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$$I(X,Y) := H(X) + H(Y) - H(X,Y)$$

$$\shortparallel$$

$$I(X,Y) := H(Y) - H(Y|X)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$H(Y|X) = -\sum_{x,y} p(x,y) \log \frac{p(x,y)}{p(x)}$$

$$I(X,Y) := H(X) + H(Y) - H(X,Y)$$

$$\parallel$$

$$I(X,Y) := H(Y) - H(Y|X)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$H(X) = -\sum_{x} P(x) \operatorname{Log} P(x)$$

$$H(Y|X) = -\sum_{x \in X,\, y \in Y} p(x,y) \operatorname{Log} \frac{p(x,y)}{p(x)}$$

$$J_A(\hat{\rho}_{AB}) := S(B) - \underbrace{S(\rho_B | \rho_A)}_{\min\limits_{\{\Pi_j^A\}} S(\rho_B | \{\Pi_j^A\})}$$

$$D_A(\rho_{AB}) := I(\rho_{AB}) - J_A(\rho_{AB}) \leftarrow \text{Quantum Discord}$$

1212.3275.pdf

https://arxiv.org/pdf/1212.3275.pdf

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Purified discord and multipartite entanglement
Abstract
I Introduction
II Entanglement structure and discord
III The case of qubits
IV Why Gaussian states require quantum correlation to have classical correlation
V Remote activation of entanglement
VI Conclusions
VII acknowledgements
References

including a relative phase between the two terms and by assuming that $|\alpha\rangle$ and $|\beta\rangle$ are *real* combinations of the basis vectors. This exclusion will not affect the primary result presented here, as will be explained below.

We now ask under what circumstances ρ_{AB} contains discord. Recall that the discord $D(A,B)$ when B performs the required measurement is not generally equivalent to the discord $D(B,A)$ when A performs the measurement. Indeed one can be zero while the other is

Let us further consider the separability of ρ_{AC}, because the two different conditions for separability mean two very different things. We see that the first condition, $|\alpha\rangle = \pm|0\rangle$, coincides with the A system being a product onto the BC system; under this condition the 3-qubit state takes the product form $|\psi\rangle_{ABC} = |0\rangle \otimes |\psi\rangle_{BC}$. Of course in this case ρ_{AB} is separable, as we have seen, but it also clearly has zero discord and we see that this

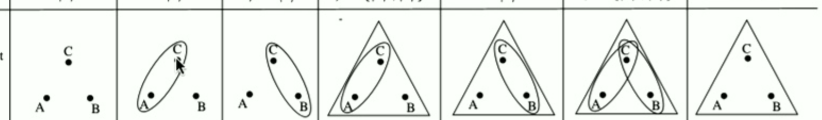
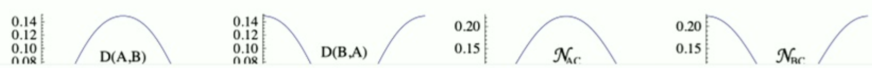
$\lvert\alpha\rangle$	$\pm\lvert 0\rangle$	$\neq\pm\lvert 0\rangle$	$\pm\lvert 0\rangle$	$\pm\lvert 1\rangle$	$\neq\pm\{\lvert 0\rangle,\lvert 1\rangle\}$	$\neq\pm\{\lvert 0\rangle,\lvert 1\rangle\}$	$\pm\lvert 1\rangle$
$\lvert\beta\rangle$	$\pm\lvert 0\rangle$	$\pm\lvert 0\rangle$	$\neq\pm\lvert 0\rangle$	$\neq\pm\{\lvert 0\rangle,\lvert 1\rangle\}$	$\pm\lvert 1\rangle$	$\neq\pm\{\lvert 0\rangle,\lvert 1\rangle\}$	$\pm\lvert 1\rangle$
Entanglement structure							
$D(A,B)$	0	0	0	>0	0	>0	0
$D(B,A)$	0	0	0	0	>0	>0	0

FIG. 1. The relationship between the entanglement structure of $|\psi\rangle_{ABC}$ and the discord in ρ_{AB}. For given conditions on $|\alpha\rangle$ and $|\beta\rangle$ we display the resulting entanglement structure and the results for the discords $D(A,B)$ and $D(B,A)$. In the structure diagrams an ellipse represents the presence of bipartite entanglement while a triangle represents the presence of tripartite entanglement.



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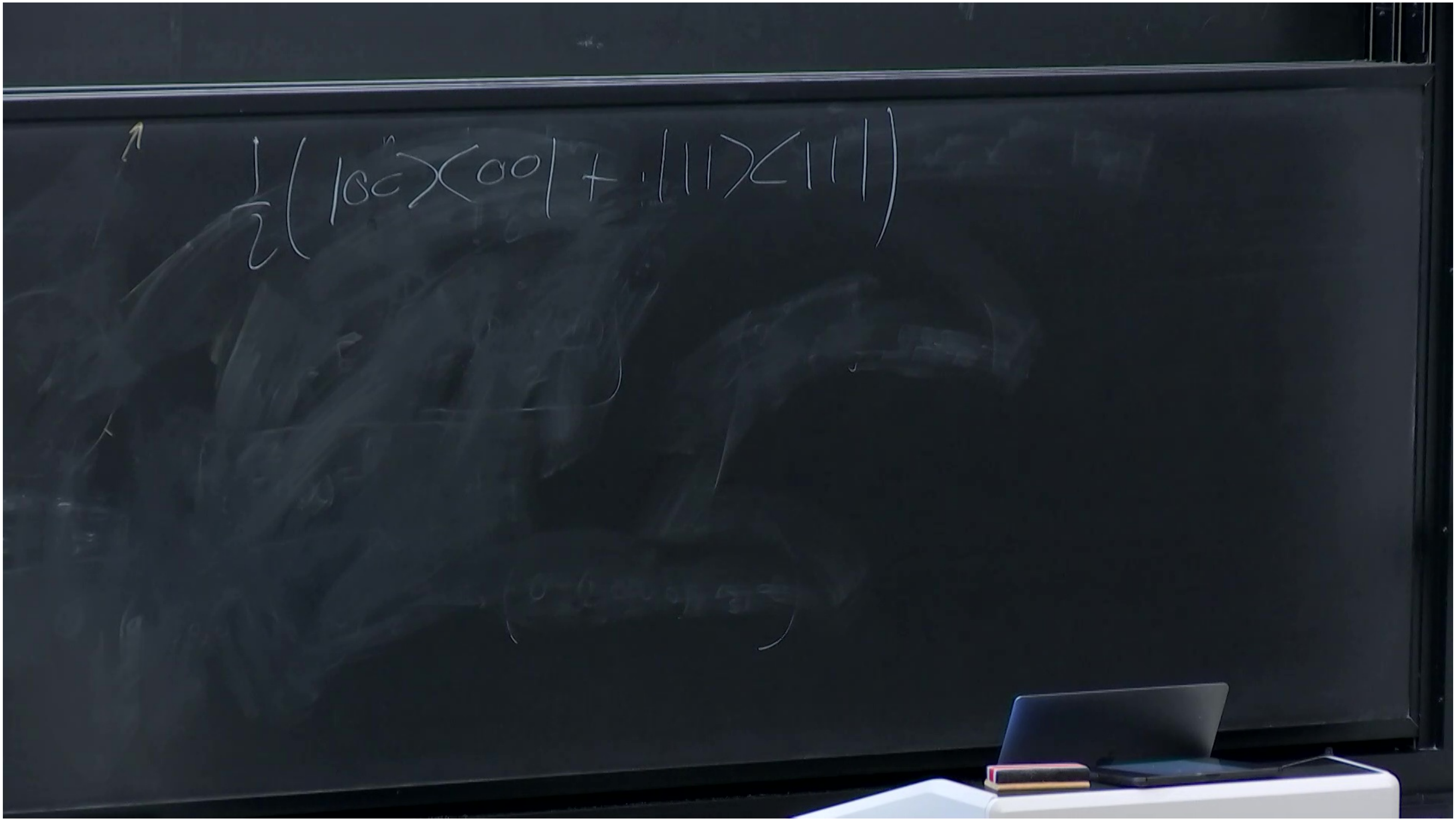

$\frac{1}{2}(|00\rangle\langle 00| + |11\rangle\langle 11|)$

and the partial systems are $\rho_A = \mathrm{Tr}_B(\rho_{AB})$, $\rho_B = \mathrm{Tr}_A(\rho_{AB})$.

Mutual information accounts for both, classical and quantum correlations, so that it can be used together with an entanglement measure to distinguish the behaviour of classical correlations: in a system which has no quantum correlations, mutual information accounts exclusively for classical correlations.

3.5 An application of entanglement: Quantum Teleportation

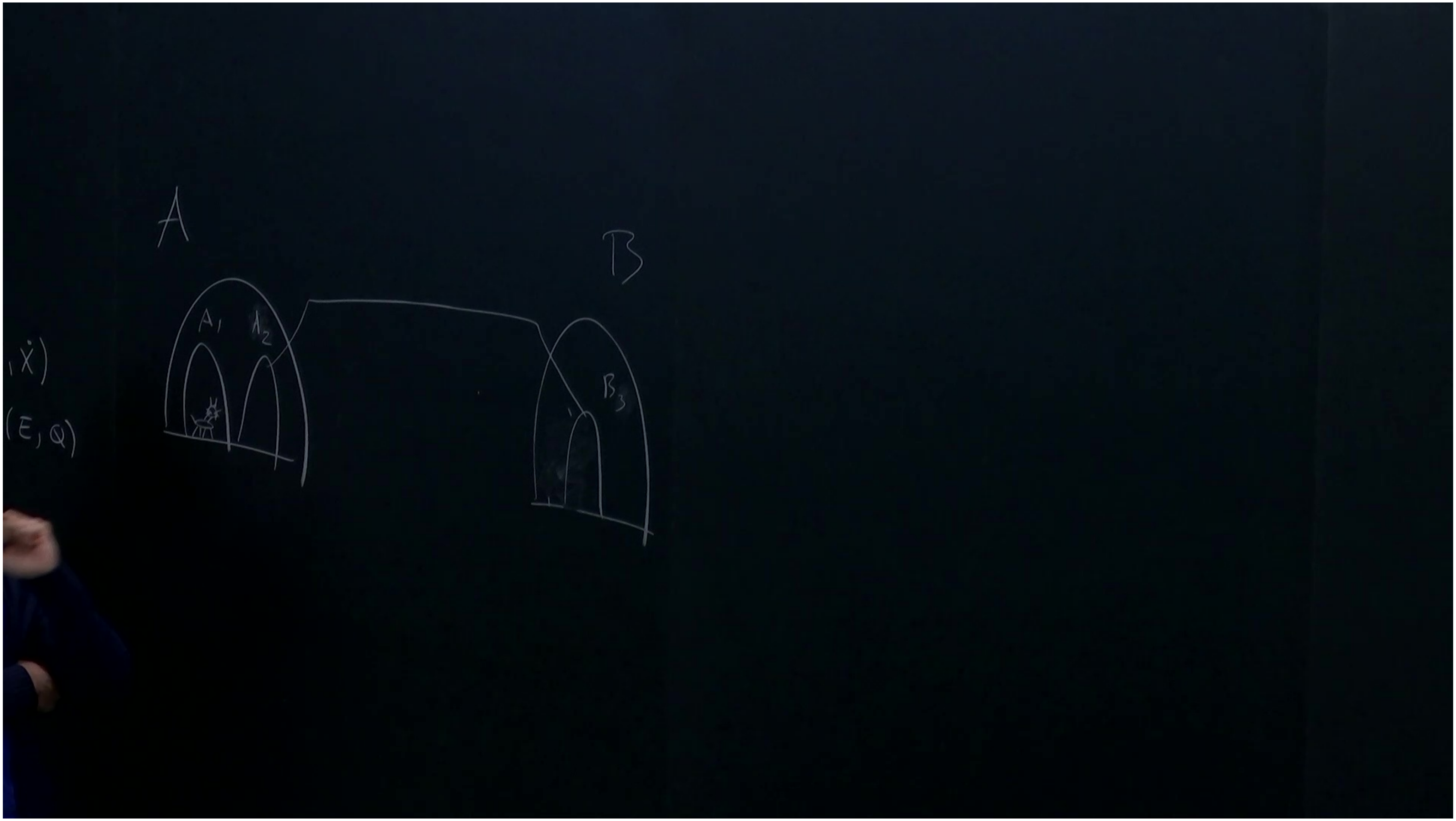
Entanglement is known to be a resource for quantum computing, cryptography and communication. In particular, we are going to see a protocol of quantum communication known as 'quantum teleportation'. Quantum teleportation is a protocol by which quantum information (e.g. the exact quantum state of an atom, photon, etc) can be transmitted from one location to another, with the help of classical communication and previously shared quantum entanglement between the sending and receiving location. It was first proposed in 1993 by Bennet, Brassard, Crepeau, Jozsa, Peres and Wootters [?], and first experimentally tested in 1997 [?].

Let us analyze the protocol step by step for qubits:

Step 0 - A and B share a Bell pair, for example

$$|\Phi^+\rangle_{A_2B_3} = \frac{1}{\sqrt{2}}\Big[|00\rangle_{A_2B_3} + |11\rangle_{A_2B_3}\Big]. \tag{3.5.1}$$

A and B prepare this bell pair and then B goes away. After that Alice is given a qubit $|\varphi\rangle_{A_1}$ that she wants to send to Bob:


A
A1
A2
B
B3
(E, Q)

$$|\varphi\rangle_{A_1} = \alpha_0|0\rangle + \alpha_1|1\rangle. \tag{3.5.2}$$

Step 1- Alice will make a joint measurement of her two qubits (the one she wants to teleport, A_1, and her own half of the entangled pair, A_2) in a Bell basis. For that we first write the tripartite system of the three qubits:

$$\begin{aligned}|\psi\rangle_{A_1A_2B_3} &= |\varphi\rangle_{A_1} \otimes |\Phi^+\rangle_{A_2B_3}\\ &= \Big(\alpha_0|0\rangle_{A_1} + \alpha_1|1\rangle_{A_1}\Big) \otimes \frac{1}{\sqrt{2}}\Big[|00\rangle_{A_2B_3} + |11\rangle_{A_2B_3}\Big]\\ &= \frac{\alpha_0}{\sqrt{2}}\Big(|000\rangle_{A_1A_2B_3} + |011\rangle_{A_1A_2B_3}\Big) + \frac{\alpha_1}{\sqrt{2}}\Big(|100\rangle_{A_1A_2B_3} + |111\rangle_{A_1A_2B_3}\Big).\end{aligned} \tag{3.5.3}$$

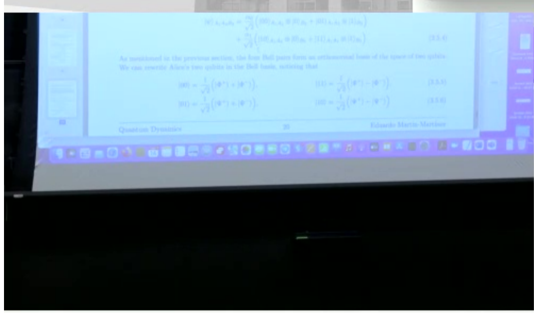
If we factor out explicitly Alice's two qubits we get

$$\begin{aligned}|\psi\rangle_{A_1A_2B_3} = &\frac{\alpha_0}{\sqrt{2}}\Big(|00\rangle_{A_1A_2} \otimes |0\rangle_{B_3} + |01\rangle_{A_1A_2} \otimes |1\rangle_{B_3}\Big)\\ &+ \frac{\alpha_1}{\sqrt{2}}\Big(|10\rangle_{A_1A_2} \otimes |0\rangle_{B_3} + |11\rangle_{A_1A_2} \otimes |1\rangle_{B_3}\Big).\end{aligned} \tag{3.5.4}$$

As mentioned in the previous section, the four Bell pairs form an orthonormal basis of the space of two qubits. We can rewrite Alice's two qubits in the Bell basis, noticing that

$$|00\rangle = \frac{1}{\sqrt{2}}\Big(|\Phi^+\rangle + |\Phi^-\rangle\Big), \qquad |11\rangle = \frac{1}{\sqrt{2}}\Big(|\Phi^+\rangle - |\Phi^-\rangle\Big), \tag{3.5.5}$$

$$|01\rangle = \frac{1}{\sqrt{2}}\Big(|\Psi^+\rangle + |\Psi^-\rangle\Big), \qquad |10\rangle = \frac{1}{\sqrt{2}}\Big(|\Psi^+\rangle - |\Psi^-\rangle\Big) \tag{3.5.6}$$



$$|00\rangle = \frac{1}{\sqrt{2}}\Big(|\Phi^+\rangle + |\Phi^-\rangle\Big), \qquad |11\rangle = \frac{1}{\sqrt{2}}\Big(|\Phi^+\rangle - |\Phi^-\rangle\Big), \tag{3.5.5}$$

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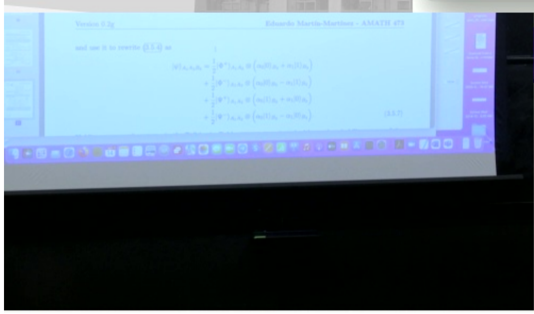
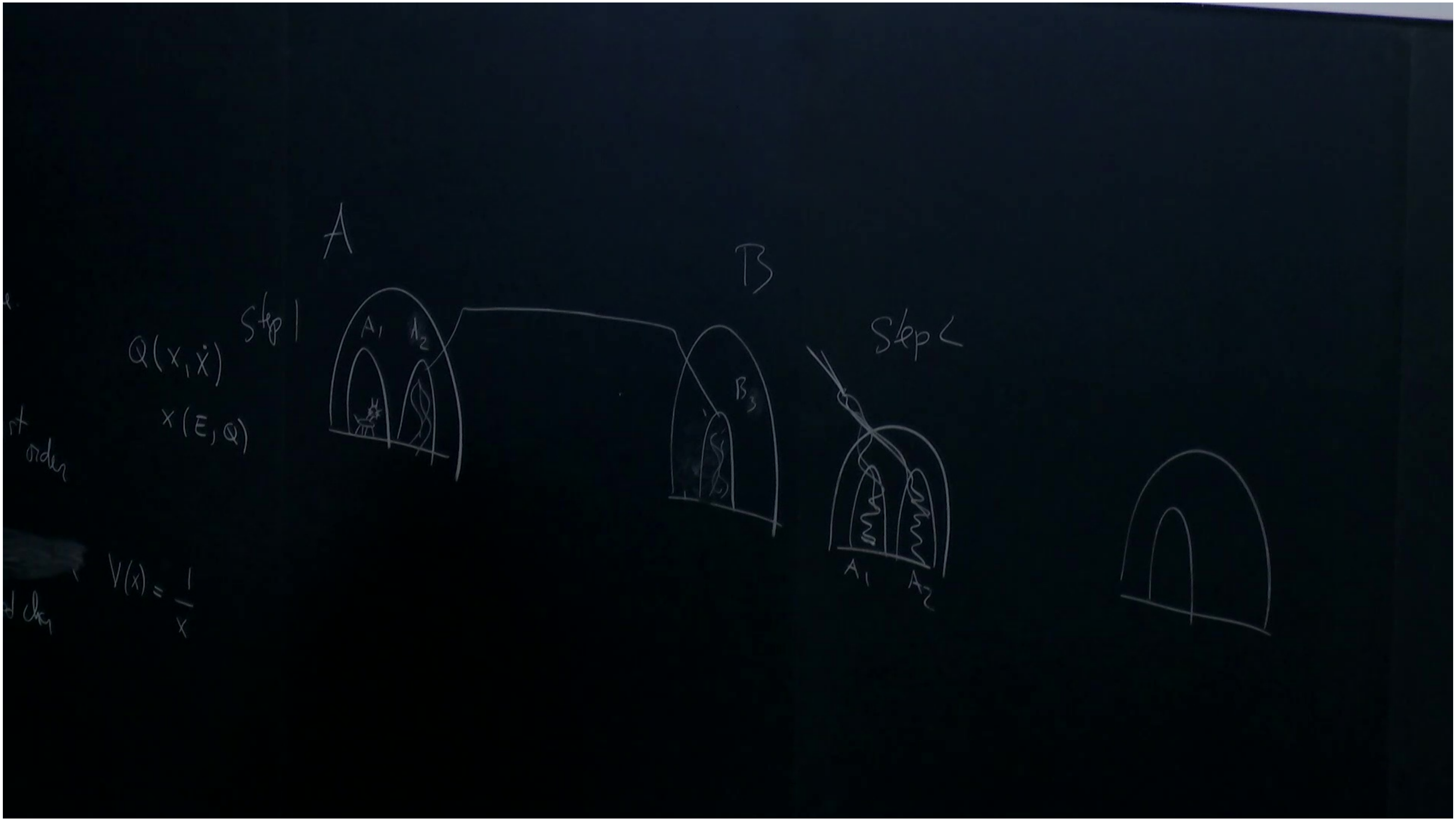
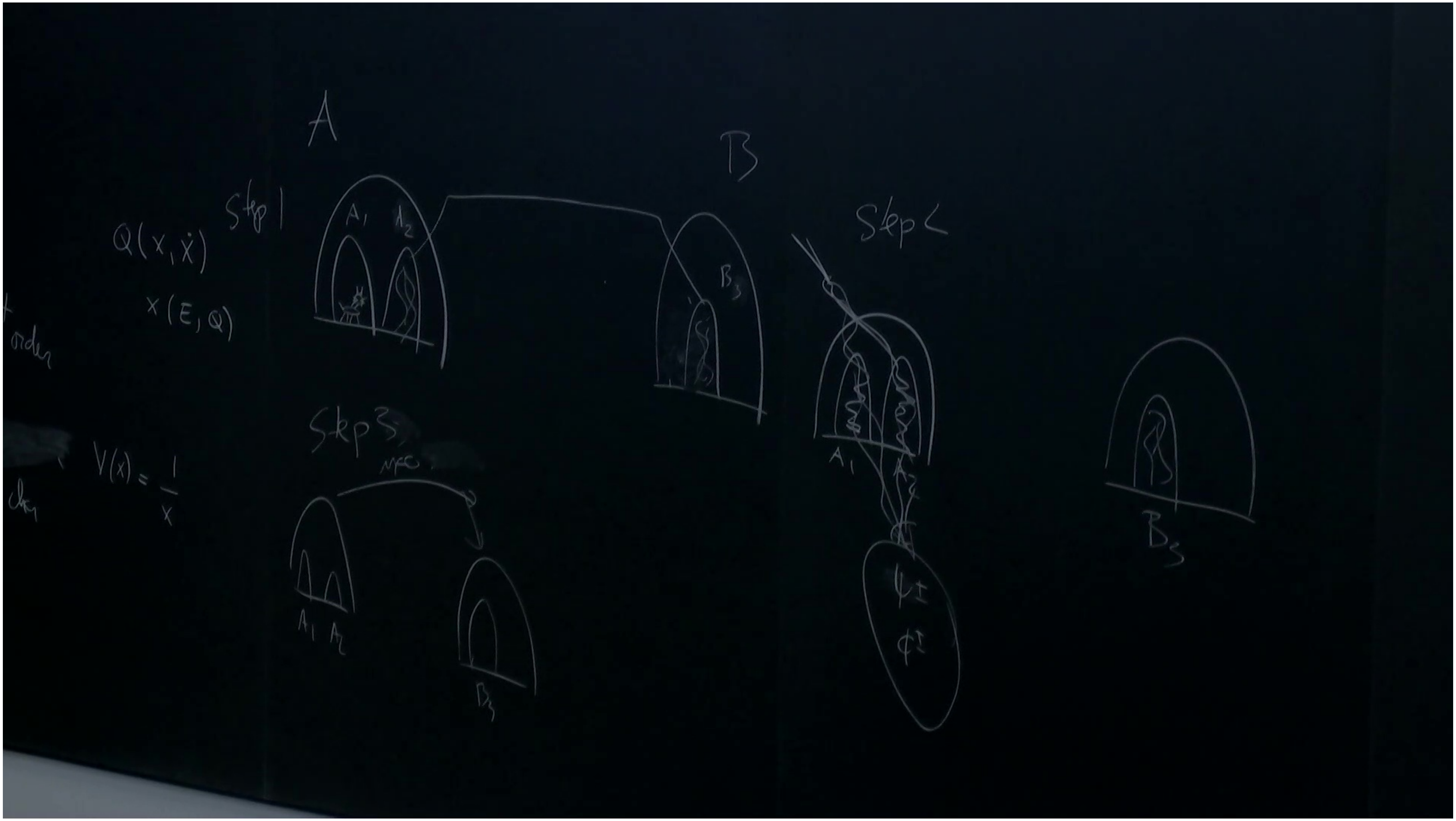
and use it to rewrite (3.5.4) as

$$\begin{aligned}|\psi\rangle_{A_1A_2B_3} = &\frac{1}{2}|\Phi^+\rangle_{A_1A_2} \otimes \Big(\alpha_0|0\rangle_{B_3} + \alpha_1|1\rangle_{B_3}\Big)\\ &+ \frac{1}{2}|\Phi^-\rangle_{A_1A_2} \otimes \Big(\alpha_0|0\rangle_{B_3} - \alpha_1|1\rangle_{B_3}\Big)\\ &+ \frac{1}{2}|\Psi^+\rangle_{A_1A_2} \otimes \Big(\alpha_0|1\rangle_{B_3} + \alpha_1|0\rangle_{B_3}\Big)\\ &+ \frac{1}{2}|\Psi^-\rangle_{A_1A_2} \otimes \Big(\alpha_0|1\rangle_{B_3} - \alpha_1|0\rangle_{B_3}\Big).\end{aligned} \tag{3.5.7}$$




A
Step 1
Q(x, ẋ)
x(E, Q)
V(x) = 1/x
A1
A2
B
B3
Step 2
A1
A2


A
B
Step 1
Step 2
Step 3
Q(x, ẋ)
x(E, Q)
V(x) = 1/x
A1
A2
B3
ψI
φI

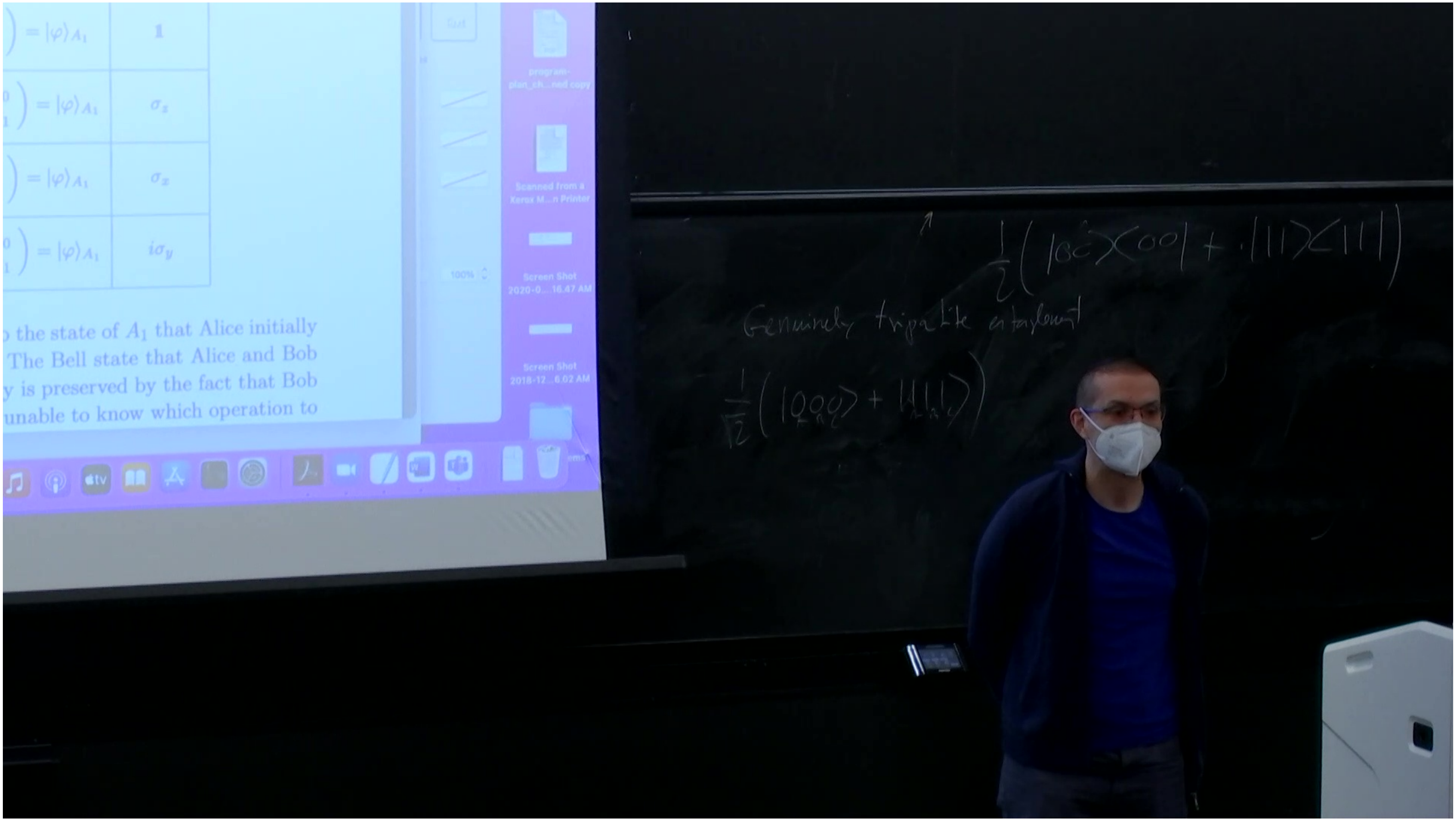
$$|\varphi\rangle_{B_3} = \left(\alpha_1\right), \left(-\alpha_1\right), \left(\alpha_0\right), \left(\alpha_0\right). \quad (3.5.8)$$

They are related with the original qubit that Alice wanted to teleport $|\varphi\rangle_{A_1}$ by simple local operations.

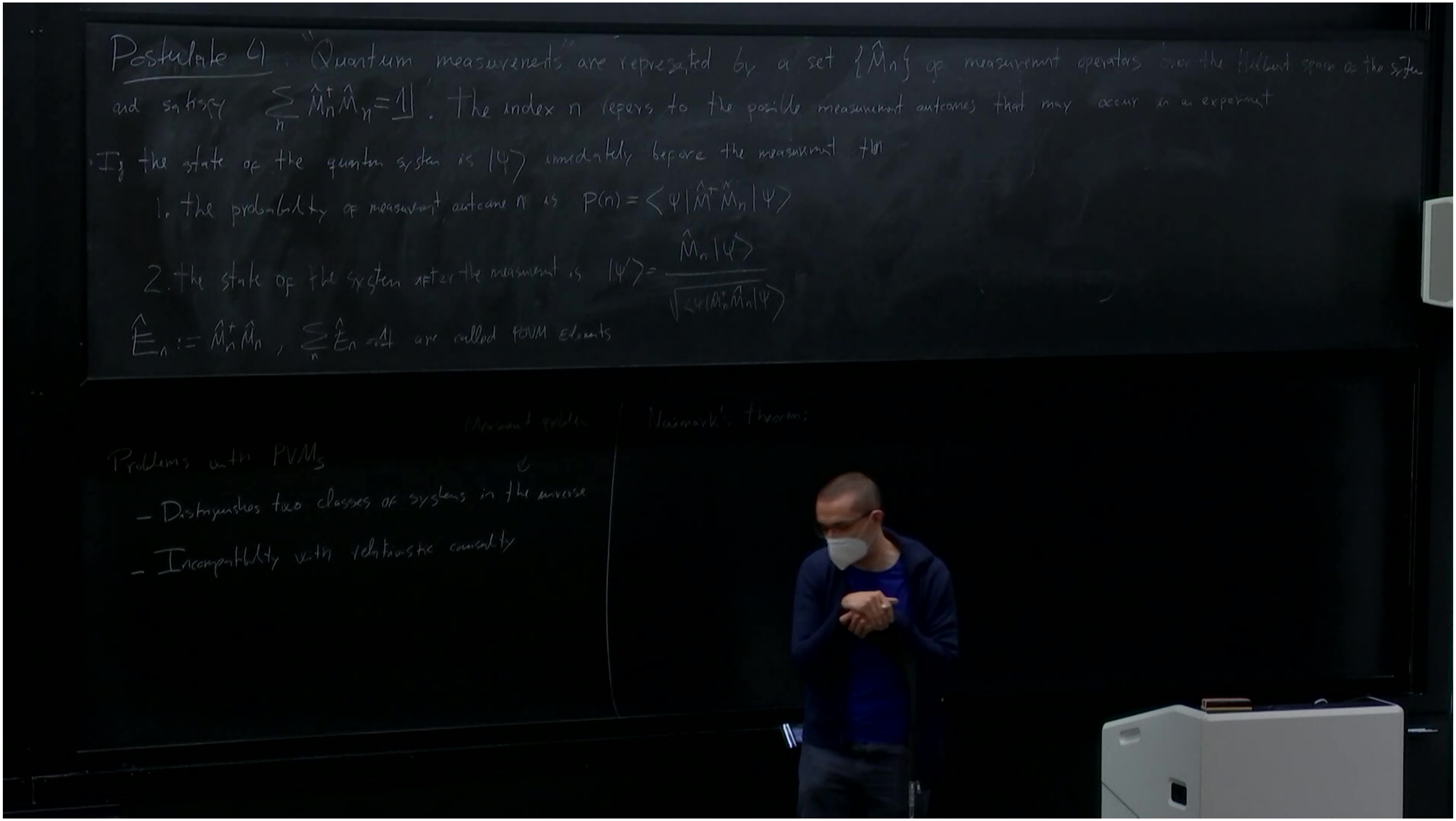
Step 3- Alice announces the result of her measurement to Bob through a classical channel (2 classical bits). With the obtained information, Bob can recover, through local unitary operations, the quantum state that Alice wanted to teleport. In particular

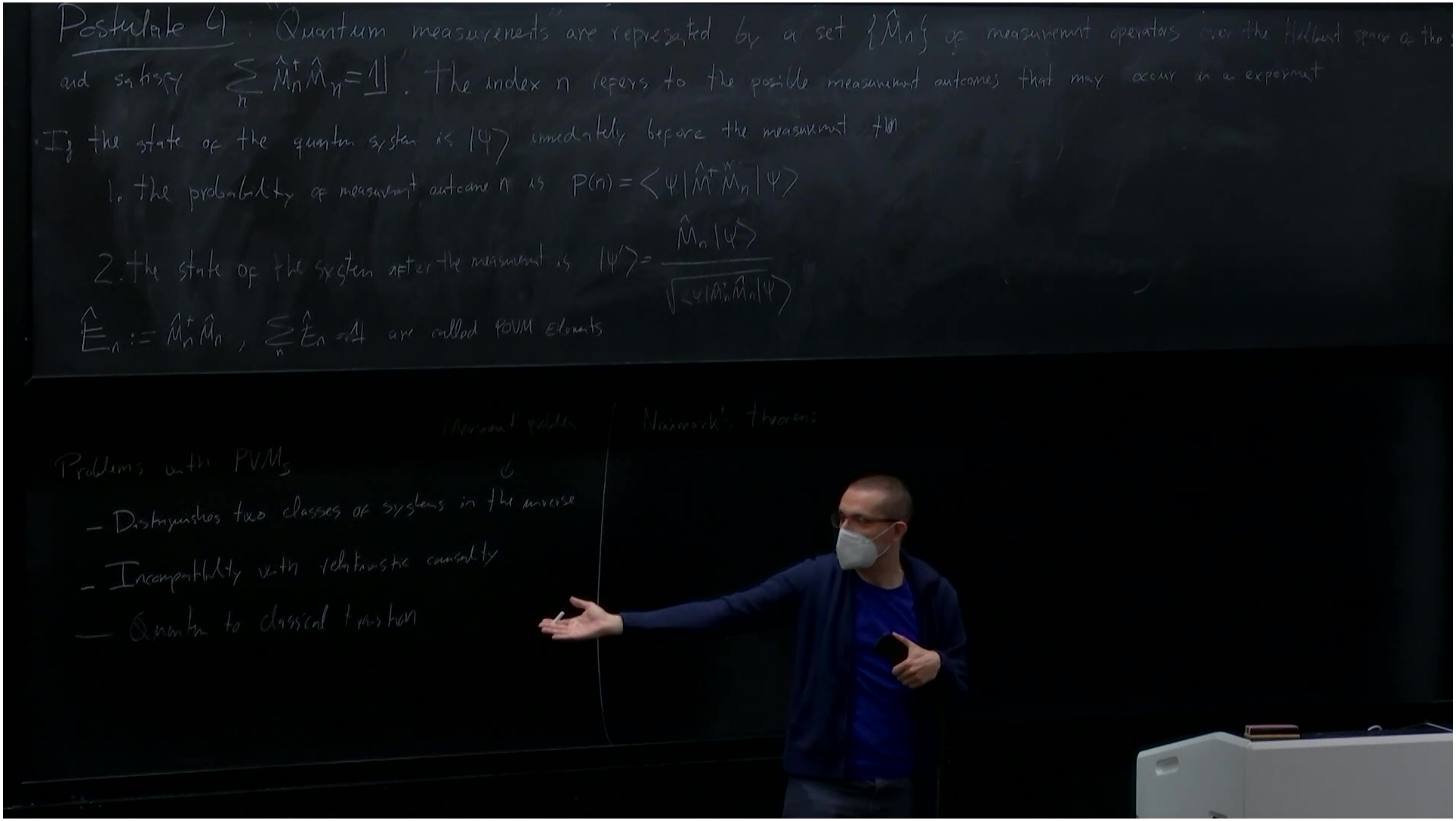
A measured	B has	B does	Local operation used
$\lvert\Phi^+\rangle$	$\lvert\varphi\rangle_{B_3} = \begin{pmatrix}\alpha_0\\ \alpha_1\end{pmatrix}$	$\lvert\varphi'\rangle_{B_3} = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}\alpha_0\\ \alpha_1\end{pmatrix} = \begin{pmatrix}\alpha_0\\ \alpha_1\end{pmatrix} = \lvert\varphi\rangle_{A_1}$	$\mathbb{1}$
$\lvert\Phi^-\rangle$	$\lvert\varphi\rangle_{B_3} = \begin{pmatrix}\alpha_0\\ -\alpha_1\end{pmatrix}$	$\lvert\varphi'\rangle_{B_3} = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}\alpha_0\\ -\alpha_1\end{pmatrix} = \begin{pmatrix}\alpha_0\\ \alpha_1\end{pmatrix} = \lvert\varphi\rangle_{A_1}$	σ_z
$\lvert\Psi^+\rangle$	$\lvert\varphi\rangle_{B_3} = \begin{pmatrix}\alpha_1\\ \alpha_0\end{pmatrix}$	$\lvert\varphi'\rangle_{B_3} = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}\alpha_1\\ \alpha_0\end{pmatrix} = \begin{pmatrix}\alpha_0\\ \alpha_1\end{pmatrix} = \lvert\varphi\rangle_{A_1}$	σ_x
$\lvert\Psi^-\rangle$	$\lvert\varphi\rangle_{B_3} = \begin{pmatrix}-\alpha_1\\ \alpha_0\end{pmatrix}$	$\lvert\varphi'\rangle_{B_3} = \begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}\begin{pmatrix}-\alpha_1\\ \alpha_0\end{pmatrix} = \begin{pmatrix}\alpha_0\\ \alpha_1\end{pmatrix} = \lvert\varphi\rangle_{A_1}$	$i\sigma_y$

At the end of the protocol, Bob ends up with a state which is identical to the state of A_1 that Alice initially had. What happened is that the subsystem B_3 has acquired the state A_1. The Bell state that Alice and Bob shared is destroyed applying this protocol, as it is the state of A_1. Causality is preserved by the fact that Bob needs the information input about the outcome of Alice, or otherwise he is unable to know which operation to



the state of A_1 that Alice initially
The Bell state that Alice and Bob
y is preserved by the fact that Bob
unable to know which operation to
Genuinely tripartite entanglement


Postulate 4: "Quantum measurements" are represented by a set $\{\hat{M}_n\}$ of measurement operators over the Hilbert space of the system
and satisfy $\sum_n \hat{M}_n^\dagger \hat{M}_n = \mathbb{1}$. The index n refers to the possible measurement outcomes that may occur in an experiment
If the state of the quantum system is $|\psi\rangle$ immediately before the measurement then
1. the probability of measurement outcome n is $p(n) = \langle\psi|\hat{M}_n^\dagger \hat{M}_n|\psi\rangle$
2. the state of the system after the measurement is $|\psi'\rangle = \frac{\hat{M}_n|\psi\rangle}{\sqrt{\langle\psi|\hat{M}_n^\dagger \hat{M}_n|\psi\rangle}}$
$\hat{E}_n := \hat{M}_n^\dagger \hat{M}_n$, $\sum_n \hat{E}_n = \mathbb{1}$ are called POVM Elements
Measurement problem
Naimark's theorem:
Problems with PVMs
- Distinguishes two classes of systems in the universe
- Incompatibility with relativistic causality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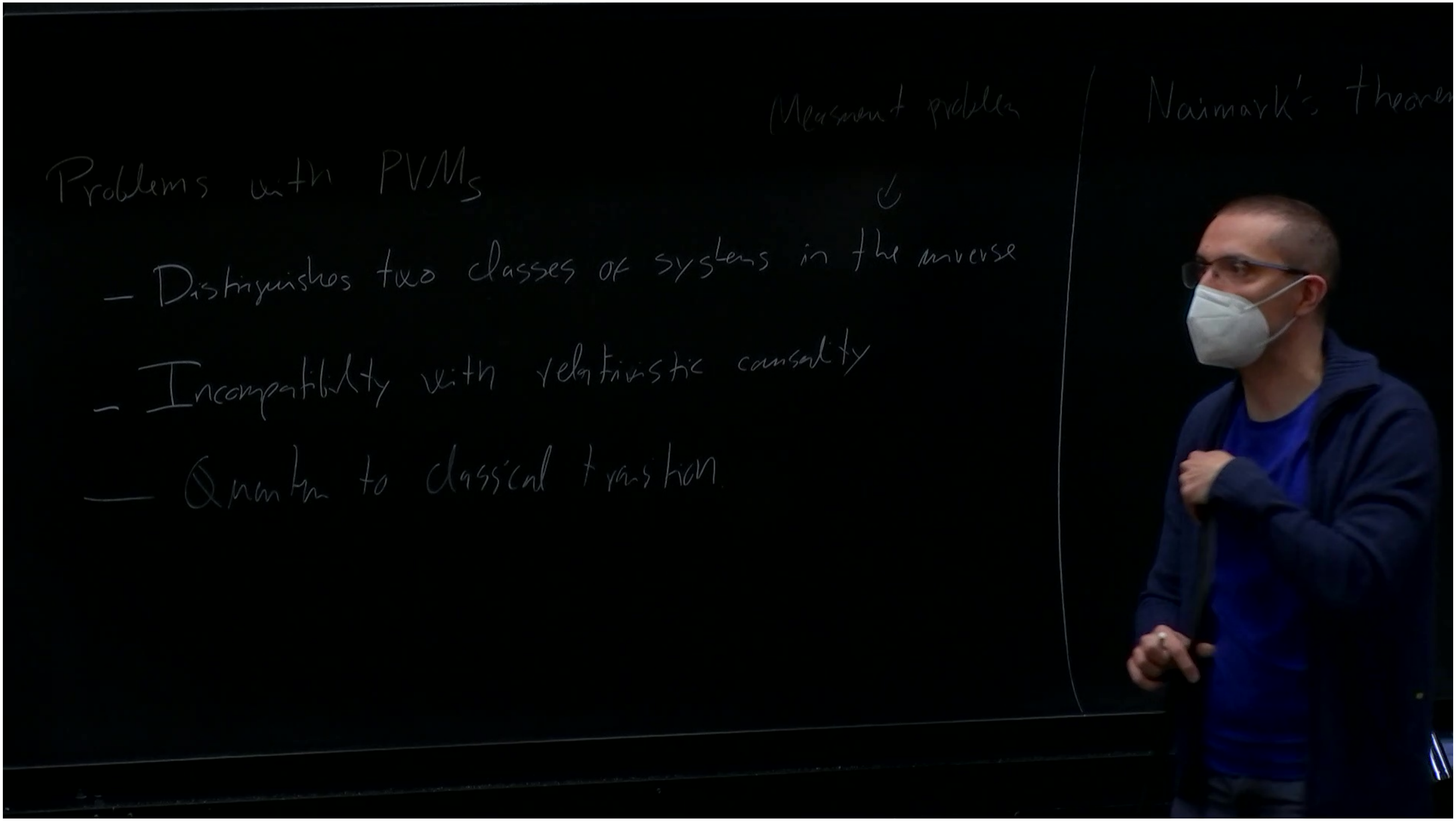
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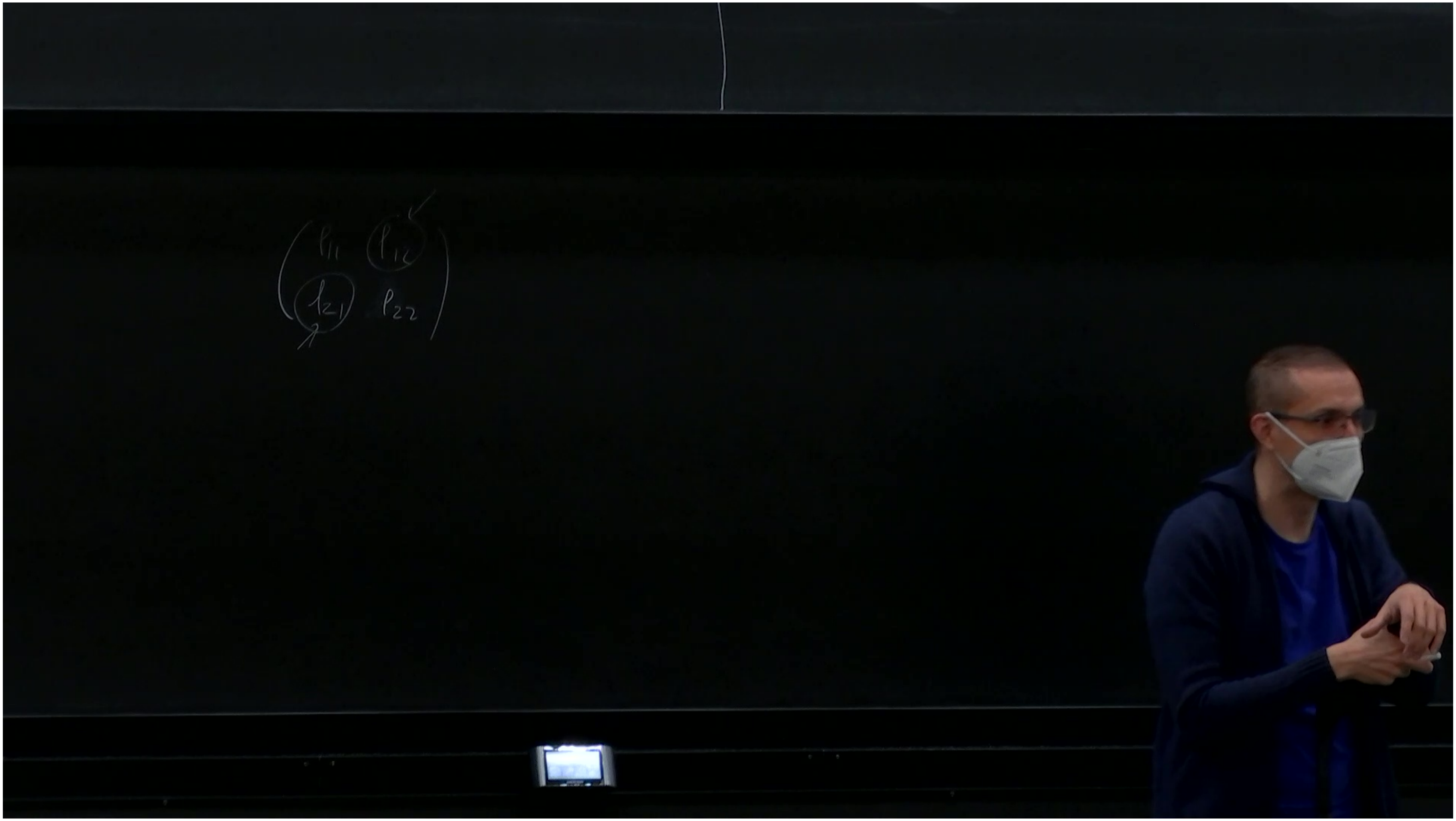
Naimark's theore


$\begin{pmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{pmatrix}$

<u>Postulate 4</u>: "Quantum measurements" are represented by a set $\{\hat{M}_n\}$ of meas

and satisfy $\sum_n \hat{M}_n^\dagger \hat{M}_n = \mathbb{1}$. The index n refers to the possible measurement outcomes th

• If the state of the quantum system is $|\psi\rangle$ immediately before the measurement then

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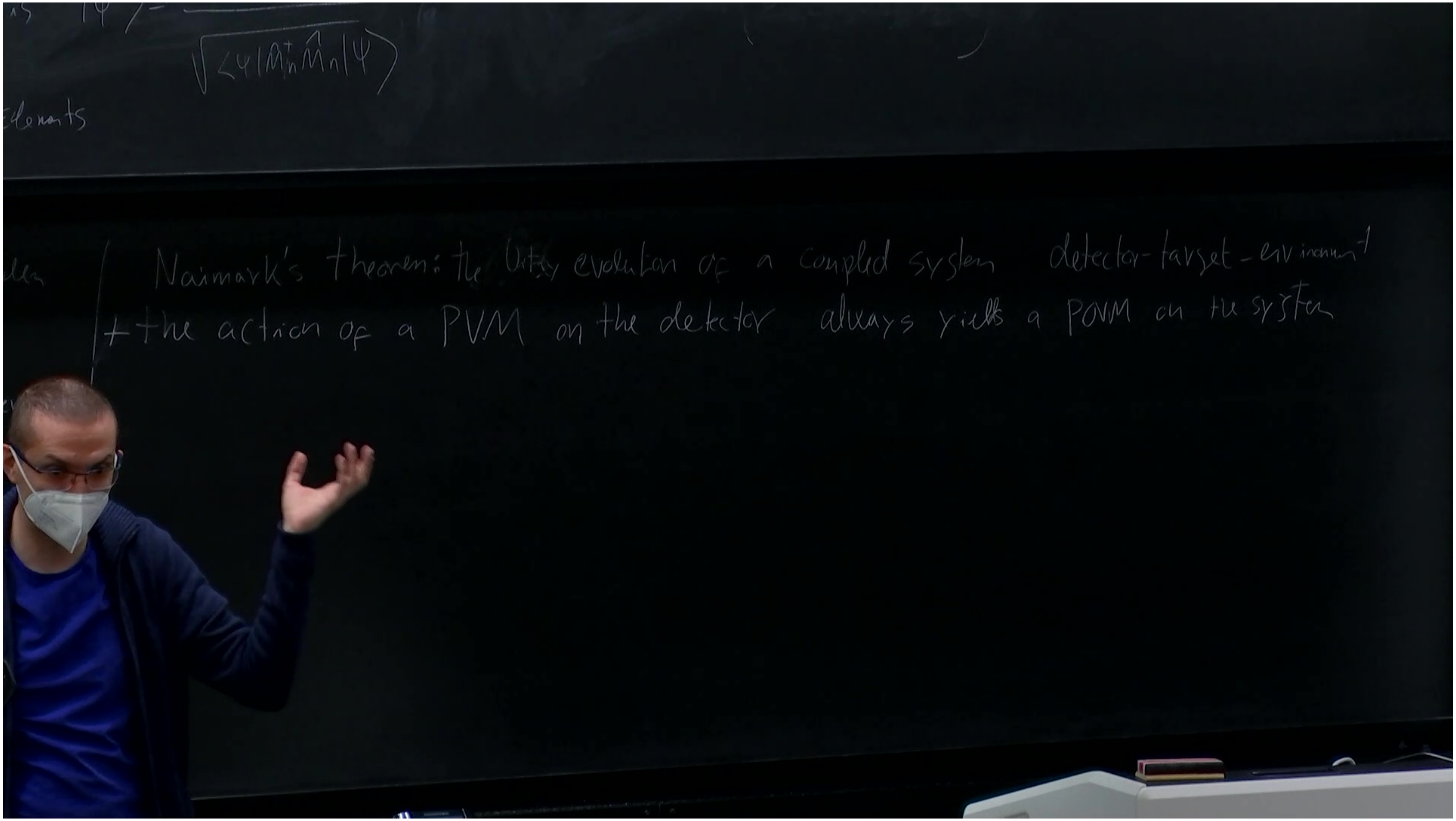

Elements
Naimark's theorem: the unitary evolution of a coupled system detector-target-environment
+ the action of a PVM on the detector always yields a POVM on the system

$$\sqrt{\langle\psi|\hat{M}_n^\dagger \hat{M}_n|\psi\rangle}$$

Elements

$$\hat{E}_n = \hat{P}_n$$

Naimark's theorem: the Unitary evolution of a coupled system detector-target-environment + the action of a PVM on the detector always yields a POVM on the system

Heisenberg cut

$$H(Y|X) = -\sum_{x\in X, y \in Y} p(x,y) \, \mathrm{Log} \frac{p(x,y)}{p(x)}$$

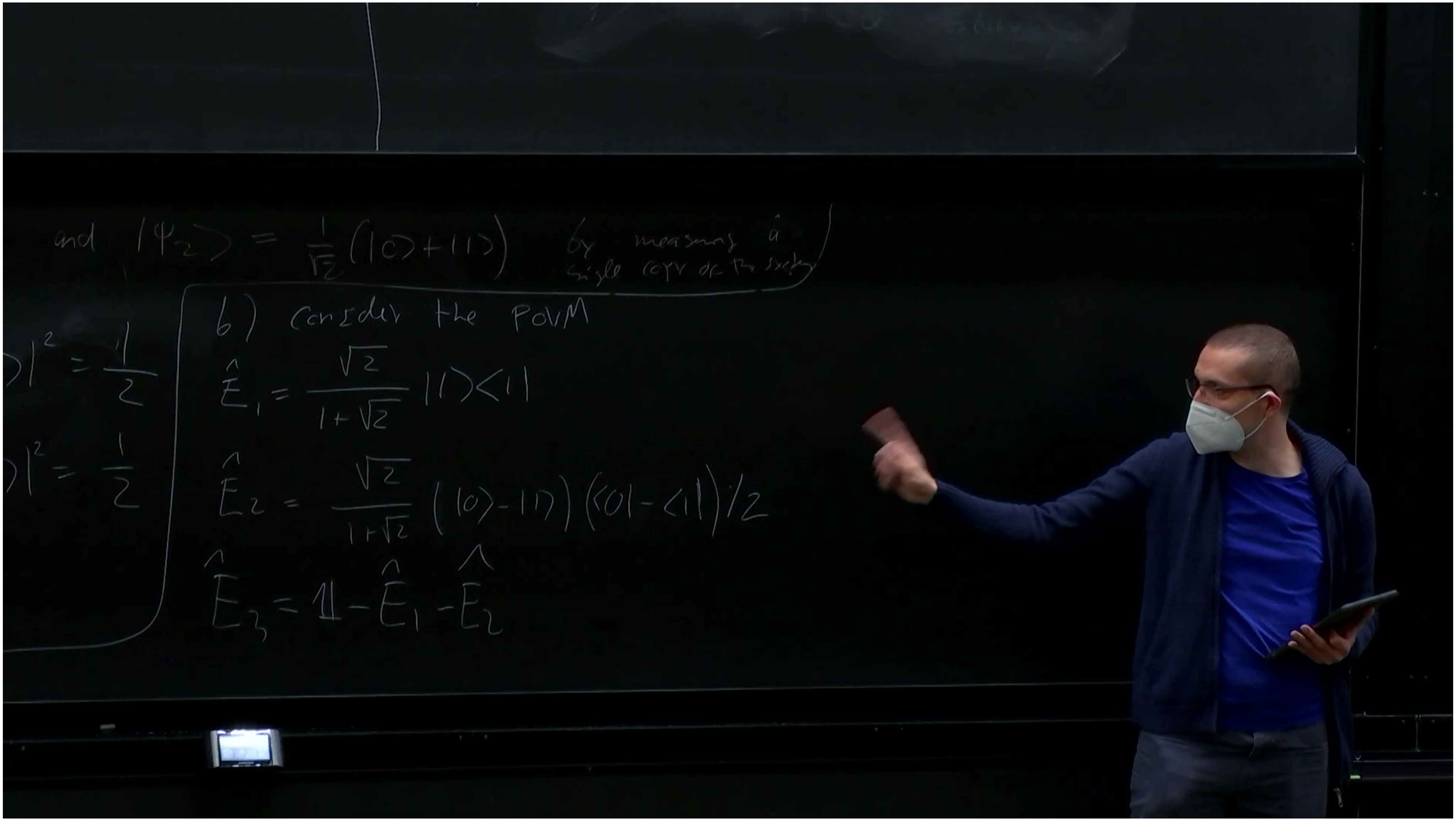
$$D_A(\rho_{AB}) := I(\rho_{AB}) - J_A(\rho_{AB})$$

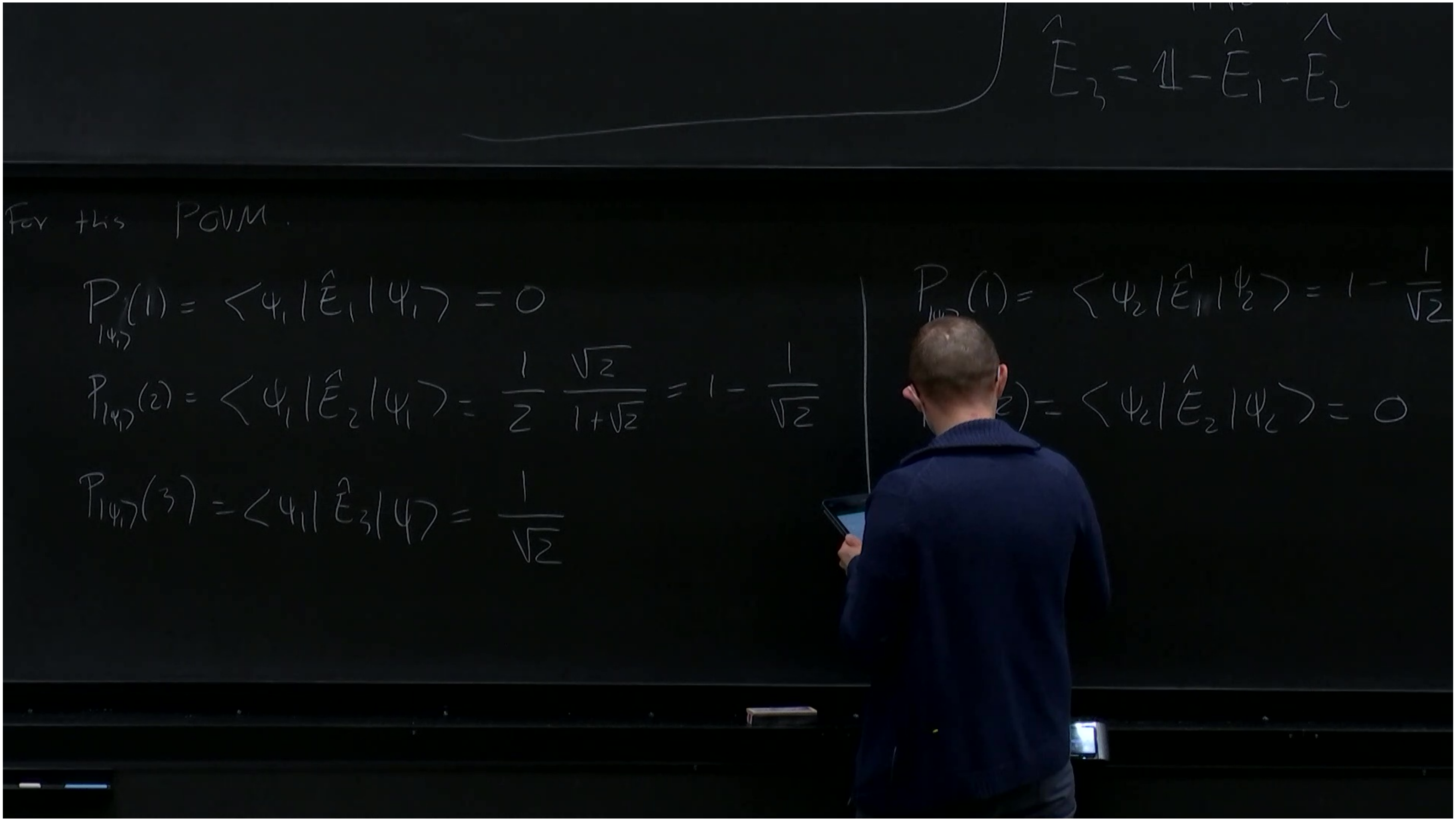
Can you distinguish between $|\psi_1\rangle = |0\rangle$ and $|\psi_2\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ by measuring $\hat{\sigma}_z$

a) Would PVMs work?

$$P_{0\,|\psi_1\rangle} = |\langle\psi_1|0\rangle|^2 = 1 \qquad P_{0\,|\psi_2\rangle} = |\langle\psi_2|0\rangle|^2 = \frac{1}{2}$$

$$P_{1\,|\psi_1\rangle} = |\langle\psi_1|1\rangle|^2 = 0 \qquad P_{1\,|\psi_2\rangle} = |\langle\psi_2|1\rangle|^2 = \frac{1}{2}$$


and $|\psi_2\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$
b) consider the POVM
$\hat{E}_1 = \frac{\sqrt{2}}{1+\sqrt{2}} |1\rangle\langle 1|$
$\hat{E}_2 = \frac{\sqrt{2}}{1+\sqrt{2}} (|0\rangle - |1\rangle)(\langle 0| - \langle 1|)/2$
$\hat{E}_3 = \mathbb{1} - \hat{E}_1 - \hat{E}_2$



$\hat{E}_3 = \mathbb{1} - \hat{E}_1 - \hat{E}_2$
For this POVM.
$P_{|\psi_1\rangle}(1) = \langle \psi_1 | \hat{E}_1 | \psi_1 \rangle = 0$
$P_{|\psi_1\rangle}(2) = \langle \psi_1 | \hat{E}_2 | \psi_1 \rangle = \frac{1}{2} \frac{\sqrt{2}}{1+\sqrt{2}} = 1 - \frac{1}{\sqrt{2}}$
$P_{|\psi_1\rangle}(3) = \langle \psi_1 | \hat{E}_3 | \psi \rangle = \frac{1}{\sqrt{2}}$
$P_{|\psi_2\rangle}(1) = \langle \psi_2 | \hat{E}_1 | \psi_2 \rangle = 1 - \frac{1}{\sqrt{2}}$
$(2) = \langle \psi_2 | \hat{E}_2 | \psi_2 \rangle = 0$

$$\hat{E}_3 = \mathbb{1} - \hat{E}_1 - \hat{E}_2$$

$$= 1 - \frac{1}{\sqrt{2}}$$

$$P_{|\psi_2\rangle}(1) = \langle \psi_2 | \hat{E}_1 | \psi_2 \rangle = 1 - \frac{1}{\sqrt{2}}$$

$$P_{|\psi_2\rangle}(2) = \langle \psi_2 | \hat{E}_2 | \psi_2 \rangle = 0$$

$$P_{|\psi_2\rangle}(3) = \langle \psi_2 | \hat{E}_3 | \psi_2 \rangle = \frac{1}{\sqrt{2}}$$

$$\hat{E}_3 = \mathbb{1} - \hat{E}_1 - \hat{E}_2$$

$$P_{|\psi_2\rangle}(1) = \langle \psi_2 | \hat{E}_1 | \psi_2 \rangle = 1 - \frac{1}{\sqrt{2}}$$

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$$P_{|\psi_2\rangle}(3) = \langle \psi_2 | \hat{E}_3 | \psi_2 \rangle = \frac{1}{\sqrt{2}}$$