Title: AdS/CFT

Speakers: Gang Xu, Pedro Vieira

Collection: AdS/CFT 2021/2022

Date: March 30, 2022 - 9:00 AM

URL: https://pirsa.org/22030100

$$\beta_{O(N)\text{ non-linear }\sigma\text{-model}} \equiv \frac{\partial e_0^2}{\partial \log \Lambda} = - \frac{N-2}{2\pi}\, e_0^4 \rightsquigarrow m = \Lambda e^{-\frac{2\pi}{(N-2)e_0^2}} \simeq \underbrace{\Lambda e^{-\frac{2\pi}{N e_0^2}}}_{\sqrt{\langle \lambda \rangle}}$$

Large N

Lagrange multiplier

$$\beta_{YM} \equiv \frac{\partial g}{\partial \log \Lambda} = - \frac{11}{3} N \frac{g^3}{16\pi^2}$$

N - 1 massless particles (sym broken)

N massive particles, symmetry restored

$|a\rangle$, $a = 1 \ldots N$

empty (gapped)

$$\beta_{\substack{O(N)\text{ non-linear}\\ \sigma\text{-model}}} \equiv \frac{\partial e_0^2}{\partial \log \Lambda} = - \frac{N-2}{2\pi} e_0^4 \rightsquigarrow m = \Lambda e^{-\frac{2\pi}{(N-2)e_0^2}} \underset{\text{Large } N}{\simeq} \Lambda e^{-\overbrace{\frac{2\pi}{N e_0^2}}^{}} \sqrt{\langle \lambda \rangle}$$

Lagrange multiplier

$$\beta_{YM} \equiv \frac{\partial g}{\partial \log \Lambda} = - \frac{11}{3} N \frac{g^3}{16\pi^2}$$

$1 \ldots N$

$|a\rangle$

N-1 massless particles (sym broken)

N massive particles, symmetry restores

empty (gapped)

$$\frac{\partial}{\partial \log \Lambda} \underbrace{(N e_0^2)}_{\substack{\text{natural} \\ \text{double scaling limit:} \\ e_0 \to 0 \\ N \to \infty,\ e_0^2 N \text{ fixed}}} \overset{\text{large } N}{=} - \frac{(N e_0^2)^2}{2\pi}$$

$$\frac{\partial g^2}{\partial \log \Lambda} = - \frac{11}{3}$$

$$\frac{\partial g^2}{\partial \log \Lambda} = -\frac{11}{3}\frac{1}{8\pi^2} N g^4$$

Natural limit

$N \to \infty$

$g \to 0$

$$\frac{\partial(g^2 N)}{\partial \log \Lambda} = -\frac{11}{3}\frac{1}{8\pi^2}(N g^2)^2$$

Natural limit

$N \to \infty$

$g \to 0$, $N g^2 \equiv \lambda$

't Hooft coupling

$g^2)^2$

Hooft
upling

Large N non-abelian gauge theories are String theories.

$$\underbrace{(A_\mu(x))_{ij}}_{1 \ldots N} \;,\; \underbrace{(F_{\mu\nu})_{ij}}_{\text{non commuting matrices.}} \quad \text{etc}$$

0-dim gauge theories

$$[A_\mu(x)]_{ij} \longrightarrow M_{ij} \leftarrow \text{hermitian}$$

$$\int \mathcal{D}A \longrightarrow \int \mathcal{D}M = \int \prod_{i=1}^{N} d\,\mathrm{Re}M_{ii} \prod_{i<j} d\,\mathrm{Re}M_{ij}\, d\,\mathrm{Im}M_{ij}$$

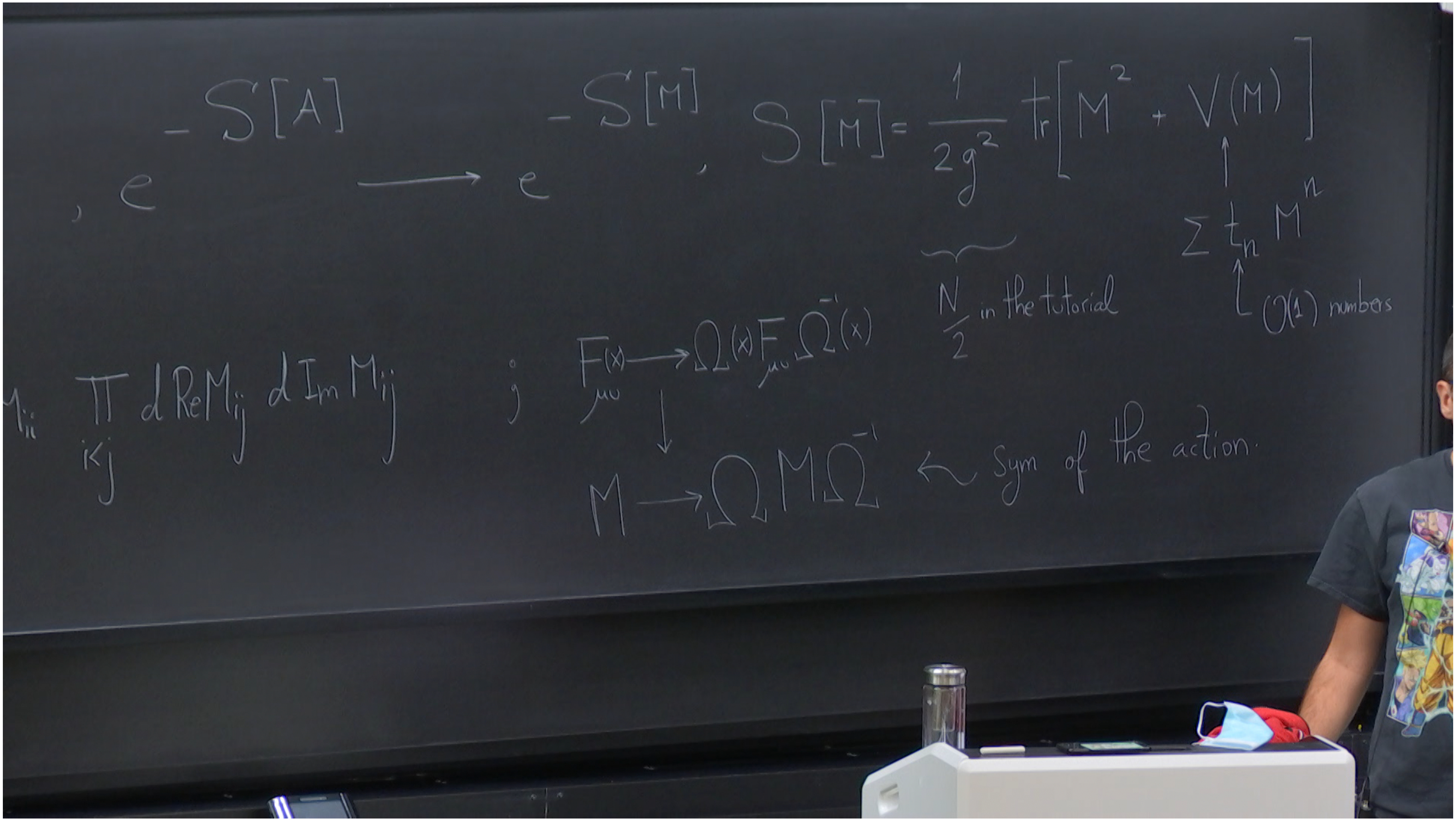
$$, \; e^{-S[A]} \longrightarrow e^{-S[M]}, \quad S[M] = \underbrace{\frac{1}{2g^2}}_{\frac{N}{2} \text{ in the tutorial}}$$

$$\prod_{i<j} d\,\mathrm{Re}M_{ij} \, d\,\mathrm{Im}\,M_{ij}$$

$$, \; e^{-S[A]} \longrightarrow e^{-S[M]}, \quad S[M] = \underbrace{\frac{1}{2g^2}}_{\frac{N}{2} \text{ in the tutorial}} \operatorname{tr}\left[M^2 + V(M)\right]$$

$$V(M) \leftarrow \sum t_n M^n, \quad t_n \leftarrow \mathcal{O}(1) \text{ numbers}$$

$$M_{ii} \prod_{i<j} d\,\mathrm{Re} M_{ij} \, d\,\mathrm{Im} M_{ij}$$


, $e^{-S[A]} \longrightarrow e^{-S[M]}$, $S[M] = \frac{1}{2g^2} \mathrm{tr}\left[M^2 + V(M)\right]$
$\sum t_n M^n$
$\frac{N}{2}$ in the tutorial
$\mathcal{O}(1)$ numbers
M_{ii} $\prod_{i<j} d\,\mathrm{Re}\,M_{ij}\; d\,\mathrm{Im}\,M_{ij}$
; $F_{\mu\nu}(x) \longrightarrow \Omega(x) F_{\mu\nu} \Omega^{-1}(x)$
$M \longrightarrow \Omega M \Omega^{-1}$
Sym of the action.

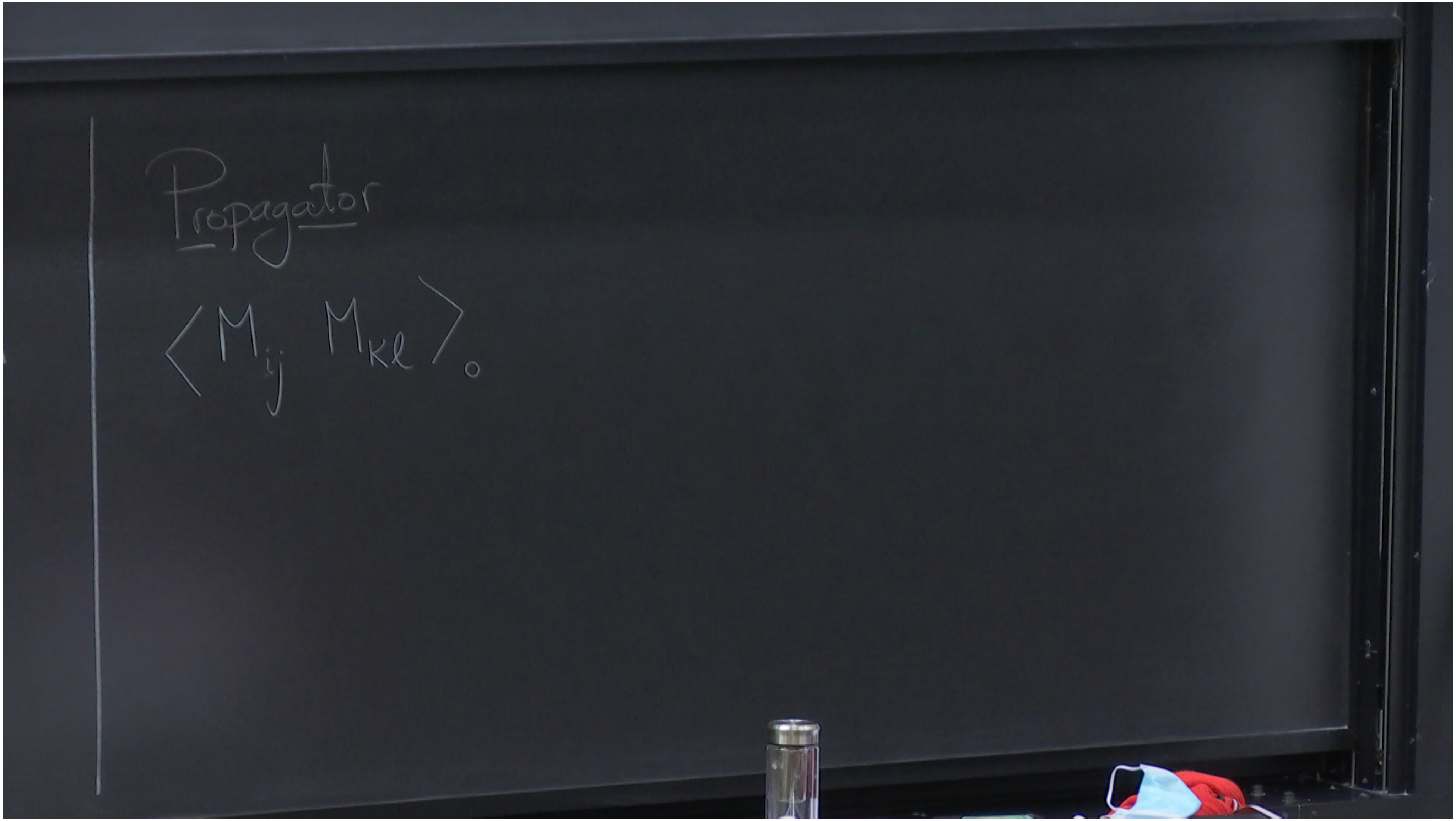
1 Matrix model

$$\langle \ldots \rangle \equiv \int \mathcal{D}M \ldots e^{-S[M]} \Big/ \int \mathcal{D}M \, e^{-S[M]}$$

1 Matrix model

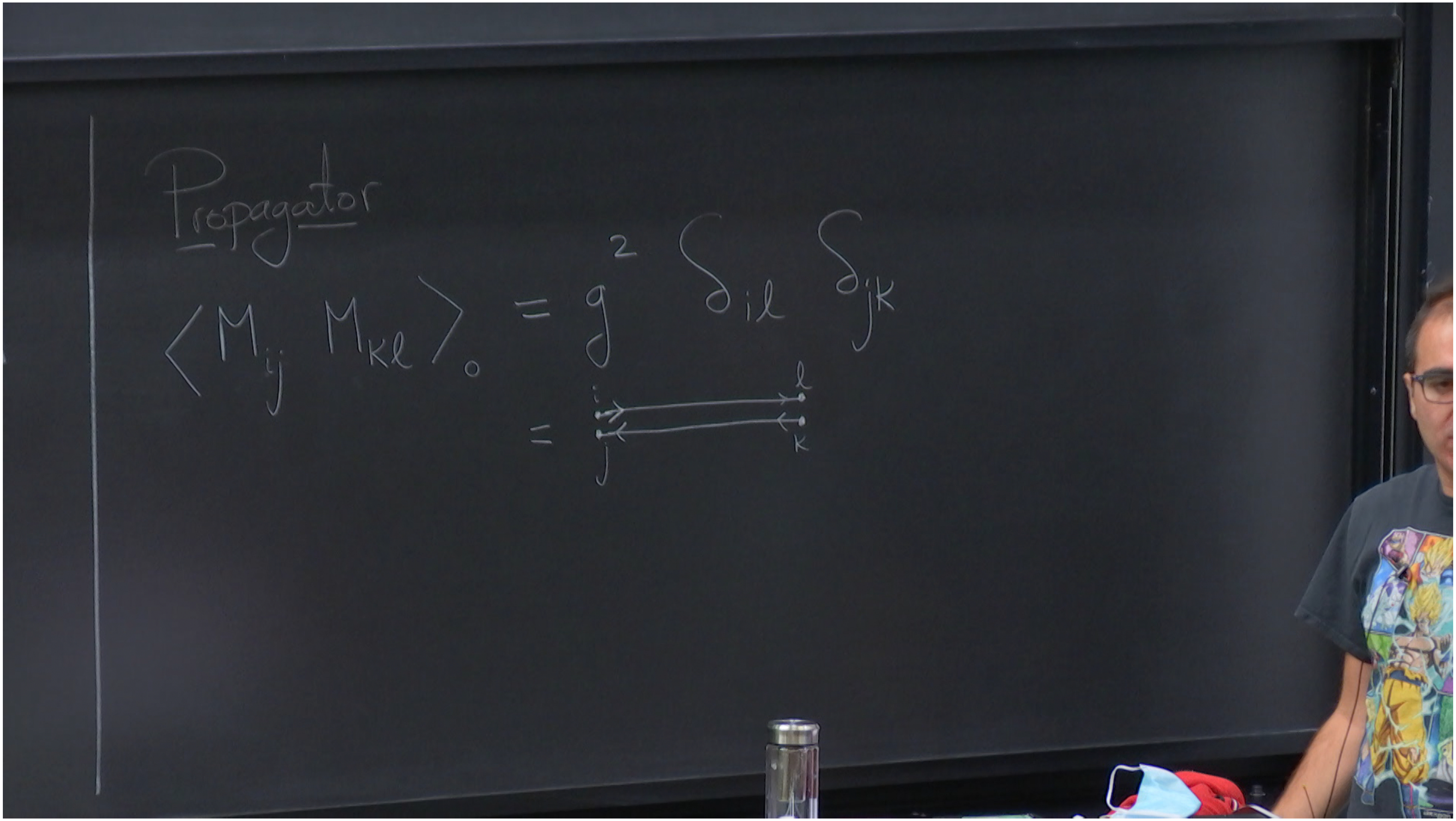
$$\langle \ldots \rangle \equiv \int \mathcal{D}M \, \ldots \, e^{-S[M]} \Big/ \int \mathcal{D}M \, e^{-S[M]}$$

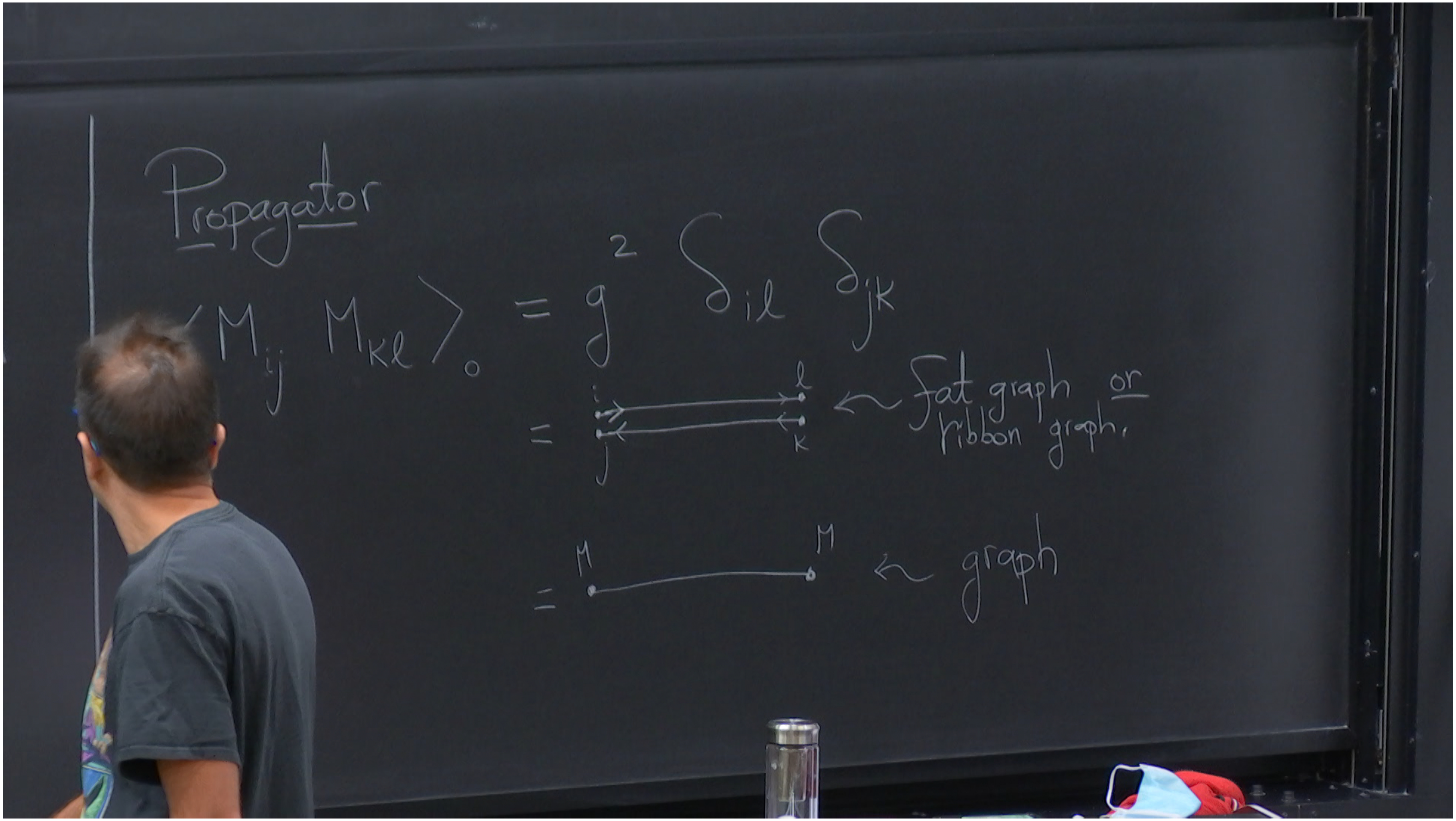
$$\langle \ldots \rangle_0 = \text{Same with } V \to 0 \qquad \text{(Perturbation theory)}$$


Propagator
$\langle M_{ij} \, M_{kl} \rangle_0$

$$\mathrm{tr}\, M^2 = M_{ij} M_{ji}$$

$$= \sum_{i=1}^{N} \underbrace{M_{ii}\, M_{ii}}_{(\mathrm{Re}\, M_{ii})^2} + \sum_{i \neq j} \underbrace{\left(\mathrm{Re}\, M_{ij} + i\, \mathrm{Im}\, M_{ij}\right)\left(\quad - \quad \right)}_{(\mathrm{Re}\, M_{ij})^2 + (\mathrm{Im}\, M_{ij})^2}$$

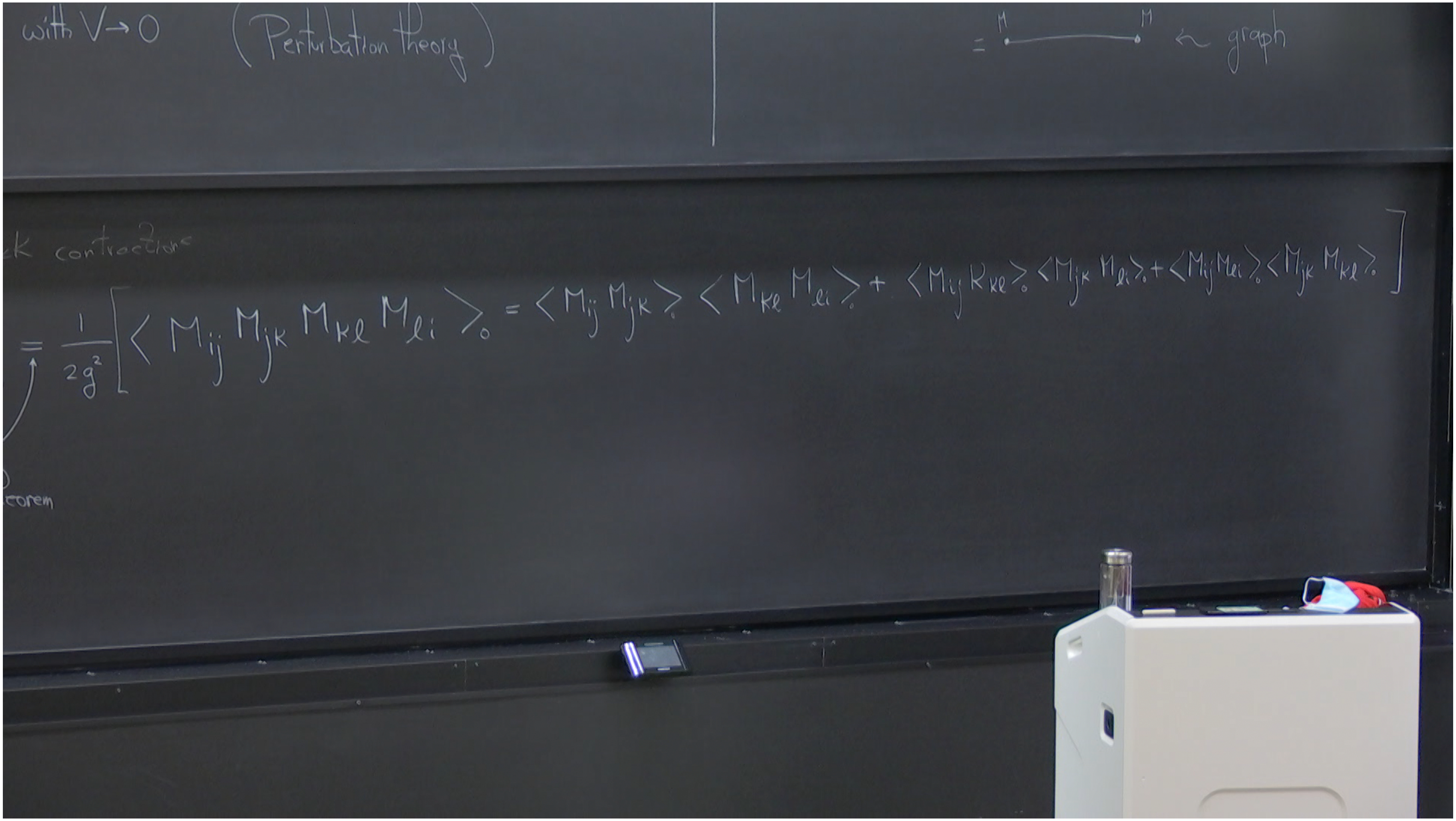

Propagator
$\langle M_{ij} M_{kl} \rangle_0 = g^2 \delta_{il} \delta_{jk}$
i l
=
j k

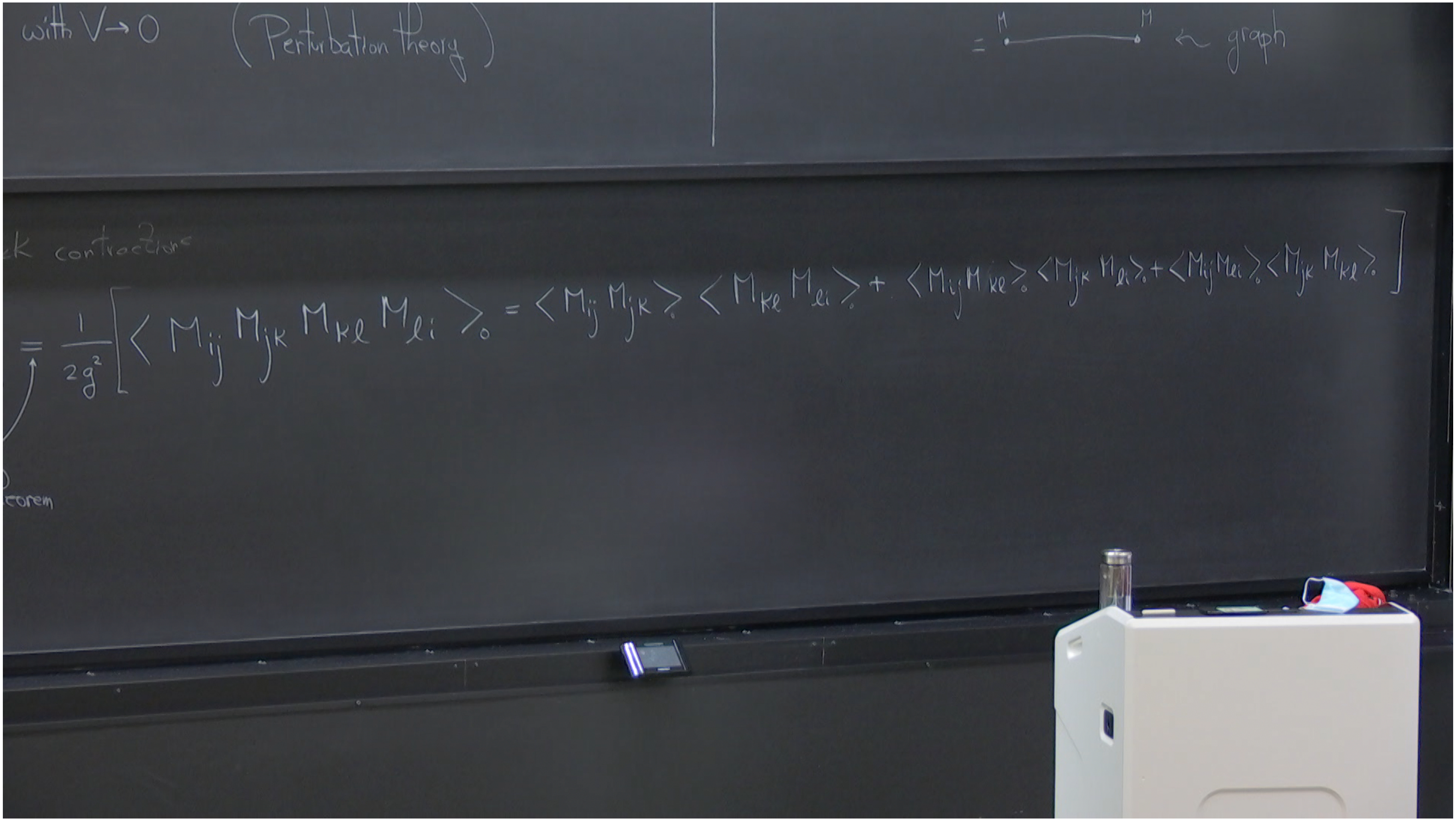

Propagator
$\langle M_{ij} M_{kl} \rangle_0 = g^2 \delta_{il} \delta_{jk}$
i
l
j
k
fat graph or
ribbon graph.
M
M
graph

② interactions by Wick contractions

$$\left\langle \frac{1}{2g^2} \operatorname{tr} M^4 \right\rangle_0 =$$

Wick theorem


with V → 0
(Perturbation theory)
= M —— M ← graph
k contractions?
= 1/(2g²) [⟨ M_ij M_jk M_kl M_li ⟩₀ = ⟨ M_ij M_jk ⟩₀ ⟨ M_kl M_li ⟩₀ + ⟨ M_ij M_kl ⟩₀ ⟨ M_jk M_li ⟩₀ + ⟨ M_ij M_li ⟩₀ ⟨ M_jk M_kl ⟩₀]
eorem


with $V \to 0$ (Perturbation theory)
$= \overset{M}{\bullet}\!\!-\!\!-\!\!\overset{M}{\bullet}$ ← graph
k contractions
$= \frac{1}{2g^2}\Big[\langle M_{ij} M_{jk} M_{kl} M_{li} \rangle_0 = \langle M_{ij} M_{jk} \rangle_0 \langle M_{kl} M_{li} \rangle_0 + \langle M_{ij} M_{kl} \rangle_0 \langle M_{jk} M_{li} \rangle_0 + \langle M_{ij} M_{li} \rangle_0 \langle M_{jk} M_{kl} \rangle_0 \Big]$
eorem

by Wick contractions

$$\langle \ldots^4 \rangle_0 = \frac{1}{2g^2}\Big[\langle M_{ij} M_{jk} M_{kl} M_{li} \rangle_0 = \langle M_{ij} M_{jk} \rangle_0 \langle$$

$$=$$

...ick theorem

contractions

$$\frac{1}{g^2}\Big[\langle M_{ij} M_{jk} M_{kl} M_{li}\rangle_0 = \langle M_{ij} M_{jk}\rangle_0 \langle M_{kl} M_{li}\rangle_0 + \langle M_{ij} M_{kl}\rangle_0 \langle M_{jk} M_{li}\rangle_0 + \Big]$$

$$= g^2\Big[(\delta_{ik}\,\delta_{jj})(\delta_{ki}\,\delta_{ll}) + (\delta_{il}\,\delta_{jk})(\delta_{ji}\,\delta_{kl}) + (\delta_{ii}\,\delta_{jl})(\delta_{jl}\,\delta_{kk})\Big]$$

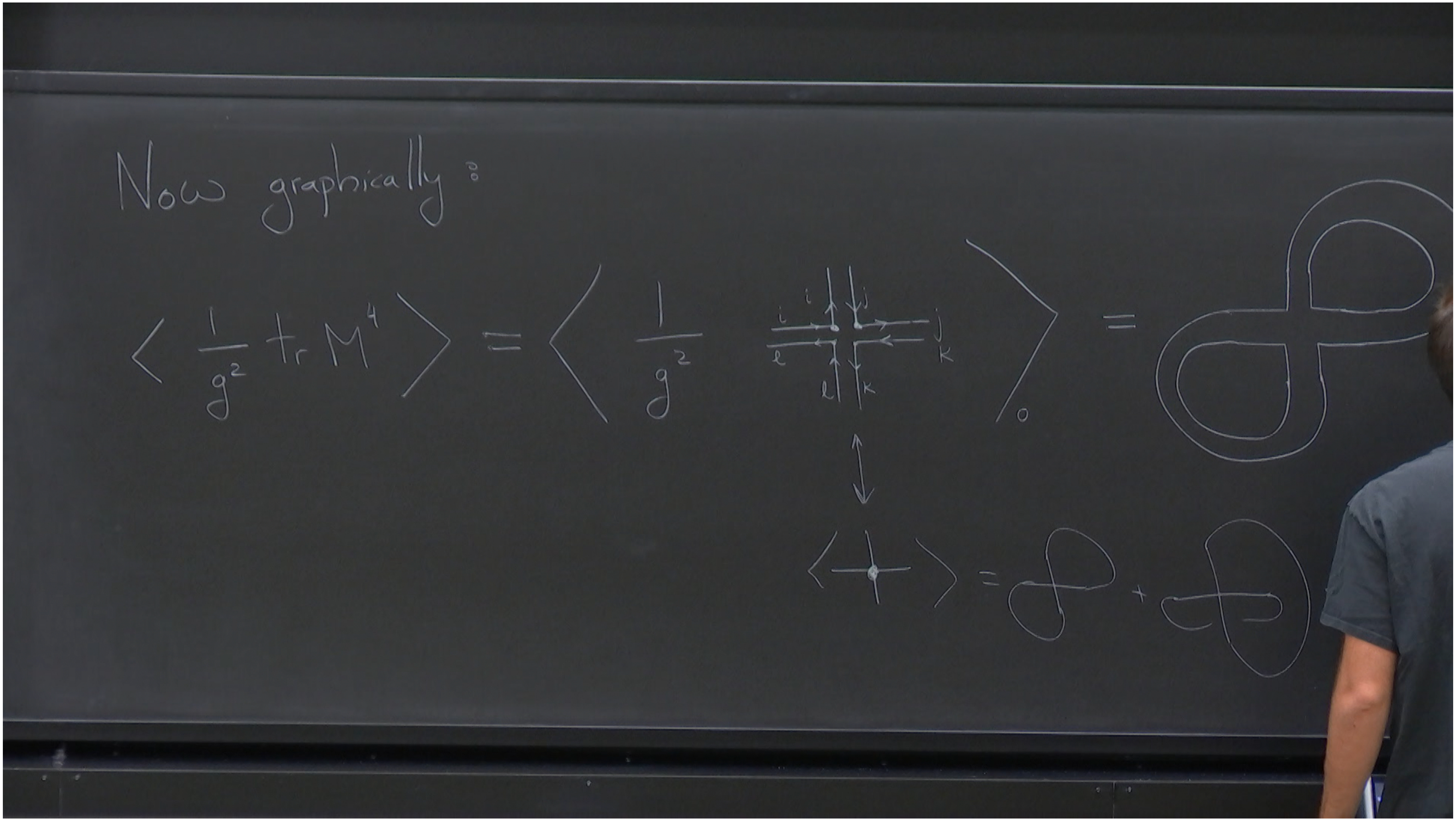
k contractions

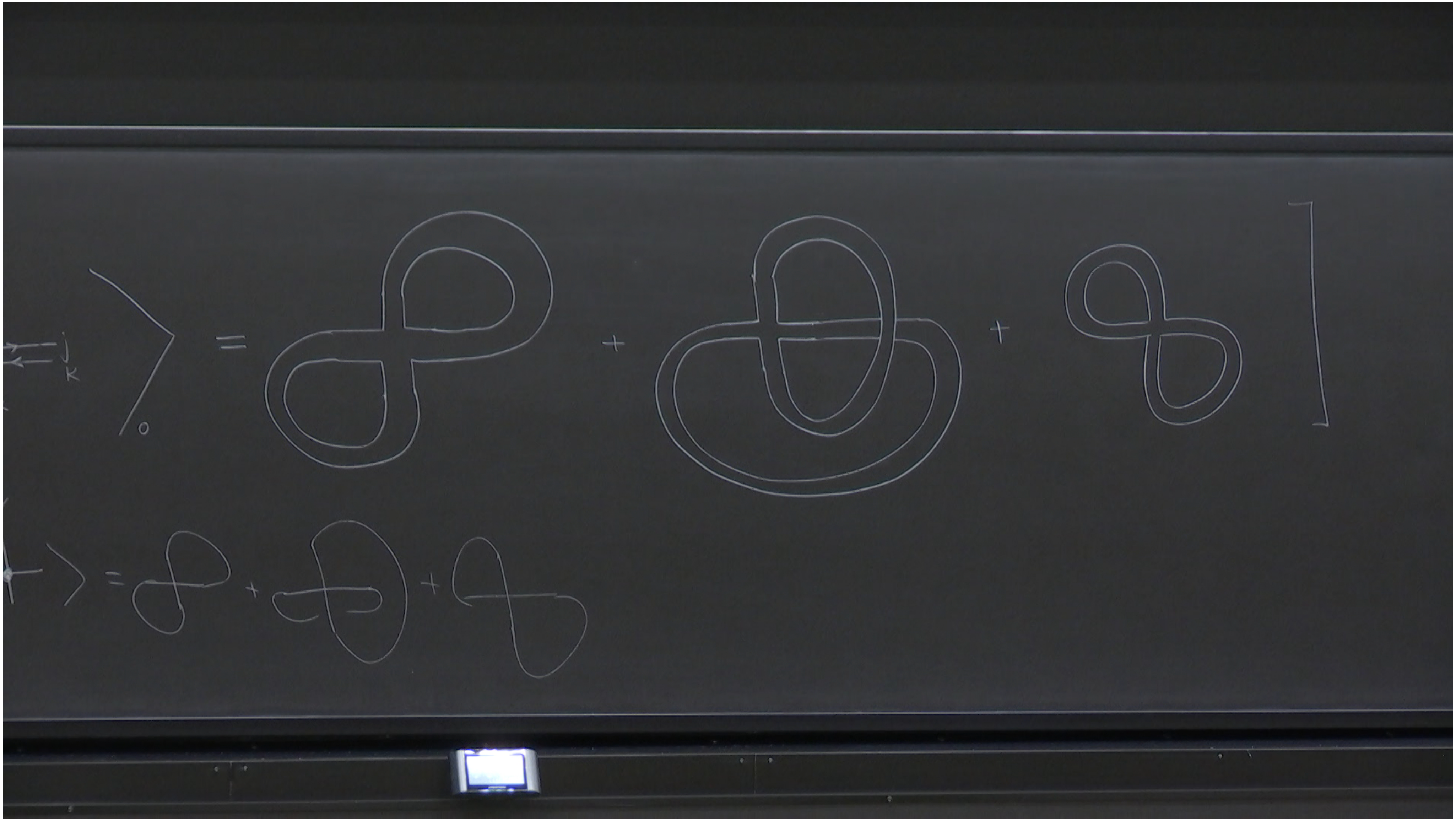
$$= \frac{1}{g^2}\Big[\langle M_{ij} M_{jk} M_{k\ell} M_{\ell i} \rangle_0 = \langle M_{ij} M_{jk} \rangle_0 \langle M_{k\ell} M_{\ell i} \rangle_0 + \langle M_{ij} M_{k\ell} \rangle_0 \langle M_{jk} M_{\ell}$$

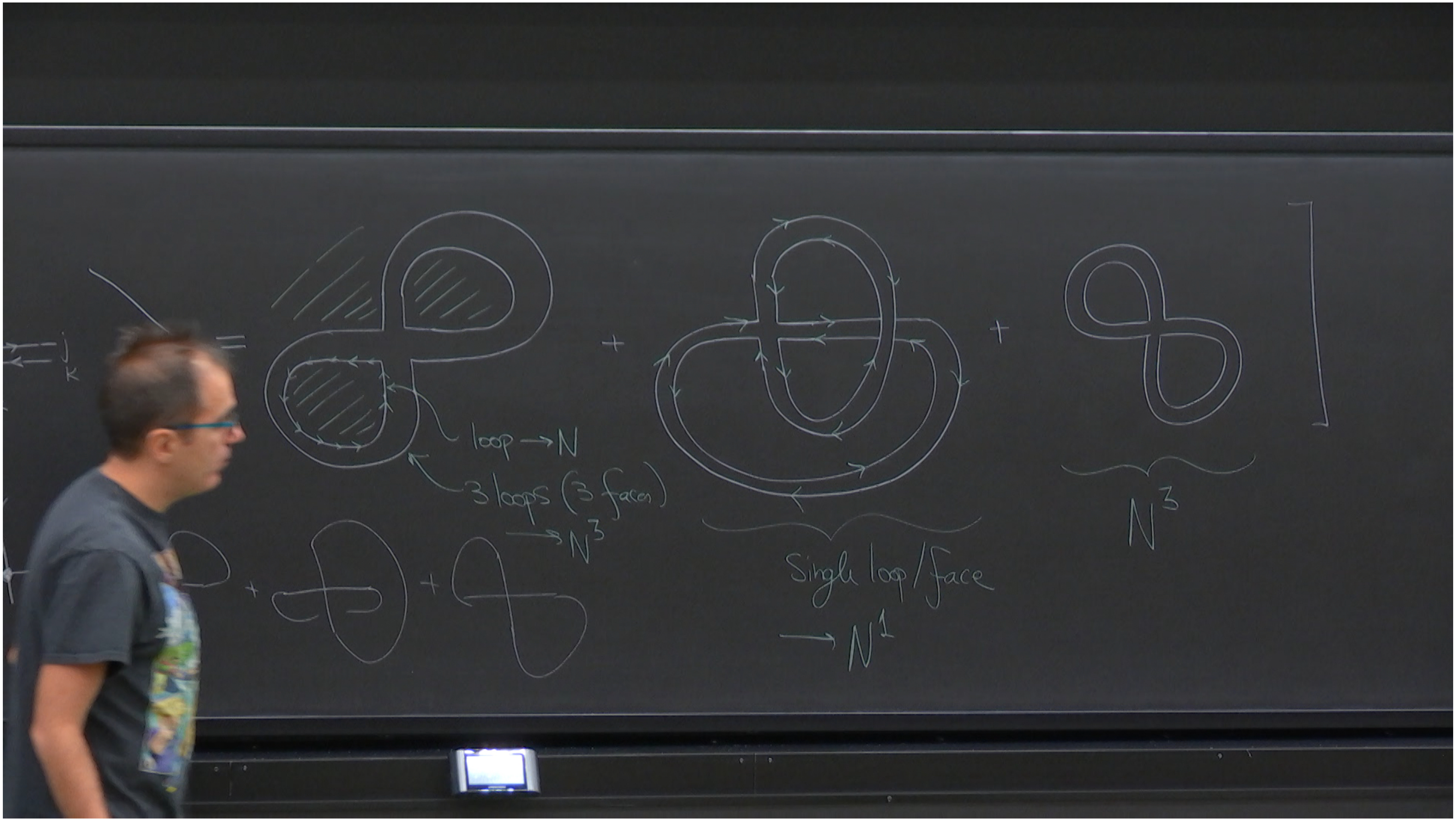
$$= g^2 \Big[(\delta_{ik}\,\delta_{jj})(\delta_{ki}\,\delta_{\ell\ell}) + (\delta_{i\ell}\,\delta_{jk})(\delta_{ji}\,\delta_{k\ell}) + (\delta_{ii}\,\delta_{j\ell})(\delta_{j\ell}\,\delta_{kk})\Big]$$

orem

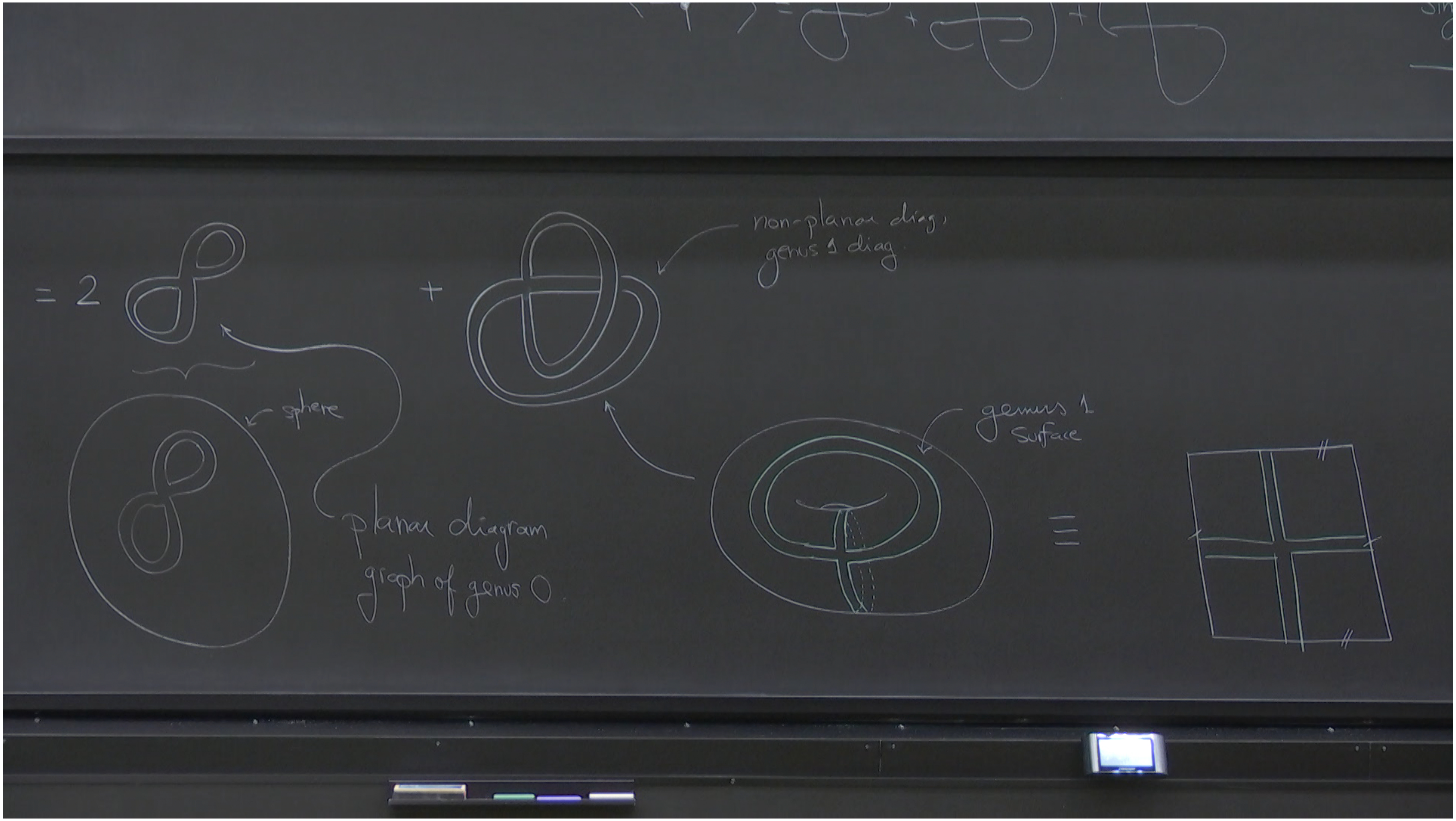
$$= g^2 \left[N^3 + N + N^3\right]$$

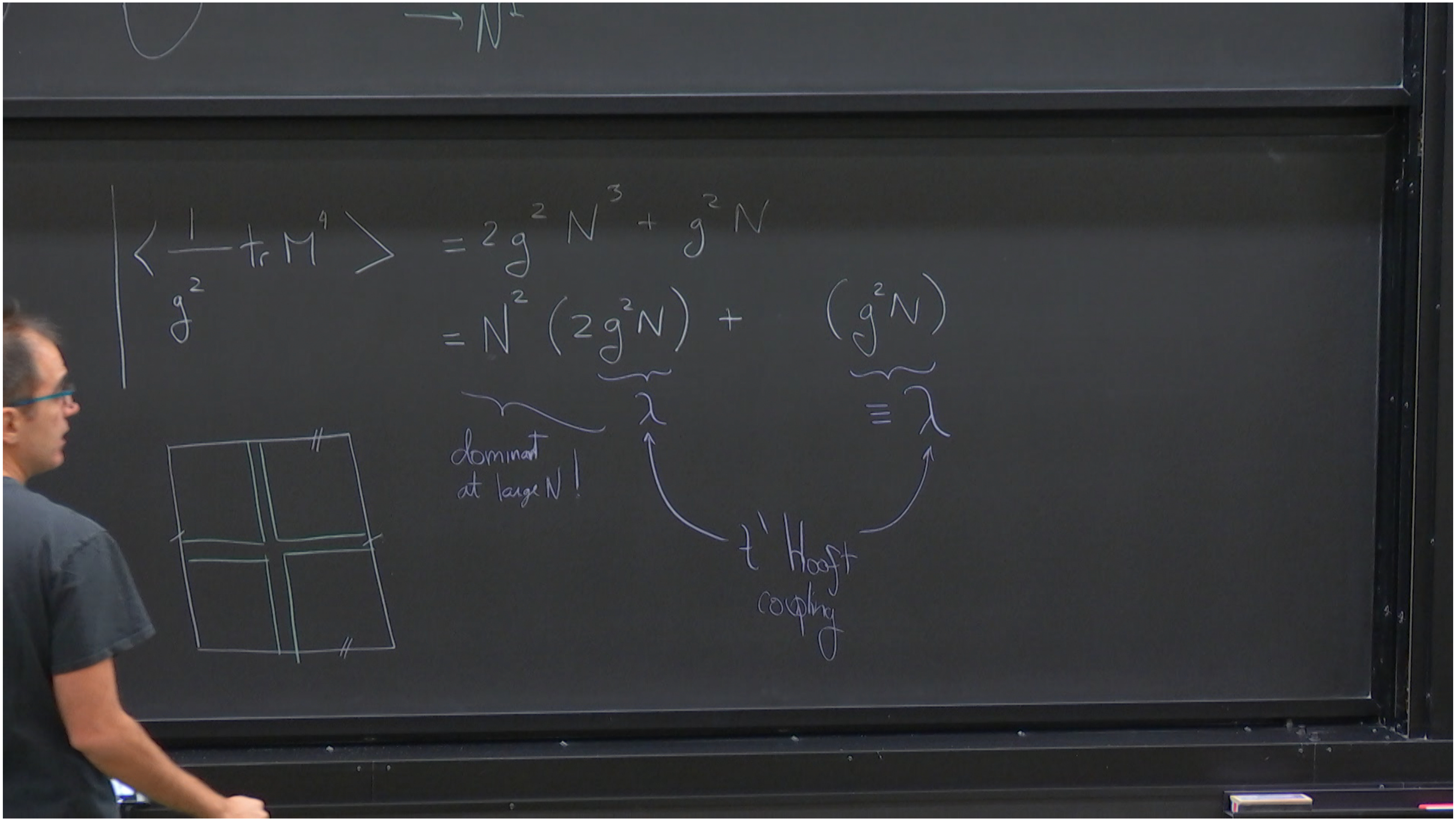

Now graphically :
$\langle \frac{1}{g^2} \mathrm{tr} M^4 \rangle = \langle \frac{1}{g^2} \rangle_0 =$
i j k l
$\langle \rangle = +$



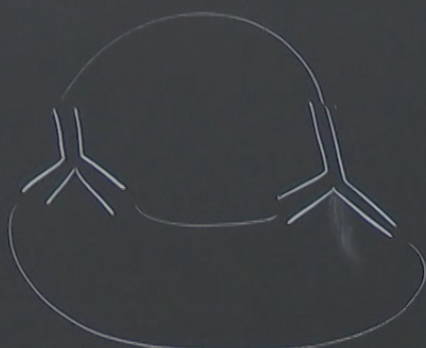
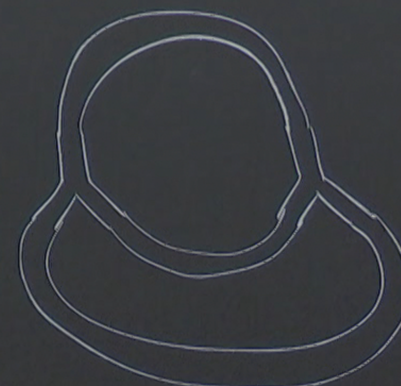


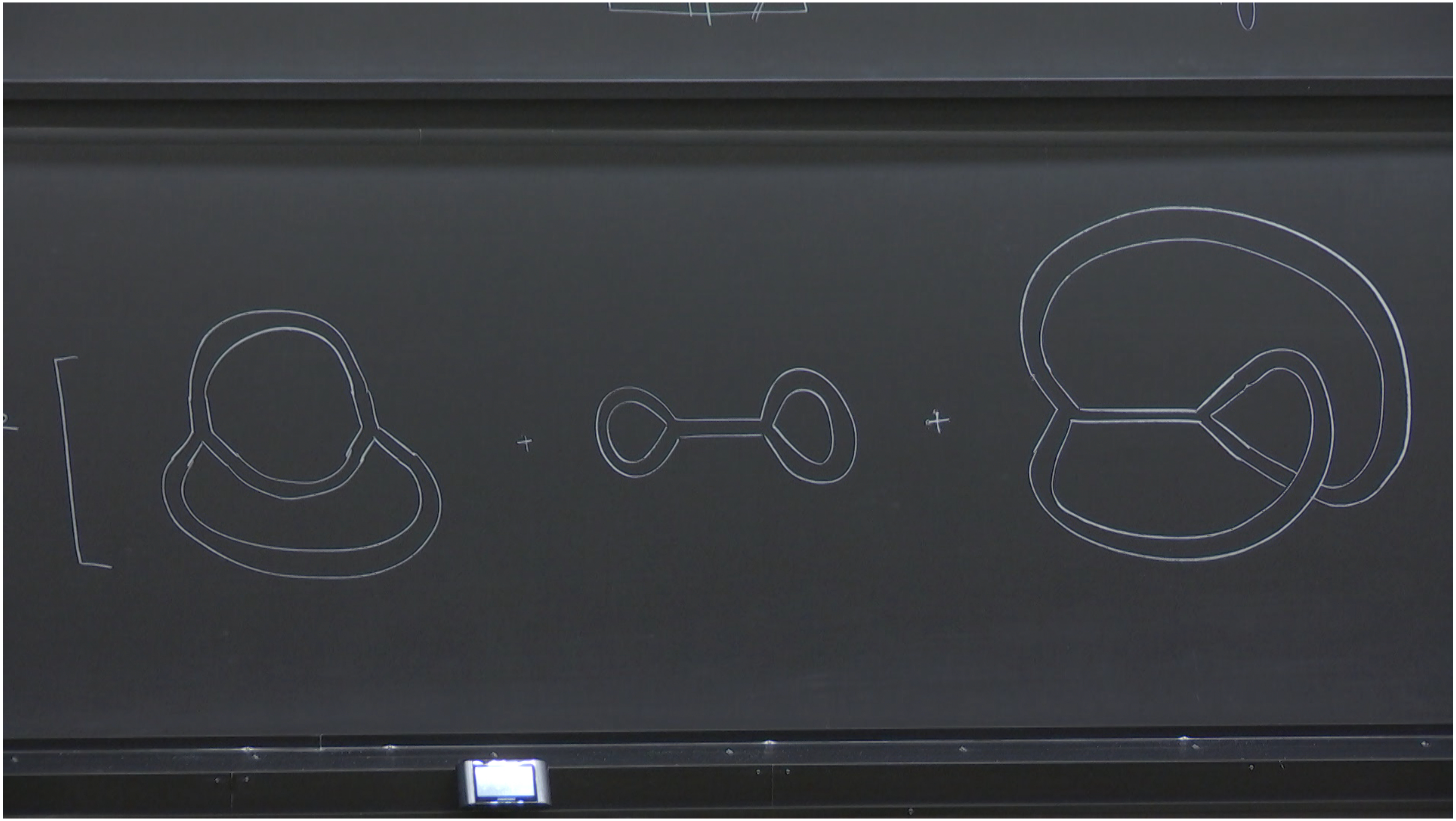
loop → N
3 loops (3 faces)
→ N^3
Single loop/face
→ N^1
N^3

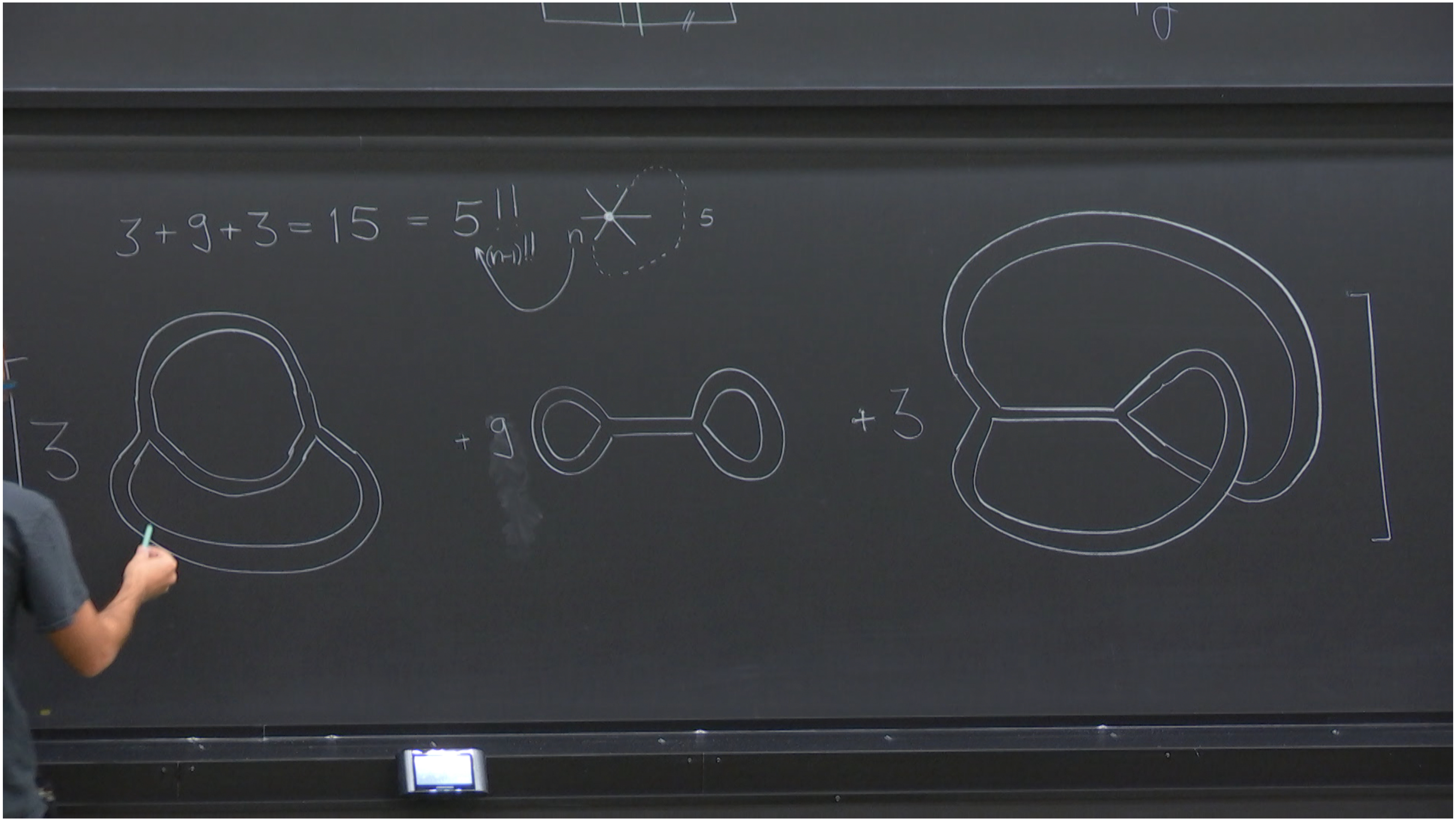

= 2
+
non-planar diag,
genus 1 diag
sphere
planar diagram
graph of genus 0
genus 1
Surface
≡

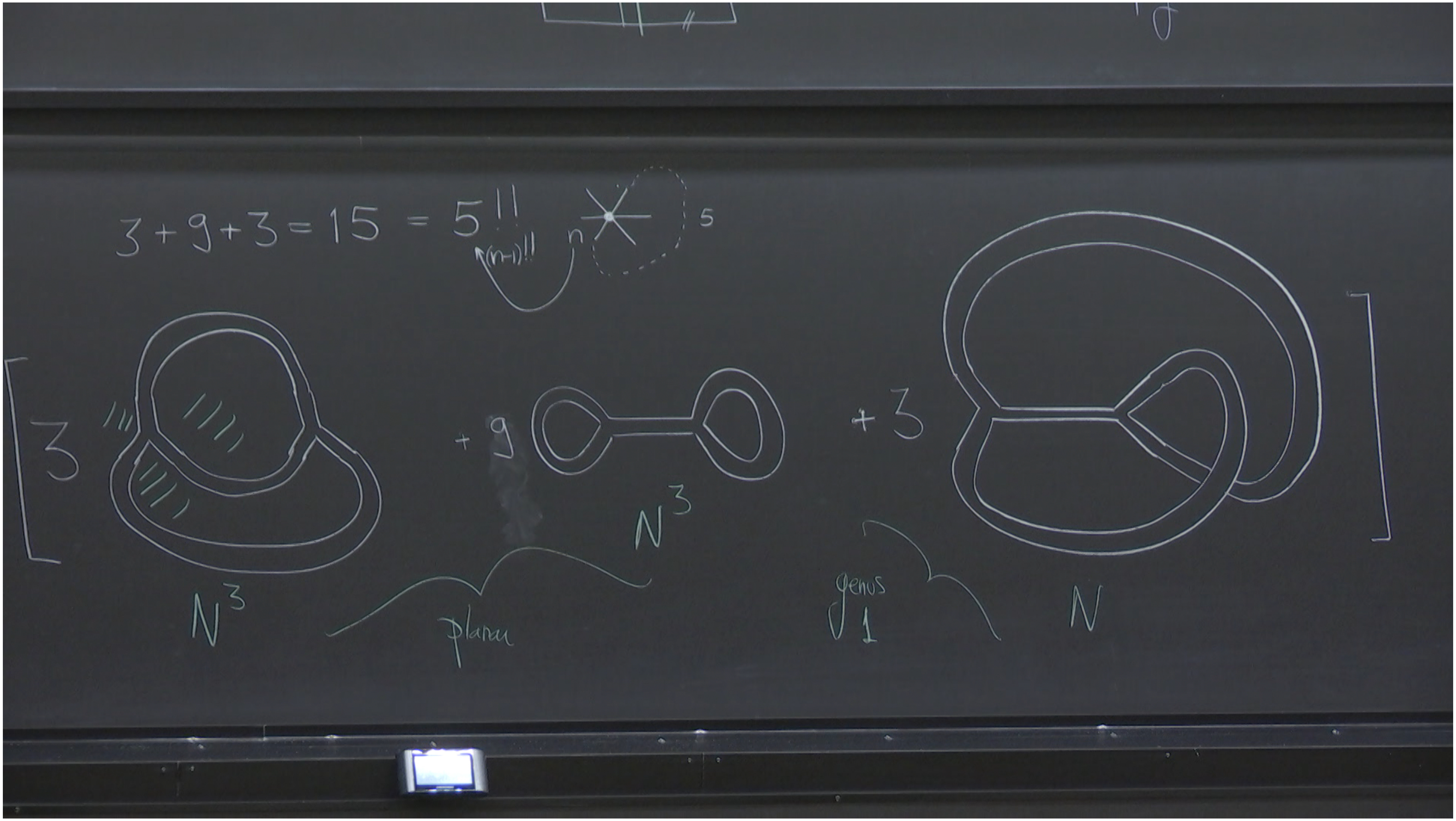

$\langle \frac{1}{g^2} \mathrm{tr} M^4 \rangle = 2g^2N^3 + g^2N$
$= N^2 (\underbrace{2g^2N}_{\lambda}) + (\underbrace{g^2N}_{\equiv \lambda})$
dominant at large N!
t' Hooft coupling

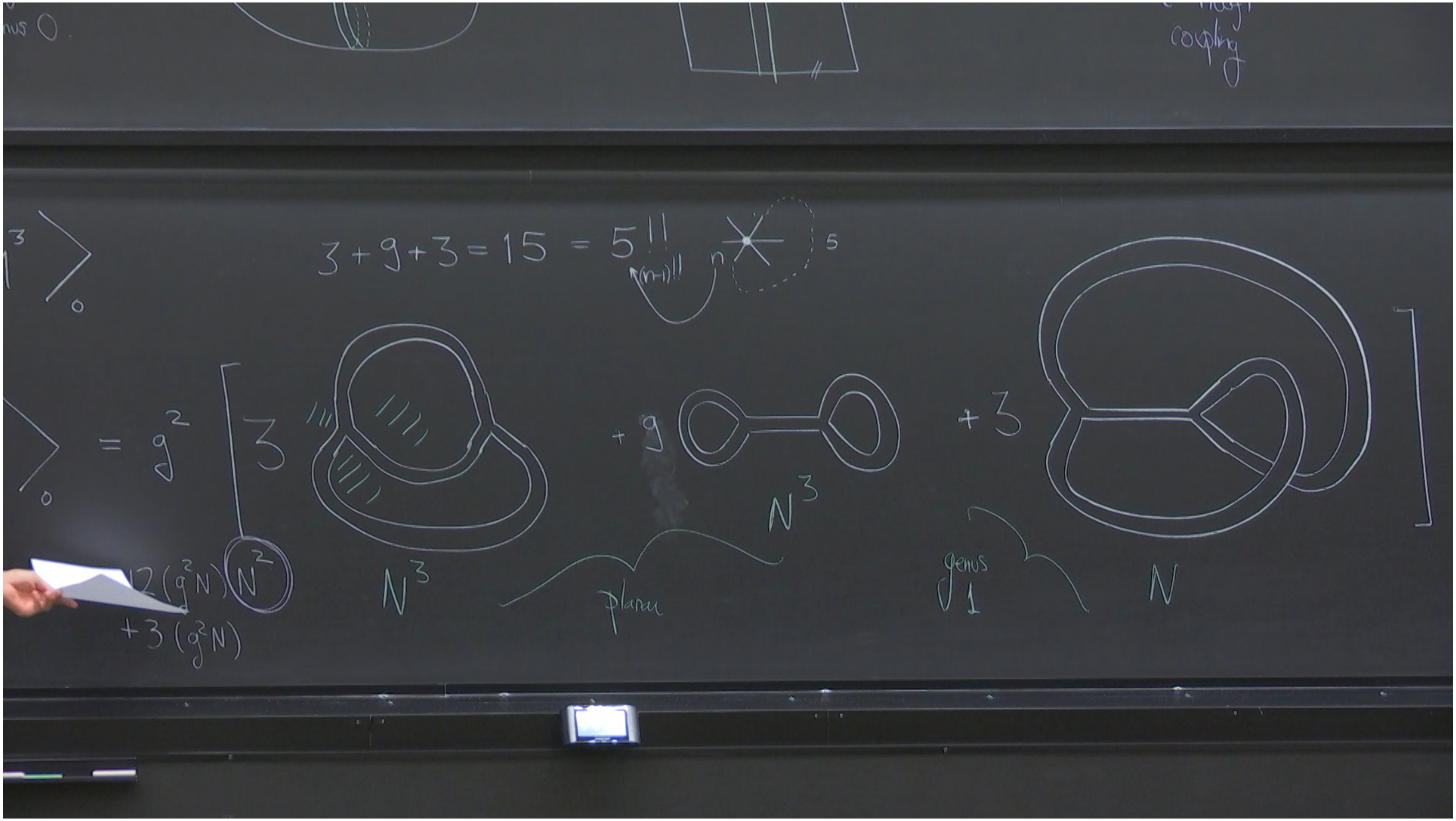
$$\left\langle \frac{1}{g^2}\operatorname{tr} M^3 \quad \frac{1}{g^2}\operatorname{tr} M^3 \right\rangle_0$$

$\frac{1}{g^4}\Big\langle$  $\Big\rangle_0 = g^2 \Big[$  $+$

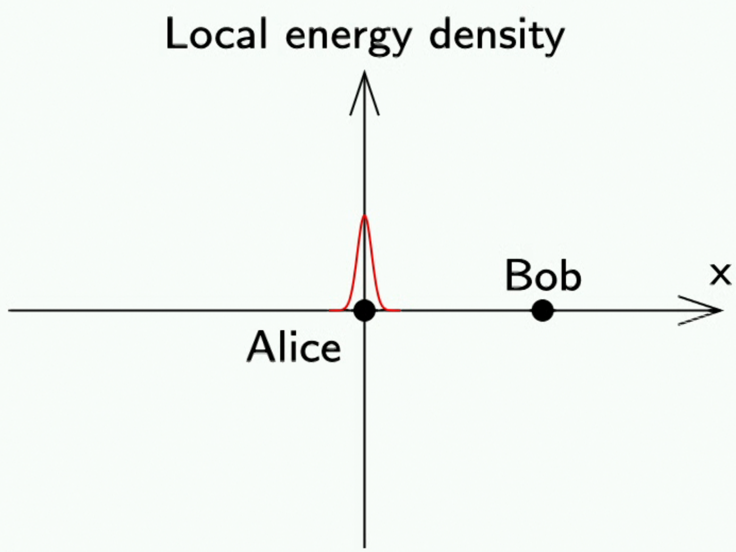



3+9+3=15 = 5!!
(n-1)!!
n
5
3
+ 9
+ 3


3+9+3=15 = 5!!
(n-1)!!
n
5
3
N^3
+9
N^3
planar
+3
genus 1
N

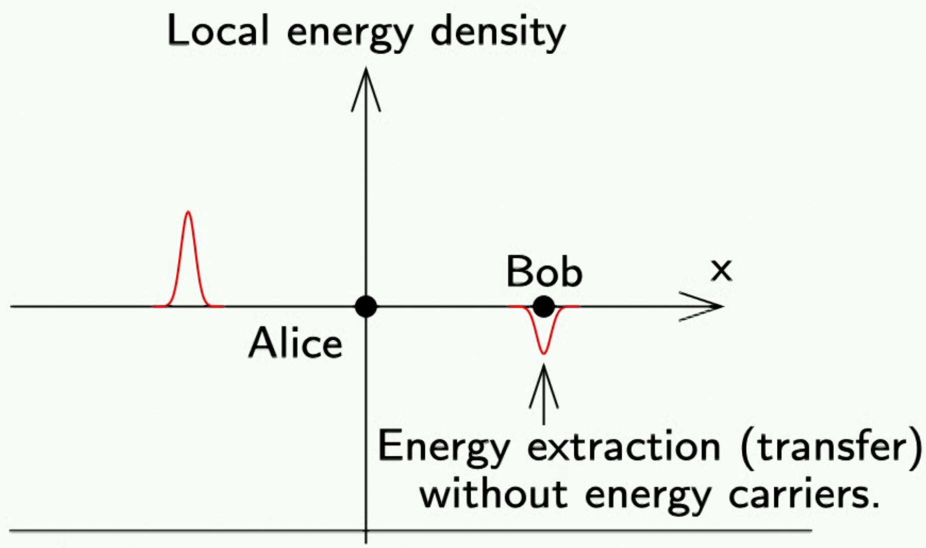

coupling
3+9+3=15 = 5!!
(n-1)!!
n
5
= g2 [3
+ 9
+ 3
N3
N3
planar
genus 1
N
12(g2N) N2
+3 (g2N)

Breaking Strong Local Passivity
Quantum Energy Teleportation



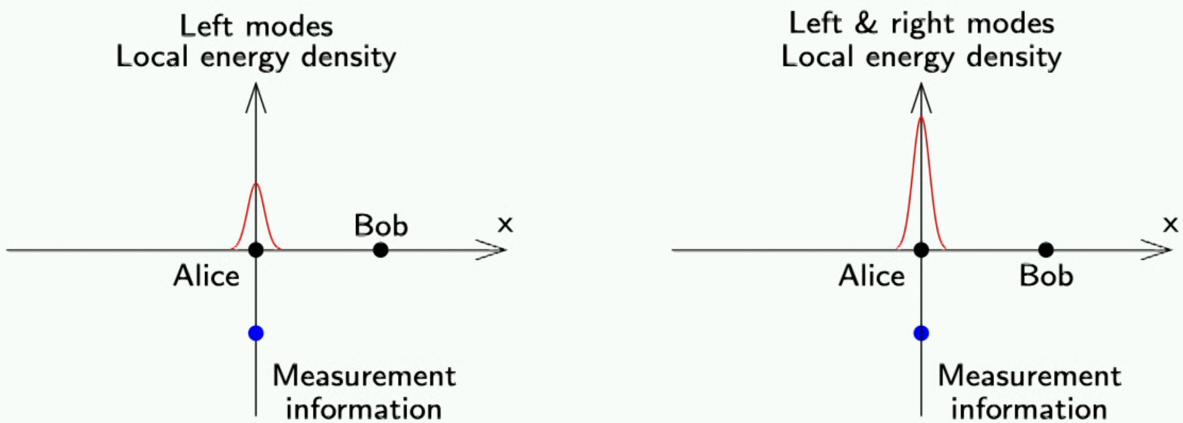
[1] Hotta, *Phys. Rev. D.*, 78.045006, Aug 2008

Breaking Strong Local Passivity Quantum Energy Teleportation

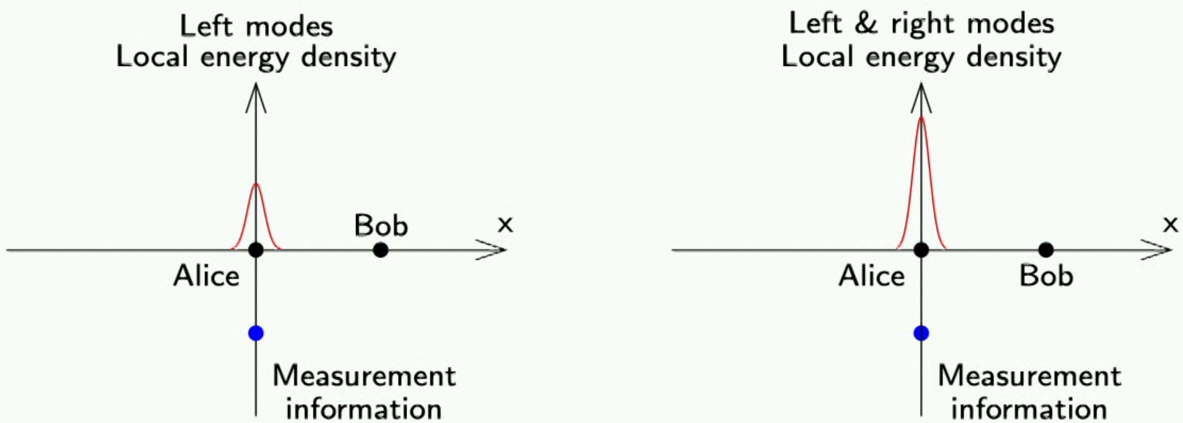


[1] Hotta, *Phys. Rev. D.*, 78.045006, Aug 2008
[2] Hotta, *J. Phys. A: Math. Theor.*, 43,105305 (2010)

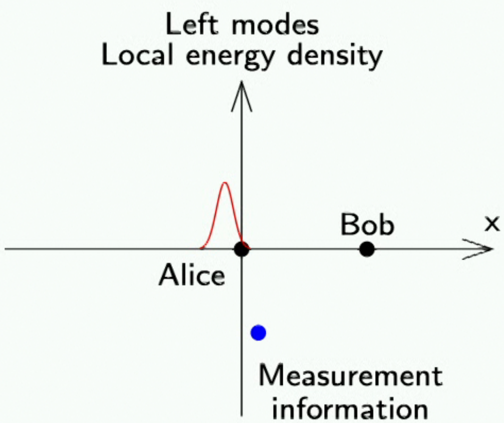
Breaking Strong Local Passivity
Quantum Energy Teleportation

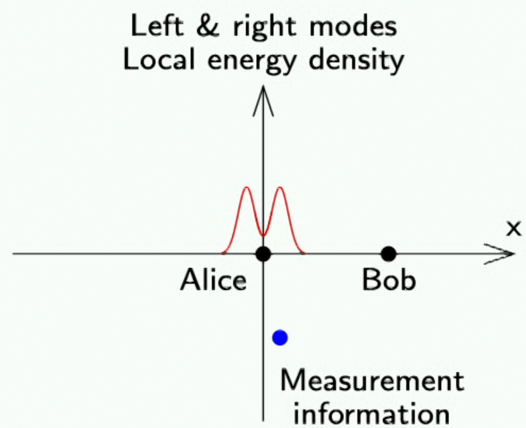


Breaking Strong Local Passivity
Quantum Energy Teleportation

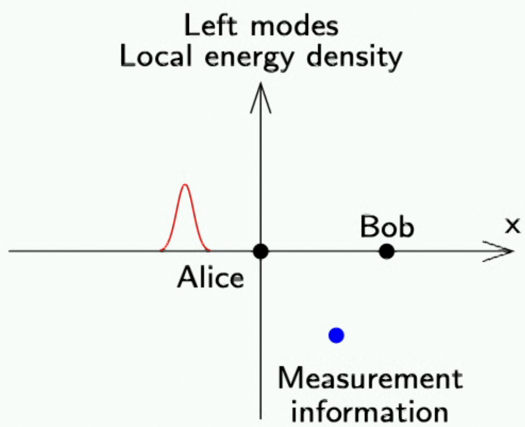


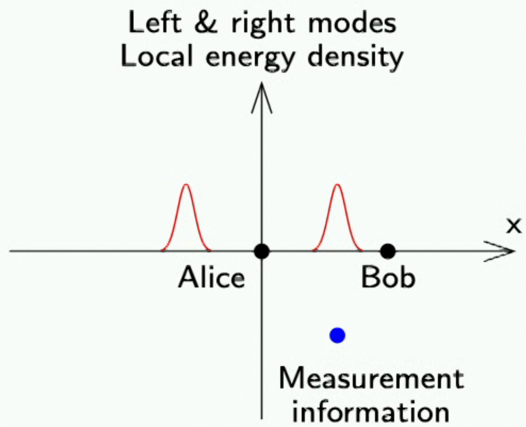
Breaking Strong Local Passivity
Quantum Energy Teleportation



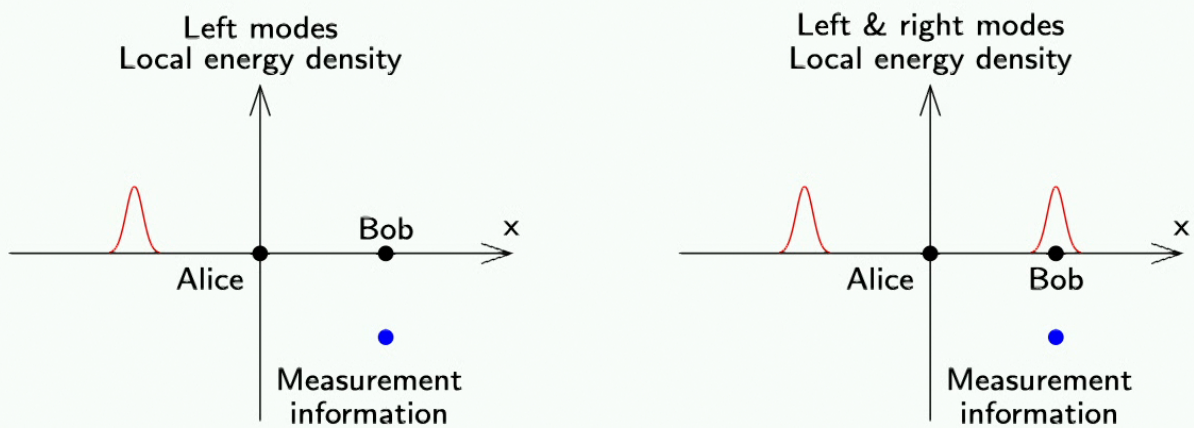


Breaking Strong Local Passivity
Quantum Energy Teleportation

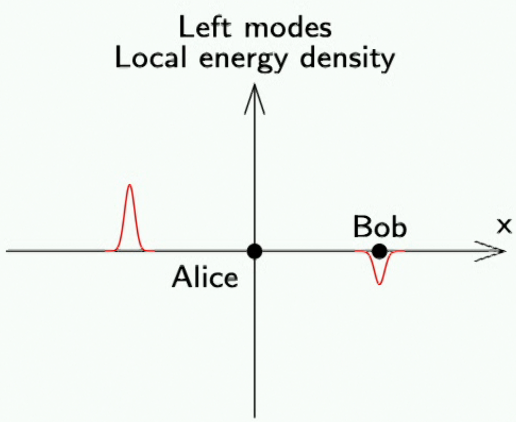


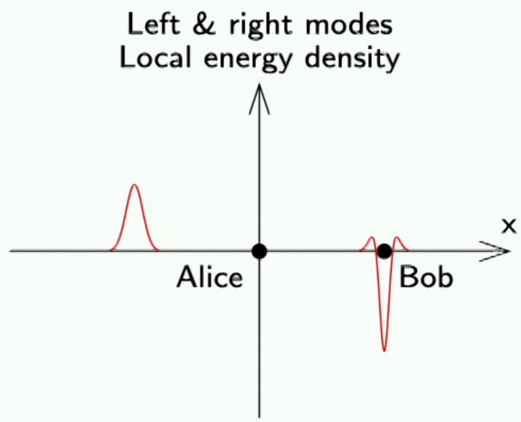


Breaking Strong Local Passivity
Quantum Energy Teleportation



Breaking Strong Local Passivity
Quantum Energy Teleportation





So what's the plan?

What we will do:

1-We will focus on the state of the field

2-We will focus on 3+1D

So what's the plan?

What we will do:

1-We will focus on the state of the field

2-We will focus on 3+1D

3-Protocol made concrete (Using atomic probes, not idealized systems)

Protocol

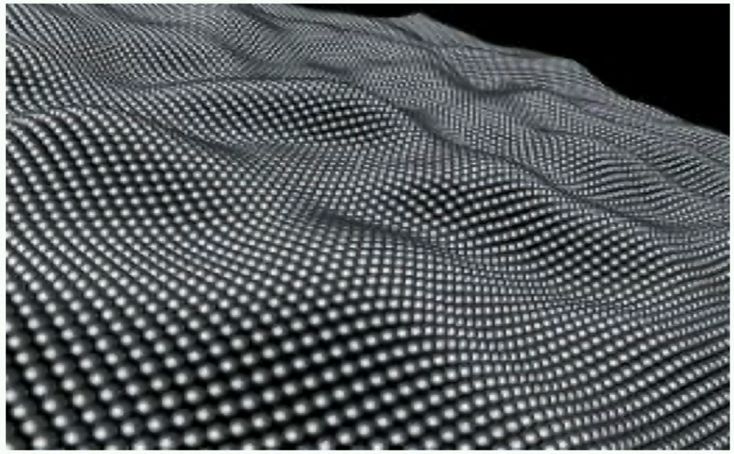
1-Alice measures the field by coupling an atom (or array fo atoms) to it

2-Alice measures her non-relativistic atom

3-Alice broadcasts the result of the measurement to an agency of Bob's

4-Bob's agents use that information to prepare atoms and couple to the field

Engineering negative energy densities



Preparation of states of spacetime

Engineering negative stress-energy densities with quantum energy teleportation
N. Funai, E. Martín-Martínez, Phys. Rev. D 96, 025014 (2017)

Let's see how effective this protocol is!

Let's try to deliver what I promised. First with a toy model.

A toy first: 1+1D QET stress-energy engineering

The system is initially in a separable state (qubit state $|A_0\rangle$, field vacuum state $|0\rangle$):

$$|\psi(t<0)\rangle = |A_0\rangle \otimes |0\rangle \, .$$

Alice interacts via:

$$\hat{H}_{\text{int}} = \delta(t)\hat{\sigma}_x \otimes \int \mathrm{d}^{n-1}\boldsymbol{r}\, \lambda(\boldsymbol{r})\hat{\pi}(\boldsymbol{r}).$$

Alice teleports her qubit to Bob, interacting via:

$$\hat{H}_{\text{int}} = \delta(t-T)\hat{\sigma}_z \otimes \int \mathrm{d}\boldsymbol{r}\, \mu(\boldsymbol{r})\hat{\phi}(\boldsymbol{r}).$$

LOQC variant of QET Phys. Rev. A 93, 022308 (2016)

A toy first: 1+1D QET stress-energy engineering

The system is initially in a separable state (qubit state $|A_0\rangle$, field vacuum state $|0\rangle$):

$$|\psi(t<0)\rangle = |A_0\rangle \otimes |0\rangle \, .$$

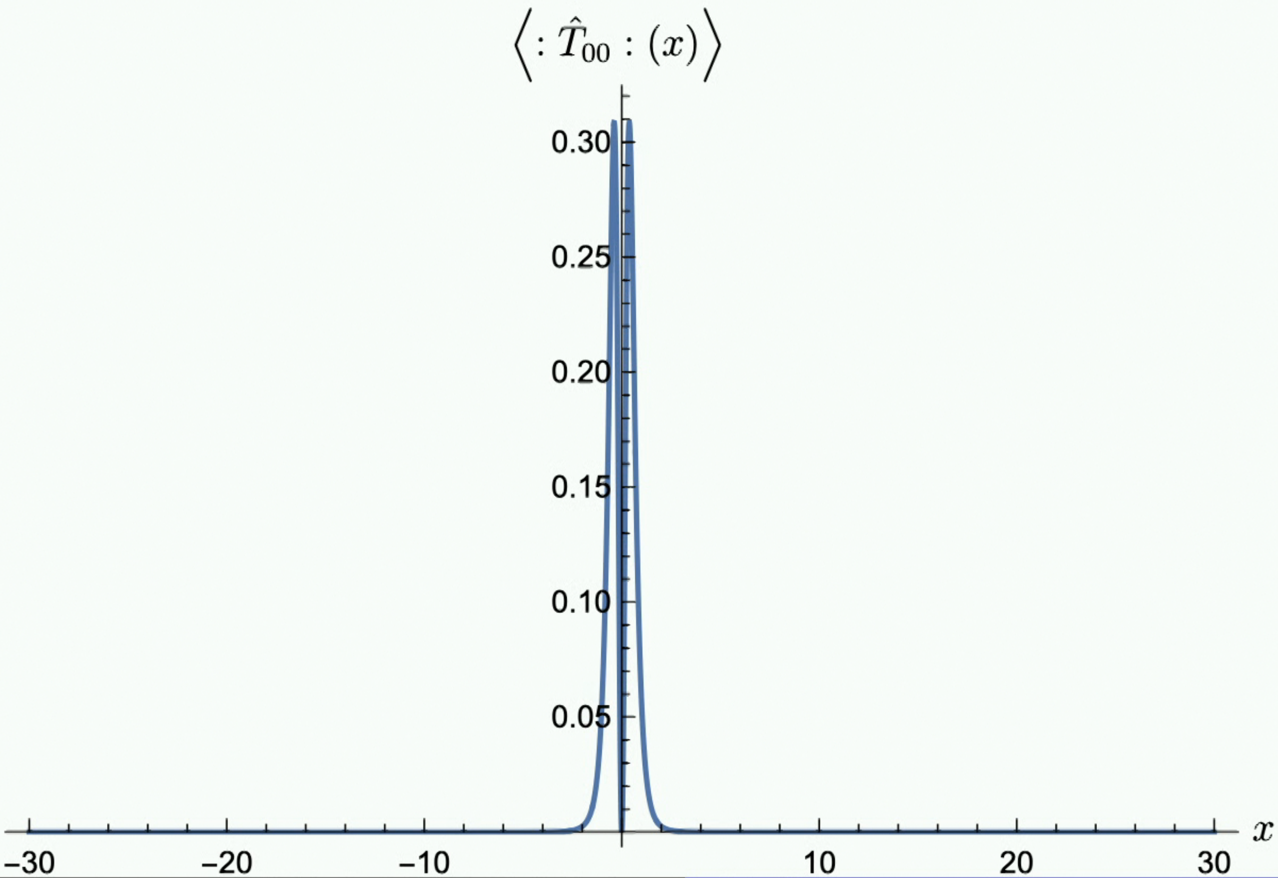
When $n=2$ the resulting stress-energy density is given by:

$$\Big\langle :\hat{T}_{00}(x,t): \Big\rangle = \overbrace{\frac{(\lambda'(x-t))^2}{4} + \frac{(\lambda'(x+t))^2}{4}}^{\text{Alice's energy contribution}} + \overbrace{\frac{(\mu(x-(t-T)))^2}{4} + \frac{(\mu(x+(t-T)))^2}{4}}^{\text{Bob's energy contribution}}$$
$$+ \underbrace{\frac{e^{-2\|\alpha\|}\,\langle A_0|\hat{\sigma}_y|A_0\rangle}{2\pi}\mu(x-(t-T))\int dy\lambda'(y)\frac{P.P}{y-x+t}}_{\text{Right moving QET term}}$$
$$+ \underbrace{\frac{e^{-2\|\alpha\|}\,\langle A_0|\hat{\sigma}_y|A_0\rangle}{2\pi}\mu(x+(t-T))\int dy\lambda'(y)\frac{P.P}{y-x-t}}_{\text{Left moving QET term}} \, .$$

1+1 D QET

Energy density immediately following Alice's interaction.

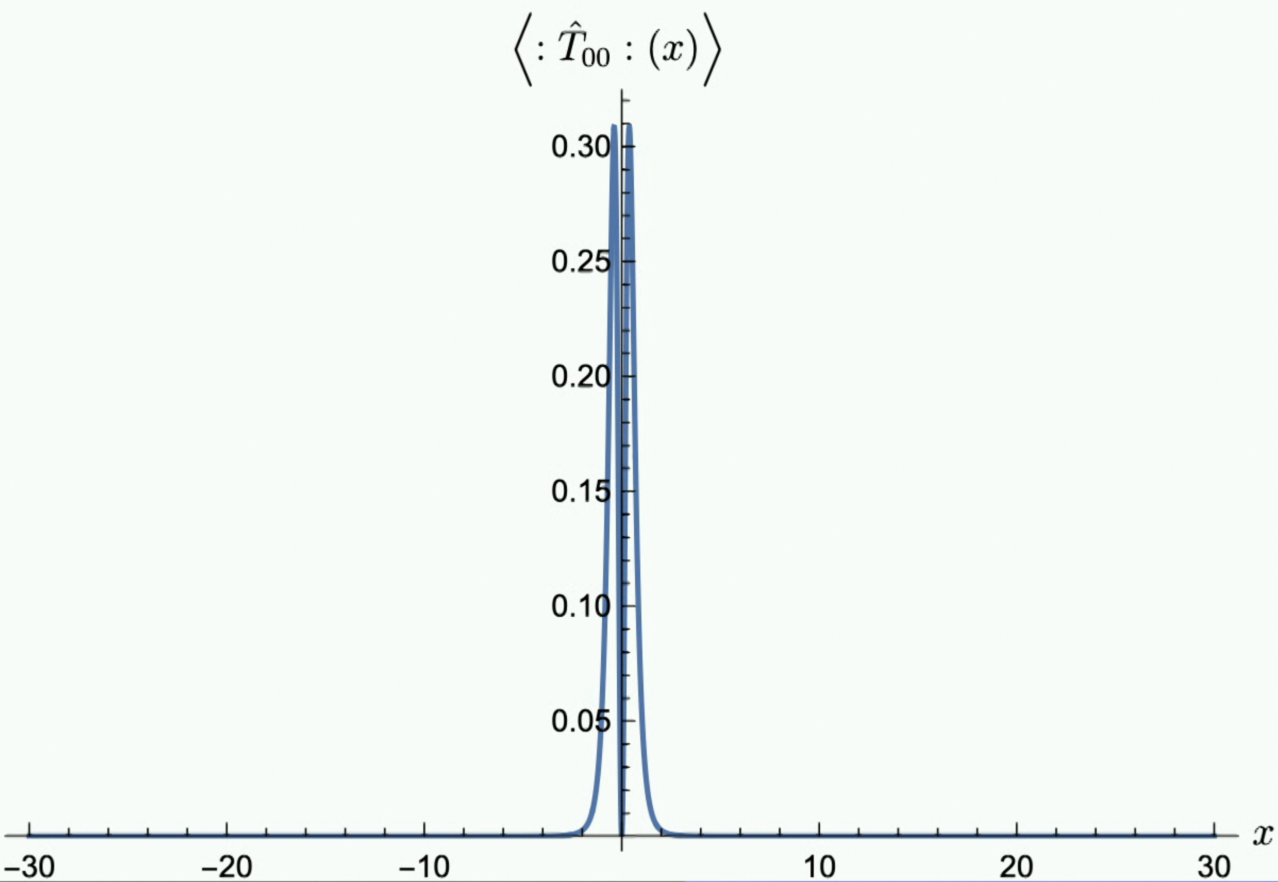
Lorentzian smearing.



1+1 D QET

Energy density immediately following Alice's interaction.

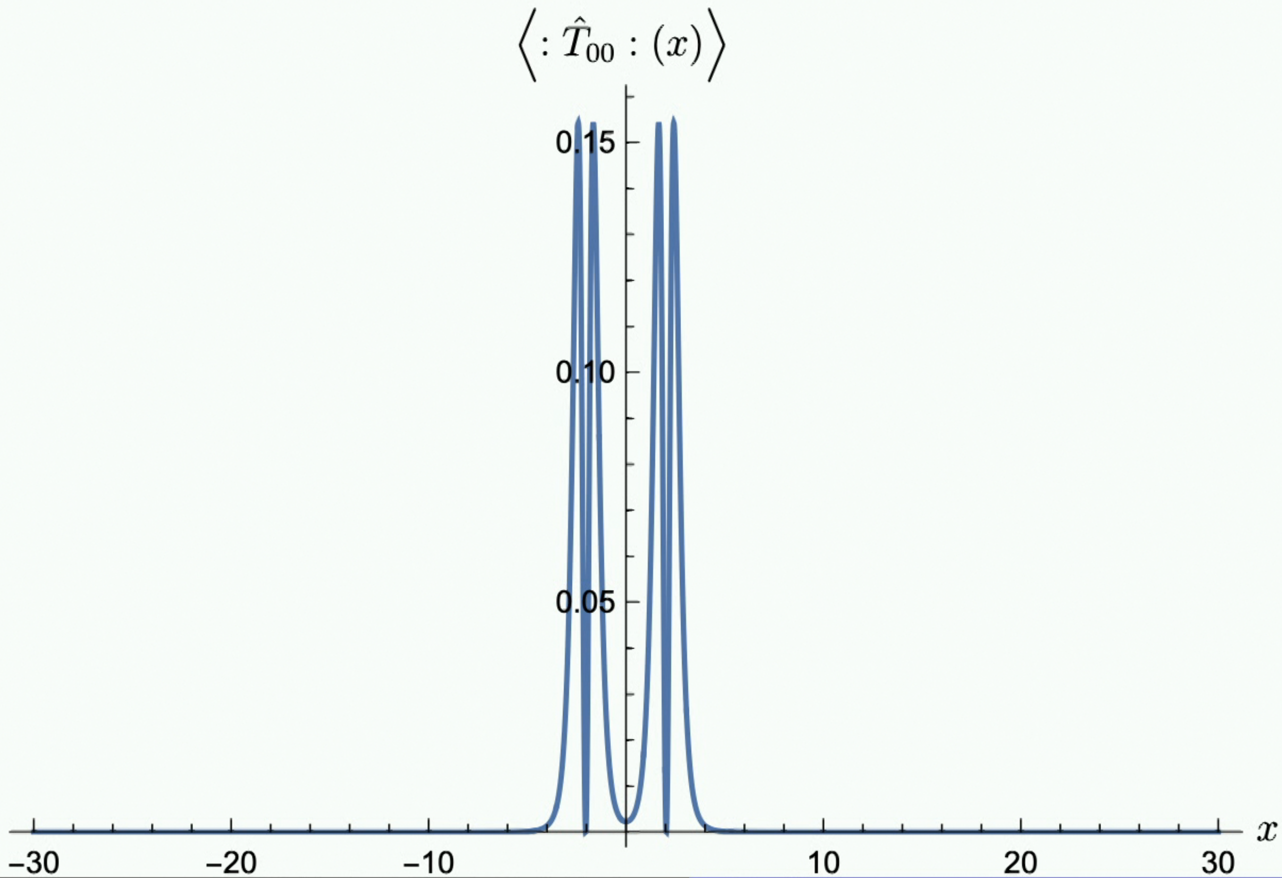
Lorentzian smearing.



1+1 D QET

Energy density after Alice's interaction.

Lorentzian smearing.

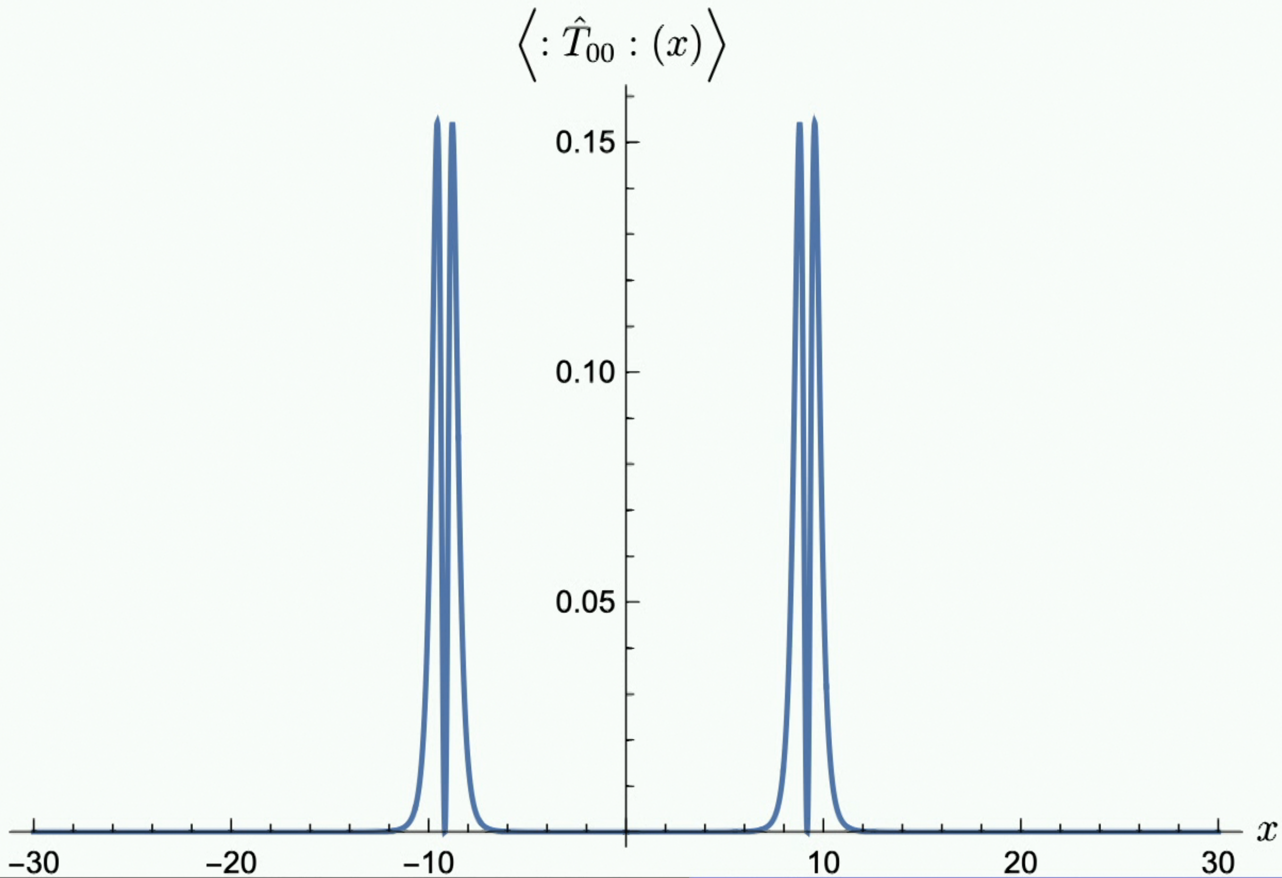


Nicholas Funai Eduardo Martín-Martínez Using QET to generate exotic spacetime geometries

1+1 D QET

Energy density after Alice's interaction.

Lorentzian smearing.



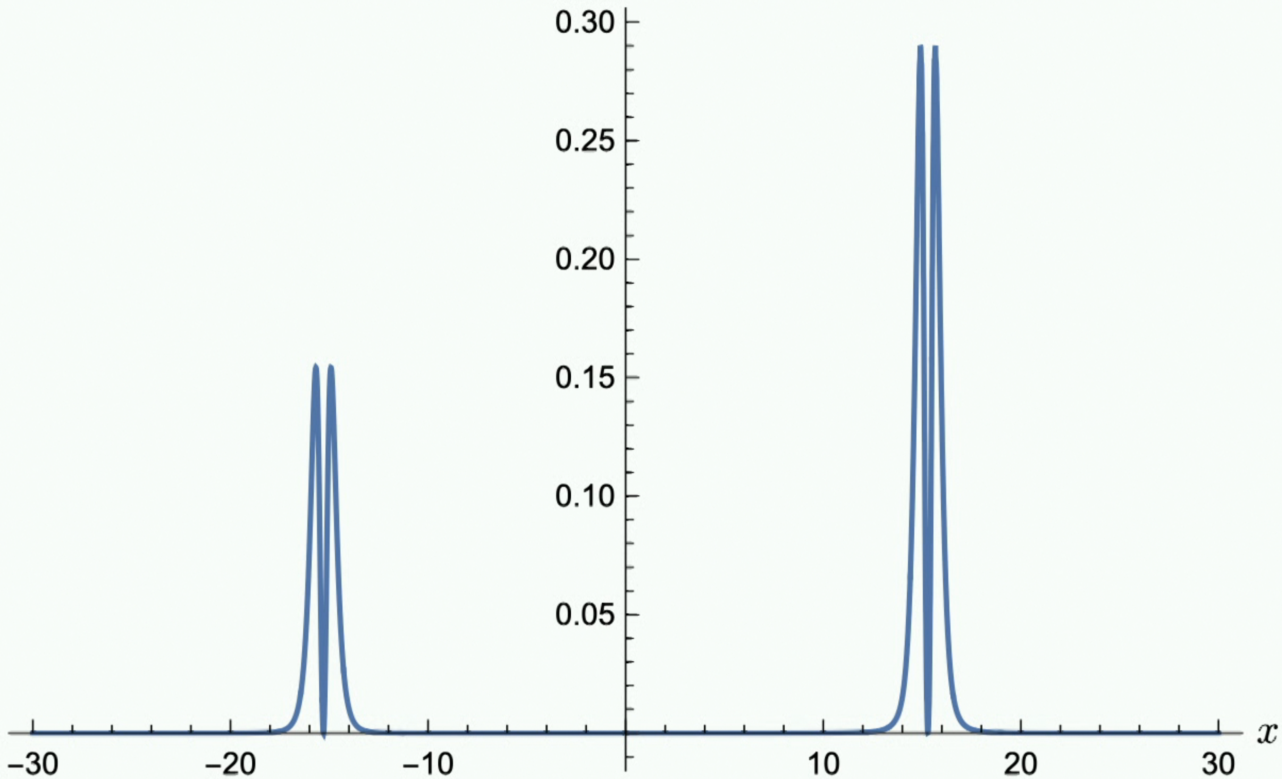
Nicholas Funai Eduardo Martín-Martínez | Using QET to generate exotic spacetime geometries

1+1 D QET

Energy density immediately following Bob's interaction.

Lorentzian smearing.

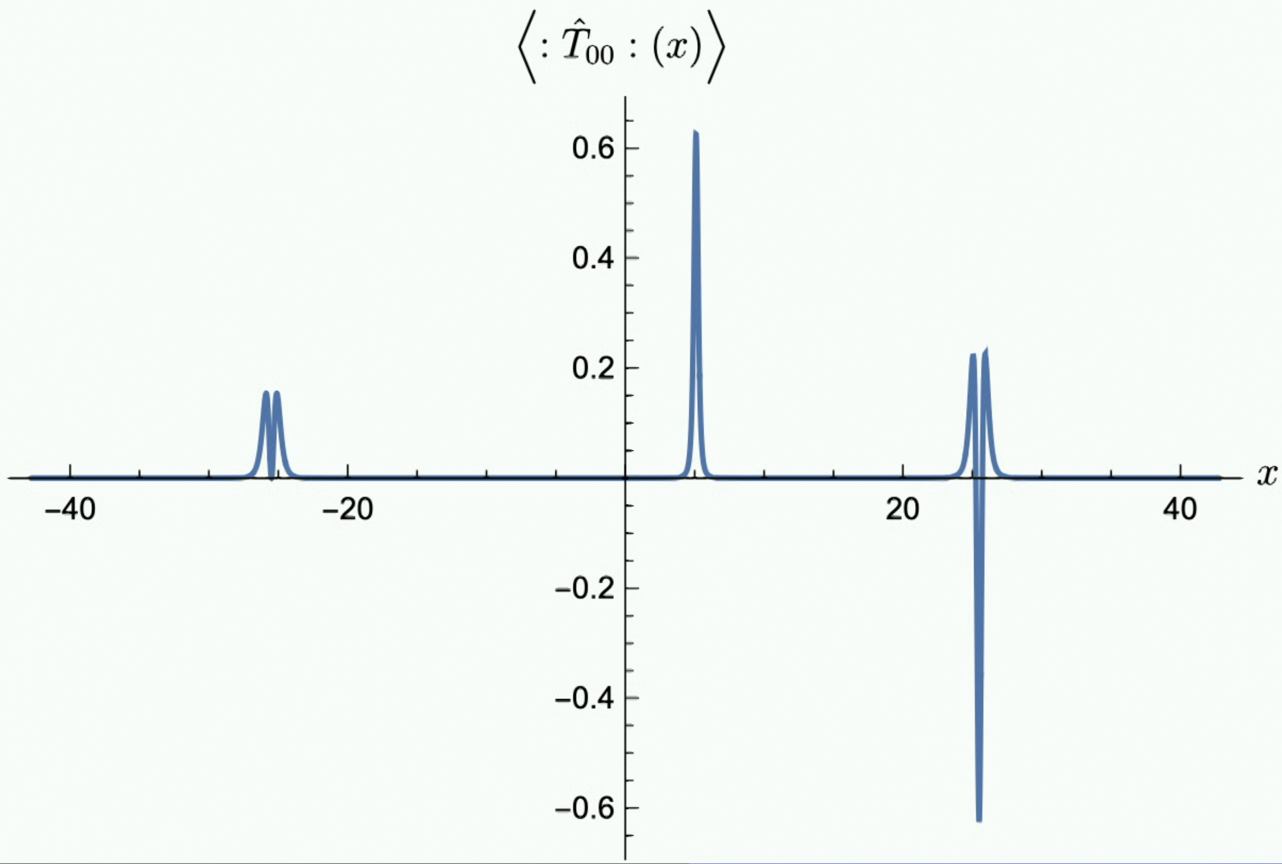
$$\left\langle : \hat{T}_{00} : (x) \right\rangle$$



1+1 D QET

Energy density ΔT after Bob's interaction.

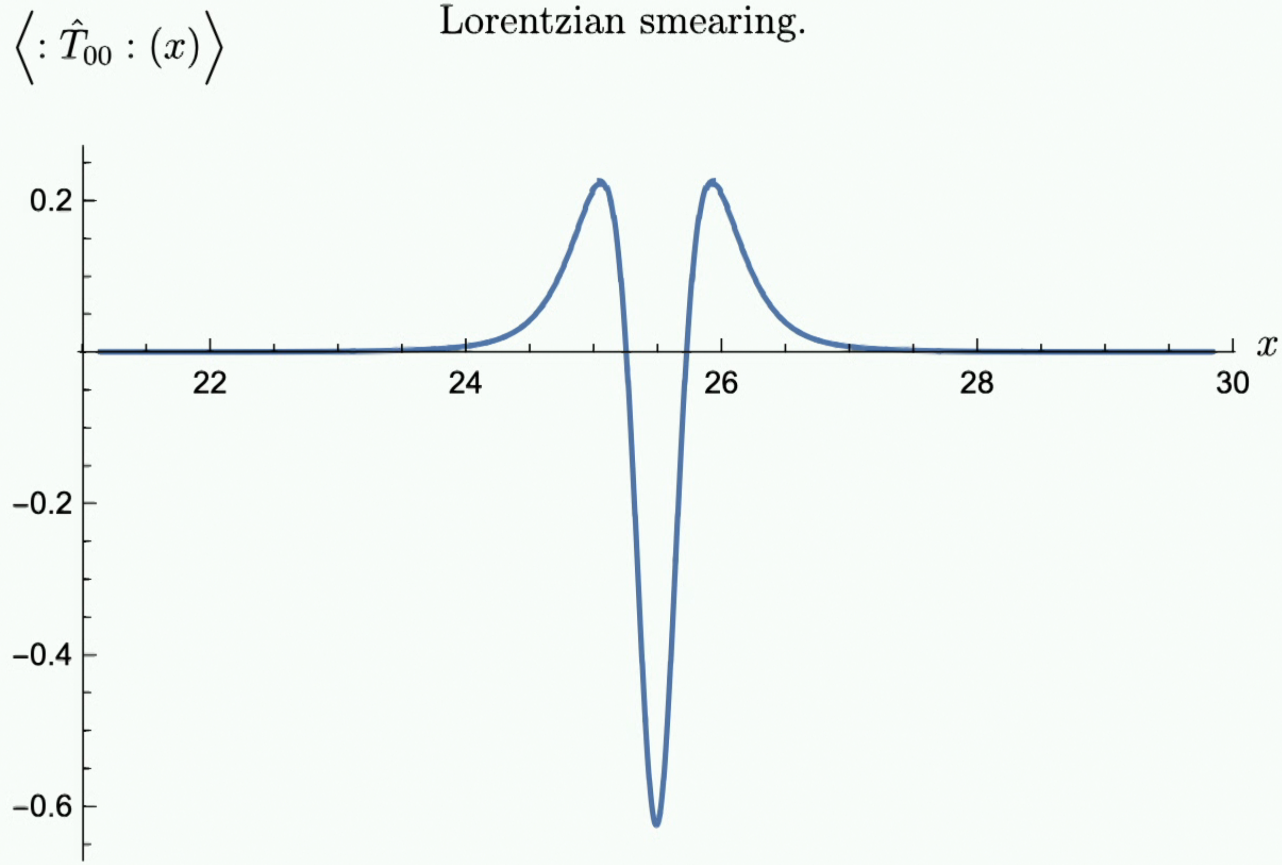
Lorentzian smearing.



1+1 D QET

Energy density ΔT after Bob's interaction.

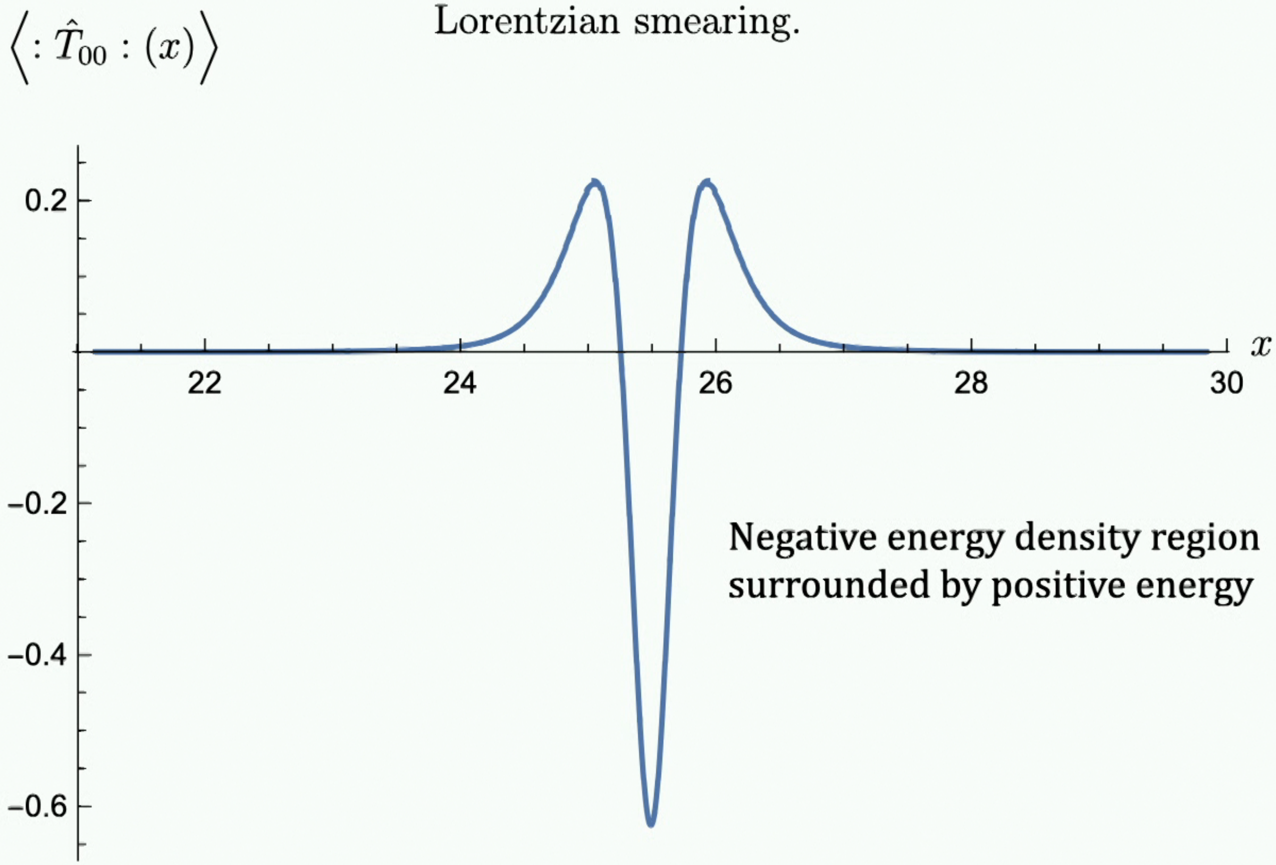
Lorentzian smearing.



1+1 D QET

Energy density ΔT after Bob's interaction.

Lorentzian smearing.



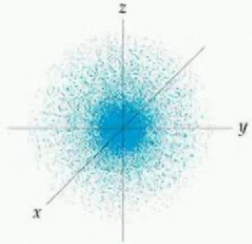
Nicholas Funai Eduardo Martín-Martínez

Using QET to generate exotic spacetime geometries

3+1D Case

Gravity needs more dimensions….

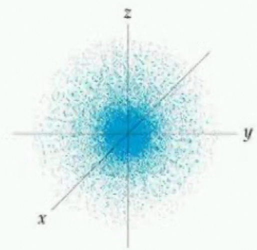
We need a distribution of Alices and a distribution of Bobs.



3+1D Case

Gravity needs more dimensions….

We need a distribution of Alices and a distribution of Bobs.

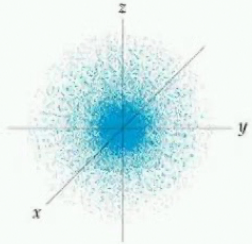


We cannot teleport the state of Alice's detectors to every Bob

3+1D Case

Gravity needs more dimensions….

We need a distribution of Alices and a distribution of Bobs.

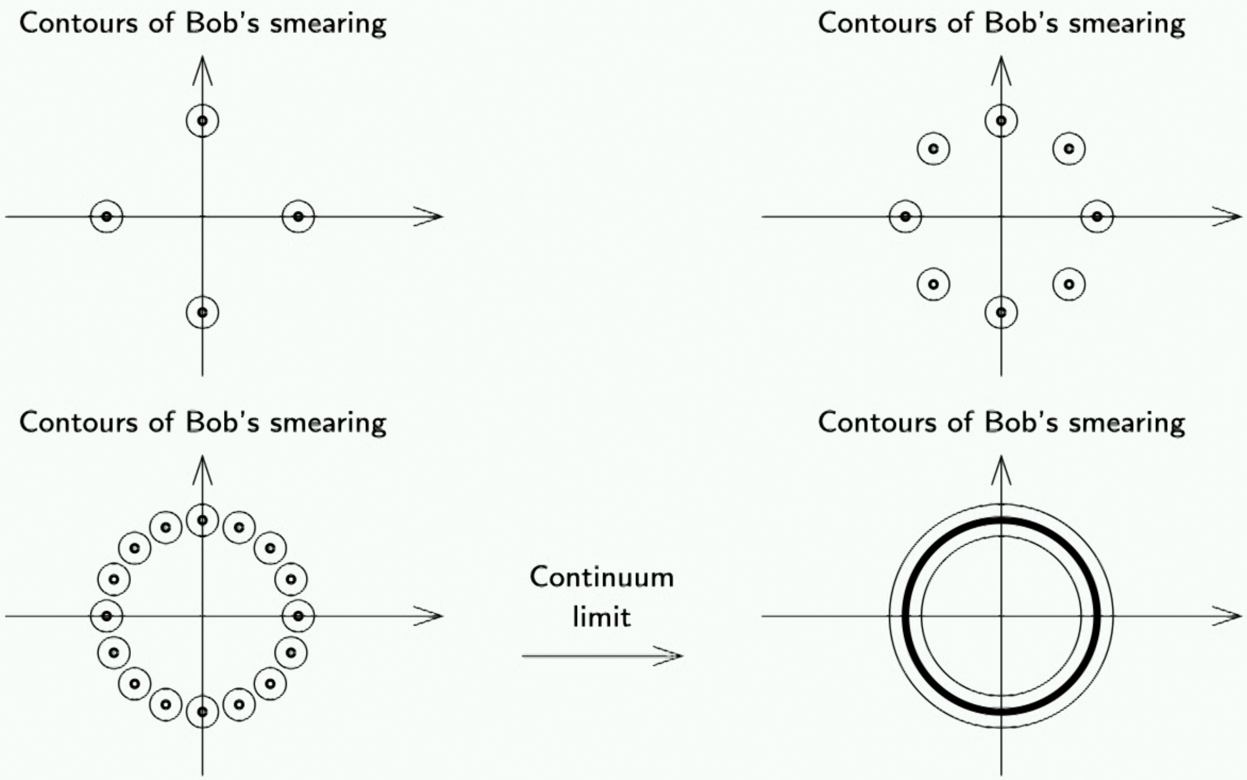


We cannot teleport the state of Alice's detectors to every Bob

NO CLONING!

Math gets much more complicated…

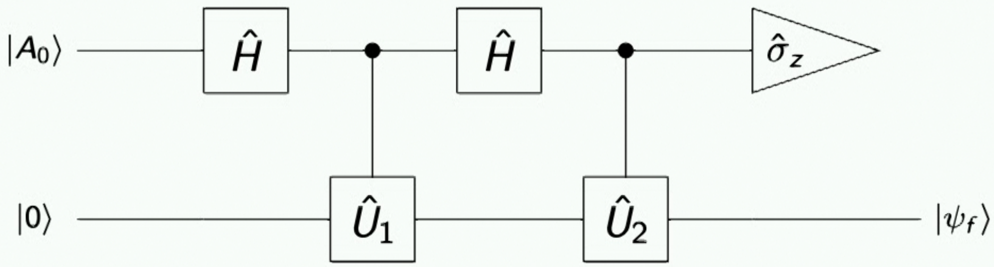
3+1D Case


Contours of Bob's smearing
Contours of Bob's smearing
Contours of Bob's smearing
Continuum
limit
Contours of Bob's smearing

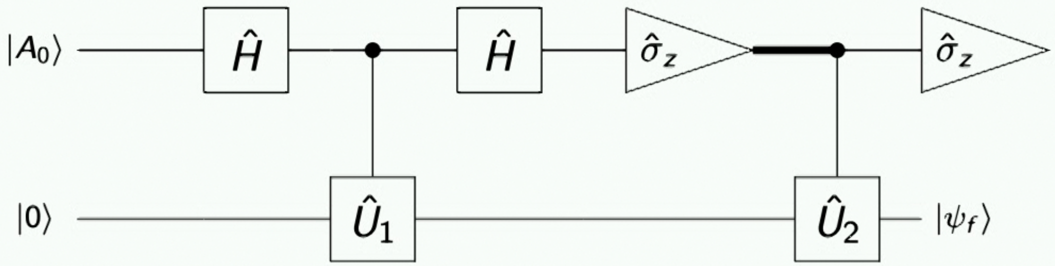
3+1D Case

If $|A_0\rangle$ is an eigenstate of $\hat{\sigma}_y$ then the LOCC protocol gives the same final state ($|\psi_f\rangle$) as LOQC, provided the final measurement $\hat{\sigma}_z$ give the same result.

LOQC



LOCC



3+1D Case

1-Alice measures the field by coupling an atom to it

2-Alice measures her non-relativistic atom

3-Alice broadcasts the result of the measurement to an agency of Bob's

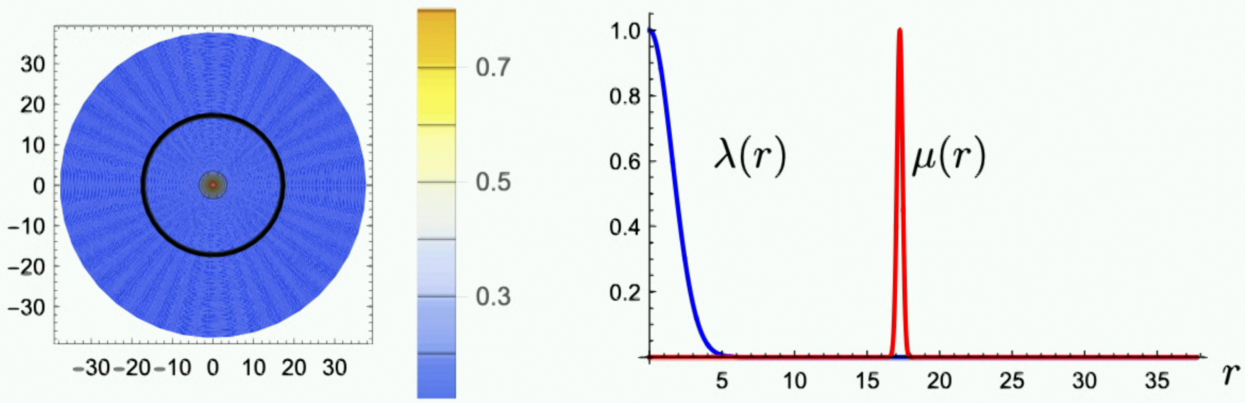
4-Bob's agents use that information to prepare atoms and couple to the field

$$\langle\psi(t)|:\hat{T}_{\mu\nu}:(\boldsymbol{x})|\psi(t)\rangle = \Bigg\{ \overbrace{\left(I^1_\mu I^1_\nu - \eta_{\mu\nu}\frac{I^1_\lambda I^{1,\lambda}}{2}\right)}^{\text{Bob's energy contribution}} - \overbrace{\left(I^2_\mu I^2_\nu - \eta_{\mu\nu}\frac{I^2_\lambda I^{2,\lambda}}{2}\right)}^{\text{Alice's energy contribution}}$$

$$\underbrace{- \langle A_0|\hat{\sigma}_y|A_0\rangle e^{-2\|\alpha\|}\left(\left(I^1_\mu I^3_\nu - \eta_{\mu\nu}\frac{I^1_\lambda I^{3,\lambda}}{2}\right) + \left(I^3_\mu I^1_\nu - \eta_{\mu\nu}\frac{I^1_\lambda I^{3,\lambda}}{2}\right)\right)}_{\text{QET term}}\Bigg\}$$

3+1 D QET

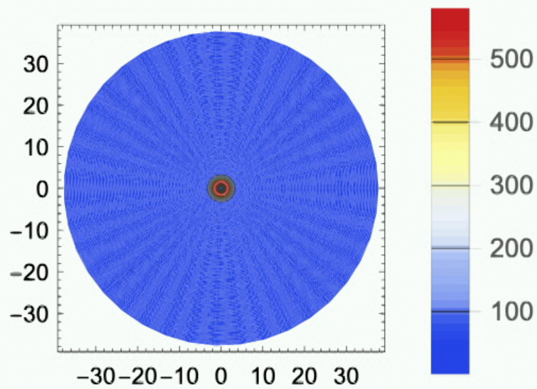
Gaussian smearing functions used.



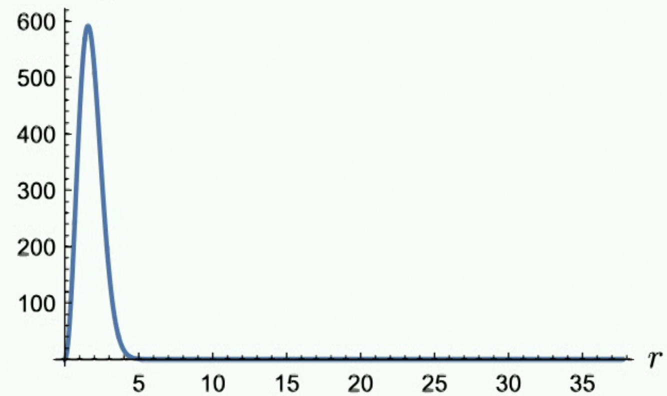
3+1 D QET

Energy density immediately following Alice's interaction.

Gaussian smearing.



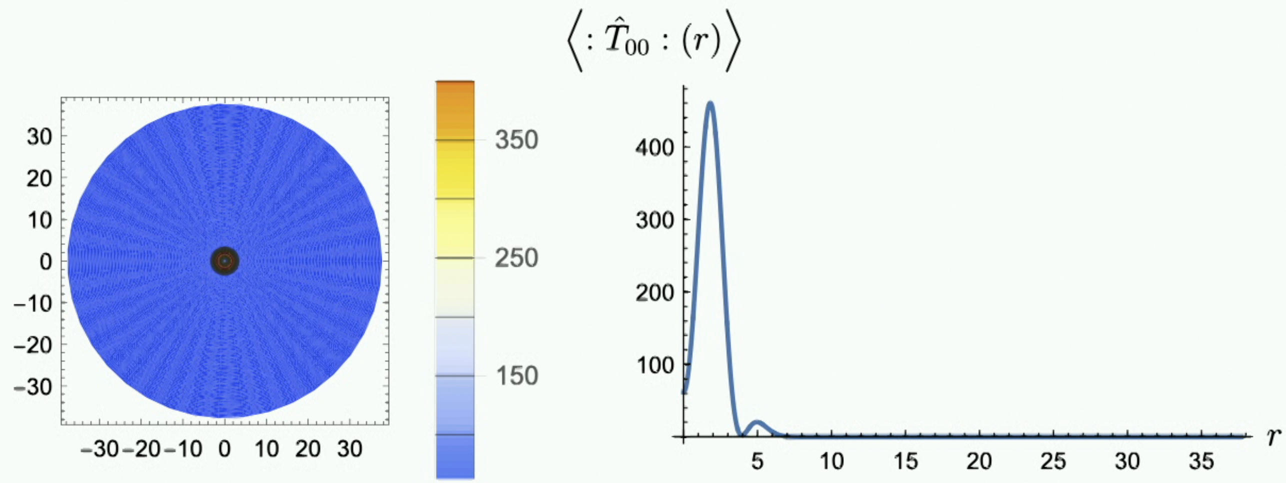
$\left\langle : \hat{T}_{00} : (r) \right\rangle$



3+1 D QET

Energy density after Alice's interaction.

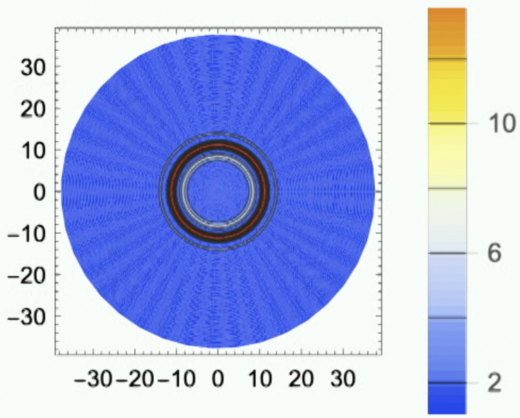
Gaussian smearing.

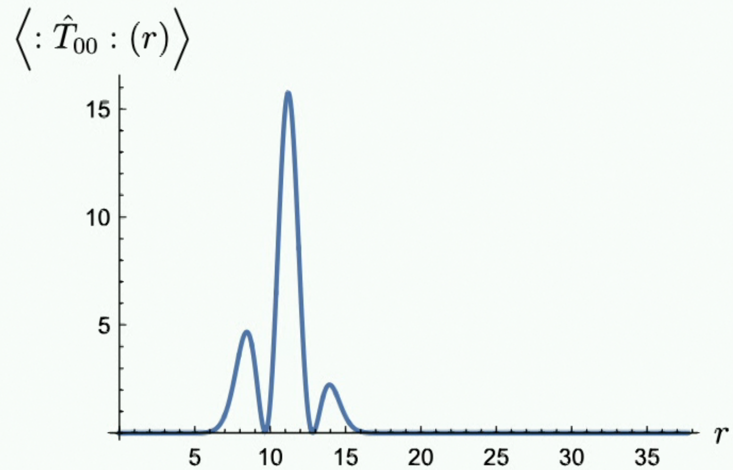


3+1 D QET

Energy density after Alice's interaction.

Gaussian smearing.

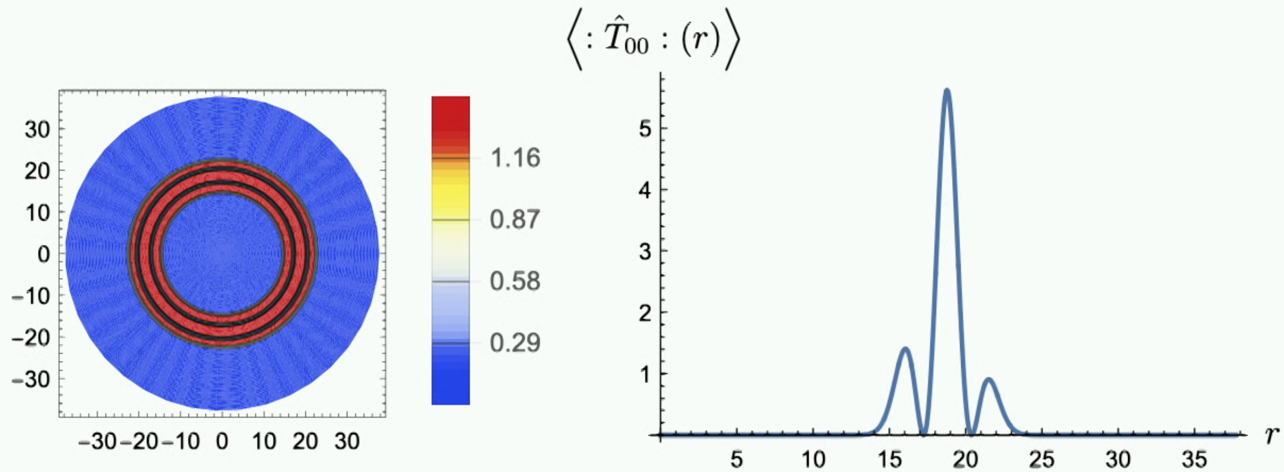




3+1 D QET

Energy density immediately prior to Bob's interaction.

Gaussian smearing.

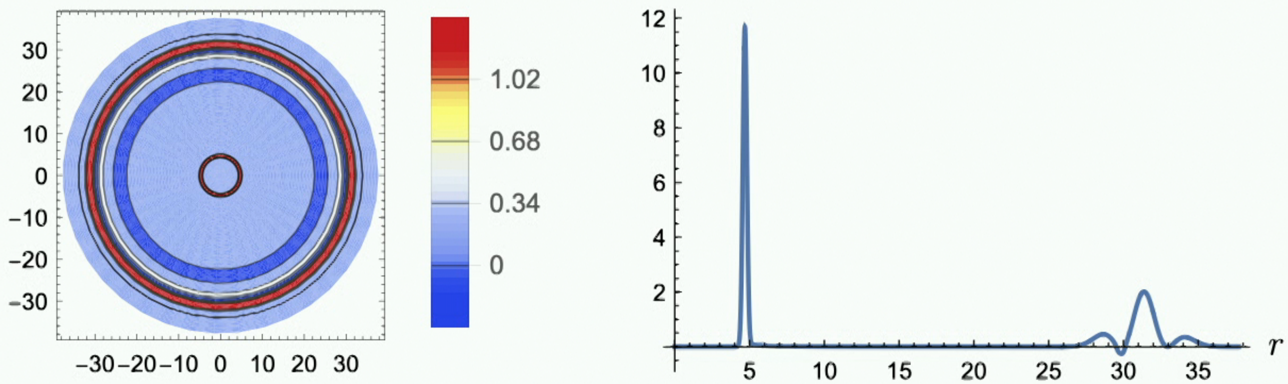


3+1 D QET

Energy density ΔT after to Bob's interaction.

Gaussian smearing.

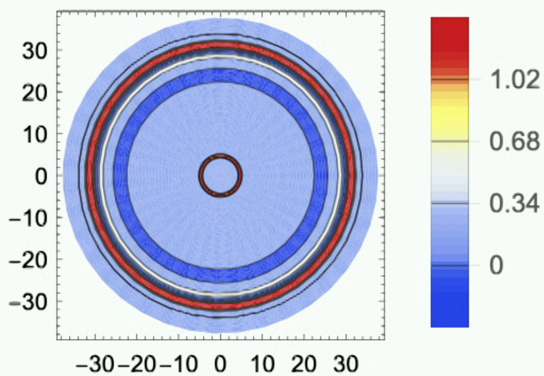
$$\left\langle : \hat{T}_{00} : (r) \right\rangle$$

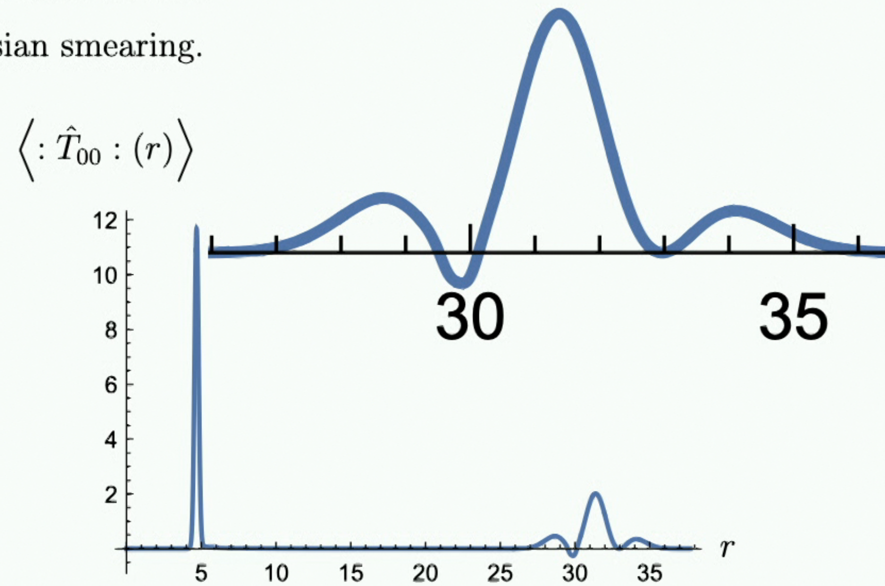


3+1 D QET

Energy density ΔT after to Bob

Gaussian smearing.





Nicholas Funai Eduardo Martín-Martínez | Using QET to generate exotic spacetime geometries

Scaling negative energy

Total amount of negative energy:

$$\Phi_E = \int\limits_{\langle : \hat{T}_{00}(\boldsymbol{x},\tau) : \rangle < 0} \langle : \hat{T}_{00} : (\boldsymbol{x},\tau) \rangle \, d\tau$$

$$\lambda(\boldsymbol{x}) \to \sigma^{\frac{n-2}{2}} \lambda(\sigma \boldsymbol{x}),$$

$$\mu(\boldsymbol{x}) \to \sigma^{\frac{n}{2}} \mu(\sigma \boldsymbol{x}),$$

$$\left\langle : \hat{T}_{\mu\nu}(\boldsymbol{x}, t) : \right\rangle \to \sigma^{n} \left\langle : \hat{T}_{\mu\nu}(\sigma \boldsymbol{x}, \sigma t) : \right\rangle$$

$$\Phi_E \to \sigma^{n-1} \Phi_E .$$

Scaling negative energy

Total amount of negative energy:

$$\Phi_E = \int_{\langle :\hat{T}_{00}(\boldsymbol{x},\tau): \rangle < 0} \langle :\ \hat{T}_{00}\ : (\boldsymbol{x},\tau) \rangle \, d\tau$$

$$\lambda(\boldsymbol{x}) \to \sigma^{\frac{n-2}{2}} \lambda(\sigma \boldsymbol{x}),$$

$$\mu(\boldsymbol{x}) \to \sigma^{\frac{n}{2}} \mu(\sigma \boldsymbol{x}),$$

$$\left\langle :\ \hat{T}_{\mu\nu}(\boldsymbol{x},t)\ : \right\rangle \to \sigma^{n} \left\langle :\ \hat{T}_{\mu\nu}(\sigma \boldsymbol{x}, \sigma t)\ : \right\rangle$$

$$\Phi_E \to \sigma^{n-1} \Phi_E .$$

Violates quantum inequalities optimally
and saturates QI conjecture

Summary

Almost arbitrary energy density distributions with arrays of Alices and Bobs **close to the limits of the quantum energy conditions**.

Backreaction on spacetime!!