Title: A Celestial Matrix Model

Speakers: Charles Marteau

Series: Quantum Fields and Strings

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Abstract: Since the advent of holography, a lot of progress has been made in our understanding of quantum gravity in AdS. However less is known of its flat space counterpart, even though quantum gravity was certainly first formulated in flat space. Various programs have emerged since, like Celestial Holography or Carrollian Holography, both formulated in four and higher dimensions which is the closest to reality but makes the computation of highly quantum quantities difficult. I will present a 2d model of flat space gravity in which these difficulties can be tackled. This model is dubbed the "Cangemi -- Jackiw model" after the authors of arXiv:9203056. It is a model of flat space gravity that can be reformulated in terms of a boundary action whose solutions are Rindler patches. I will explain how this "boundary graviton" reformulation gives access to fully quantum Euclidean results. In particular we will compute the exact spectrum of the Bondi Hamiltonian and show that it can be non-perturbatively completed by a matrix model. I will also comment on scrambling in flat space.

Based on "From black holes to baby universes in CGHS gravity" with Victor Godet (https://arxiv.org/abs/2103.13422) and an upcoming paper with Arjun Kar, Lampros Lamprou and Felipe Rosso.

Zoom Link: https://pitp.zoom.us/j/95709808325?pwd=QVdFUFZiTHVvMWlDb211U3kxQ0ZkZz09

A Celestial Matrix Model

Based on arxiv: 2005.08999 with V. Godet
and upcoming paper with
A. Kar, L. Lamprou and F. Rosso
Perimeter Institute seminar
March 1st 2022

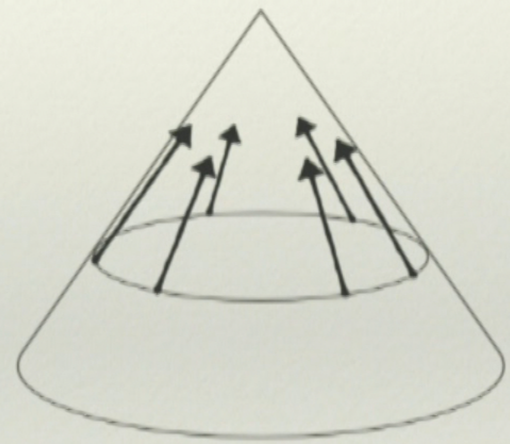
Charles Marteau, University of British Columbia

Introduction: Low-dimensional gravity

- Simple models of quantum gravity.
- Breaking of the Virasoro symmetry → JT gravity. [Maldacena, Stanford, Yang][Jensen]
- Simplest non trivial gravitational theory in bulk dimension $D = 2$.
- Exact gravitational path integral, including all topologies: duality with ensemble. [Saad, Shenker, Stanford]
- Non perturbative observables: partition function, correlators of local operators.
- Chaos: early time (scrambling, $t \sim \log S$) and late time (eigenvalue repulsion, $t \sim e^{S}$).

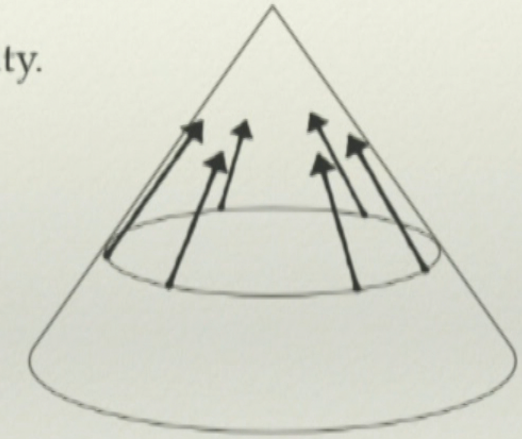
Introduction: Flat gravity

- Asymptotically flat gravity: BMS group. [Bondi, Metzner, Sachs]
- Vacuum degeneracy: soft modes.
- Vacuum transition: memory effect.
- Perturbative quantum gravity: Weinberg's soft theorems.
- More recently: Celestial holography. [Strominger & al]
- Could provide non-perturbative definition of gravity S-matrix.
- Asymptotic symmetry ↔ Operator (Ward identity).
- Bondi Hamiltonian generates time evolution on Scri.



Introduction: Today

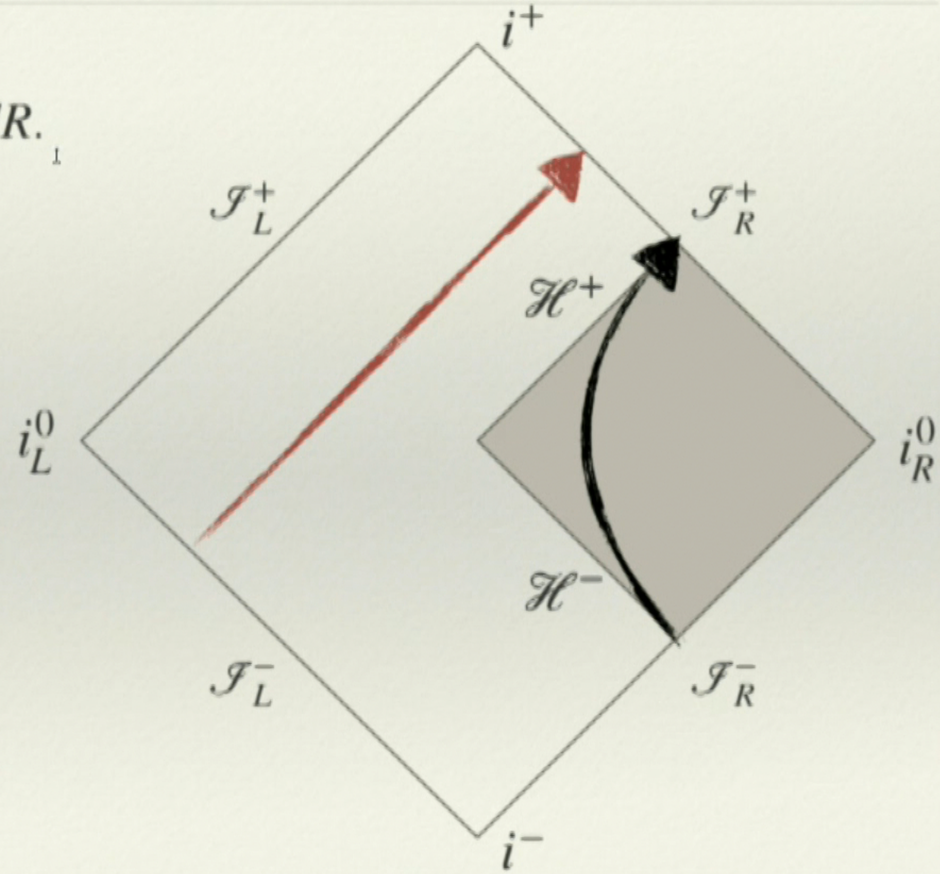
- Mix both approaches to learn about non-perturbative aspects of a celestial operator: *the Bondi Hamiltonian*.
- Our model: Cangemi—Jackiw gravity, 2d model of flat space pure gravity.
- Possesses ∞-dimensional symmetry, a 2d version of BMS.
- Like in JT, dynamics controlled by symmetry breaking.
- No bulk gravitons, only boundary soft modes.
- Allow for exact computation of Euclidean path integral.
- Non-perturbative completion by Matrix Model.
- Gives access to non-perturbative spectral properties of the Bondi Hamiltonian (late time chaos?).
- Study of scrambling (early time chaos?)



Preliminaries: Minkowski and Rindler

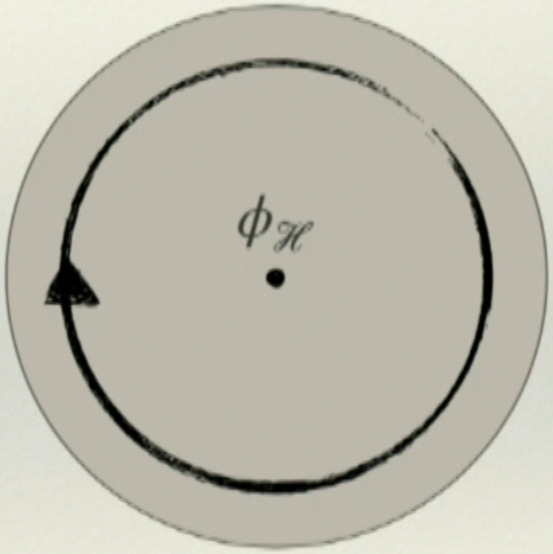
- Global Minkowski: $ds^2 = -\,dU^2 - 2dUdR.$
- Rindler: $ds^2 = -\,\frac{4\pi r}{\beta}du^2 - 2dudr.$
- Acceleration: $a = \frac{2\pi}{\beta}.$
- Boost symmetry: $u \to u + cst, \quad r \to r.$
- Boost = Rindler's Bondi Hamiltonian.

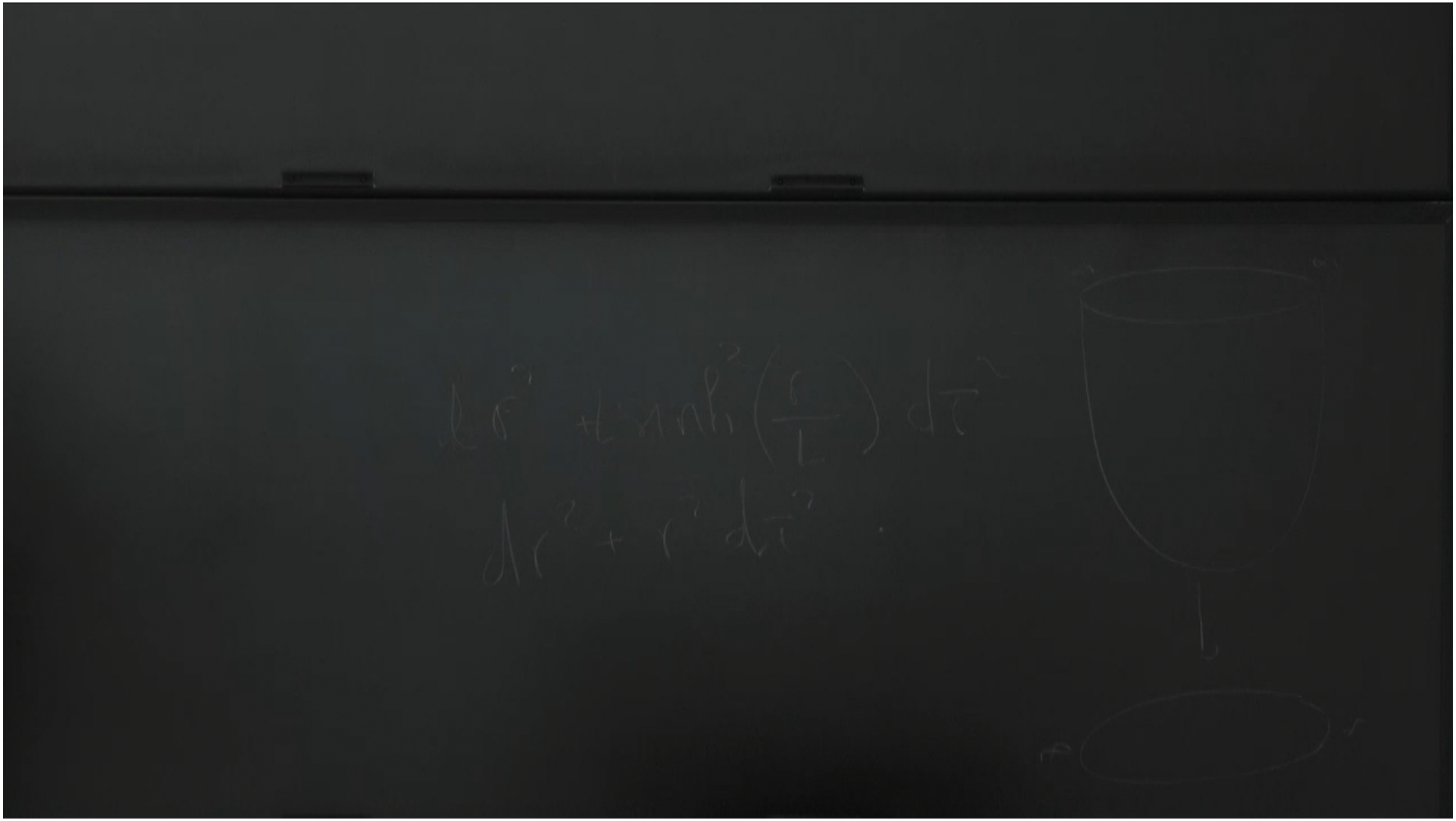
$$B = H^+ = \partial_u$$



Flat Jackiw–Teitelboim gravity

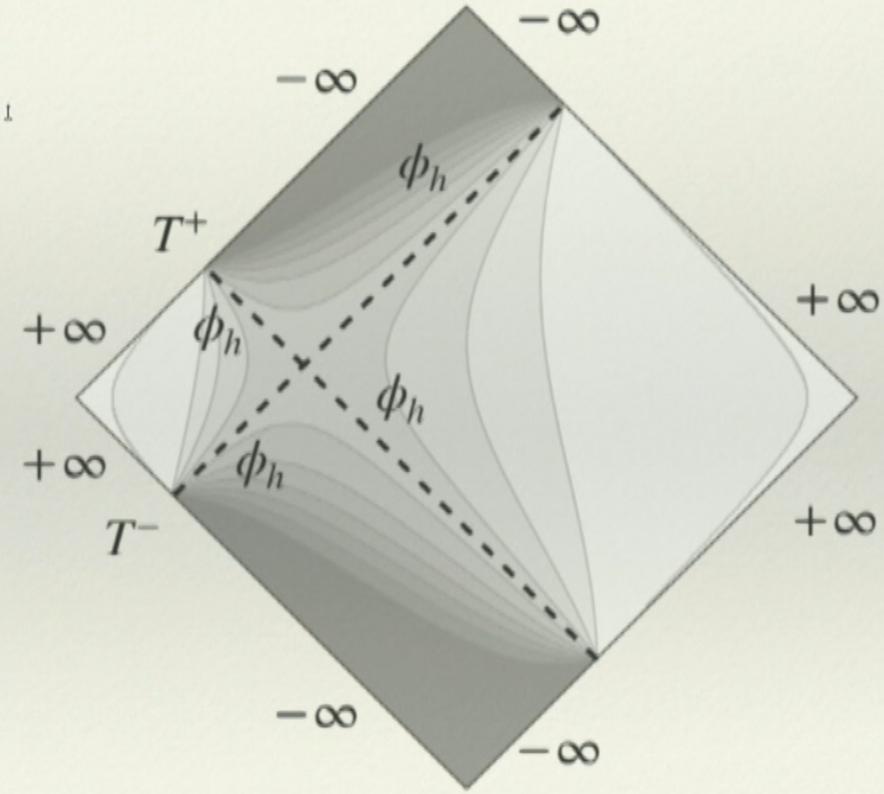
- Einstein-Hilbert: topological.
- Simplest dilaton gravity: $\int d^2x\sqrt{g}(\phi R - 2\Lambda)$.
- Related to CGHS without matter by Weyl rescaling.
- EOM: $R = 0, \quad \nabla_\mu \nabla_\nu \Phi - g_{\mu\nu} \Box \Phi = g_{\mu\nu} \Lambda$.
- Solution on thermal Rindler (disk): $\phi = \frac{\beta \Lambda r}{2\pi} + O(1)$.
- Dirichlet: $\phi \sim \phi_r r \ \to$ temperature is fixed! $\beta = \frac{2\pi\phi_r}{\Lambda}$.
- Non-trivial $Z(\beta)$: make Λ dynamical.
- Integrate in a gauge field to ensure Λ constant on-shell.




$d\rho^2 + \tanh^2\left(\frac{\rho}{L}\right) d\tau^2$
$dr^2 + r^2 d\tau^2$

A theory of Rindler wedges

- Most general solution on Minkowski:
$$\phi = \frac{\Lambda}{2}(U + \cot T_-)(V + \cot T_+) + \phi_h \,.$$
- Four integration constants.
- Impose boundary conditions: $\phi_\partial = \dfrac{\phi_r}{\epsilon}$.
- Allows to locate boundary.



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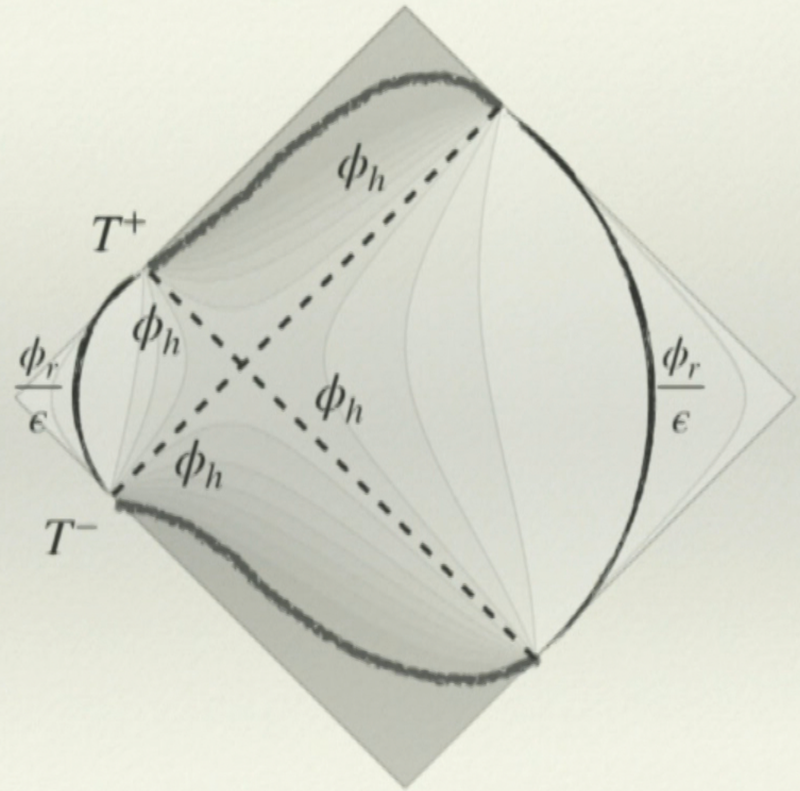
- Solutions = pair of Rindler wedges.
- T_+, T_-: location of bifurcation horizon.
- ϕ_h: horizon value of dilation.
- Λ: acceleration of the boundary.



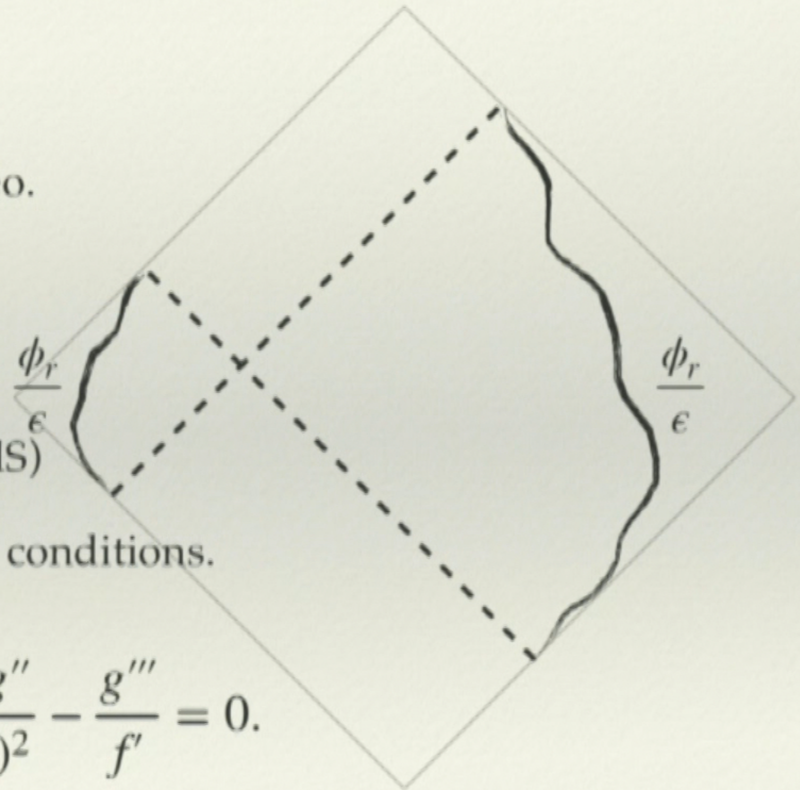
A theory of boundary gravitons

- We look for off-shell configurations.
- $R = 0$ is a *constraint*.
- All configurations are related by (large gauge) diffeo.
- Preserve Bondi gauge:

$$U = f(u), \quad R = \frac{1}{f'(u)}(r + g'(u))\,.$$

- Infinite-dimensional group: BMS_2. (Virasoro in AdS)
- Apply diffeo to general solution, impose boundary conditions.
- Obtain two EOMs:

$$\frac{f''}{f'} + \frac{\Lambda}{\phi_r} = 0, \quad -f'' + \frac{f''g''}{(f')^2} - \frac{g'''}{f'} = 0.$$



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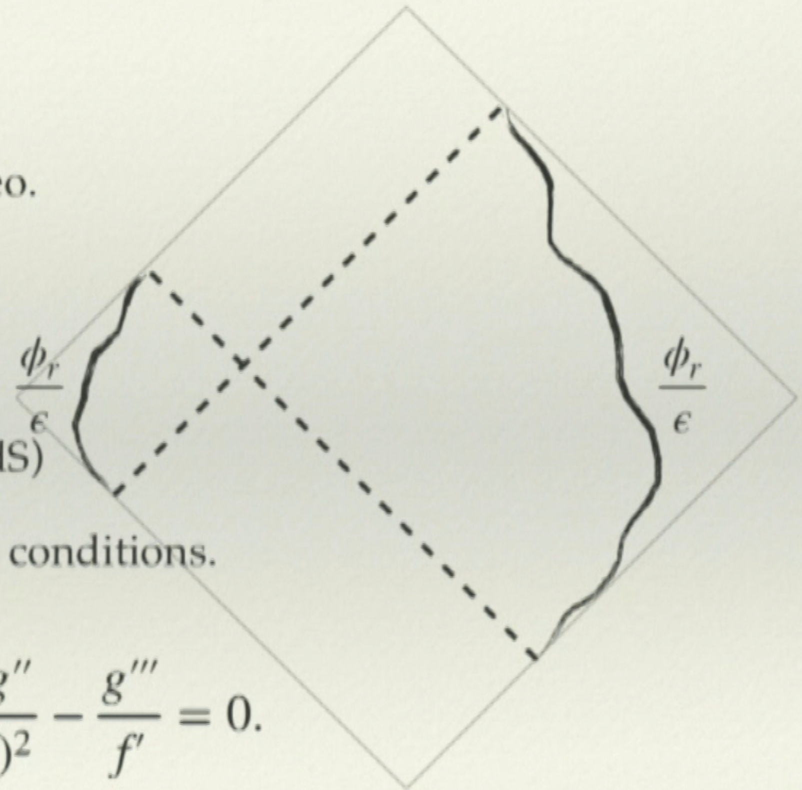
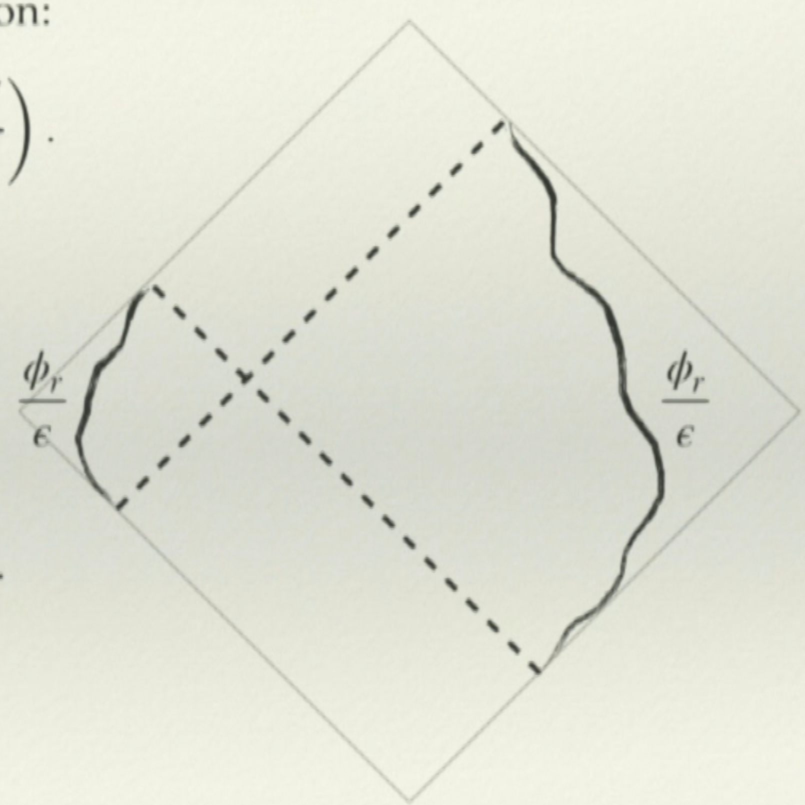


A theory of boundary gravitons

- These two EOMs are reproduced by boundary action:

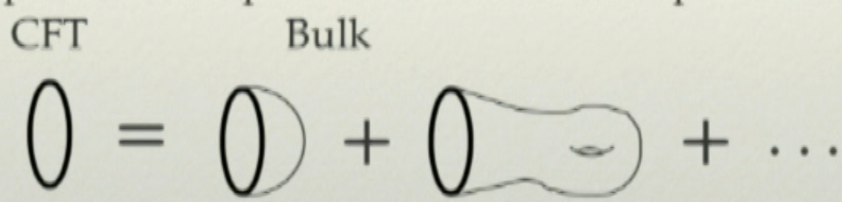
$$I_\partial = \int_\partial \sqrt{h}\left(\phi K - \frac{1}{2}n^\mu \partial_\mu \phi\right) = \phi_r \int du \left(\frac{1}{2}(f')^2 - \frac{g' f''}{f'}\right).$$

- Equivalent of the Schwarzian.
- Coadjoint action of BMS_2.
- Invariant under $ISO(2) \times \mathbb{R}$.
- Controls symmetry breaking $BMS_2 \to ISO(2) \times \mathbb{R}$.
- (f, g) Goldtone modes.
- Bulk term *vanishes* on this configuration space.

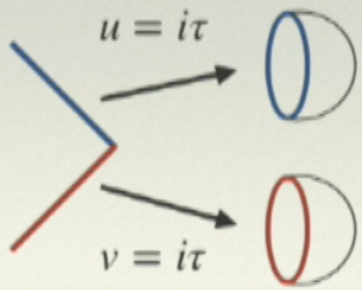


Euclidean path integral: the Bondi Hamiltonian

- Boundary action reformulation allows for exact computations.
- Which observable can we compute in flat space?
- Use AdS/CFT prescription to compute observables: set up boundary path integral and fill it with bulk.
- e.g. $\mathrm{Tr}\, e^{-\beta H} = Z(\beta)$:



- We trust this prescription: analytically continue *boundary time*.



$$= \mathrm{Tr}\, e^{-\beta H_+} = Z_+(\beta)$$

$$= \mathrm{Tr}\, e^{-\beta H_-} = Z_-(\beta)$$

2 different analytic continuations

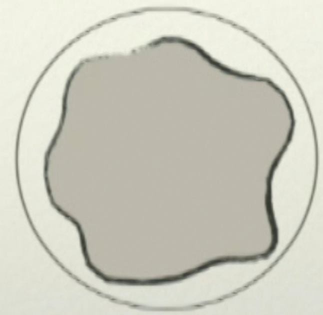
Disk and Cylinders

Boundary modes   One path integral for each boundary

- Path integral reduces to boundary:

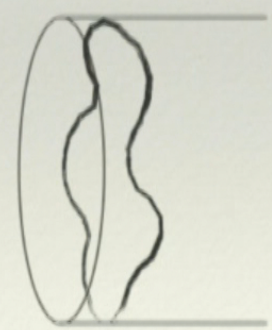
$$Z_{\text{grav}} = \int D\phi Dg DA D\Lambda \, e^{-I_{\text{top}} - I_{\text{CJ}} - I_\partial} = \ldots = \sum_{\text{moduli}}^{1} \int Df Dg \, e^{-I_\partial[f,g]}$$

$\delta(R=0)$ $\delta(F=1)$ Measures Euler χ Vanishes if $R=0$ and $F=1$



Disk: no modulus

- Only 2 allowed geometries with 1 boundary : disk and half-cylinder.
- f, g Goldstone modes belong to $\text{Diff}(S^1) \ltimes C^\infty(S^1)/\text{ISO}(2) \times \mathbb{R}$.
- Coadjoint orbit method, one-loop exact!
- $$Z^{\text{disk}}(\beta) \sim \frac{e^{S_0}}{\beta^2} \;\rightarrow\; \rho_+(E) \sim e^{S_0} E, \qquad Z^{\text{half-cyl}}(\beta, b) \sim \frac{1}{\beta} e^{-\frac{b^2}{2\beta}}$$

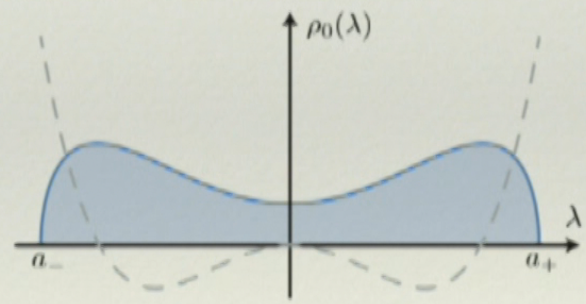


Half-wormhole: circumference b

Non-perturbative completion

- Negativities are cured by non-perturbative corrections:

$$\langle Z_+(\beta)\rangle = \underbrace{\frac{e^{S_0}}{\beta^2}}_{\text{Gravity}} + \underbrace{O(e^{-e^{S_0}})}_{\text{Matrix Model}}, \quad \langle Z_+(\beta_1) Z_+(\beta_2)\rangle_{\text{conn.}} = \frac{1}{\beta_1+\beta_2} + O(e^{-e^{S_0}}), \quad \underbrace{\langle Z_+(\beta_1)\dots Z_+(\beta_n)\rangle_{\text{conn.}} = 0 + O(e^{-e^{S_0}})}_{\text{Non gaussian thanks to corrections}}.$$



- Corrections are generated by Matrix Model.
- Consider double scaled, double cut MM,
- measure: $dM\, e^{-N\mathrm{Tr}V(M)}$, $\quad V(M) = -M^2 + \frac{1}{4}M^4$, M^2 Hermitian.
- ***Central result***: make the following identification

$$\boxed{Z_+(\beta) = \int_{-\infty}^{+\infty} dp \,\mathrm{Tr}\, e^{-\beta(M^2+p^2)}}$$

Non-perturbative completion

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Gravity   Matrix Model   Non gaussian thanks to corrections

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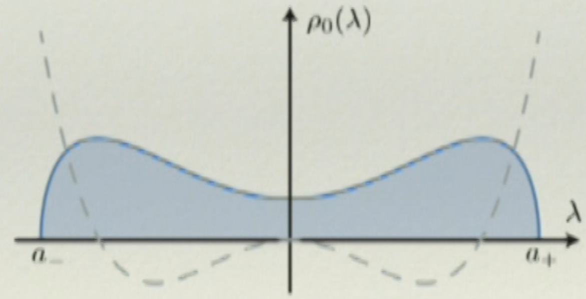

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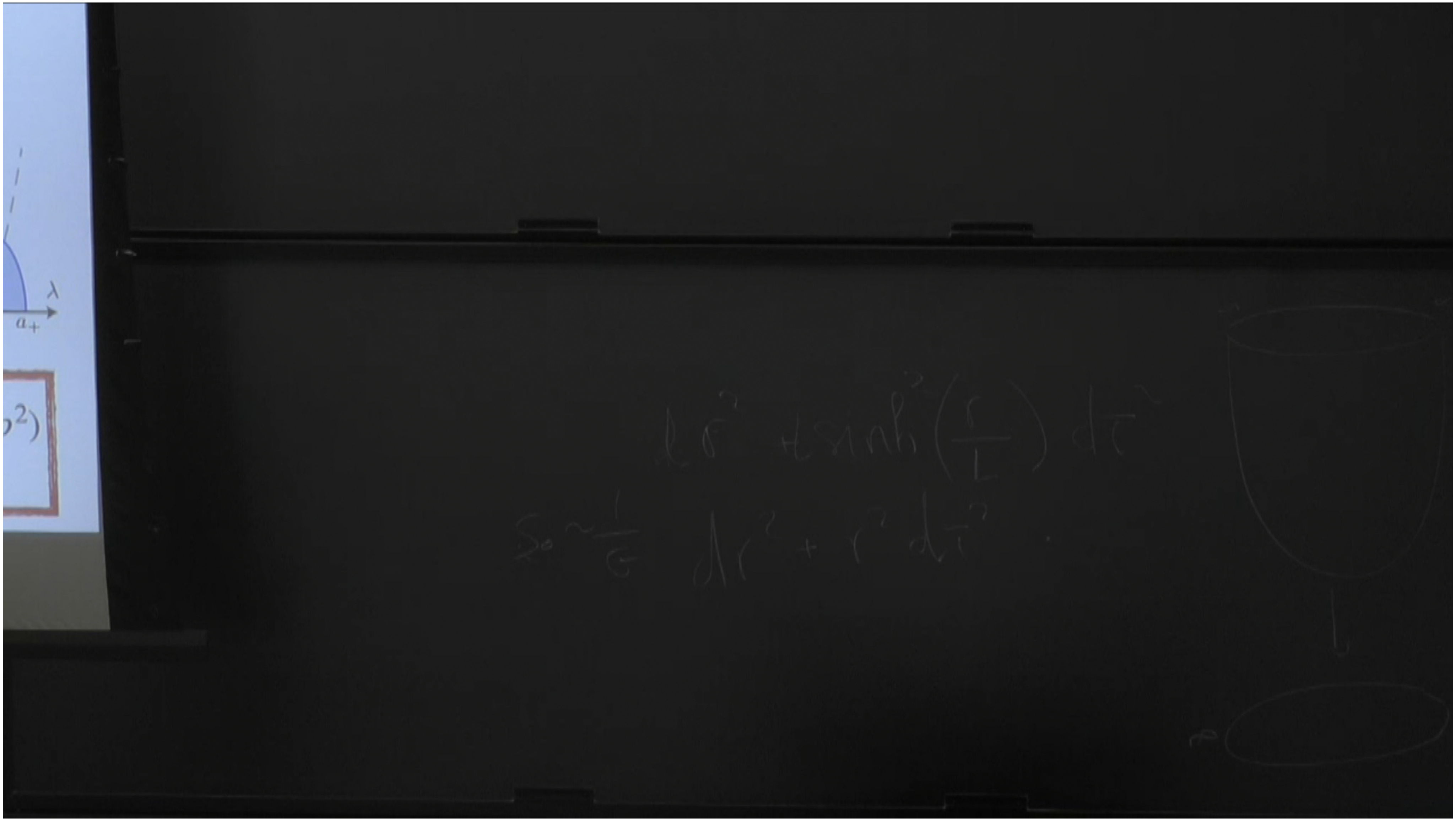
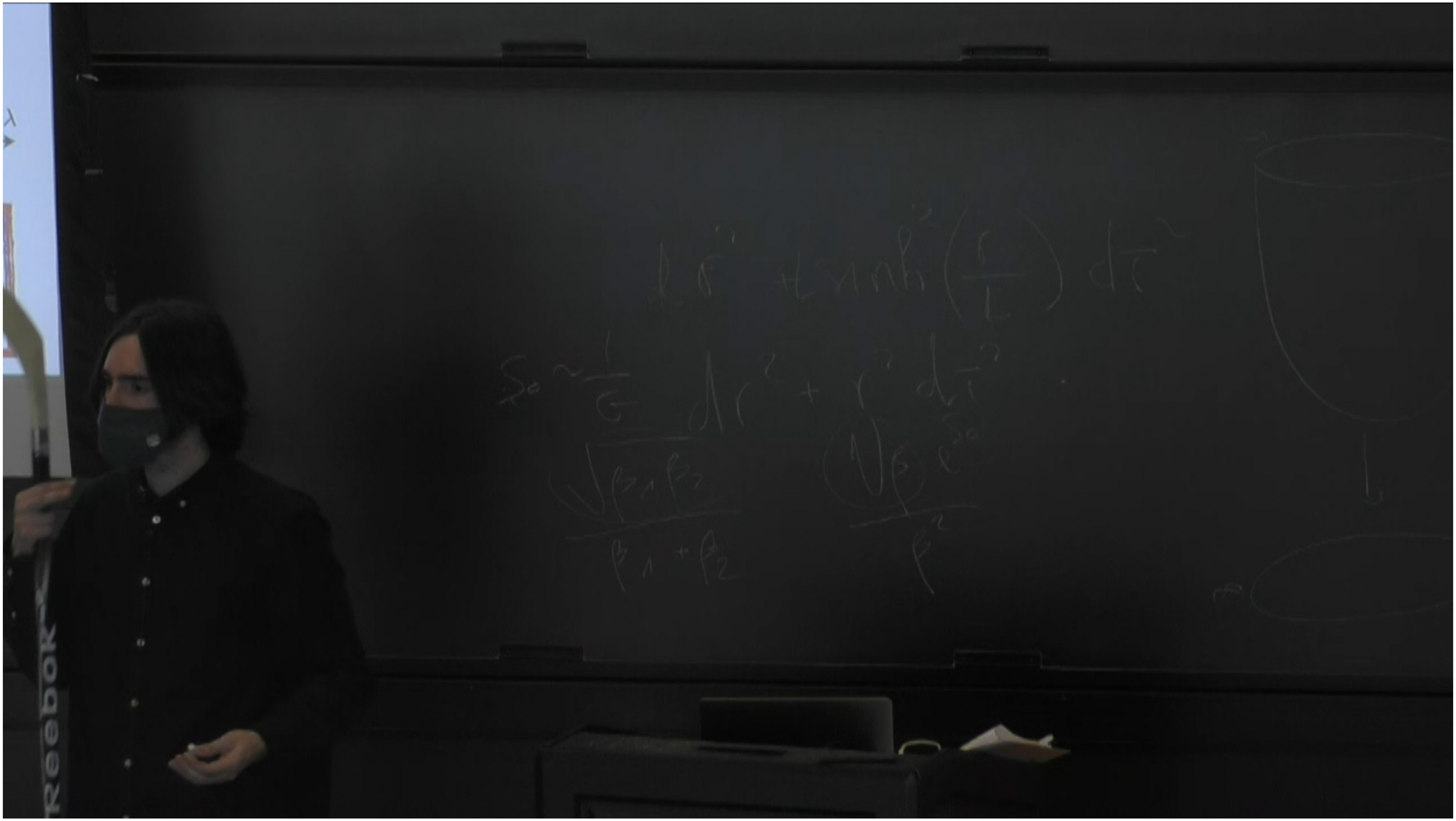
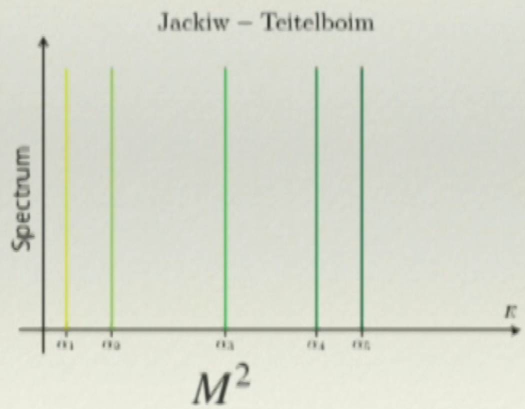


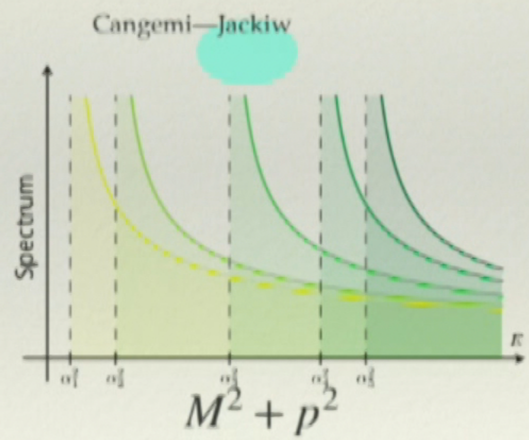

Reebok

Non-perturbative completion

$$Z_+(\beta) = \int_{-\infty}^{+\infty} dp \, \mathrm{Tr}\, e^{-\beta(M^2+p^2)}$$

- Correlators reproduce perturbative (in e^{-S_0}) QG results.
- Hamiltonian: $M^2 + p^2$, contains *discrete* and *continuous* part.
- Continuous part: free 1d particle (density of state $1/\sqrt{E}$), same for all M, not sensitive to the averaging.
- Free particle blurs spectrum: $\longrightarrow$
- Origin of continuous part mysterious.
- Feature of flat space? (infinite volume)
- Non-perturbative density of states.

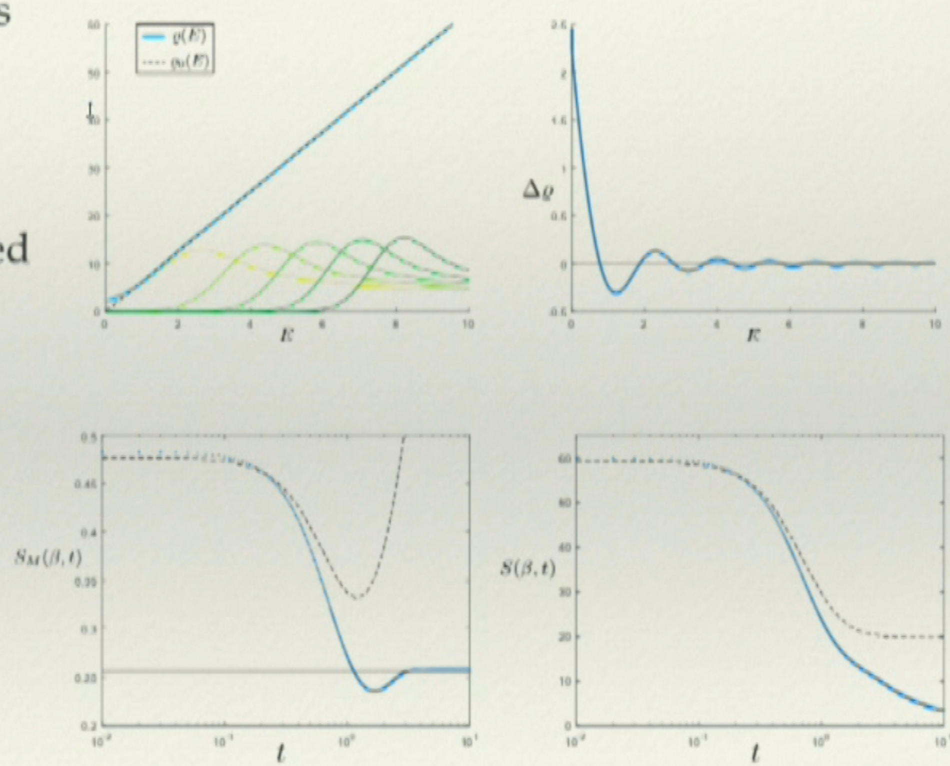




Non-perturbative completion: Results

$$Z_+(\beta) = \int_{-\infty}^{+\infty} dp \, \mathrm{Tr}\, e^{-\beta(M^2+p^2)}$$

- NP density of state: average density of state is approx. linear.
- Small oscillations, period e^{-S_0}.
- Signature of chaos in the spectrum? Diagnosed by behaviour of spectral form factor (SFF).
- MM: $S_M(\beta, t) = \langle \mathrm{Tr}\, e^{-(\beta+it)M^2} \mathrm{Tr}\, e^{-(\beta-it)M^2} \rangle$.
- CJ gravity: $S(\beta, t) = \langle Z_+(\beta+it) Z_+(\beta-it) \rangle$.
- Continuous component erases ramp and plateau.
- No eigenvalue repulsion, no discreteness.



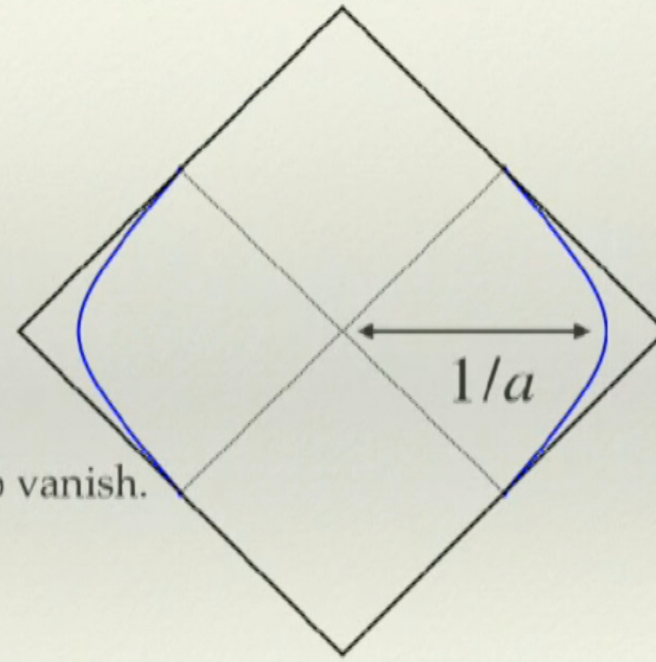
Scrambling in CJ gravity

- Measure how fast a system forgets initial state.
- Diagnosed with OTOC.
- Problem: no holographic dictionary in flat space.
- In AdS, extrapolate dictionary: $\langle O(x_1)O(x_2)\rangle \sim e^{-ML_{\text{geodesic}}(x_1,x_2)}$ for heavy fields.
- We will assume this formula can be imported.
- To compute OTOC, need to couple theory to matter.
- Simplified by mapping to charged particle.

Scrambling: boundary particle

- EOMs of the boundary action reproduced by those of charged particle in constant electric field:

$$\ddot{x}^\mu + a\,F^\mu_{\ \nu}\dot{x}^\nu = 0, \quad x^\mu(a\tau) = \left(f(a\tau), \frac{\varepsilon^{-1} + g'(a\tau)}{f'(a\tau)} \right).$$



- Acceleration: $a = \dfrac{q}{m} = \sqrt{\dfrac{\Lambda}{2\phi_r/\epsilon}}$, implies $a \underset{\epsilon\to 0}{\to} 0$.
- Other boundary: opposite charge.
- Relative placement: Poincaré symmetry is *gauged*, ask total charge to vanish.

$$P^\mu = m\dot{x} + qA^\mu, \quad M^{\mu\nu} = x^\mu(P^\nu - \frac{q}{2}A^\nu) - x^\nu(P^\mu - \frac{q}{2}A^\mu).$$

$$P_L + P_R = 0, \quad M_L + M_R = 0.$$

Page 19 of 23

Scrambling: coupling to matter

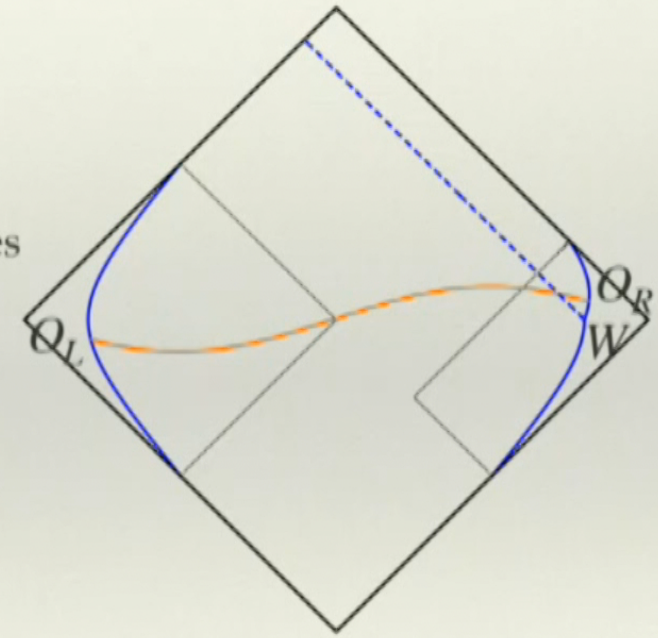
- Diagnose Left/Right correlation at time s after perturbation W:

$$\frac{\langle \Psi | W^{\dagger}(0) O_L(-s) O_R(s) W(0) | \Psi \rangle}{\langle \Psi | W^{\dagger}(0) W(0) | \Psi \rangle} \sim e^{-ML_{\text{pert}}(s)}.$$

- Need to compute backreaction: operator W inserts shock who carries Poincaré charges.
- Boundary particle is kicked:

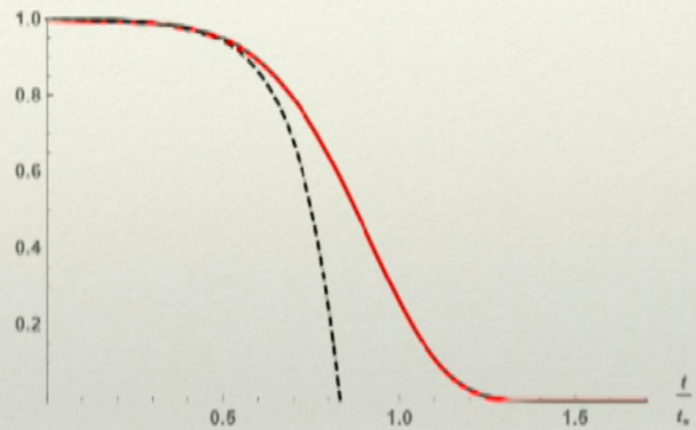
$$P_L + P_{R,\text{pert}} + P_{\text{shock}} = 0, \quad M_L + M_{R,\text{pert}} + M_{\text{shock}} = 0.$$

- In boundary particle language: emission of massless particle.
- 2d equivalent of the vacuum transition in 4d.



Scrambling in flat space

- We find $\delta E \ll m$: $\langle O_L(-s) O_R(s) \rangle_W \sim \left(1 - \frac{\delta E}{2\pi m}\left(e^{\frac{2\pi s}{\beta}} - 1\right)\right)^{M\beta}$.
- Scrambling time: $t_\star = \beta \log m/\delta E$.
- After replacing with CJ parameters: $t_\star = \beta \log\left(\frac{\phi_r}{\epsilon} - \phi_h\right)$.



- Scrambling time divergent: consistent with flat space, it takes ∞ *boundary time* to reach horizon (not true in AdS).
- Argument of log is not $S_{\text{grav}} = \phi_h$, instead S_{grav} is a correction to universal divergent piece.

Conclusion

- Define a 2d flat space model of gravity: *CJ gravity*
- Non trivial partition function.
- Admits a reformulation in terms of *boundary gravitons*: flat space version of the Schwarzian.
- Dynamics controlled by symmetry breaking $\mathrm{BMS}_2 \to \mathrm{ISO}(2) \times \mathbb{R}$.
- *Conjecture*: analytic continuation $u = i\tau$, $\tau \sim \tau + \beta$, of path integral computes $Z_+(\beta) = \mathrm{Tr}\, e^{-\beta H_+}$.
- Wormhole contribution → duality with ensemble: $Z_{\text{grav}}(\beta_1, \beta_2) = \langle Z_+(\beta_1) Z_+(\beta_2) \rangle$.
- Correlations non-pertubatively completed by matching with operator in a Matrix Model.
- Trace contains mysterious continuous part that *destroys signatures of long-term chaos.*
- Short-term chaos: infinite scrambling time (becomes finite at finite cutoff).
- Gravitational entropy plays a different role in formula for scrambling time than in AdS.