

Title: Quantum Fields and Strings - Lecture 2

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Anomalous Non-conservation Equation

$$\Delta A_1 = \frac{2\pi}{L} \rightarrow \Delta Q_5 = -2$$

$$\boxed{\partial_\mu J_5^\mu = ?}$$

$$\frac{\Delta Q_5}{\Delta t} = -\frac{L}{2\pi} \frac{\Delta A_1}{\Delta t}$$

$$\dot{Q}_5 = -\frac{L}{2\pi} \dot{A}_1$$

the state $|\hat{n}\rangle$

$$Q_5 = \int d\mathbf{x}$$

the state $\hat{p}_1$ and $\hat{p}_2$ of the system

$$Q_5 = \int dx J_5^0$$

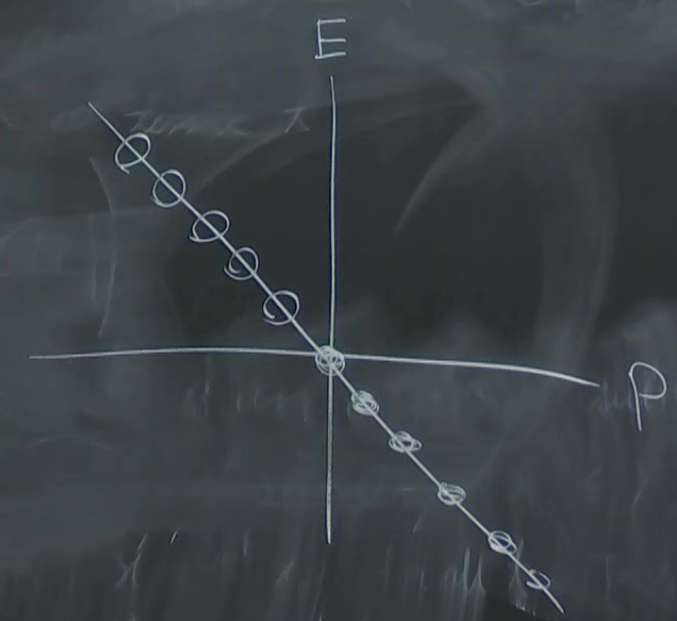
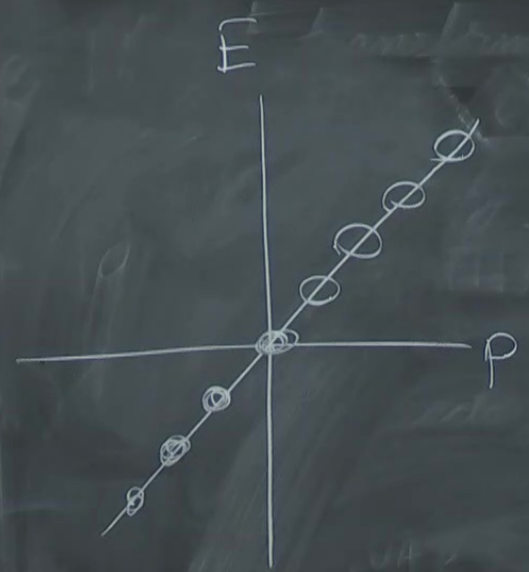
$$L A_1 = \int dx A_1 = \int dx \epsilon^{0\mu} A_\mu dx$$

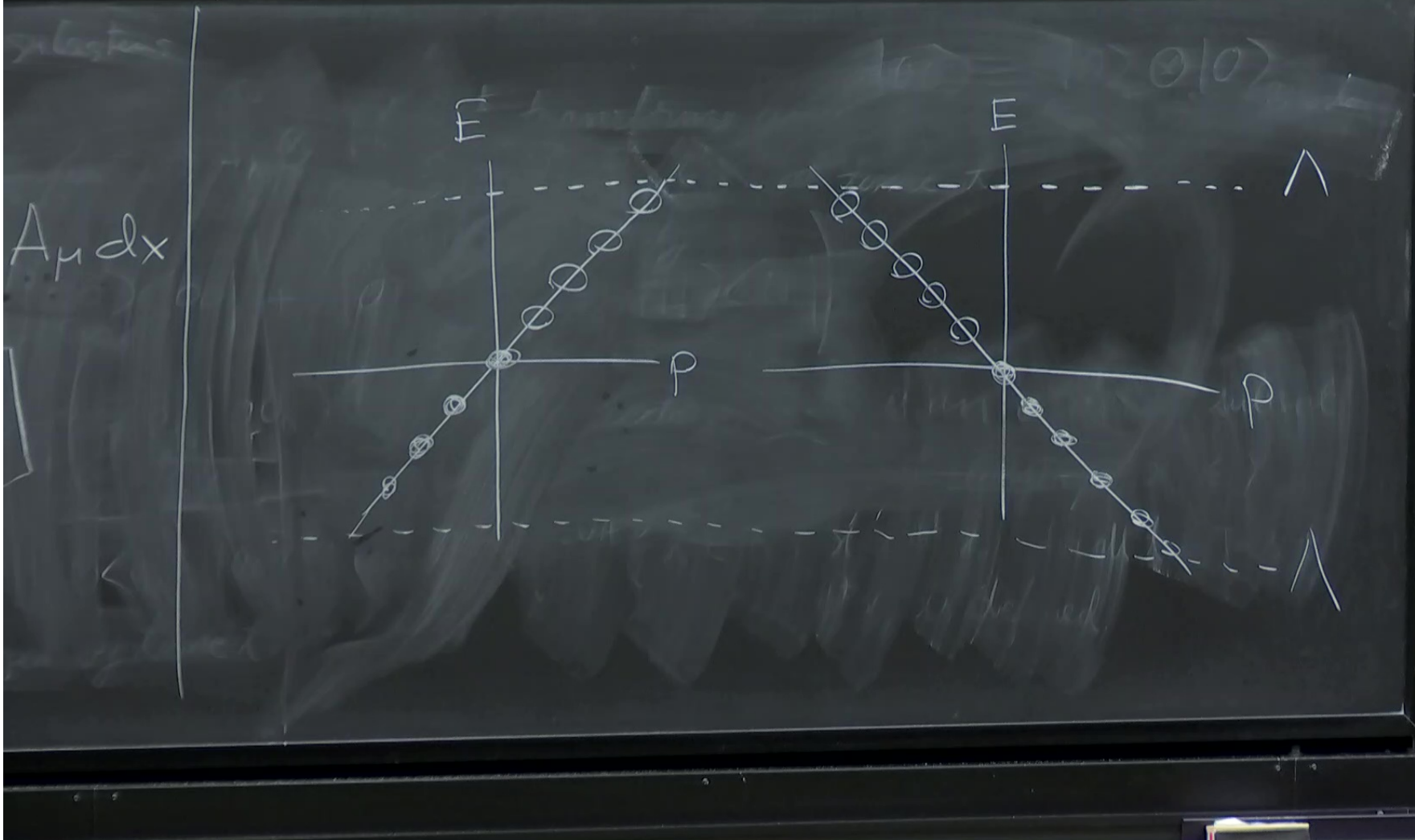
$$\partial_\mu J_5^\mu = \frac{1}{i} \epsilon^{\mu\nu} F_{\mu\nu}$$

of the system

$dx \in {}^{\text{on}} A_\mu dx$

$F_{\mu\nu}$



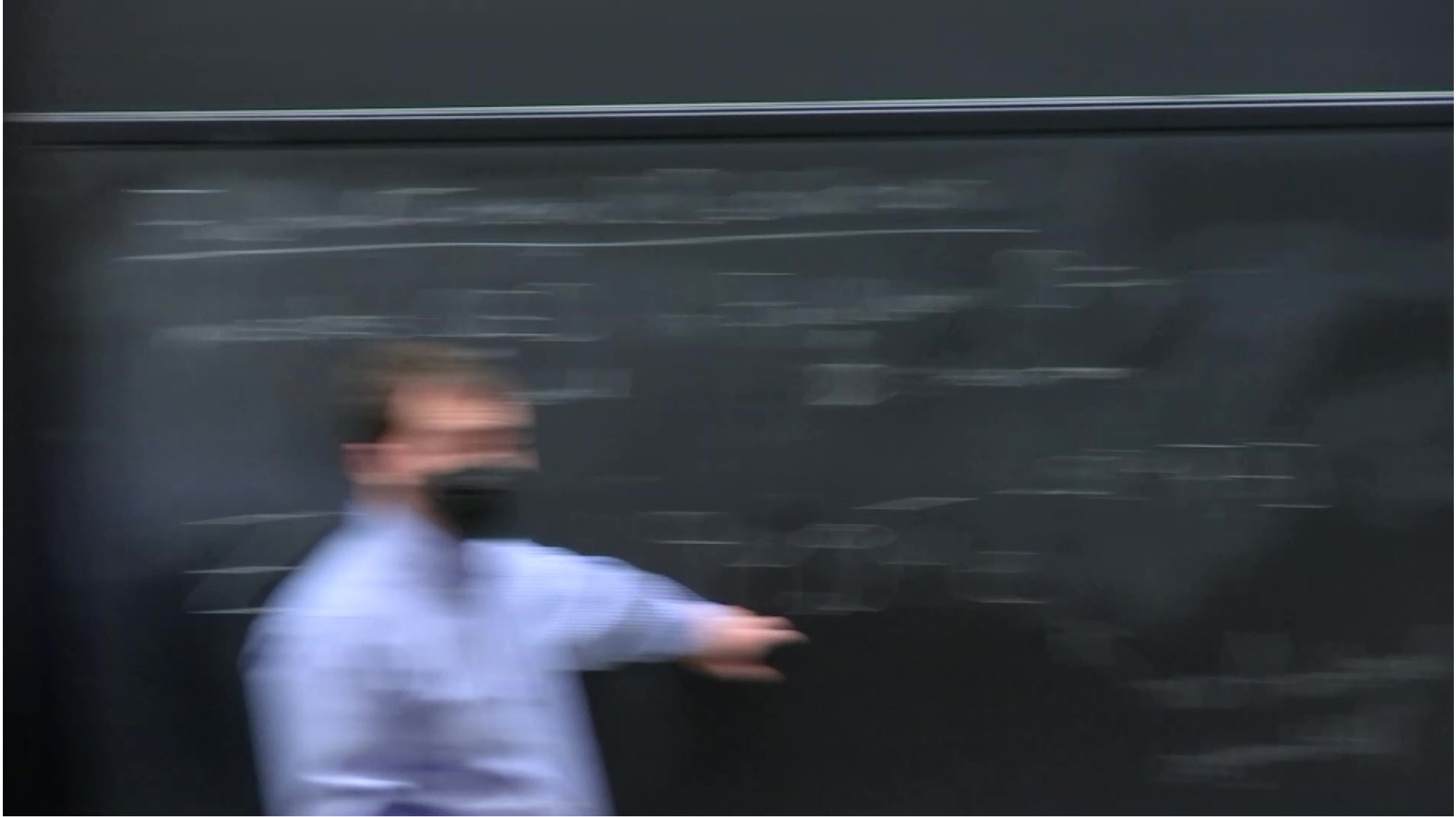


Functional Integral Derivation

massless QED in Euclidean $D=4$
not essential $i\cancel{D}$ is Hermitian

$$Z[A] = \int \mathcal{D}\psi \mathcal{D}\bar{\psi} e^{-S[\psi, \bar{\psi}, A]}$$

no A integral



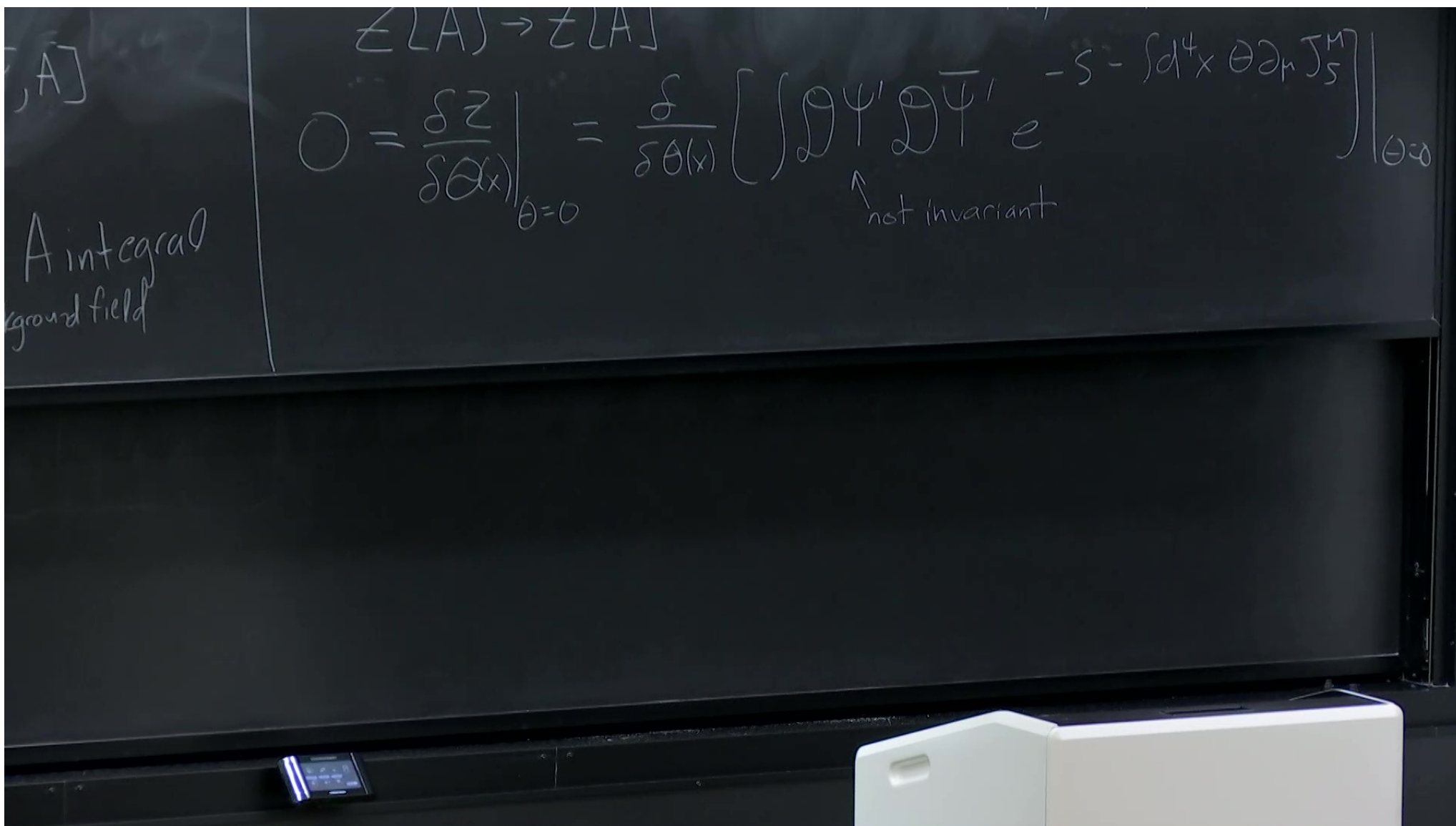
Local chiral transformation

$$S \rightarrow S + \int d^4x \, \Theta(x) \partial_\mu J_5^\mu$$

$$Z[A] \rightarrow Z[A]$$

$$\psi \rightarrow \psi e^{i\Theta(x)\gamma^5}$$

$$A_\mu \rightarrow A_\mu$$



trace

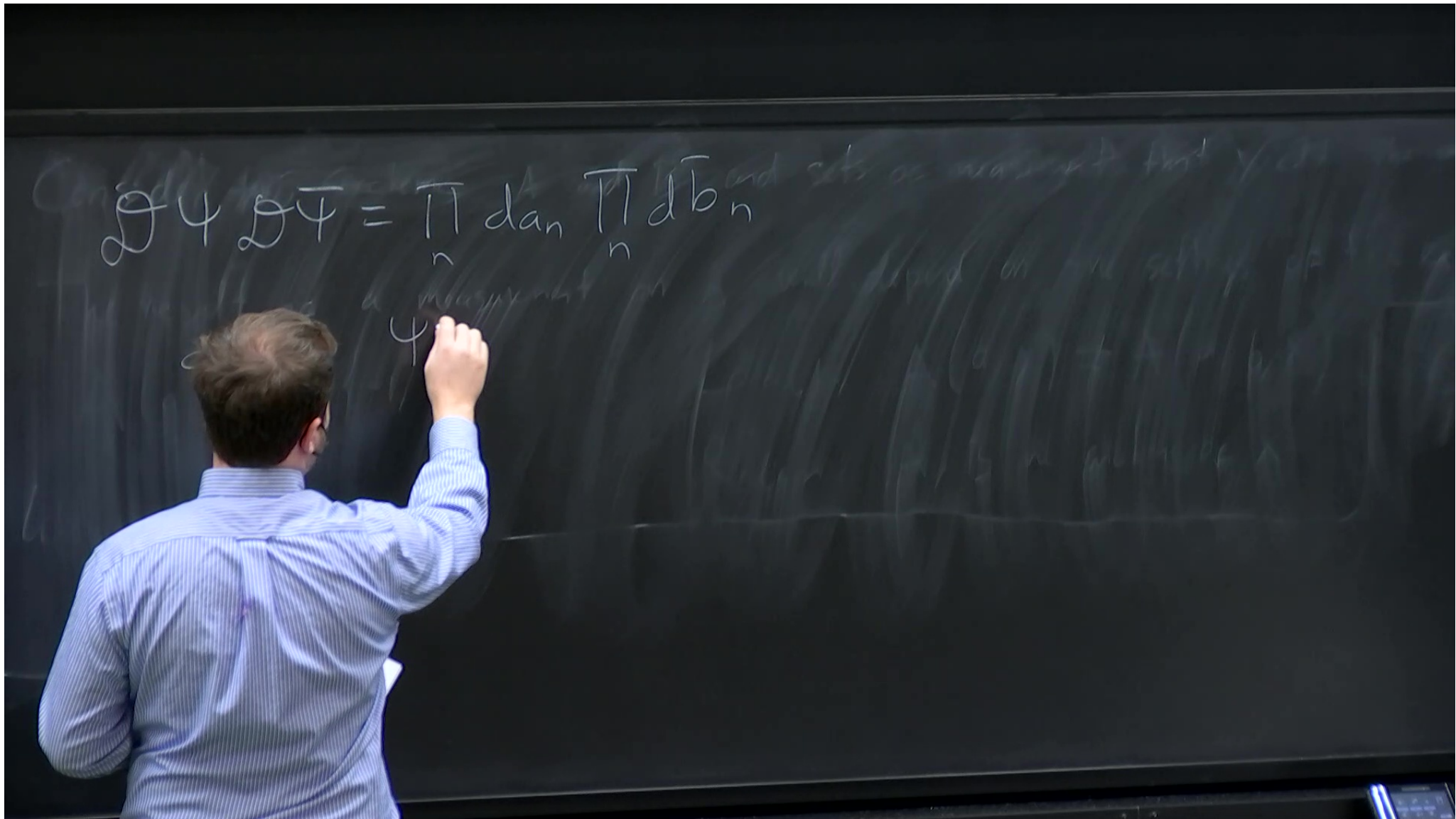
Define measure

$$i \not{D} \phi_n = \lambda_n \phi_n$$

↑ ↖
eigenfunctions real eigenvalues

ϕ_n orthonormal
complete

$$\int d^4x \phi_m^\dagger \phi_n = \delta_{nm}$$
$$\sum_n \phi_{n\alpha}^\dagger(x) \phi_{n\beta}(y) = \delta(x-y) \delta_{\alpha\beta}$$



$$\mathcal{D}\Psi \mathcal{D}\bar{\Psi} = \prod_n da_n \prod_n d\bar{b}_n$$

$$\begin{aligned} a_n' &= \langle \phi_n | \Psi \rangle \\ &= \langle \phi_n | e^{i\theta \gamma^5} | \Psi \rangle \\ &= \sum_m a_m \underbrace{\langle \phi_n | e^{i\theta \gamma^5} | \phi_m \rangle}_{C_{nm}} \end{aligned}$$

$$\begin{aligned} \prod da_n' &= (\det C)^{-1} \prod da_n \\ &= e^{-\text{Tr} \log C} \prod da_n \\ &= e^{i \text{Tr} \theta \gamma^5} \prod da_n \end{aligned}$$

$$S \rightarrow S + \int d^4x \theta(x) \partial_\mu J_5^\mu$$

$$A_\mu \rightarrow A_\mu$$

$$Z[A] \rightarrow Z[A]$$

$$0 = \left. \frac{\delta Z}{\delta \theta(x)} \right|_{\theta=0} = \frac{\delta}{\delta \theta(x)} \left[\int \mathcal{D}\psi' \mathcal{D}\bar{\psi}' e^{-S - \int d^4x \theta \partial_\mu J_5^\mu} \right] \Big|_{\theta=0}$$

↑ not invariant

$$= \frac{\delta}{\delta \theta(x)} \left[\int \mathcal{D}\psi \mathcal{D}\bar{\psi} e^{-S - \theta \cdot \partial_\mu J_5^\mu - 2i \text{Tr} \theta \gamma^5} \right]$$

$$\text{Tr } \Theta \gamma^5 = \int d^4x \phi_{n\alpha}^+(x) \gamma_{\alpha\beta}^5 \phi_{n\beta}(x) \Theta(x)$$

$$= \int d^4x \delta^{(4)}(x-x) \Theta(x) \text{tr } \gamma^5$$

↖ matrix trace

$$\text{Tr } \Theta \gamma^5 = \int d^4x \phi_{n\alpha}^+(x) \gamma_{\alpha\beta}^5 \phi_{n\beta}(x) \Theta(x)$$

$$= \int d^4x \delta^{(4)}(x-x) \Theta(x) \text{tr } \gamma^5$$

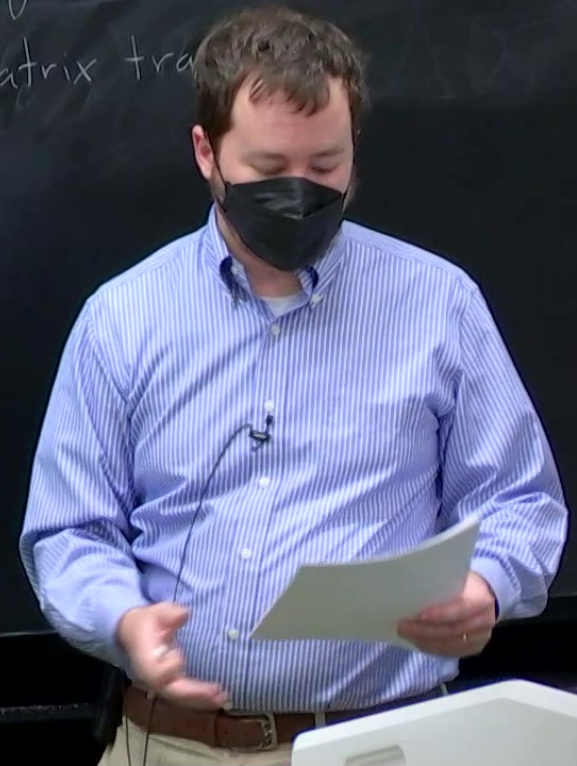
↑ matrix trace

$$= \infty \cdot 0$$

regulator: $f(k^2)$

$$f(0) = 1$$

$$f(\infty) = f'(\infty) = \dots = 0$$



$$= e^{i\theta} \frac{1}{\text{dan}}$$

$$\Theta \gamma^5 = \lim_{\lambda \rightarrow \infty} \int d^4x \sum_n \Theta \phi_n^\dagger \gamma^5 f\left(-\frac{\lambda_n^2}{\lambda^2}\right) \phi_n$$

eigenvalues are
gauge-invariant $\rightarrow$ electric charge conserved

$$= e^{i \text{Tr} \Theta \gamma^5} \frac{1}{\text{Tr} 1} \text{Tr} 1$$

$$\text{Tr} \Theta \gamma^5 = \lim_{\Lambda \rightarrow \infty} \int d^4x \sum_n \Theta \phi_n^\dagger \gamma^5 f\left(-\frac{\lambda_n^2}{\Lambda^2}\right) \phi_n$$

eigenvalues are gauge-invariant $\rightarrow$ electric charge

$\vdots$ algebra, using completeness + orthonormality of eigenfunctions

$$= \int d^4x \Theta(\text{anomaly})$$

$$\langle \partial_\mu J_5^\mu \rangle = \frac{e^2}{16\pi^2} \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma}$$

$$f\left(-\frac{\lambda_n^2}{\Lambda^2}\right) \phi_n = f\left(-\frac{(i\not{D})^2}{\Lambda^2}\right) \phi_n$$

massless QED in Euclidean is not essential

$i\cancel{D}$ is Hermitian

$Z[A] = \int \mathcal{D}\Psi \mathcal{D}\bar{\Psi} e^{-S[\Psi, \bar{\Psi}, A]}$

no A integral background field

$S \rightarrow S + \int d^4x \theta(x) \partial_\mu J_5^\mu$
 $Z[A] \rightarrow Z[A]$
 $\Psi \rightarrow \Psi e^{i\theta}$
 $A_\mu \rightarrow A_\mu$

$0 = \frac{\delta Z}{\delta \theta(x)} \Big|_{\theta=0} = \frac{\delta}{\delta \theta(x)} \left[\int \mathcal{D}\Psi' \mathcal{D}\bar{\Psi}' e^{-S - \int d^4x \theta \partial_\mu J_5^\mu} \right] \Big|_{\theta=0}$
 $= \frac{\delta}{\delta \theta(x)} \left[\int \mathcal{D}\Psi \mathcal{D}\bar{\Psi} e^{-S - \theta \partial_\mu J_5^\mu - 2i \text{Tr} \theta \gamma^5} \right] \Big|_{\theta=0}$

not invariant

$\text{Tr} \theta \gamma^5 = \lim_{\lambda \rightarrow \infty} \int d^4x \sum_n \theta \phi_n^\dagger \gamma^5 f\left(\frac{-\lambda_n^2}{\Lambda^2}\right) \phi_n$

= eigenvalues are gauge-invariant $\rightarrow$ electric charge conserved

$\vdots$ algebra, using completeness + orthonormality of eigenfunctions

$= \int d^4x \theta (\text{anomaly})$

$\langle \partial_\mu J_5^\mu \rangle = \frac{e^2}{16\pi^2} \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma}$

$f\left(\frac{-\lambda_n^2}{\Lambda^2}\right) \phi_n = f\left(\frac{-(i\cancel{D})^2}{\Lambda^2}\right) \phi_n$
 $\text{tr} \gamma^5 = \text{tr} \gamma^5 \chi \chi^\dagger$