Title: Axion Cavity Experiments for High-Frequency Gravitational Waves

Speakers: Raffaele Dâ€™Agnolo

Series: Particle Physics

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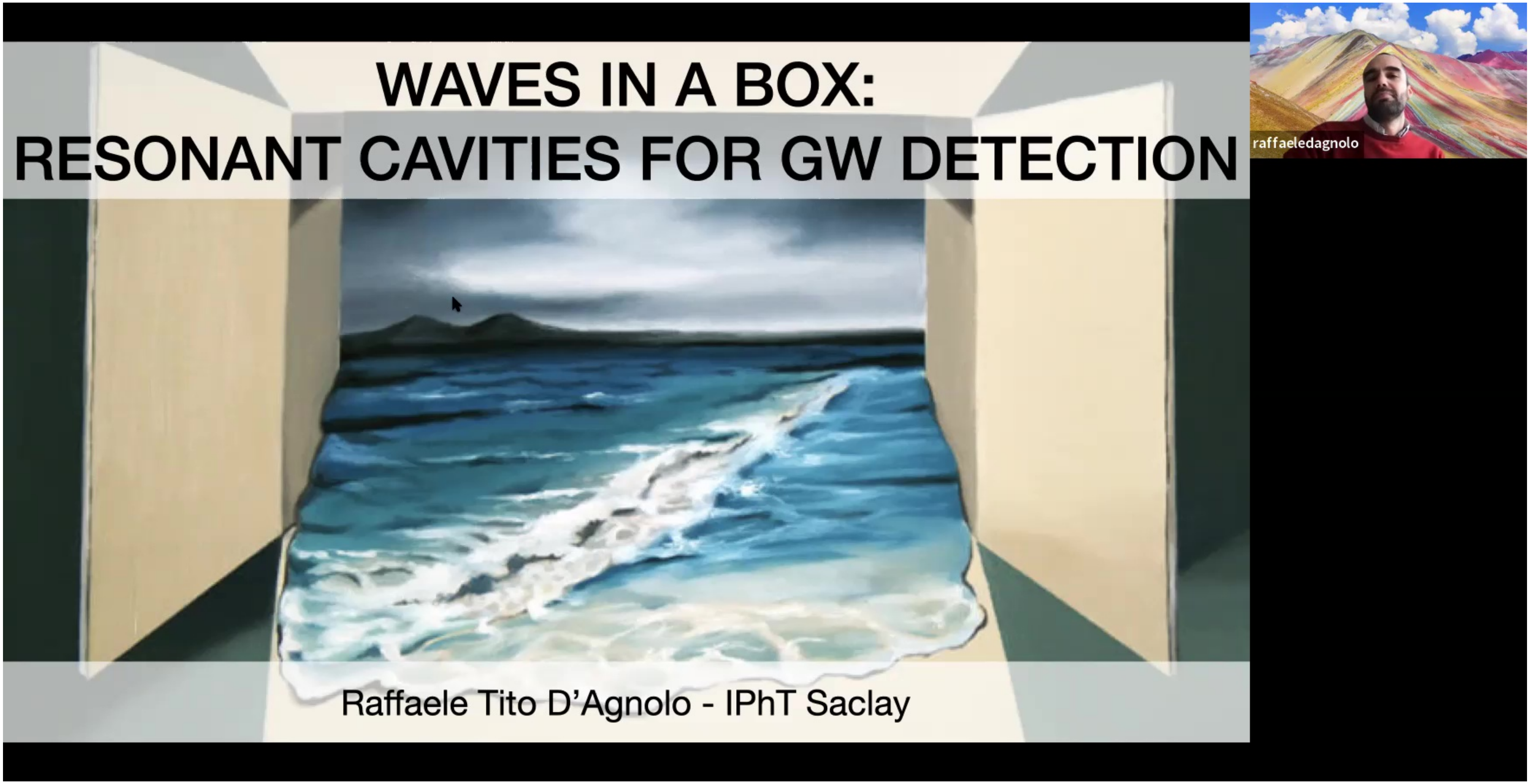
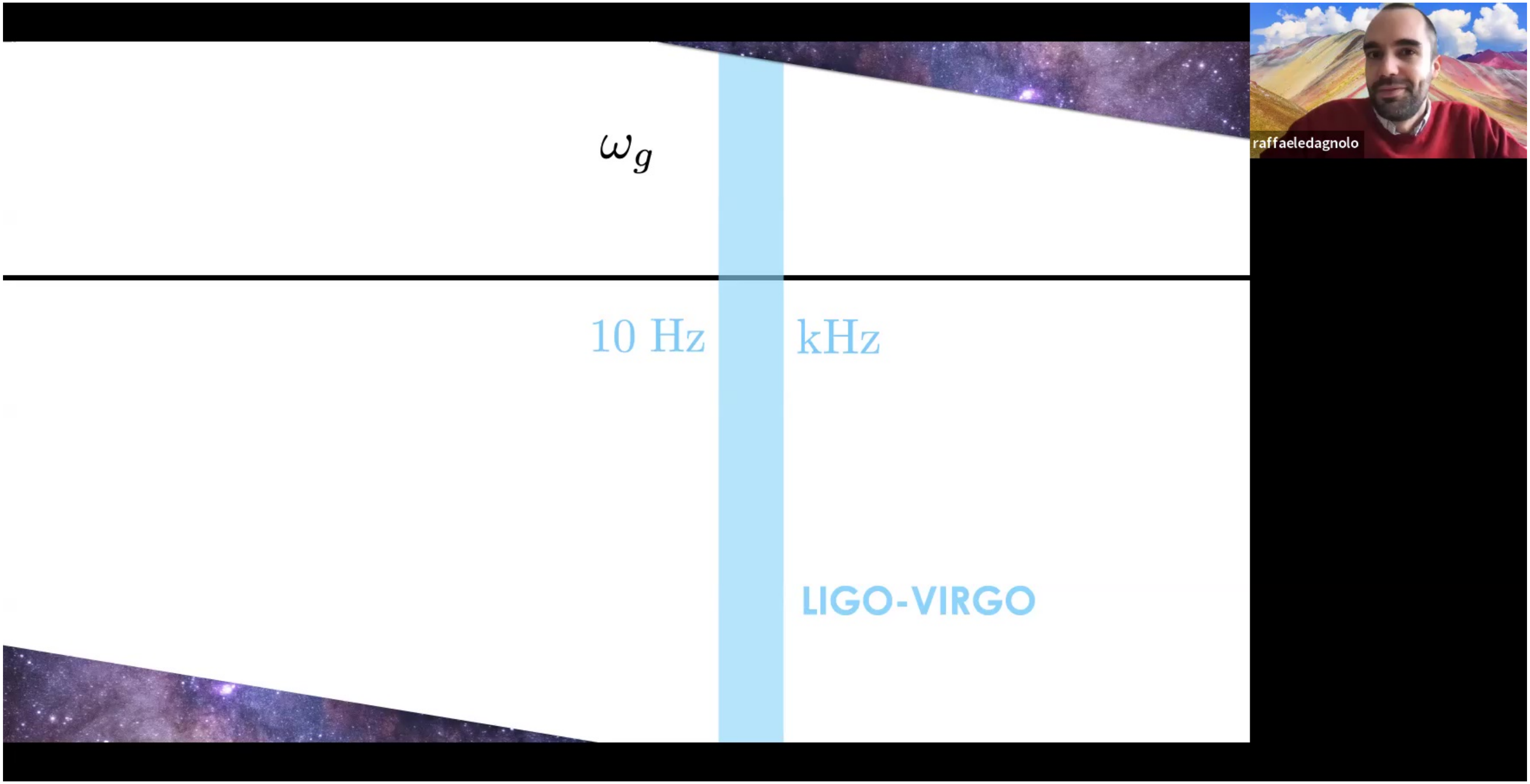
Abstract: I will discuss electromagnetic signals generated by gravitational waves (GWs) in resonant cavity experiments. Experiments designed for the detection of axion dark matter only need to reanalyze existing data to search for GWs in the GHz range with strains as small as h ~ 10^{-22} - 10^{-21}. This is still far from the BBN bound on primordial sources, but it is the best direct constraint that we can currently obtain in the laboratory. To get to this result I will correct some long standing misconceptions present in the literature.

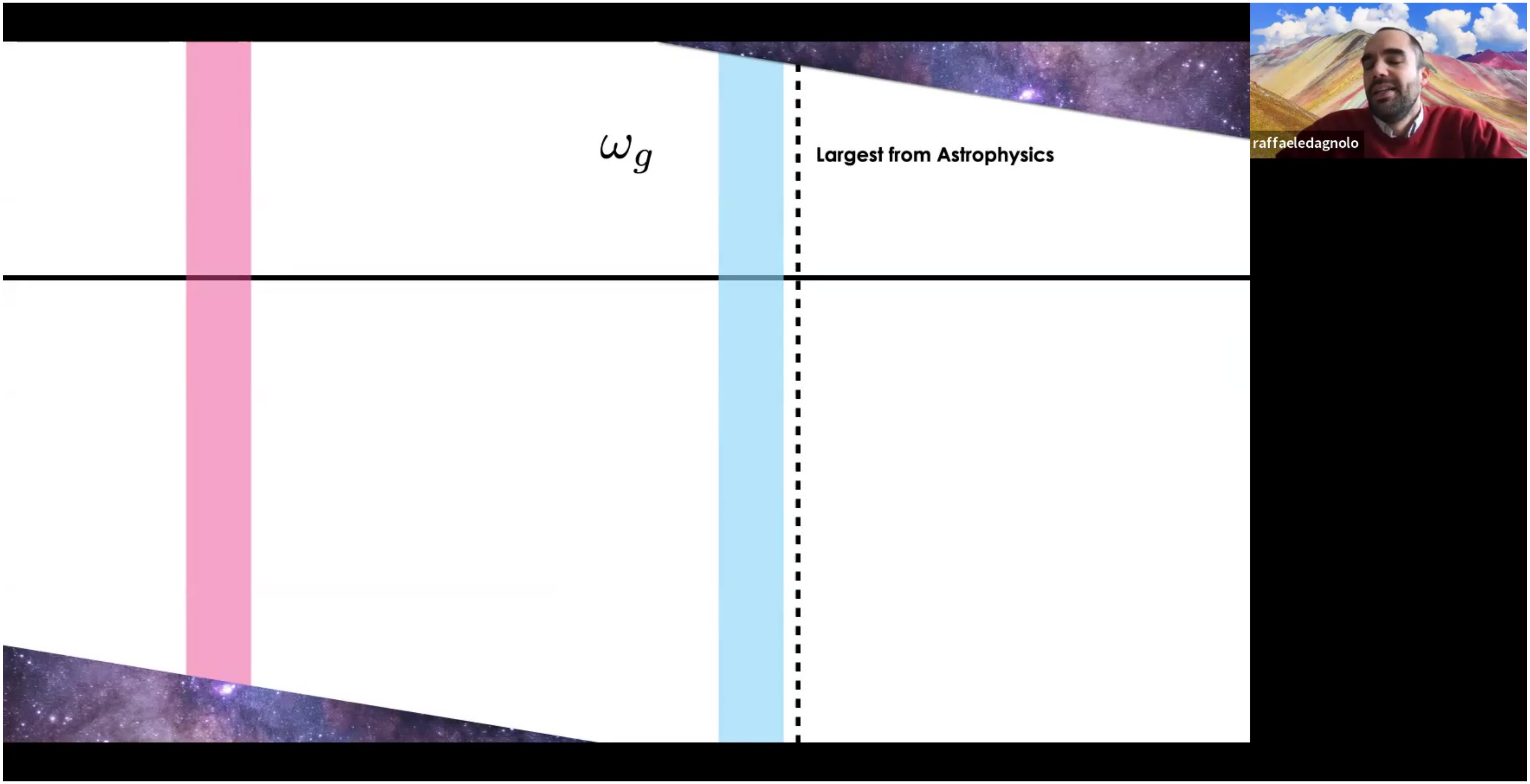
Zoom Link: https://pitp.zoom.us/j/94313100053?pwd=bWk1eHh3NWM3NnB1TjR5d0xrY3BIUT09


WAVES IN A BOX:
RESONANT CAVITIES FOR GW DETECTION
raffaeledagnolo
Raffaele Tito D'Agnolo - IPhT Saclay

$$\omega_g$$


ω_g
10 Hz
kHz
LIGO-VIRGO
raffaeledagnolo

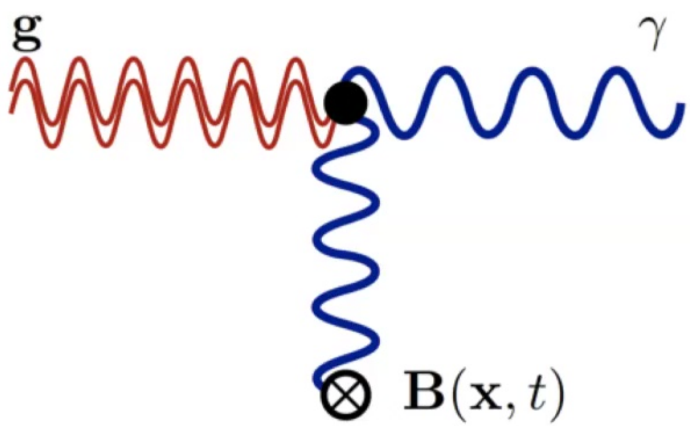

ω_g
nHz
μHz
10 Hz
kHz
PULSAR TIMING
LIGO-VIRGO
raffaeledagnolo


ω_g
Largest from Astrophysics
raffaeledagnolo

$$\omega_g$$

$$\lambda(T_*) < \frac{1}{H(T_*)}$$ Causality

$$\omega_0(T_*) = \omega(T_*)\frac{a(T_*)}{a_0} \gtrsim \boxed{100\ \mathrm{MHz}}\left(\frac{T_*}{10^{15}\mathrm{GeV}}\right)\left(\frac{g_*(T_*)}{100}\right)^{1/6}$$

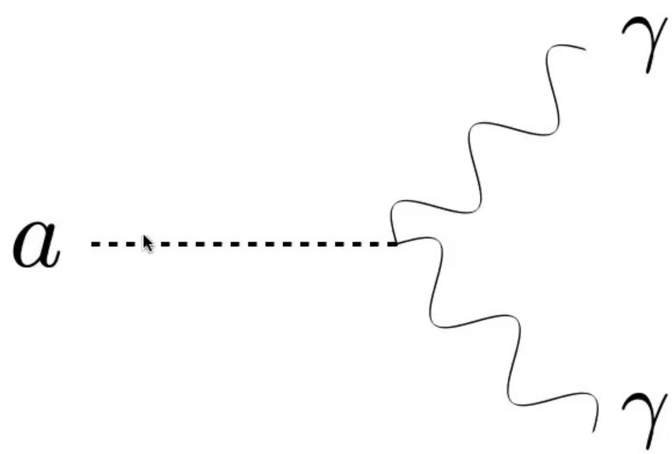

g
γ
$\mathbf{B}(\mathbf{x}, t)$


raffaeledagnolo

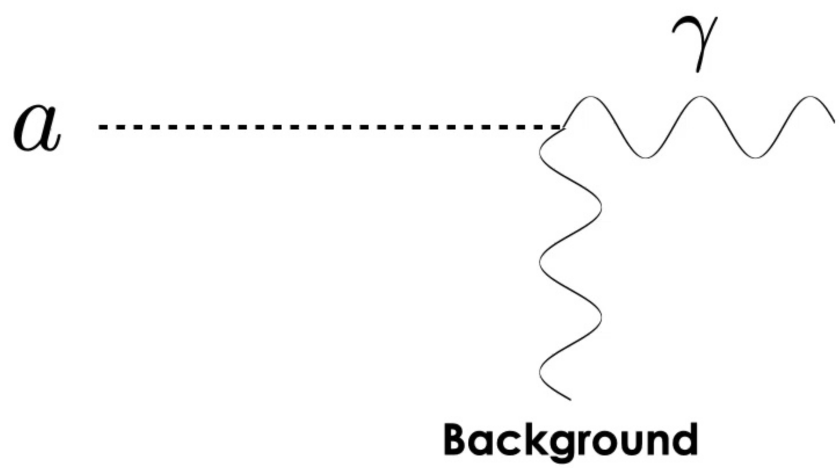

ANALOGY WITH
AXION DETECTION
Asimina Arvanitaki


ANALOGY WITH
AXION DETECTION
raffaeledagnolo

ALP DARK MATTER DETECTION



ALP DARK MATTER DETECTION



$$\sim \frac{a}{f_a} E_{\rm bkg} \simeq \boxed{10^{-21} E_{\rm bkg}}$$

but you know exactly the waveform
and the signal is always there

Dark Matter Particles in a de Broglie Volume **Today**

Galaxy: $$N_{\mathrm{DM}} \simeq 10^3 \left(\frac{\mathrm{eV}}{m_{\mathrm{DM}}}\right)$$

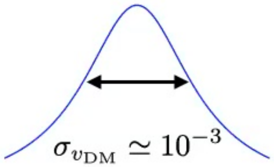
ALP DARK MATTER IN THE LAB

Produced Colder than the SM

$$E_a \approx m_a$$

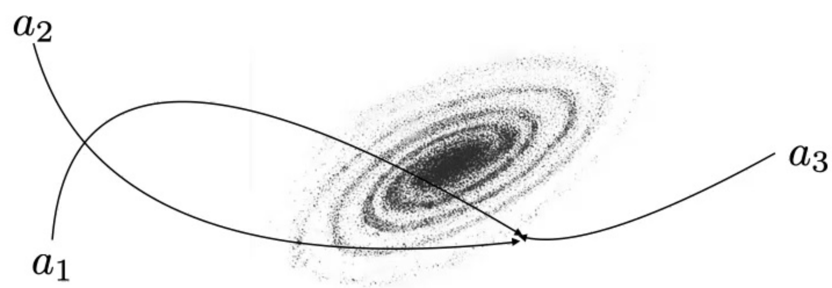
It acquires a **small velocity dispersion** from virialization **in the Milky Way**

$$E_a \simeq m_a \left(1 + \frac{v_{\rm DM}^2}{2}\right)$$



ALP DARK MATTER IN THE LAB

In each experimental bin we are summing over a multitude of plane waves with different phases



$$a(t) = a_0 \left[\cos\left(m_a\left(1+\frac{v_1^2}{2}\right)t+\phi_1\right)+\cos\left(m_a\left(1+\frac{v_2^2}{2}\right)t+\phi_2\right)+\ldots\right]$$

$$\simeq a_0 \cos(m_a t+\phi)\left[\cos(\delta\omega_a t+\phi')+\ldots\right]$$

$$\delta\omega_a \simeq \frac{1}{m_a\langle v_{\rm DM}^2\rangle} \simeq \frac{10^6}{m_a}$$

Effectively: very **slow modulation of** an approximately **monochromatic field**

ALP DARK MATTER IN THE LAB

$$a(t) \simeq \frac{\sqrt{2\rho_{\rm DM}}}{m_a} \cos(\omega_a t + \phi)$$

ALP DARK MATTER IN THE LAB

$$a(t) \simeq \frac{\sqrt{2\rho_{\mathrm{DM}}}}{m_a} \cos(\omega_a t + \phi)$$

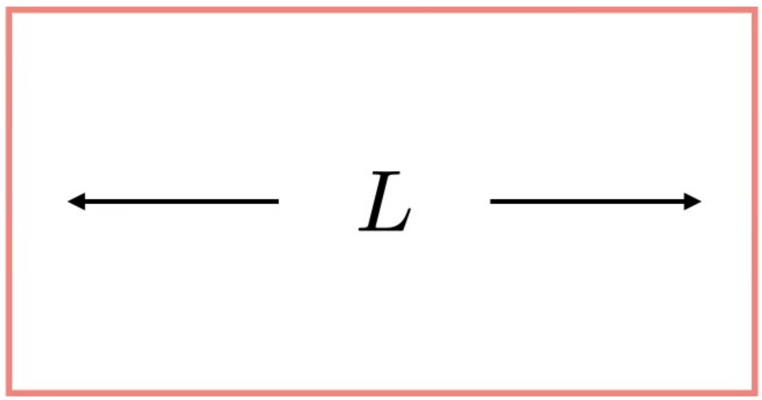
Frequency: $\omega_a \simeq \mathrm{GHz}\, \frac{m_a}{10^{-6}\ \mathrm{eV}}$

Coherence: $\tau_a \simeq \mathrm{ms}\, \frac{10^{-6}\ \mathrm{eV}}{m_a}$

Max Exp. Size: $\lambda_a \simeq 200\ \mathrm{m}\, \frac{10^{-6}\ \mathrm{eV}}{m_a}$

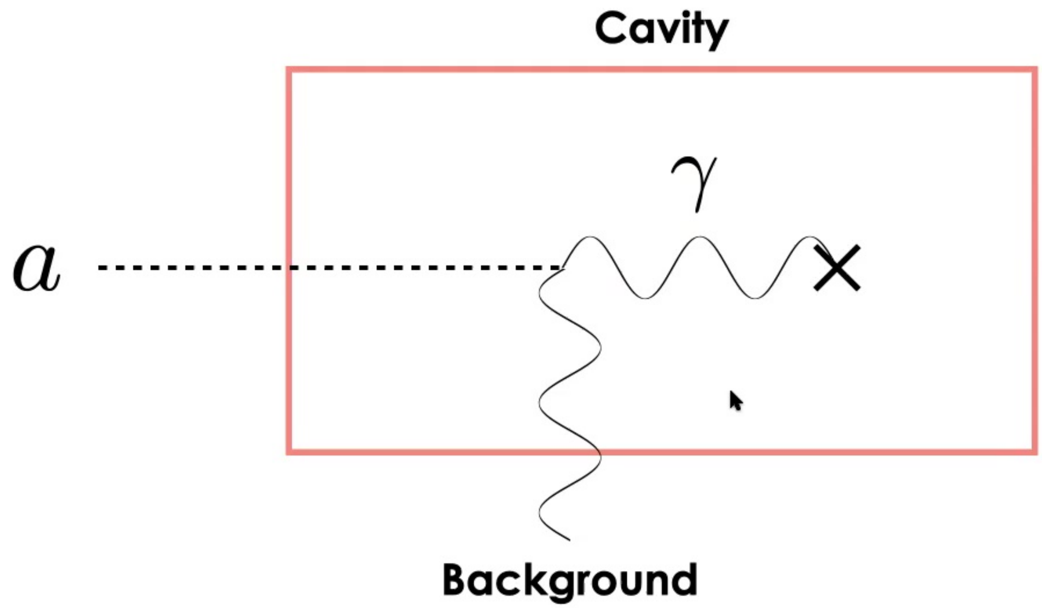
AXION DARK MATTER DETECTION

Cavity



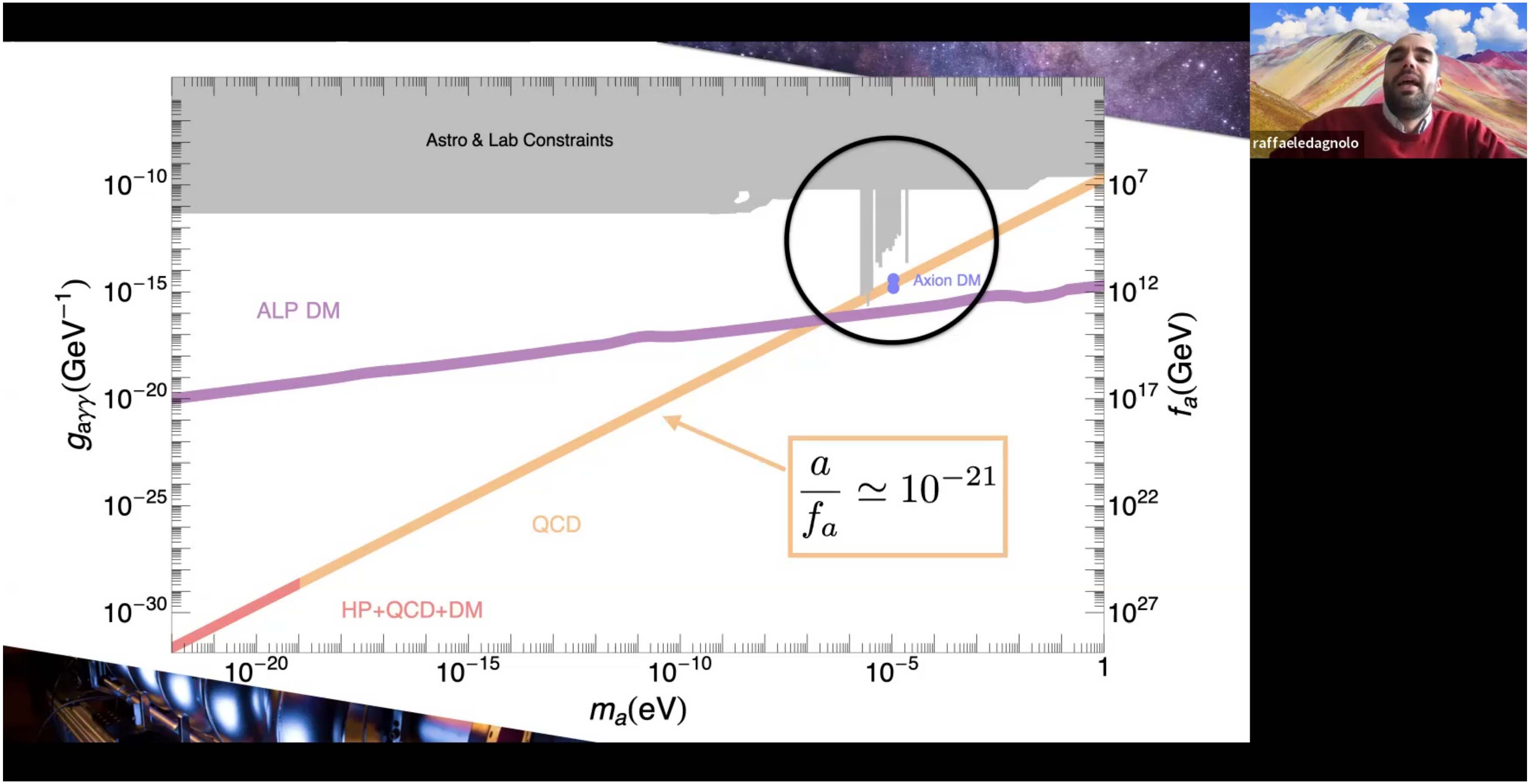
$$m_\gamma \simeq \frac{1}{L}$$

AXION DARK MATTER DETECTION



Resonance

$$m_a \simeq \frac{1}{L}$$


Astro & Lab Constraints
10^{-10}
10^{-15}
10^{-20}
10^{-25}
10^{-30}
$g_{a\gamma\gamma}(\mathrm{GeV}^{-1})$
ALP DM
Axion DM
QCD
HP+QCD+DM
$\frac{a}{f_a} \simeq 10^{-21}$
10^{7}
10^{12}
10^{17}
10^{22}
10^{27}
$f_a(\mathrm{GeV})$
10^{-20}
10^{-15}
10^{-10}
10^{-5}
1
$m_a(\mathrm{eV})$
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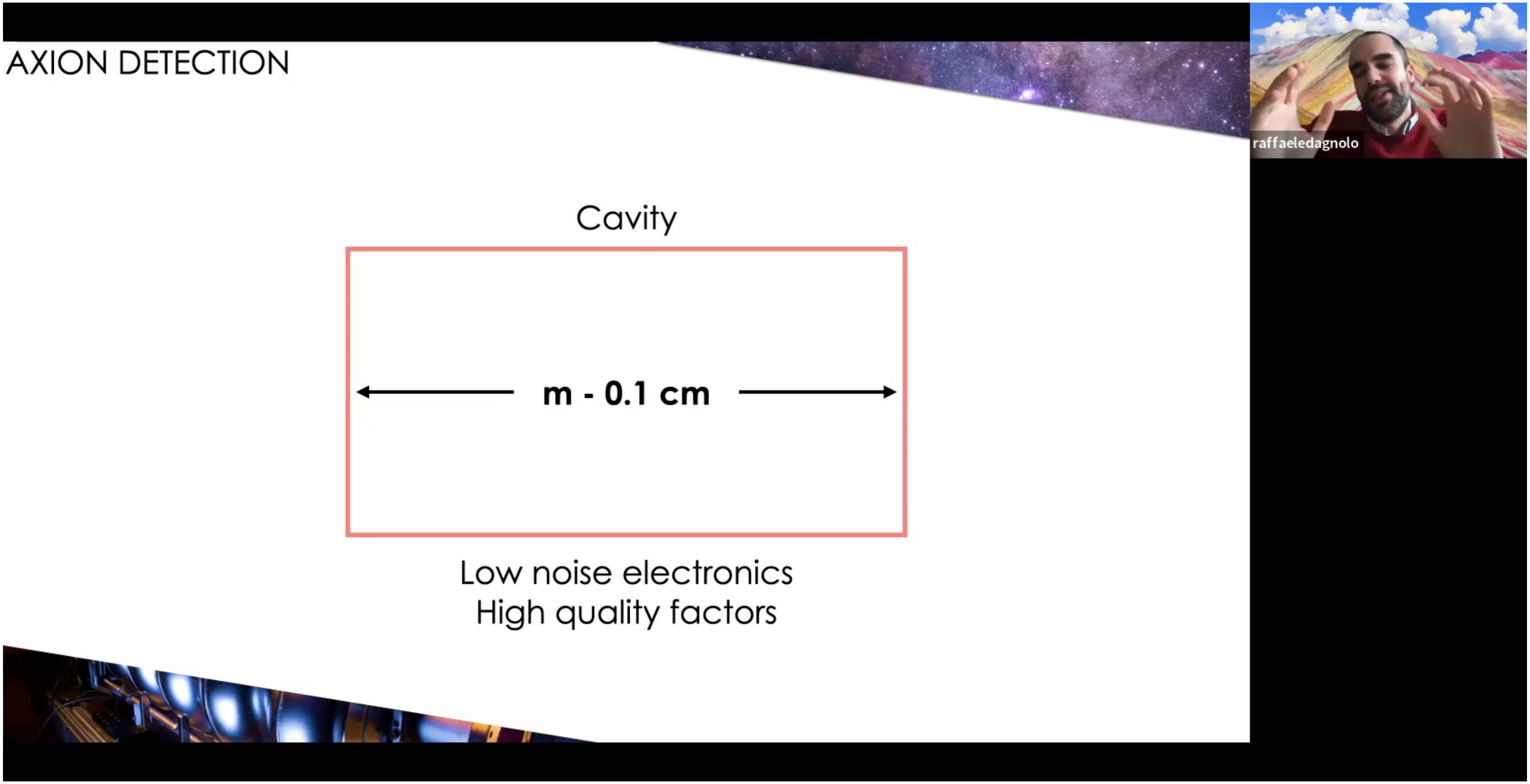
AXION DETECTION
Cavity
m - 0.1 cm
Low noise electronics
High quality factors
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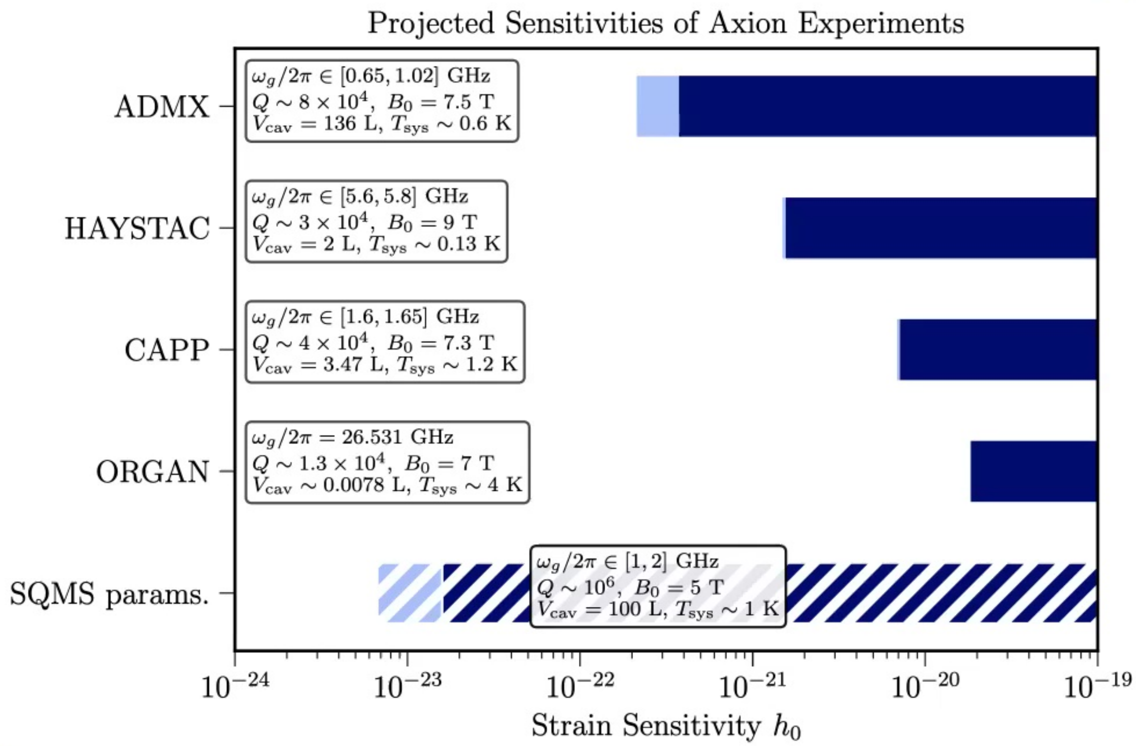
GRAVITATIONAL WAVE DETECTION



Berlin, Blas, D'Agnolo, Ellis, Harnik, Kahn, Schutte-Engel '21

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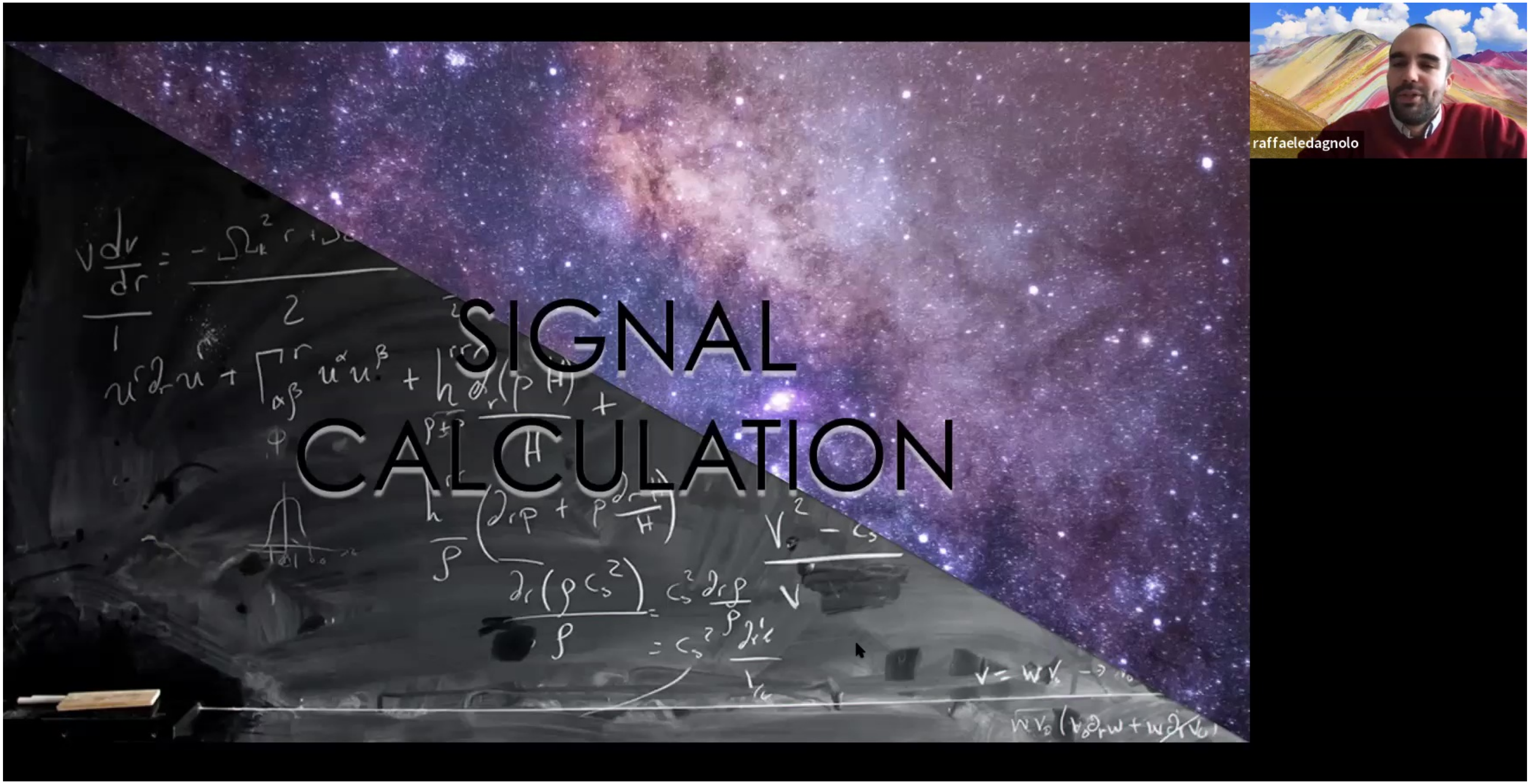
SIGNAL
CALCULATION
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$$S \supset -\frac{1}{2} \int d^4x \; j^{\mu}_{\text{eff}} \, A_{\mu}$$

GW

$$\underline{j^{\mu}_{\text{eff}}} = \partial_{\nu} \left(\frac{1}{2} \, h \, F^{\mu\nu} + h^{\nu}{}_{\alpha} \, F^{\alpha\mu} - h^{\mu}{}_{\alpha} \, F^{\alpha\nu} \right)$$

Axion

$$\underline{j^{\mu}_{\text{eff}}} = \epsilon^{\mu\nu\rho\sigma} \partial_{\nu} (a F_{\rho\sigma})$$

Not a covariant vector

$$S \supset -\frac{1}{2}\int d^4x\, j^{\mu}_{\mathrm{eff}}\, A_{\mu}$$

GW

$$j^{\mu}_{\mathrm{eff}} = \partial_{\nu}\left(\frac{1}{2}\,h\,\underline{F^{\mu\nu}} + h^{\nu}{}_{\alpha}\,\underline{F^{\alpha\mu}} - h^{\mu}{}_{\alpha}\,\underline{F^{\alpha\nu}}\right)$$

Axion

$$j^{\mu}_{\mathrm{eff}} = \epsilon^{\mu\nu\rho\sigma}\partial_{\nu}(a\underline{F_{\rho\sigma}})$$

depends on the background field in the laboratory

Proper Detector Frame (PDF) Vs **Transverse Traceless (TT) Frame**

(Fermi Normal Coordinates)

$$j^{\mu}_{\text{eff}} = \partial_{\nu}\left(\frac{1}{2}\, h\, F^{\mu\nu} + h^{\nu}{}_{\alpha}\, F^{\alpha\mu} - h^{\mu}{}_{\alpha}\, F^{\alpha\nu}\right)$$

If you do the calculation in the reference frame of the laboratory (**proper detector frame** = Fermi normal coordinates)

1. **What you compute is what you measure**

2. The applied fields and all other aspects of the **experimental apparatus** are **simple**

3. The metric of the **GW** is **not simple**

$$j^{\mu}_{\text{eff}} = \partial_{\nu}\left(\frac{1}{2}\,h\,F^{\mu\nu} + h^{\nu}{}_{\alpha}\,F^{\alpha\mu} - h^{\mu}{}_{\alpha}\,F^{\alpha\nu}\right)$$

If you do the calculation in the **reference frame of the wave** (**TT gauge**)

1. **What you compute is NOT what you measure** (unless you choose a gauge invariant quantity)

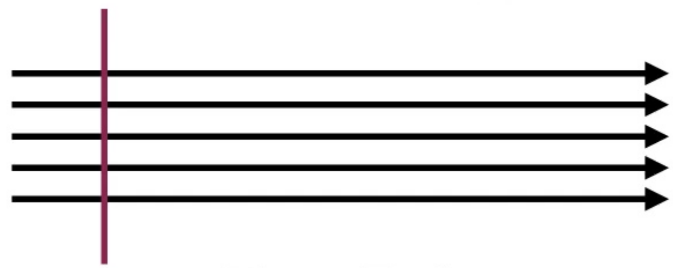
2. The applied fields and all other aspects of the **experimental apparatus** are **very complicated**

3. The metric of the **GW** is **very simple**

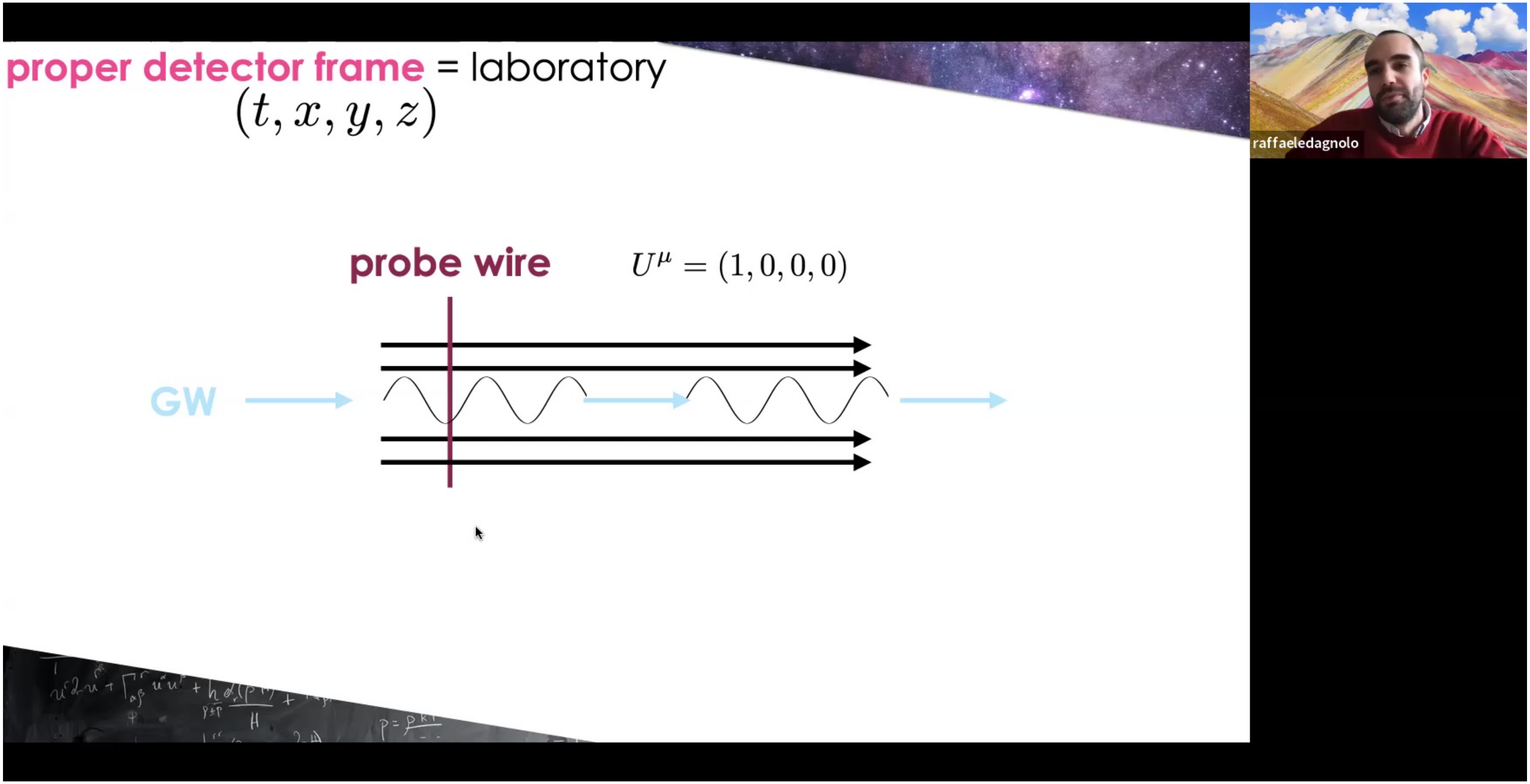
proper detector frame = laboratory

$$(t, x, y, z)$$

probe wire $\quad U^\mu = (1,0,0,0)$



$$\mathbf{B} = B_0 \hat{z}$$


proper detector frame = laboratory
(t, x, y, z)
probe wire
$U^{\mu} = (1, 0, 0, 0)$
GW
raffaeledagnolo

proper detector frame = laboratory

$$(t, x, y, z)$$

GW $\longrightarrow$ $\hat{z}$

$$h_{00} = -\omega_g^2\, h_{ab}^{\mathrm{TT}}\, x^a\, x^b \left[-\frac{i}{\omega_g z} + \frac{1 - e^{-i\omega_g z}}{(\omega_g z)^2} \right]$$

$$h_{ij} = \omega_g^2 \left[\left(\delta_{iz}\, h_{ja}^{\mathrm{TT}} + \delta_{jz}\, h_{ia}^{\mathrm{TT}} \right) z\, x^a - h_{ij}^{\mathrm{TT}}\, z^2 - \delta_{iz}\, \delta_{jz}\, h_{ab}^{\mathrm{TT}}\, x^a\, x^b \right] \left[-\frac{1 + e^{-i\omega_g z}}{(\omega_g z)^2} - 2i\, \frac{1 - e^{-i\omega_g z}}{(\omega_g z)^3} \right]$$

$$h_{0i} = -\omega_g^2 \left(h_{ia}^{\mathrm{TT}}\, z\, x^a - \delta_{iz}\, h_{ab}^{\mathrm{TT}}\, x^a\, x^b \right) \left[-\frac{i}{2\, \omega_g z} - \frac{e^{-i\omega_g z}}{(\omega_g z)^2} - i\, \frac{1 - e^{-i\omega_g z}}{(\omega_g z)^3} \right]$$

TT gauge = comoving with the wave

$$(t_{\mathrm{TT}}, x_{\mathrm{TT}}, y_{\mathrm{TT}}, z_{\mathrm{TT}})$$

GW $\longrightarrow$ $\hat{z}$

$$h_{00}^{\mathrm{TT}} = h_{0i}^{\mathrm{TT}} = 0$$

$$h_{ij}^{\mathrm{TT}} = e^{i\omega_g(t-z)} \begin{pmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

TT gauge = comoving with the wave

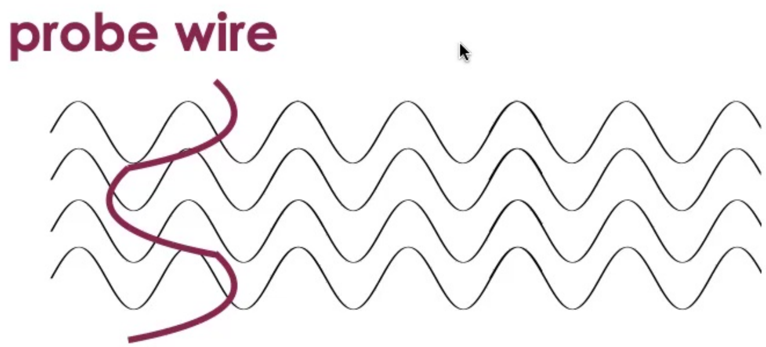
$$(t_{\rm TT}, x_{\rm TT}, y_{\rm TT}, z_{\rm TT})$$

$$t_{\rm TT} \simeq t - \frac{i}{4}\,\omega_g\,(x^2 - y^2)\,h_+\,e^{i\omega_g t}\ ,\quad x_{\rm TT} \simeq x - \frac{1}{2}\,x\,(1 - i\omega_g z)\,h_+\,e^{i\omega_g t}$$

$$y_{\rm TT} \simeq y + \frac{1}{2}\,y\,(1 - i\omega_g z)\,h_+\,e^{i\omega_g t}\ ,\quad z_{\rm TT} \simeq z - \frac{i}{4}\,\omega_g\,(x^2 - y^2)\,h_+\,e^{i\omega_g t}$$

TT gauge = comoving with the wave

$$(t_{\mathrm{TT}}, x_{\mathrm{TT}}, y_{\mathrm{TT}}, z_{\mathrm{TT}})$$



$$\mathbf{B} = B_0 \hat{z} + \frac{i}{2}(h_+ B_0) e^{i\omega_g t}(\omega_g x, -\omega_g y, 0) + \mathcal{O}(h^2)$$

TT gauge = comoving with the wave

$$(t_{\mathrm{TT}}, x_{\mathrm{TT}}, y_{\mathrm{TT}}, z_{\mathrm{TT}})$$

$$\mathbf{B} = B_0 \hat{z}$$

Theorem: $j^{\mu}_{\mathrm{eff,TT}} = 0$

TT gauge = comoving with the wave

$$(t_{\mathrm{TT}}, x_{\mathrm{TT}}, y_{\mathrm{TT}}, z_{\mathrm{TT}})$$

Wrong Conclusion

$\boxed{\text{Theorem: } j^{\mu}_{\mathrm{eff,TT}} = 0}$ → No signal

Doubly Wrong:

1. Impossible to prepare a uniform B-field in the TT frame
2. Even if you could do it, there would still be a signal (wire moving)

OUR CALCULATION

Proper Detector Frame

$$S \supset -\frac{1}{2} \int d^4x \; j^{\mu}_{\text{eff}} \, A_{\mu}$$

OUR CALCULATION

Proper Detector Frame

$$S \supset -\frac{1}{2} \int d^4x \; j^{\mu}_{\rm eff} \, A_{\mu}$$

$$\mathbf{E}(\vec{x},t) = \sum_n e_n(t) \, \mathbf{E}_n(\vec{x})$$

Cavity

OUR CALCULATION

Proper Detector Frame

$$\left(\partial_t^2 + \frac{\omega_n}{Q_n}\,\partial_t + \omega_n^2\right) e_n(t) = -\,\frac{\int_{V_{\rm cav}} d^3x\ \mathbf{E}_n^* \cdot \partial_t \mathbf{j}_{\rm eff}}{\int_{V_{\rm cav}} d^3x\ |\mathbf{E}_n|^2}$$

OUR CALCULATION

Proper Detector Frame

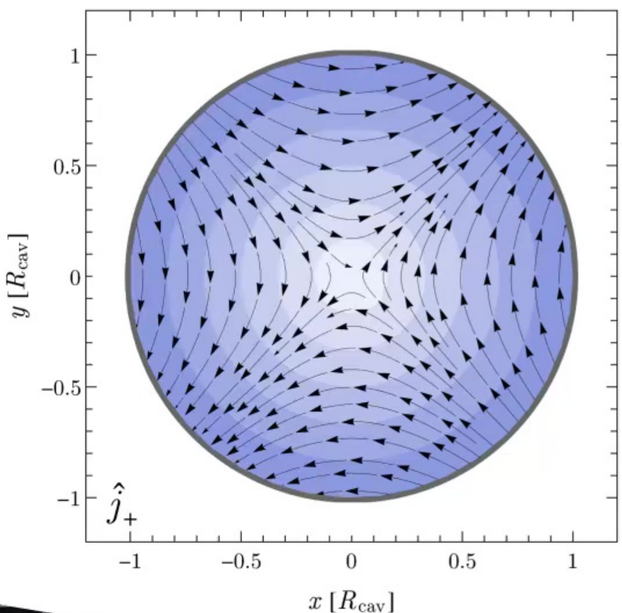
$$\left(\partial_t^2 + \frac{\omega_n}{Q_n}\,\partial_t + \omega_n^2\right) e_n(t) = -\,\frac{\int_{V_{\rm cav}} d^3x\ \mathbf{E}_n^* \cdot \partial_t \mathbf{j}_{\rm eff}}{\int_{V_{\rm cav}} d^3x\ |\mathbf{E}_n|^2}$$

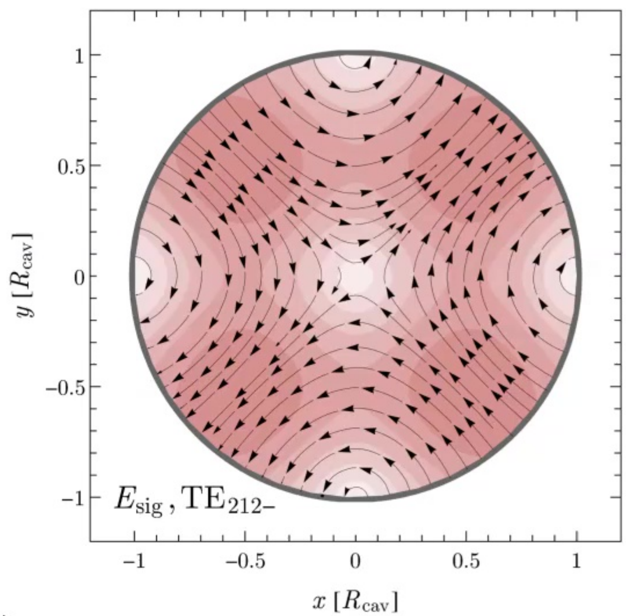
$$\eta_n \equiv \frac{\left|\int_{V_{\rm cav}} d^3x\ \mathbf{E}_n^* \cdot \mathbf{j}_{+,\times}\right|}{V_{\rm cav}^{1/2}\left(\int_{V_{\rm cav}} d^3x\ |\mathbf{E}_n|^2\right)^{1/2}}$$

Geometry

OUR CALCULATION

Selection rules of SO(3) breaking
by the GW and the B-field

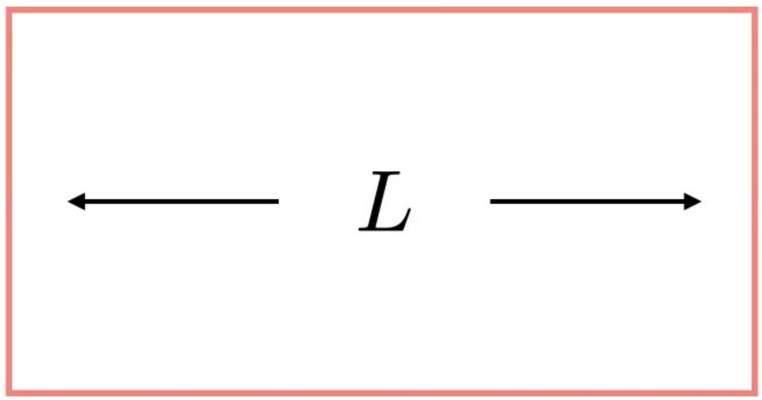




$$\eta_n = \mathcal{O}(1)$$

OUR CALCULATION

Cavity



$$\lambda_g \simeq L$$

OUR CALCULATION

Resummed Metric in the Detector Frame

$$h_{00} = -\omega_g^2\, h_{ab}^{\mathrm{TT}}\, x^a\, x^b \left[-\frac{i}{\omega_g z} + \frac{1 - e^{-i\omega_g z}}{(\omega_g z)^2}\right]$$

$$h_{ij} = \omega_g^2 \left[\left(\delta_{iz}\, h_{ja}^{\mathrm{TT}} + \delta_{jz}\, h_{ia}^{\mathrm{TT}}\right) z\, x^a - h_{ij}^{\mathrm{TT}}\, z^2 - \delta_{iz}\, \delta_{jz}\, h_{ab}^{\mathrm{TT}}\, x^a\, x^b\right] \left[-\frac{1 + e^{-i\omega_g z}}{(\omega_g z)^2} - 2i\, \frac{1 - e^{-i\omega_g z}}{(\omega_g z)^3}\right]$$

$$h_{0i} = -\omega_g^2 \left(h_{ia}^{\mathrm{TT}}\, z\, x^a - \delta_{iz}\, h_{ab}^{\mathrm{TT}}\, x^a\, x^b\right) \left[-\frac{i}{2\,\omega_g z} - \frac{e^{-i\omega_g z}}{(\omega_g z)^2} - i\, \frac{1 - e^{-i\omega_g z}}{(\omega_g z)^3}\right]$$

OUR CALCULATION

Metric in the Detector Frame

$$h_{00} \simeq -R_{0i0j}\, x^i\, x^j + \mathcal{O}(\omega_g x)^3$$

$$h_{ij} \simeq -\frac{1}{3}\, R_{ikjl}\, x^k\, x^l + \mathcal{O}(\omega_g x)^3$$

$$h_{0i} \simeq -\frac{2}{3}\, R_{0jik}\, x^j\, x^k + \mathcal{O}(\omega_g x)^3$$

OUR CALCULATION

Resummed Metric in the Detector Frame

$$h_{00} = -\omega_g^2\, h_{ab}^{\mathrm{TT}}\, x^a\, x^b \left[-\frac{i}{\omega_g z} + \frac{1 - e^{-i\omega_g z}}{(\omega_g z)^2}\right]$$

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OUR CALCULATION

ADMX, HAYSTAC, ...

$$\mathbf{j}_\times \cdot \hat{z} \propto \sin\phi \, \sin\underline{\beta}\left(-1 + \frac{2i}{3}\,\omega_g\,(z\cos\underline{\beta} - r\sin\phi\sin\underline{\beta}) + \mathcal{O}(\omega_g^2\, L_{\rm det}^2)\right)$$

Angle between GW and B-field

OUR CALCULATION

ADMX, HAYSTAC, ...

$$\mathbf{j}_\times \cdot \hat{z} \propto \sin\phi \, \sin\beta \left(-1 + \frac{2i}{3}\,\omega_g \,(z\cos\beta - r\sin\phi\sin\beta) + \mathcal{O}(\omega_g^2\, L_{\rm det}^2) \right)$$

The volume integral of the leading order piece vanishes

OUR CALCULATION

ADMX, HAYSTAC, ...

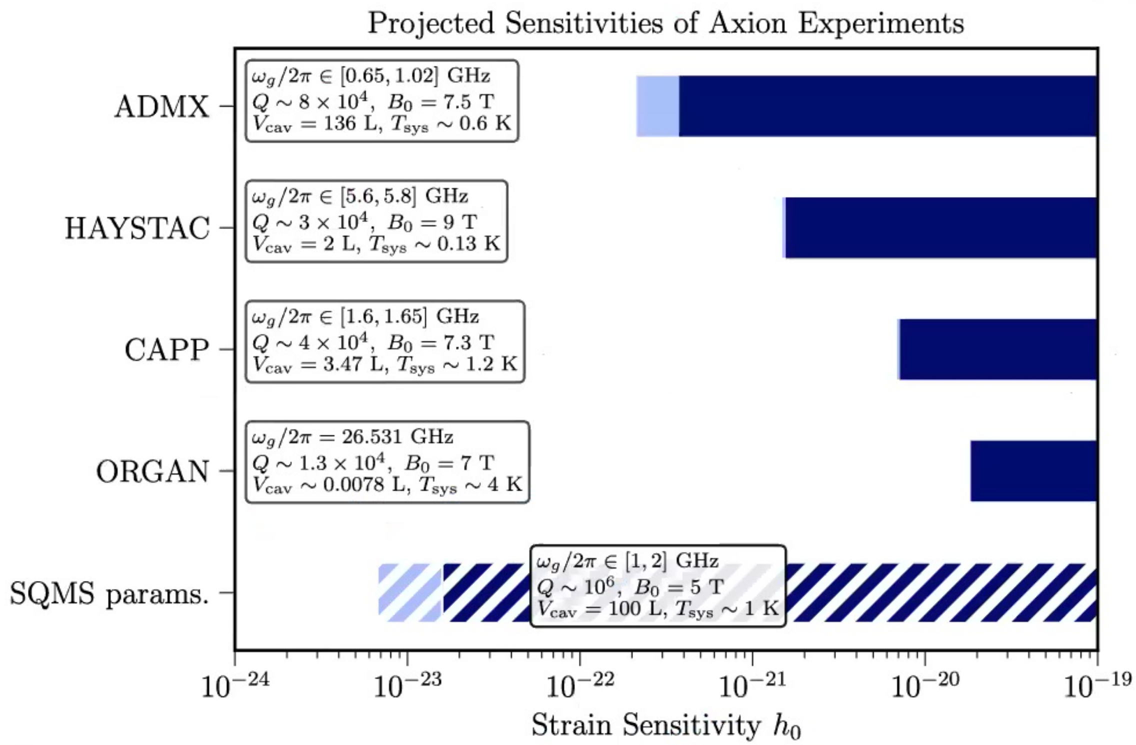
$$\mathbf{j}_\times \cdot \hat{z} \propto \sin\phi\, \sin\beta \left(-1 + \frac{2i}{3}\,\omega_g\,(z\cos\beta - r\sin\phi\sin\beta) + \mathcal{O}(\omega_g^2\, L_{\rm det}^2) \right)$$

For the orthogonal polarization
need to go to

$$\mathcal{O}(\omega_g^4\, L_{\rm det}^4)$$

to see that it's non-zero

GRAVITATIONAL WAVE DETECTION



OUR CALCULATION

ADMX, HAYSTAC, ...

$$\mathbf{j}_\times \cdot \hat{z} \propto \sin\phi\, \sin\beta \left(-1 + \frac{2i}{3}\,\omega_g\,(z\cos\beta - r\sin\phi\sin\beta) + \mathcal{O}(\omega_g^2\, L_{\rm det}^2) \right)$$

For the orthogonal polarization
need to go to

$$\mathcal{O}(\omega_g^4\, L_{\rm det}^4)$$

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OUR CALCULATION

Proper Detector Frame

$$\left(\partial_t^2 + \frac{\omega_n}{Q_n}\,\partial_t + \omega_n^2\right) e_n(t) = -\,\frac{\int_{V_{\rm cav}} d^3x\ \mathbf{E}_n^* \cdot \partial_t \mathbf{j}_{\rm eff}}{\int_{V_{\rm cav}} d^3x\ |\mathbf{E}_n|^2}$$

$$\eta_n \equiv \frac{\left|\int_{V_{\rm cav}} d^3x\ \mathbf{E}_n^* \cdot \mathbf{j}_{+,\times}\right|}{V_{\rm cav}^{1/2}\left(\int_{V_{\rm cav}} d^3x\ |\mathbf{E}_n|^2\right)^{1/2}}$$

Geometry

$$S \supset -\frac{1}{2}\int d^4x\; j^{\mu}_{\rm eff}\, A_{\mu}$$

GW

$$\underline{j^{\mu}_{\rm eff}} = \partial_{\nu}\left(\frac{1}{2}\, h\, F^{\mu\nu} + h^{\nu}{}_{\alpha}\, F^{\alpha\mu} - h^{\mu}{}_{\alpha}\, F^{\alpha\nu}\right)$$

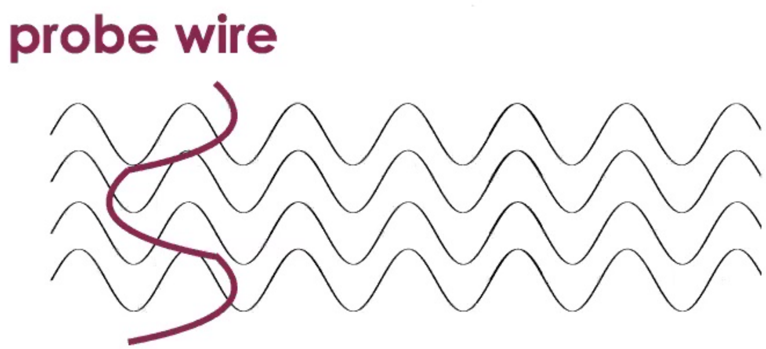
Axion

$$\underline{j^{\mu}_{\rm eff}} = \epsilon^{\mu\nu\rho\sigma}\partial_{\nu}(aF_{\rho\sigma})$$

Not a covariant vector

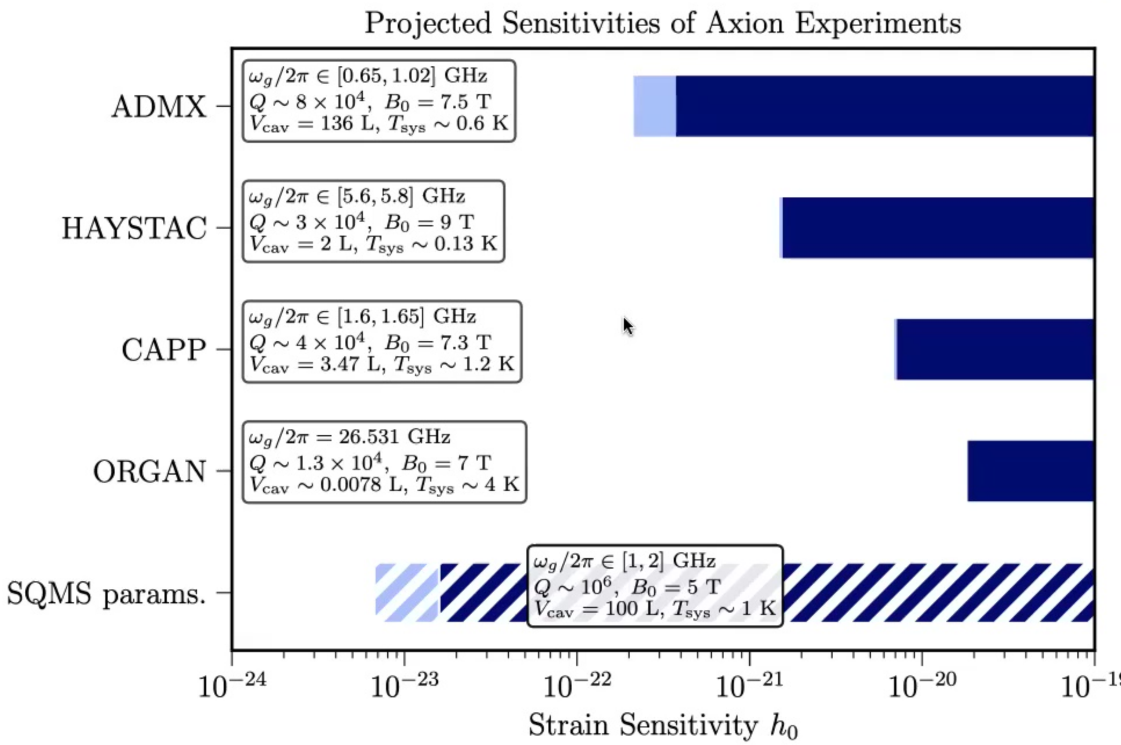
TT gauge = comoving with the wave

$$(t_{\mathrm{TT}}, x_{\mathrm{TT}}, y_{\mathrm{TT}}, z_{\mathrm{TT}})$$

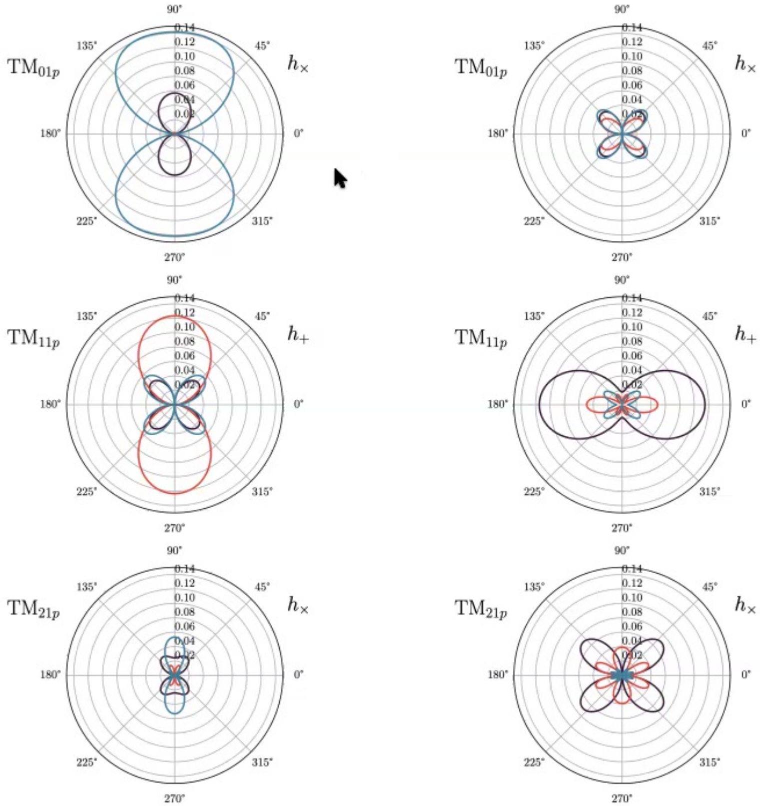


$$\mathbf{B} = B_0\hat{z} + \frac{i}{2}(h_+ B_0)e^{i\omega_g t}(\omega_g x, -\omega_g y, 0) + \mathcal{O}(h^2)$$

GRAVITATIONAL WAVE DETECTION



GRAVITATIONAL WAVE DETECTION



LOWER FREQUENCIES

Proper Detector Frame

$$\left(\partial_t^2 + \frac{\omega_n}{Q_n}\,\partial_t + \omega_n^2\right) e_n(t) = -\,\frac{\int_{V_{\mathrm{cav}}} d^3x\ \mathbf{E}_n^* \cdot \partial_t \mathbf{j}_{\mathrm{eff}}}{\int_{V_{\mathrm{cav}}} d^3x\ |\mathbf{E}_n|^2}$$

Proper Detector Frame

$$\left(\partial_t^2 + \frac{\omega_n}{Q_n}\,\partial_t + \omega_n^2\right) e_n(t) = -\,\frac{\int_{V_{\rm cav}} d^3x\ \mathbf{E}_n^* \cdot \partial_t \mathbf{j}_{\rm eff}}{\int_{V_{\rm cav}} d^3x\ |\mathbf{E}_n|^2}$$

$$\partial_t j_{\rm eff} \sim \partial_t (RB) \sim \omega_g^3 B$$

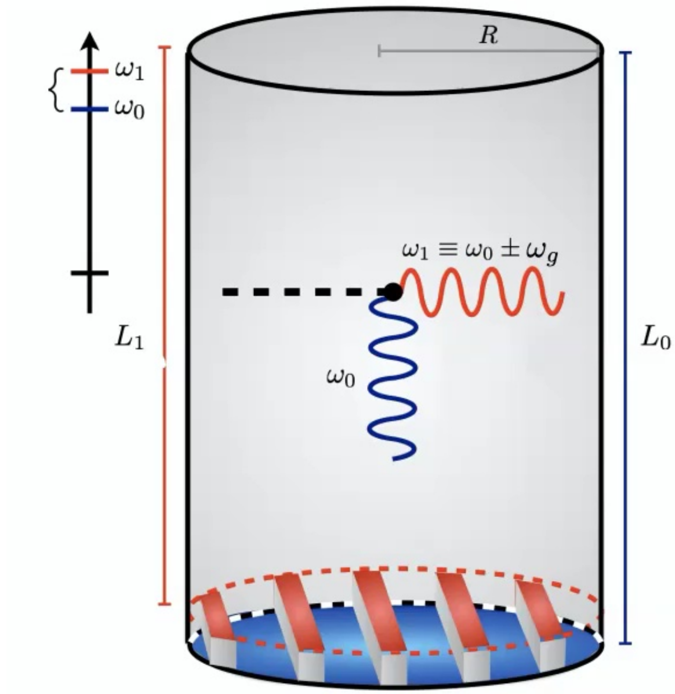
LOWER FREQUENCIES

Heterodyne Detection

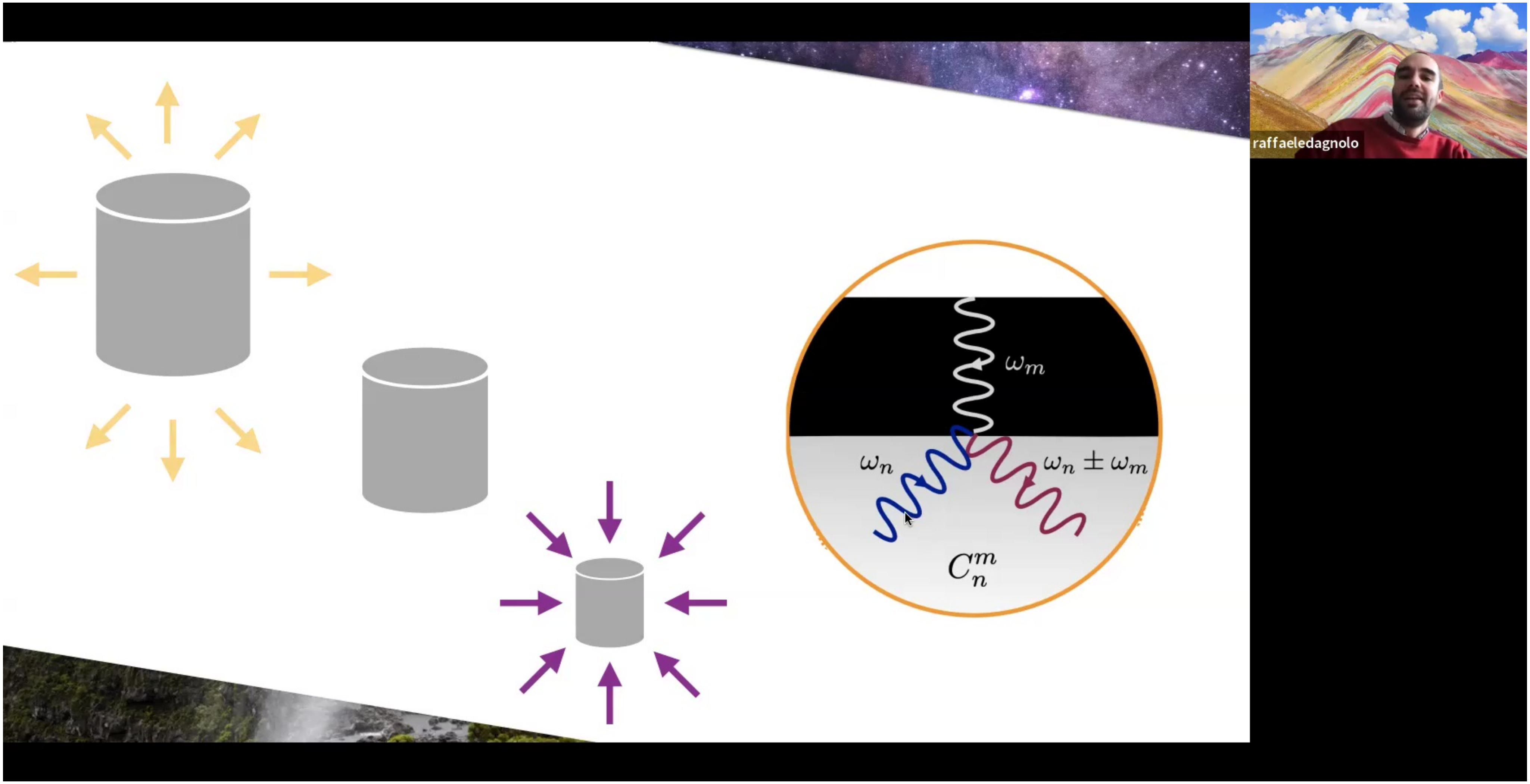
$$\partial_t j_{\text{eff}} \sim \partial_t (RB) \sim \omega_g^3 B$$

$$\partial_t j_{\text{eff}} \sim \partial_t (RB(t)) \sim \omega_g^2 \dot{B}(t)$$

[Caves '78], [Radicati, Pegoraro, Picasso '78]

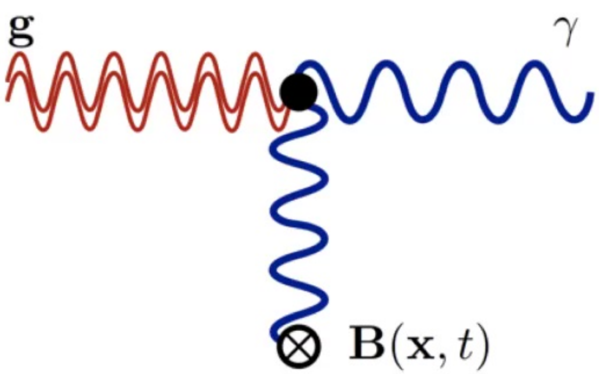

ω_1
ω_0
R
$\omega_1 \equiv \omega_0 \pm \omega_g$
L_1
L_0
ω_0


raffaeledagnolo



raffaeledagnolo
ω_m
ω_n
$\omega_n \pm \omega_m$
C_n^m

ELECTROMAGNETIC



Oscillating

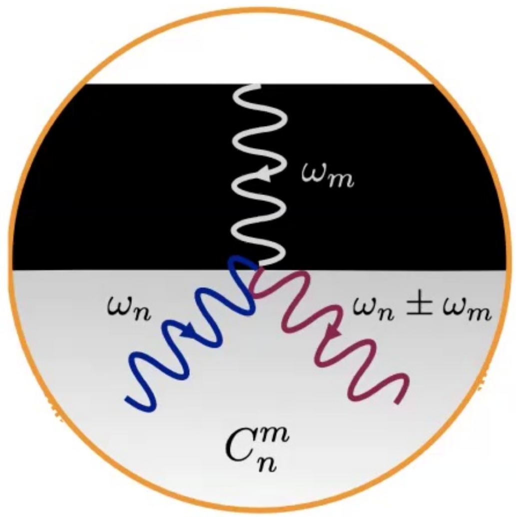
$$\Delta B_{\rm sig} \sim h B_0 \sqrt{Q}\, \min\left[\frac{\omega_{\rm GW}^2}{\omega_{\rm res}^2}, 1\right]$$

Static [RF cavity]

$$\Delta B_{\rm sig} \sim h B_0 \sqrt{Q}\, \min\left[\frac{\omega_{\rm GW}^2}{\omega_{\rm res}^2}\frac{\omega_{\rm GW}}{\omega_{\rm res}}, 1\right]$$

$$\Delta B_{\rm sig} \sim h B_0 \sqrt{Q}$$

MECHANICAL



[Caves '78], [Radicati, Pegoraro, Picasso '78]

CONCLUSION

- We clarified a few aspects of GW interactions with EM fields in the laboratory that were often treated incorrectly in the literature

- Axion dark matter experiments can re-analyze their data and be the most sensitive probe of GWs at GHz frequencies

- Stay tuned for heterodyne detection

GRAVITATIONAL WAVE DETECTION