Title: Contextuality, Fine-tuning and Teleological Explanation

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Series: Quantum Foundations

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Abstract: In this talk I will assess various proposals for the source of the intuition that there is something problematic about contextuality, and argue that contextuality is best thought of in terms of fine-tuning. I will suggest that as with other fine-tuning problems in quantum mechanics, this behaviour can be understood as a manifestation of teleological features of physics. I will also introduce several formal mathematical frameworks that have been used to analyse contextuality and discuss how their results should be interpreted.

Contextuality, Fine-Tuning and Teleological Explanation[1]

Emily Adlam

March 11, 2022

[1] E. Adlam. "Contextuality, Fine-Tuning and Teleological Explanation". In: *Found Phys* 51 (2021). URL: https://arxiv.org/pdf/2110.15898.pdf.

Plan

- Definitions
- What's the problem with contextuality?
- Contextuality as fine-tuning
- How can we explain fine-tuning?
- Other frameworks: quantum logic and nonclassical probabilities

Kochen-Specker Contextuality

- Each measurement outcome associated with a definite property
- Assign values 1 or 0 to each possible outcome such that every possible measurement has exactly one outcome with value 1 (and that is the value that will definitely occur)
- Kochen Specker theorem: there exist sets of measurements (in > 2 dimensions) where this can't be achieved[2]

[2] Simon Kochen and E.P. Specker. "The Problem of Hidden Variables in Quantum Mechanics". In: *The Logico-Algebraic Approach to Quantum Mechanics*. Ed. by C.A. Hooker. The University of Western Ontario Series in Philosophy of Science. Springer Netherlands, 1975, pp. 293–328.

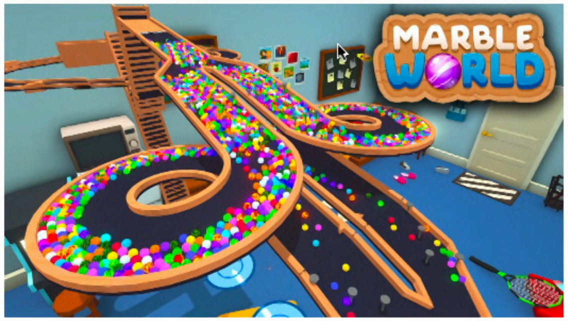
Spekkens Contextuality

- Ontological models framework: each possible preparation is associated with a probability distribution μ_P over ontic states and each measurement M and outcome k is associated with a response function $\xi^{k,M}$.
- Preparation non-contextuality: all preparations that prepare the same quantum state are represented by the same probability distribution
- Measurement non-contextuality: a quantum mechanical measurement operator is always represented by the same response function
- Spekkens: there is no preparation non-contextual ontological model for quantum mechanics[3]

[3] R. W. Spekkens. "Contextuality for preparations, transformations, and unsharp measurements". In: *Physical Review A* 71.5, 052108 (May 2005), p. 052108. DOI: 10.1103/PhysRevA.71.052108. eprint: quant-ph/quant-ph/0406166.

Gleason's Property

- What would a non-contextual model look like?
- Rudolph's marble-world proposal: each measurement outcome is a vector on a sphere, state is a 'marble' which is 'attracted' to the nearest outcome[5]
- BUT Gleason's property: in quantum mechanics, for a given state, the probability that we obtain a given measurement outcome is always the same, regardless of which measurement we're performing



[5] Terry Rudolph. "Ontological models for quantum mechanics and the Kochen-Specker theorem". In: *arXiv preprint quant-ph/0608120* (2006).

Fine-Tuning

- Must choose probability distributions such that for any given ontic state the probability distribution depends on context, but when we average over ontic states the dependence disappears
- Cavalcanti - uses the framework of causal models to show that any set of experiments violating a Kochen-Specker inequality must be represented by a fine-tuned causal model[6]
  - Describe causal influences between the variables involved, and/or some set of 'latent' variables, in terms of a causal model represented as a directed acyclic graph
  - A causal model is fine-tuned if its causal graph has a causal connection between two variables which are conditionally independent at the level of the operational statistics
- I show in this paper that preparation contextuality is likewise a form of fine-tuning

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[6] Eric G. Cavalcanti. "Classical Causal Models for Bell and Kochen-Specker Inequality Violations Require Fine-Tuning". In: *Physical Review X* 8.2 (2018). ISSN: 2160-3308. DOI: 10.1103/physrevx.8.021018. URL: http://dx.doi.org/10.1103/PhysRevX.8.021018.

Biased and unbiased counterfactual outcomes

- Deterministic case: define 'counterfactual outcome' = vector $\vec{c}$ such that the entry in position i of specifies the set of outcomes that we will definitely obtain if we perform the set of measurements belonging to the context labelled by i.
- Probability distribution associated with preparation procedure is 'unbiased' if it is the case that for each i, the marginal probability distribution induced by this distribution over the possible values of c_i is independent of the values of the other entries in the vector $\vec{c}$.

Biased and unbiased counterfactual outcomes

$$M_1 = \{|0\rangle\langle 0|, |1\rangle\langle 1|\} \qquad P_1 \to |0\rangle$$
$$M_2 = \{(\tfrac{1}{2}|0\rangle + \tfrac{\sqrt{3}}{2}|1\rangle)(\tfrac{1}{2}\langle 0| + \tfrac{\sqrt{3}}{2}\langle 1|), \mathbb{I} - (\tfrac{1}{2}|0\rangle + \tfrac{\sqrt{3}}{2}|1\rangle)(\tfrac{1}{2}\langle 0| + \tfrac{\sqrt{3}}{2}\langle 1|) \qquad P_2 \to \tfrac{1}{2}|0\rangle + \tfrac{\sqrt{3}}{2}|1\rangle$$
$$M_3 = \{(\tfrac{1}{2}|0\rangle - \tfrac{\sqrt{3}}{2}|1\rangle)(\tfrac{1}{2}\langle 0| - \tfrac{\sqrt{3}}{2}\langle 1|), \mathbb{I} - (\tfrac{1}{2}|0\rangle - \tfrac{\sqrt{3}}{2}|1\rangle)(\tfrac{1}{2}\langle 0| - \tfrac{\sqrt{3}}{2}\langle 1|)\} \qquad P_3 \to \tfrac{1}{2}|0\rangle - \tfrac{\sqrt{3}}{2}|1\rangle$$

- μ_{P_1} assigns probability 0 to all $\vec{c}$ with $c_1 = 1$; μ_{P_2} assigns probability 0 to all $\vec{c}$ with $c_2 = 1$; μ_{P_3} assigns probability 0 to all $\vec{c}$ with $c_3 = 1$.
- Equal mixture of P_1, P_2, P_3: probability of obtaining the result 1 to any of these measurements is $\frac{1}{2}$. But there is no way to obtain a counterfactual outcome where we are certain to get the result 1 to all three of these measurements; so $\mu_{mix;0}$ must assign probability 0 to all counterfactual outcomes with $[c1, c2, c3] = [1, 1, 1]$, i.e. $\mu_{mix;0}(c_1 = 1|c_2 = 1, c_3 = 1) = 0$. However, $\mu_{mix;0}(c_1 = 1) = \frac{1}{2}$, and therefore $\mu_{mix;0}$ is biased.
- Equal mixture of $|0\rangle$ and $|1\rangle$: no reason to think that a preparation of $|1\rangle$ can't prepare a counterfactual outcome with $[c1, c2, c3] = [1, 1, 1]$. So presumably $\mu_{mix;1}$ will not satisfy $\mu_{mix;1}(c_1 = 1|c_2 = 1, c_3 = 1) = 0$.

Explaining Fine-Tuning

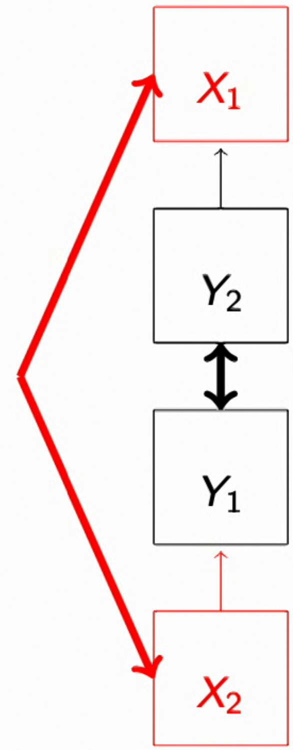
- Different types of fine-tuning arguments
- Acceptance: superdeterminism
- Denial: antirealism
- Two realist options:
  - Equilibration
  - Teleological explanation

Teleological explanation?

- Suppose we have two maximal contexts (measurements) $\{A, B, C\}$ and $\{A, D, E\}$ such that the probability for obtaining a positive result to the measurement $\{A, \mathbb{I} - A\}$ is different in the two contexts.
- Suppose that $\{A, \mathbb{I} - A\}$ is performed first and the decision about whether to proceed with $\{\{B, \mathbb{I} - B\}, \{C, \mathbb{I} - C\}\}$ or $\{\{D, \mathbb{I} - D\}, \{E, \mathbb{I} - E\}\}$ is made later.
- Then the probability that we obtain the result A to the measurement $\{A, \mathbb{I} - A\}$ depends on a future decision about which additional measurements to perform, so the result of that measurement is a 'signal' from the future

What's wrong with signalling backwards in time?

- Bilking
- Violations of determinism



Other approaches to contextuality: Quantum Logic

- We can derive from classical logic an upper bound on the sum of the probabilities for various sets of measurement outcomes; non-local and Kochen-Specker correlations violate this[7]
- Two ways of thinking about this:
  - Classical logic doesn't work in the quantum world
  - Measurement outcomes are not the sort of thing to which it is appropriate to describe as boolean variables

[7] Samson Abramsky and Lucien Hardy. "Logical Bell inequalities". In: *Physical Review A* 85.6 (2012). ISSN: 1094-1622. DOI: 10.1103/physreva.85.062114. URL: http://dx.doi.org/10.1103/PhysRevA.85.062114.

Other approaches to contextuality: Nonclassical probabilities

- Negative probabilities, or complex probabilities, or relax the requirement that we should be able to assign probabilities to all conjunctions of events, or upper probability spaces which are subadditive rather than additive on disjoint measurable sets, or quantum measures which are not additive on pairs of events but which are additive on triples of events
- Subjective probabilities? Would violate rationality constraints[8]
- Objective probabilities? Probably not, because of the Principal Principle
- In the paper, I show how to obtain negative probabilities by taking a contextual model with normal probabilities and 'compressing' it to a model on fewer ontic states which is non-contextual but has negative probabilities

[8] Benjamin H. Feintzeig and Samuel C. Fletcher. "On Noncontextual, Non-Kolmogorovian Hidden Variable Theories". In: *Foundations of Physics* 47.2 (2017), 294–315. ISSN: 1572-9516. DOI: 10.1007/s10701-017-0061-z. URL: http://dx.doi.org/10.1007/s10701-017-0061-z.

Other approaches to contextuality

- Graph theory (Cabello, Severini and Winter)
- Sheaf-theoretic contextuality (Abramsky and Brandenburger)