Title: Classical and Quantum Chaos 2021/2022 - Lecture 7

Speakers: Meenu Kumari

Collection: Classical and Quantum Chaos 2021/2022

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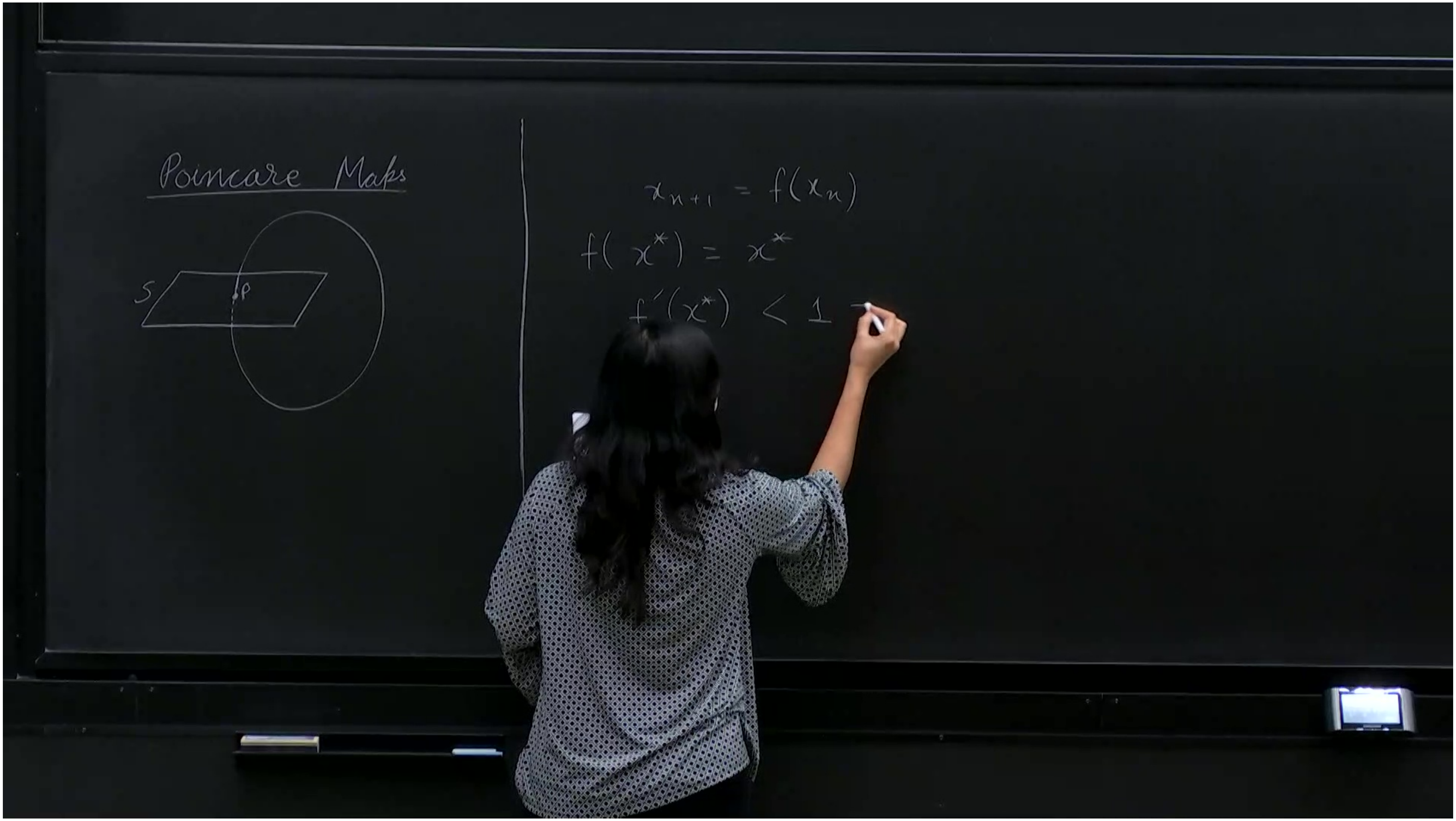
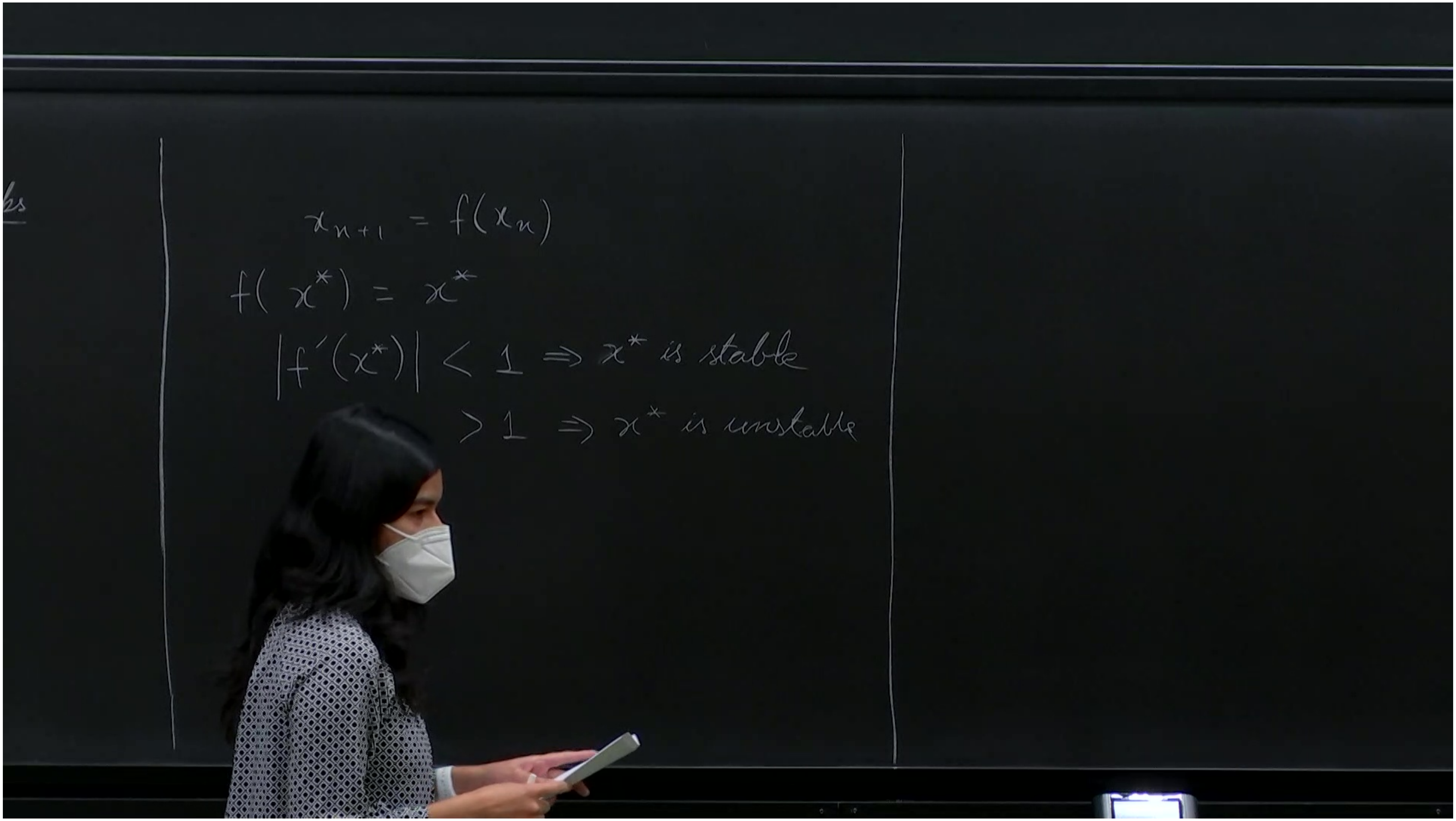
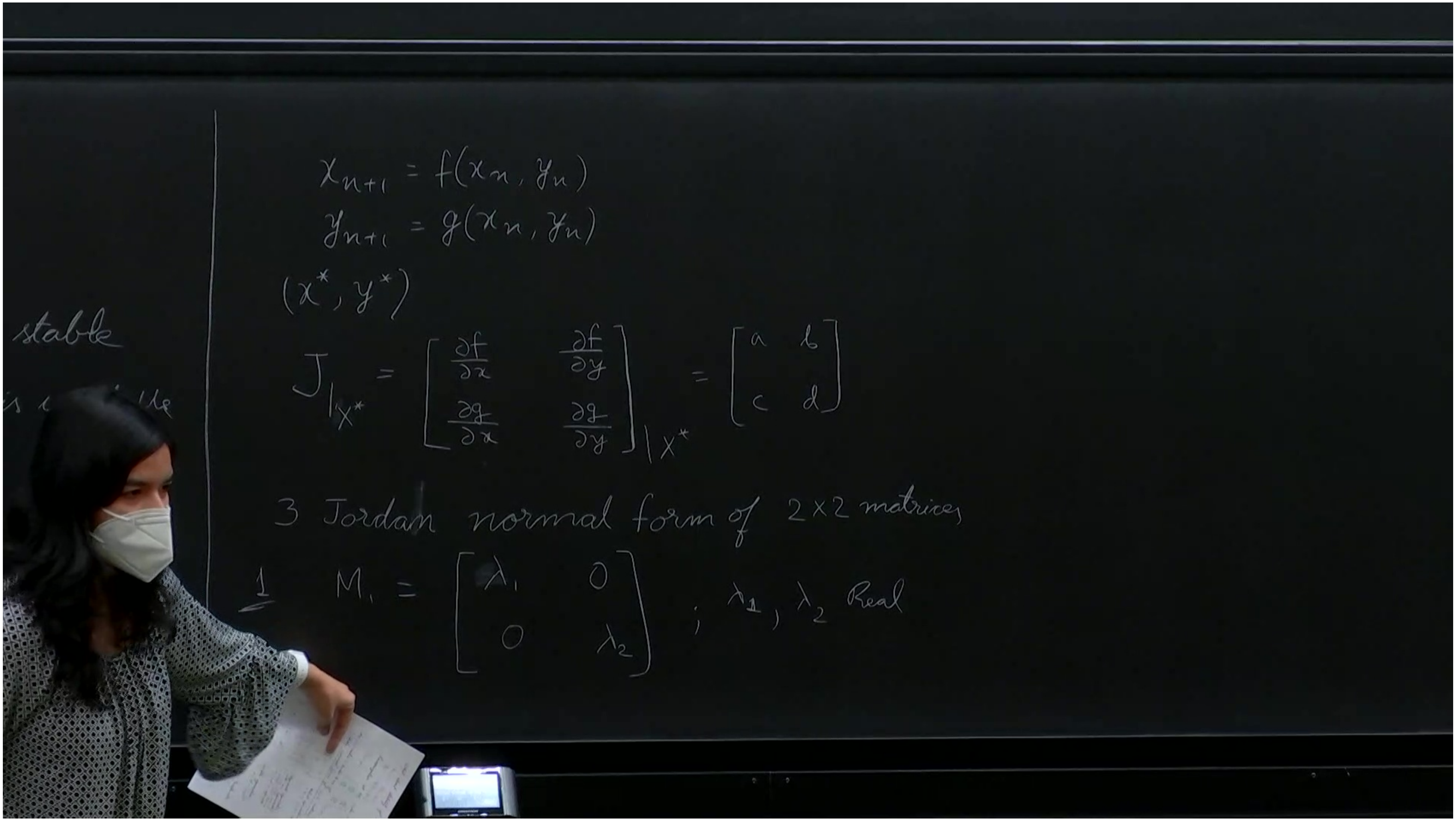
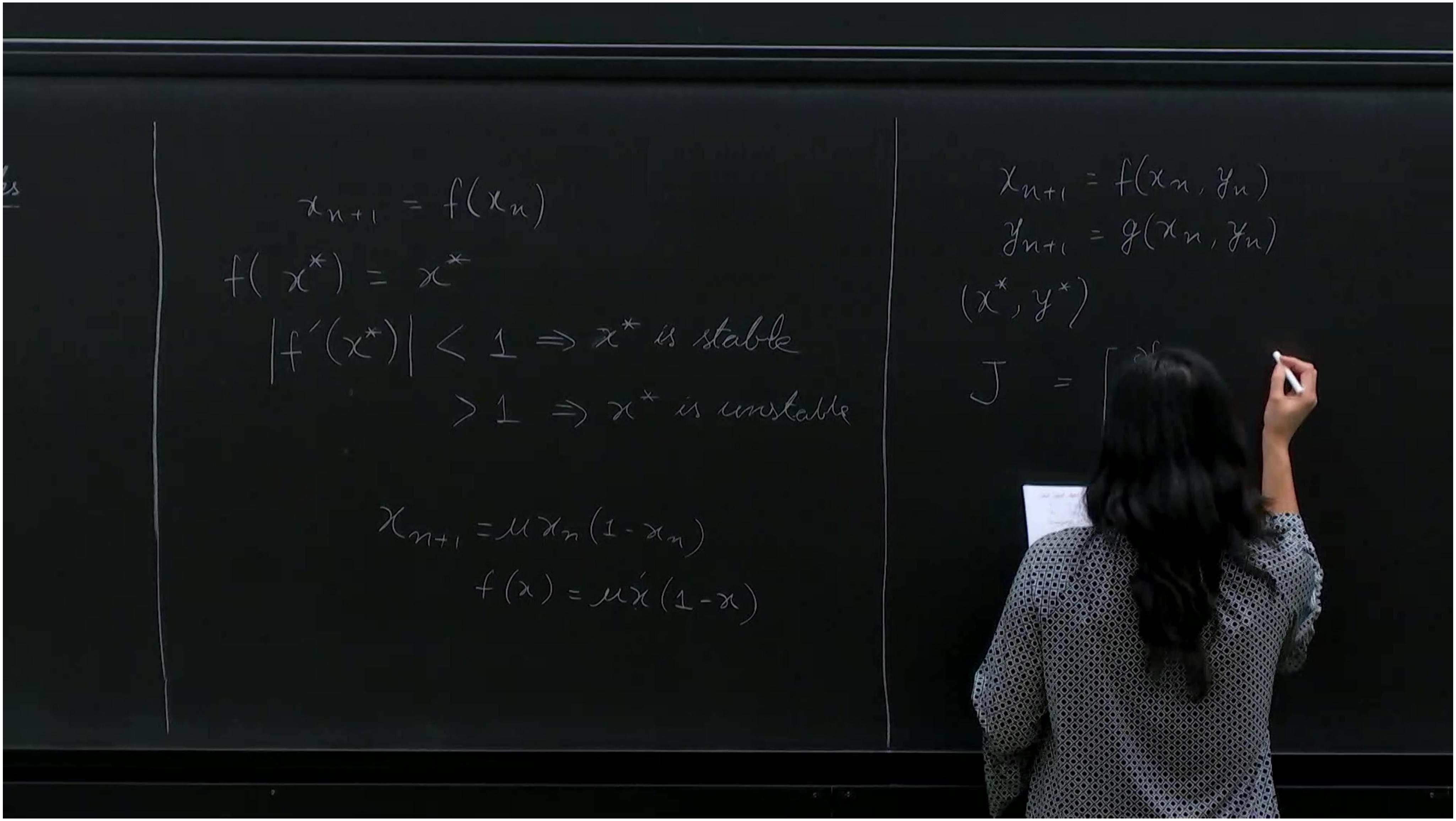
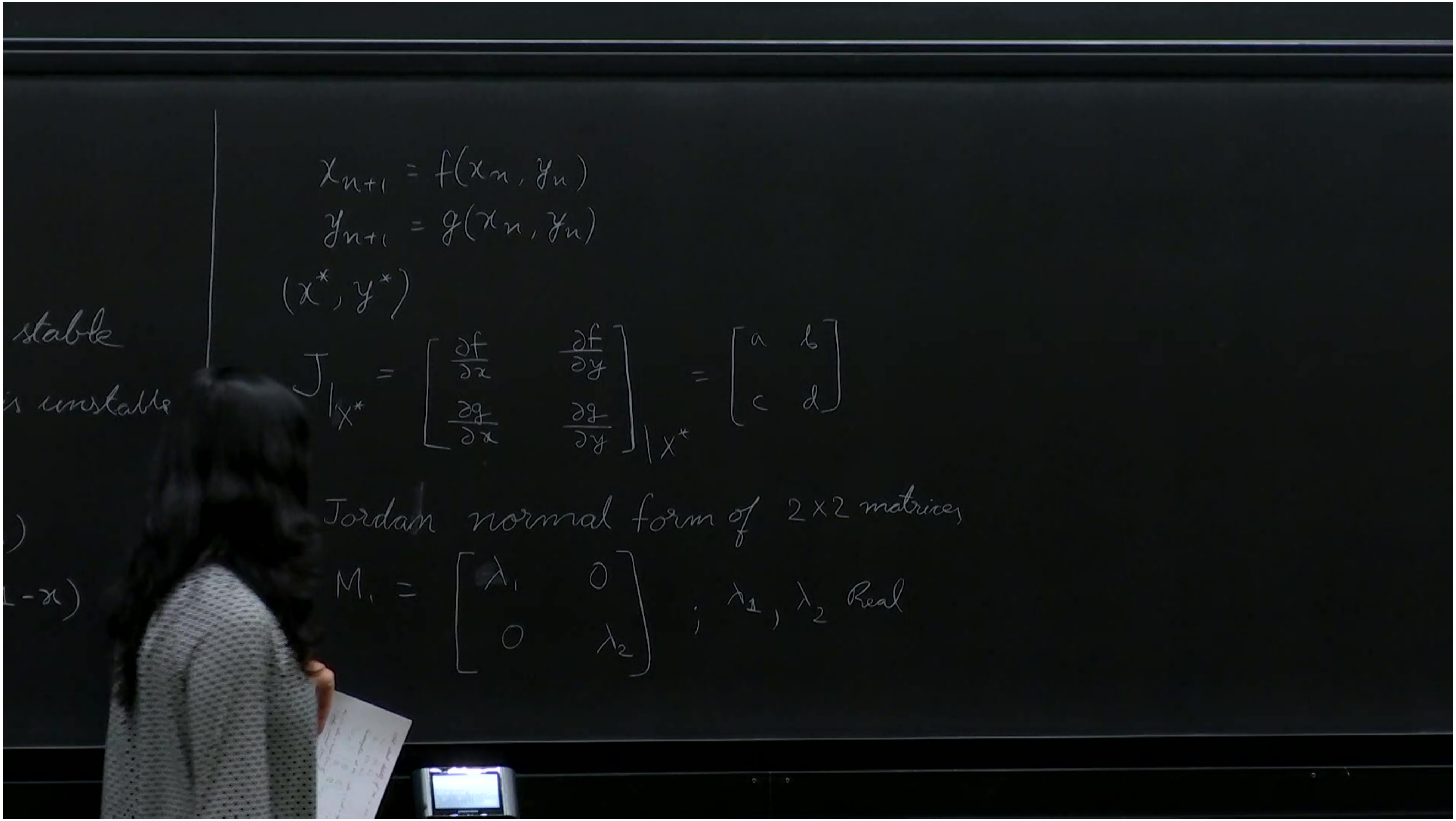

Poincare Maps
S
P
$x_{n+1} = f(x_n)$
$f(x^*) = x^*$
$f'(x^*) < 1$


$x_{n+1} = f(x_n)$
$f(x^*) = x^*$
$|f'(x^*)| < 1 \Rightarrow x^*$ is stable
$> 1 \Rightarrow x^*$ is unstable


stable
$x_{n+1} = f(x_n, y_n)$
$y_{n+1} = g(x_n, y_n)$
(x^*, y^*)
$J|_{X^*} = \begin{bmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \\ \frac{\partial g}{\partial x} & \frac{\partial g}{\partial y} \end{bmatrix}_{|X^*} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$
3 Jordan normal form of 2×2 matrices
1 $M_1 = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$; λ_1, λ_2 Real


$x_{n+1} = f(x_n)$
$f(x^*) = x^*$
$|f'(x^*)| < 1 \Rightarrow x^*$ is stable
$> 1 \Rightarrow x^*$ is unstable
$x_{n+1} = \mu x_n (1 - x_n)$
$f(x) = \mu x (1 - x)$
$x_{n+1} = f(x_n, y_n)$
$y_{n+1} = g(x_n, y_n)$
(x^*, y^*)
$J =$



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Jordan normal form of 2×2 matrices
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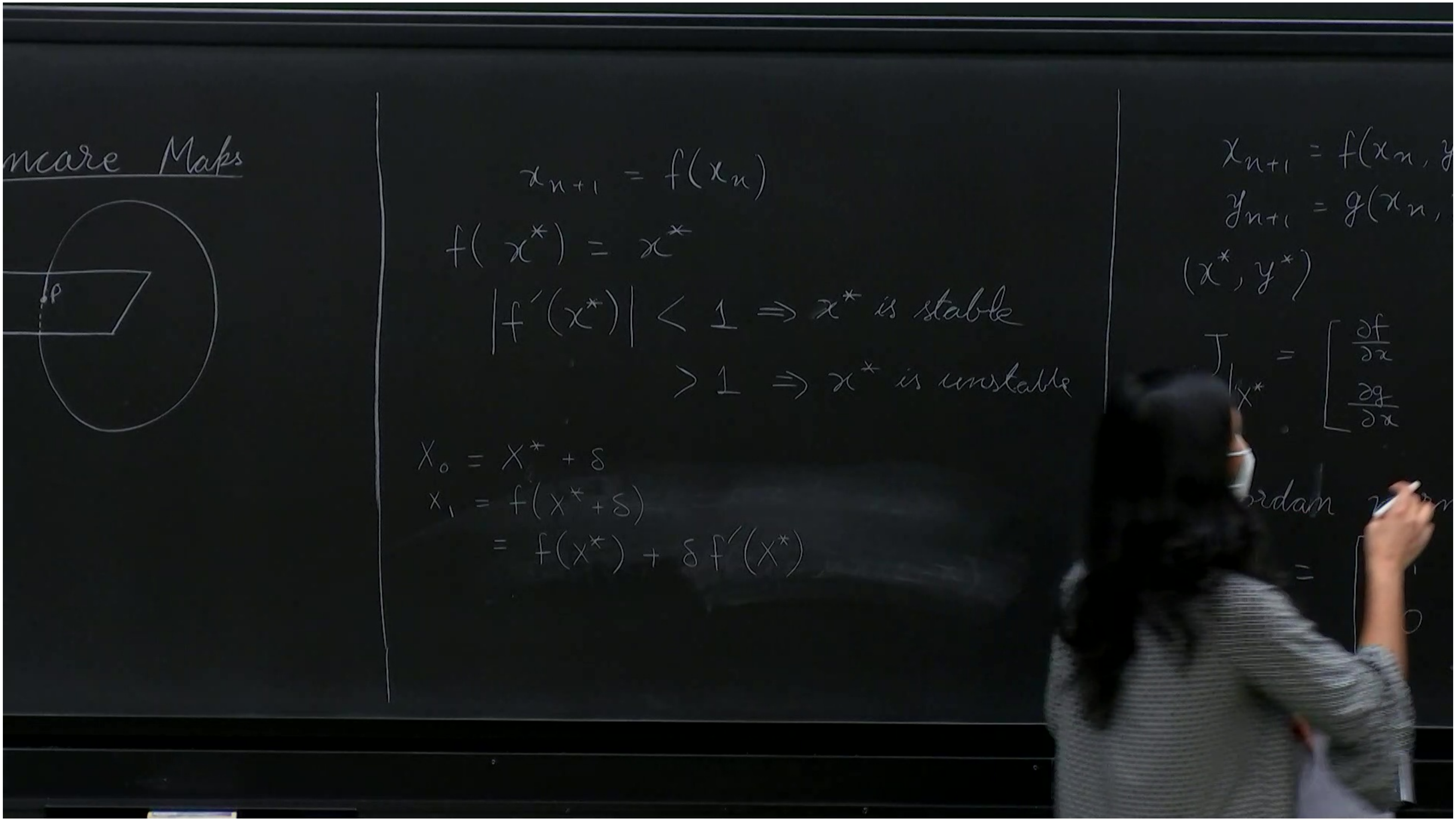
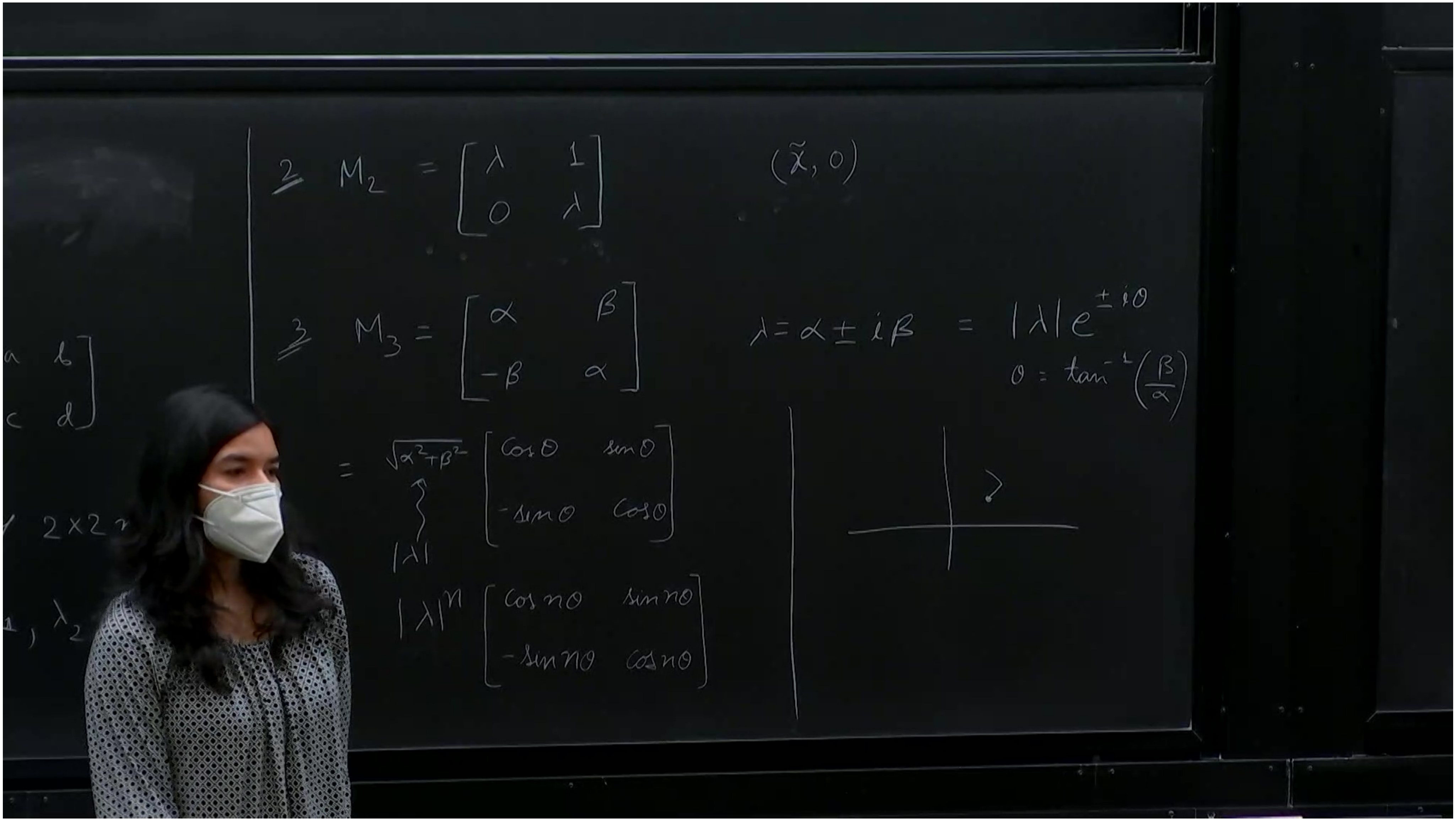

ncare Maps
P
$x_{n+1} = f(x_n)$
$f(x^*) = x^*$
$|f'(x^*)| < 1 \Rightarrow x^*$ is stable
$> 1 \Rightarrow x^*$ is unstable
$X_0 = X^* + \delta$
$X_1 = f(X^* + \delta)$
$= f(X^*) + \delta f'(X^*)$
$x_{n+1} = f(x_n, y$
$y_{n+1} = g(x_n,$
(x^*, y^*)
$J|_{X^*} = \Big[\frac{\partial f}{\partial x} \quad \frac{\partial g}{\partial x}$
ordan


2 $M_2 = \begin{bmatrix} \lambda & 1 \\ 0 & \lambda \end{bmatrix}$
$(\tilde{x}, 0)$
3 $M_3 = \begin{bmatrix} \alpha & \beta \\ -\beta & \alpha \end{bmatrix}$
$\lambda = \alpha \pm i\beta = |\lambda| e^{\pm i\theta}$
$\theta = \tan^{-1}\left(\frac{\beta}{\alpha}\right)$
$= \sqrt{\alpha^2+\beta^2} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$
$|\lambda|$
$|\lambda|^n \begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}$
2x2

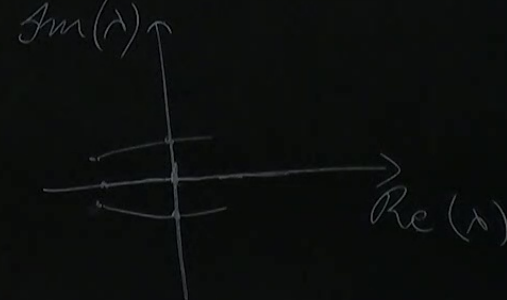
Stability of FPs in discrete maps

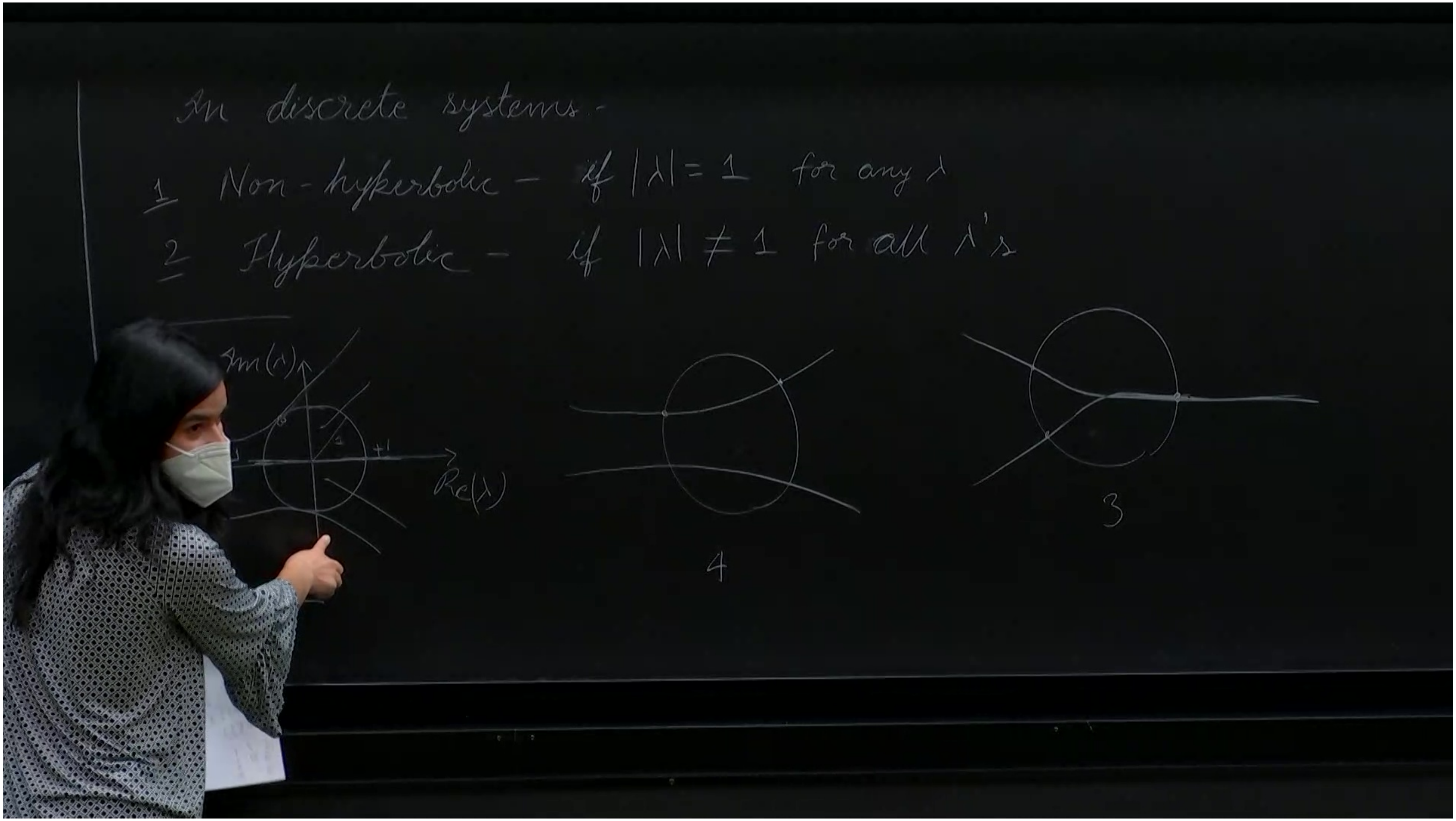
$|\lambda| < 1$ for all λ's

If $|\lambda| > 1$ for any λ, then x^* is unstable

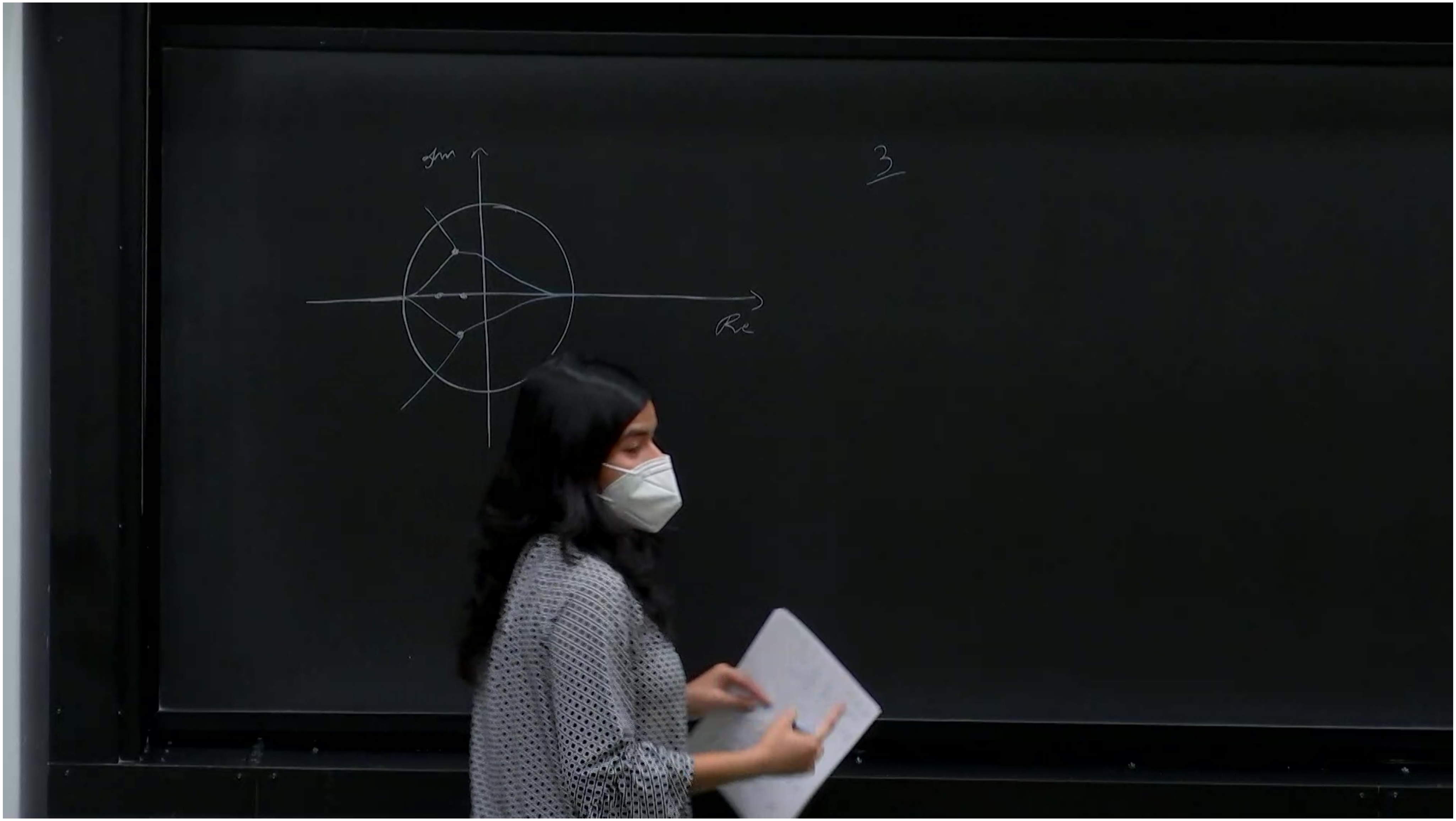
Hyperbolic = $Re(\lambda) \neq 0$ of all λ's

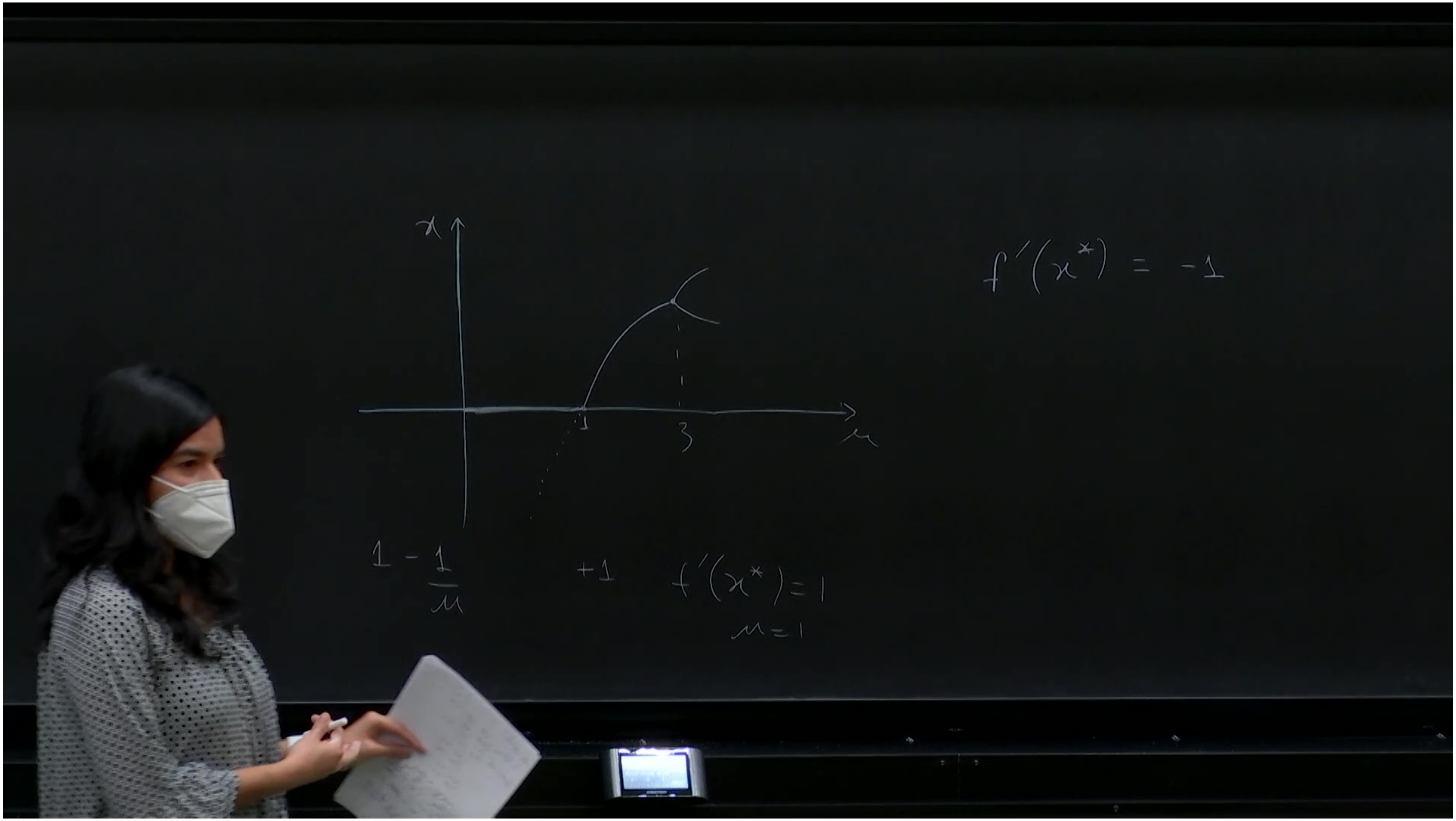
Non-hyperbolic



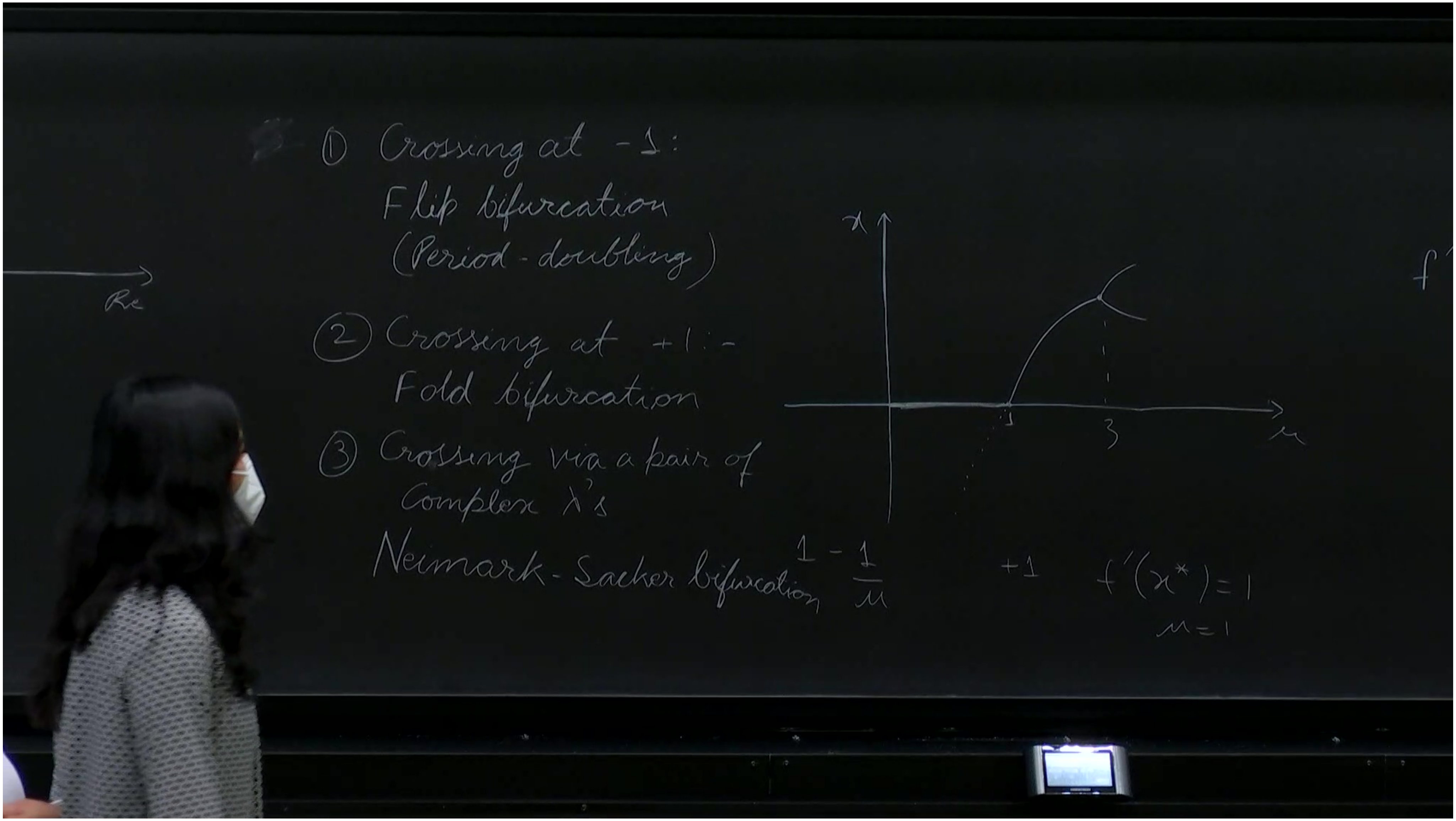


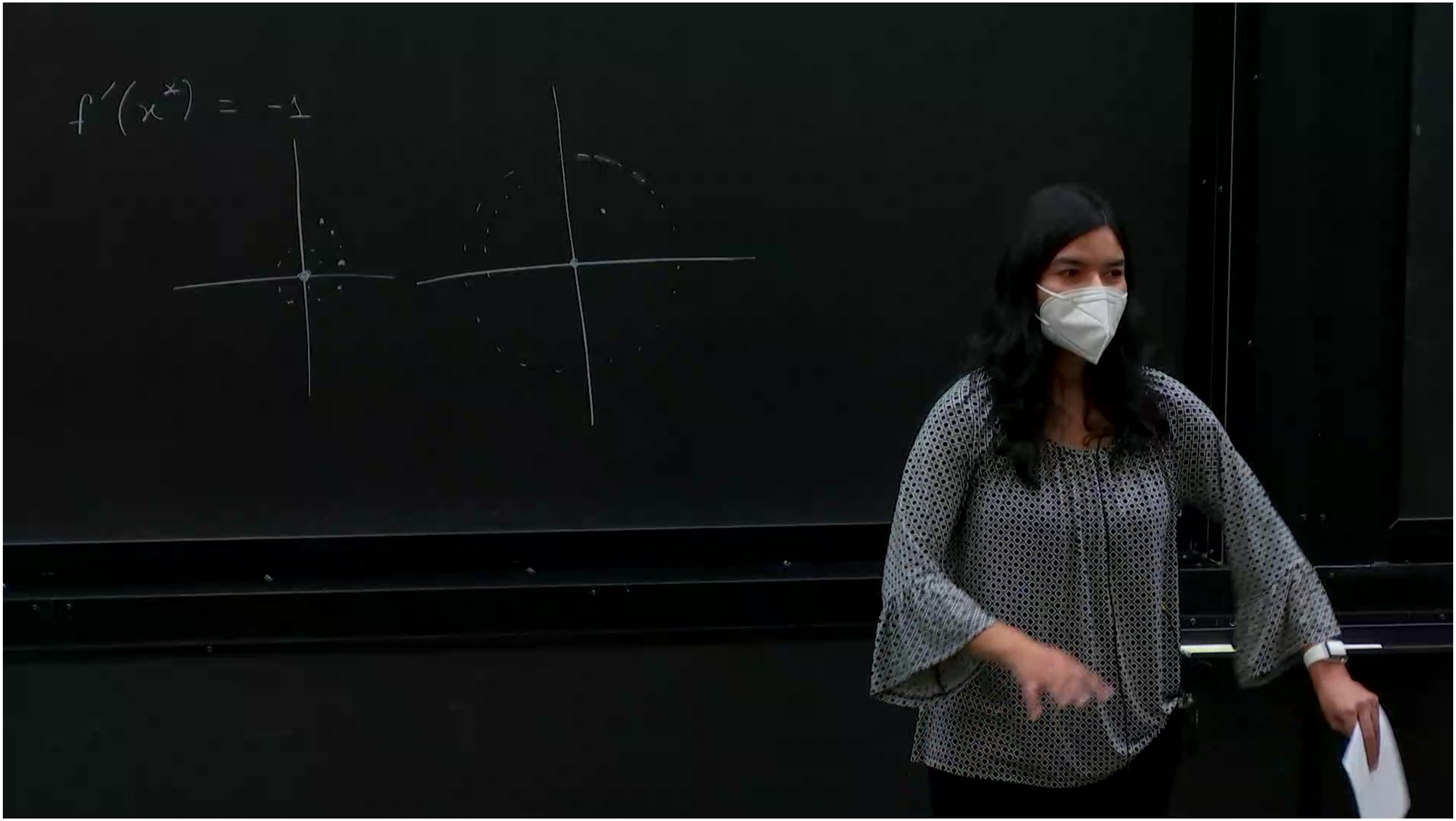
In discrete systems:-
1 Non-hyperbolic – if |λ| = 1 for any λ
2 Hyperbolic – if |λ| ≠ 1 for all λ's
Im(λ)
+1
Re(λ)
4
3

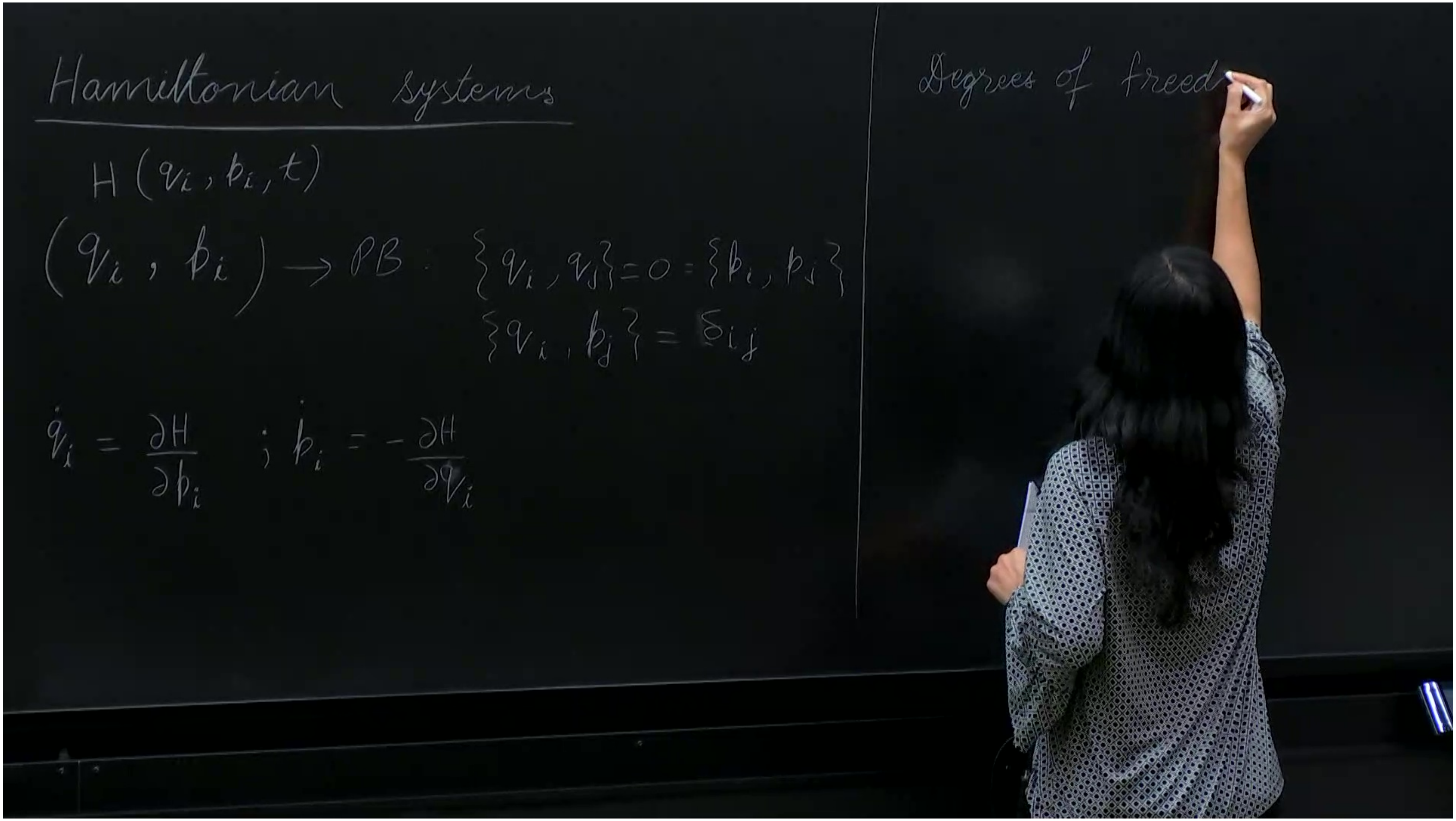

Im
Re
3

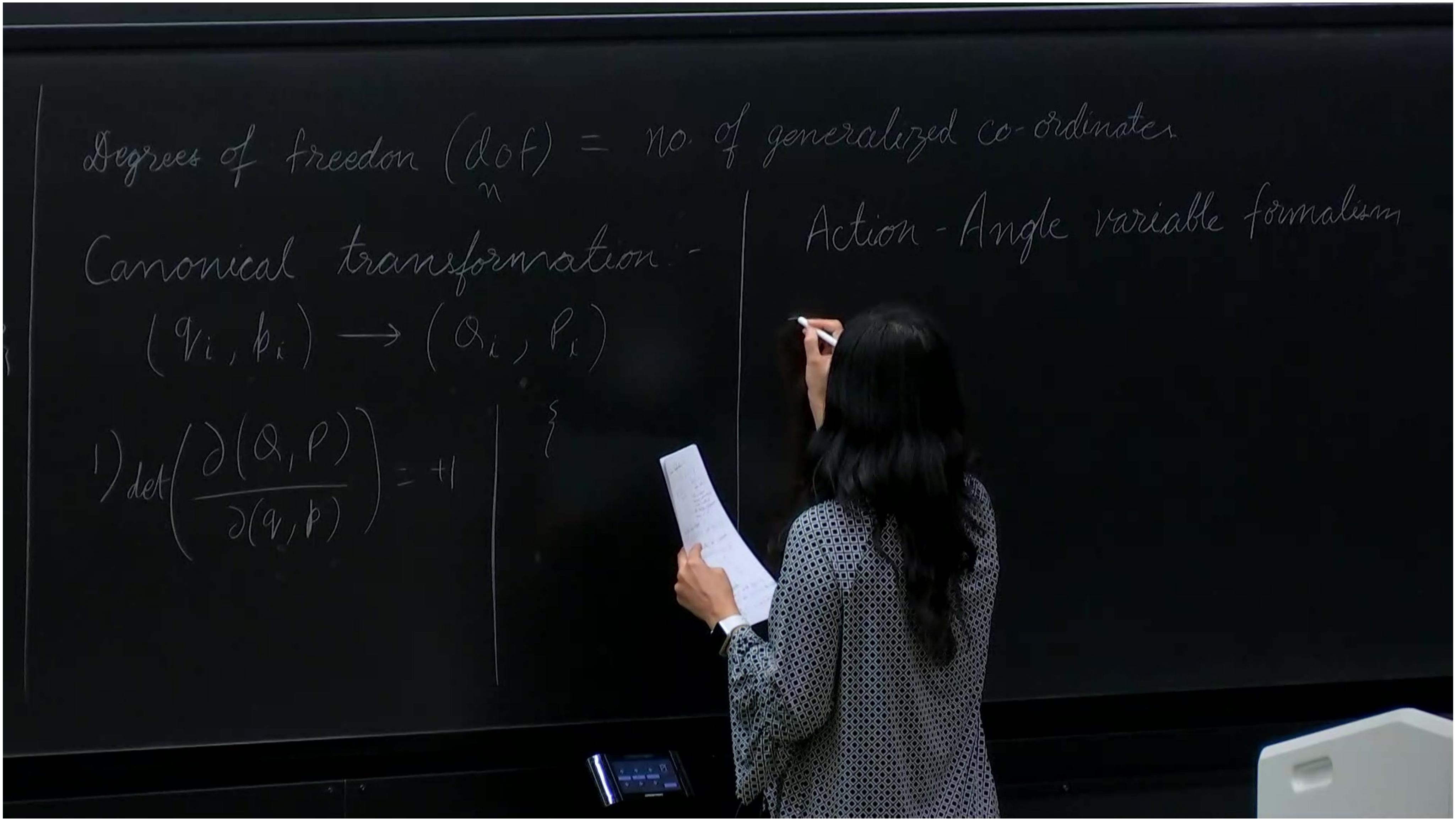


x
1
3
μ
f'(x*) = -1
1 - 1/μ
+1
f'(x*) = 1
μ = 1


Re
① Crossing at -1:
Flip bifurcation
(Period-doubling)
② Crossing at +1:-
Fold bifurcation
③ Crossing via a pair of
Complex λ's
Neimark-Sacker bifurcation
x
μ
1
3
1 - 1/μ
+1
f′(x*) = 1
μ = 1


$f'(x^*) = -1$

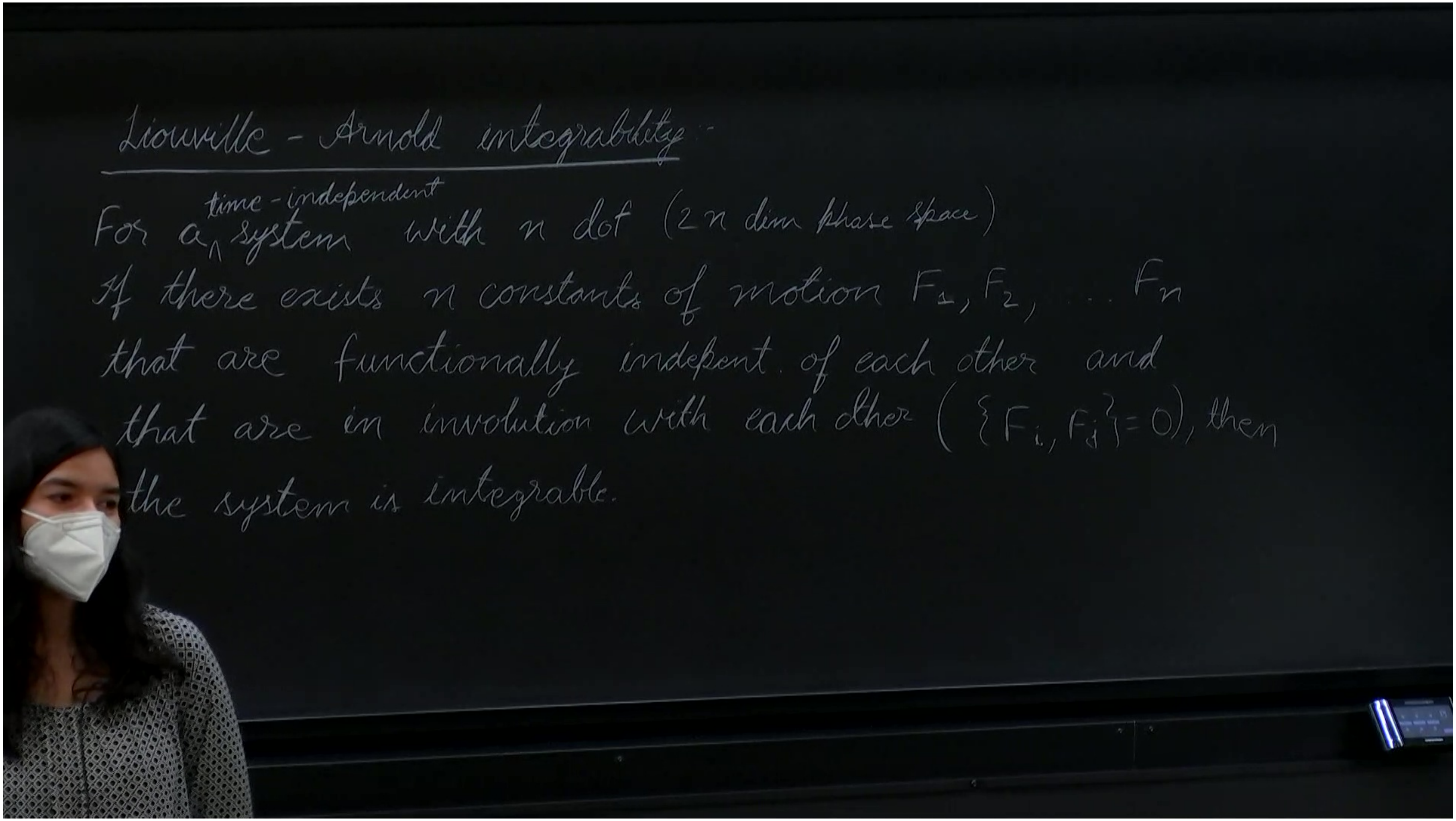

Hamiltonian systems
$H(q_i, p_i, t)$
$(q_i, p_i) \rightarrow PB : \{q_i, q_j\} = 0 = \{p_i, p_j\}$
$\{q_i, p_j\} = \delta_{ij}$
$\dot{q}_i = \frac{\partial H}{\partial p_i} \; ; \; \dot{p}_i = -\frac{\partial H}{\partial q_i}$
Degrees of freed


Degrees of freedom (dof) = no. of generalized co-ordinates
n
Canonical transformation :-
$(q_i, p_i) \longrightarrow (Q_i, P_i)$
1) $\det\left(\frac{\partial(Q,P)}{\partial(q,p)}\right) = +1$
Action - Angle variable formalism

f generalized co-ordinates

Action - Angle variable formalism

If there exists a CT such that $\tilde{H}(\theta, J)$ is only a func^{n} of J, then θ is cyclic co-ordinates


Liouville - Arnold integrability :
For a time-independent system with n dof (2n dim phase space)
If there exists n constants of motion $F_1, F_2, \ldots F_n$
that are functionally indepent. of each other and
that are in involution with each other ($\{F_i, F_j\} = 0$), then
the system is integrable.