

Title: Causality Constraints on Gravitational Effective Field Theories

Speakers: Jun Zhang

Series: Strong Gravity

Date: March 31, 2022 - 1:00 PM

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Abstract: In this talk, I would like to discuss gravitational waves in gravitational effective field theories. In particular, I will focus on gravitational waves propagating on a black hole background, and will show that the coefficients of the higher-dimension operators in the EFT can be constrained by causality considerations. I will also comment on observability of these higher-dimension operators in gravitational wave detections.

Zoom Link: <https://pitp.zoom.us/j/94226476006?pwd=MEdHWG9oQ2dwQ2hXMUd2ZEFFQnRjZz09>



# Causality Constraints on Gravitational Effective Field Theories

张君 [Zhang, Jun]

based on 2005.13923, 2112.05054

in collaboration with Claudia de Rham, Jérémie Francfort and Andrew Tolley

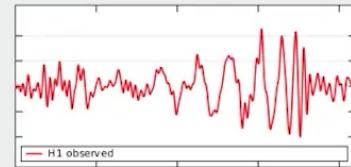
03/31/2022 @ Strong Gravity Seminar, Perimeter Institute





## GR as a gravitational EFT

$$\mathcal{L}_{\text{EFT}} = \frac{M_{\text{Pl}}^2}{2} \left( R + \frac{\mathcal{L}_{\text{D4}}}{\Lambda^2} + \frac{\mathcal{L}_{\text{D6}}}{\Lambda^4} + \frac{\mathcal{L}_{\text{D8}}}{\Lambda^6} + \dots \right)$$





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- Insprialling GWs

[Endlich, Gorbenko, Huang & Senatore, JHEP (2017)]

[Accettulli Huber, Brandhuber, Angelis & Travaglini, PRD (2021)]

...

- QNMs

→ [Cardoso, Kimura, Maselli & Senatore, PRL (2018)]

[de Rham, Francfort & JZ, PRD (2020)]

[Cano, Fransen, Hertog & Maenaut, arXiv:2110.11378]

...

- Constraints from LIGO-Virgo

[Sennett, Brito, Buonanno, Gorbenko & Senatore, PRD (2020)]



## GR as a gravitational EFT

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- Constraints from theoretical considerations

[Bellazzini, Cheung & Remmen, PRD (2016)]

[Bern, Kosmopoulos & Zhiboedov, J. Phys. A (2021)]

[Gruzinov & Kleban, CQG (2007)]

...

- In this talk

Causality, when scattering GWs on BHs.



# Outline



GWs in the presence of a particular dim-8 operator

- BH perturbations
- scattering phase shift and time delay
- causality in curved space

Causality constraints on the generic gravitational EFT

Detecting higher dimension operators with GWs



# GWs in the dim-8 EFT

The dim-8 EFT  $S_{\text{D}8}^{(1)} = \int d^4x \sqrt{-g} \frac{M_{\text{Pl}}^2}{2} \left[ R + \frac{c_1}{\Lambda^6} (R_{abcd} R^{abcd})^2 \right]$   $c_1 = \pm 1$

Field equations  $\mathcal{E}_{\text{GR}}(g) + \epsilon \mathcal{E}_{\text{D}8}(g) = 0$   $\epsilon = \frac{1}{(GMA)^6}$

The deformed Schwarzschild solution

[Cardoso, Kimura, Maselli & Senatore [PLB](#) (2018)]

$$ds^2 = -[f(r) + \epsilon \delta f_t(r)] dt^2 + \frac{1}{f(r) + \epsilon \delta f_r(r)} dr^2 + r^2 d\Omega^2$$

$$\delta f_t = -1024 \left( \frac{GM}{r} \right)^9 + 1408 \left( \frac{GM}{r} \right)^{10}$$

$$\delta f_r = -4608 \left( \frac{GM}{r} \right)^9 + 8576 \left( \frac{GM}{r} \right)^{10}$$



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GWs as metric perturbations  $g_{\mu\nu} = \bar{g}_{\mu\nu} + \frac{1}{M_{\text{Pl}}} h_{\mu\nu}^- + \frac{1}{M_{\text{Pl}}} h_{\mu\nu}^+$

[Cardoso, Kimura, Maselli & Senatore PRL (2018)]

## In GR

Regge-Wheeler-Zerilli Eqs.  $\frac{d^2 \Psi_{\omega\ell m}^\pm}{dr_*^2} = - [\underbrace{\omega^2 - V_{\text{GR}}^\pm(r; \ell)}_{\lambda}] \Psi_{\omega\ell m}^\pm$

$$V_{\text{GR}}^- = \frac{f}{r_g^2} \left( \frac{\lambda+2}{x^2} - \frac{3}{x} \right) \quad r_g = 2GM \quad x = r/r_g \quad \lambda = \ell(\ell+1) - 2$$

$$V_{\text{GR}}^+ = \frac{f}{r_g^2} \left[ \frac{\lambda^2(\lambda+2)x^3 + 3\lambda^2x^2 + 9\lambda x + 9}{x^3(\lambda x + 3)^2} \right]$$



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[Cardoso, Kimura, Maselli & Senatore PRL (2018)]

$$\mathcal{E}_{\text{GR}}(\partial^2 h, \partial h, h) = \epsilon \mathcal{E}_{\text{D8}}(\partial^8 h, \dots, h) + \mathcal{O}(\epsilon^2)$$

- Remove higher- $\partial$  terms 

$$\mathcal{E}_{\text{GR}}(\partial^2 h, \partial h, h) = \epsilon \mathcal{E}_{\text{D8}}(\partial h, h) + \mathcal{O}(\epsilon^2)$$

The corrected RWZ Eqs.  $\frac{d^2 \Psi_{\omega\ell m}^\pm}{dr_*^2} = - [\omega^2 - V_{\text{GR}}^\pm(r; \ell) - c_1 \epsilon \underset{\lambda}{\textcircled{\lambda}} V^\pm(r; \ell, \omega)] \Psi_{\omega\ell m}^\pm$



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- Remove higher- $\partial$  terms   $\partial^i [\mathcal{E}_{\text{GR}}(\partial^2 h, \partial h, h) + \mathcal{O}(\epsilon)] = 0$

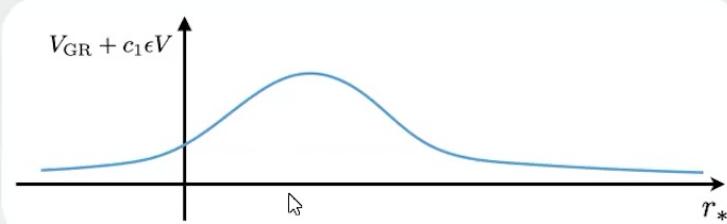
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# GW Scattering on a BH

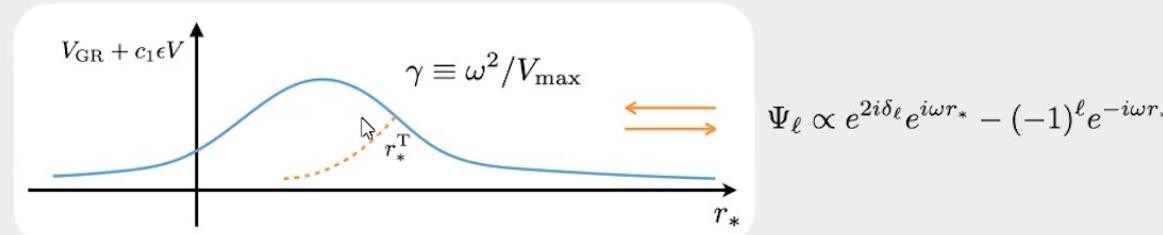
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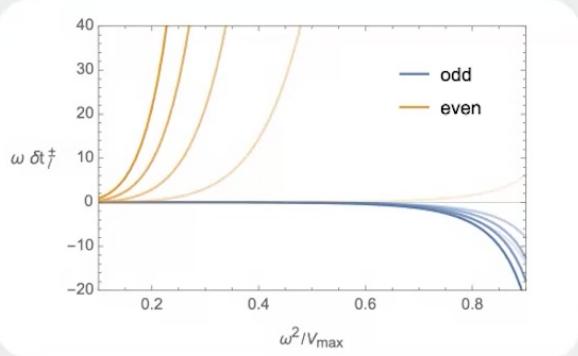


- Phase shift in WKB  $\delta_\ell = \int_{r_*^T}^{\infty} dr_* \left( \sqrt{\omega^2 - V_{\text{GR}} - c_1\epsilon V} - \omega \right) - \omega r_*^T + \frac{\pi}{2} \left( \ell + \frac{1}{2} \right)$

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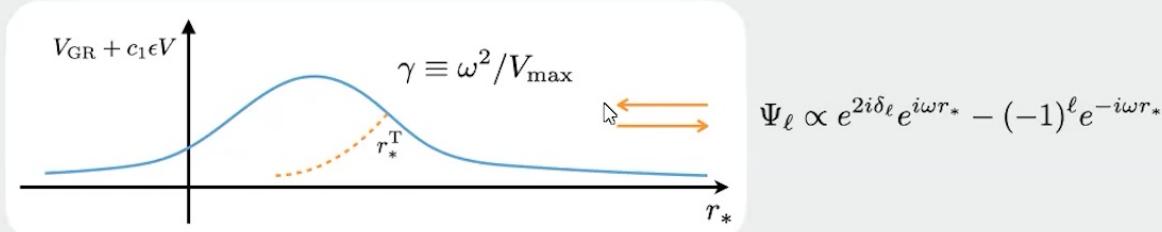
- Time delay  $T_\ell = T_\ell^{\text{GR}} + \delta T_\ell + \mathcal{O}(\epsilon^2)$



$$S_{\text{D8}}^{(1)} = \int d^4x \sqrt{-g} \frac{M_{\text{Pl}}^2}{2} \left[ R + \frac{c_1}{\Lambda^6} (R_{abcd}R^{abcd})^2 \right]$$
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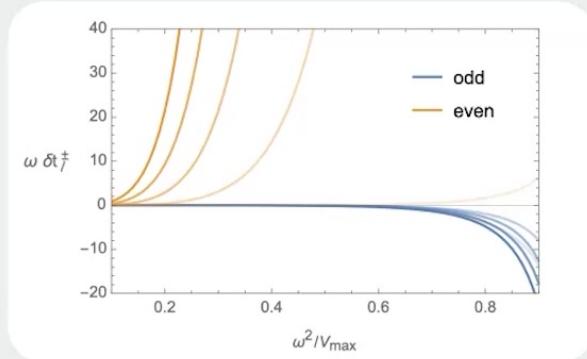
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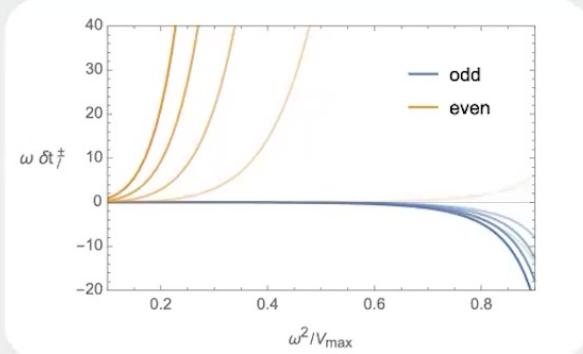
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Inevitable time advance



# A Lesson from QED

EFT of QED in curved spacetime

[Drummond & Hathrell, PRD (1980)]

$$\mathcal{L}_{\text{EFT}}^{\text{QED}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\alpha}{M_e^2}R_{\mu\nu\sigma\rho}F^{\mu\nu}F^{\sigma\rho} + \dots$$

↓

Transverse EM waves on a Schwarzschild background

$$c_s^2 = 1 + \frac{\beta_P}{M_e^2} \frac{r_g}{r^3} + \mathcal{O}\left(\frac{r_g^3}{M_e^4 r^6}\right)$$

- Radial polarization       $\beta_P > 0$
- Angular polarization     $\beta_P < 0$



Time Advance  $\rightleftharpoons$  Acausality



# Causality in Curved Spacetime

Time delay in the EFT  $T_\ell = T_\ell^{\text{GR}} + \delta T_\ell + \mathcal{O}(\epsilon^2)$

- Asymptotic causality  $T_\ell$

$\downarrow_{\mathfrak{f}}$

In flat spacetime

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Relativistic causality: No faster than light.



# Causality in Curved Spacetime

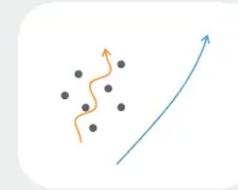
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## In flat spacetime

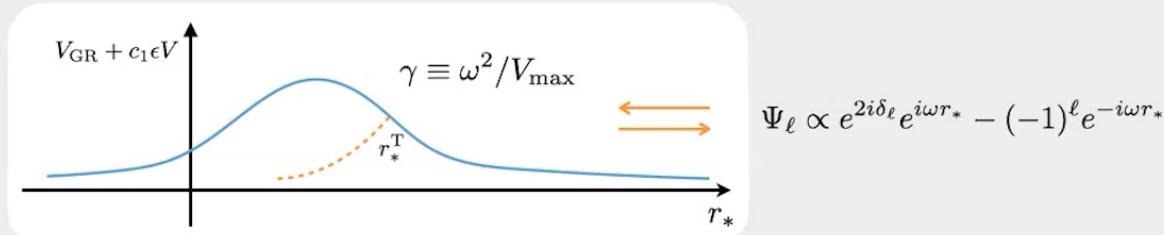
Relativistic causality: The retarded propagator has no support outside the lightcone set by the high-energy modes.

- Infrared causality  $\delta T_\ell$



# GW Scattering on a BH

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- Phase shift in WKB  $\delta_\ell = \int_{r_*^T}^{\infty} dr_* \left( \sqrt{\omega^2 - V_{\text{GR}} - c_1\epsilon V} - \omega \right) - \omega r_*^T + \frac{\pi}{2} \left( \ell + \frac{1}{2} \right)$
- Time delay  $T_\ell = 2 \frac{d\delta_\ell}{d\omega} \quad T_\ell = T_\ell^{\text{GR}} + \delta T_\ell + \mathcal{O}(\epsilon^2)$

# Infrared Causality

Time delay in the EFT       $T_\ell = T_\ell^{\text{GR}} + \delta T_\ell + \mathcal{O}(\epsilon^2)$

Causal violating time advance       $\frac{1}{-c_1 \omega \delta t_\ell} \lesssim \epsilon \ll \left( \frac{\ell + 1/2}{\omega^2 G^2 M^2} \right)^3 \quad \delta T_\ell^\pm = c_1 \epsilon \delta t_\ell^\pm$



- Resolvability       $-\delta T_\ell > 1/\omega$
- Calculable within the EFT validity regime

## Background & GWs

$$\omega \ll \Lambda^2 r_b \quad r_b = (\ell + 1/2)/\omega \quad \text{Tr}[A^n] \ll \Lambda^{4n} \quad A^a{}_b \equiv W^a{}_{cbd} k^c k^d$$

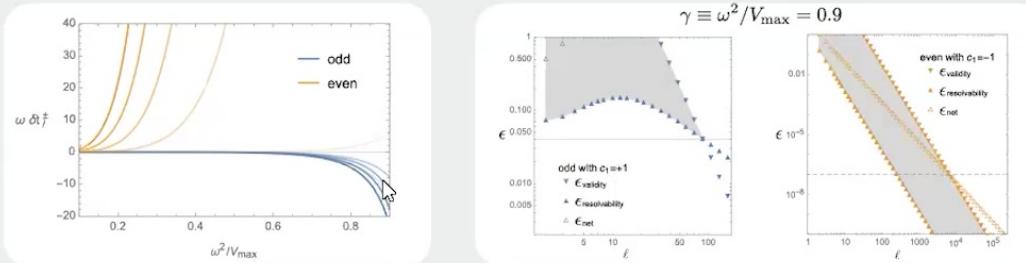
$$((k^\mu \nabla_\mu) A^{ab}) ((k^\nu \nabla_\nu)_p A_{ab}) \ll \Lambda^{8+4p}$$



# Causality Violation in the Scattering

Time delay in the EFT  $T_\ell = T_\ell^{\text{GR}} + \delta T_\ell + \mathcal{O}(\epsilon^2)$   $\delta T_\ell^\pm = c_1 \epsilon \delta t_\ell^\pm$

Infrared acausality  $\frac{1}{-c_1 \omega \delta t_\ell} \lesssim \epsilon \ll \left( \frac{\ell + 1/2}{\omega^2 G^2 M^2} \right)^3$



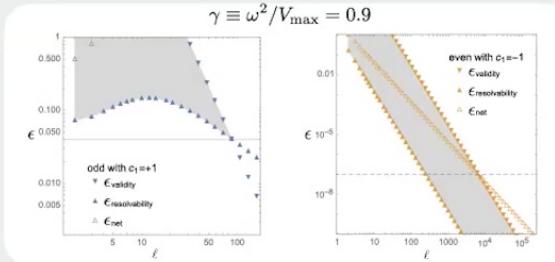
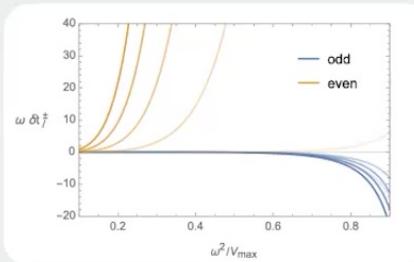
$$\delta T_\ell = 2 \int_{r^T + \delta r^T}^{\infty} dr \frac{1}{f + \epsilon \delta f} \left( \frac{2\omega - c_1 \epsilon \frac{\partial V}{\partial \omega}}{2\sqrt{\omega^2 - V_{\text{GR}} - c_1 \epsilon V}} \right) - 2 \int_{r^T}^{\infty} dr \frac{1}{f} \left( \frac{\omega}{\sqrt{\omega^2 - V_{\text{GR}}}} \right)$$



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$$\delta t_\ell = -2 \int_{r^T}^{\infty} dr \mathcal{A} \left( \frac{\delta \mathcal{A}}{\mathcal{A}'} \right)' \quad \mathcal{A} \equiv \frac{\omega}{f \sqrt{\omega^2 - V_{\text{GR}}}} \quad \delta \mathcal{A} \equiv \mathcal{A} \left[ \frac{V}{2(\omega^2 - V_{\text{GR}})} - \frac{1}{2\omega} \frac{\partial V}{\partial \omega} - \frac{\delta f}{f} \right]$$

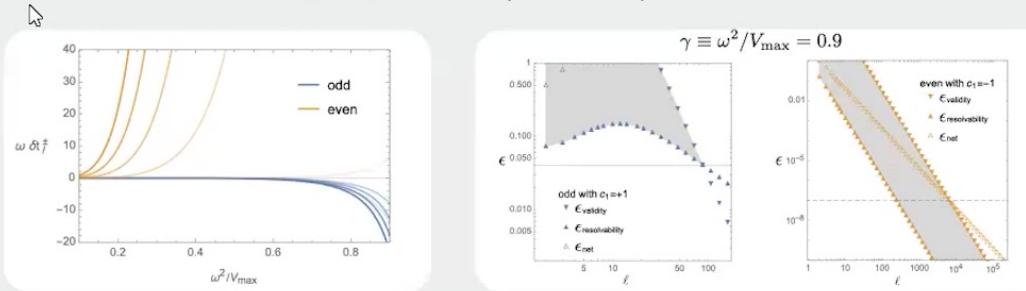
$$V^- \sim \ell^2 \quad V^+ \sim \ell^4 \quad \omega^2 \sim V_{\text{GR}} \sim \ell^2$$



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Infrared causality  $0 < c_1 \epsilon < 0.04$



# Causality Constraints on the dim-8 EFT

The dim-8 EFT  $S_{\text{D}8}^{(1)} = \int d^4x \sqrt{-g} \frac{M_{\text{Pl}}^2}{2} \left[ R + \frac{c_1}{\Lambda^6} (R_{abcd} R^{abcd})^2 \right]$

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- v.s. other theoretical considerations

[Metsaev & Tseytlin, PLB (1987)]

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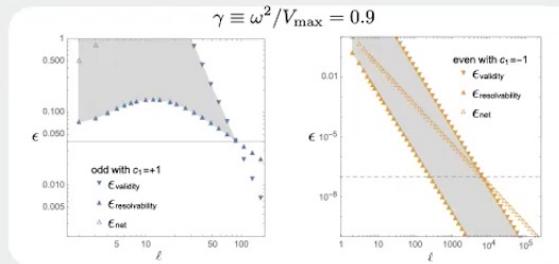
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Infrared causality  $0 < c_1 \epsilon < 0.04$   $\epsilon = \frac{1}{(G M \Lambda)^6}$

- v.s. asymptotical causality

$$\frac{T_\ell^{\text{GR}} + \omega^{-1}}{-c_1 \delta t_\ell} < \epsilon \ll \left( \frac{\ell + 1/2}{\omega^2 G^2 M^2} \right)^3$$





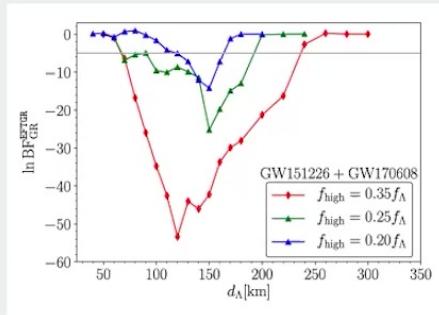
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Infrared causality  $0 < c_1 \epsilon < 0.04$   $\epsilon = \frac{1}{(G M \Lambda)^6}$

- v.s. LIGO observations

[Sennett, Brito, Bonsu, Gorbenko & Senatore, PRD (2020)]



$$\Lambda \sim 10^{-13} \text{ eV}$$

Given the smallest known BH ( $M \simeq 3M_\odot$ ), infrared causality indicates  $\Lambda > 7 \times 10^{-11} \text{ eV}$ .

# EFT with dim-x operators

If parity preserving,

$$\frac{d^2\Psi_{\omega\ell m}^\pm}{dr_*^2} = - [\omega^2 - V_{\text{GR}}^\pm(r; \ell) - c \epsilon V^\pm(r; \ell, \omega)] \Psi_{\omega\ell m}^\pm \quad \epsilon = (GM\Lambda)^{-2m}$$
$$m = \frac{x}{2} - 1$$





Jun Zhang

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$$\frac{d^2\Psi_{\omega\ell m}^\pm}{dr_*^2} = - [\omega^2 - V_{\text{GR}}^\pm(r; \ell) - c \epsilon V^\pm(r; \ell, \omega)] \Psi_{\omega\ell m}^\pm \quad \epsilon = (GM\Lambda)^{-2m}$$
$$m = \frac{x}{2} - 1$$

At large  $\ell$        $V \sim \ell^n$

Infrared acausality

$$\frac{1}{-c_1 \omega \delta t_\ell} \lesssim \epsilon \ll \left( \frac{\ell + 1/2}{\omega^2 G^2 M^2} \right)^3 \xrightarrow[\omega^2 = \gamma V_{\max}]{} \ell^{-n+1} < \epsilon \ll 27^m \ell^{-m}$$
$$\delta t_\ell = -2 \int_{r_T}^{\infty} dr \mathcal{A} \left( \frac{\delta \mathcal{A}}{\mathcal{A}} \right)' \quad \mathcal{A} \equiv \frac{\omega}{f \sqrt{\omega^2 - V_{\text{GR}}}}$$
$$\delta \mathcal{A} \equiv \mathcal{A} \left[ \frac{V}{2(\omega^2 - V_{\text{GR}})} - \frac{1}{2\omega} \frac{\partial V}{\partial \omega} - \frac{\delta f}{f} \right]$$

- If  $n \geq m + 1$ , the coefficient has to be sign-definite, if the operator contributes to the scattering time delay.





# EFT with the other dim-8 operators

The dim-8 EFT  $S_{\text{D8}}^{(1)} = \int d^4x \sqrt{-g} \frac{M_{\text{Pl}}^2}{2} \left[ R + \frac{c_1}{\Lambda^6} (R_{abcd} R^{abcd})^2 \right]$

The other dim-8 operators  $\mathcal{L}_{\text{D8}} = c_1 C^2 + c_2 \tilde{C}^2 + c_3 C \tilde{C}$      $C = R_{abcd} R^{abcd}$      $\tilde{C} = \epsilon^{ab}{}_{ef} R_{abcd} R^{efcd}$

- The  $c_2$ -term

$$V^+ = 0$$

$$V^- \sim \ell^4 \quad \delta t_\ell^- < 0 \quad c_2 < 0$$

Infrared acausality at large  $\ell$

$$\ell^{-n+1} < \epsilon \ll 27^m \ell^{-m}$$

Infrared causality requires  $c_2 > 0$





# The Generic Gravitational EFT

$$\mathcal{L}_{\text{EFT}} = \frac{M_{\text{Pl}}^2}{2} \left( R + \frac{\mathcal{L}_{\text{D4}}}{\Lambda^2} + \frac{\mathcal{L}_{\text{D6}}}{\Lambda^4} + \frac{\mathcal{L}_{\text{D8}}}{\Lambda^6} + \dots \right)$$

The dim-4 operators  $\mathcal{L}_{\text{D4}} = a_{R^2} R^2 + a_{W^2} W_{\mu\nu\alpha\beta}^2 + a_{\text{GB}} R_{\text{GB}}^2$ ,

$$\mathcal{L}_{\text{D4}} = a_1 R^2 + a_2 R_{\mu\nu} R^{\mu\nu} \quad a_1 = a_{R^2} - \frac{2}{3} c_{W^2}, \quad a_2 = 2a_{W^2}$$

- At 0th order (in GR)  $R_{\mu\nu} = \delta R_{\mu\nu} = 0$

The dim-4 operators do not contribute to linearized GWs propagating on a Ricci-flat background at  $\mathcal{O}(1/\Lambda^4)$ .



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The dim-6 operators

$$\begin{aligned}\mathcal{L}_{\text{D6}} = & b_1 R_{\mu\nu}^{\alpha\beta} R_{\alpha\beta}^{\gamma\sigma} R_{\gamma\sigma}^{\mu\nu} + b_2 R^{\mu\nu} R_{\mu\alpha\beta\gamma} R_\nu^{\alpha\beta\gamma} + b_3 R R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} + b_4 R_\mu^{\alpha} \nabla^\beta R_\alpha^{\gamma} \beta^\sigma R_\gamma^{\mu} \sigma^\nu \\ & + b_5 R \square R + b_6 R_{\mu\nu} \square R^{\mu\nu} + b_7 R^3 + b_8 R R_{\mu\nu}^2 + b_9 R_{\mu\nu}^3 + b_{10} R^{\mu\nu} R^{\alpha\beta} R_{\mu\nu\alpha\beta}\end{aligned}$$



# The Generic Gravitational EFT

$$\mathcal{L}_{\text{EFT}} = \frac{M_{\text{Pl}}^2}{2} \left( R + \frac{\mathcal{L}_{\text{D4}}}{\Lambda^2} + \frac{\mathcal{L}_{\text{D6}}}{\Lambda^4} + \frac{\mathcal{L}_{\text{D8}}}{\Lambda^6} + \dots \right)$$

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- Field redefinition

$$g_{\mu\nu} \rightarrow g_{\mu\nu} - \delta g_{\mu\nu} \quad \delta(\sqrt{-g} R) = \sqrt{-g} R^{\mu\nu} \left( \delta g_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \delta g \right)$$



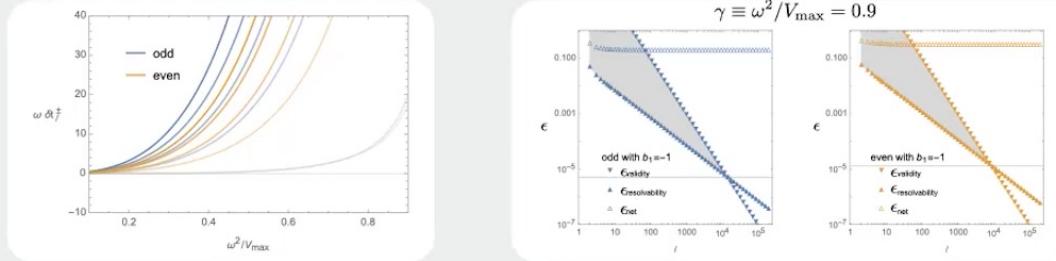
Jun Zhang

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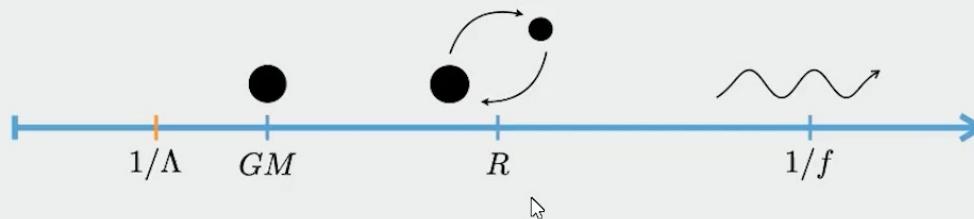
$$\epsilon = (G M \Lambda)^{-4} \quad V^\pm \sim \ell^2$$



Infrared causality requires  $\Lambda \gtrsim 10^{-11} \text{ eV} (M_\odot/M)$ , if  $b_1 = -1$ .



# Probe Higher-Dim Operators



with BH ringdown

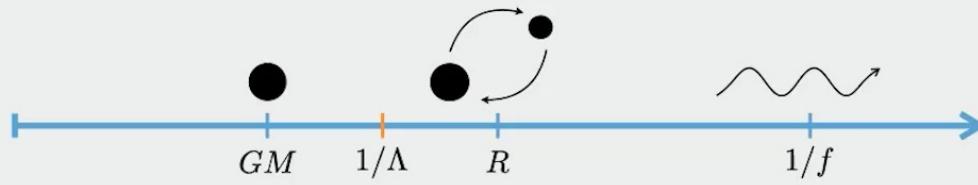
$$\delta\omega_{D6} \sim (GM\Lambda)^{-4}\omega_{GR} \quad \Lambda_{\max} \sim 1/(\delta\hat{f}^{1/4}GM)$$

with finite size effects in inspiralling GWs

$$\delta\Psi_{D6}^{\text{SPA}} \sim (GM\Lambda)^{-4}v^{10}$$



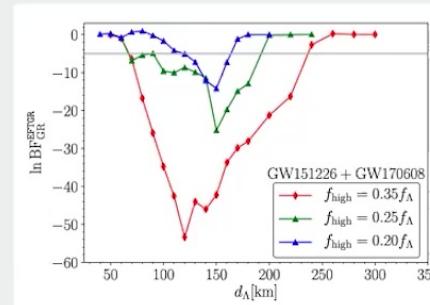
# Probe Higher-Dim Operators



with early inspiraling GWs

$$\mathcal{L}_{\text{EFT}}^{\text{D8}} = \frac{M_{\text{Pl}}^2}{2} \left( R + \frac{\mathcal{L}_{\text{D8}}}{\Lambda^6} + \dots \right)$$

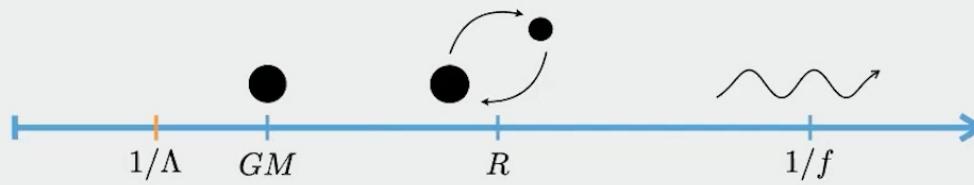
$$\delta\Psi_{\text{D8}}^{\text{SPA}} \sim (GM\Lambda)^{-6} v^{16}$$



[Sennett, Brito, Buonanno,  
Gorbenko & Senatore, PRD (2020)]



# Probe Higher-Dim Operators



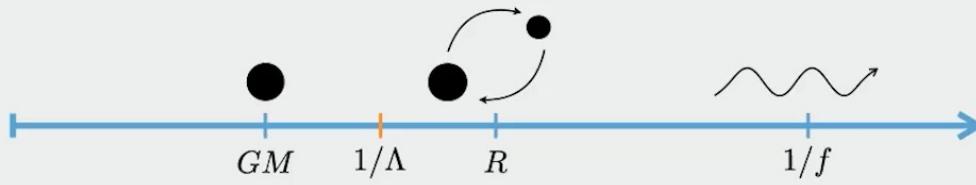
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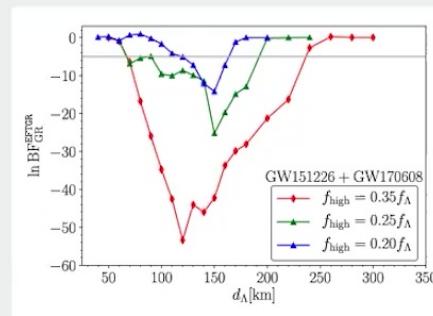
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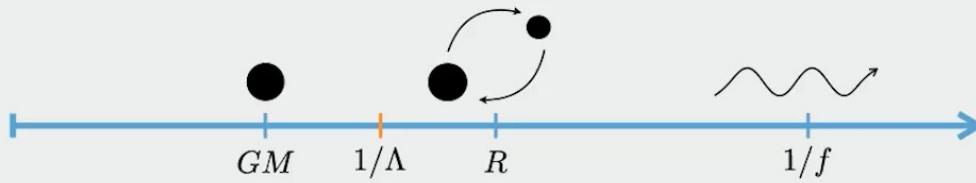
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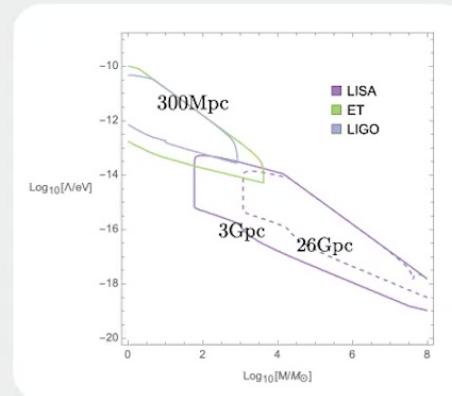


with early inspiraling GWs

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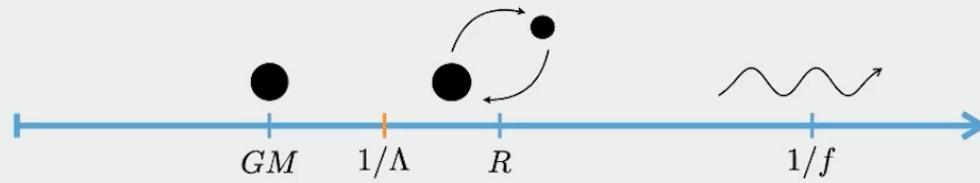
$$\delta\Psi_{\text{D6}}^{\text{SPA}} \sim (GMA)^{-4}v^{10}$$

$$\delta\Psi_{\text{D8}}^{\text{SPA}} \sim (GMA)^{-6}v^{16}$$





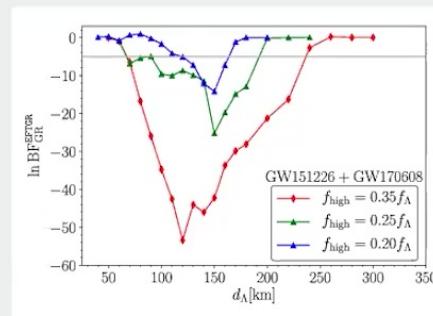
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with early inspiraling GWs

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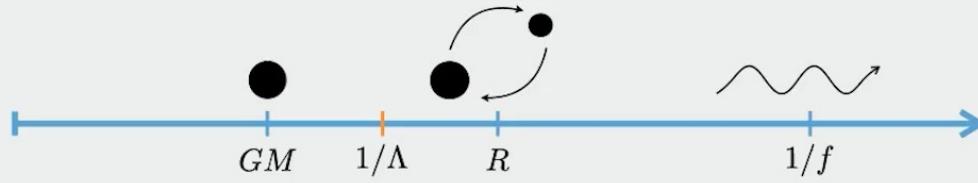
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