Title: Quantum Fields and Strings 2021/2022 - Lecture 3

Speakers: Dan Wohns

Collection: Quantum Fields and Strings 2021/2022

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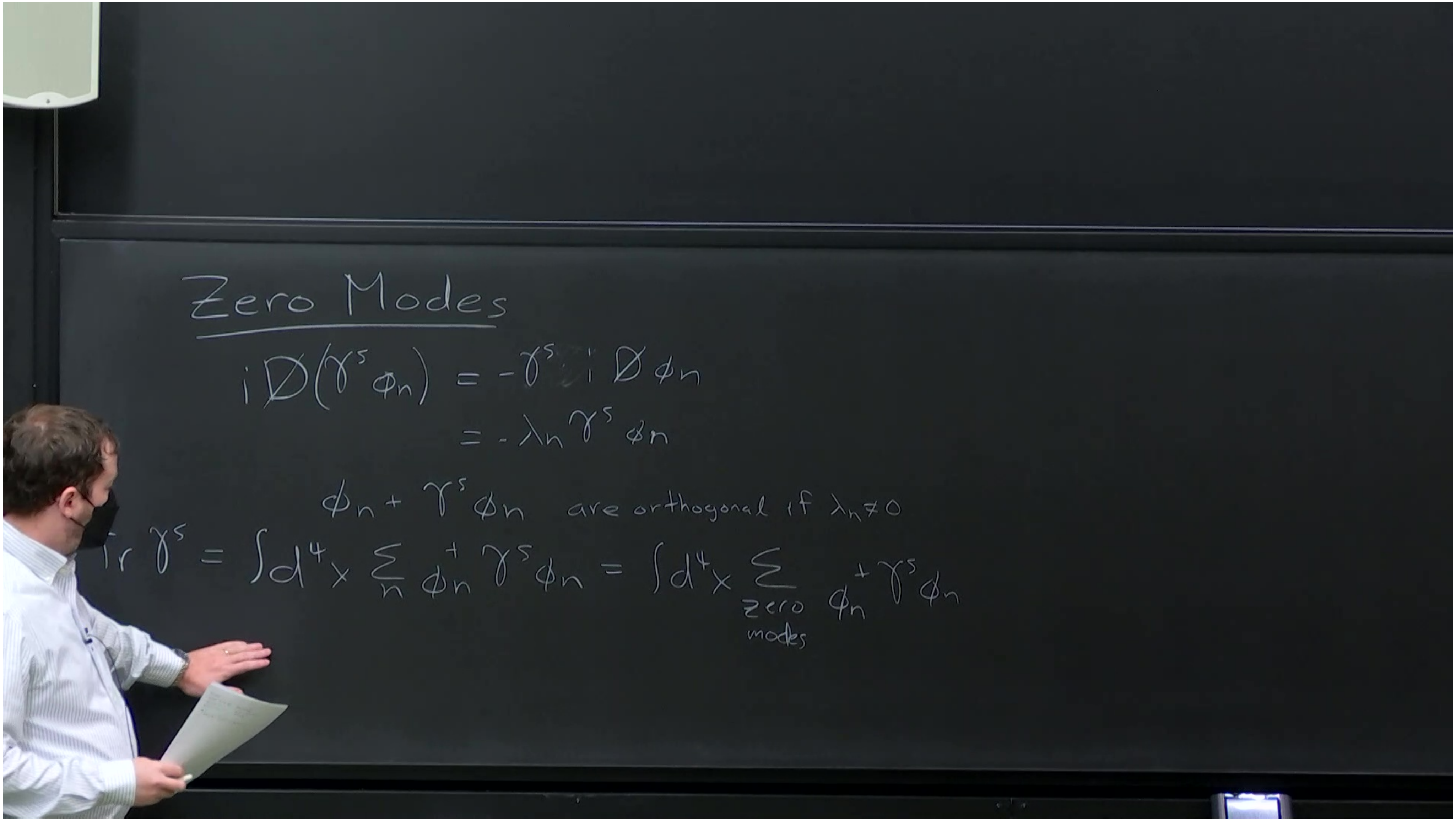
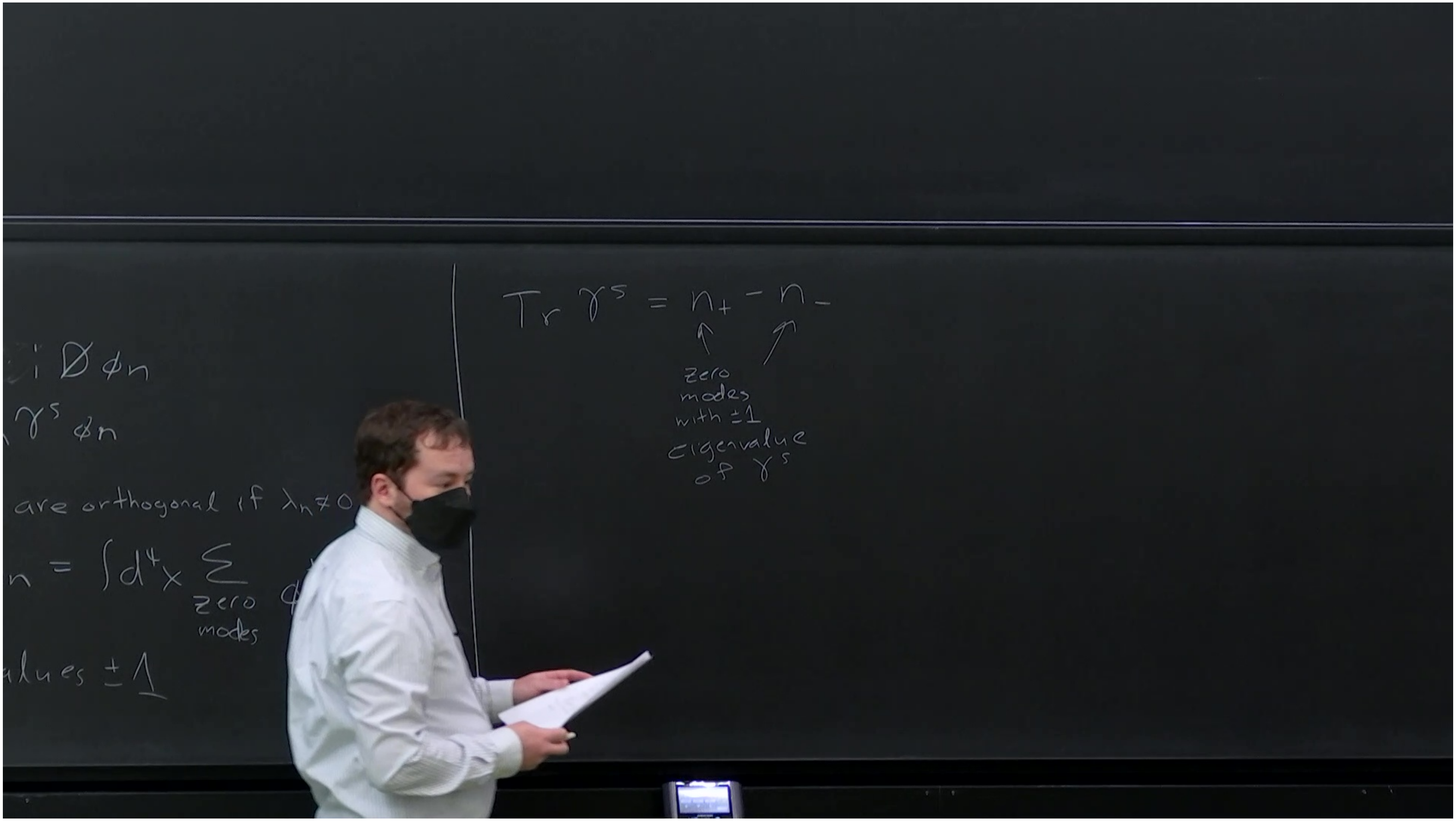
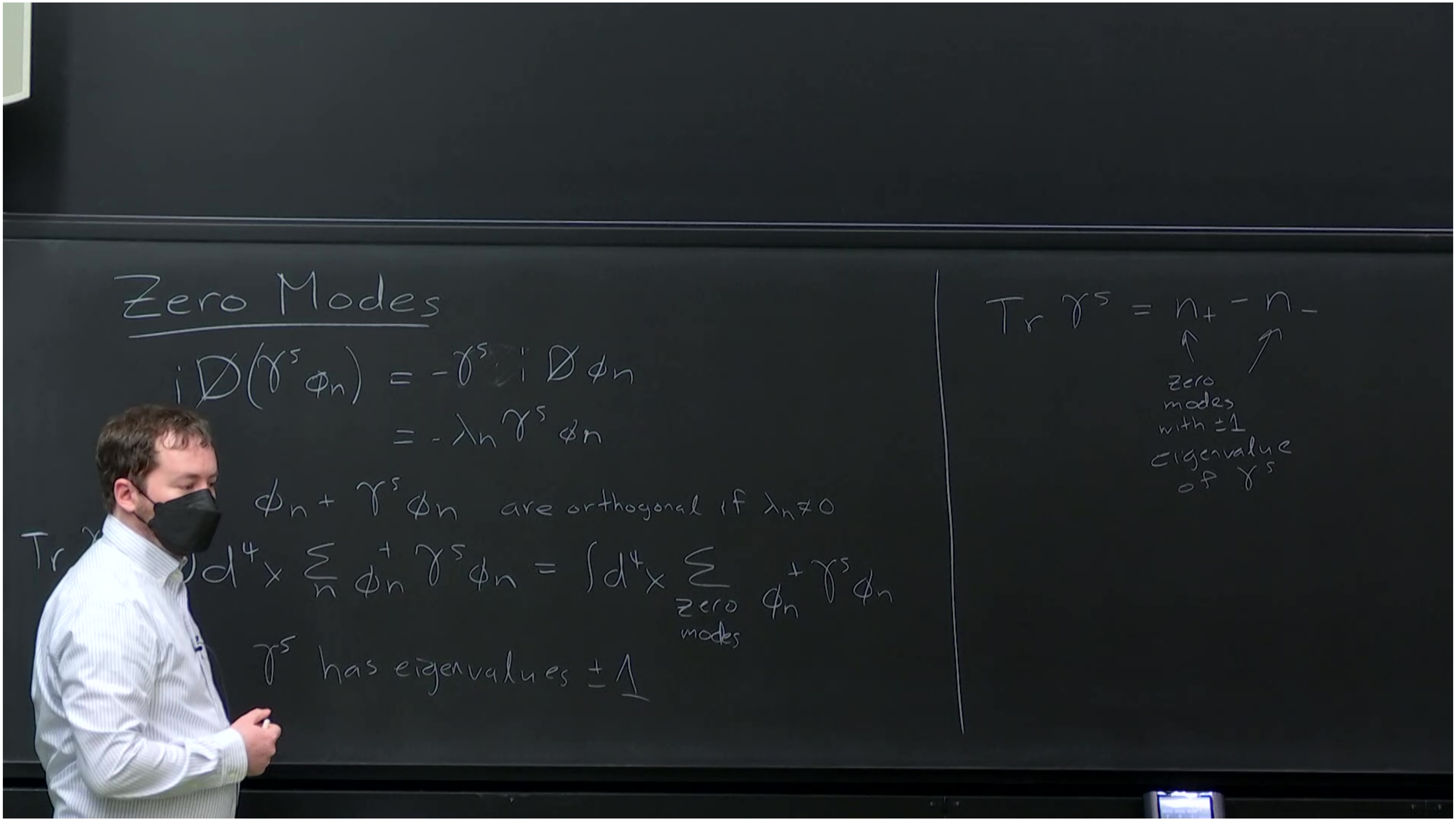

Zero Modes
$i\not{D}(\gamma^5 \phi_n) = -\gamma^5 i \not{D} \phi_n$
$= -\lambda_n \gamma^5 \phi_n$
ϕ_n + $\gamma^5 \phi_n$ are orthogonal if $\lambda_n \neq 0$
$\mathrm{Tr}\, \gamma^5 = \int d^4x \sum_n \phi_n^\dagger \gamma^5 \phi_n = \int d^4x \sum_{\text{zero modes}} \phi_n^\dagger \gamma^5 \phi_n$


Tr γ5 = n+ − n−
zero
modes
with ±1
eigenvalue
of γ5
are orthogonal if λn ≠ 0
zero
modes
lues ± 1


Zero Modes
$i\not{D}(\gamma^5\phi_n) = -\gamma^5 i \not{D}\phi_n$
$= -\lambda_n \gamma^5 \phi_n$
ϕ_n + $\gamma^5\phi_n$ are orthogonal if $\lambda_n \neq 0$
$\mathrm{Tr}\,\gamma^5 = \int d^4x \sum_n \phi_n^\dagger \gamma^5 \phi_n = \int d^4x \sum_{\text{zero modes}} \phi_n^\dagger \gamma^5 \phi_n$
γ^5 has eigenvalues ± 1
$\mathrm{Tr}\,\gamma^5 = n_+ - n_-$
zero modes with ± 1 eigenvalue of γ^5

$1 = (P_L + P_R)$

$\gamma^5 \; i \not{D} \phi_n$

$\lambda_n \gamma^5 \phi_n$

n are orthogonal if $\lambda_n \neq 0$

$\gamma^5 \phi_n = \int d^4x \sum_{\text{zero modes}} \phi_n^+ \gamma^5 \phi_n$

genvalues $\underline{\pm 1}$

$\mathrm{Tr}\, \gamma^5 = n_+ - n_-$

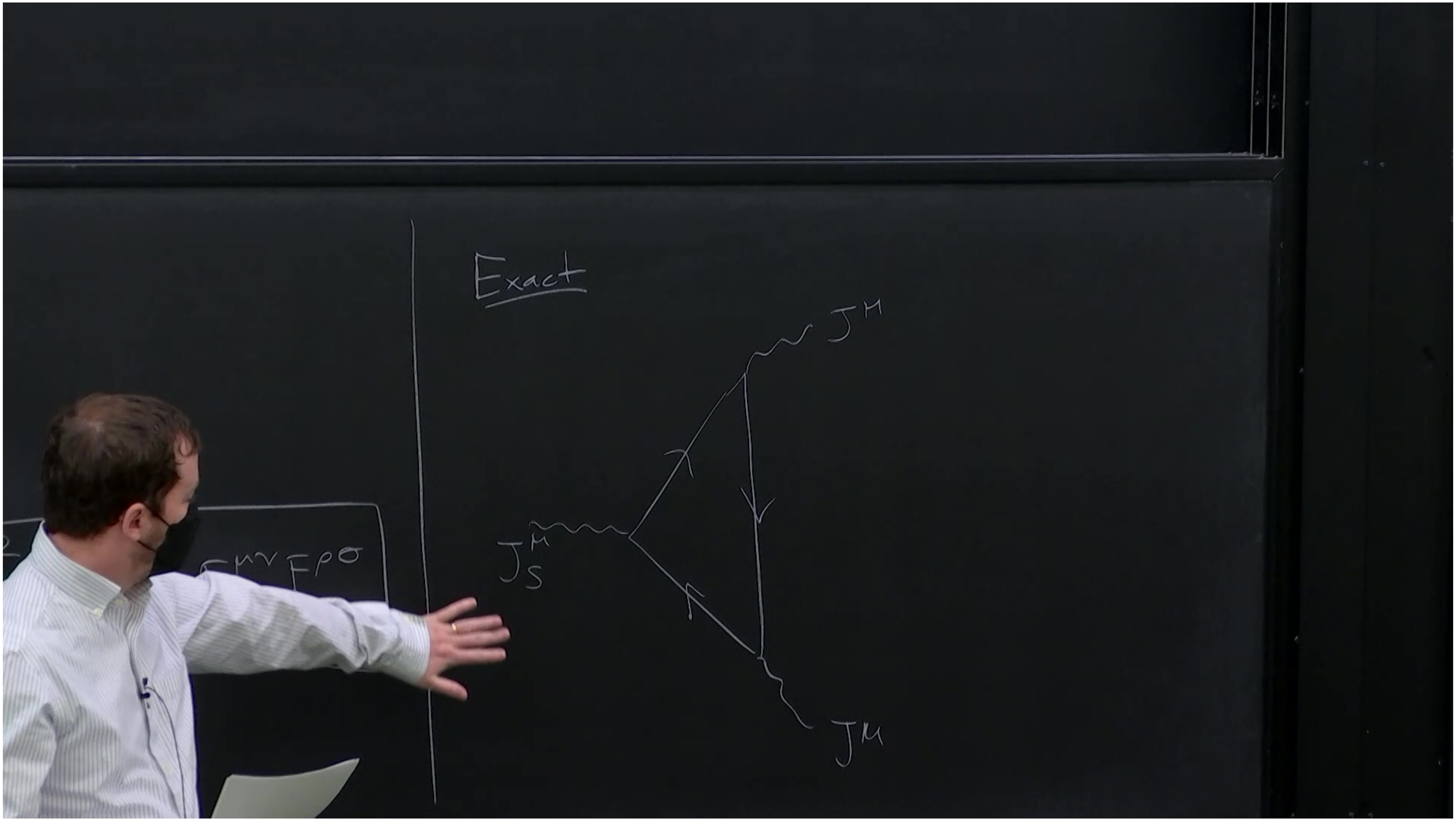
zero modes with ± 1 eigenvalue of γ^5

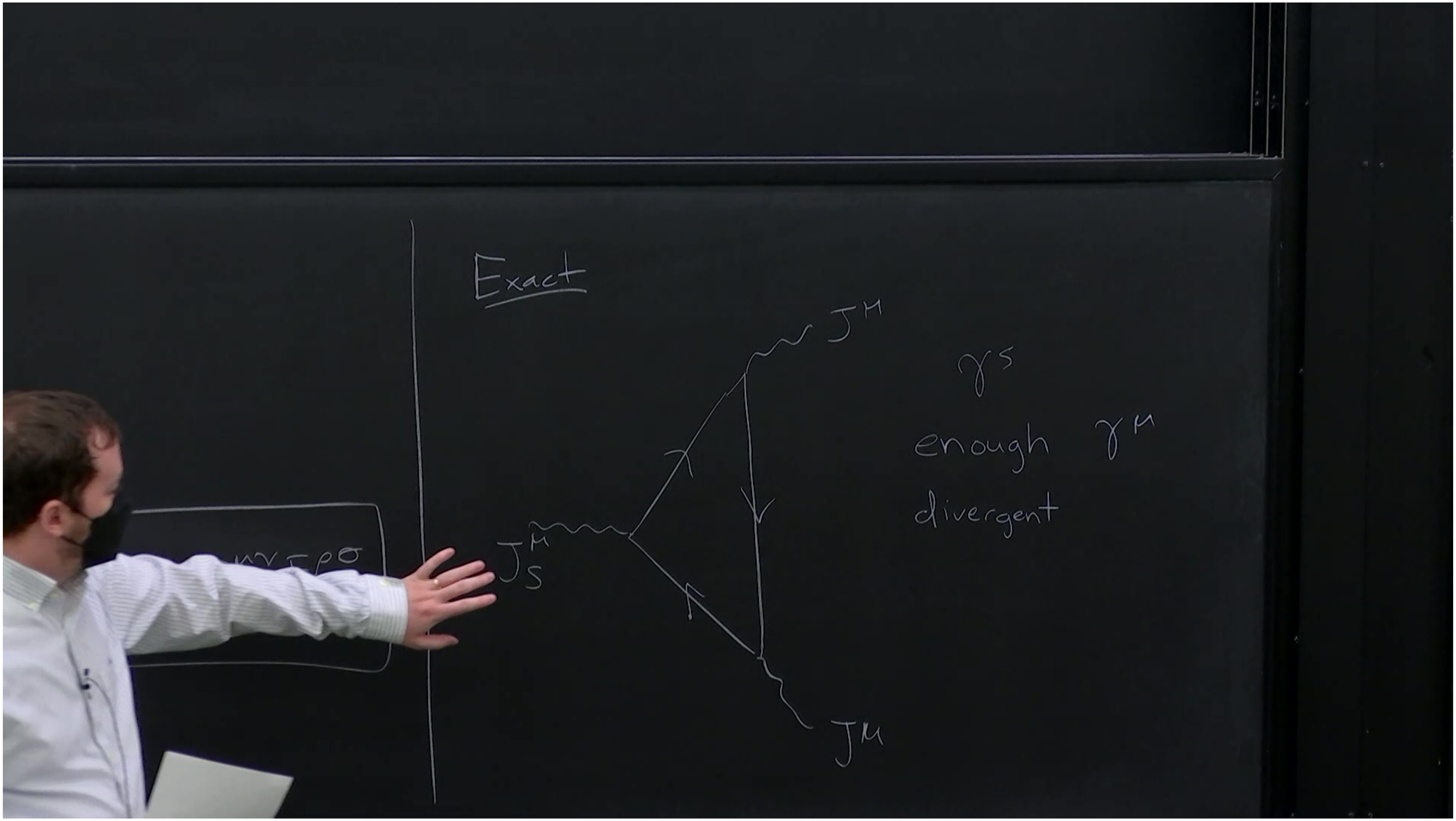
integer! $n_+ - n_-$

$$\boxed{\mathrm{Index}\; i\not{D} = \frac{e^2}{32\pi^2} \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma}}$$

Atiyah-Singer index theorem

Exact


Exact
J^μ
J_S^μ
J^μ


Exact
J^μ
γ^5
enough γ^μ
divergent
J_S^μ
J^μ

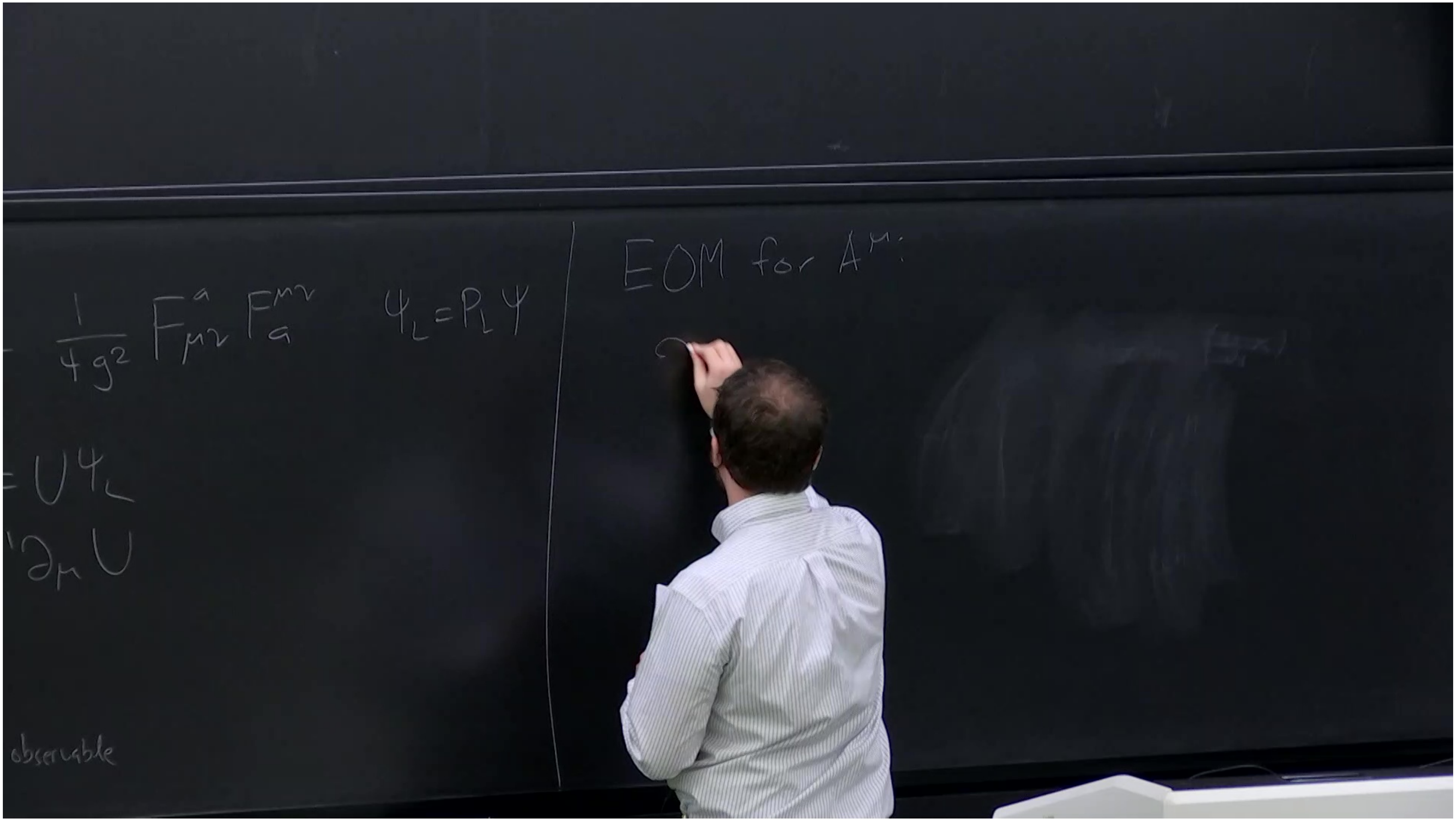
Gauge anomalies

$$\mathcal{L} = \bar{\psi}_L^i \left(i\partial\!\!\!/ + i A^a (T_a)_{ij} \right) \psi_L^j - \frac{1}{4g^2} F^a_{\mu\nu} F_a^{\mu\nu} \qquad \psi_L = P_L \psi$$

Gauge invariance

$$\psi_L \to e^{i\beta^i(x) T^i} \psi_L = U \psi_L$$

$$A_\mu \to U A_\mu U^{-1} - U^{-1} \partial_\mu U$$

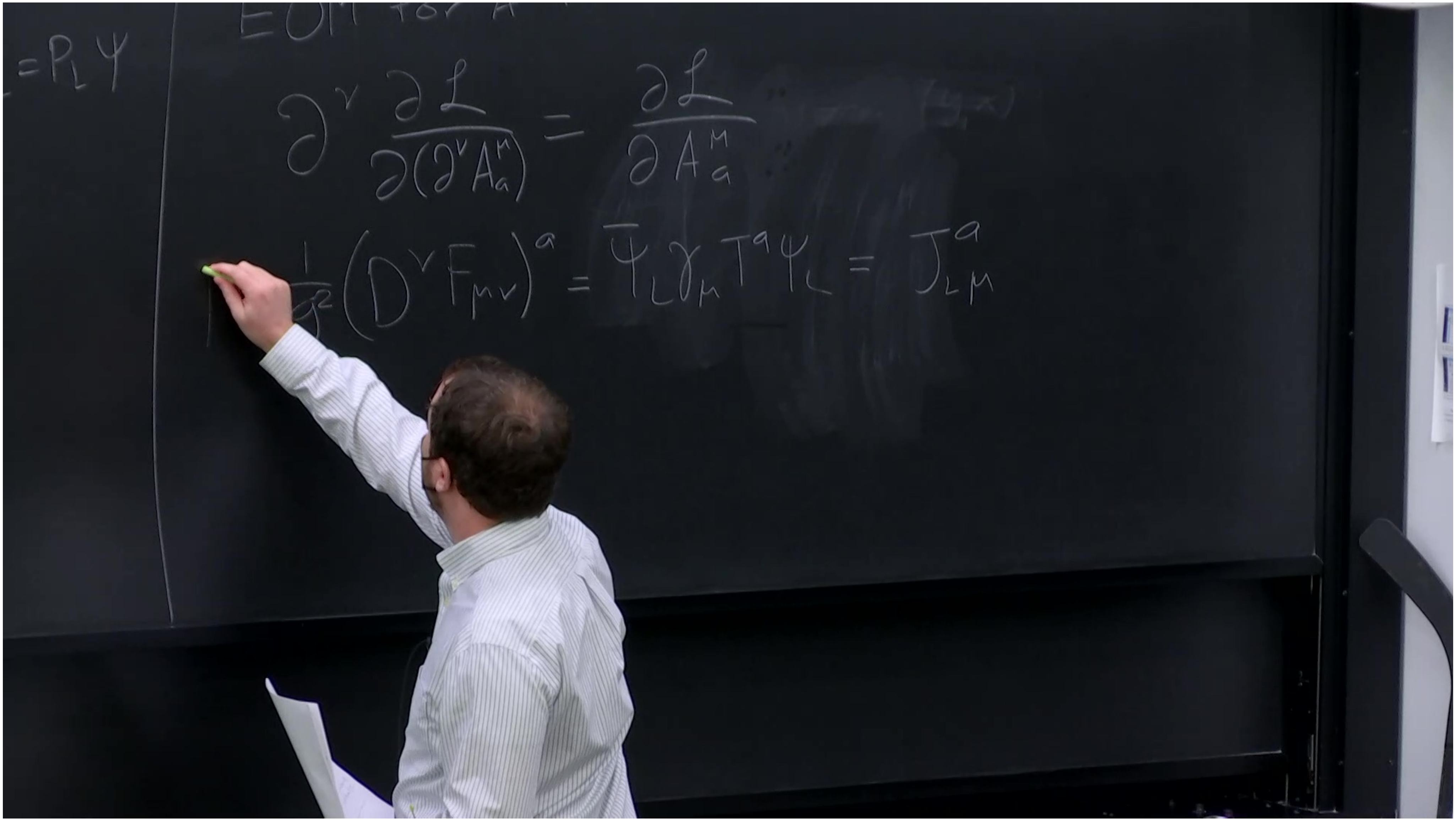

$\frac{1}{4g^2} F^a_{\mu\nu} F_a^{\mu\nu}$
$\psi_L = P_L \psi$
$= U \psi_L$
$\partial_\mu U$
observable
EOM for A^μ:

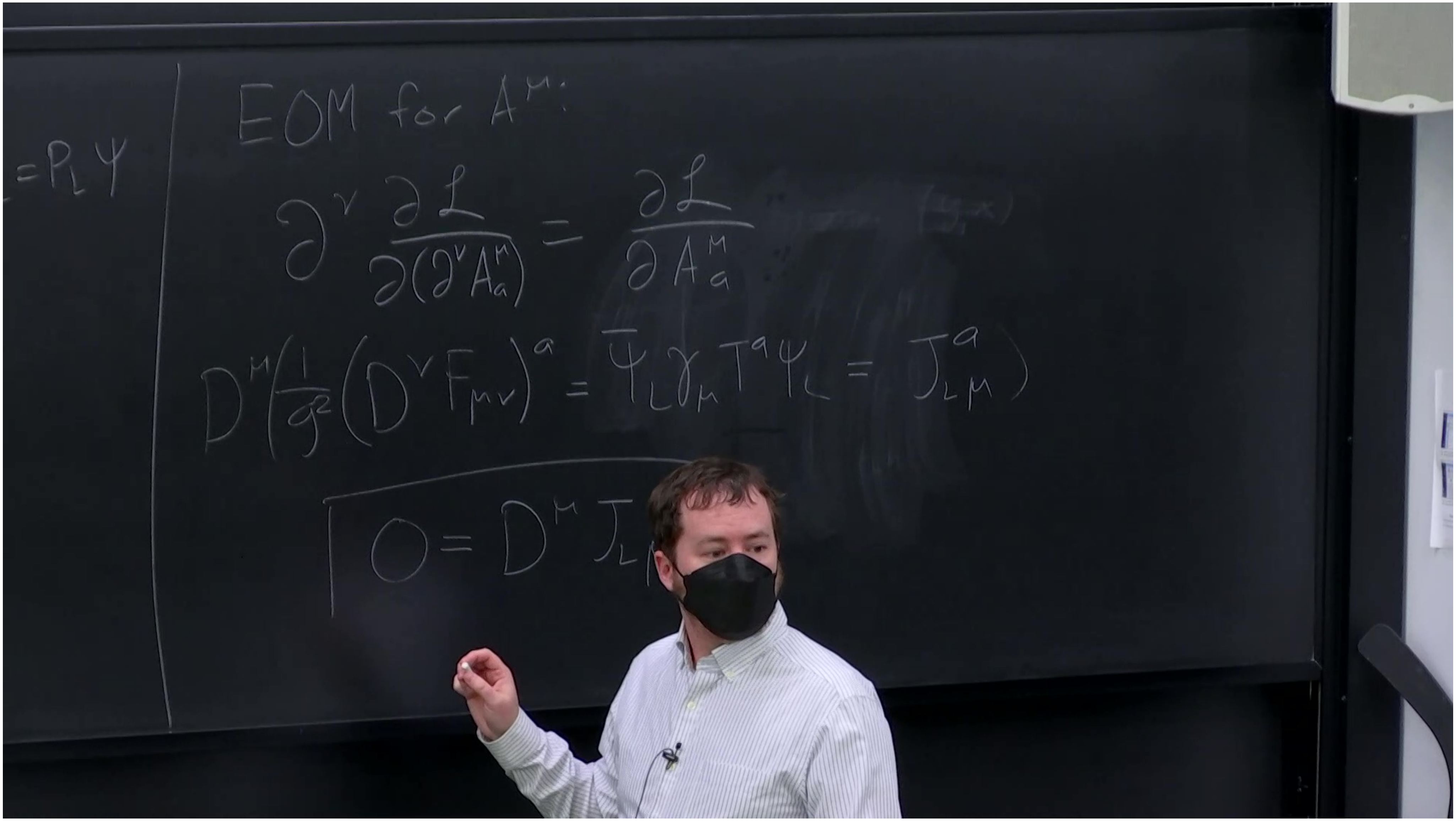
$\psi_L = P_L \psi$

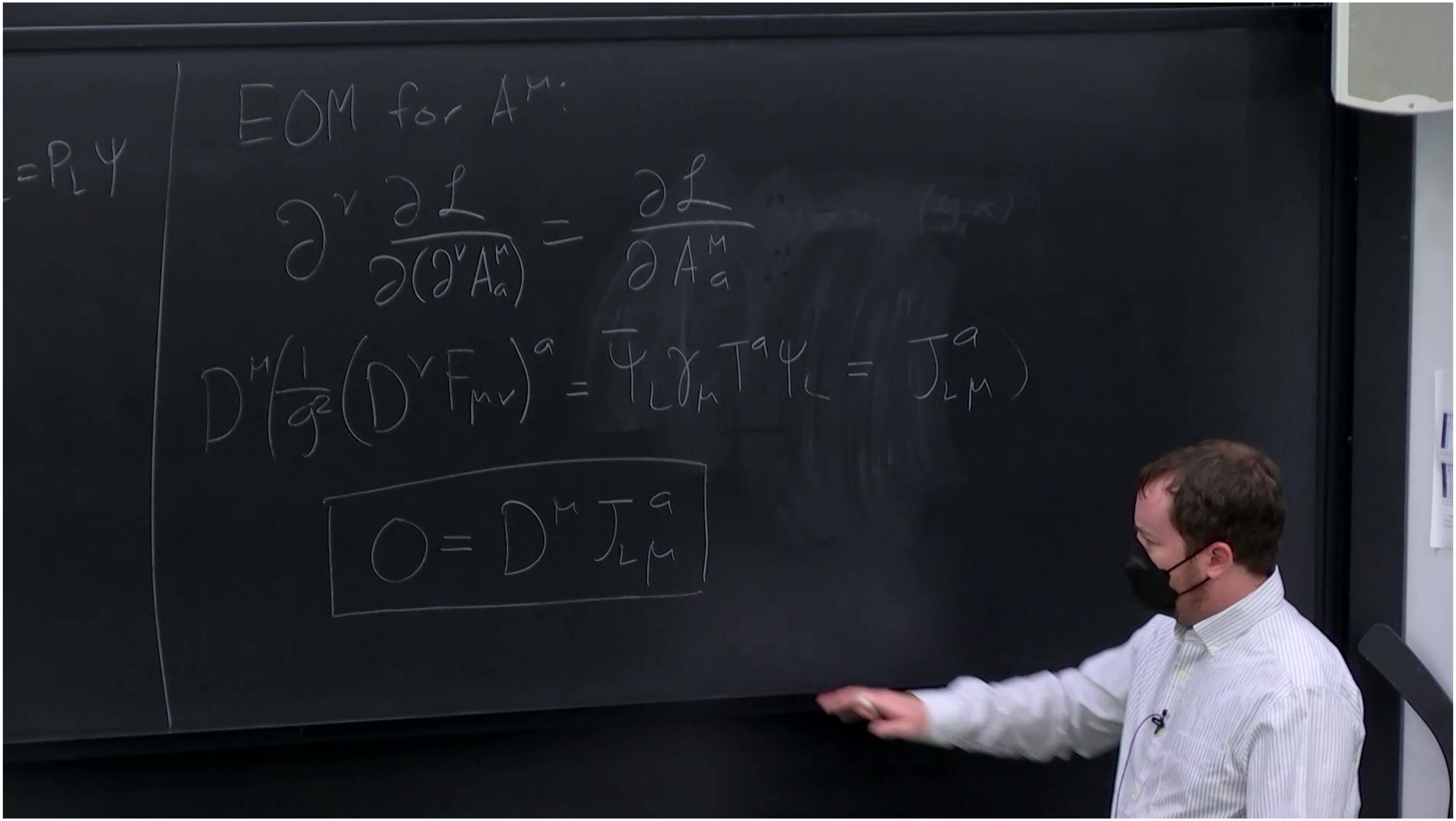
EOM for A^μ:

$$\partial^\nu \frac{\partial \mathcal{L}}{\partial(\partial^\nu A^\mu_a)} = \frac{\partial \mathcal{L}}{\partial A^\mu_a}$$

$$\frac{1}{g^2}\left(D^\nu F_{\mu\nu}\right)^a = \bar{\psi}_L \gamma_\mu T^a \psi_L$$


EOM for A
$= P_L \psi$
$\partial^\nu \frac{\partial \mathcal{L}}{\partial(\partial^\nu A_a^\mu)} = \frac{\partial \mathcal{L}}{\partial A_a^\mu}$
$\frac{1}{g^2}(D^\nu F_{\mu\nu})^a = \bar{\psi}_L \gamma_\mu T^a \psi_L = J_{L\mu}^a$


EOM for A^μ:
$\partial^\nu \frac{\partial \mathcal{L}}{\partial(\partial^\nu A^\mu_a)} = \frac{\partial \mathcal{L}}{\partial A^\mu_a}$
$D^\mu \left(\frac{1}{g^2}(D^\nu F_{\mu\nu})^a = \bar{\Psi}_L \gamma_\mu T^a \Psi_L = J^a_{L\mu}\right)$
$0 = D^\mu J_{L\mu}$
$= P_L \Psi$


$= P_L \psi$
EOM for A^μ:
$\partial^\nu \frac{\partial \mathcal{L}}{\partial(\partial^\nu A^\mu_a)} = \frac{\partial \mathcal{L}}{\partial A^\mu_a}$
$D^\mu \left(\frac{1}{g^2} (D^\nu F_{\mu\nu})^a = \bar{\psi}_L \gamma_\mu T^a \psi_L = J^a_{L\mu} \right)$
$\boxed{0 = D^\mu J^a_{L\mu}}$

$= P_L \psi$

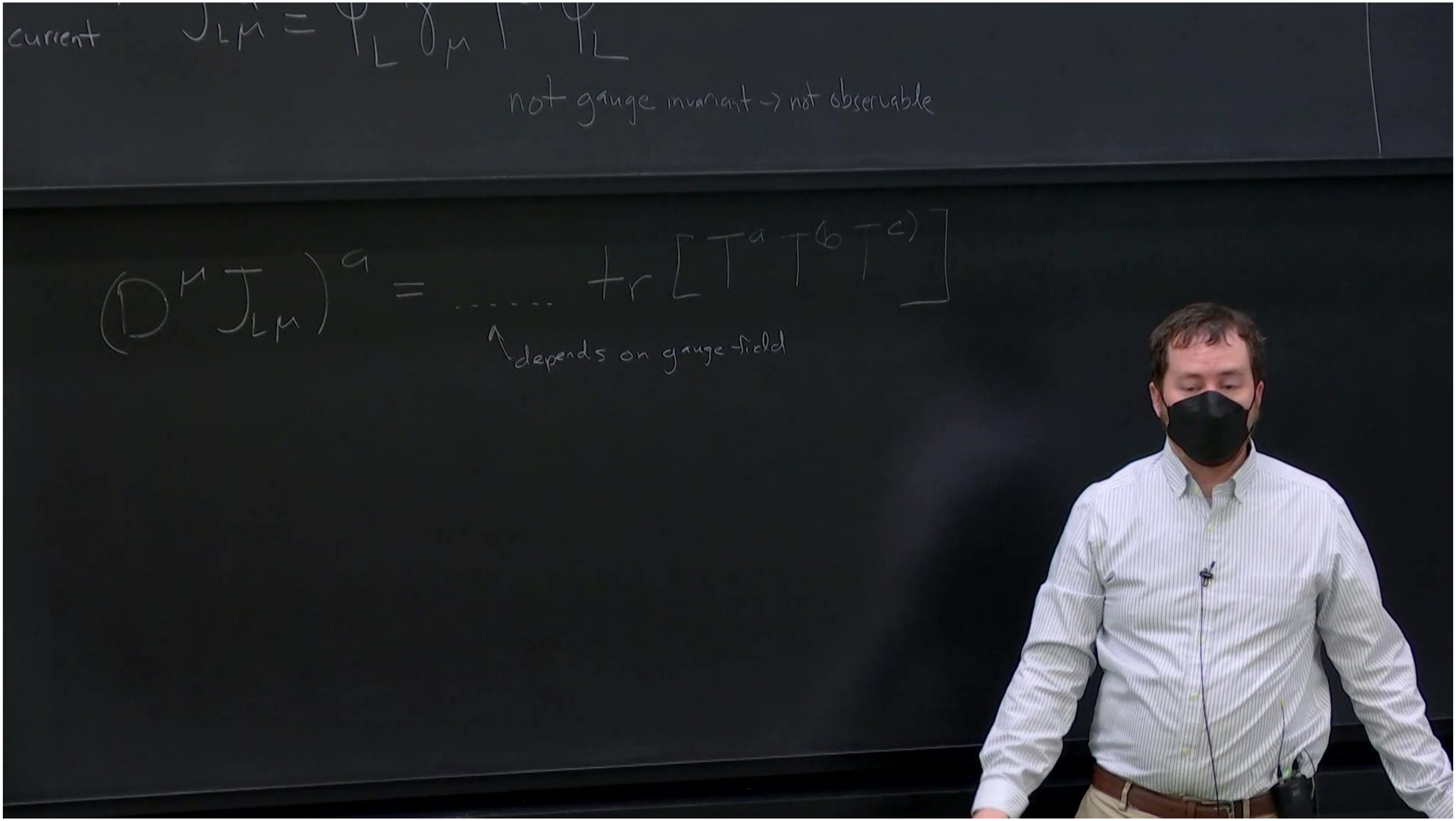
EOM for A^μ:

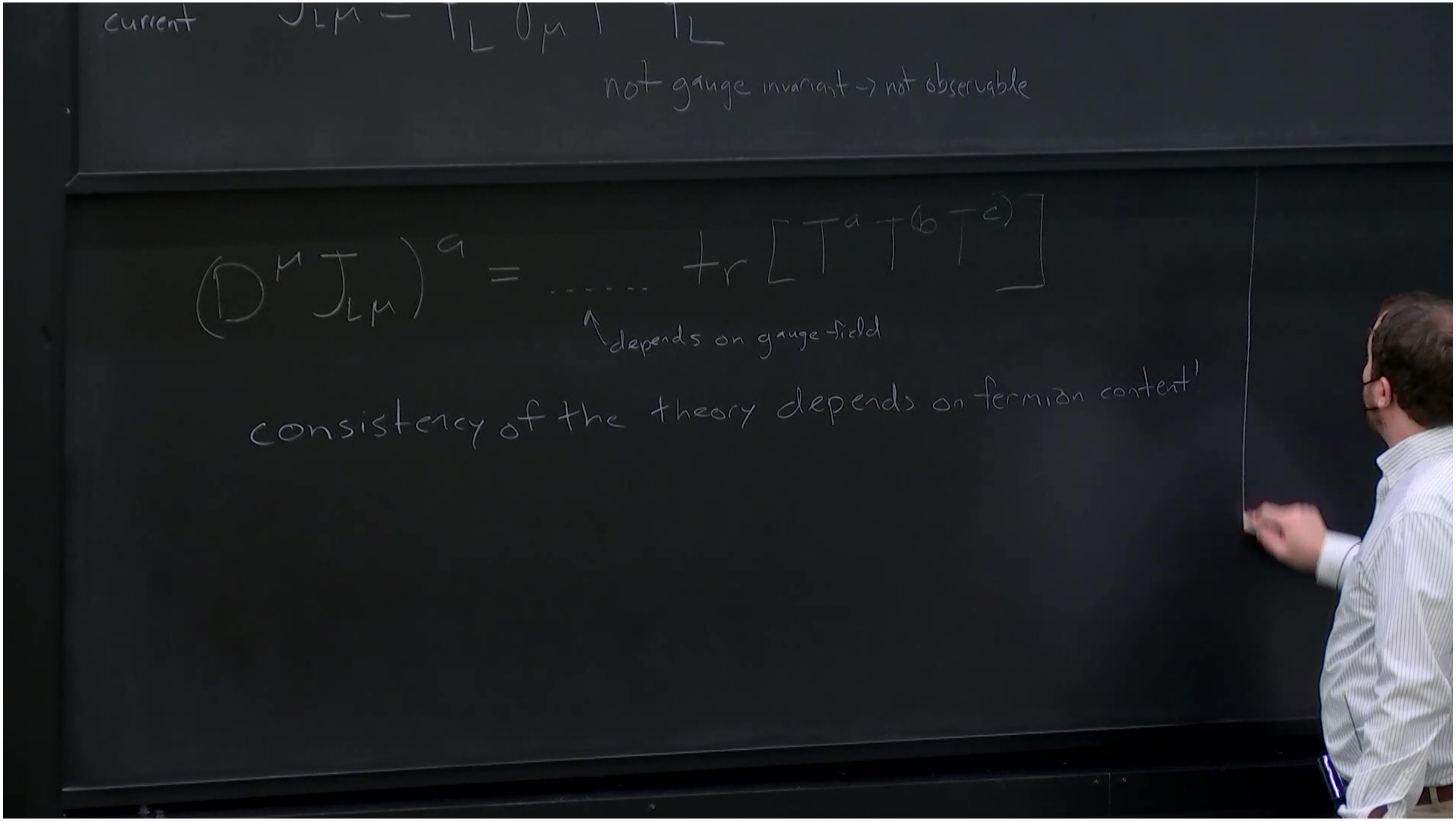
$$\partial^\nu \frac{\partial \mathcal{L}}{\partial(\partial^\nu A^\mu_a)} = \frac{\partial \mathcal{L}}{\partial A^\mu_a}$$

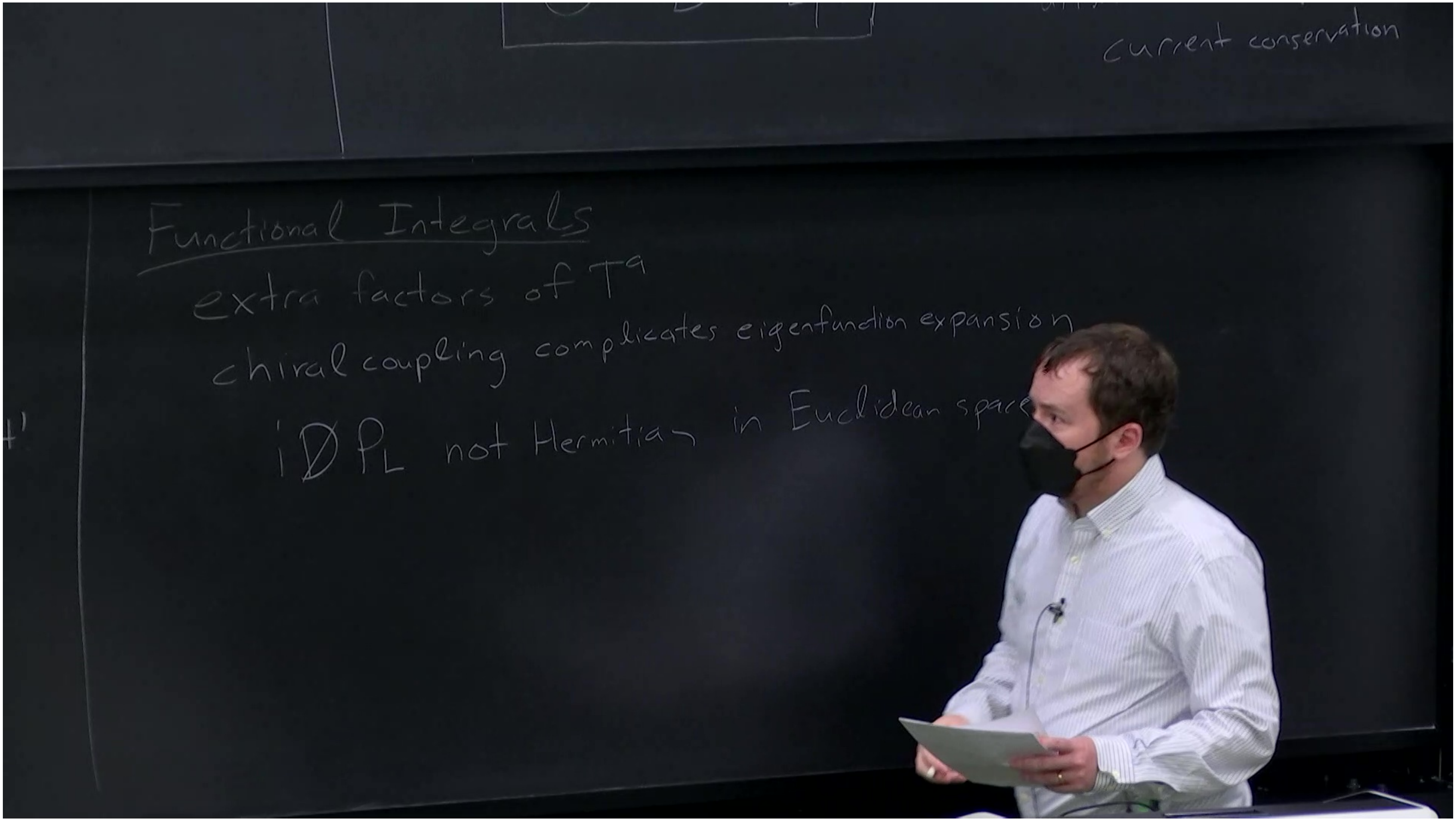
$$D^\mu\left(\frac{1}{g^2}\left(D^\nu F_{\mu\nu}\right)^a = \bar{\psi}_L \gamma_\mu T^a \psi_L = J^a_{L\mu}\right)$$

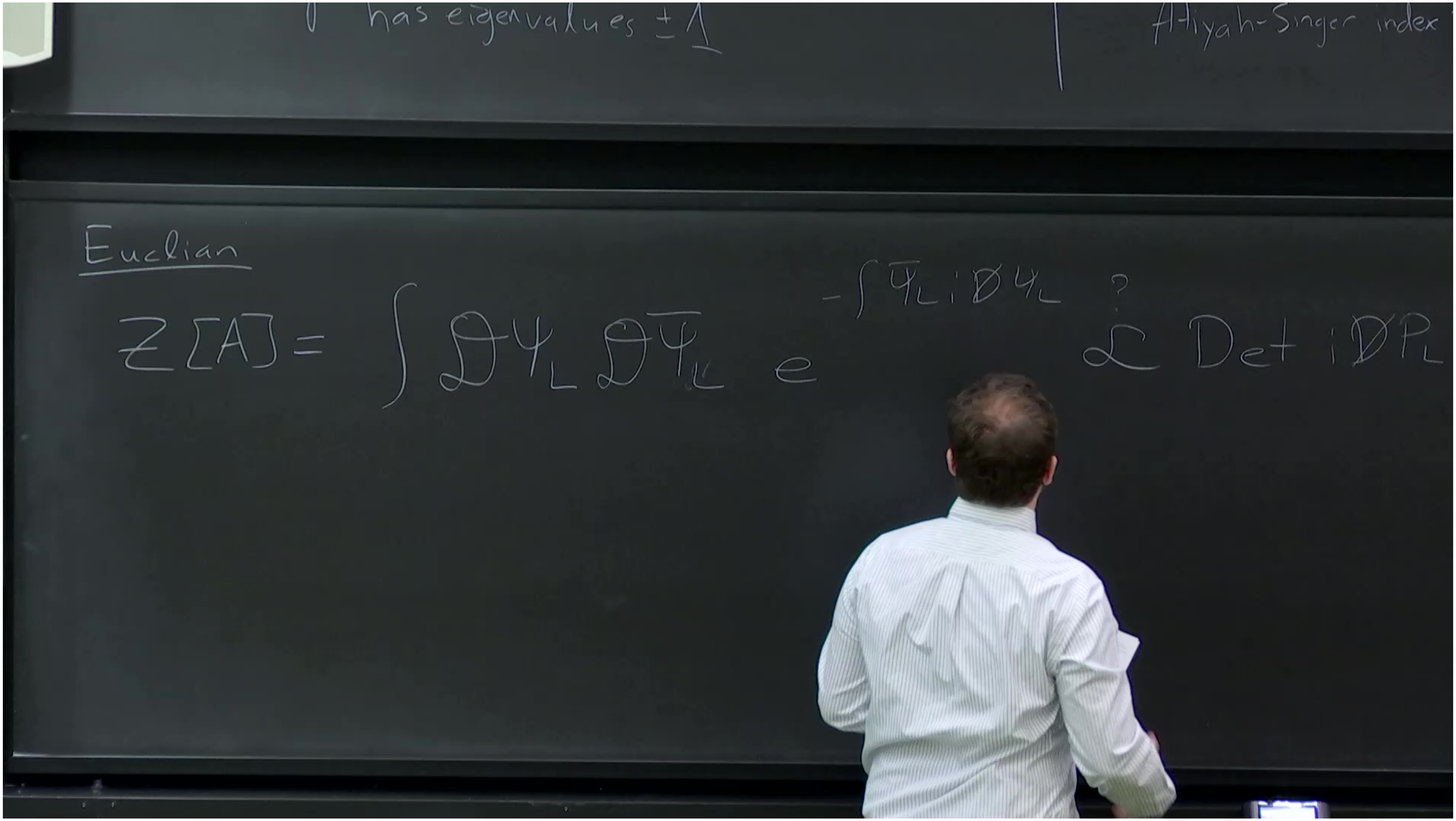
$$\boxed{0 = D^\mu J^a_{L\mu}}$$

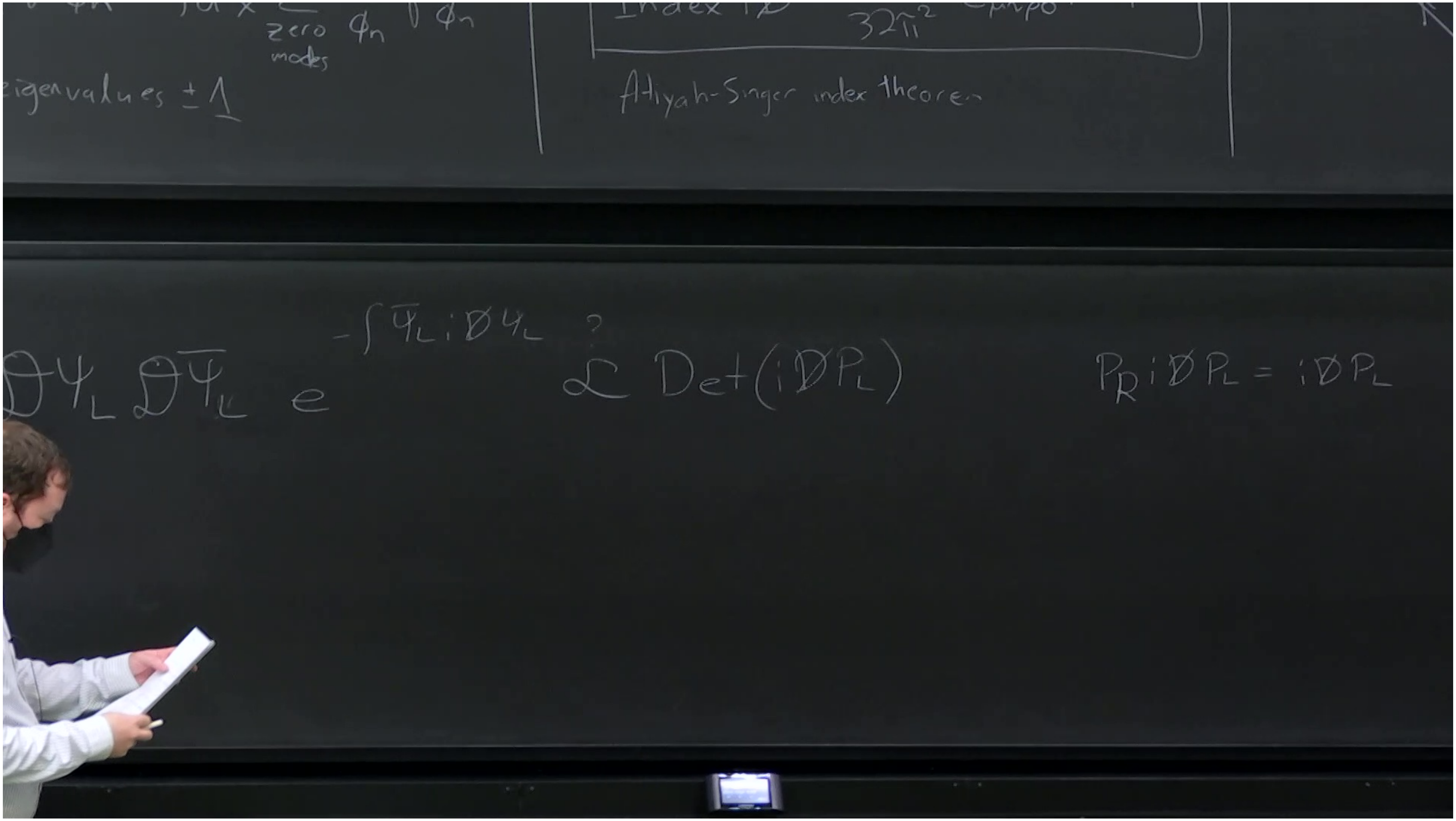
essential for consistency
unlike chiral
current conservation

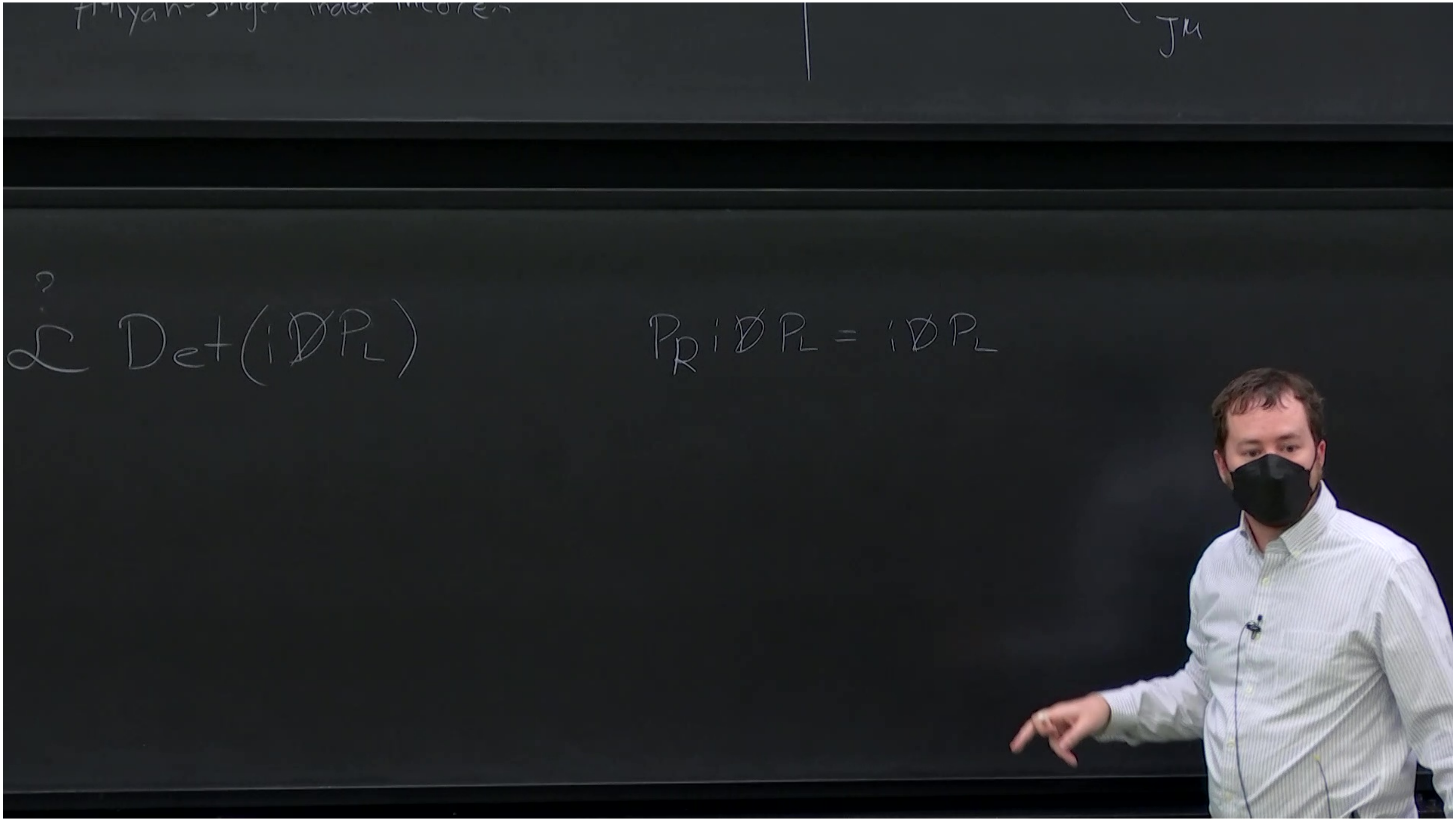

current $J_{L\mu} = \psi_L \gamma_\mu T \psi_L$
not gauge invariant -> not observable
$(D^\mu J_{L\mu})^a = \ldots\ldots \mathrm{tr}[T^a T^{(b} T^{c)}]$
depends on gauge field

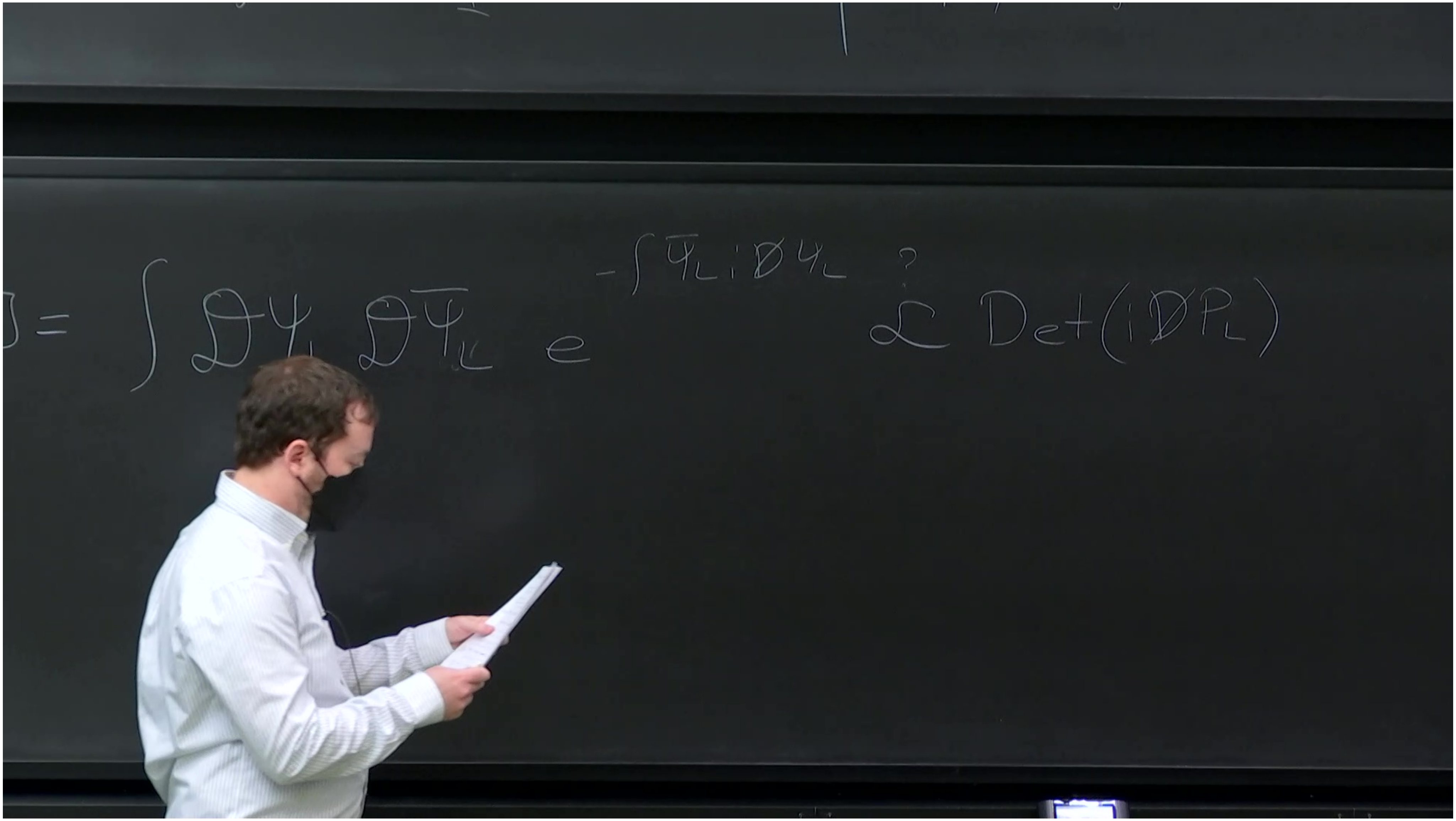

current
not gauge invariant -> not observable
(D^μ J_Lμ)^a = ... tr[T^a T^(b T^c)]
depends on gauge field
consistency of the theory depends on fermion content!

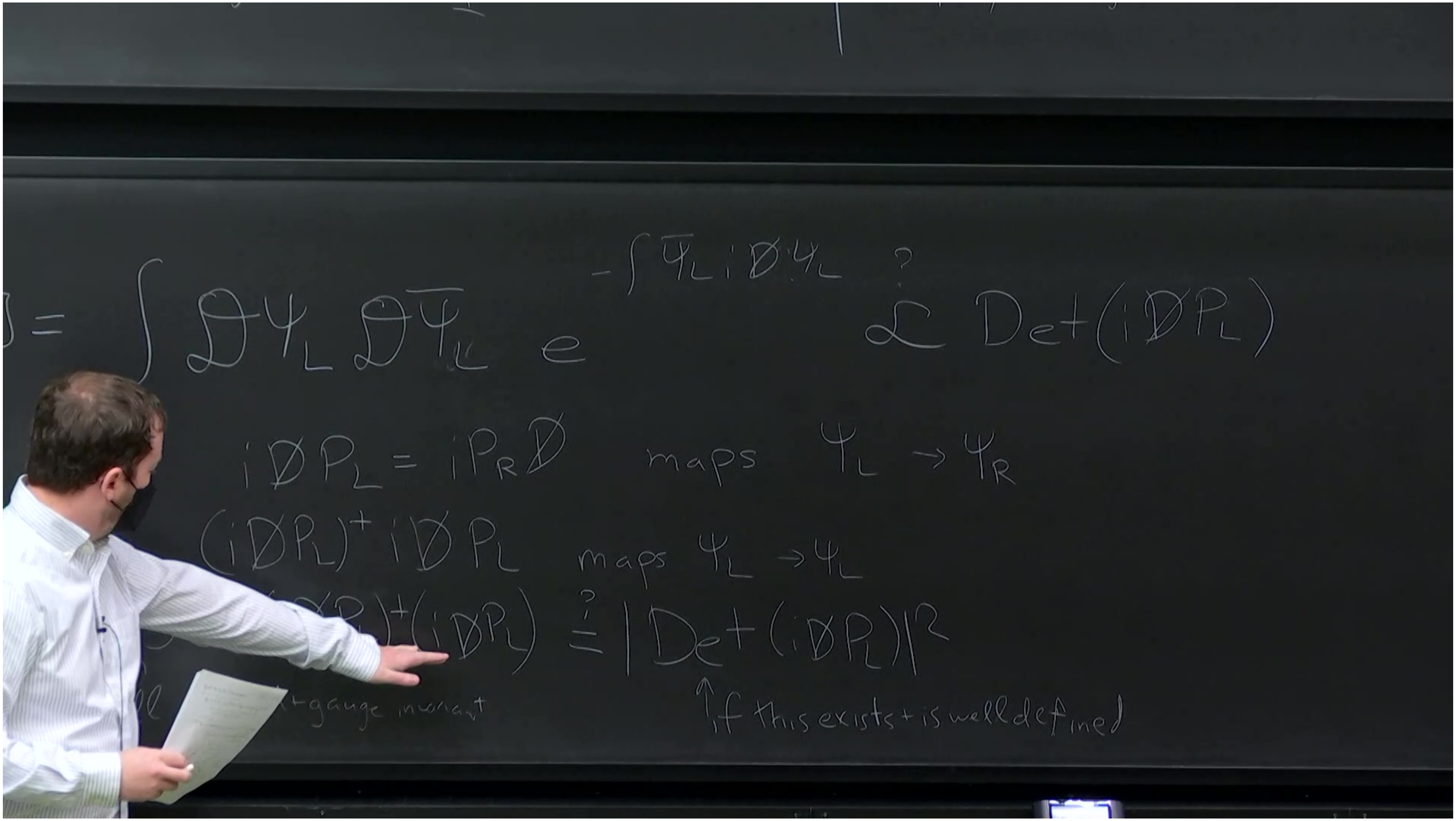

current conservation
Functional Integrals
extra factors of T^a
chiral coupling complicates eigenfunction expansion
$i\not{D} P_L$ not Hermitian in Euclidean space

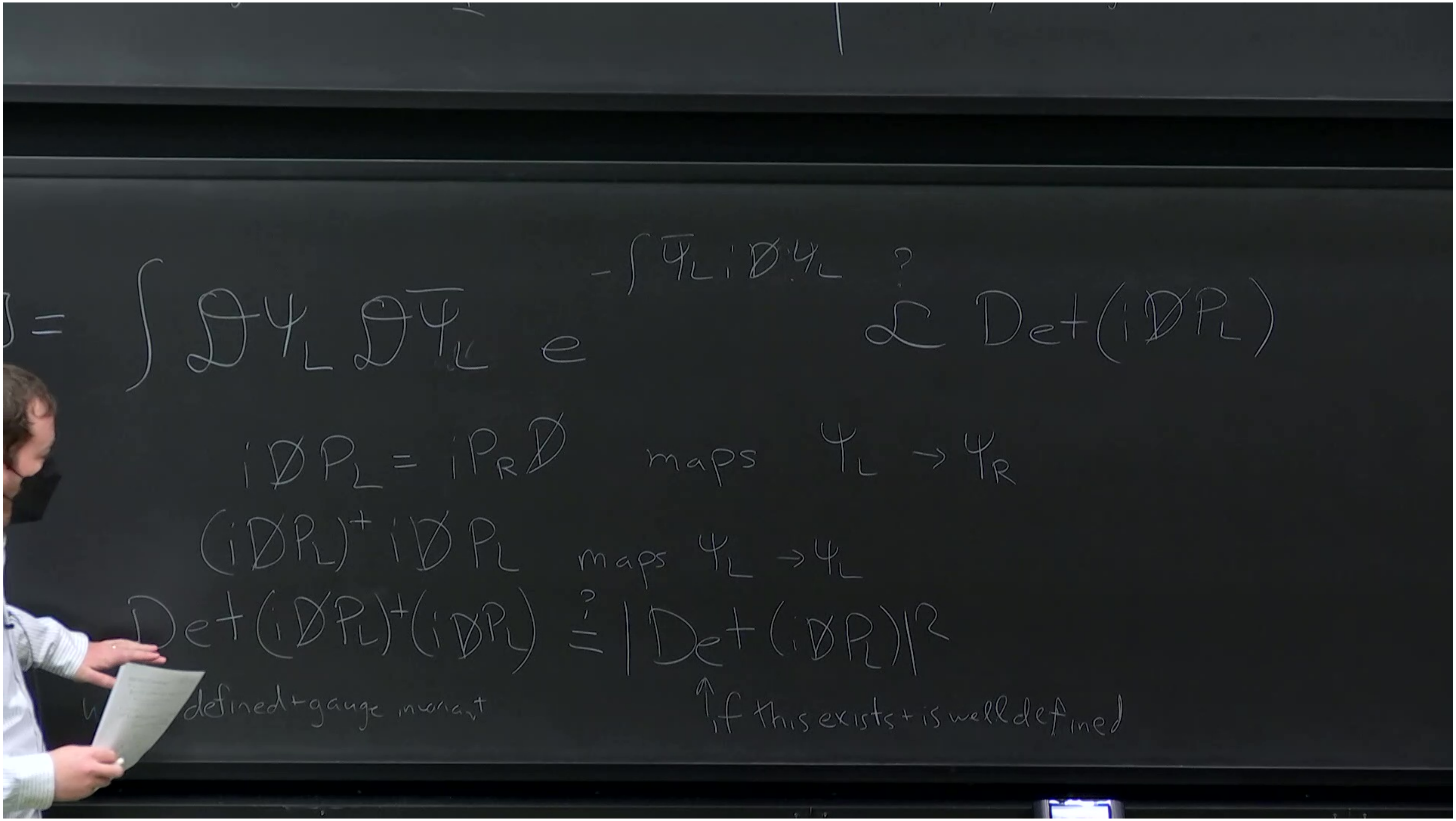

has eigenvalues ±1
Atiyah-Singer index
Euclian
$Z[A] = \int \mathcal{D}\Psi_L \mathcal{D}\bar{\Psi}_L \, e^{-\int \bar{\Psi}_L i \not{D} \Psi_L} \overset{?}{\propto} \mathrm{Det}\, i\not{D} P_L$

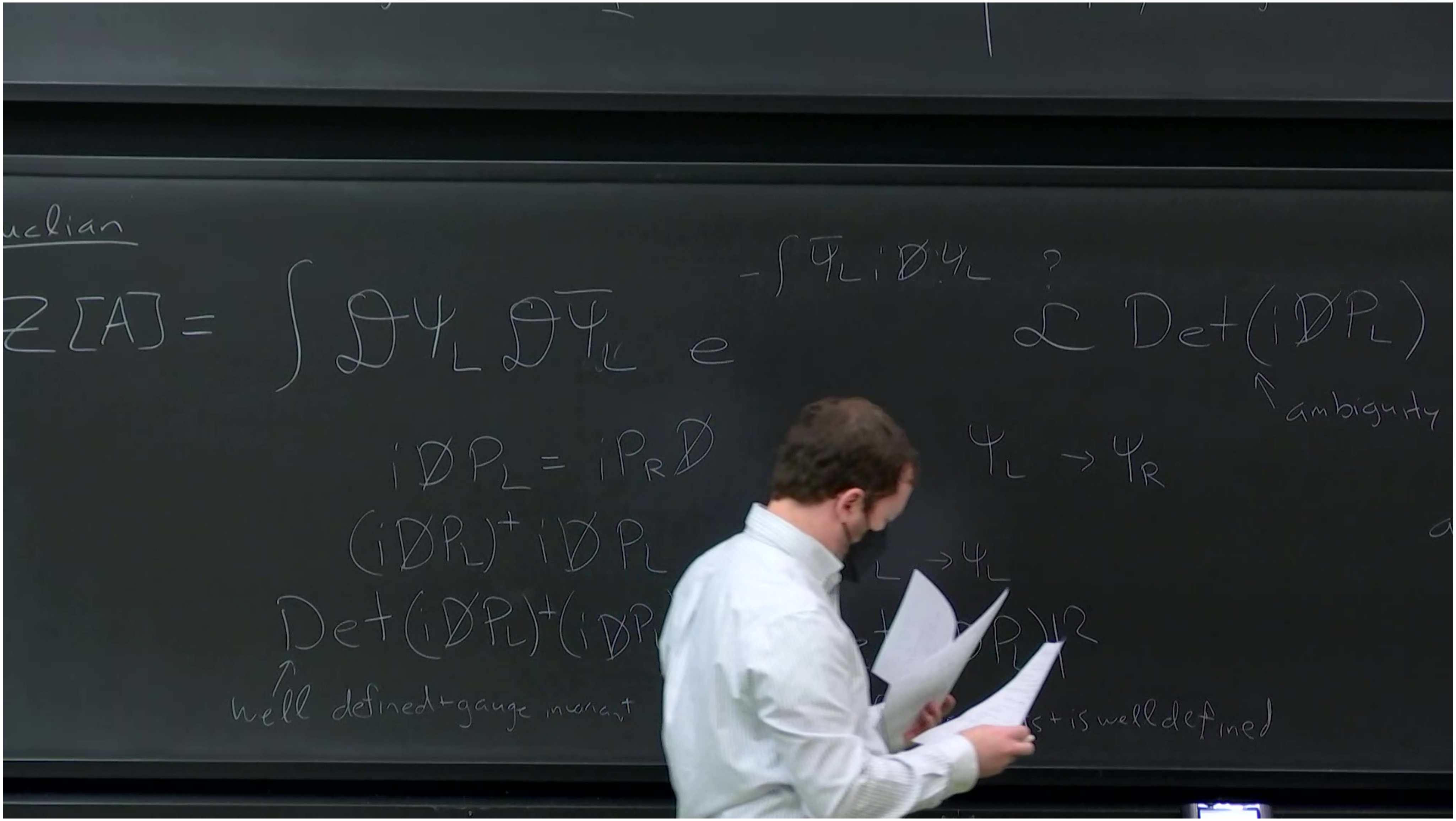

zero modes
eigenvalues ± 1
32π²
Atiyah-Singer index theorem
$\mathcal{D}\psi_L \mathcal{D}\bar\psi_L \, e^{-\int \bar\psi_L i \not{D} \psi_L} \overset{?}{\propto} \mathrm{Det}(i\not{D}P_L)$
$P_R i \not{D} P_L = i \not{D} P_L$


$\overset{?}{\mathcal{L}}$ $\mathrm{Det}(i\not{D}P_L)$
$P_R i\not{D}P_L = i\not{D}P_L$
J^μ


$Z = \int \mathcal{D}\Psi_L \, \mathcal{D}\bar{\Psi}_L \, e^{-\int \bar{\Psi}_L i \not{D} \Psi_L} \overset{?}{\propto} \mathrm{Det}(i \not{D} P_L)$


$= \int \mathcal{D}\Psi_L \mathcal{D}\bar{\Psi}_L \, e^{-\int \bar{\Psi}_L i\not{D} \Psi_L} \overset{?}{\mathcal{L}} \mathrm{Det}(i\not{D} P_L)$
$i\not{D} P_L = i P_R \not{D}$ maps $\Psi_L \to \Psi_R$
$(i\not{D} P_L)^\dagger i\not{D} P_L$ maps $\Psi_L \to \Psi_L$
$(i\not{D} P_L)^\dagger (i\not{D} P_L) \overset{?}{=} |\mathrm{Det}(i\not{D} P_L)|^2$
+ gauge invariant
if this exists + is well defined

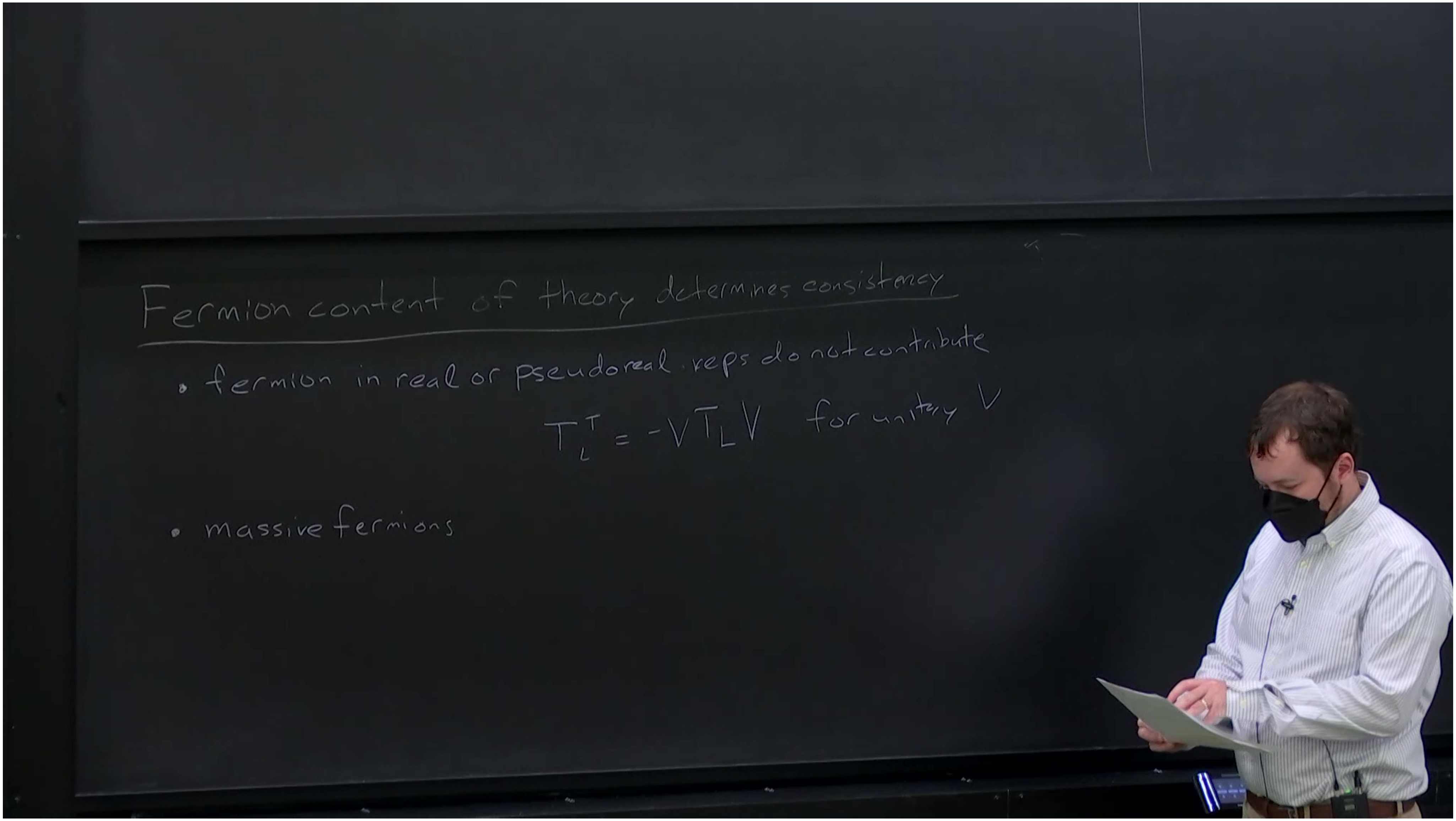

$Z = \int \mathcal{D}\Psi_L \mathcal{D}\bar{\Psi}_L \, e^{-\int \bar{\Psi}_L i \not{D} \Psi_L} \overset{?}{\mathcal{L}} \, \mathrm{Det}(i\not{D}P_L)$
$i\not{D}P_L = iP_R\not{D}$ maps $\Psi_L \to \Psi_R$
$(i\not{D}P_L)^\dagger i\not{D}P_L$ maps $\Psi_L \to \Psi_L$
$\mathrm{Det}(i\not{D}P_L)^\dagger(i\not{D}P_L) \overset{?}{=} |\mathrm{Det}(i\not{D}P_L)|^2$
defined + gauge invariant
if this exists + is well defined

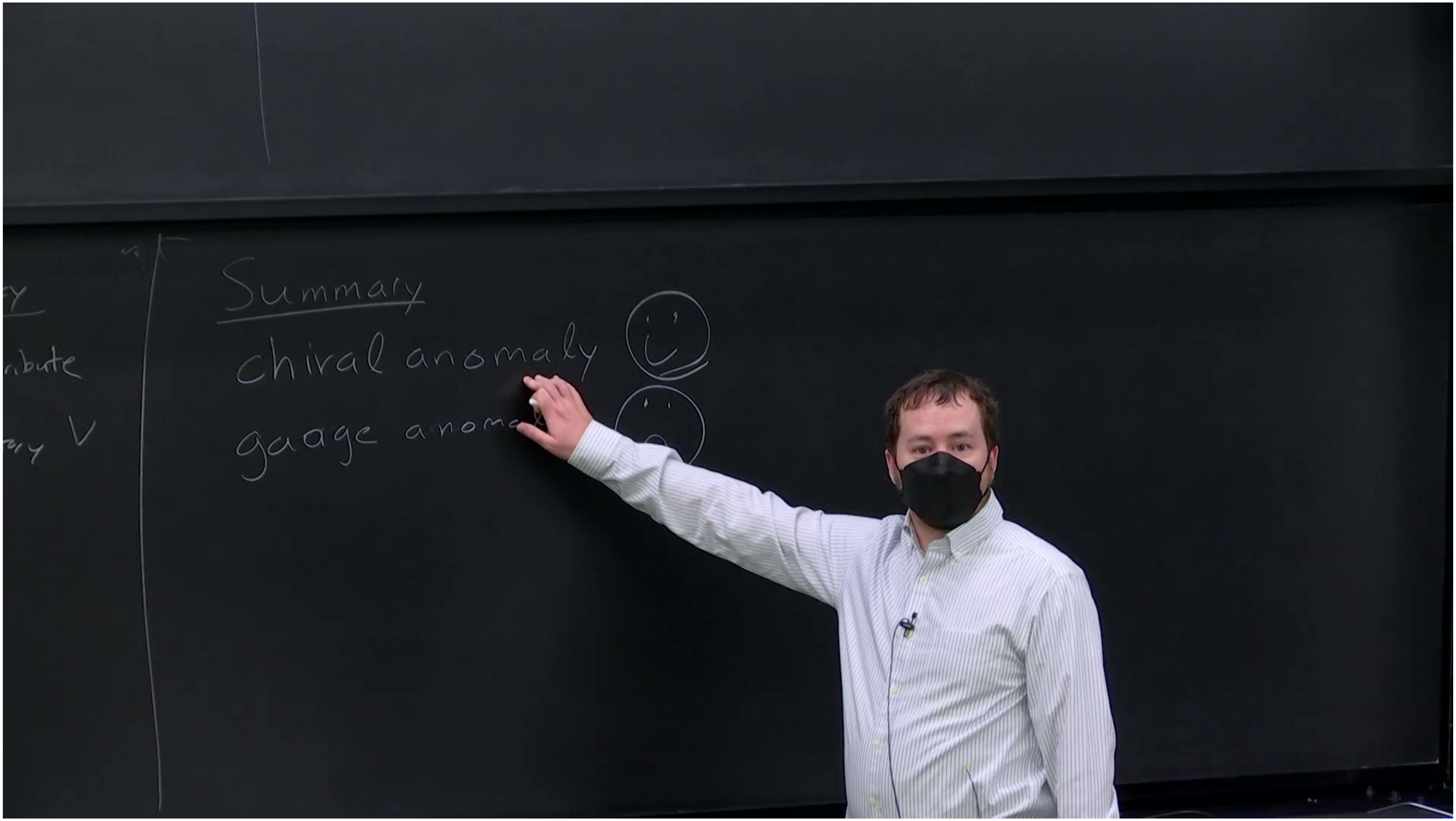

uclian
Z[A] = ∫ DΨ_L DΨ̄_L e^{-∫ Ψ̄_L i D̸ Ψ_L} ?= Det(iD̸P_L)
ambiguity
iD̸P_L = iP_R D̸
Ψ_L → Ψ_R
(iD̸P_L)^+ iD̸P_L
→ Ψ_L
Det(iD̸P_L)^+(iD̸P
well defined + gauge invariant
is well defined

Fermion content of theory determines consistency

- fermion in real or pseudoreal reps do not contribute

$$T_L^T = -V T_L V \quad \text{for unitary } V$$


Fermion content of theory determines consistency
• fermion in real or pseudoreal reps do not contribute
$T_L^T = -V T_L V$ for unitary V
• massive fermions


Summary
chiral anomaly
gauge