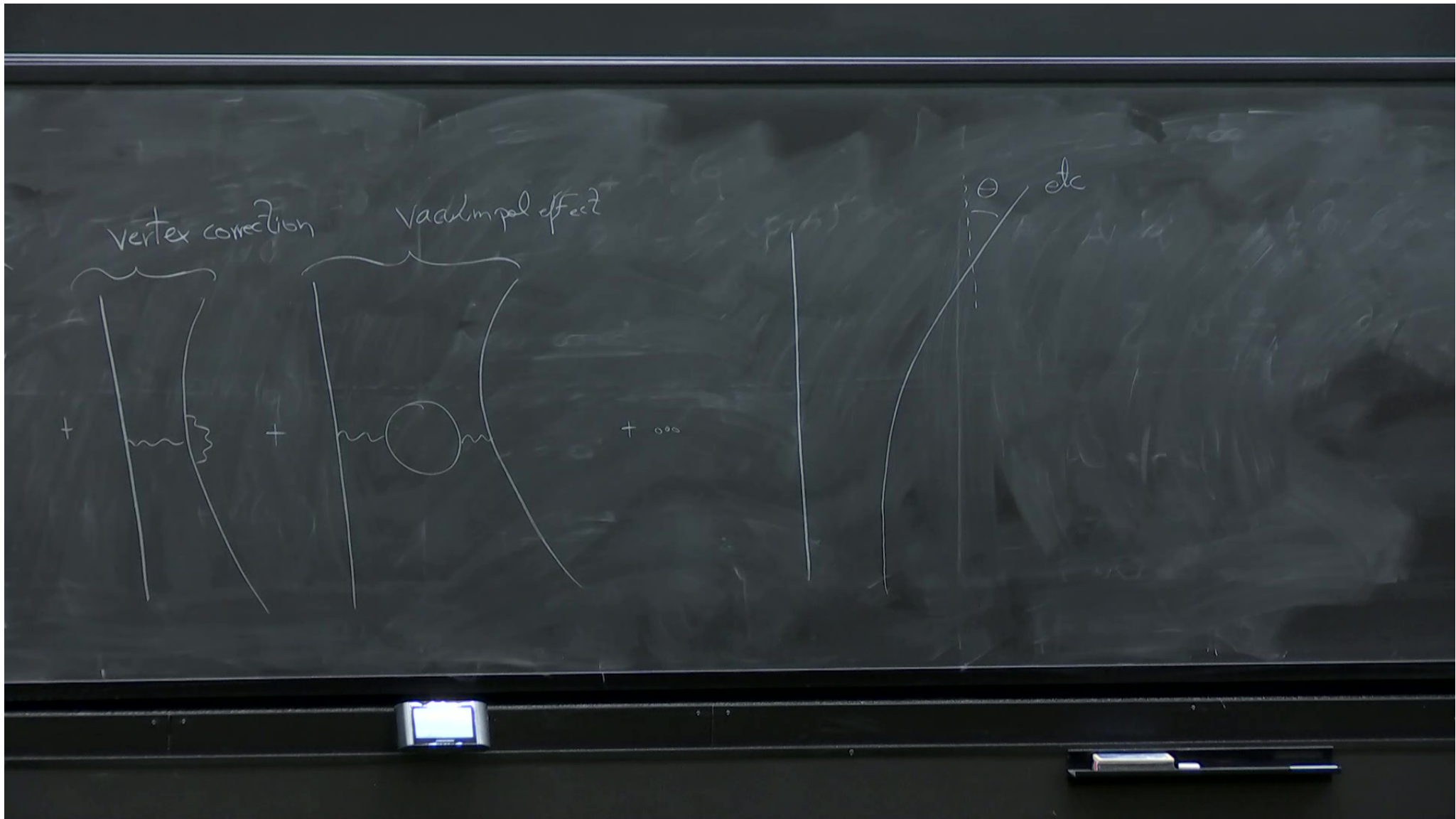
from $[\phi_n, \phi_m]^2$ → Energy = 0 for $t \in \mathbb{Z}$
 from all other interactions
 IRABLE! ↑ 10D origin

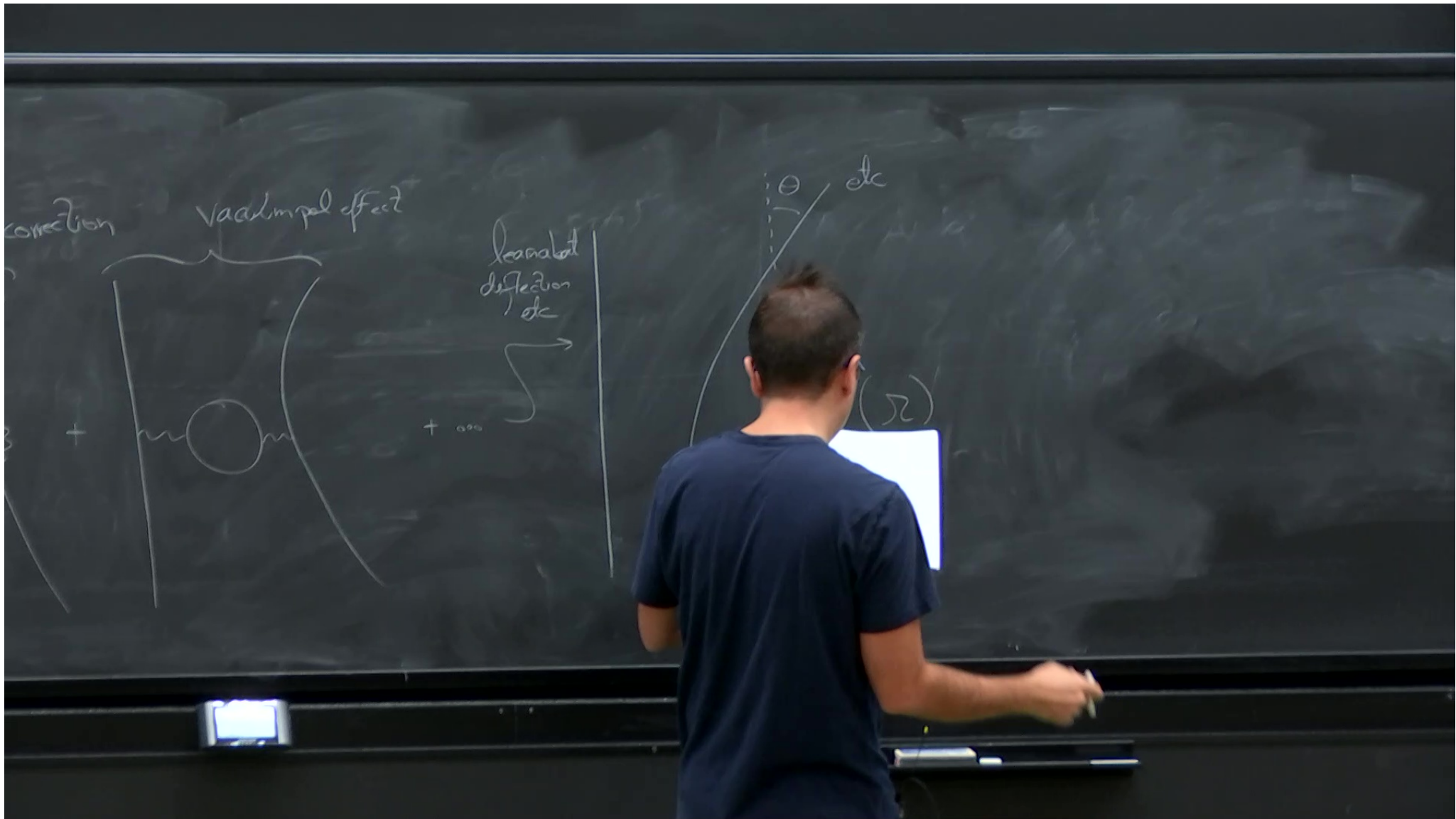
$\Delta(\lambda)$ for single trace ops is known!
 Classical Dim

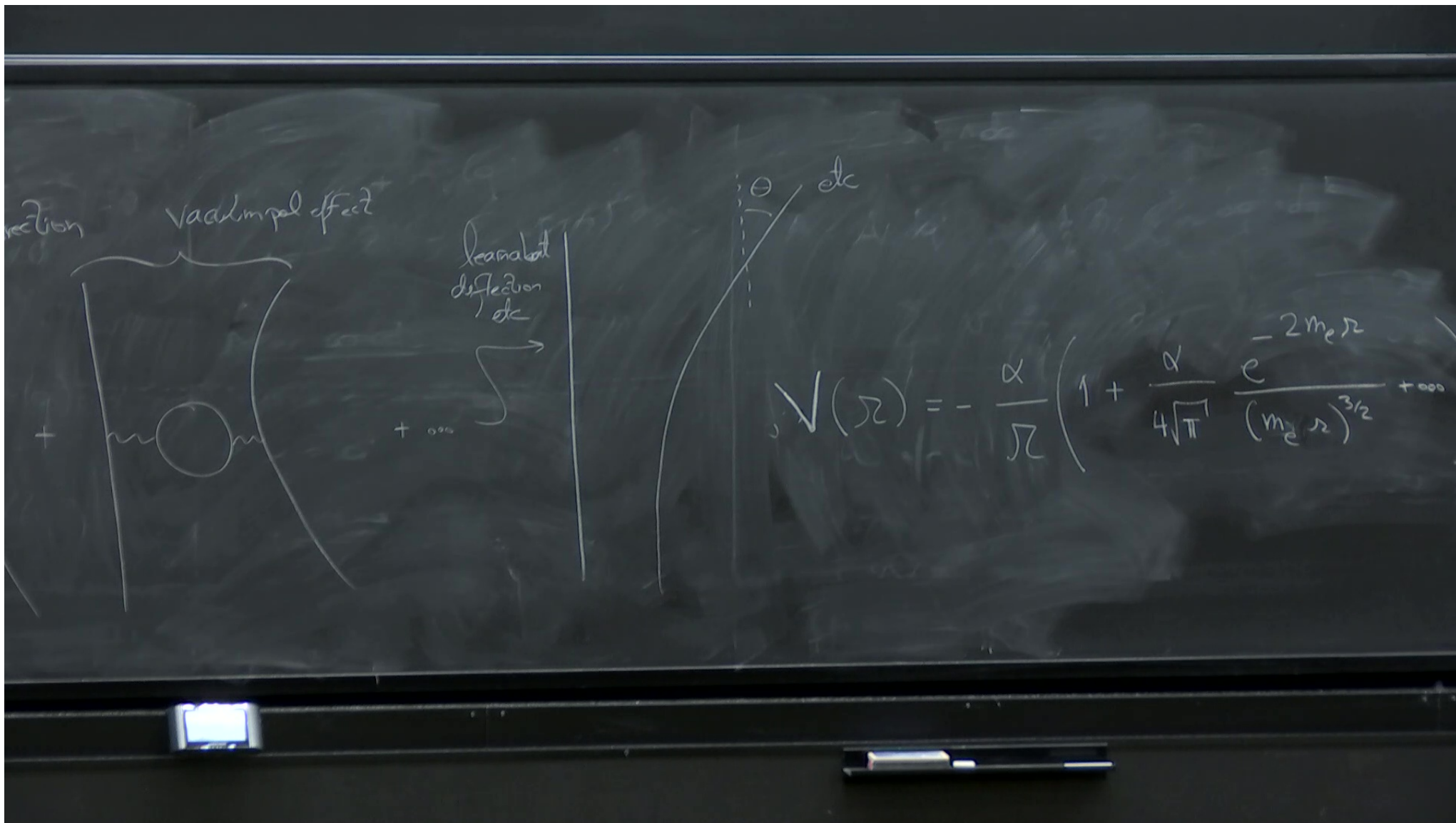
Graph of $\Delta(\lambda)$ vs λ :
 - Curve starts at origin and increases, labeled "diverge at $\lambda \rightarrow \infty$ ".
 - Horizontal line at $\Delta = J$ labeled " $\Delta = J$ for protected ops".
 - Vertical segment from origin to $\Delta = J$ labeled "loop slope".

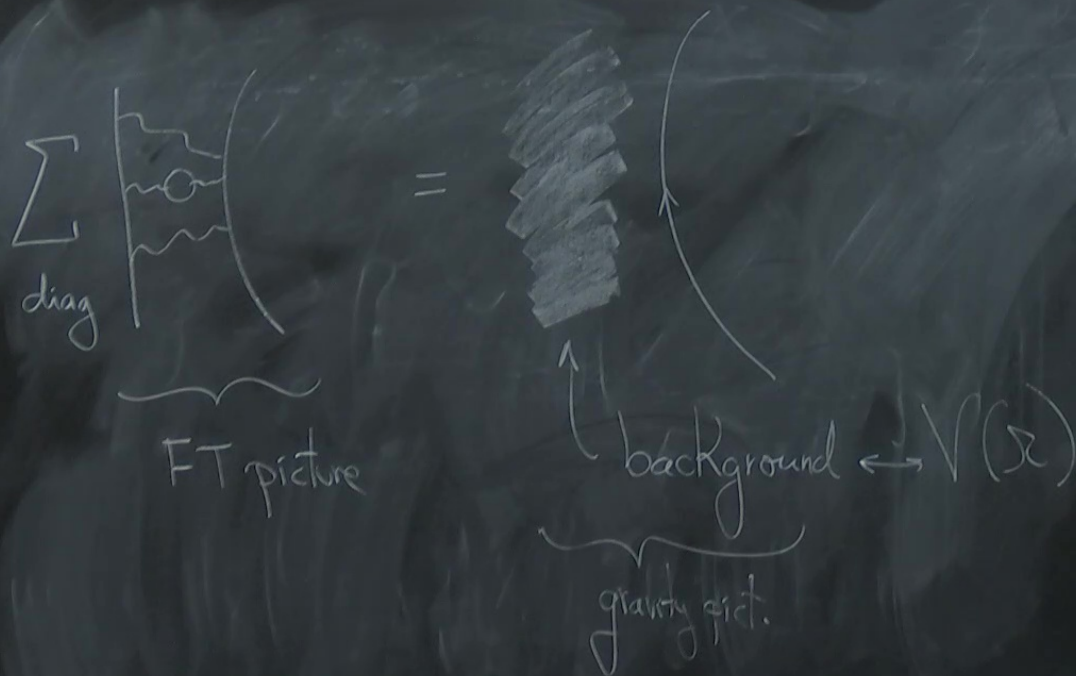
Decoupling Argument

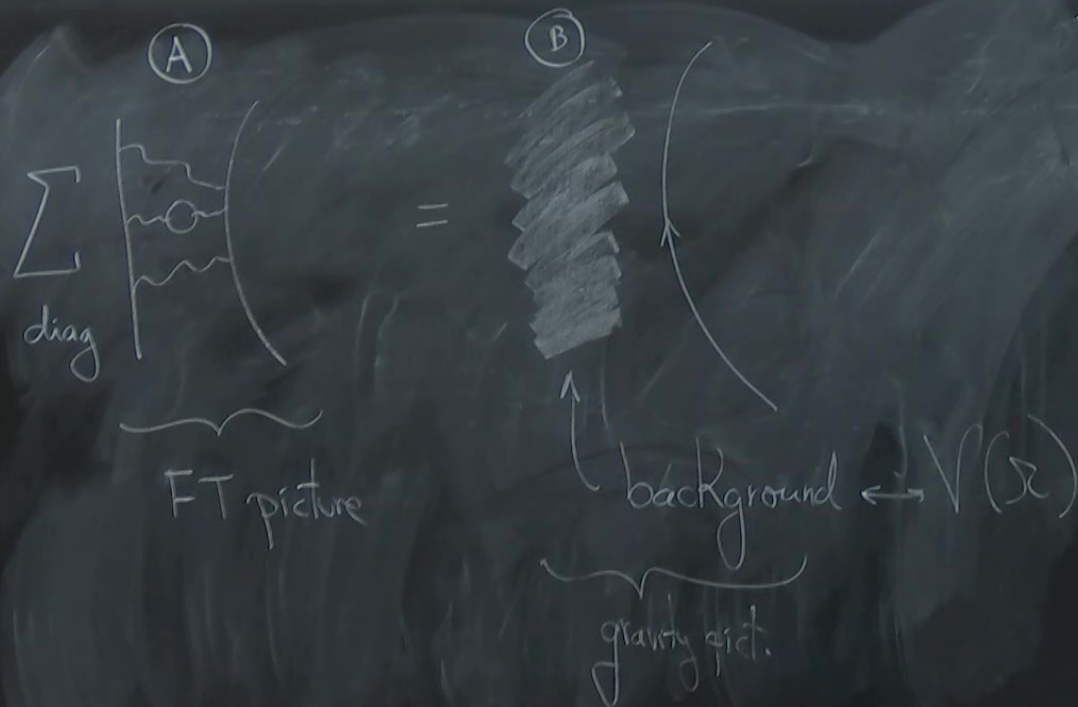


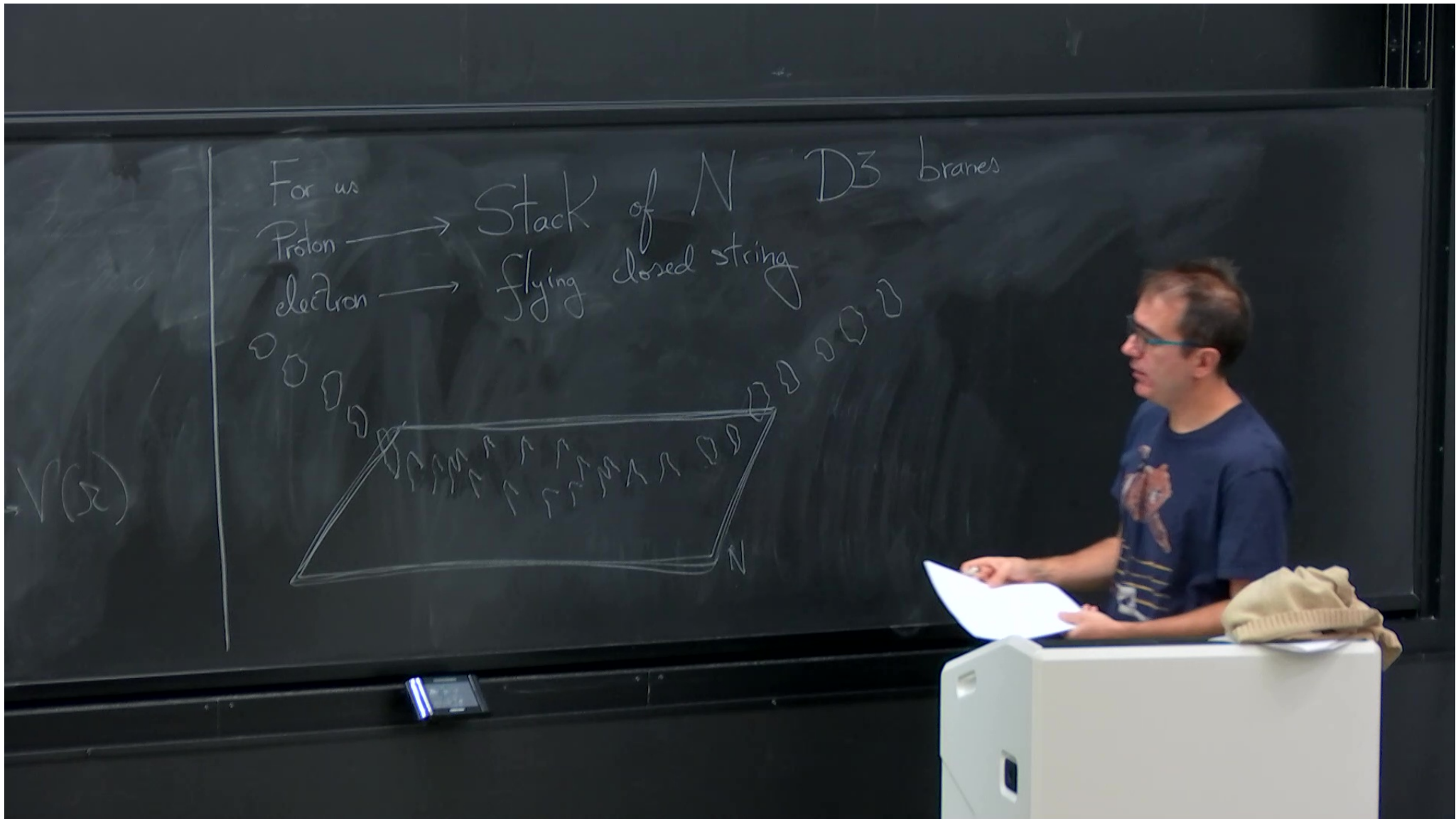




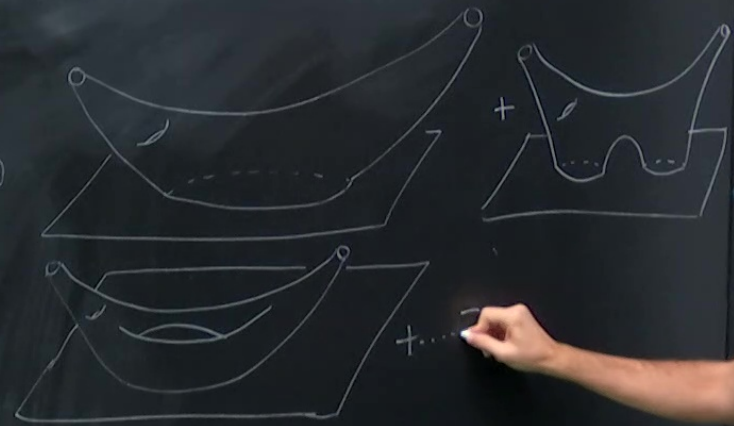
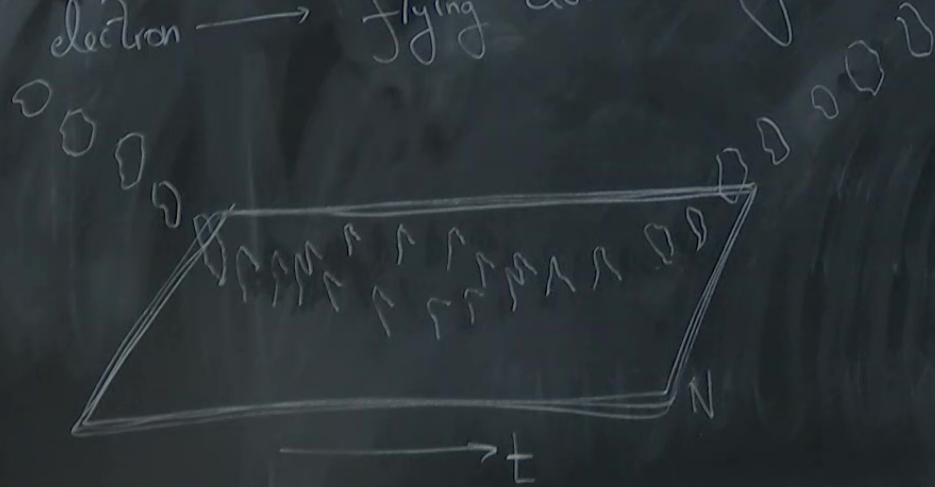


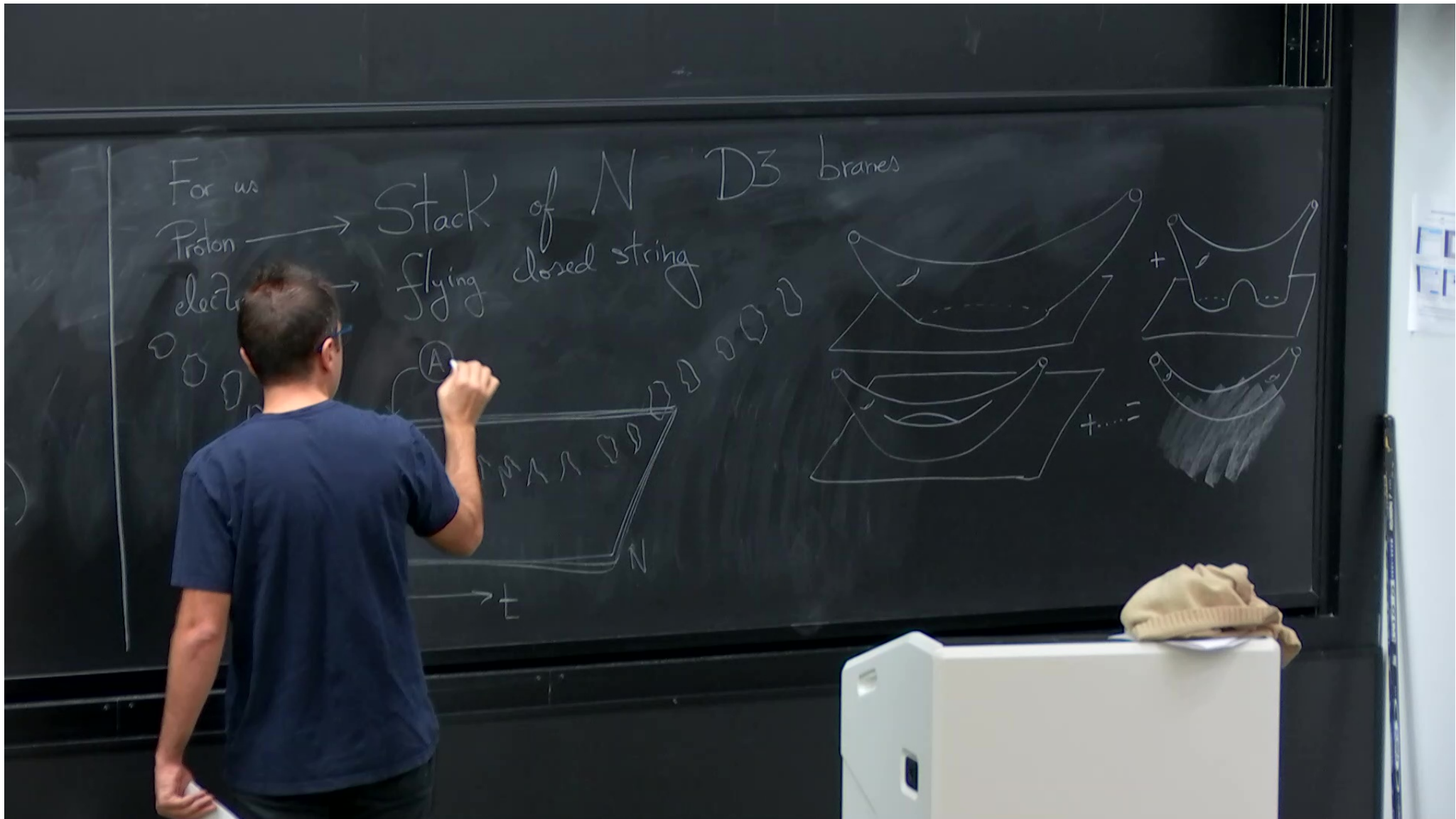






For us
 Proton $\rightarrow$ Stack of N D3 branes
 electron $\rightarrow$ flying closed string



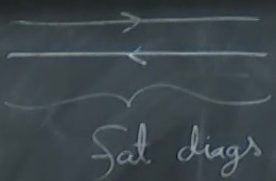


gravity dict.

$\rightarrow t$

in (A) $\rightarrow$
We have open strings

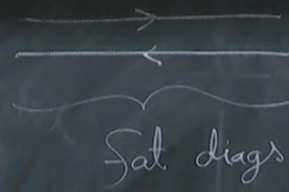
β t



YM + massive open strings

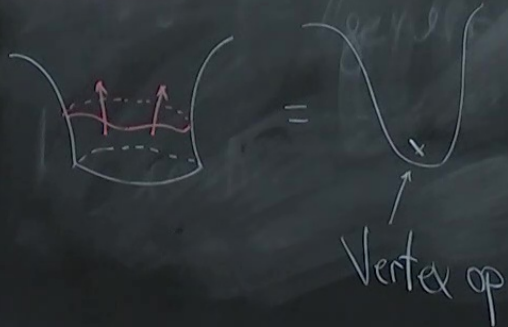
in (A) $\longrightarrow$
 We have open strings

β^t

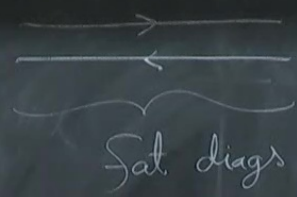


YM + massive open string

in (B)



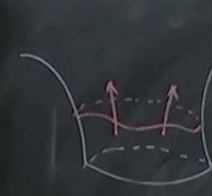
in (A) $\longrightarrow$ β^t
 We have open strings



Sat. diags

YM + massive open string
 condensate of gravitons

in (B)



Vertex op

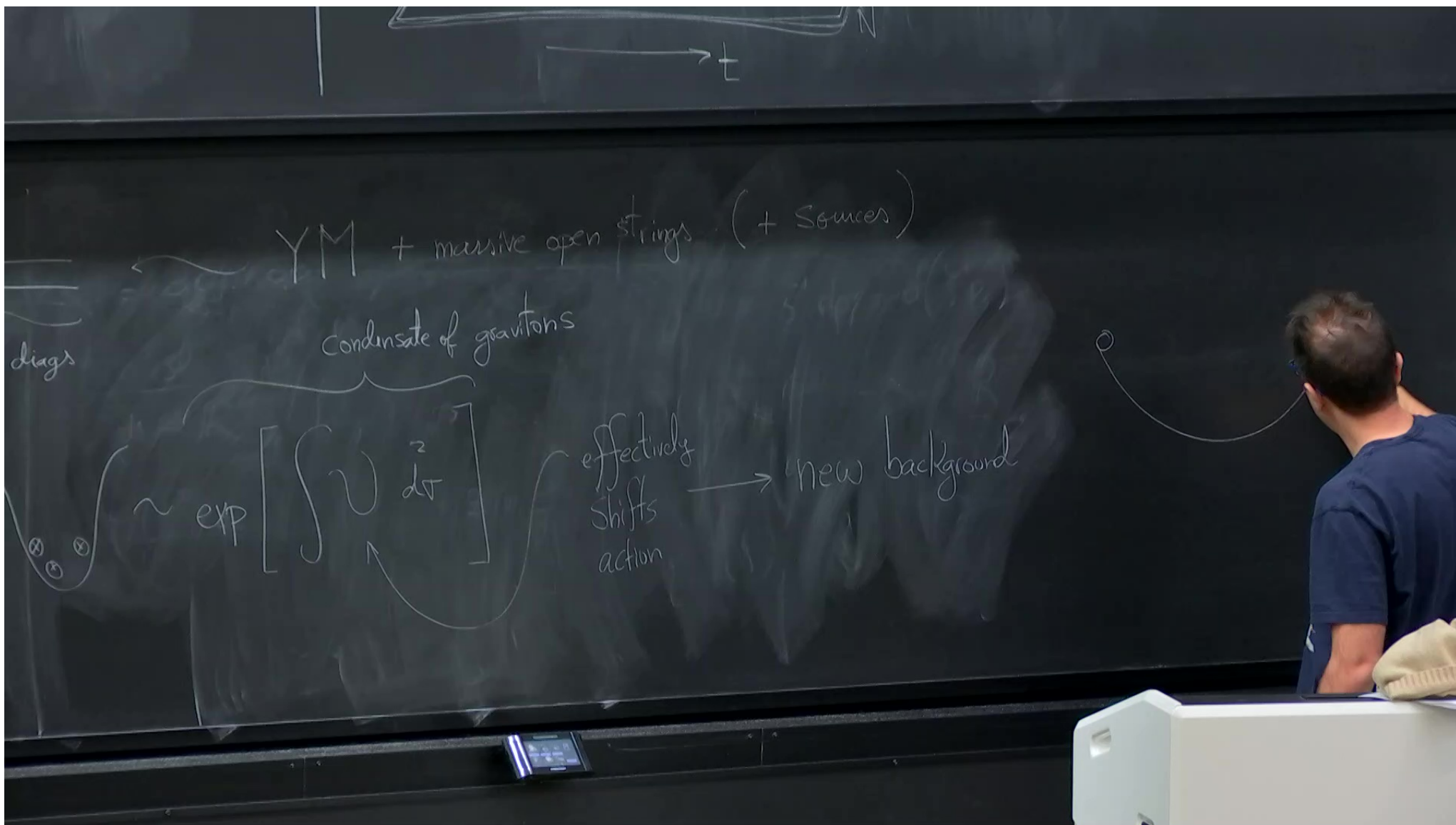
and

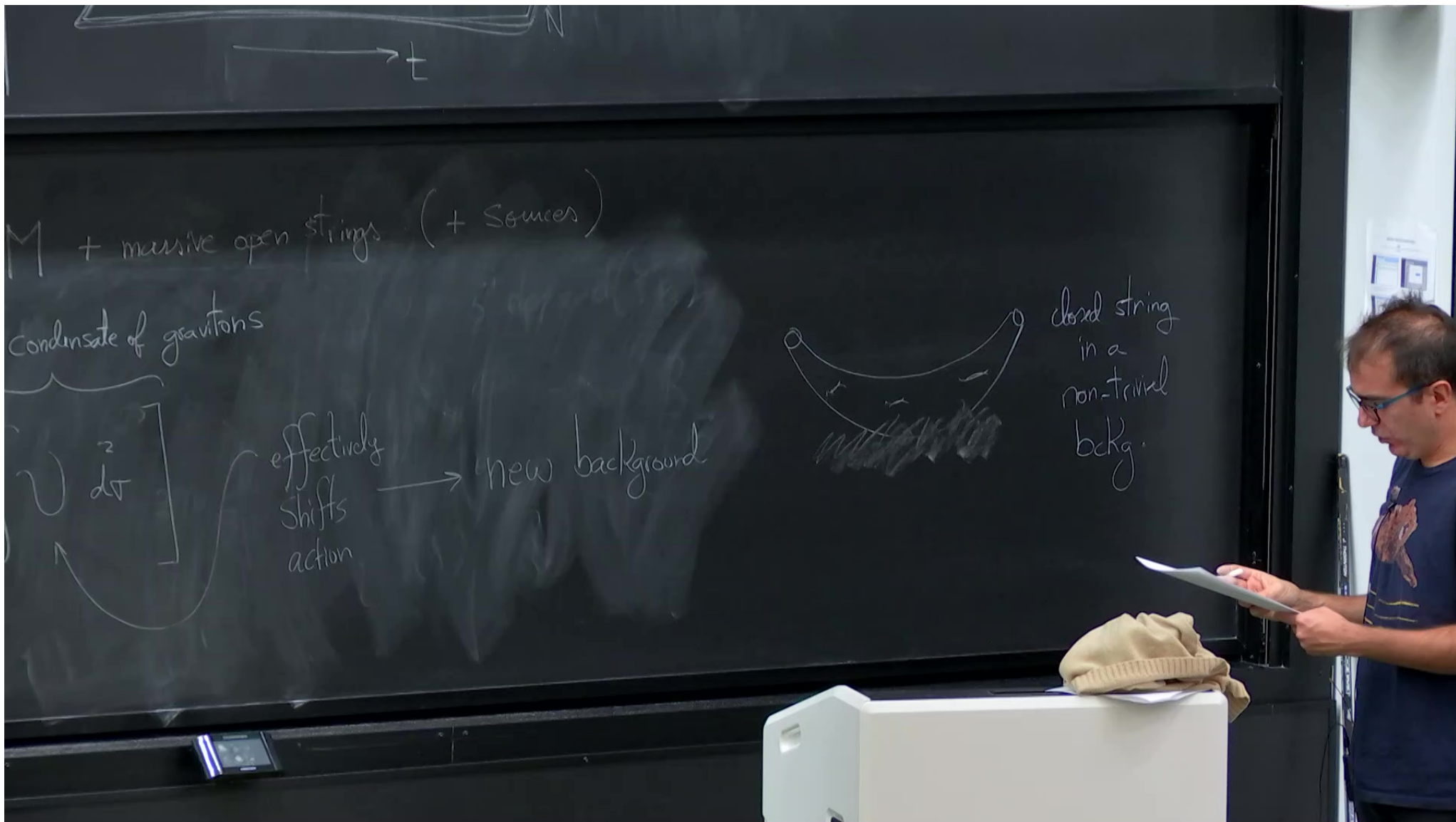
Σ

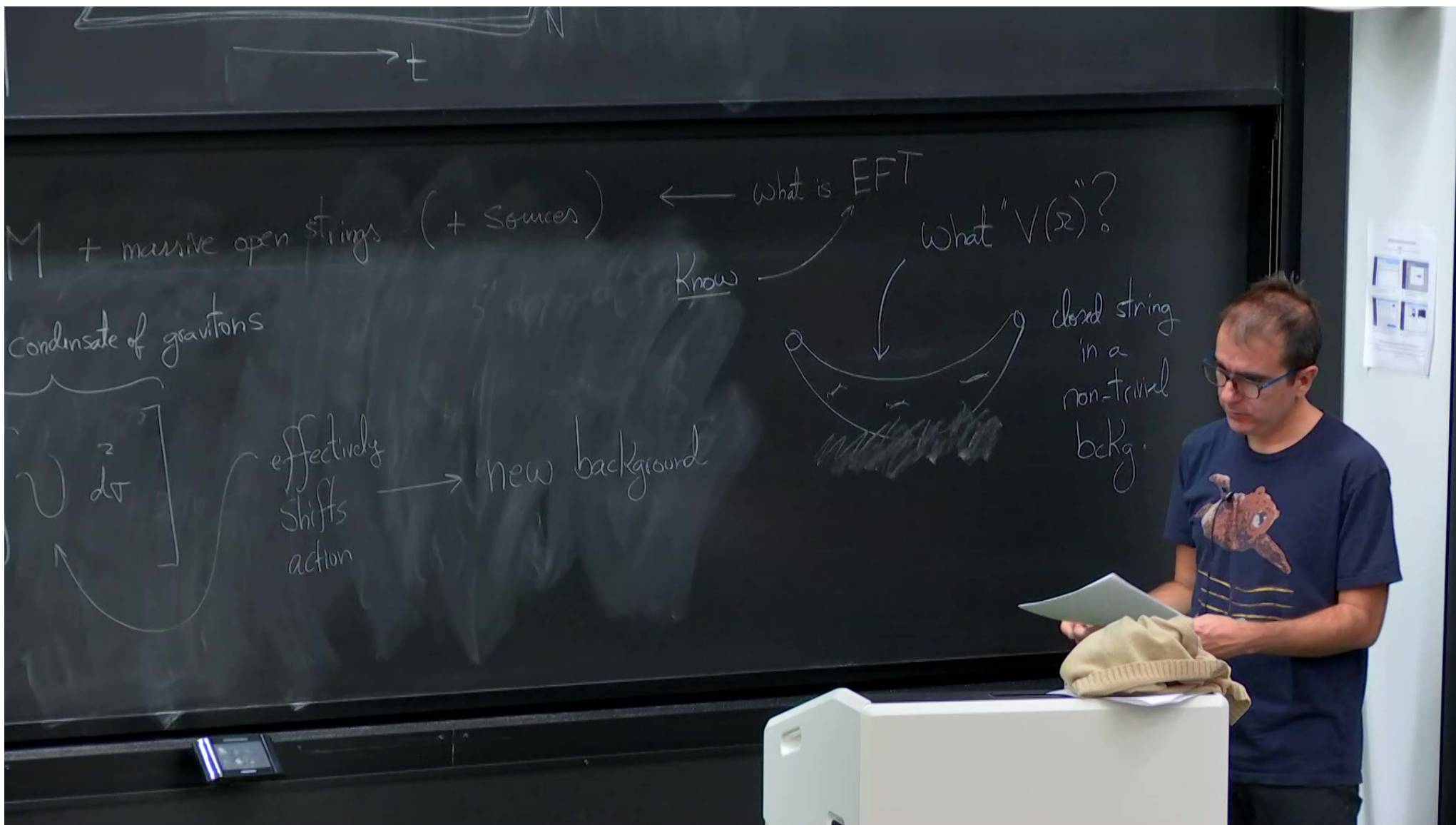


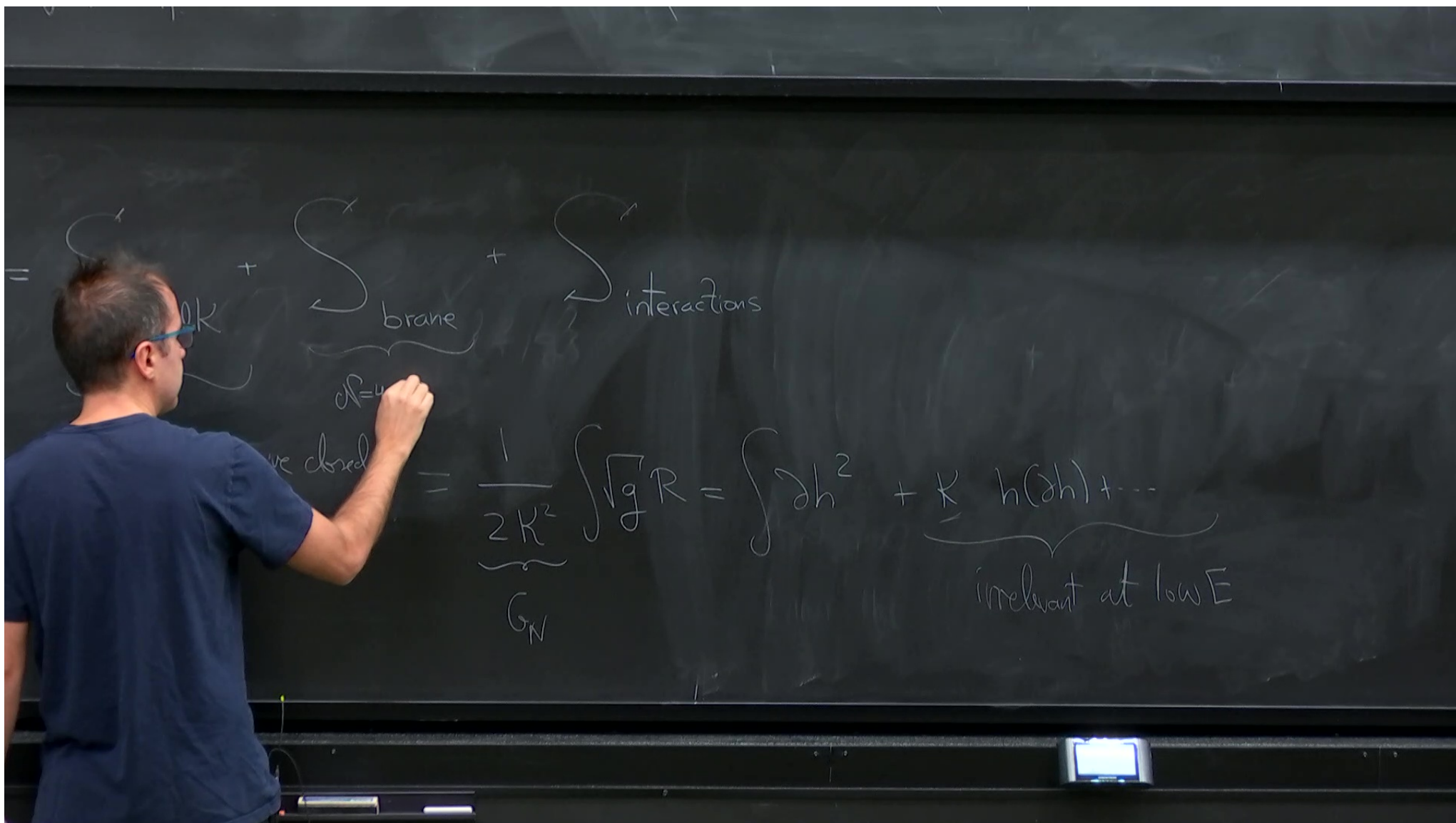
$\sim \exp$

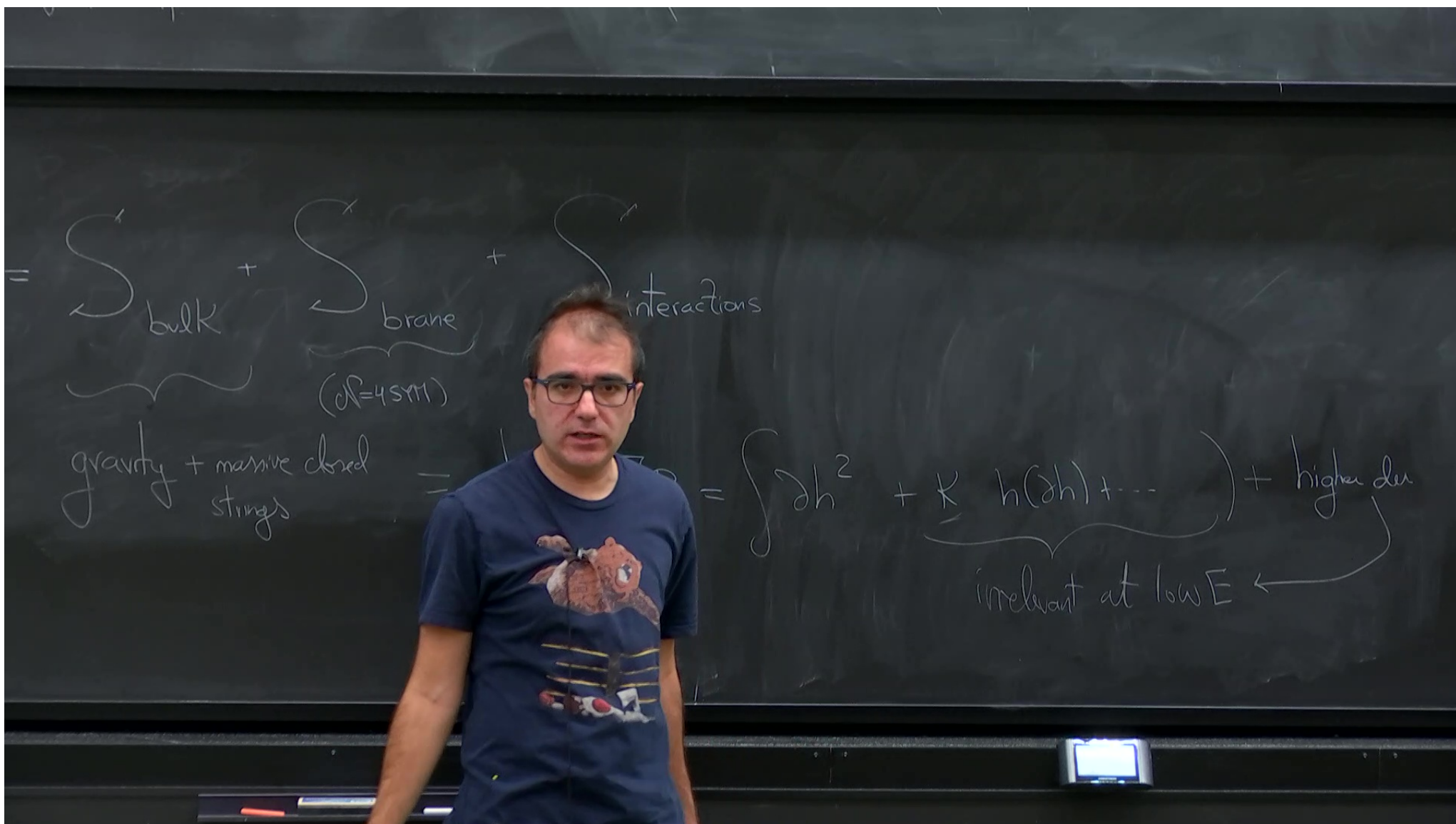
$$\left[\int \mathcal{L}^2 d\tau \right]$$

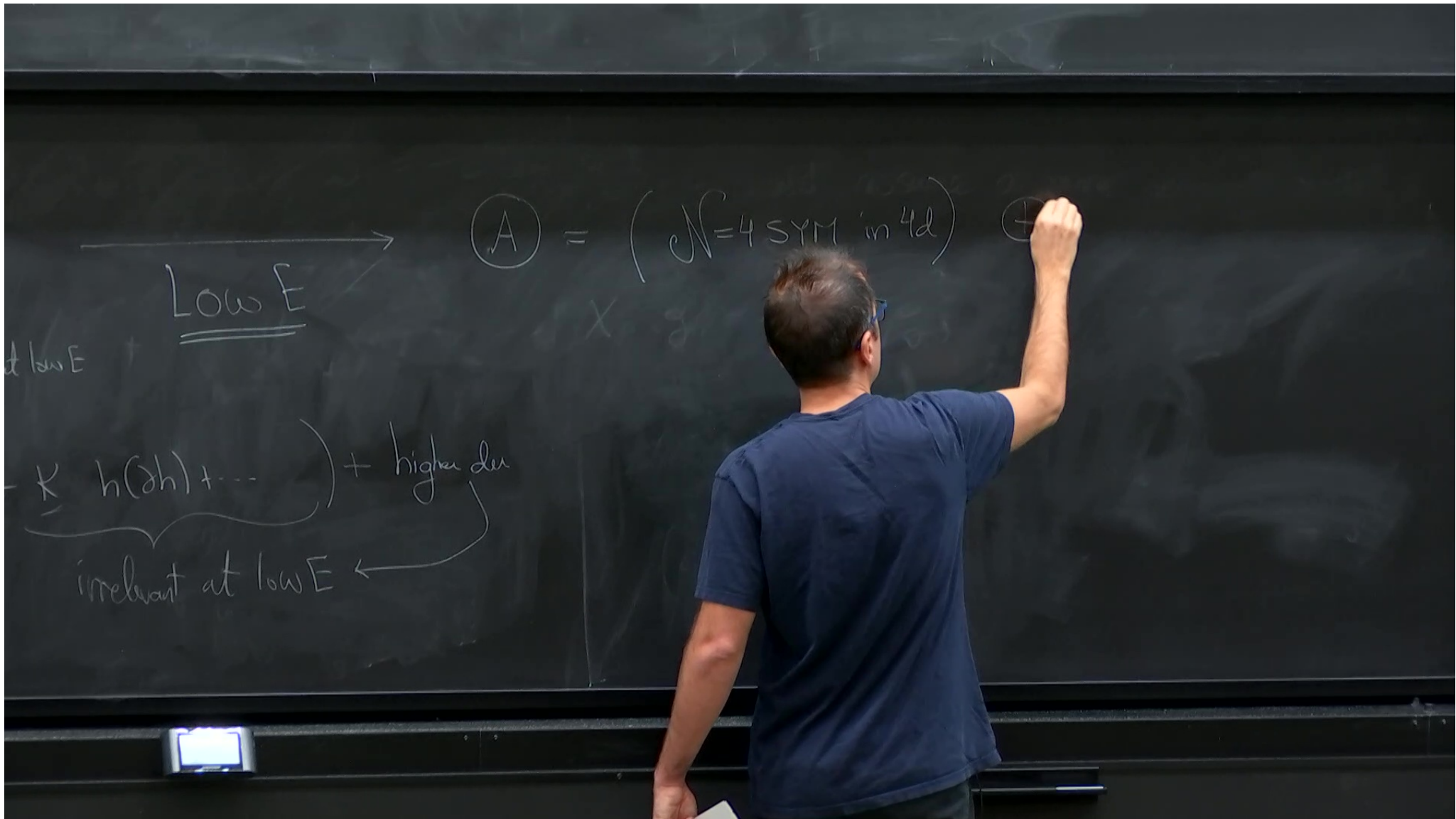


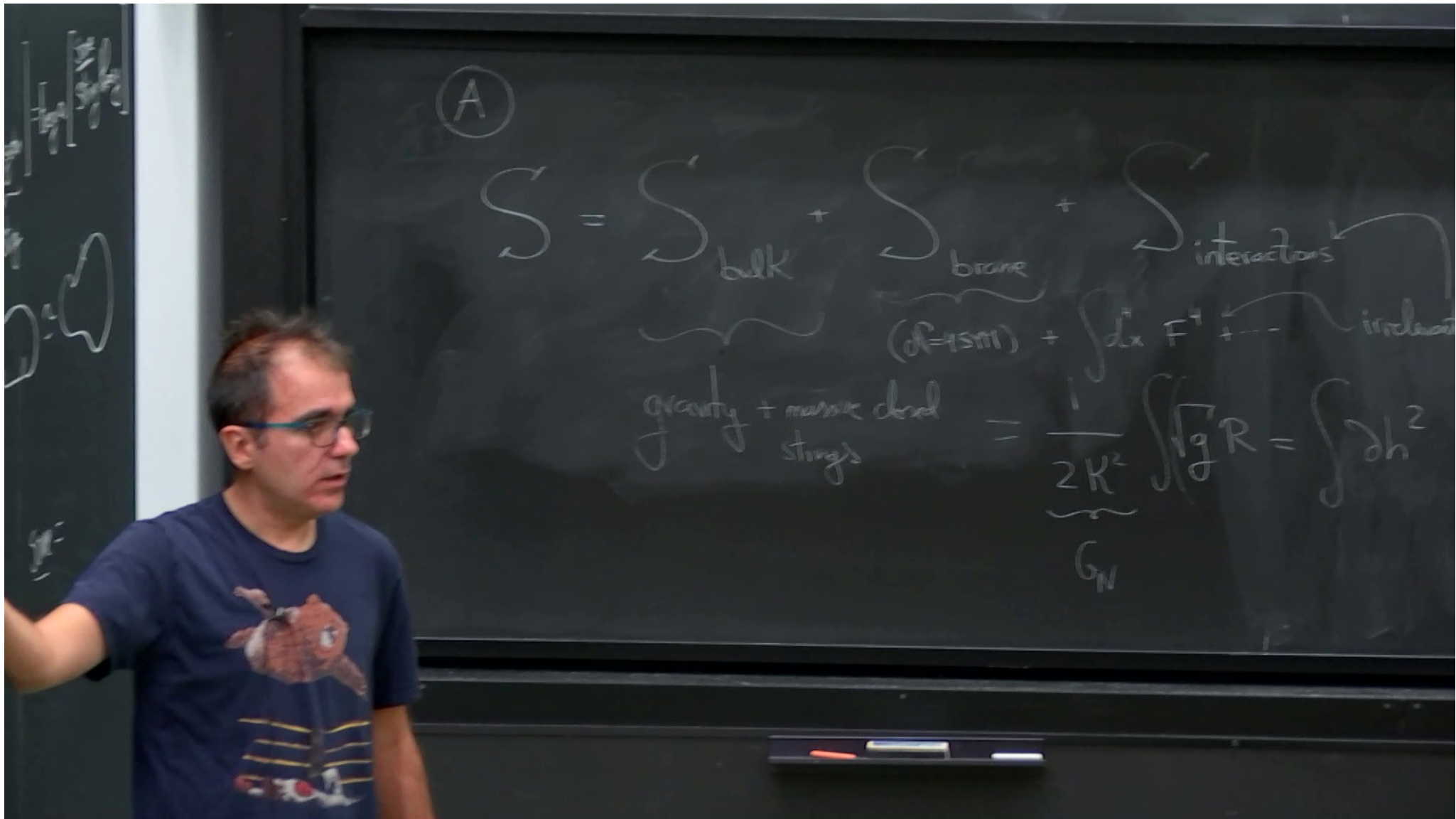










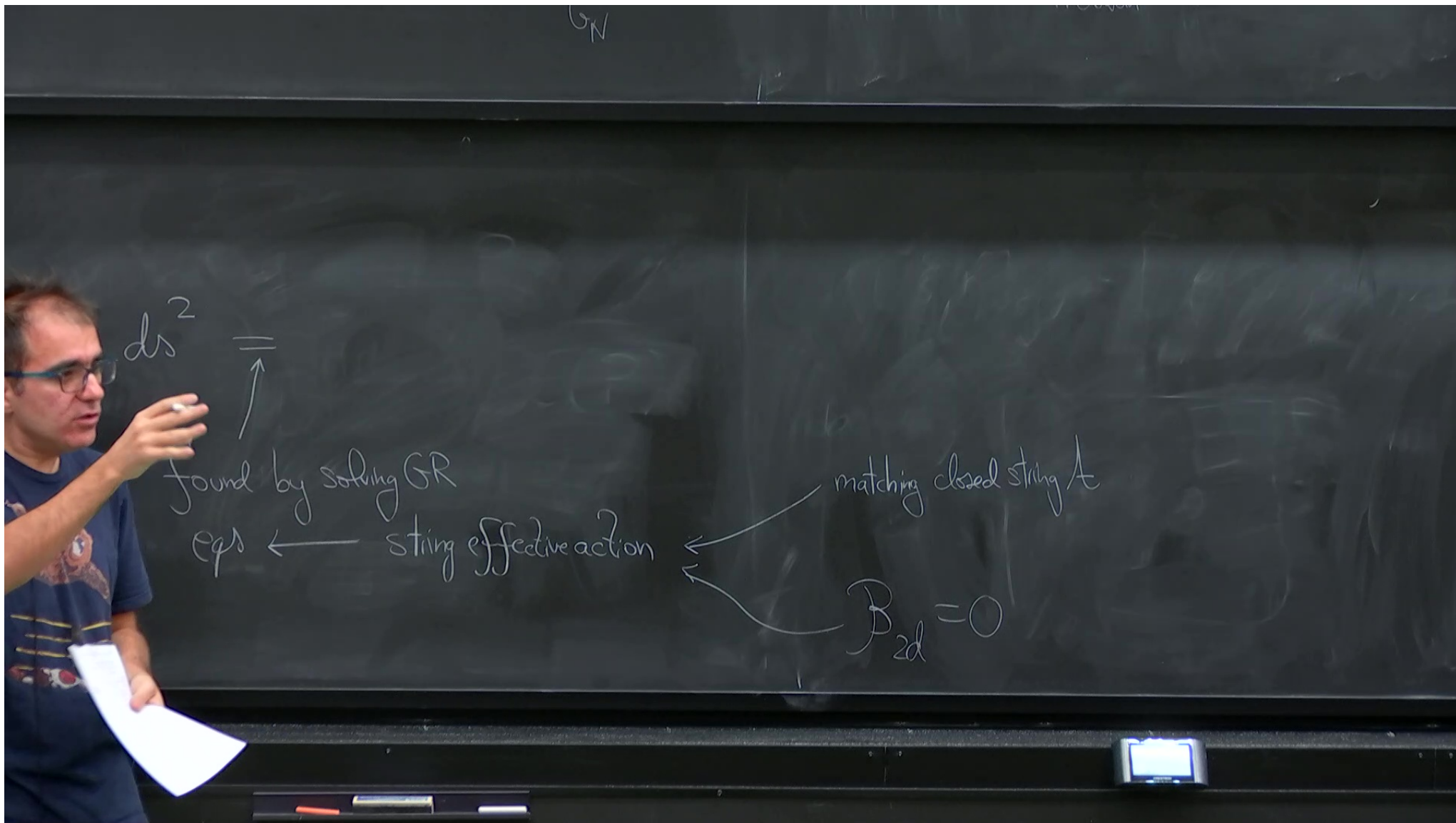


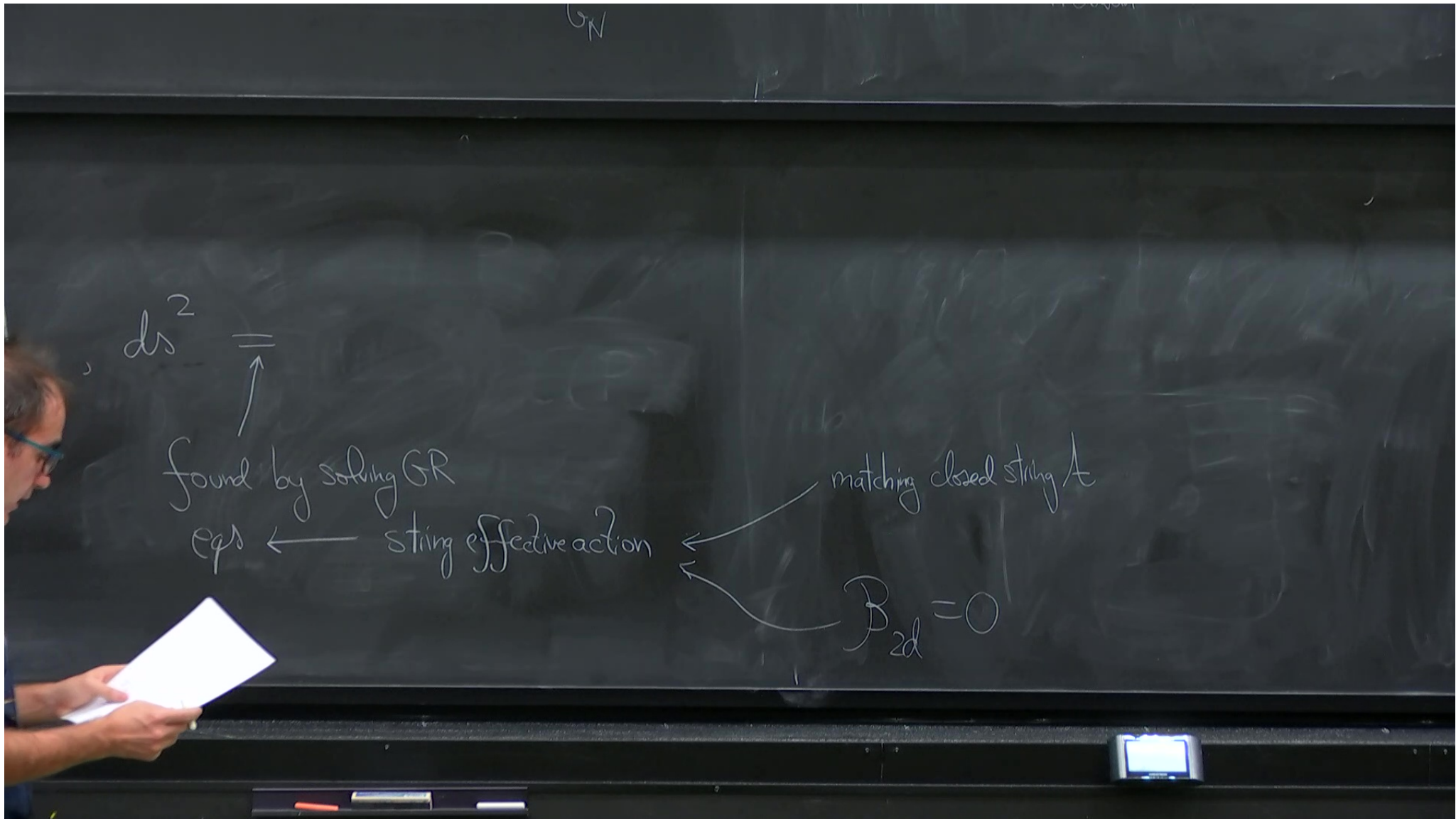
low E →

$$\textcircled{A} = \left(\mathcal{N}=4 \text{ SYM in } 4d \right) \oplus \left(10d \text{ SUGRA} \right)$$

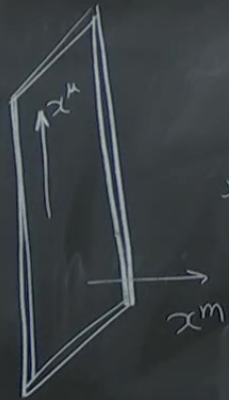
↑ easiest for $\lambda \ll 1$

$(1 + \dots) + \text{higher der}$
 at low E ←





(B)



ds^2

$$ds^2 = \frac{\eta_{\mu\nu} dx^\mu dx^\nu}{\sqrt{1 + \frac{L^4}{\Omega^4}}} + \sqrt{1 + \frac{L^4}{\Omega^2}} dx^m dx^m$$

Annotations: An arrow points from $\eta_{\mu\nu} dx^\mu dx^\nu$ to $0..3$. Another arrow points from $dx^m dx^m$ to $1...6$.

found by solving GR

eqs ← string effective action

string A

Where $\sigma = \sqrt{X^m X^m}$
 and $L^4 = g_s^2 N 4\pi \alpha'^2$

$g_s \rightarrow \infty$
 $g_{YM}^2 \rightarrow 0$
 λ fixed

easiest for
 $\lambda \gg 1$

L is large

Where $\sigma = \sqrt{X^m X^m}$
 and $L^4 = g_s N 4\pi \alpha'^2$

$g_s \equiv g_{YM}^2$ λ fixed

$0 \leftarrow$

easiest for
 $\lambda \gg 1$

L is large
 (L is many string lengths)

$\pi \gg L$ (far from branes)

$\pi \ll L$ (close to branes)

$$ds^2 \simeq \frac{\pi^2}{L^2} \eta_{\mu\nu} dx^\mu dx^\nu + \frac{L^2}{\pi^2} dr^2 + L^2 d\Omega_2^2$$

→ Flat 10 d

$r \gg L$ (far from branes) $\longrightarrow$ Flat 10 d

$r \ll L$ (close to branes) $\xrightarrow{6d}$

$$ds^2 \simeq \frac{r^2}{L^2} dt^2 + \frac{L^2}{r^2} dr^2 + \underbrace{L^2 d\Omega_5^2}_{S^5}$$

$$r = \frac{L}{z}$$

$r \gg L$ (far from branes) $\longrightarrow$ Flat 10 d

$r \ll L$ (close to branes) $\overset{6d}{\curvearrowright}$

$$ds^2 \simeq \frac{r^2}{L^2} \eta_{\mu\nu} dx^\mu dx^\nu + \frac{L^2}{r^2} dr^2 + L^2 d\Omega_5^2$$

$r = \frac{L}{z}$

$$\frac{\eta_{\mu\nu} dx^\mu dx^\nu + dz^2 L^2}{z^2} = \text{AdS}_5 \times S^5$$

(far from branes) →

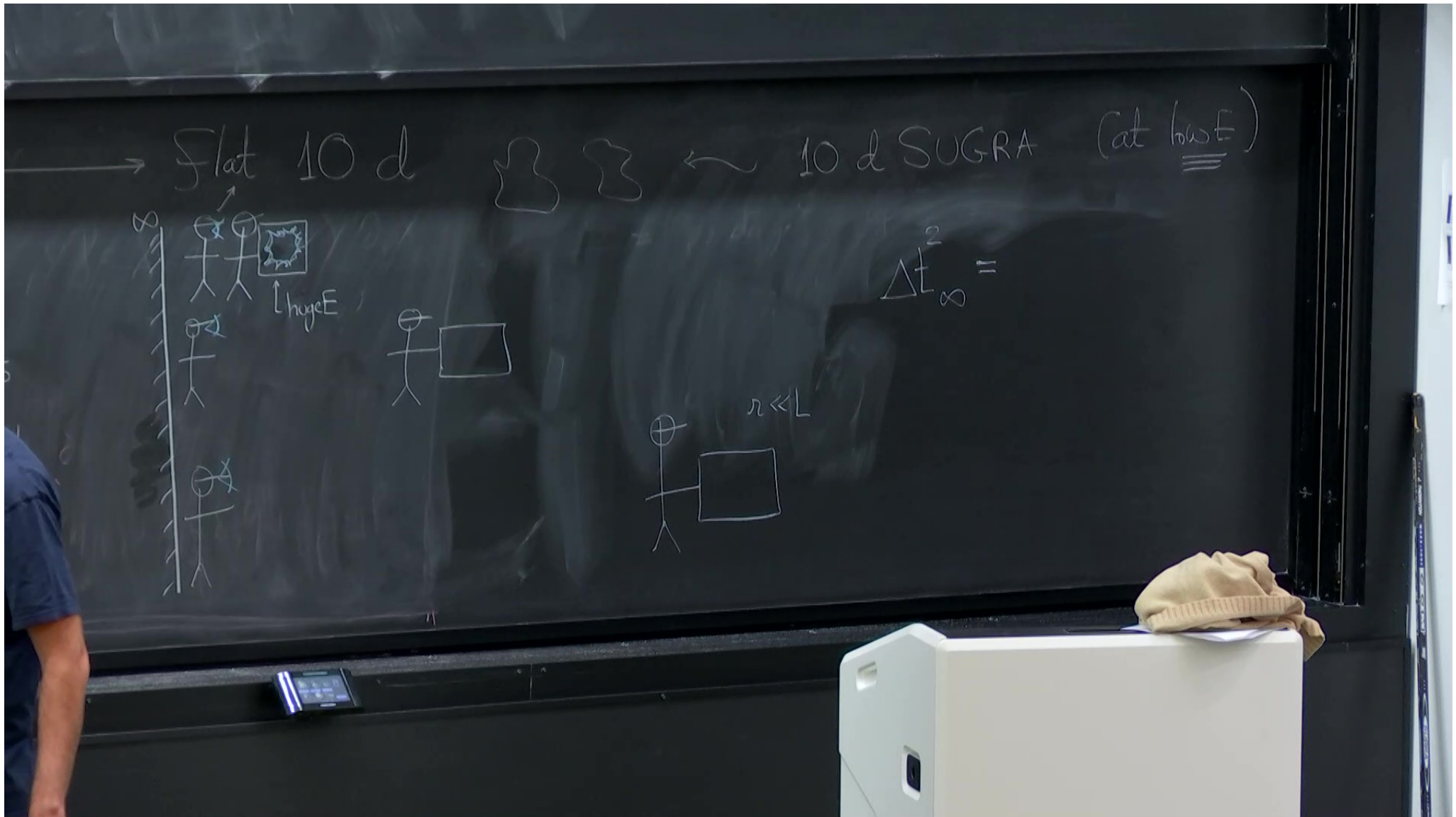
(close to branes)

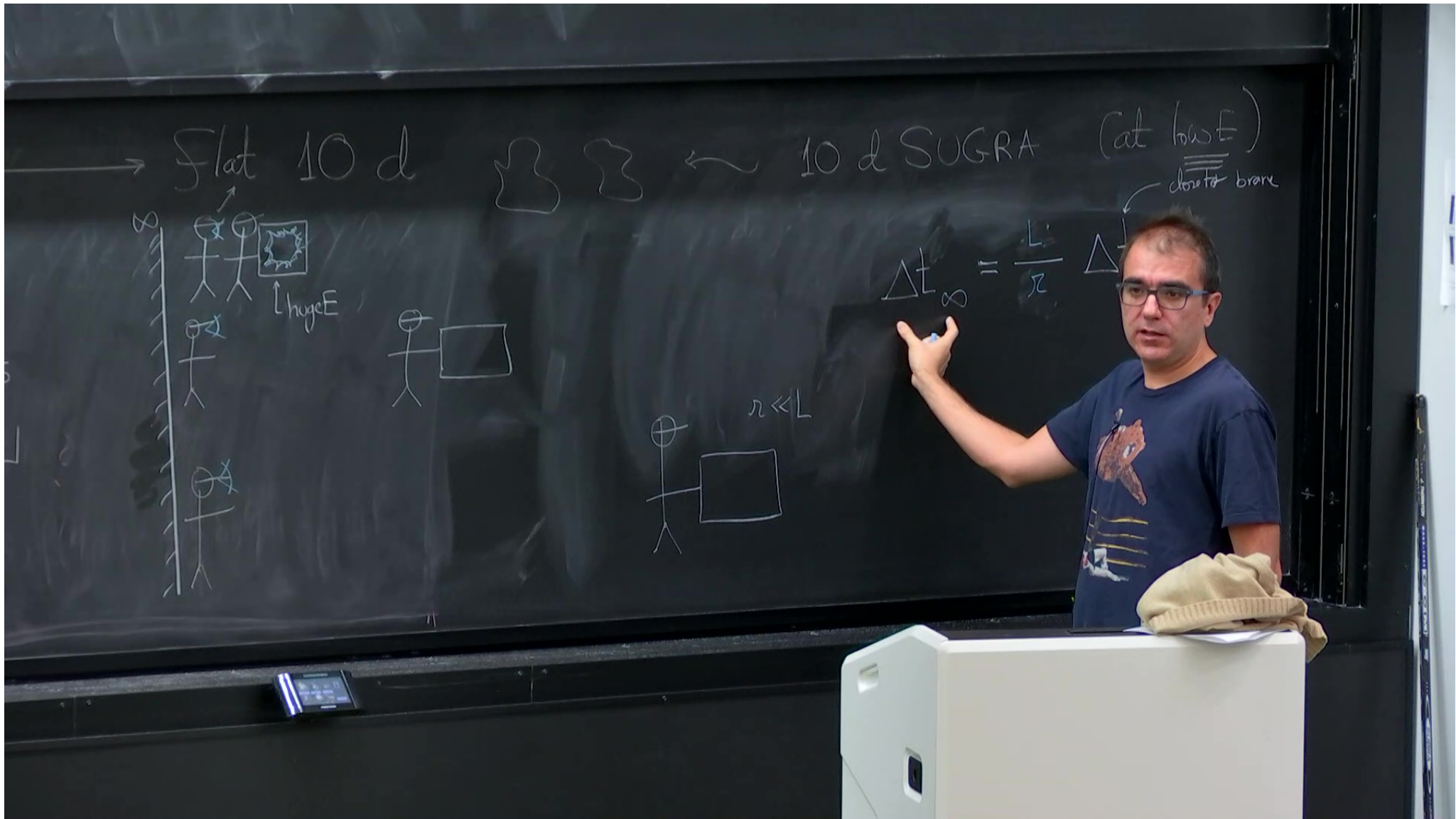
$$\frac{\pi^2}{L^2} \eta_{\mu\nu} dx^\mu dx^\nu + \frac{L^2}{\pi^2} dr^2 + L^2 d\Omega_5^2$$

$$\frac{\eta_{\mu\nu} dx^\mu dx^\nu + dz^2 L^2}{z^2} = \text{AdS}_5 \times S^5$$

Flat 10 d

Diagram showing two vertical lines representing branes. Between them is a box with a jagged top edge, labeled 'huge E' with an upward arrow. Above the box are two circles. To the right of the box is a small cloud-like shape.



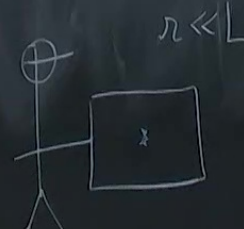
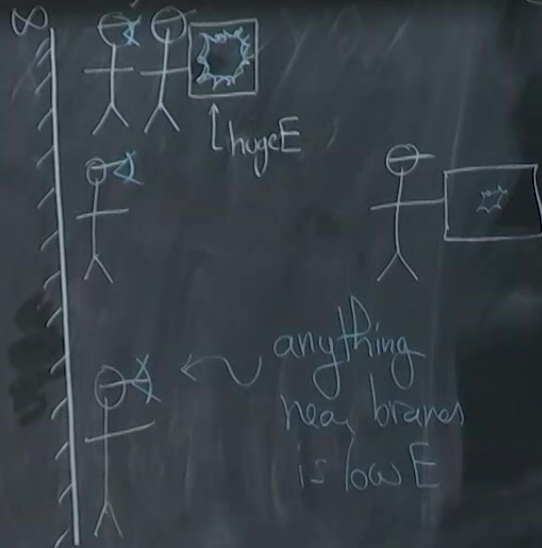


→ Flat 10 d



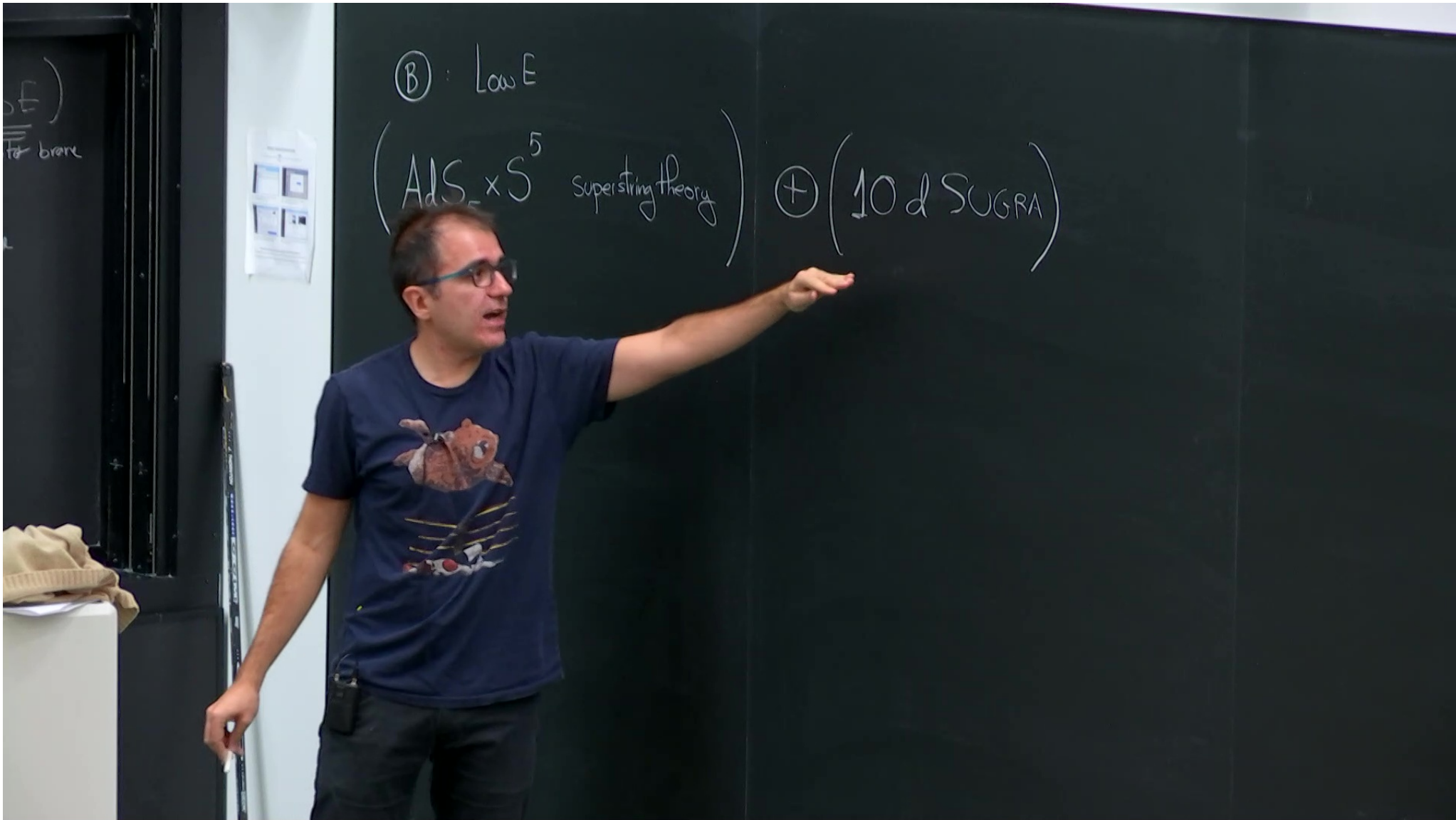
← 10 d SUGRA

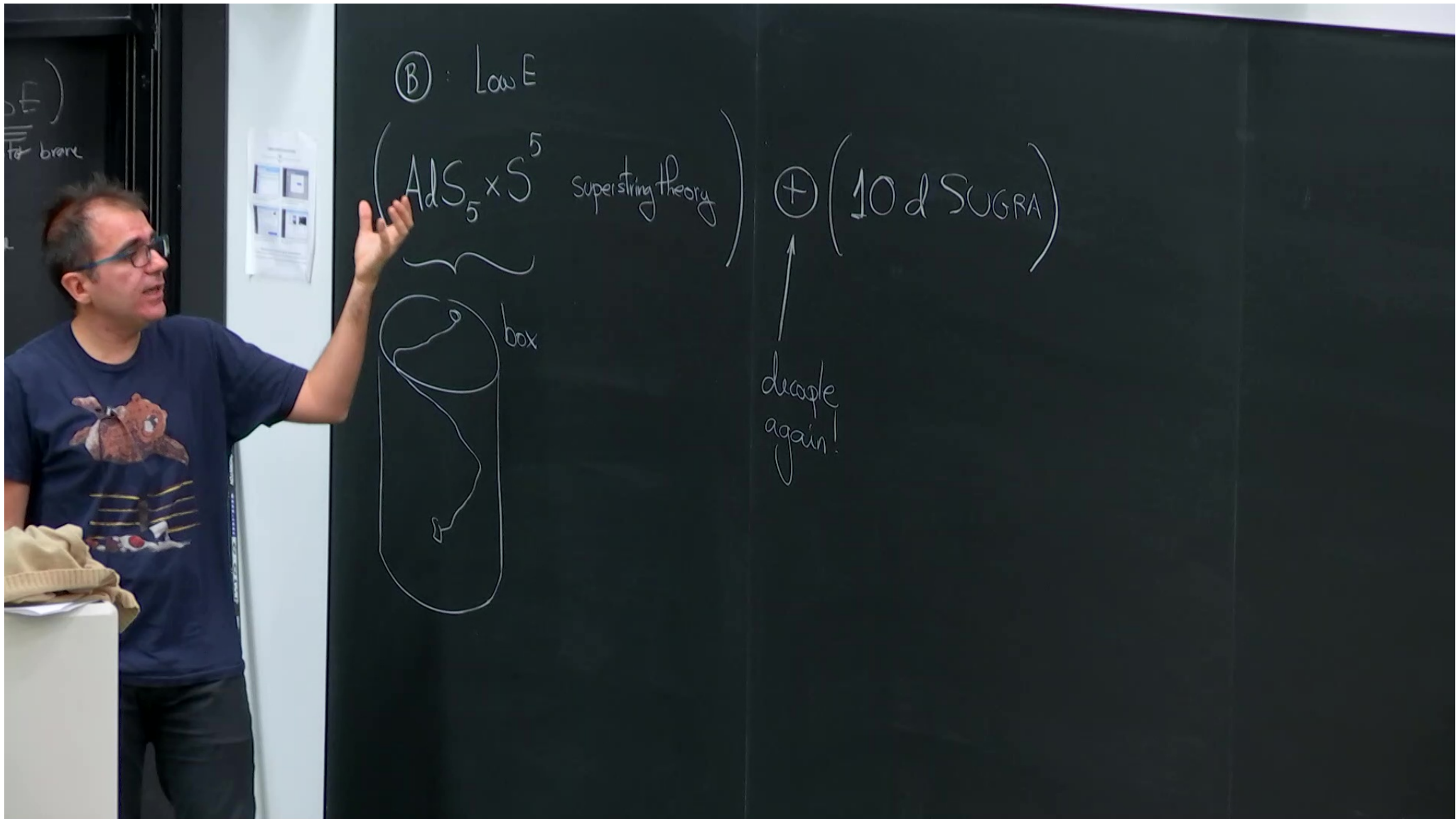
(at low E)
dense brane



$$\Delta t_{\infty} = \frac{L}{r} \Delta t_{\text{proper}}$$

$$E_{\infty} = \frac{r}{L} E_{\text{proper}}$$





Low E →

$$(A) = (\mathcal{N}=4 \text{ SYM in } 4d) \oplus (10d \text{ SUGRA})$$

↑ easiest for $\lambda \ll 1$

... + higher der
at low E ←

$$4d \mathcal{N}=4 \text{ SYM} = \text{type IIB } 10d \text{ Superstrings in } \text{AdS}_5 \times S^5$$

↑ AdS/CFT duality.

