Title: AdS/CFT

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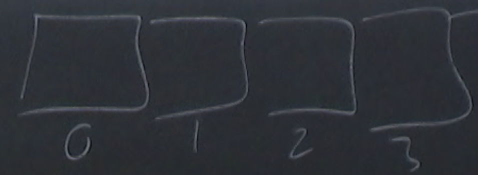
Grover's Algorithm   Unstructured search



e.g. 8 elements $|0\rangle, |1\rangle, |2\rangle, |3\rangle, |4\rangle, |5\rangle, |6\rangle, |7\rangle$. $|000\rangle, |001\rangle, |010$

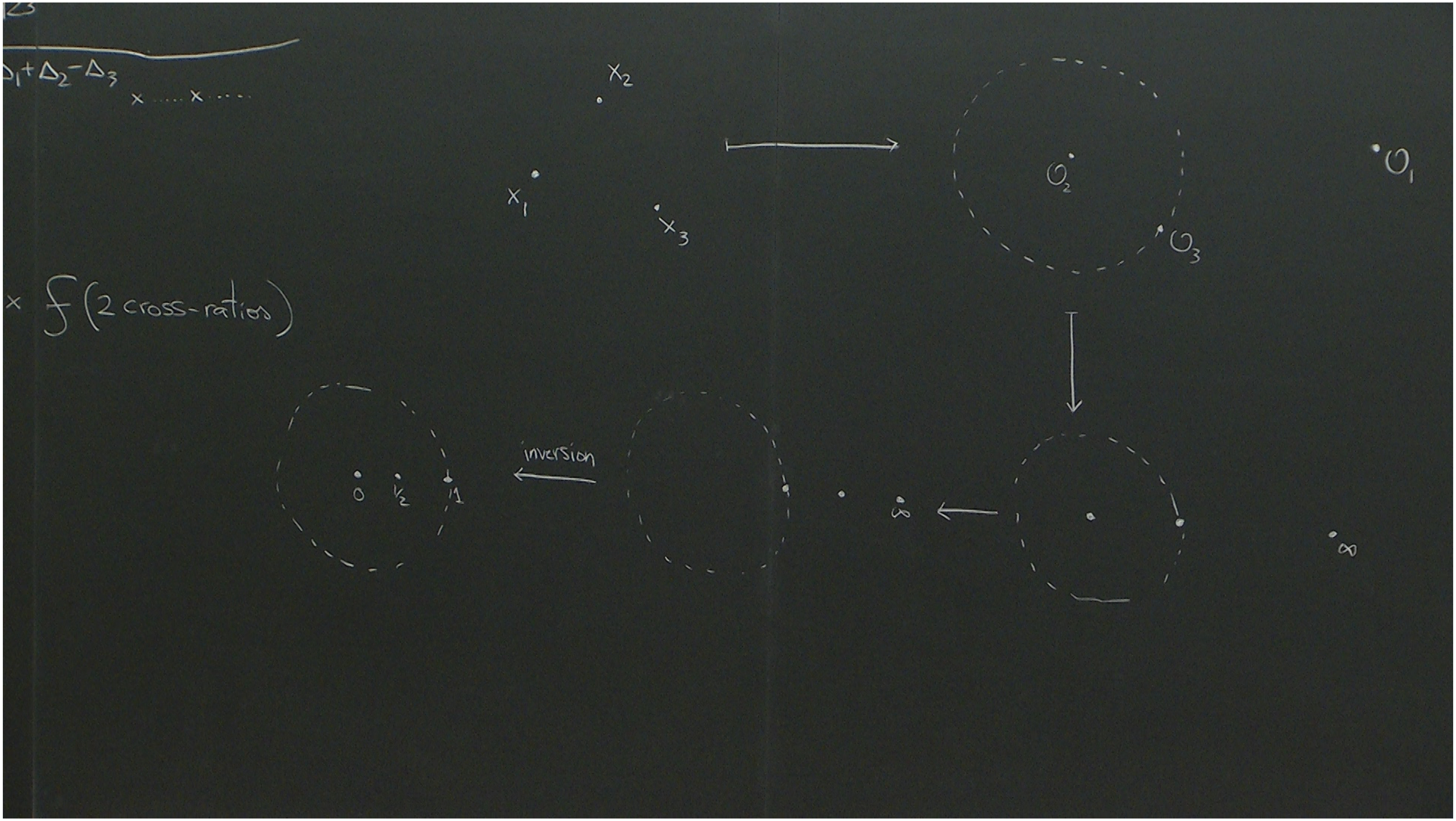
Oracle: Flips the sign of the state with the "winning" property, $|w\rangle$ e.g. $w =$

$$\hat{U}_w |x\rangle = (-1)^{f(x)} |x\rangle \qquad f(x) = \begin{cases} 0 & x \neq w \\ 1 & x = w \end{cases} \qquad \hat{U}_w |x\rangle = \begin{cases} |x\rangle & x \neq w \\ -|x\rangle & x = w \end{cases}$$

$$\cdot x_1 \quad \cdot x_2 \longmapsto \underbrace{\overset{\bullet}{0} \quad \overset{\bullet}{1}}_{1} \Rightarrow 2pt$$

$$\cdot x_1 \quad \cdot x_2 \quad \cdot x_3 \longmapsto \underbrace{\overset{\bullet}{0} \quad \overset{\bullet}{\tfrac{1}{2}} \quad \overset{\bullet}{1}}_{C_{123}} \Longrightarrow 3pt = \frac{C_{123}}{(x_1 - x_2)^{\Delta_1 + \Delta_2 - \Delta_3} \times \cdots \times \cdots}$$

$$\cdot x_1 \quad \cdot x_2 \quad \cdot x_3 \quad \cdot x_4 \longmapsto \overset{\bullet}{0} \quad \overset{\bullet}{1} \quad \cdot \overbrace{(x,y)}^{\text{2 degrees of freedom}} \quad \overset{\bullet}{\infty} \Rightarrow 4pt = \infty \times f(\text{2 cross-ratios})$$



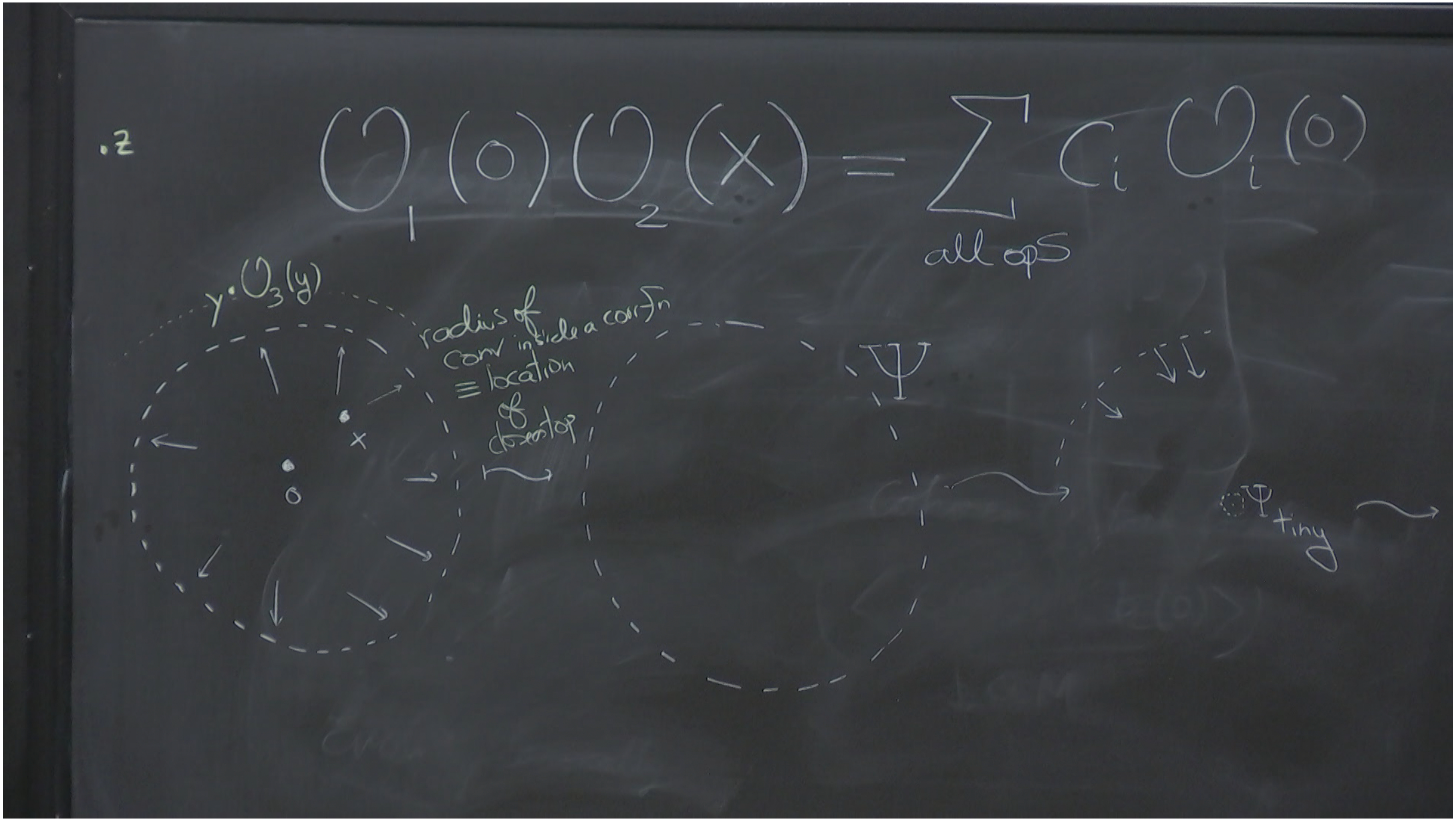
Δ1+Δ2−Δ3
x2
x1
x3
O2
O3
O1
× f(2 cross-ratios)
inversion
0
1/2
1
∞
∞

$$\leftarrow\!\!\sim \quad \tilde{X}^{\mu}_{\underline{\underline{half}}} = \underbrace{\left[\frac{X^{\mu} - X_1^{\mu}}{(X - X_1)^2} - \left(\text{same with } x^{\mu} \to x_2^{\mu}\right) \right]}_{\equiv V^{\mu}} \sqrt{\underbrace{V^{\mu} V_{\mu}}_{\displaystyle \frac{X_{23}^2}{X_{13}^2\, X_{12}^2}}}\Bigg|_{X = X_3}$$

$$\Omega_{half}(X_3) = \frac{1}{(X_3 - X_1)^2} \Big/ \sqrt{V^2}$$

$$= \sqrt{\frac{X_{12}^2}{X_{23}^2\, X_{13}^2}}$$

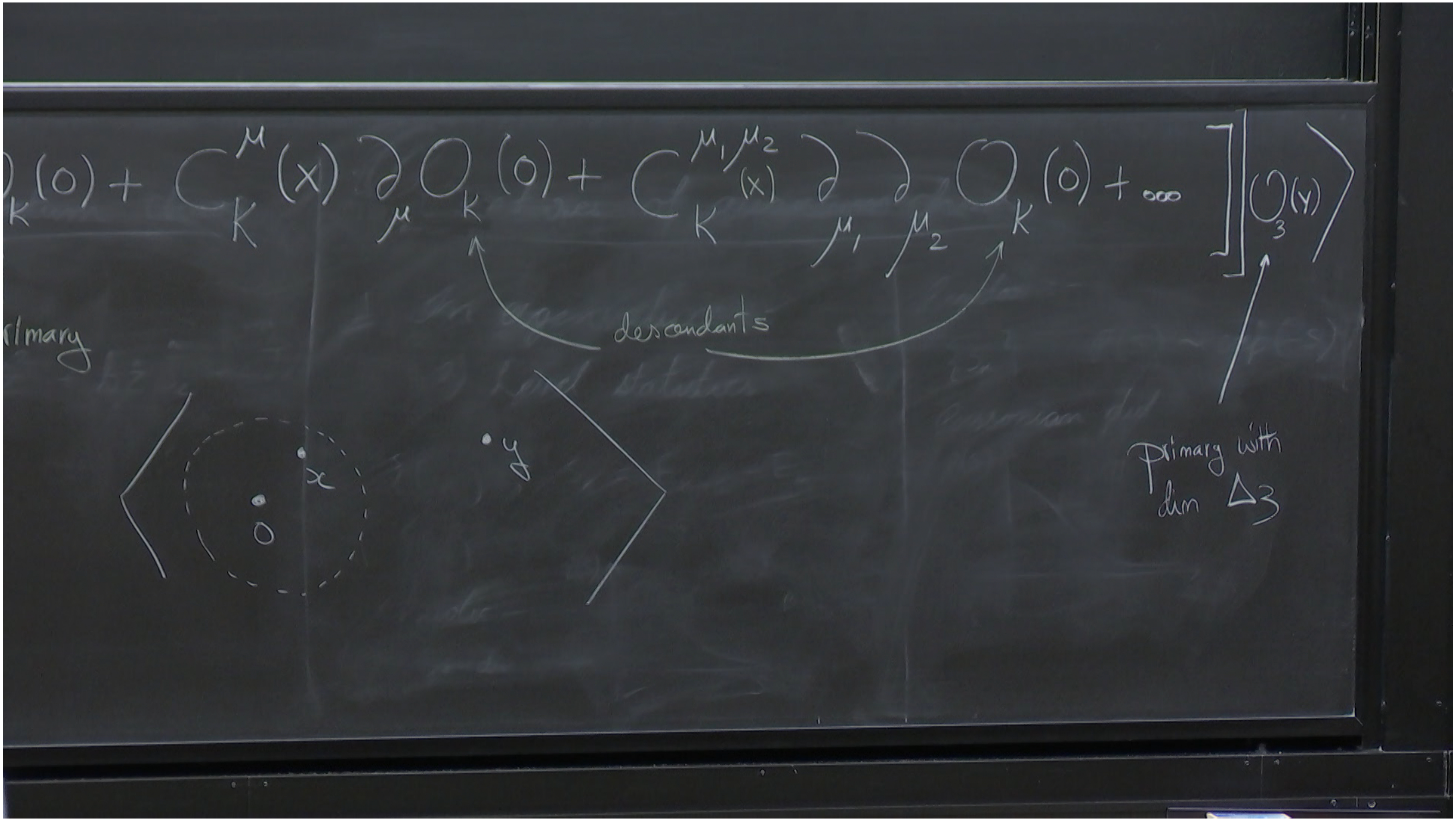
$$\Omega_{\text{second half}}\left(\tilde{X}_{half}(X_3)\right) = \text{number}$$

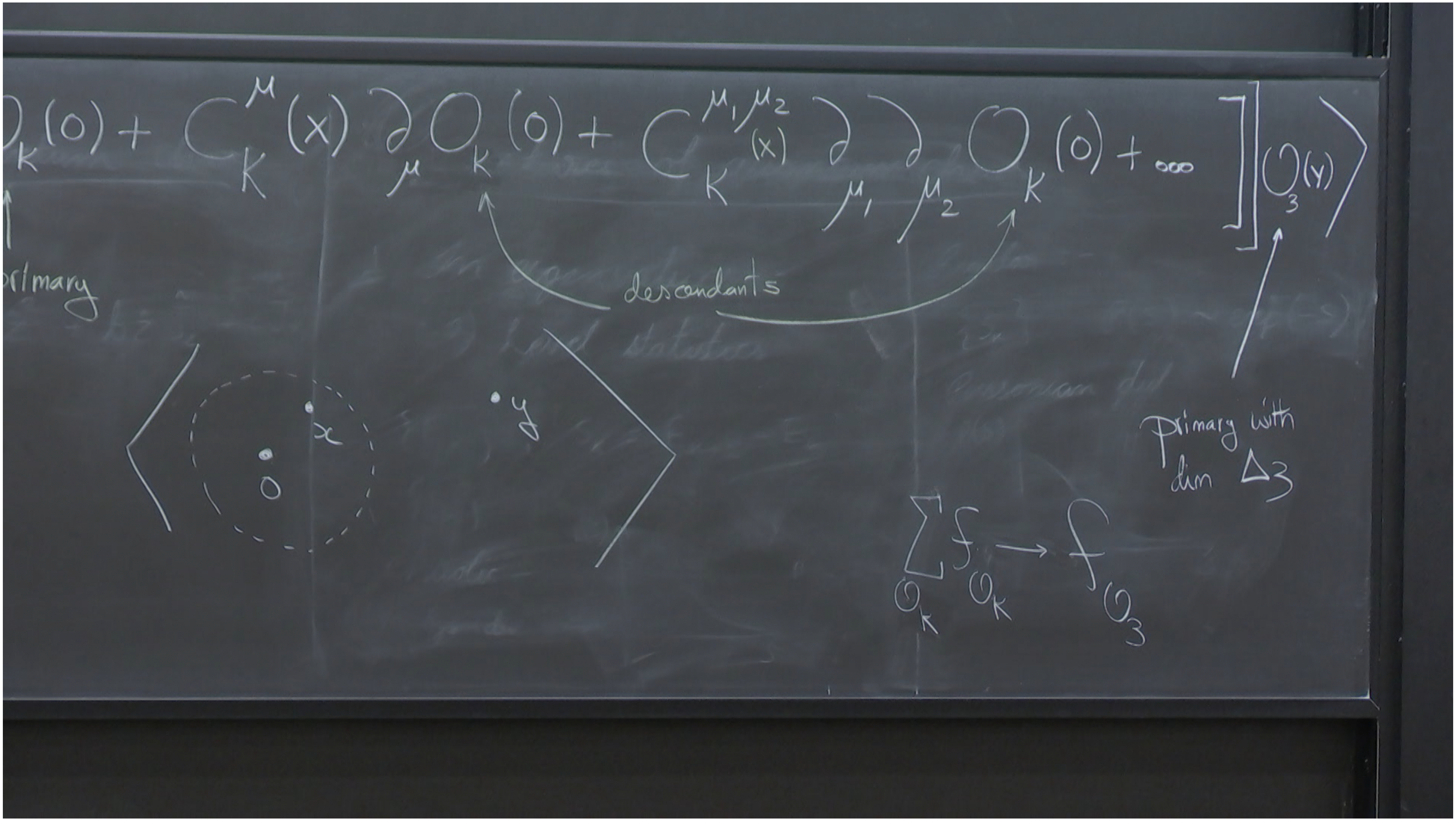

z
$\mathcal{O}_1(0)\,\mathcal{O}_2(x) = \sum_{\text{all ops}} C_i\,\mathcal{O}_i(0)$
y • $\mathcal{O}_3(y)$
radius of conv inside a corr fn ≡ location of closest op
0
x
Ψ
Ψ tiny

$$C_k(x)\,\mathcal{O}_k(0) + C_k^{\mu}(x)\,\partial_\mu \mathcal{O}_k(0) + C_k^{\mu_1\mu_2}(x)\,\partial_{\mu_1}\partial_{\mu_2}\mathcal{O}_k(0) + \dots \Big]$$

primary

descendants


$O_k(0) + C_k^{\mu}(x)\,\partial_\mu O_k(0) + C_k^{\mu_1\mu_2}(x)\,\partial_{\mu_1}\partial_{\mu_2} O_k(0) + \dots \big]\big] |O_3(y)\rangle$
descendants
rimary
primary with dim Δ_3
x
0
y


$O_K(0) + C^{\mu}_K(x)\,\partial_\mu O_K(0) + C^{\mu_1\mu_2}_K(x)\,\partial_{\mu_1}\partial_{\mu_2} O_K(0) + \ldots\,]\,O_3(y)\rangle$
primary
descendants
x
0
y
primary with dim Δ_3
$\sum_{O_K} f_{O_K} \to f_{O_3}$

$$\langle O_1(0)\, O_2(x)\, O_k(y) \rangle = C_k(x) \langle O_k(0)\, O_k(y) \rangle + C$$

$$\langle O_k\, O_{k'} \rangle = 0 \quad \text{unless } \Delta_k = \Delta_{k'}$$

$$\langle O_k\, O_{k'} \rangle = \frac{d_{kk'}}{|x|^{2\Delta}} \xrightarrow{\text{COB}} \frac{\delta_{kk'}}{|x|^{2\Delta}} \quad \text{(good basis)}$$

$$x)\,\mathcal{O}_k(y)\rangle = \frac{C_{12k}}{|x|^{\Delta_1+\Delta_2-\Delta_k}\,|y|^{\Delta_1+\Delta_2-\Delta_k}\,|x-y|^{\Delta_2+\Delta_k-\Delta_1}}$$

$$= \frac{C_{12k}}{|x|^{\Delta_1+\Delta_2-\Delta_k}\,|y|^{2\Delta_k}}\left(1-\frac{2x\cdot y}{y^2}+\frac{x^2}{y^2}\right)^{\frac{-\Delta_2-\Delta_k+\Delta_1}{2}}$$

$$\langle \mathcal{O}_1(0)\,\mathcal{O}_2(x)\,\mathcal{O}_k(y)\rangle = \frac{C_{12k}}{|x|^{\Delta_1+\Delta_2-\Delta_k} \cdots}$$

$$= \frac{C_{12k}}{|x|^{\Delta_1+\Delta_2-\Delta_k}\,|y|^{2\Delta \cdots}}$$

$$\sum = \frac{C_{12K}}{|x|^{\Delta_1+\Delta_2-\Delta_K}\,|y|^{\Delta_1+\Delta_K-\Delta_2}\,|x-y|^{\Delta_2+\Delta_K-\Delta_1}}$$

$$= \frac{C_{12K}}{|x|^{\Delta_1+\Delta_2-\Delta_K}\,|y|^{2\Delta_K}}\left(1-\frac{2x\cdot y}{y^2}+\frac{x^2}{y^2}\right)^{\frac{-\Delta_2-\Delta_K+\Delta_1}{2}}$$

$$= \frac{C_{12K}}{|x|^{\Delta_1+\Delta_2-\Delta_K}\,|y|^{2\Delta_K}}\left[1-\left(\Delta_2-\Delta_1+\Delta_K\right)\frac{x\cdot y}{y^2}+\circ\circ\circ\right]$$

$$\ldots\rangle + C_k^{\mu}(x)\,\partial_\mu \underbrace{\langle O_k(x)\,O_k(y)\rangle_{x=0}}_{\frac{1}{y^{2\Delta_k}}} + C_k^{\mu_1\mu_2}(x)\,\partial_{\mu_1}\partial_{\mu_2}\langle O_k(0)\,O_k(y)\rangle$$

$$\frac{y_\mu}{y^2}\,\frac{1}{y^{2\Delta_k}}$$

$$C_k(x) = \frac{1}{|x|^{\Delta_1+\Delta_2-\Delta_k}} \times \boxed{C_{12k}}$$

dynamics

$$C_k^{\mu}(x) = \frac{\Delta_1-\Delta_2-\Delta_k}{2\Delta_k}\, x^{\mu}\, \frac{1}{|x|^{\Delta_1+\Delta_2-\Delta_k}} \times \boxed{C_{12k}}$$

$$C_k^{\mu_1\cdots}(x) = b^{\mu_1\cdots}(x) \times C_{12k}$$

fixed!

$$C_k(x) = \boxed{\frac{1}{|x|^{\Delta_1+\Delta_2-\Delta_k}}} \times \boxed{C_{12k}}$$

dynamics

Kinematics.

$$C_k^{\mu}(x) = \boxed{\frac{\Delta_1-\Delta_2-\Delta_k}{2\Delta_k}\, x^{\mu}\, \frac{1}{|x|^{\Delta_1+\Delta_2-\Delta_k}}} \times \boxed{C_{12k}}$$

$$C_k^{\mu_1\cdots}(x) = b^{\mu_1\cdots}(x) \times C_{12k}$$

fixed!

$\{C_{ijk}, \Delta_k\} \longmapsto$ gives us all correlators

$$\langle \mathcal{O}_1(x_1)\, \mathcal{O}_2(x_2)\, \mathcal{O}_3(x_3)\, \mathcal{O}_4(x_4) \rangle = \sum_{\mathcal{O}_k}$$

$$\{C_{ijk}, \Delta_k\} \longrightarrow \text{gives us all correlators}$$

$$\langle \mathcal{O}_1(x_1)\,\mathcal{O}_2(x_2)\,\mathcal{O}_3(x_3)\,\mathcal{O}_4(x_4)\rangle = \sum_{\mathcal{O}_k} C_{12k}\, C_{34k} \left(\sum_{n=0}^{\infty} \underbrace{b^{\mu_1 \cdots \mu_n}}_{\text{fixed}} (x_4 - x_3)_{\mu_1} \cdots \right)$$

$$\underbrace{C_{34K}\left(\sum_{n=0}^{\infty} \underbrace{b^{\mu_1\cdots\mu_n}}_{\text{fixed}}(x_4-x_3)\,\overbrace{\partial_{\mu_1}\cdots\cdots\partial_{\mu_n}}^{x_3}\left[\langle O_1(x_1)\,O_2(x_2)\,O_K(x_3)\rangle / C_{12K}\right]\right)}_{\text{just Kinematics!}}$$

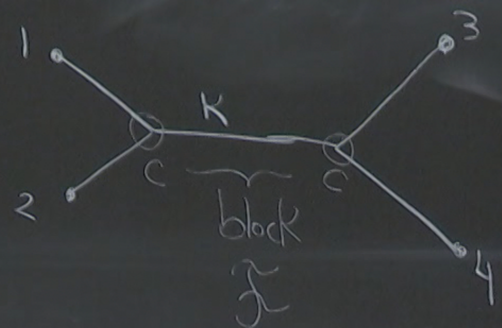
ll correlators

$$\langle \cdots (x_4) \rangle = \sum_{\mathcal{O}_K} \underbrace{C_{12K}\, C_{34K}}_{\text{dyn}} \underbrace{\left(\sum_{n=0}^{\infty} \underbrace{b^{\mu_1 \cdots \mu_n}(x_4 - x_3)}_{\text{fixed}} \overbrace{\partial_{\mu_1} \cdots \cdots \partial_{\mu_n}}^{x_3} \right) \left[\langle \mathcal{O}_1(x_1)\, \mathcal{O}_2(x_2)\, \mathcal{O}_K(x_3 \cdots \right.}_{\text{just kinematics!}}$$

$$f_{\mathcal{O}_K}(x_1 \cdots x_4) \leftarrow \text{conformal block.}$$

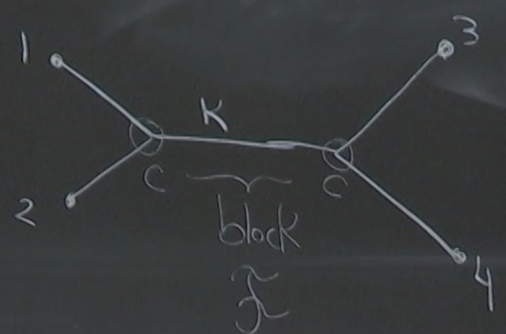
$$\{C_{ijk}, \Delta_k\} \longmapsto \text{gives us all correlators}$$

$$\langle \mathcal{O}_1(x_1)\,\mathcal{O}_2(x_2)\,\mathcal{O}_3(x_3)\,\mathcal{O}_4(x_4)\rangle = \sum_{\mathcal{O}_k} \underbrace{C_{12k}\, C_{34k}}\left(\sum_{n=0}^{\infty} \underbrace{b^{\mu_1\cdots\mu_n}(x_4-x\ldots}_{\text{fixed}}\right.$$



$$\{C_{ijk}, \Delta_k\} \longmapsto \text{gives us all correlators}$$

$$\langle \mathcal{O}_1(x_1)\,\mathcal{O}_2(x_2)\,\mathcal{O}_3(x_3)\,\mathcal{O}_4(x_4)\rangle = \sum_{\mathcal{O}_k} C_{12k}\, C_{34k} \left(\sum_{n=0}^{\infty} \underbrace{b^{\mu_1 \cdots \mu_n}}_{\text{fixed}} (x_4 - x \ldots \right.$$



$$\mathcal{F}_{\mathcal{O}_K}(x_1 \dots x_4) = \frac{1}{(x_1-x_2)^{2\Delta}(x_3-x_4)^{2\Delta}} \underbrace{\mathcal{F}(u,v)}_{//\, d=4} \quad , u =$$

$$\Delta_1 = \Delta_2 = \Delta_3 = \Delta_4 = \Delta$$

$$\frac{z\bar{z}}{z-\bar{z}} \left(h_{\frac{\Delta_K+0}{2}}(z)\, h_{\frac{\Delta_K-0-2}{2}}(\bar{z}) \right)$$

$$\mathcal{F}(x_1 \ldots x_4) = \frac{1}{(x_1-x_2)^{2\Delta}(x_3-x_4)^{2\Delta}} \underbrace{\mathcal{F}(u,v)}_{//\, d=4} \quad , u =$$

$$\mathcal{O}_K$$

$$\Delta_1 = \Delta_2 = \Delta_3 = \Delta_4 = \Delta$$

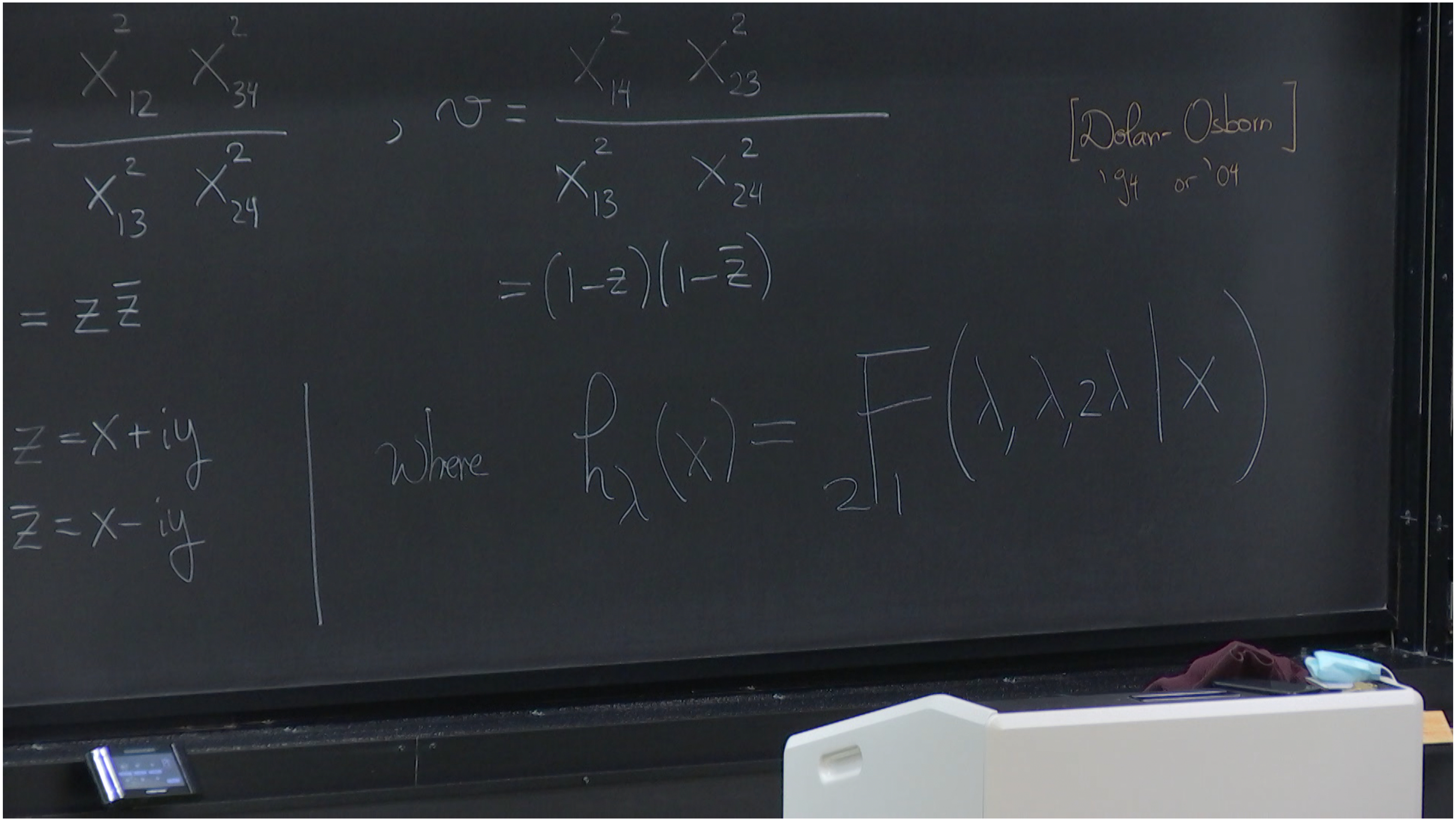
$$\frac{z\bar{z}}{z-\bar{z}}\left(h_{\frac{\Delta_K+0}{2}}(z)\, h_{\frac{\Delta_K-0-2}{2}}(\bar{z}) - (z \longleftrightarrow \bar{z}) \right)$$

$$, \ u = \frac{x_{12}^2 \, x_{34}^2}{x_{13}^2 \, x_{24}^2} = z \bar{z} \qquad , \ v = \frac{x_{14}^2 \, x_{23}^2}{x_{13}^2 \, x_{24}^2} = (1-z)(1-\bar{z})$$

$$z = x + iy$$

$$\bar{z} = x - iy$$

where $h_\lambda(x) = {}_2F_1(\lambda, \lambda, 2\lambda \,|\, x)$


$= \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}$, $v = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}$
$= z\bar{z}$
$= (1-z)(1-\bar{z})$
[Dolan-Osborn]
'94 or '04
$z = x + iy$
$\bar{z} = x - iy$
where $f_{h_\lambda}(x) = {}_2F_1(\lambda, \lambda, 2\lambda \mid x)$

$$\mathcal{F}_{\mathcal{O}_K}(x_1 \ldots x_4) = \frac{1}{(x_1-x_2)^{2\Delta}(x_3-x_4)^{2\Delta}} \underbrace{\mathcal{F}(u,v)}_{//\, d=4} \quad ; u = \frac{x \ldots}{x \ldots}$$

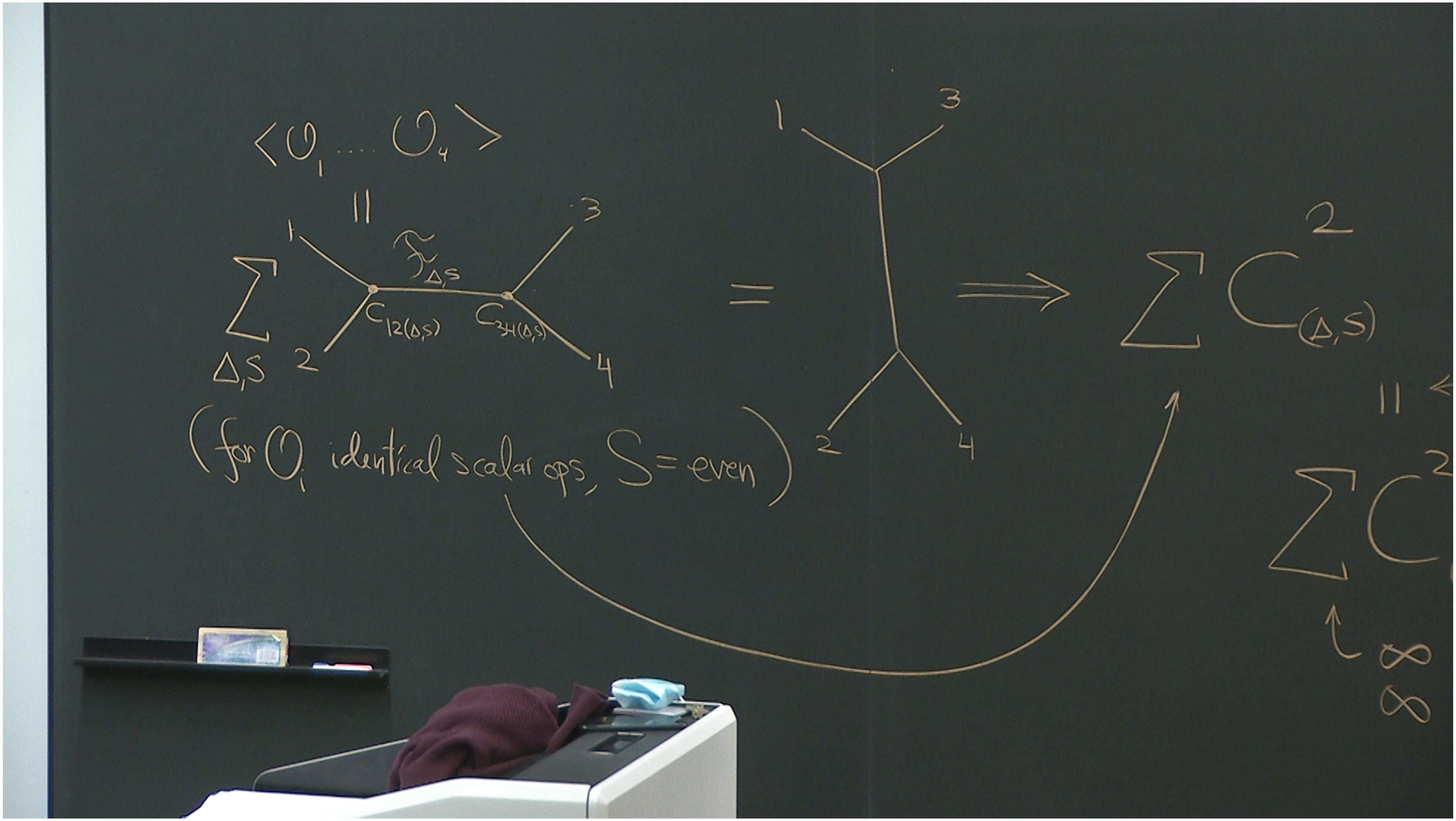
$$= z\bar{z} \ldots$$

$$\Delta_1=\Delta_2=\Delta_3=\Delta_4=\Delta$$

$$\frac{z\bar{z}}{z-\bar{z}}\left(h_{\frac{\Delta_K+\overset{S}{0}}{2}}(z)\, h_{\frac{\Delta_K-\overset{S}{0}-2}{2}}(\bar{z}) - (z \longleftrightarrow \bar{z}) \right)$$

$$z = x \ldots$$

$$\bar{z} = x \ldots$$



$\langle \mathcal{O}_1 \ldots \mathcal{O}_4 \rangle$
$=$
$\sum_{\Delta, S}$
$\mathcal{F}_{\Delta, S}$
$C_{12(\Delta,S)}$
$C_{34(\Delta,S)}$
1
2
3
4
(for $\mathcal{O}_i$ identical scalar ops, S = even)
$=$
$\Longrightarrow$
$\sum C^2_{(\Delta,S)}$

$$\Longrightarrow \sum C^2_{(\Delta,s)} \mathcal{F}_{\Delta s}(z,\bar{z})$$

Bootstrap

$$= \sum C^2_{(\Delta,s)} \mathcal{F}(1-z, 1-\bar{z})$$

∞ set of eqs for

∞ set of var.

$$\rightsquigarrow \sum C^2_{\Delta s}$$

Bootstrap

$$\rightsquigarrow \sum C^2_{\Delta s} \underbrace{\left(\mathcal{F} - \mathcal{F}\right)}_{\underbrace{\mathcal{L}\cdot H_{\Delta s}(z,\bar{z})}_{\text{if } \exists\, \mathcal{L} \text{ st } \mathcal{L}\cdot H > 0}} = 0 \to \mathcal{L}\cdot 0 = 0$$

$z, 1-\bar{z})$