Title: AdS/CFT

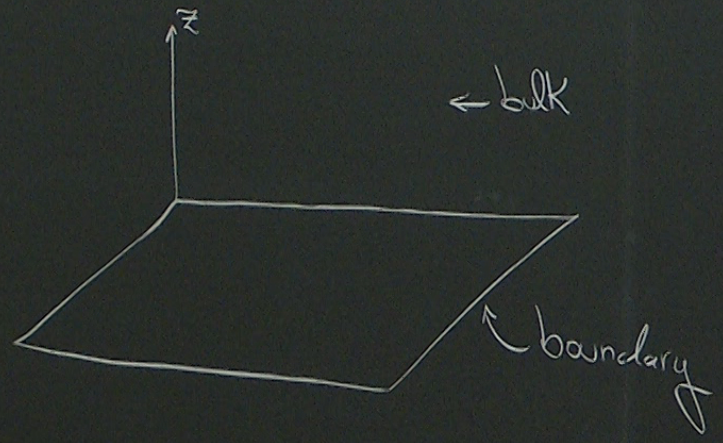
Speakers: Pedro Vieira, Gang Xu

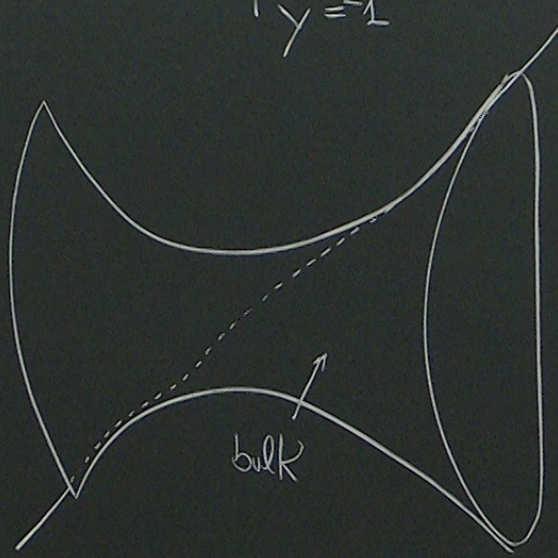
Collection: AdS/CFT 2021/2022

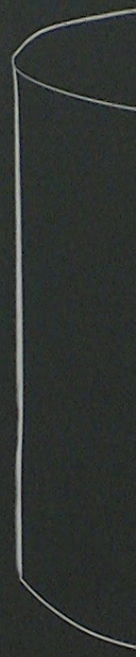
Date: March 25, 2022 - 2:00 PM

URL: https://pirsa.org/22030091

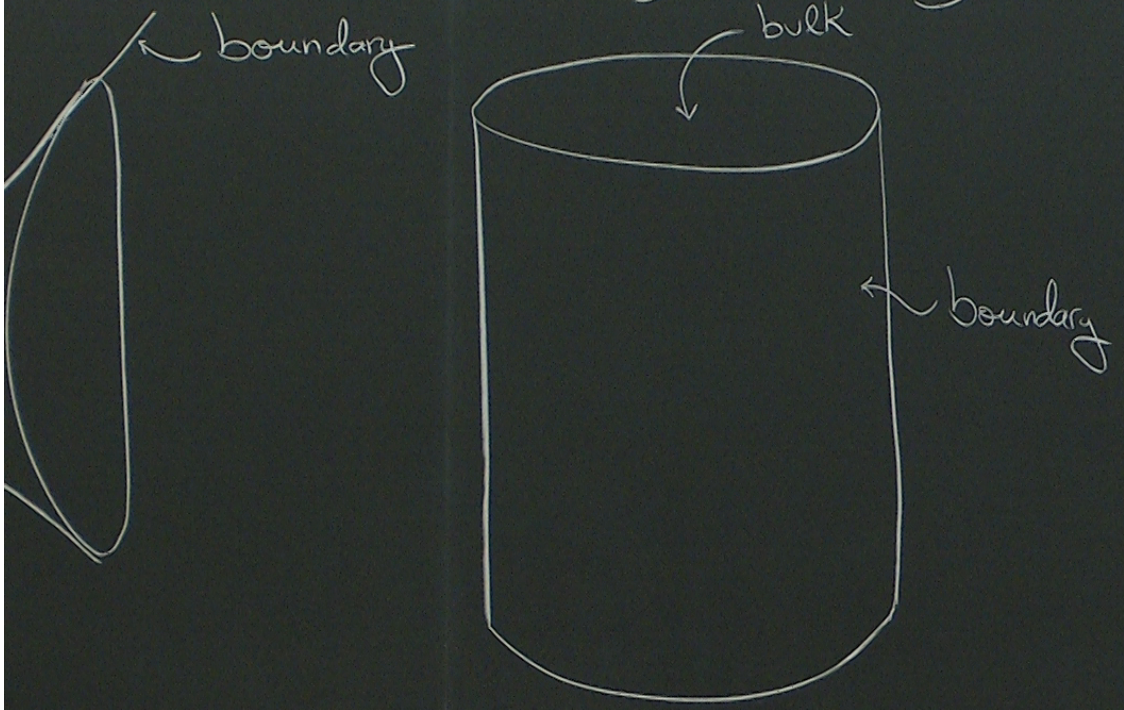
$$AdS = \frac{dz^2 + dx_\mu dx^\mu}{z^2} = d\vec{y} \cdot d\vec{y}\Big|_{\vec{y}^2 = -1} = -\cosh^2 \rho \, dt^2 +$$

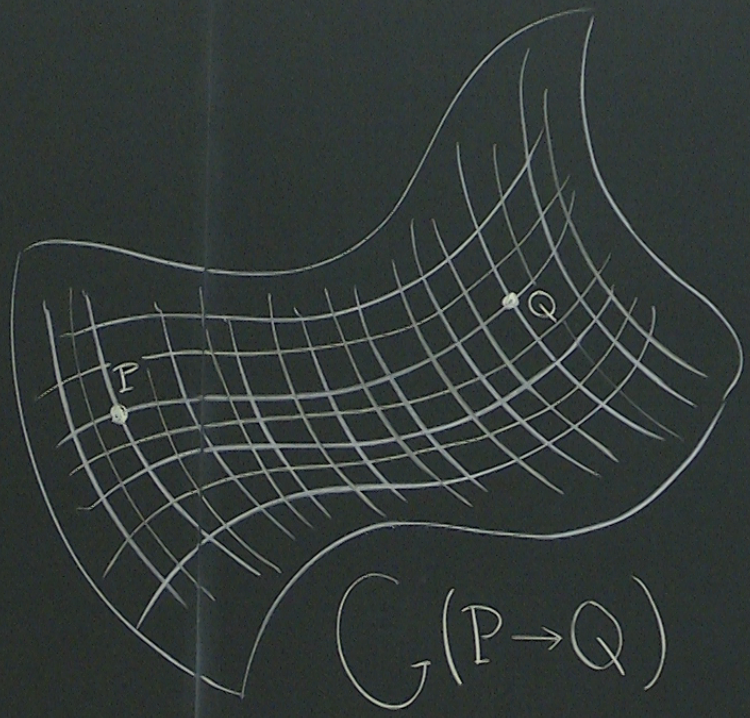


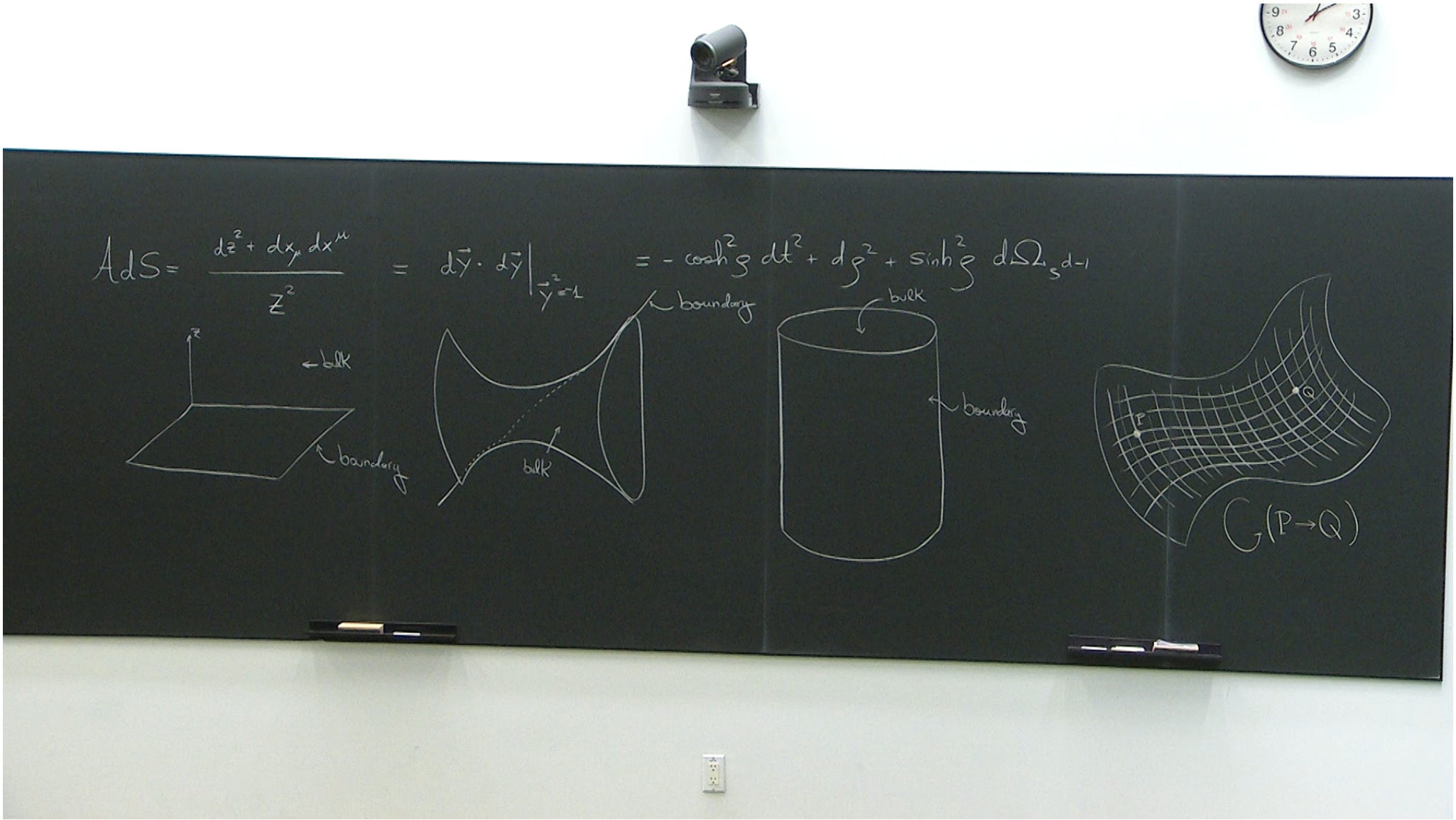


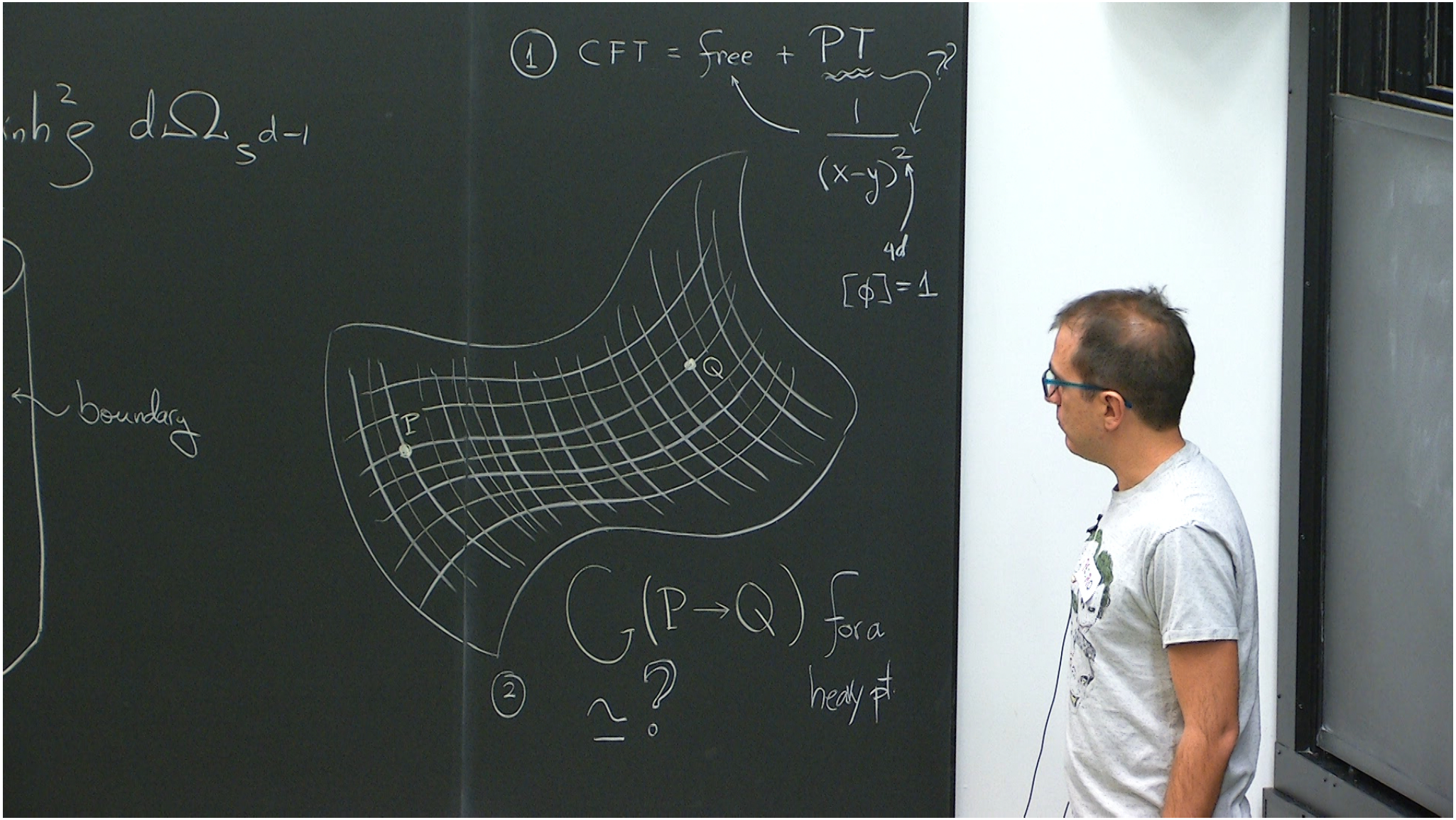


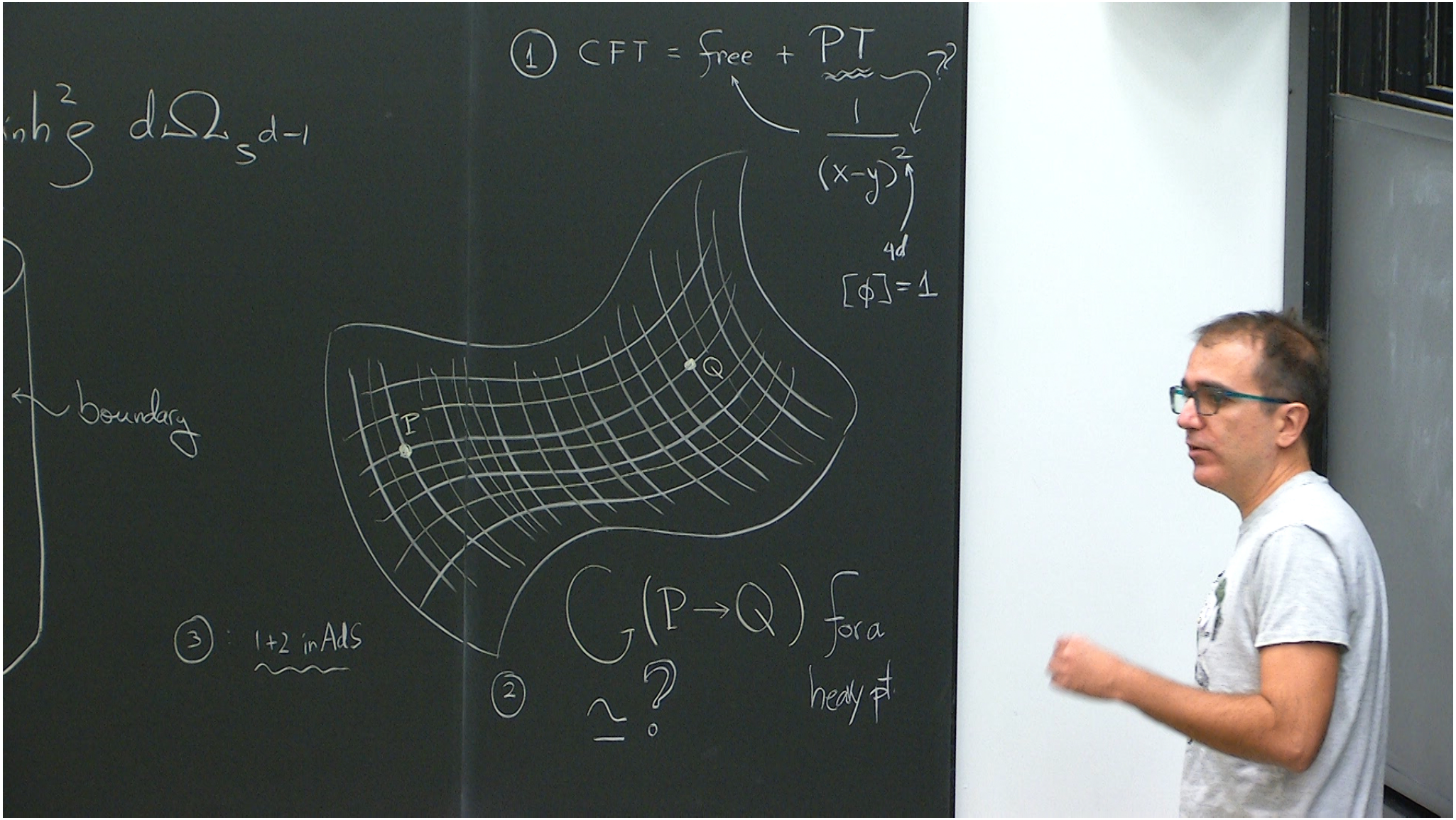
$$= -\cosh^2\rho\, dt^2 + d\rho^2 + \sinh^2\rho\, d\Omega_{S^{d-1}}$$

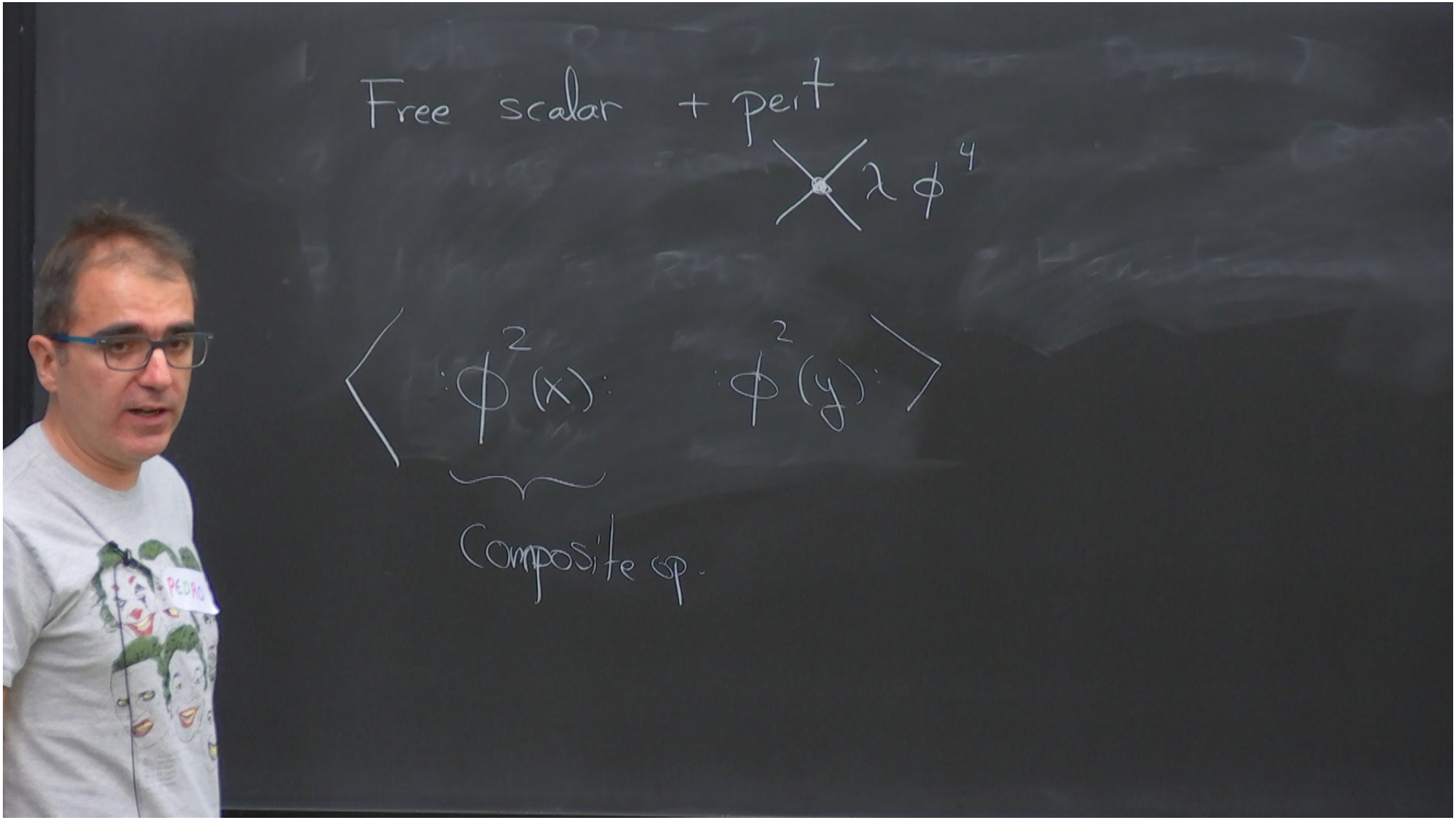





$AdS = \frac{dz^2 + dx_\mu dx^\mu}{z^2} = d\vec{y} \cdot d\vec{y}\Big|_{\vec{y}^2=1} = -\cosh^2\rho\, dt^2 + d\rho^2 + \sinh^2\rho\, d\Omega_{S^{d-1}}$
z
bulk
boundary
boundary
bulk
bulk
boundary
P
Q
$G(P \to Q)$

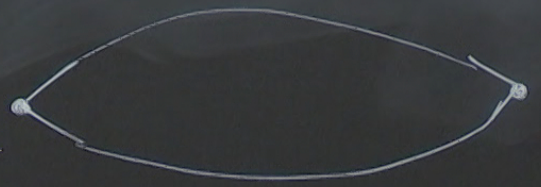

① CFT = free + PT
??
$\frac{1}{(x-y)^2}$
4d
$[\phi] = 1$
$\sinh^2 \rho \; d\Omega_{S^{d-1}}$
boundary
P
Q
$G(P \to Q)$ for a heavy pt.
②
$\sim$?


① CFT = free + PT
??
$\frac{1}{(x-y)^2}$
4d
$[\phi] = 1$
$\sinh^2\rho \; d\Omega_{S^{d-1}}$
boundary
P
Q
③ 1+2 in AdS
② $G(P \to Q)$ for a heavy pt.
$\sim$?


Free scalar + pert
$\lambda\phi^4$
$\langle :\phi^2(x): \quad :\phi^2(y): \rangle$
Composite op.

Free scalar + pert in 4d

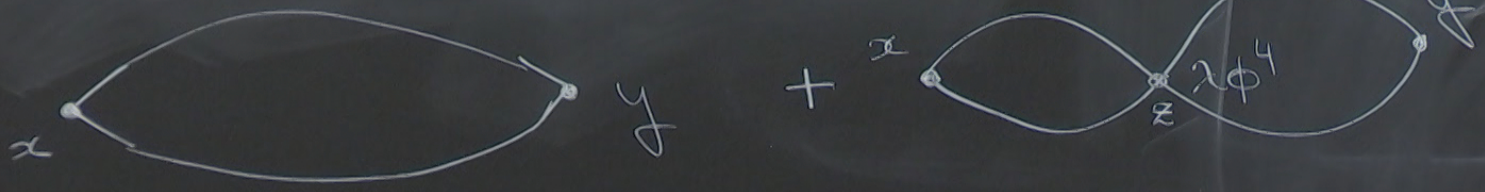
$\lambda\phi^4$

$\langle :\phi^2(x): \; :\phi^2(y): \rangle =$ 

$= \dfrac{1}{(x-y)^4}$

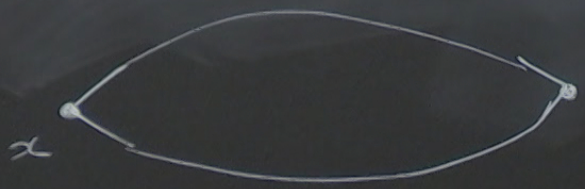
Composite op.
has classical dim $\Delta_{\mathcal{O}} = 2$

4d

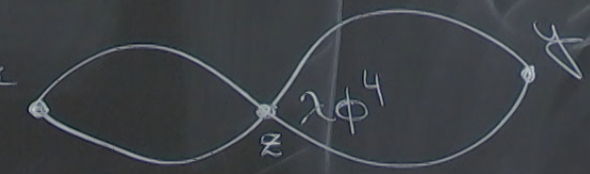


$$= \frac{1}{(x-y)^4}$$

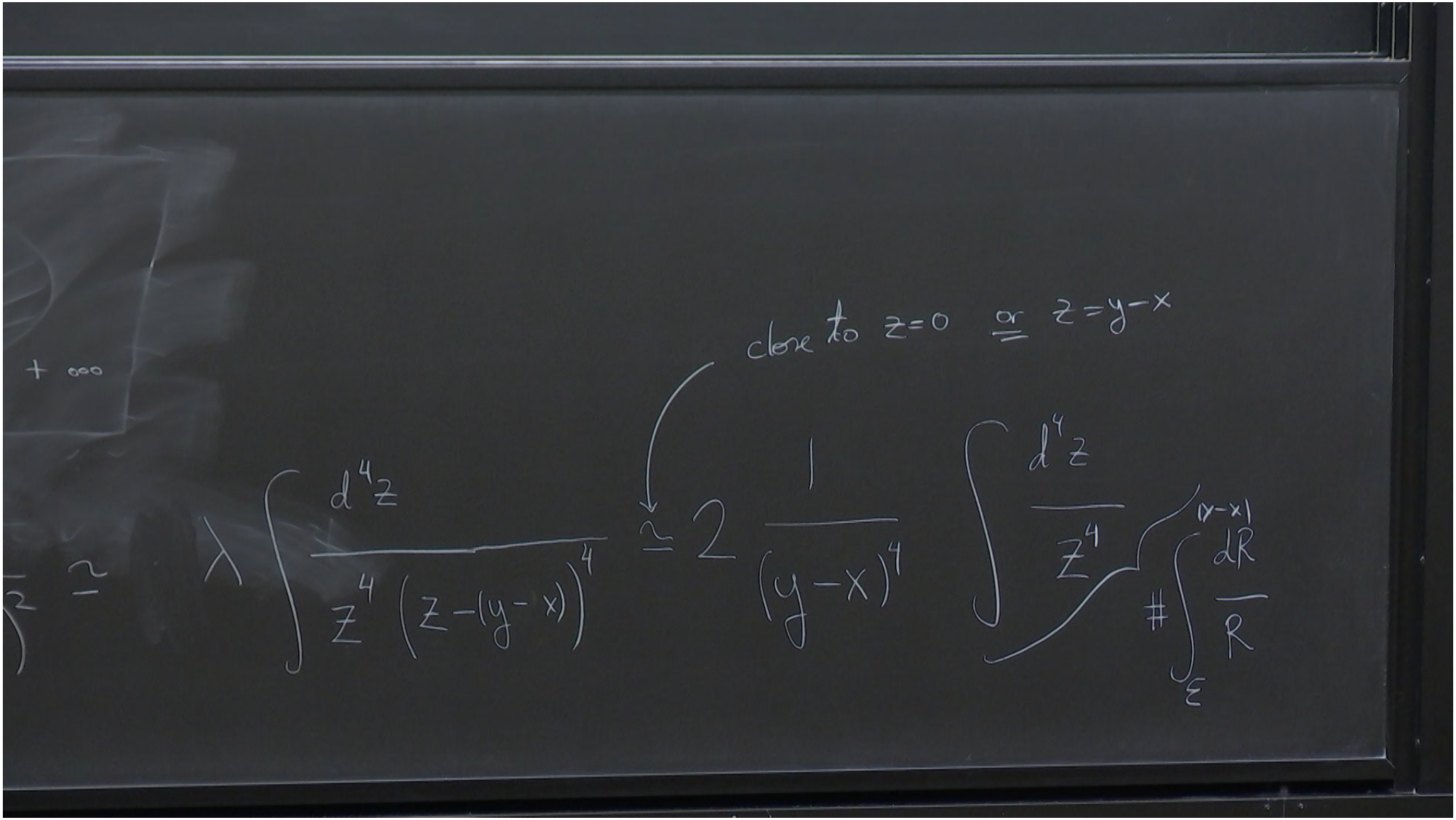
4d



$$= \frac{1}{(x-y)^4}$$



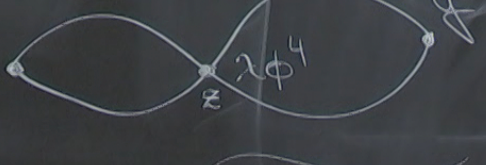
$$+ \lambda \int \frac{d^4 z}{((x-z)^2)^2 ((z-y)^2)^2} + \cdots$$


close to z=0 or z=y-x

t in 4d

$\lambda\phi^4$

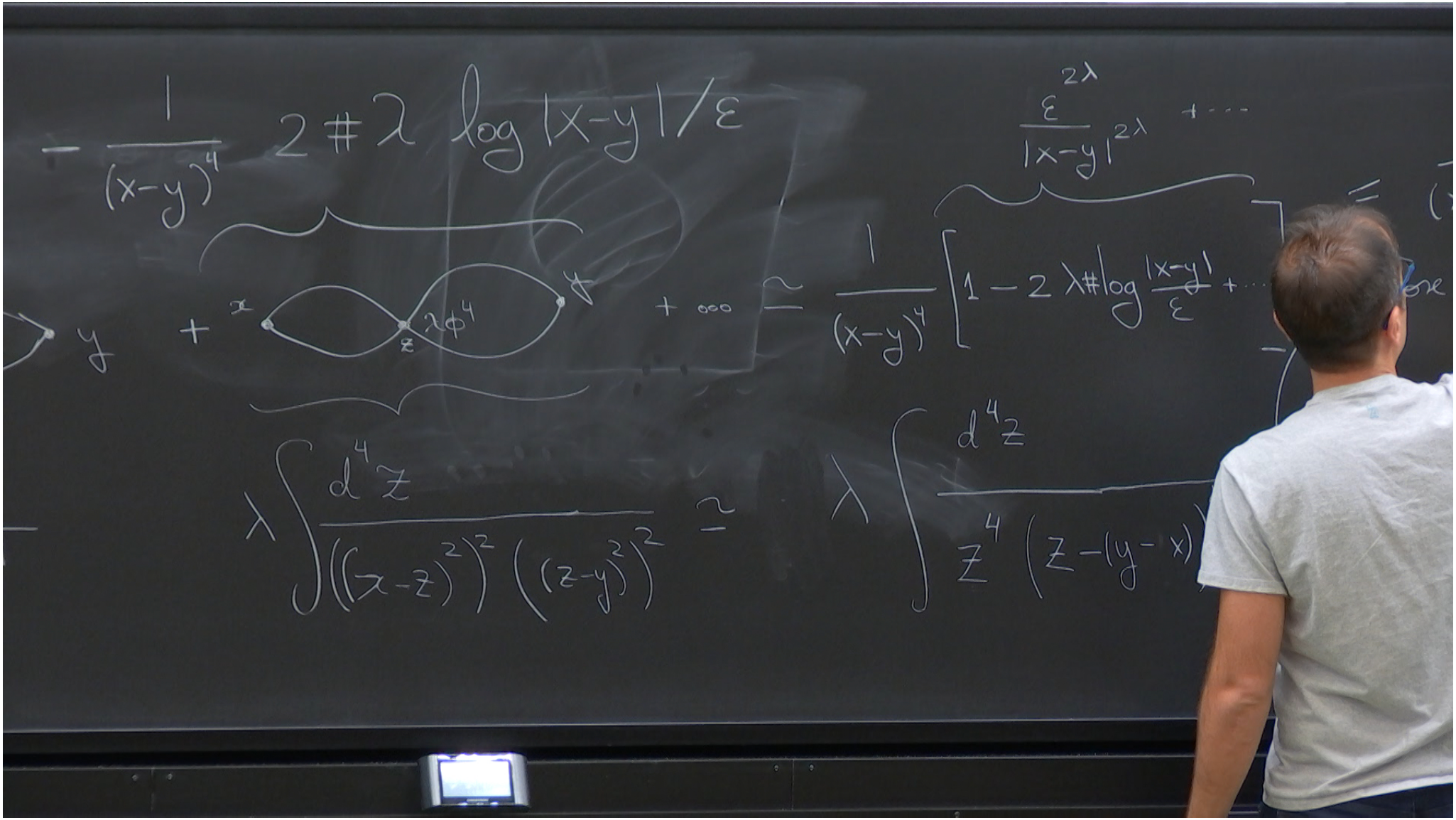
$-2\#\lambda \log|x-y|/\varepsilon$

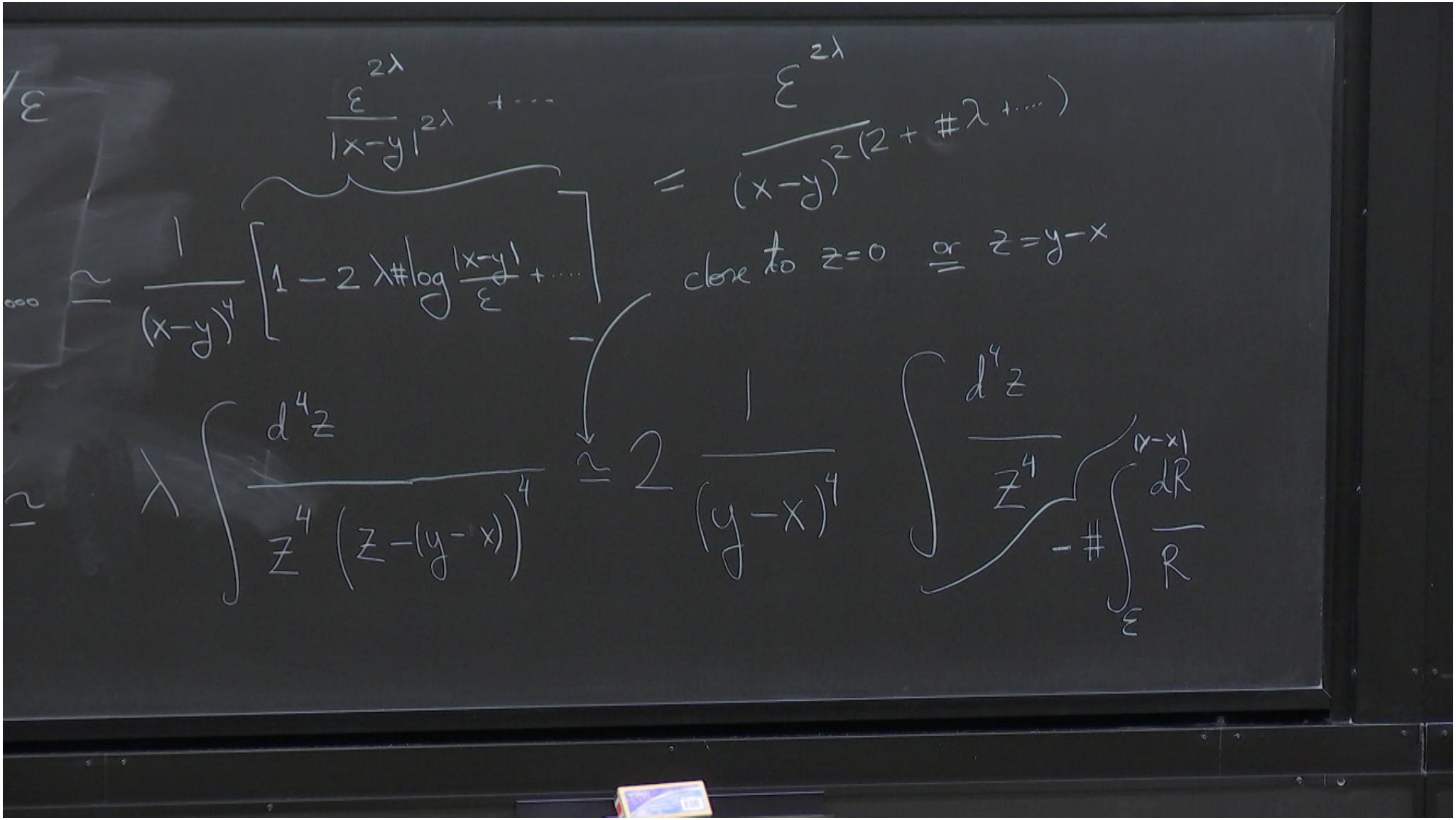
$\langle y|x\rangle =$  $+$  $+ \cdots$

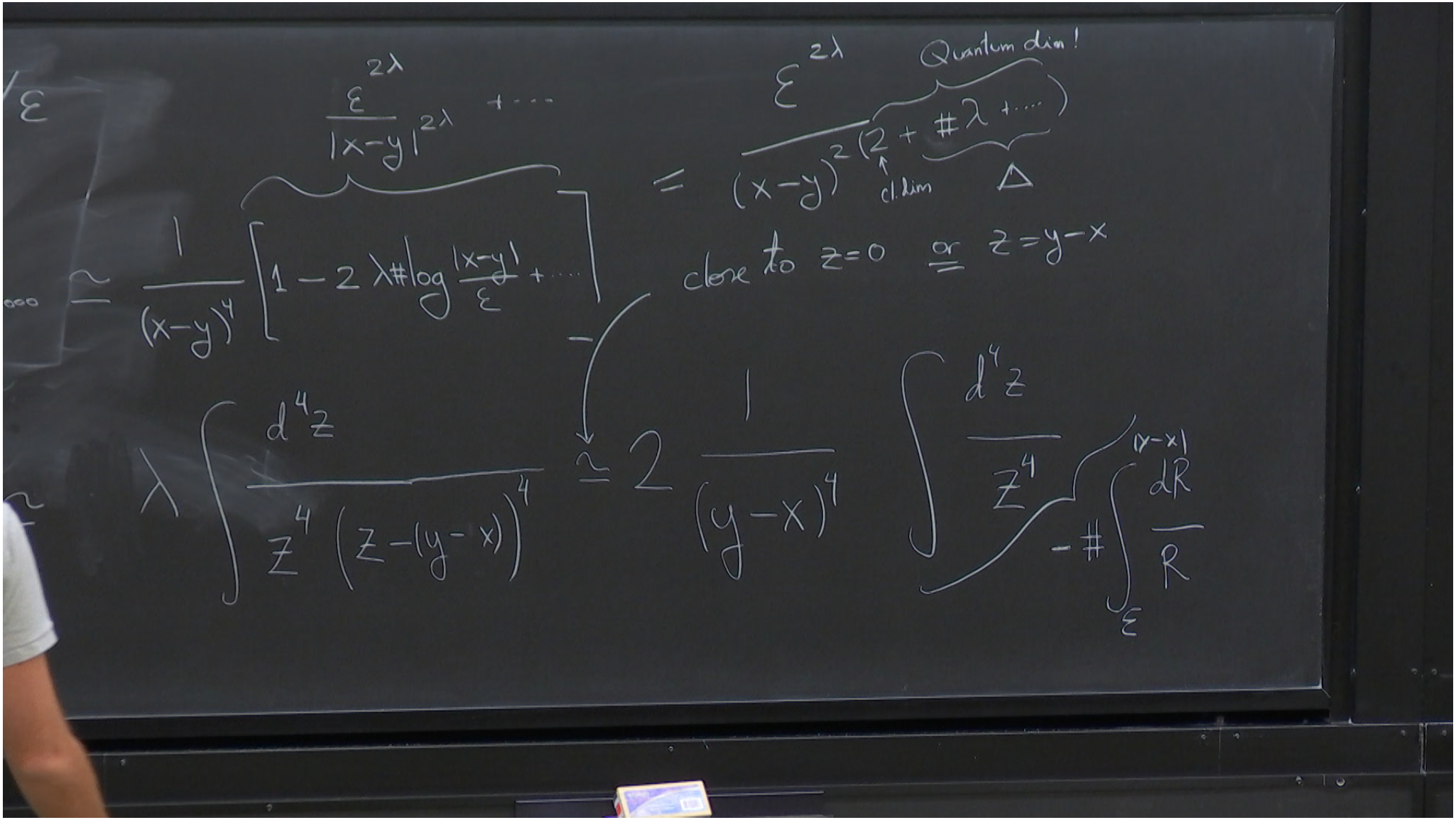
$\Delta_{\mathcal{O}}=2$

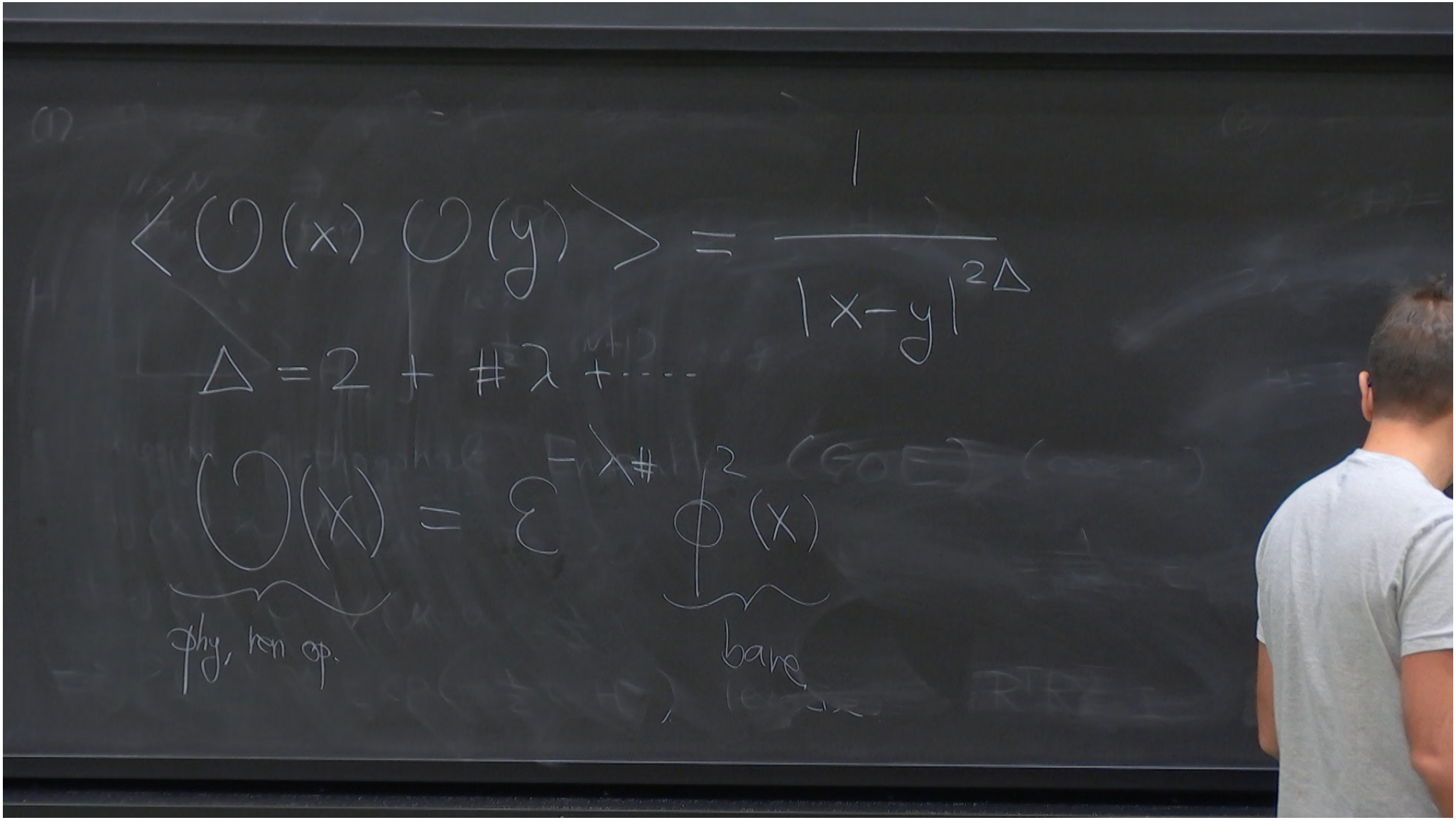
$$\frac{1}{(x-y)^4}$$

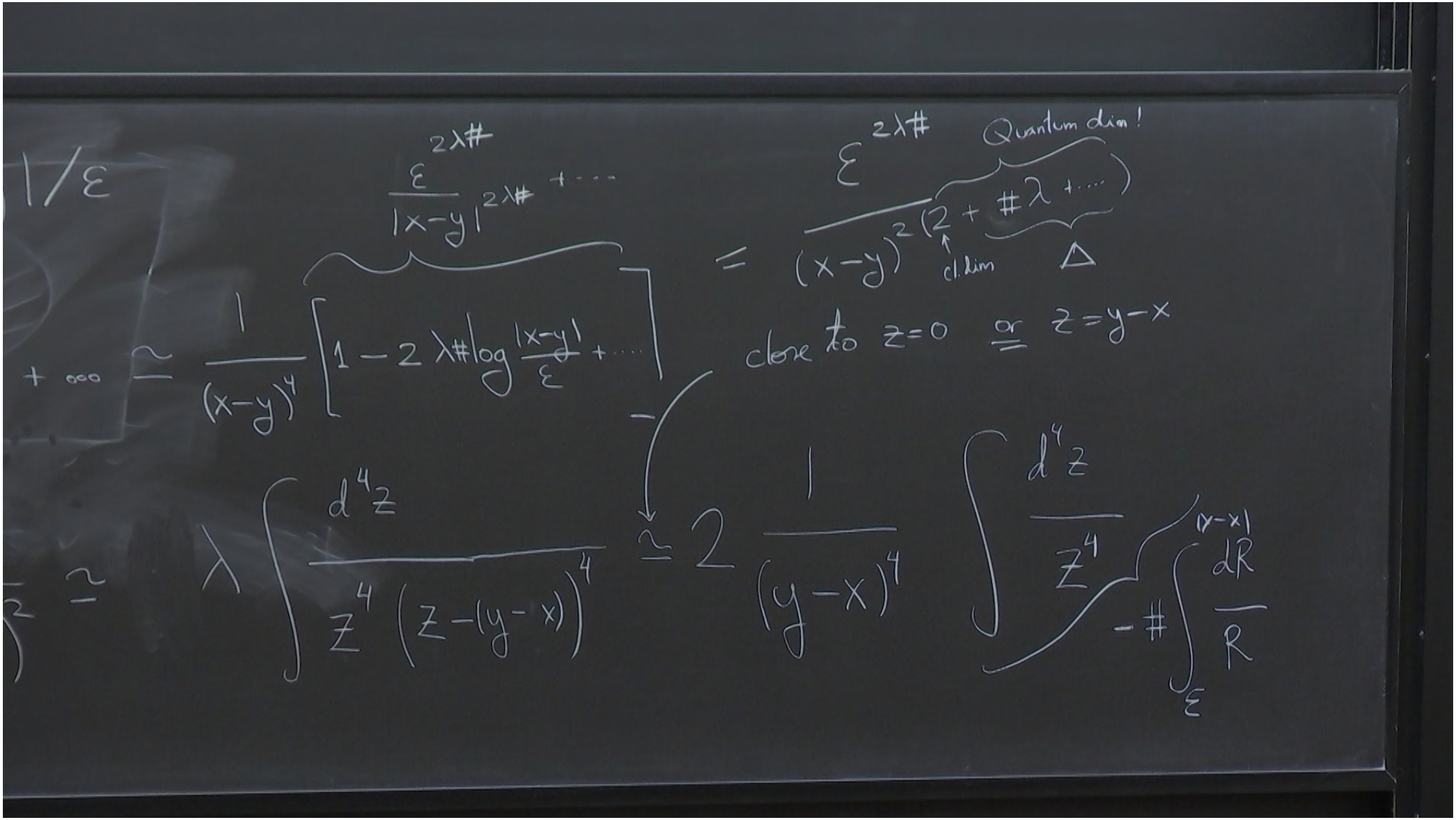
$$\lambda\int\frac{d^4z}{((x-z)^2)^2((z-y)^2)^2} = \lambda\int$$

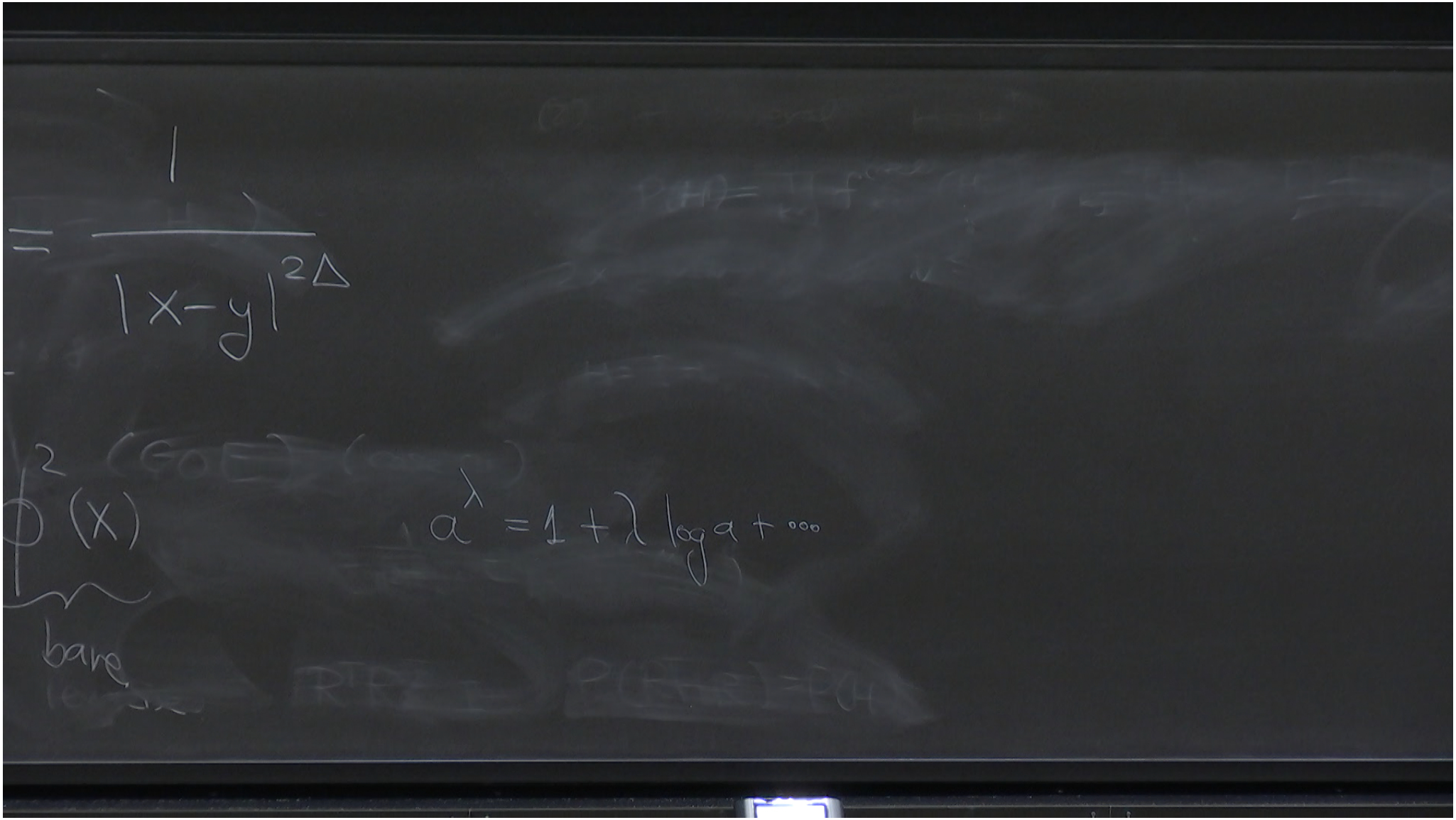

$-\frac{1}{(x-y)^4}\, 2\#\lambda \log|x-y|/\varepsilon$
$\frac{\varepsilon^{2\lambda}}{|x-y|^{2\lambda}} + \cdots$
$y + x$
z
$\lambda\phi^4$
$+ \cdots \simeq \frac{1}{(x-y)^4}\left[1 - 2\lambda\#\log\frac{|x-y|}{\varepsilon} + \cdots\right.$
$\lambda \int \frac{d^4z}{((x-z)^2)^2\,((z-y)^2)^2} \simeq \lambda \int \frac{d^4z}{z^4\,(z-(y-x)$

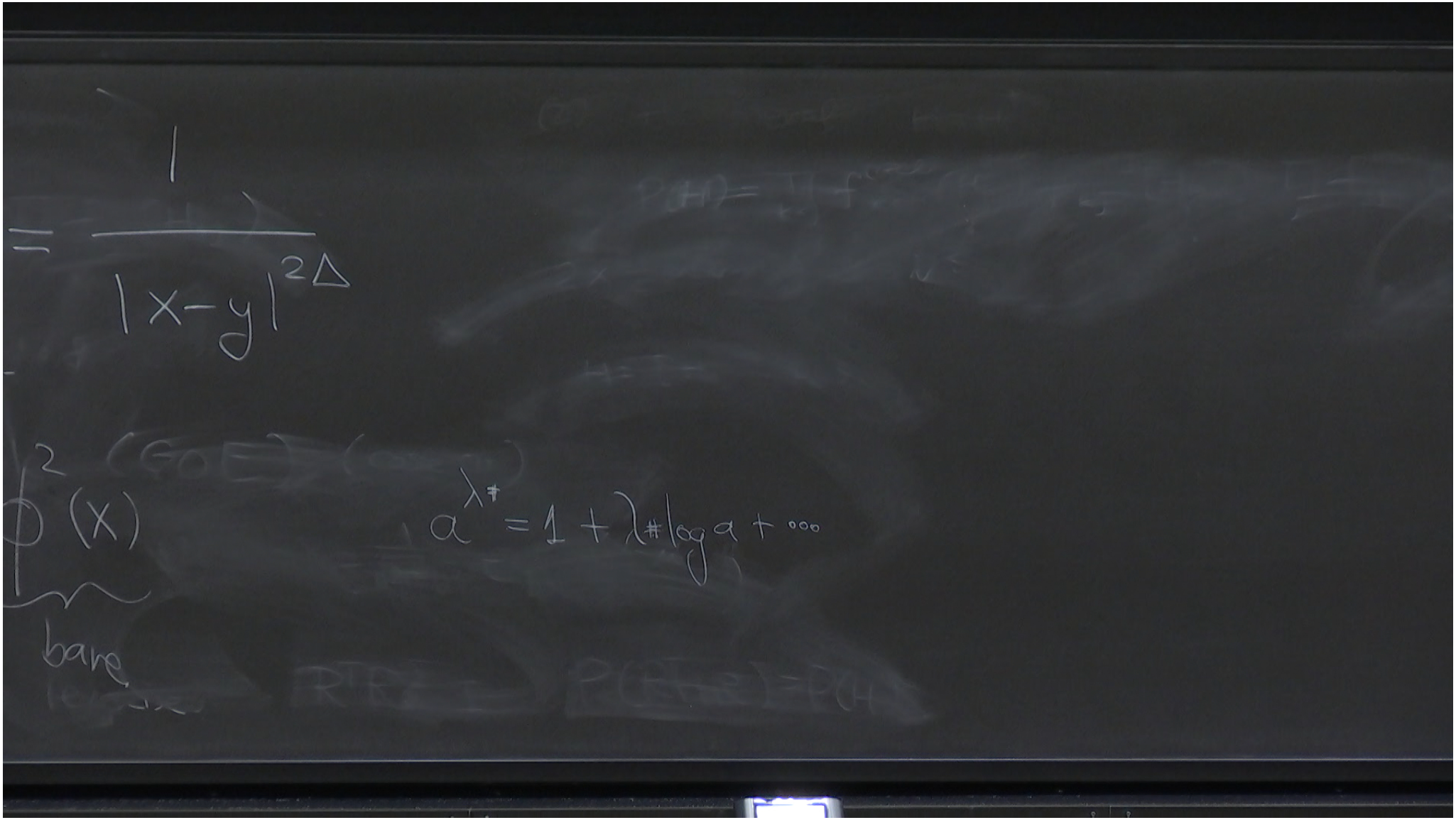

$\frac{\varepsilon^{2\lambda}}{|x-y|^{2\lambda}} + \cdots$
$= \frac{\varepsilon^{2\lambda}}{(x-y)^2 (2 + \#\lambda + \cdots)}$
$\simeq \frac{1}{(x-y)^4}\left[1 - 2\lambda \# \log \frac{|x-y|}{\varepsilon} + \cdots\right]$
close to $z=0$ or $z = y-x$
$\simeq \lambda \int \frac{d^4 z}{z^4 (z-(y-x))^4} \simeq 2 \frac{1}{(y-x)^4} \int \frac{d^4 z}{z^4} - \# \int_{\varepsilon}^{|y-x|} \frac{dR}{R}$

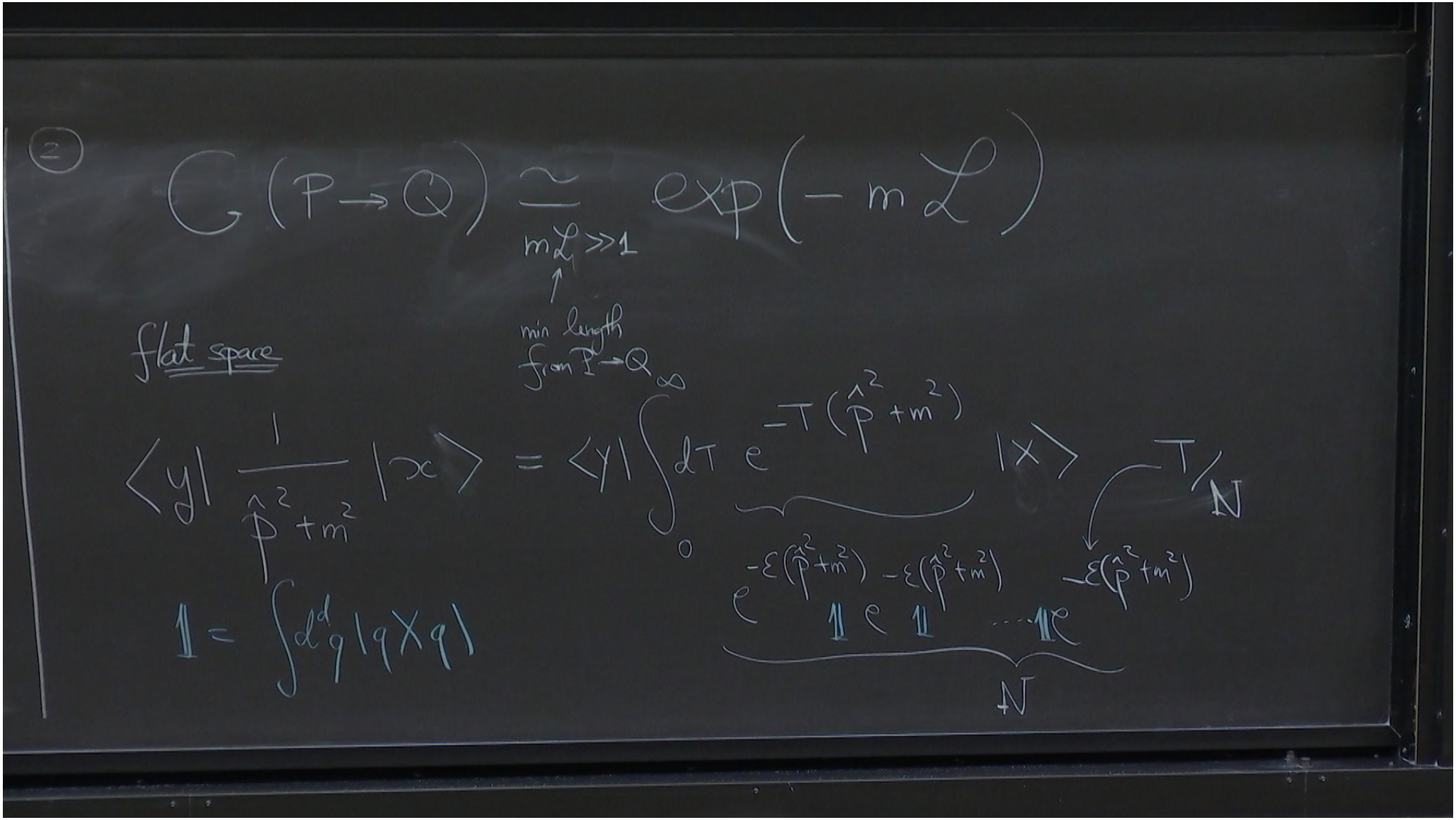

Quantum dim!
cl. dim
close to z=0 or z=y-x


$\langle \mathcal{O}(x)\,\mathcal{O}(y) \rangle = \frac{1}{|x-y|^{2\Delta}}$
$\Delta = 2 + \#\lambda + \cdots$
$\mathcal{O}(x) = \varepsilon^{-\lambda\#} \phi^2(x)$
phy. non op.
bare


1/ε
+ ...
≃
$\frac{\varepsilon^{2\lambda\#}}{|x-y|^{2\lambda\#}} + \cdots$
$= \frac{\varepsilon^{2\lambda\#}}{(x-y)^2}\left(2 + \#\lambda + \cdots\right)$
Quantum dim!
cl. dim
Δ
$\frac{1}{(x-y)^4}\left[1 - 2\lambda\# \log\frac{|x-y|}{\varepsilon} + \cdots\right]$
close to z=0 or z=y-x
$\lambda \int \frac{d^4z}{z^4\,(z-(y-x))^4}$
$\simeq 2\,\frac{1}{(y-x)^4}$
$\int \frac{d^4z}{z^4}$
$-\#\int_{\varepsilon}^{|x-y|} \frac{dR}{R}$


$= \frac{1}{|x-y|^{2\Delta}}$
$\phi^2(x)$
$a^\lambda = 1 + \lambda \log a + \cdots$
bare


$= \frac{1}{|x-y|^{2\Delta}}$
$\phi^2(x)$
$a^{\lambda_\#} = 1 + \lambda_\# \log a + \cdots$
bare


2
$G(P \to Q) \simeq \exp(-m\mathcal{L})$
$m\mathcal{L} \gg 1$
min length from $P \to Q$
flat space
$\langle y| \frac{1}{\hat{p}^2 + m^2} |x\rangle = \langle y| \int_0^\infty dT\, e^{-T(\hat{p}^2+m^2)} |x\rangle$
$T = N\varepsilon$
$\mathbb{1} = \int d^d q\, |q\rangle\langle q|$
$e^{-\varepsilon(\hat{p}^2+m^2)} \mathbb{1}\, e^{-\varepsilon(\hat{p}^2+m^2)} \mathbb{1} \cdots \mathbb{1}\, e^{-\varepsilon(\hat{p}^2+m^2)}$
N

(2)

$$G(P \to Q) \underset{m\mathcal{L} \gg 1}{\simeq} \exp(-m\mathcal{L})$$

$m\mathcal{L}$: min length from $P \to Q$

flat space

$$\langle y| \frac{1}{\hat{p}^2 + m^2} |x\rangle = \langle y| \int_0^\infty dT \, e^{-T(\hat{p}^2 + m^2)} |x\rangle$$

T/N

$$e^{-\varepsilon(\hat{p}^2+m^2)} \, e^{-\varepsilon(\hat{p}^2+m^2)} \cdots e^{-\varepsilon(\hat{p}^2+m^2)}$$

$$\mathbb{1}_1 \, e \, \mathbb{1}_2 \cdots \mathbb{1}_N \qquad N$$

$$\mathbb{1}_1 = \int d^d q_1 |q_1\rangle\langle q_1| \text{ etc}$$

$$\int_0^\infty dT \prod dx_j \langle y | e^{-\varepsilon(\hat{p}^2+m^2)} | x_1 \rangle \langle x_1 | e^{-\varepsilon(\hat{p}^2+m^2)} | x_2 \rangle \langle x_2 | \dots$$

$$\langle x_j | e^{-\varepsilon(\hat{p}^2+m^2)} | x_{j+1} \rangle = \int$$

$$1 = \int |p_j\rangle \langle p_j| \, dp_j$$

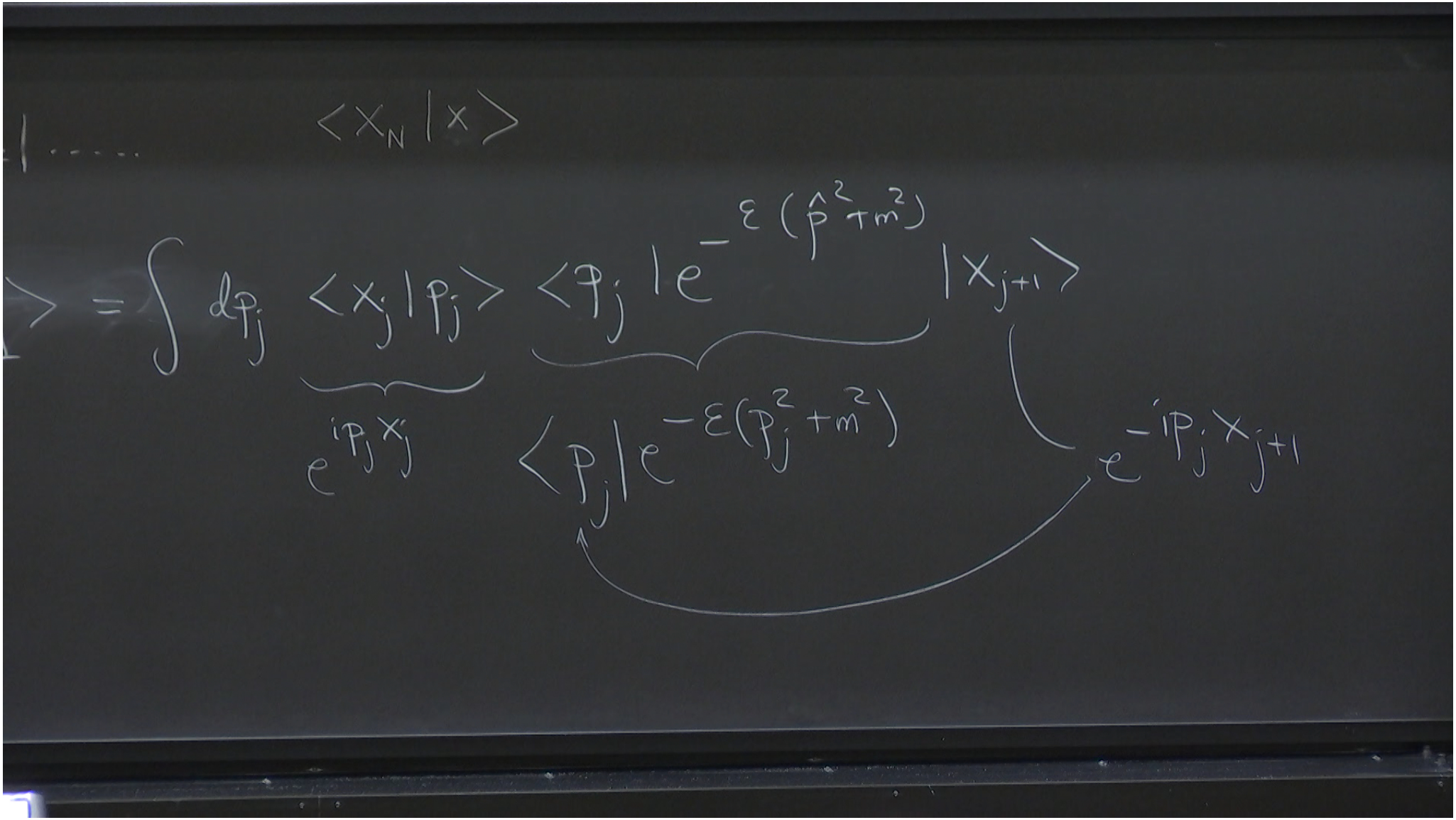
$$\langle x_1 | e^{-\varepsilon(\hat{p}^2+m^2)} | x_2 \rangle \langle x_2 | \cdots \langle x_N | x \rangle$$

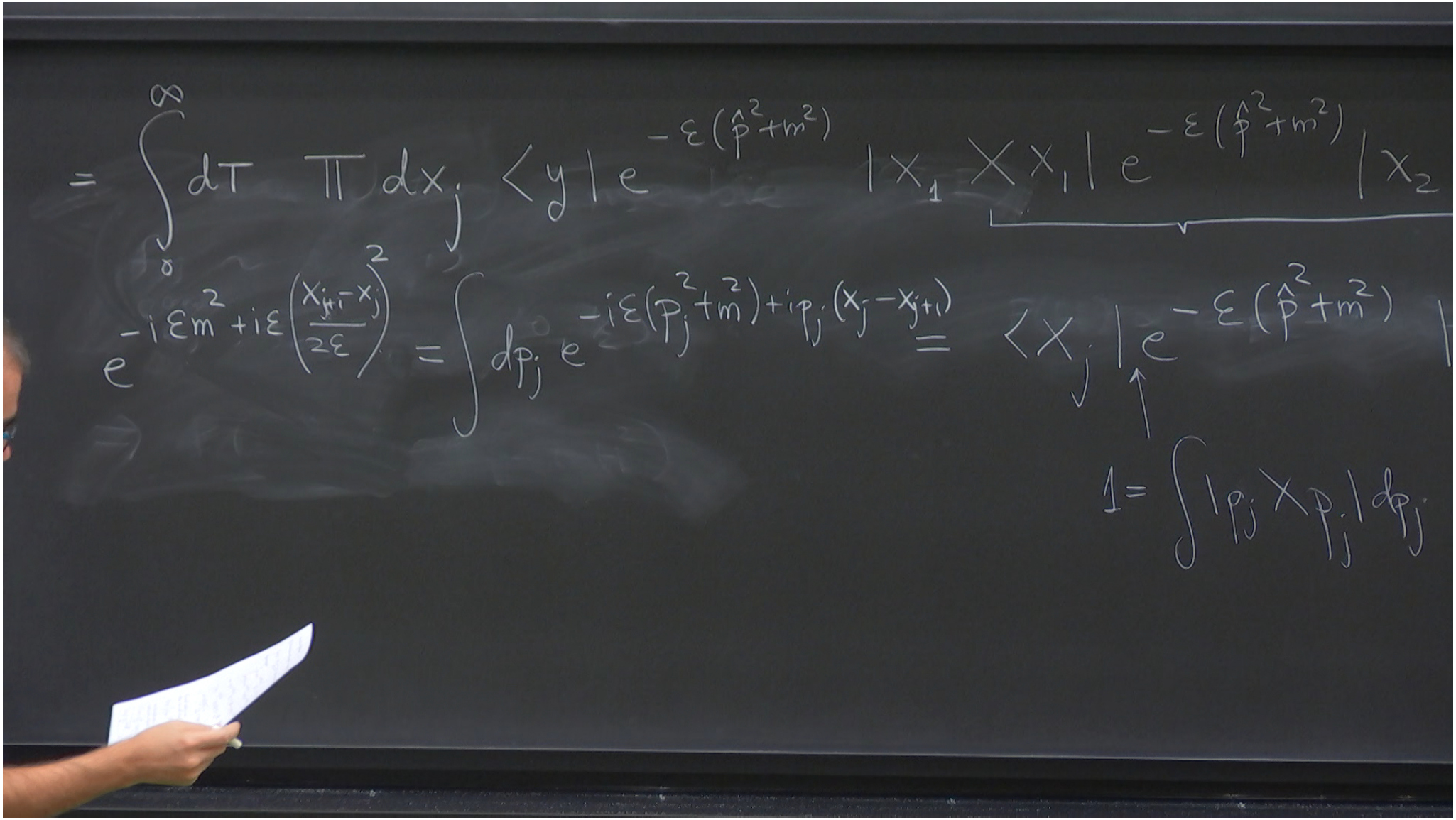
$$= \langle x_j | e^{-\varepsilon(\hat{p}^2+m^2)} | x_{j+1} \rangle = \int dp_j \, \langle x_j | p_j \rangle \langle p_j | e^{-}$$

$$\langle x_j | p_j \rangle = e^{i p_j x_j}$$

$$1 = \int | p_j \rangle \langle p_j | \, dp_j$$

$$\langle p_j | e^{-\varepsilon($$

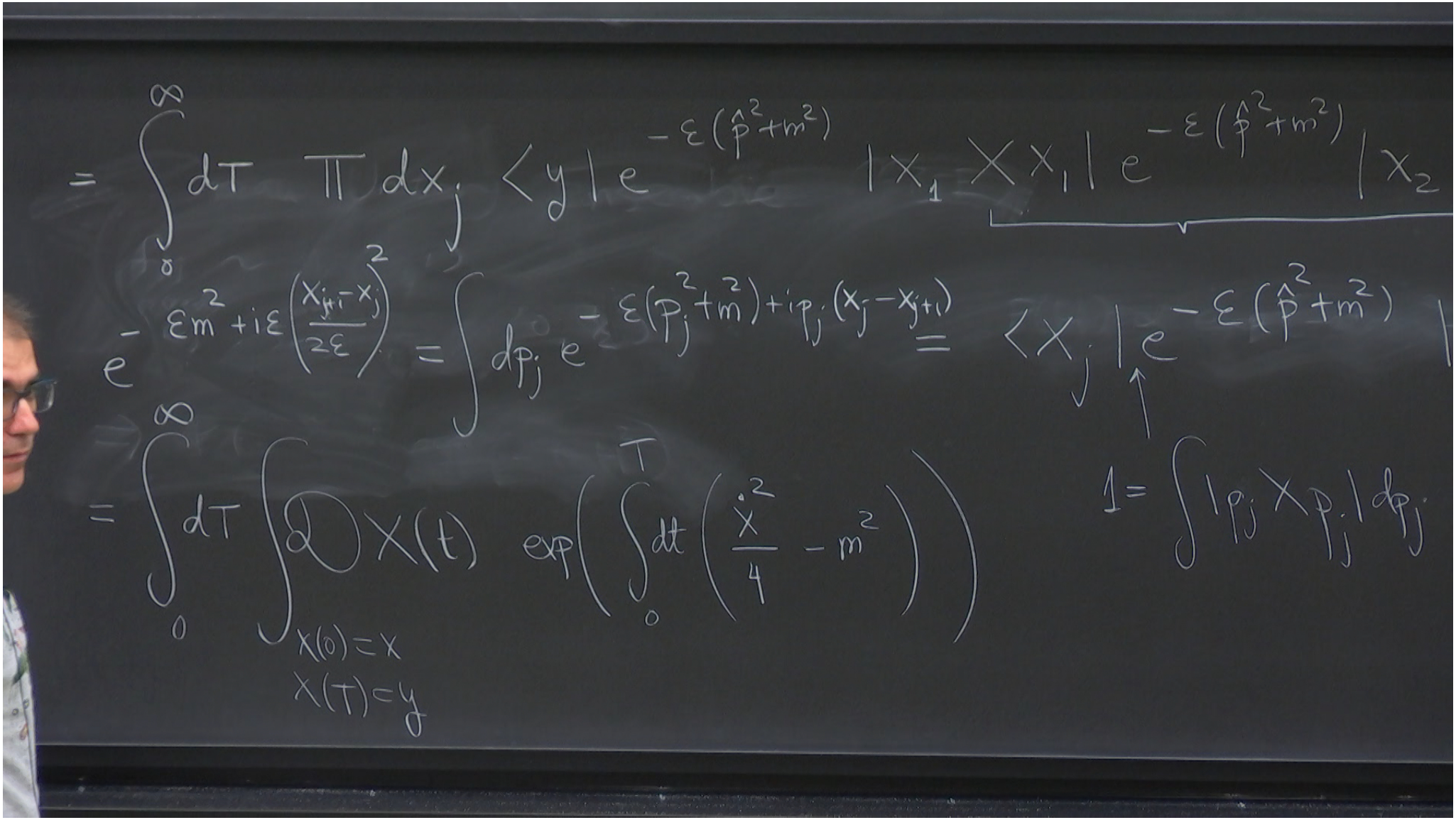

$\langle x_N | x \rangle$
$= \int dp_j \langle x_j | p_j \rangle \langle p_j | e^{-\varepsilon(\hat{p}^2 + m^2)} | x_{j+1} \rangle$
$e^{ip_j x_j}$
$\langle p_j | e^{-\varepsilon(p_j^2 + m^2)}$
$e^{-ip_j x_{j+1}}$


$= \int_0^\infty dT \prod dx_j \langle y | e^{-\varepsilon(\hat{p}^2+m^2)} | x_1 \rangle \langle x_1 | e^{-\varepsilon(\hat{p}^2+m^2)} | x_2$
$e^{-i\varepsilon m^2 + i\varepsilon\left(\frac{x_{j+1}-x_j}{2\varepsilon}\right)^2} = \int dp_j\, e^{-i\varepsilon(p_j^2+m^2)+ip_j(x_j-x_{j+1})}$
$= \langle x_j | e^{-\varepsilon(\hat{p}^2+m^2)} |$
$1 = \int |p_j\rangle\langle p_j| dp_j$

$$= \int_0^\infty dT \prod dx_j \langle y | e^{-\varepsilon(\hat{p}^2+m^2)} | x_1 \rangle \langle x_1 | e^{-\varepsilon(\hat{p}^2+m^2)} | x_2 \rangle$$

$$e^{-\varepsilon m^2 + i\varepsilon\left(\frac{x_{j+1}-x_j}{2\varepsilon}\right)^2} = \int dp_j \, e^{-\varepsilon(p_j^2+m^2)+ip_j(x_j-x_{j+1})} = \langle x_j | e^{-\varepsilon(\hat{p}^2+m^2)} | \cdots$$

$$1 = \int |p_j\rangle\langle p_j| \, dp_j$$

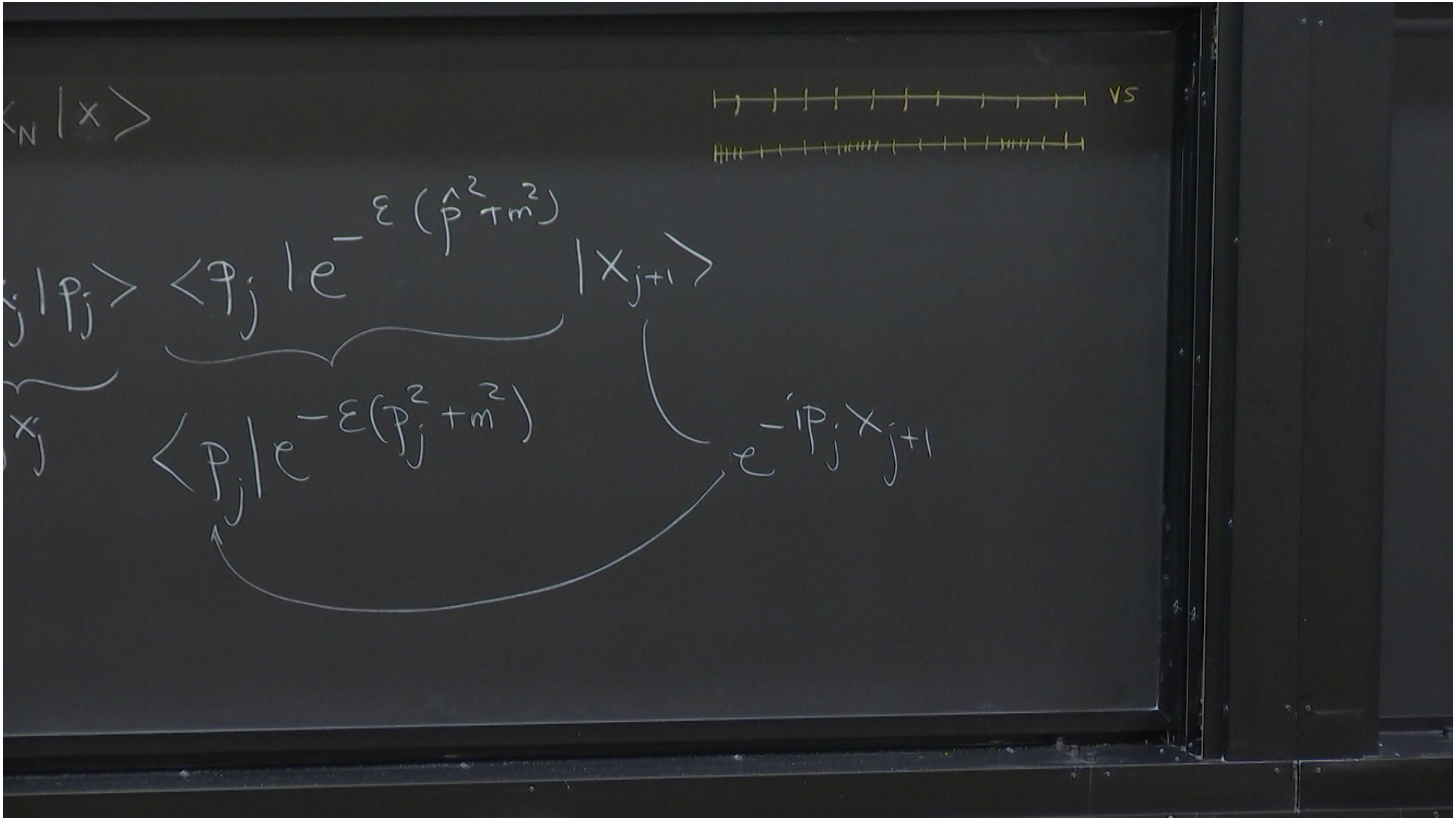

$= \int_0^\infty dT \prod dx_j \langle y | e^{-\varepsilon(\hat{p}^2+m^2)} | x_1 \rangle \langle x_1 | e^{-\varepsilon(\hat{p}^2+m^2)} | x_2$
$e^{-\varepsilon m^2 + i\varepsilon\left(\frac{x_{j+1}-x_j}{2\varepsilon}\right)^2} = \int dp_j\, e^{-\varepsilon(p_j^2+m^2)+ip_j(x_j-x_{j+1})}$
$= \langle x_j | e^{-\varepsilon(\hat{p}^2+m^2)} |$
$1 = \int |p_j\rangle\langle p_j| dp_j$
$= \int_0^\infty dT \int_{x(0)=x}^{x(T)=y} \mathcal{D}x(t)\, \exp\left(\int_0^T dt \left(\frac{\dot{x}^2}{4} - m^2\right)\right)$

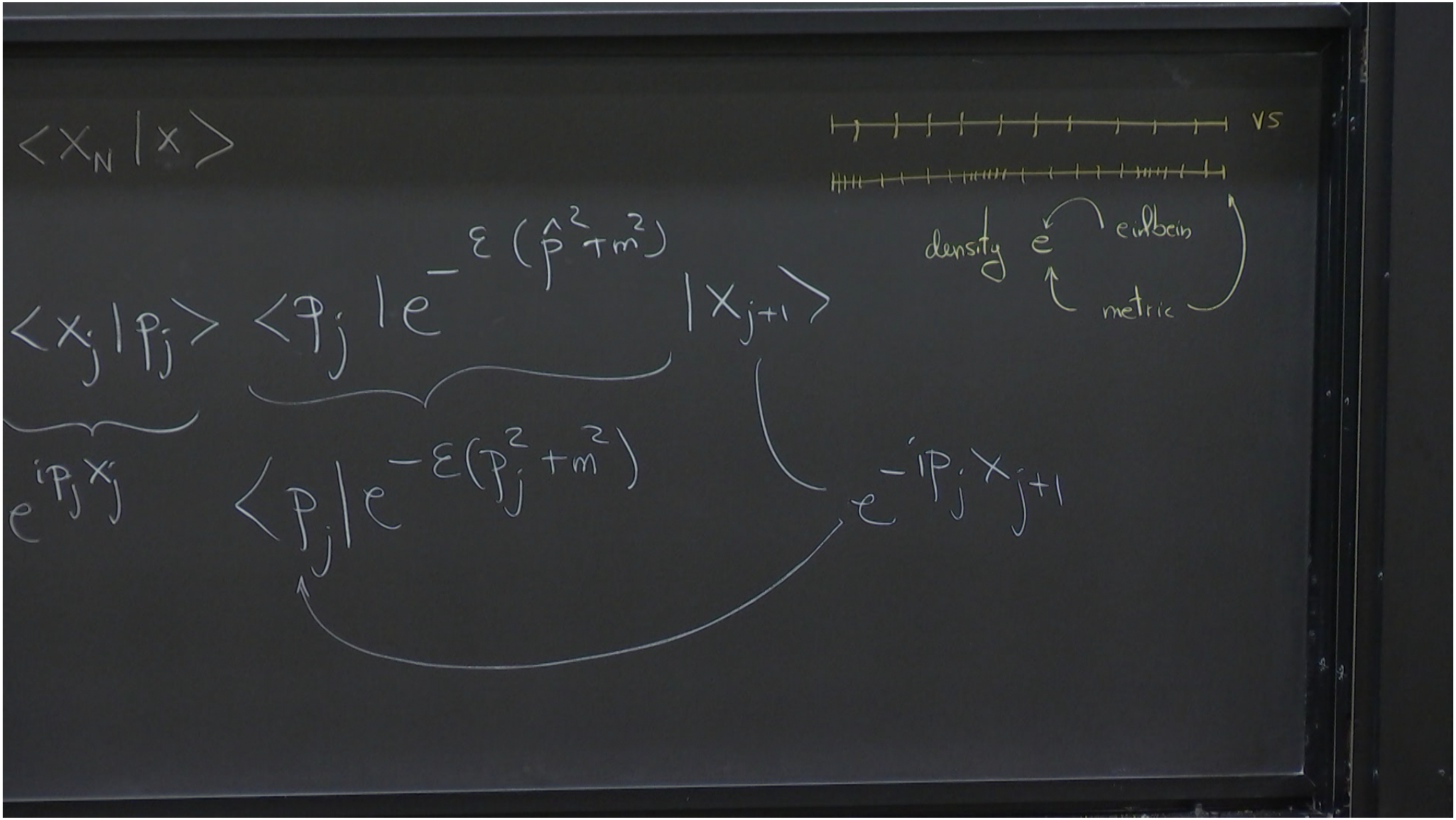
$$= \int_0^\infty dT \prod dx_j \langle y | e^{-\varepsilon(\hat{p}^2+m^2)} | x_1 \rangle \underbrace{\langle x_1 | e^{-\varepsilon(\hat{p}^2+m^2)} | x_2}$$

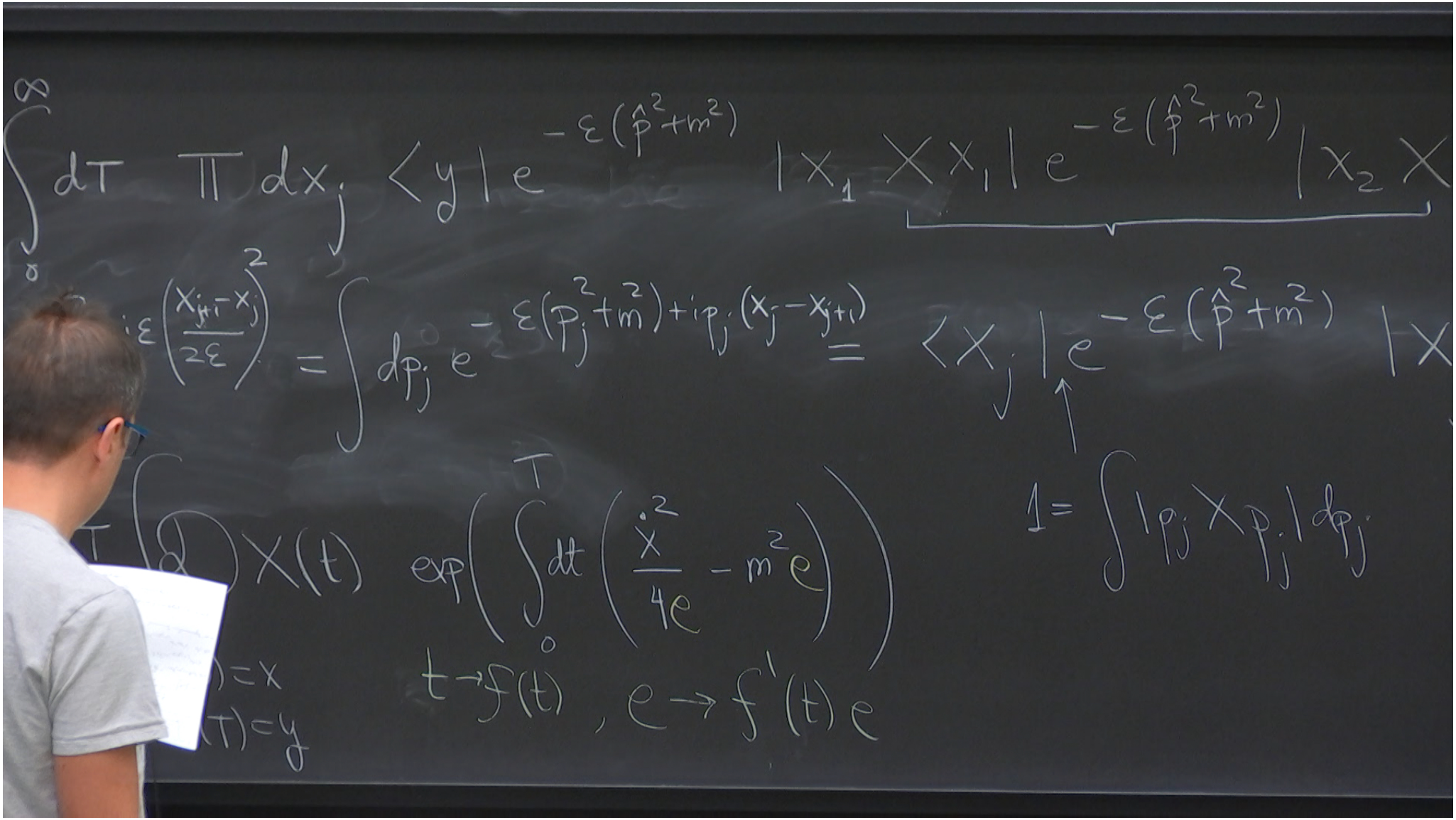
$$e^{-\varepsilon m^2 + i\varepsilon\left(\frac{x_{j+1}-x_j}{2\varepsilon}\right)^2} = \int dp_j \, e^{-\varepsilon(p_j^2+m^2)+ip_j(x_j-x_{j+1})} = \langle x_j | e^{-\varepsilon(\hat{p}^2+m^2)} |$$

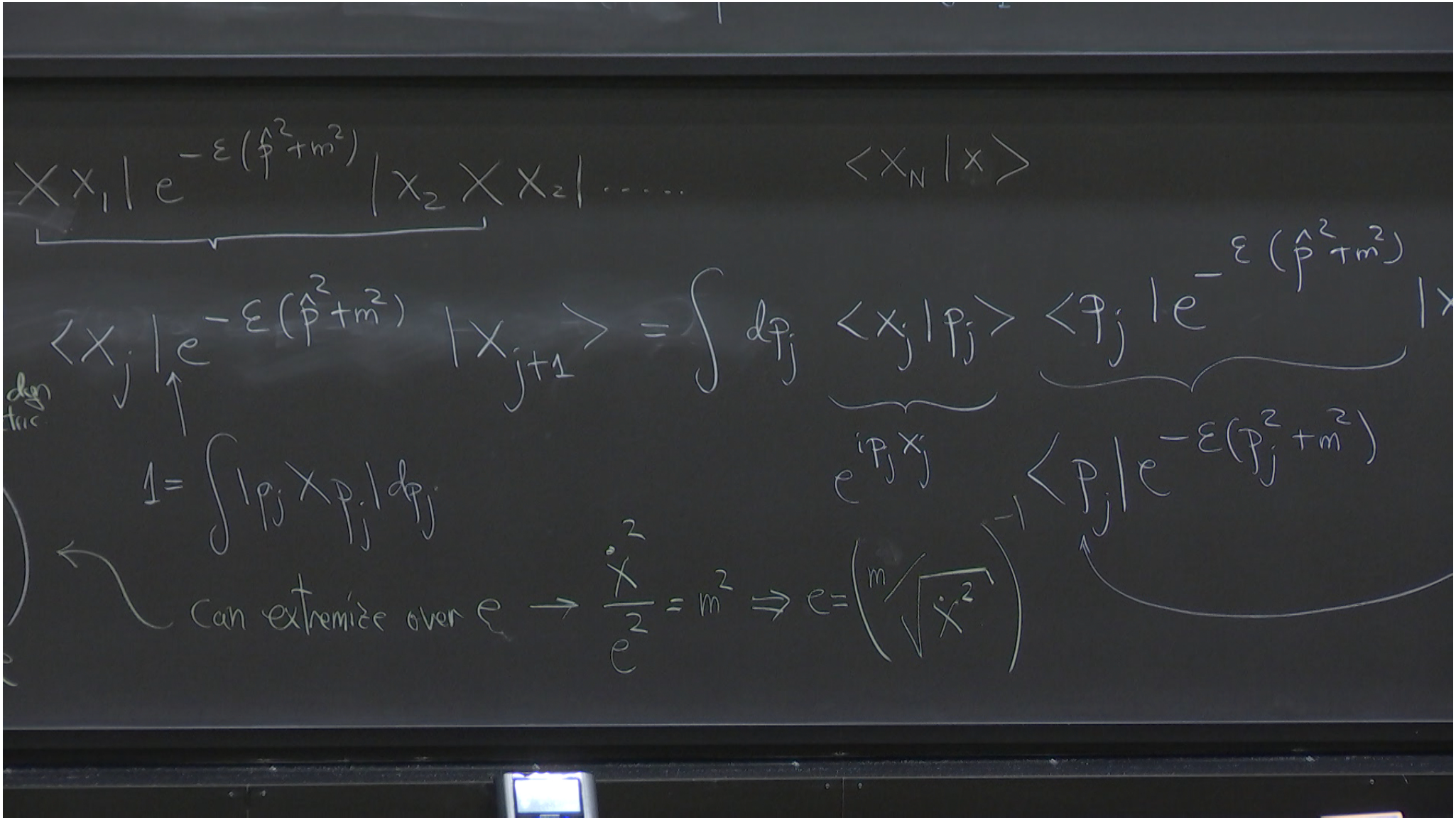
$$1 = \int |p_j\rangle\langle p_j| dp_j$$

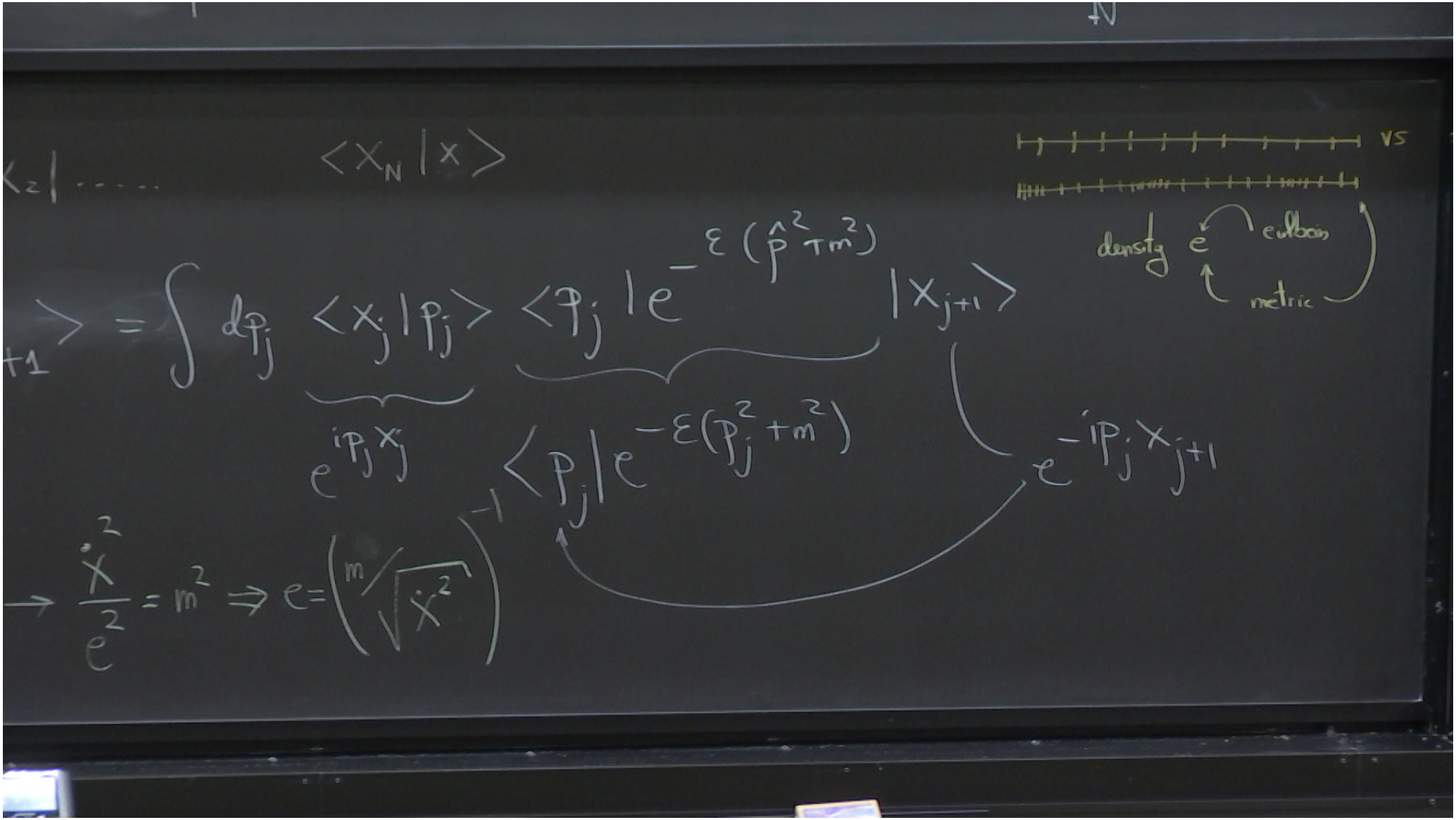
$$\langle x \to y) = \int_0^\infty dT \int_{\substack{X(0)=x \\ X(T)=y}} \mathcal{D}X(t) \exp\left(\int_0^T dt \left(\frac{\dot{X}^2}{4} - m^2 \right) \right)$$

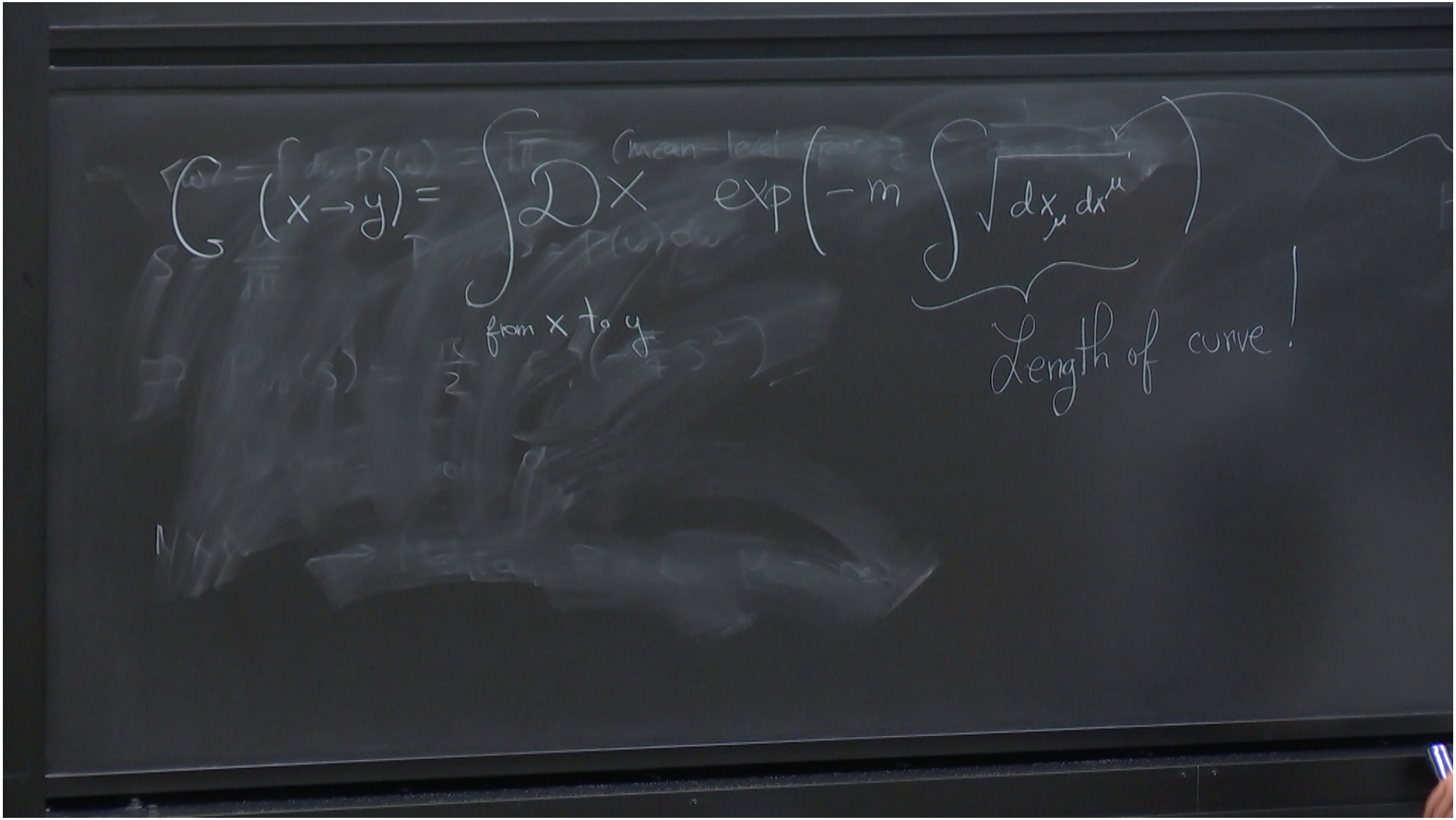

vs
$\langle x_N | x \rangle$
$\langle p_j | p_j \rangle$
$\langle p_j | e^{-\varepsilon(\hat{p}^2+m^2)} | x_{j+1} \rangle$
$\langle p_j | e^{-\varepsilon(p_j^2+m^2)}$
$e^{-ip_j x_{j+1}}$

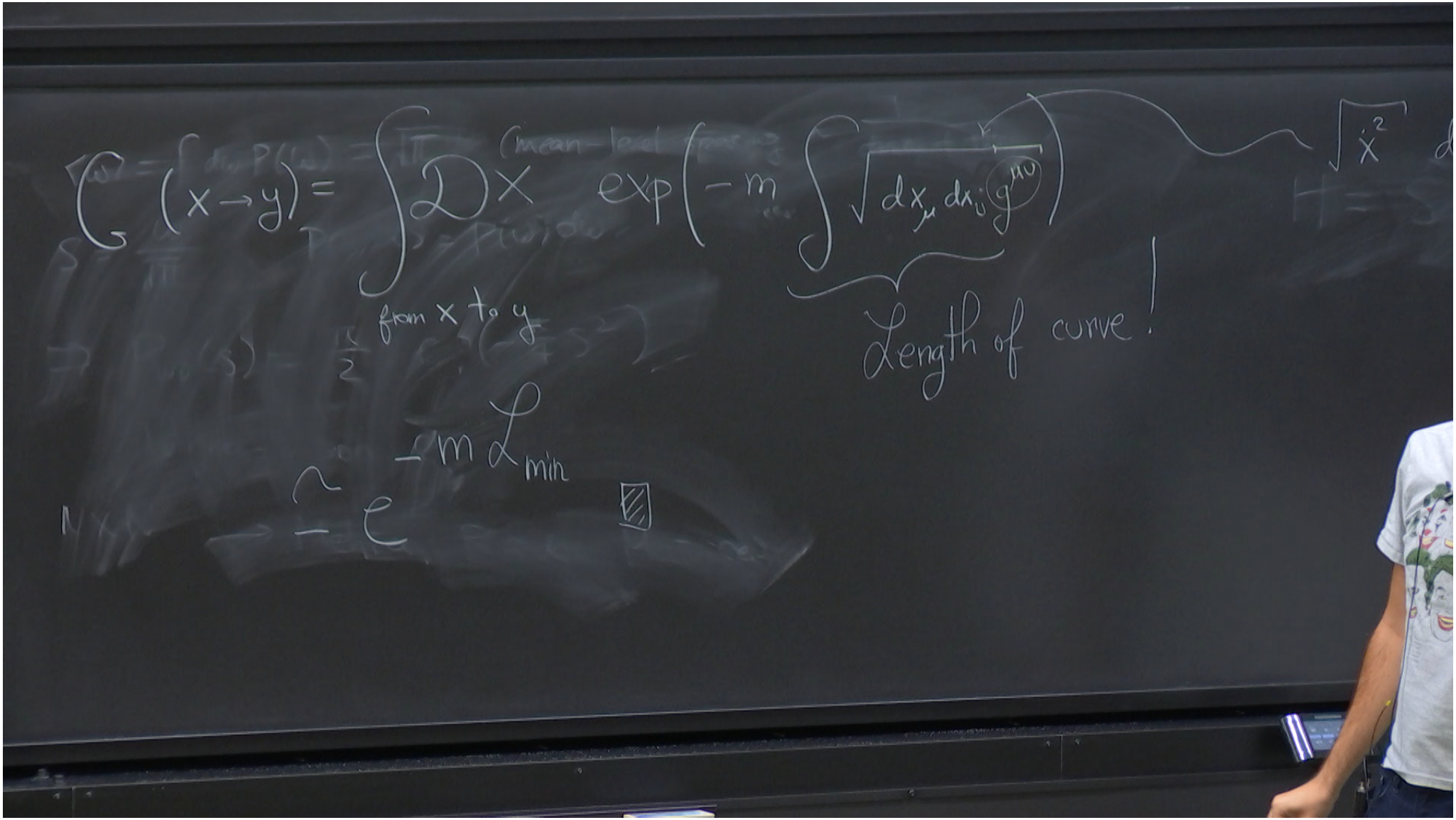

$\langle x_N | x \rangle$
$\langle x_j | p_j \rangle$
$e^{ip_j x_j}$
$\langle p_j | e^{-\varepsilon(\hat{p}^2 + m^2)} | x_{j+1} \rangle$
$\langle p_j | e^{-\varepsilon(p_j^2 + m^2)}$
$e^{-ip_j x_{j+1}}$
vs
density
e
einbein
metric

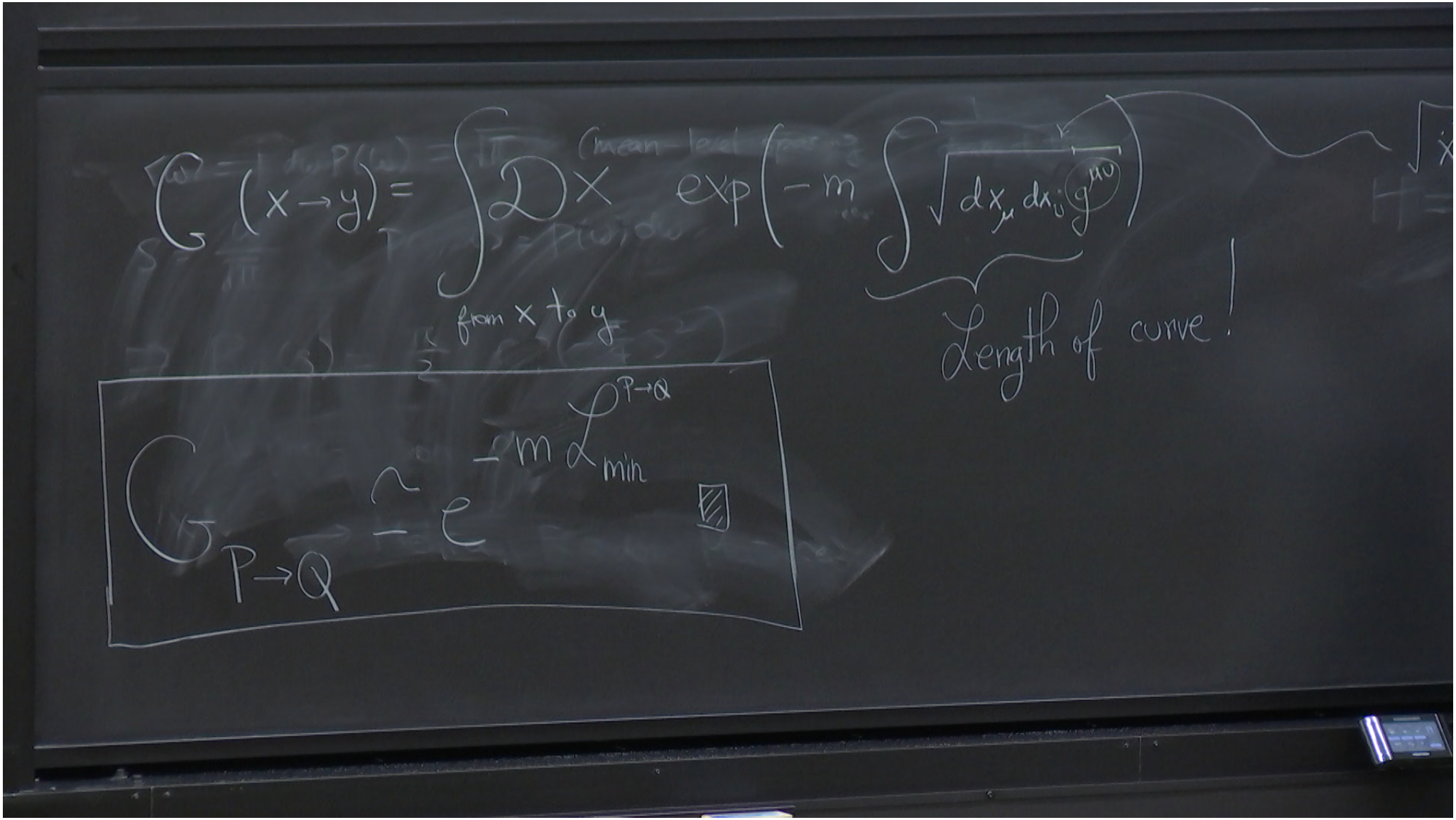

$\int_0^\infty dT \prod dx_j \langle y | e^{-\varepsilon(\hat{p}^2+m^2)} | x_1 \rangle \langle x_1 | e^{-\varepsilon(\hat{p}^2+m^2)} | x_2 \rangle$
$\varepsilon\left(\frac{x_{j+1}-x_j}{2\varepsilon}\right)^2 = \int dp_j\, e^{-\varepsilon(p_j^2+m^2)+ip_j(x_j-x_{j+1})}$
$= \langle x_j | e^{-\varepsilon(\hat{p}^2+m^2)} | x$
$1 = \int |p_j\rangle\langle p_j| dp_j$
$\int_0^T X(t) \exp\left(\int_0^T dt \left(\frac{\dot{x}^2}{4e} - m^2 e\right)\right)$
$t \to f(t), \; e \to f'(t)\, e$

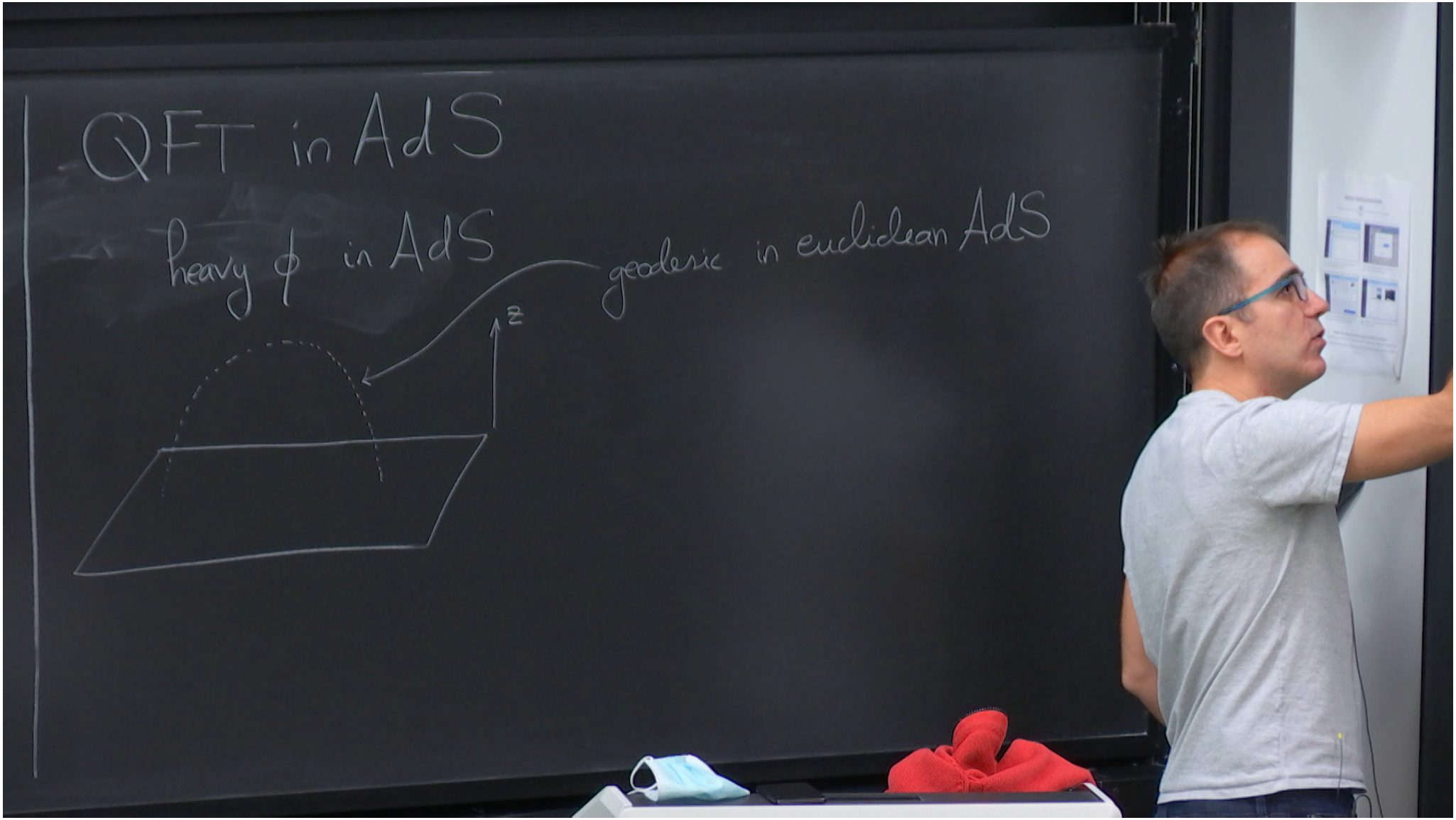

$\langle x_1 | e^{-\varepsilon(\hat{p}^2+m^2)} | x_2 \rangle \langle x_2 | \cdots \langle x_N | x \rangle$
$\langle x_j | e^{-\varepsilon(\hat{p}^2+m^2)} | x_{j+1} \rangle = \int dp_j \langle x_j | p_j \rangle \langle p_j | e^{-\varepsilon(\hat{p}^2+m^2)} | x$
$e^{ip_j x_j}$
$1 = \int |p_j\rangle\langle p_j| dp_j$
$\langle p_j | e^{-\varepsilon(p_j^2+m^2)}$
Can extremize over $e \rightarrow \frac{\dot{x}^2}{e^2} = m^2 \Rightarrow e = \left(\frac{m}{\sqrt{\dot{x}^2}}\right)^{-1}$


$\langle x_N | x \rangle$
$\int dp_j \langle x_j | p_j \rangle \langle p_j | e^{-\varepsilon(\hat{p}^2 + m^2)} | x_{j+1} \rangle$
$e^{ip_j x_j}$
$\langle p_j | e^{-\varepsilon(p_j^2 + m^2)}$
$e^{-ip_j x_{j+1}}$
$\frac{\dot{x}^2}{e^2} = m^2 \Rightarrow e = \left(\frac{m}{\sqrt{\dot{x}^2}}\right)^{-1}$
vs
density
euclean
metric


$G(x \to y) = \int_{\text{from } x \text{ to } y} \mathcal{D}x \; \exp\left(-m \int \sqrt{dx_\mu \, dx^\mu}\right)$
(mean-lived)
Length of curve!


$G(x \to y) = \int \mathcal{D}X \exp\left(-m \int \sqrt{dx_\mu dx_\nu g^{\mu\nu}}\right) \sqrt{\dot{x}^2}$
mean-level
from x to y
Length of curve!
$\sim e^{-m\mathcal{L}_{min}}$


$G(x \to y) = \int \mathcal{D}X \; \exp\left(-m \int \sqrt{dx_\mu dx_\nu g^{\mu\nu}}\right)$
from x to y
Length of curve!
$G_{P\to Q} \sim e^{-m \mathcal{L}^{P\to Q}_{min}}$

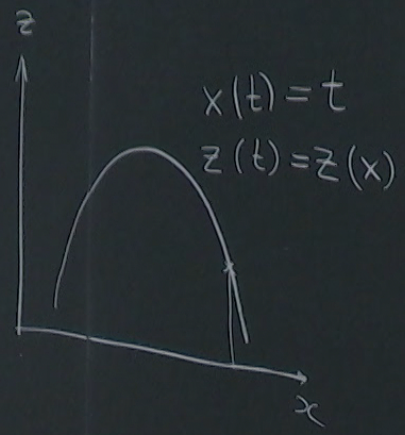

QFT in AdS
heavy φ in AdS
geodesic in euclidean AdS
z

$EAdS_2$

$$ds^2 = \frac{dz^2 + dx^2}{z^2}$$

$$\mathcal{L} = \int dt \sqrt{\frac{\dot{z}^2 + \dot{x}^2}{z^2}}$$

$$= \int dx \sqrt{\frac{z'^2 + 1}{z^2}}$$

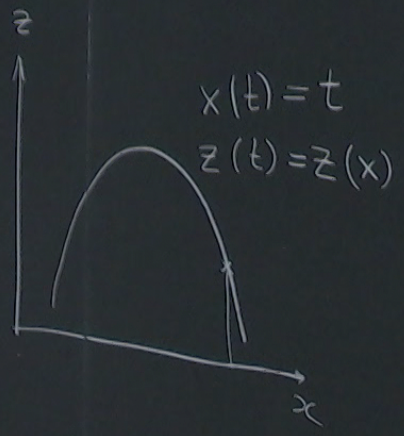


$EAdS_2$

$$ds^2 = \frac{dz^2 + dx^2}{z^2}$$

$$\mathcal{L} = \int dt \sqrt{\frac{\dot{z}^2 + \dot{x}^2}{z^2}}$$

$$= \int dx \sqrt{\frac{z'^2 + 1}{z^2}}$$



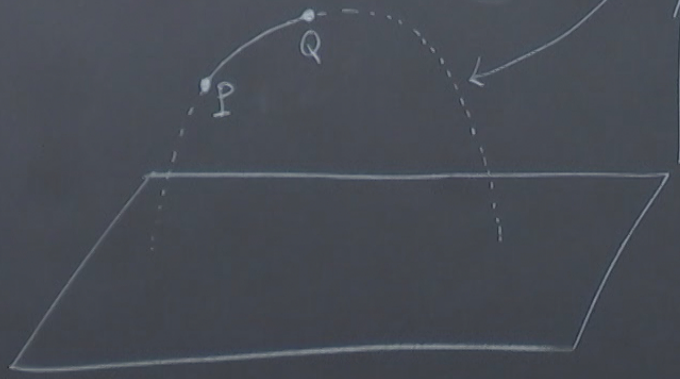
$$x(t) = t$$
$$z(t) = z(x)$$

$$\partial_x \left(\frac{z'/z^2}{\sqrt{\quad}} \right) = \partial_z \sqrt{\quad} \Rightarrow z = \sqrt{X^2 - X_0^2}$$

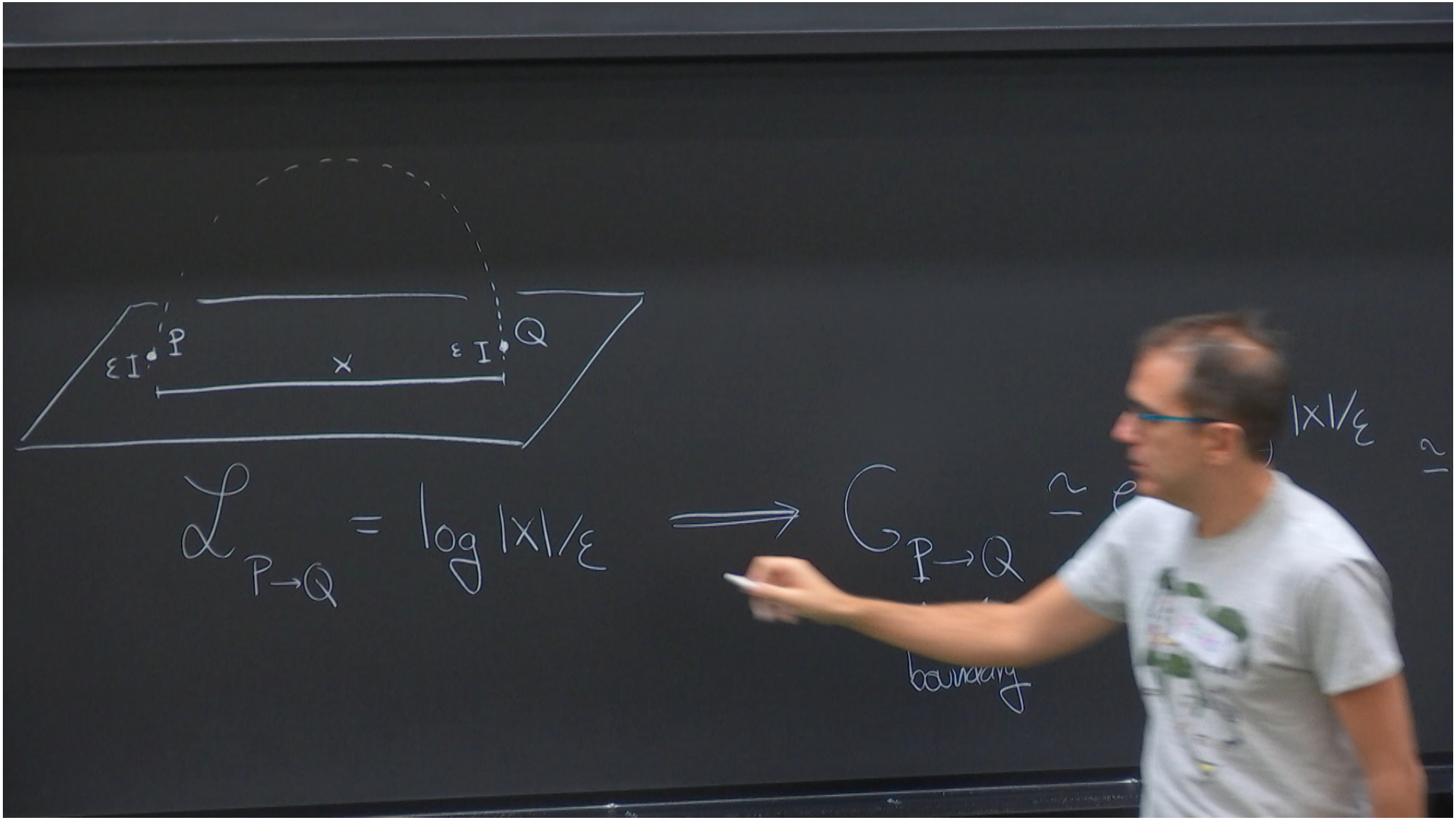
QFT in AdS

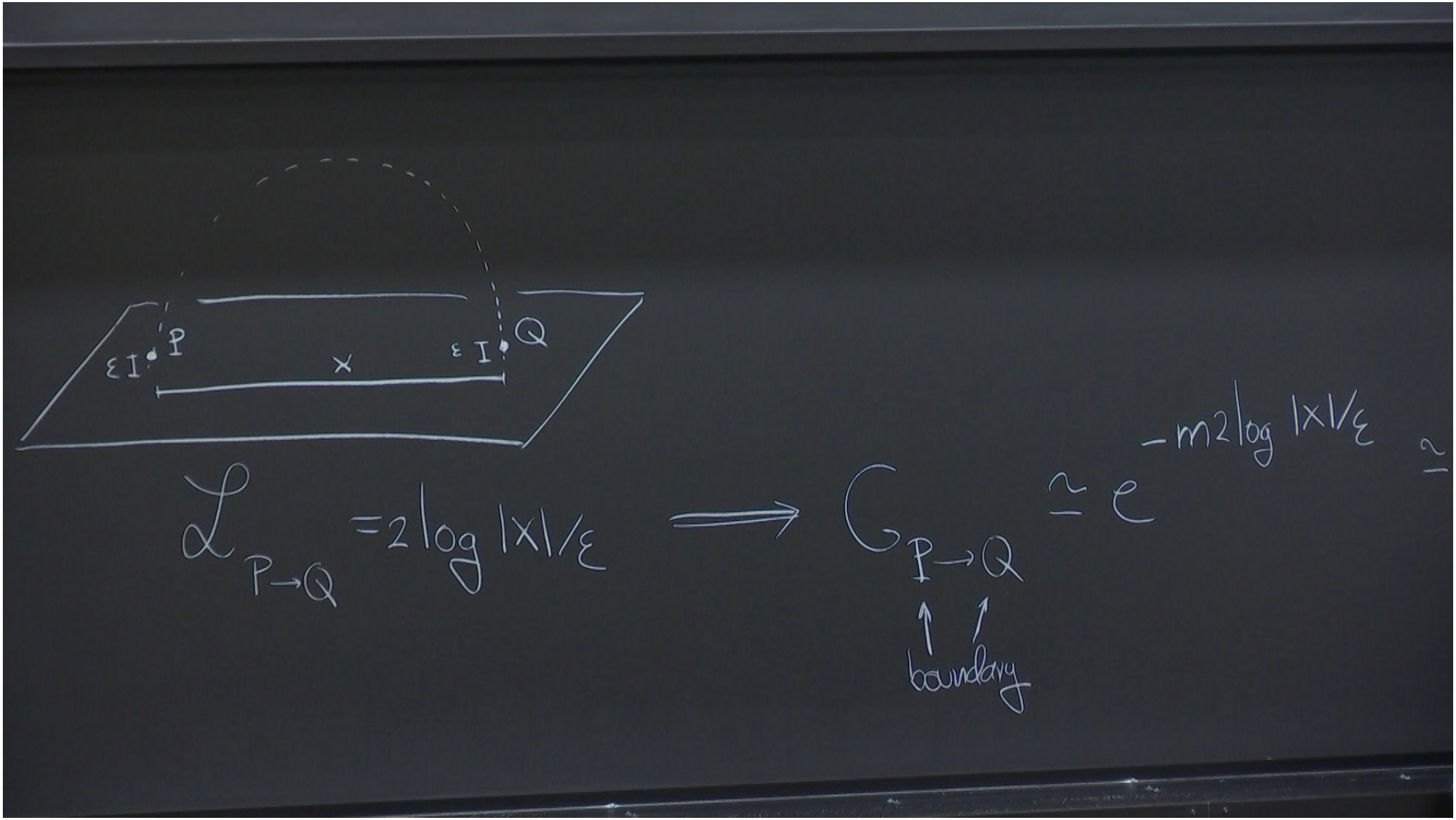
heavy ϕ in AdS

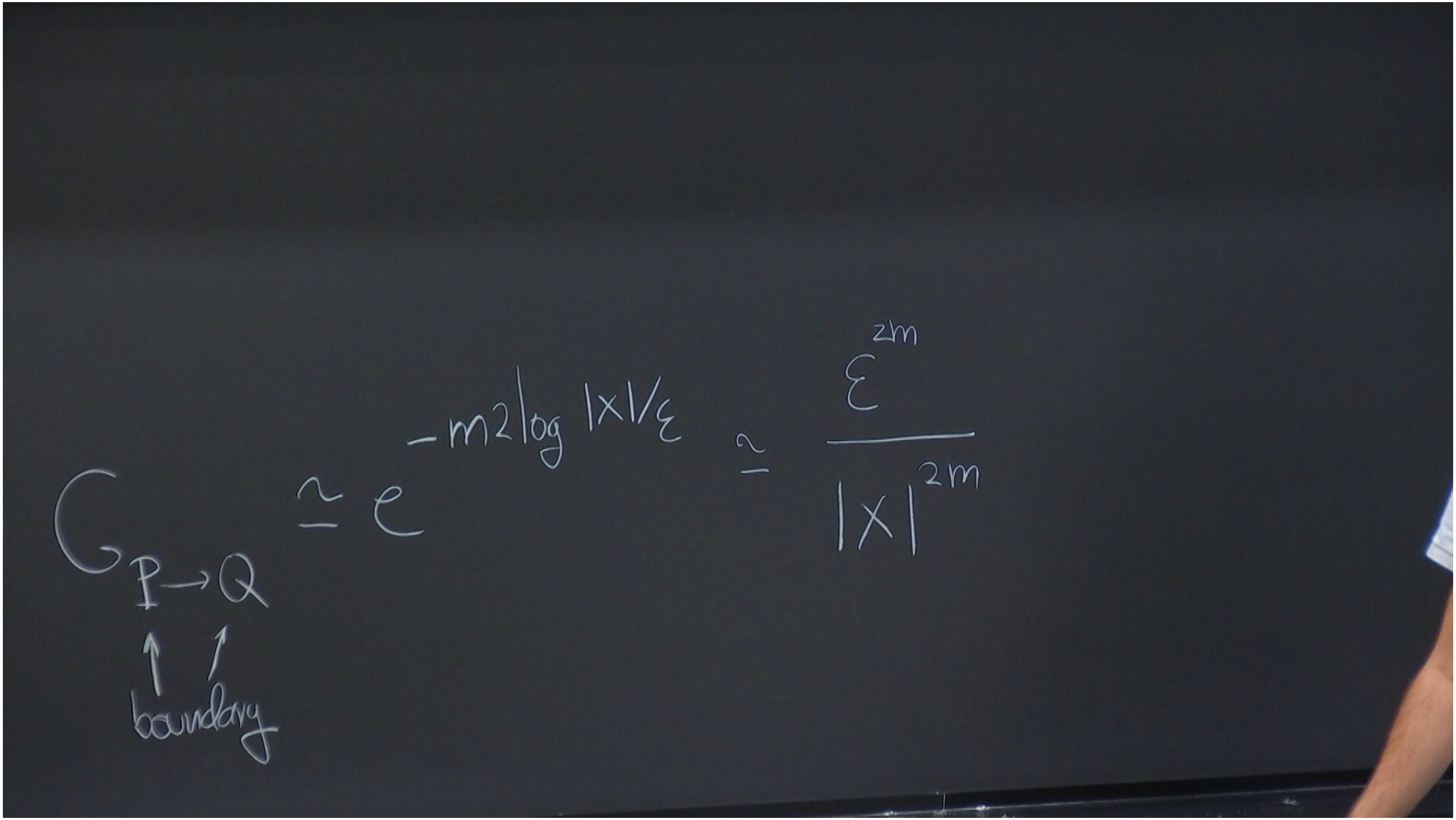
geodesic in euclidean AdS

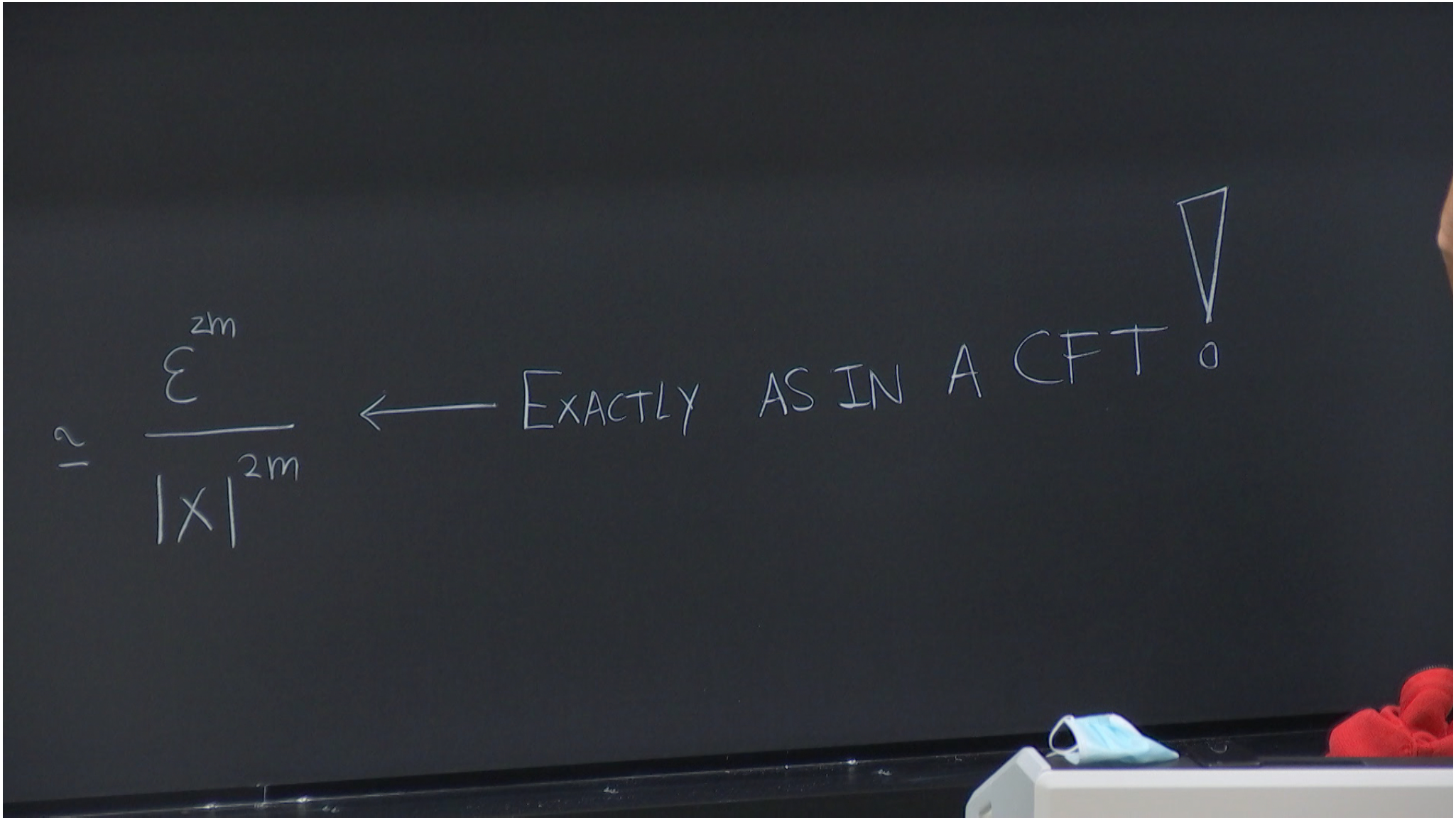


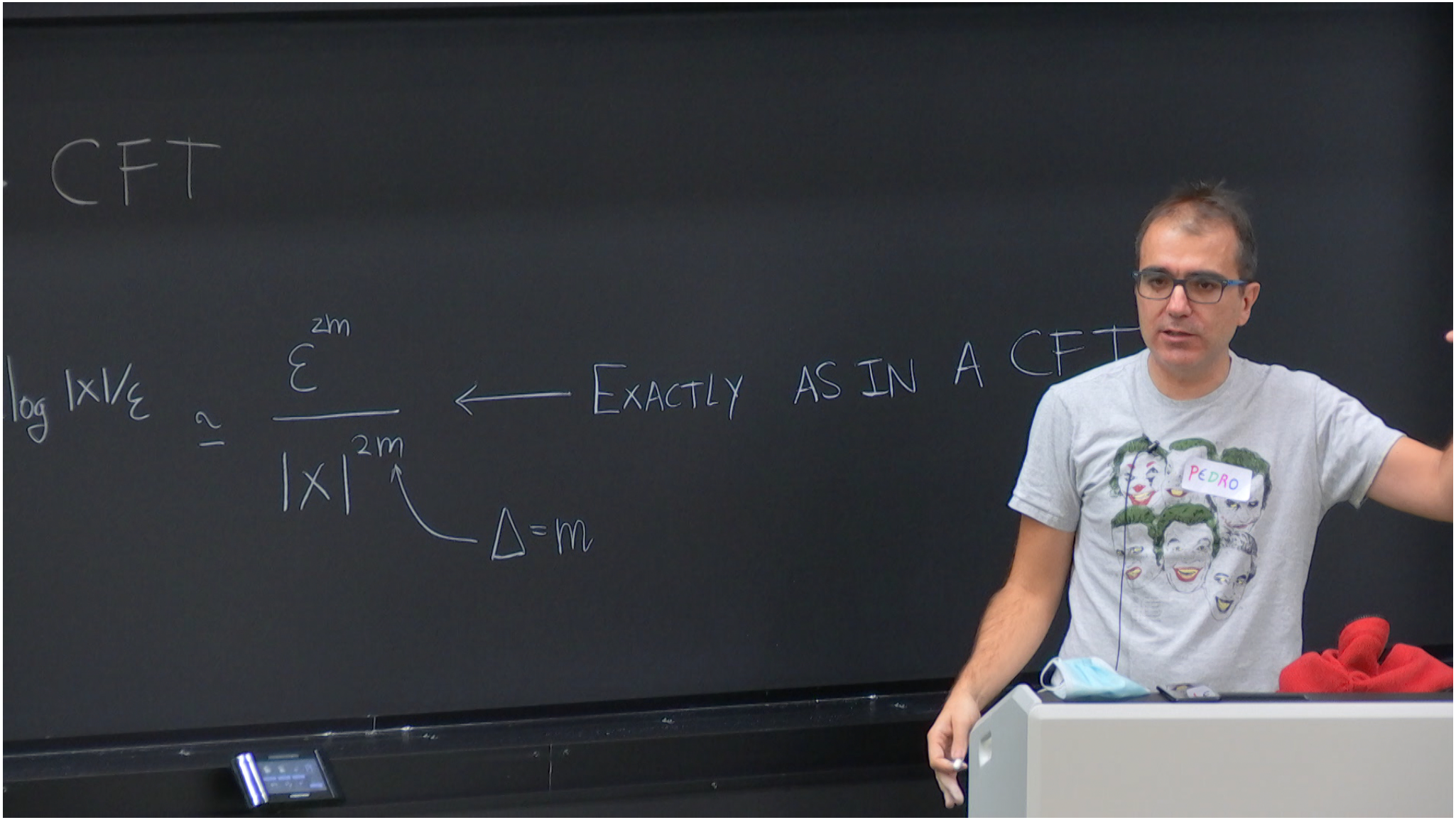
$G_{P \to Q}$

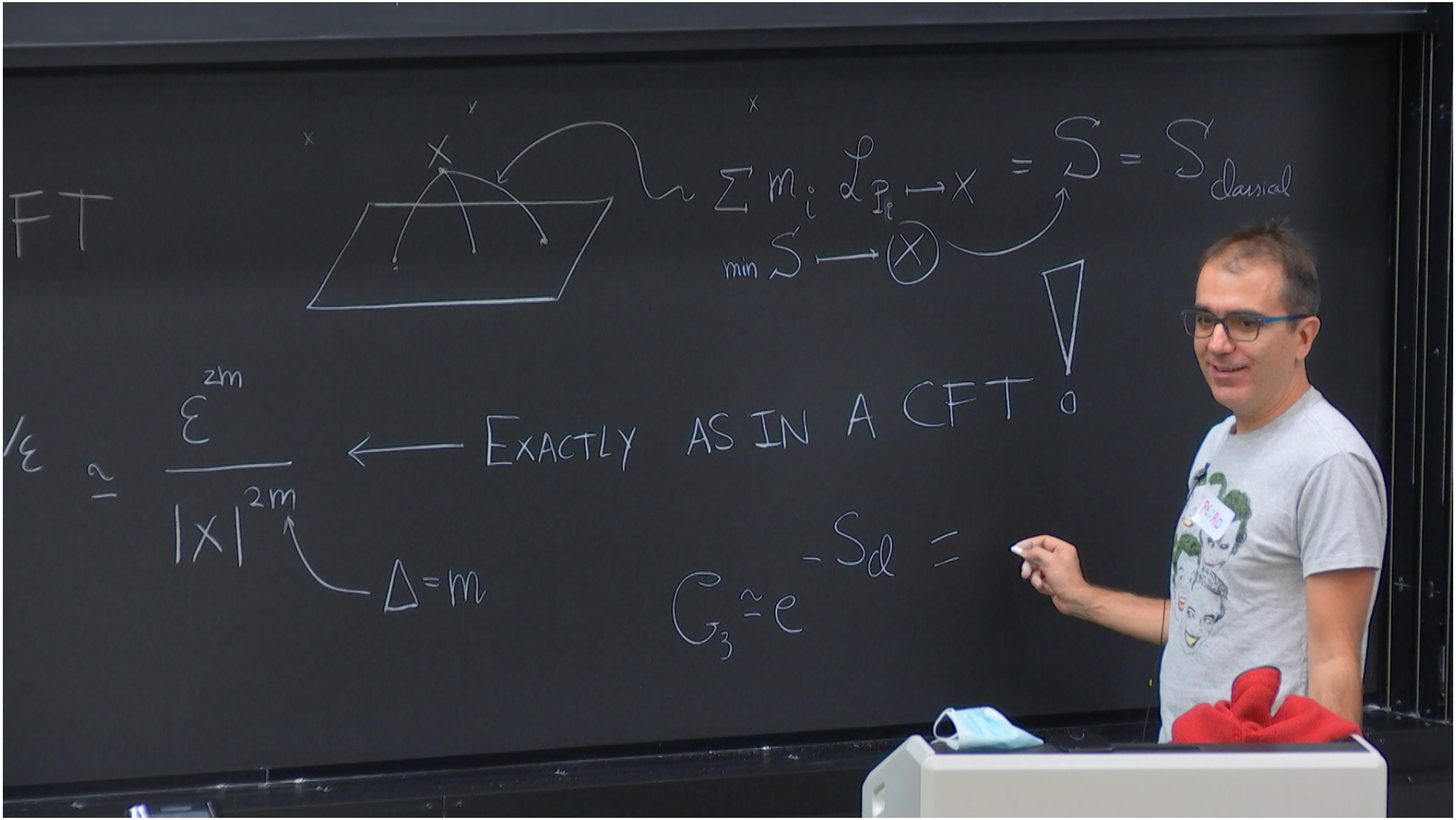

P
Q
x
$\mathcal{L}_{P\to Q} = \log |x|/\varepsilon$
$\Longrightarrow$
$G_{P\to Q}$
boundary

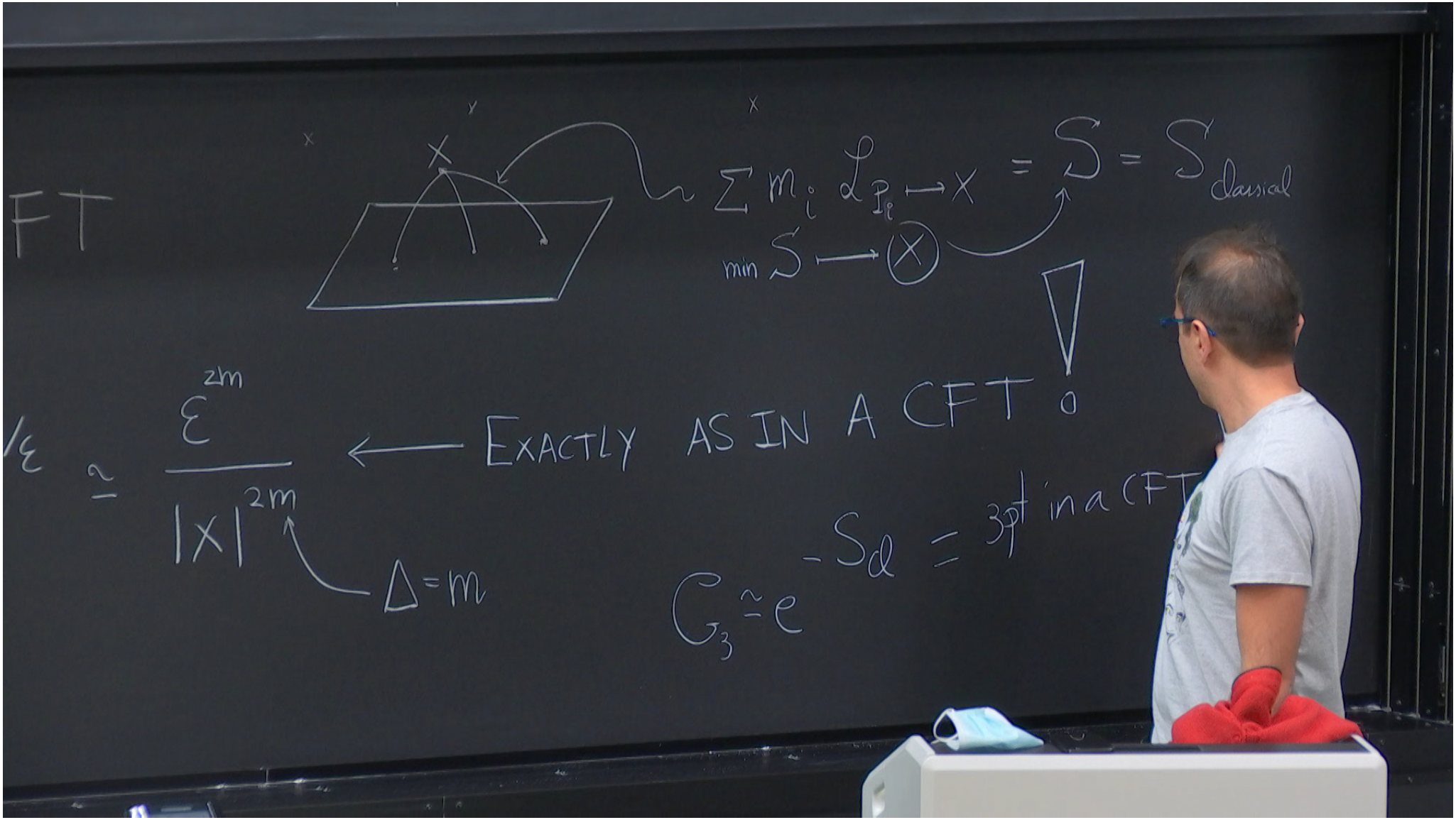

P
Q
x
ε
ε
$\mathcal{L}_{P\to Q} = 2\log|x|/\varepsilon \Longrightarrow G_{P\to Q} \simeq e^{-m2\log|x|/\varepsilon} \simeq$
boundary


$G_{P \to Q} \simeq e^{-m 2 \log |x| / \varepsilon} \simeq \frac{\varepsilon^{2m}}{|x|^{2m}}$
boundary


$\simeq \frac{\varepsilon^{2m}}{|x|^{2m}}$
EXACTLY AS IN A CFT!


CFT
log |x|/ε ≃ ε^{2m} / |x|^{2m}
EXACTLY AS IN A CFT
Δ = m
PEDRO


FT
$\Sigma m_i \mathcal{O}_{P_i \to x} = S = S_{classical}$
$\min S \longrightarrow$
$\sim \frac{\varepsilon^{2m}}{|x|^{2m}}$
EXACTLY AS IN A CFT !
$\Delta = m$
$G_3 \sim e^{-S_{cl}} =$


FT
$\Sigma m_i \mathcal{O}_{P_i} \to X = S = S_{classical}$
$\min S \to$
EXACTLY AS IN A CFT
$\Delta = m$
$G_3 \simeq e^{-S_{cl}} =$ 3pt in a CFT