Title: Quantum Information 2021/2022

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Peres criterion

$$\hat{\rho}_{AB} = \sum_i c_i \, \hat{\rho}_A \otimes \hat{\rho}_B \implies \hat{\rho}_{AB}^{pT_B} = \sum_i c_i \, \hat{\rho}_A \otimes \hat{\rho}_B^{T} \implies \hat{\rho}_{AB}^{pT_B} \geq 0$$

$\hat{\rho}_{AB}$ separable $\implies \hat{\rho}_{AB}^{pT_B}, \hat{\rho}_{AB}^{pT_A}$ are positive semidefinite

Negativity: $\mathcal{N}(\hat{\rho}_{AB}) = -\sum_{\sigma_i < 0} \sigma_i$ where σ_i are the eigenvalues of ρ_{AB}^{pT}

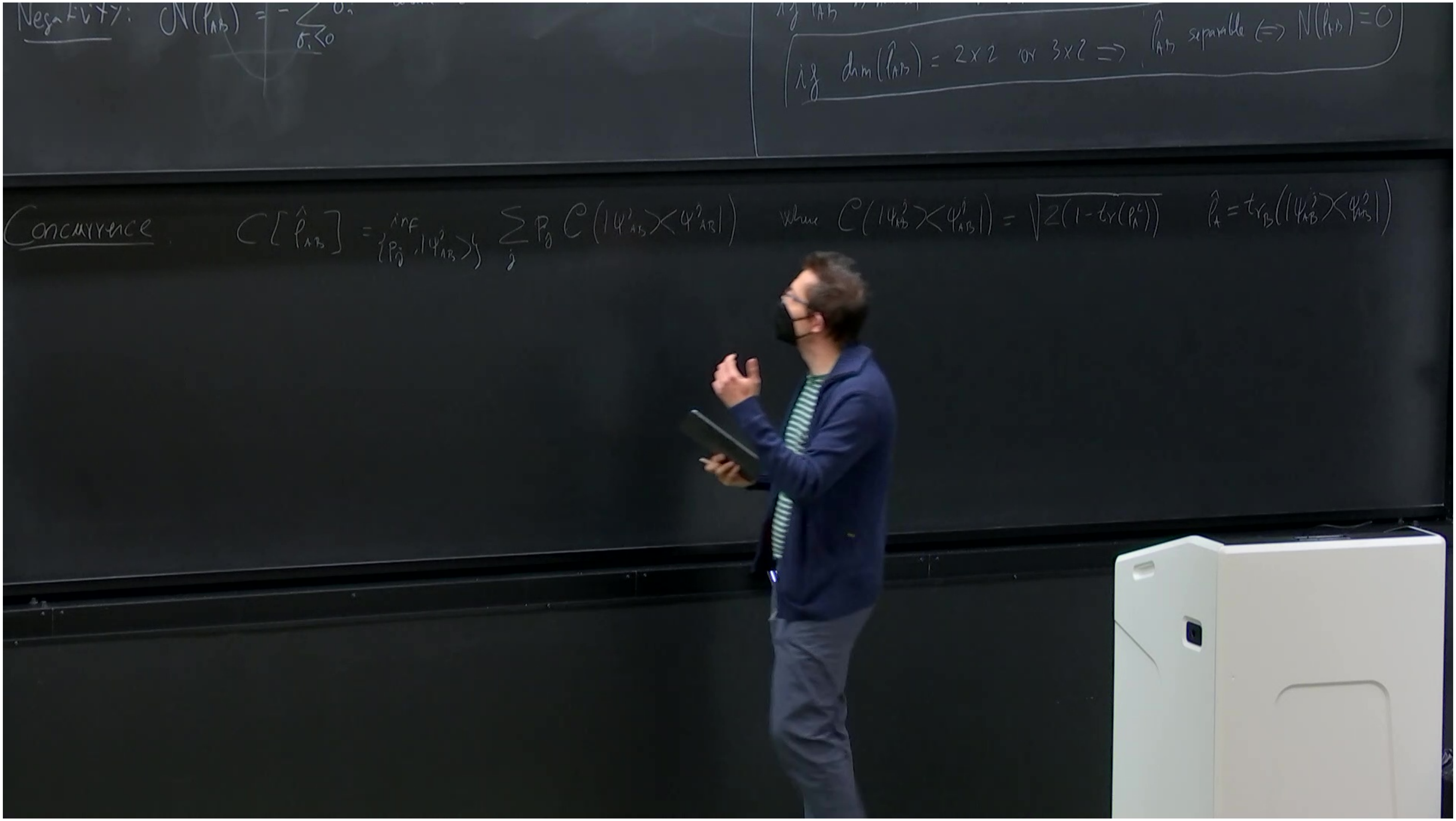
Dist

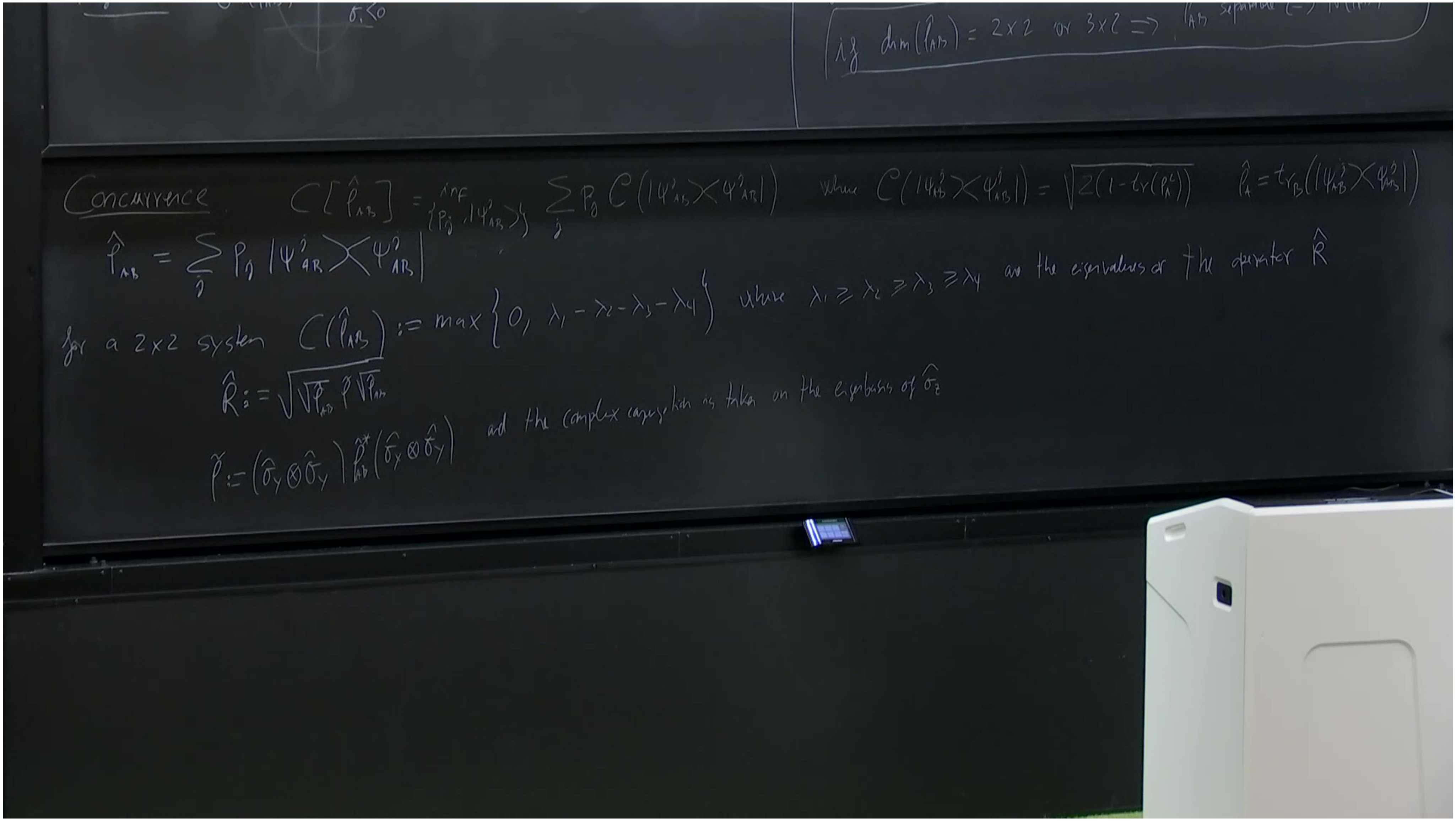
Distillable entanglement: the amount of Bell pairs that one can obtain from N copies of $\hat{\rho}_{AB}$ and LOCC.

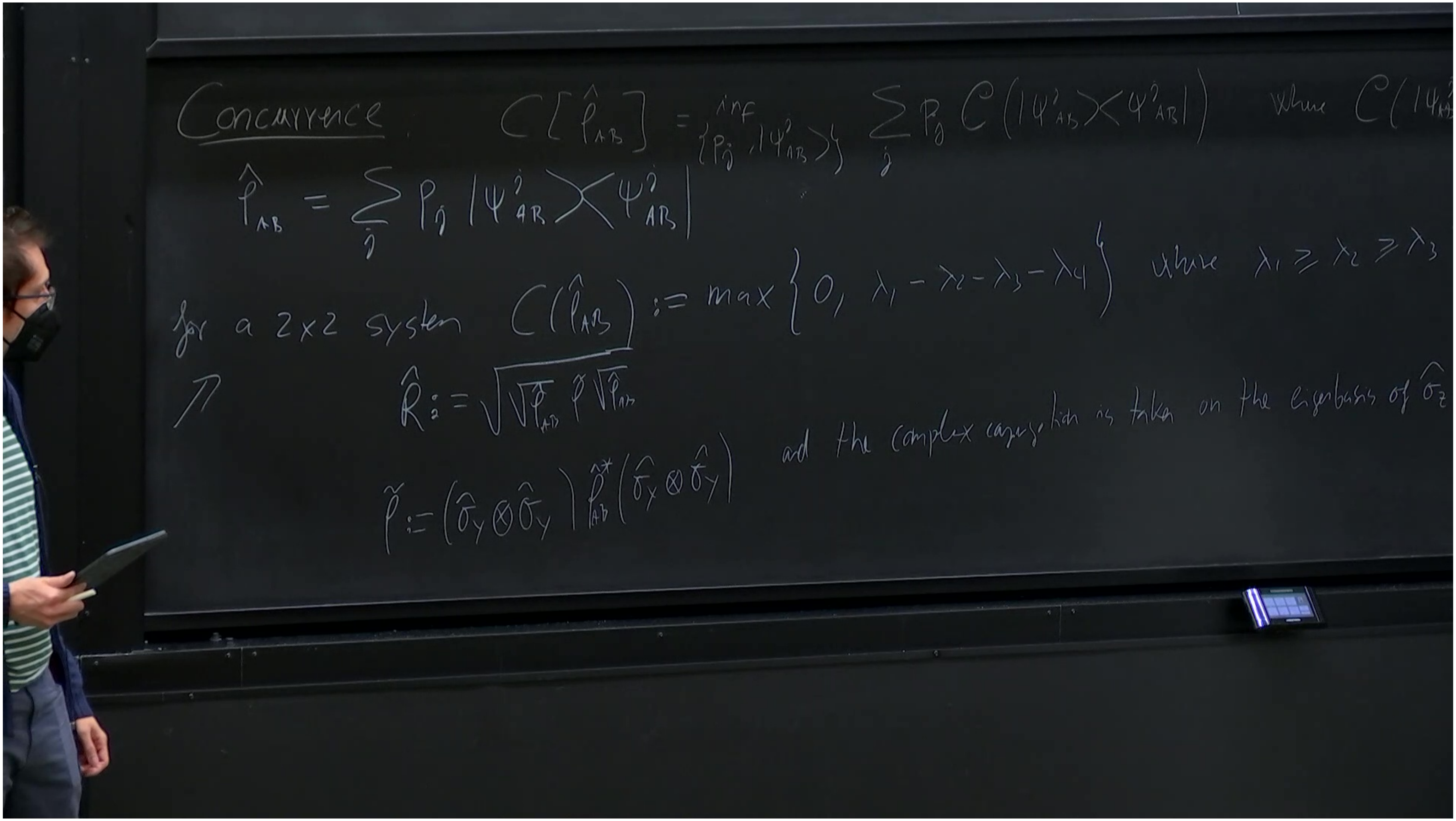
if $\hat{\rho}_{AB}$ has any distillable entanglement $\iff \mathcal{N}(\hat{\rho}_{AB}) > 0$

if $\hat{\rho}_{AB}$ is non-separable and $\mathcal{N}(\hat{\rho}_{AB}) = 0 \implies \hat{\rho}_{AB}$ has only bound entanglement

if $\dim(\hat{\rho}_{AB}) = 2\times 2$ or $3\times 2 \implies \hat{\rho}_{AB}$ separable $\iff \mathcal{N}(\hat{\rho}_{AB}) = 0$


Negativity: $\mathcal{N}(\rho_{AB}) = -\sum_{\sigma_i<0} \sigma_i$
if $\dim(\rho_{AB}) = 2\times 2$ or $3\times 2 \Rightarrow \hat{\rho}_{AB}$ separable $\Leftrightarrow N(\rho_{AB}) = 0$
Concurrence
$C[\hat{\rho}_{AB}] = \inf_{\{p_j, |\psi^j_{AB}\rangle\}} \sum_j p_j C(|\psi^j_{AB}\rangle\langle\psi^j_{AB}|)$
where $C(|\psi^j_{AB}\rangle\langle\psi^j_{AB}|) = \sqrt{2(1-\mathrm{tr}(\rho_A^2))}$
$\hat{\rho}_A = \mathrm{tr}_B(|\psi^j_{AB}\rangle\langle\psi^j_{AB}|)$


if $\dim(\hat\rho_{AB}) = 2\times 2$ or $3\times 2 \Rightarrow \hat\rho_{AB}$ separable
Concurrence $C[\hat\rho_{AB}] = \inf_{\{p_j, |\psi^j_{AB}\rangle\}} \sum_j p_j C(|\psi^j_{AB}\rangle\langle\psi^j_{AB}|)$ where $C(|\psi^j_{AB}\rangle\langle\psi^j_{AB}|) = \sqrt{2(1-\mathrm{tr}(\hat\rho_A^2))}$ $\hat\rho_A = \mathrm{tr}_B(|\psi^j_{AB}\rangle\langle\psi^j_{AB}|)$
$\hat\rho_{AB} = \sum_j p_j |\psi^j_{AB}\rangle\langle\psi^j_{AB}|$
for a 2x2 system $C(\hat\rho_{AB}) := \max\{0, \lambda_1-\lambda_2-\lambda_3-\lambda_4\}$ where $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \lambda_4$ are the eigenvalues of the operator $\hat R$
$\hat R := \sqrt{\sqrt{\hat\rho_{AB}}\,\tilde\rho\,\sqrt{\hat\rho_{AB}}}$
$\tilde\rho := (\hat\sigma_y\otimes\hat\sigma_y)\,\hat\rho^*_{AB}\,(\hat\sigma_y\otimes\hat\sigma_y)$ and the complex conjugation is taken on the eigenbasis of $\hat\sigma_z$


Concurrence: $C[\hat{\rho}_{AB}] = \inf_{\{p_j, |\psi^j_{AB}\rangle\}} \sum_j p_j C(|\psi^j_{AB}\rangle\langle\psi^j_{AB}|)$ where $C(|\psi_{AB}$
$\hat{\rho}_{AB} = \sum_j p_j |\psi^j_{AB}\rangle\langle\psi^j_{AB}|$
for a 2x2 system $C(\hat{\rho}_{AB}) := \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}$ where $\lambda_1 \geq \lambda_2 \geq \lambda_3$
$\hat{R} := \sqrt{\sqrt{\hat{\rho}_{AB}}\,\tilde{\rho}\,\sqrt{\hat{\rho}_{AB}}}$
$\tilde{\rho} := (\hat{\sigma}_y \otimes \hat{\sigma}_y)\,\hat{\rho}^*_{AB}\,(\hat{\sigma}_x \otimes \hat{\sigma}_y)$ and the complex conjugation is taken on the eigenbasis of $\hat{\sigma}_z$

$R = \sqrt{\sqrt{\hat\rho_{AB}}\,\tilde\rho\,\sqrt{\hat\rho_{AB}}}$ and the complex conjugation is taken on the eigenbasis of $\hat\sigma_z$

$$\tilde\rho := (\hat\sigma_y\otimes\hat\sigma_y)\,\hat\rho^{*}_{AB}\,(\hat\sigma_y\otimes\hat\sigma_y)$$

<u>Entanglement of formation</u>. Amount of Bell pairs needed to prepare a target state $\hat\rho_{AB}$ with LOCC

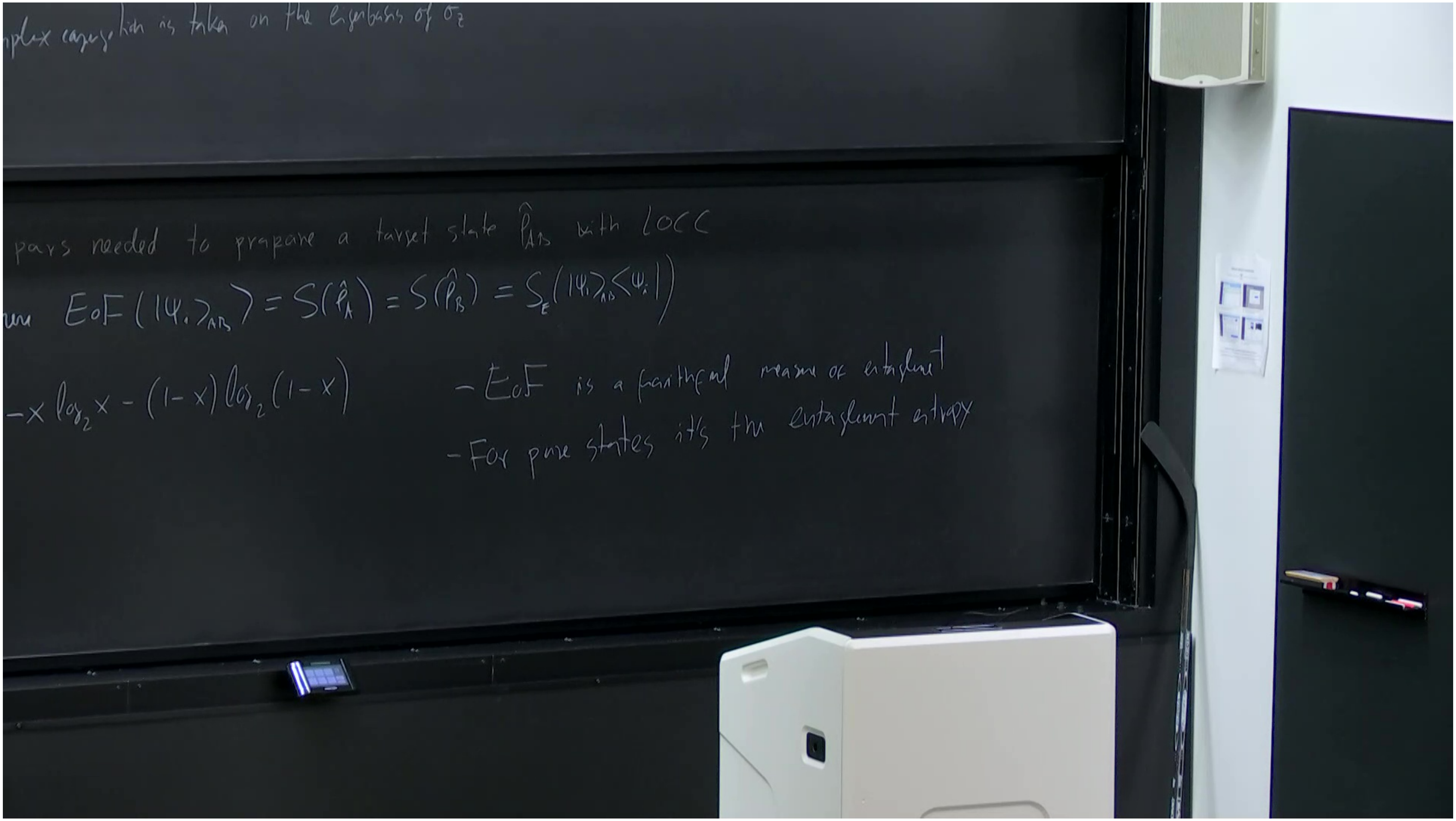
$$EoF(\hat\rho_{AB}) = \inf \sum_i p_i\, EoF(|\psi_i\rangle_{AB})$$

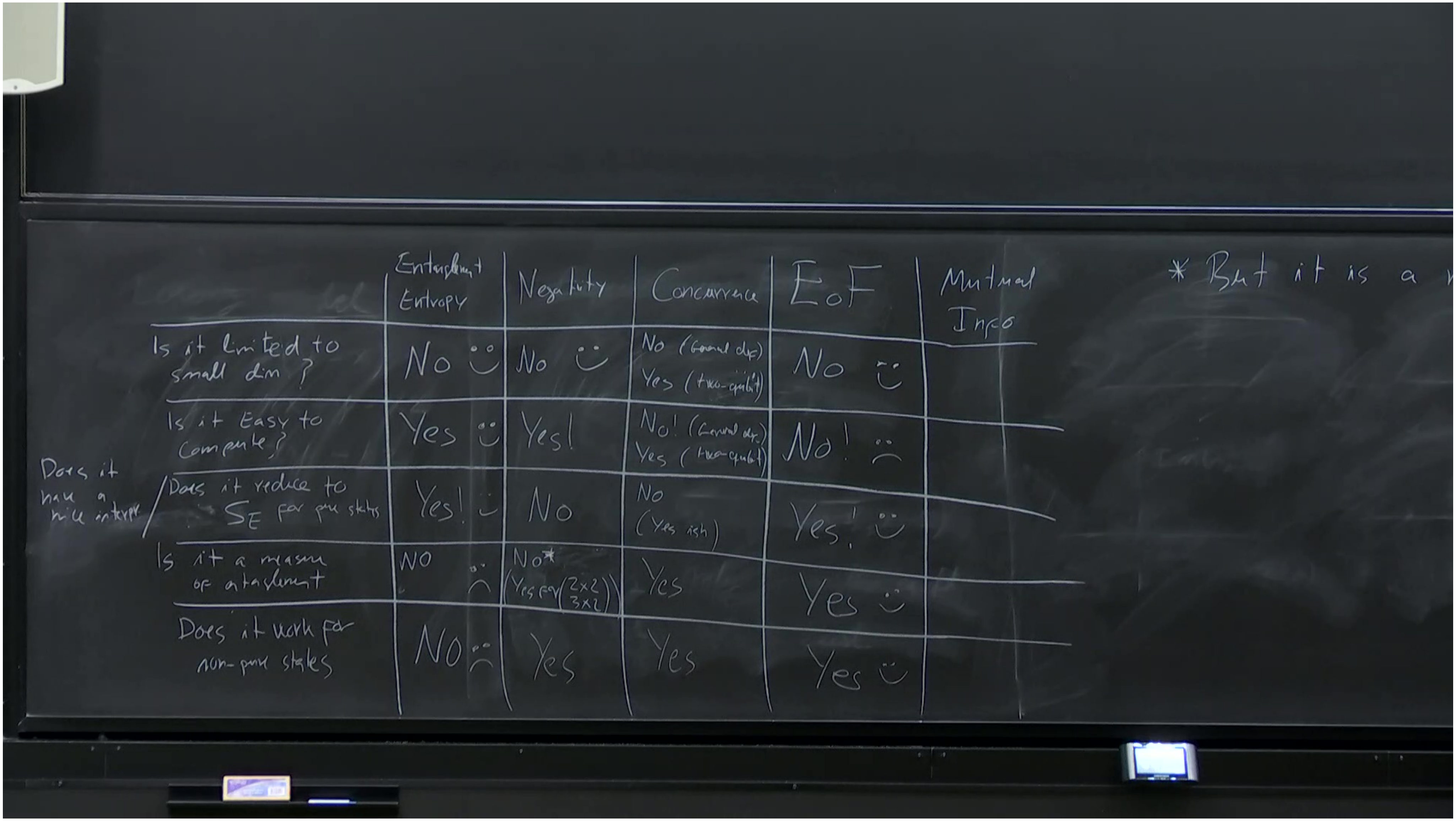
, where $EoF(|\psi_i\rangle_{AB}) = S(\hat\rho_A) = S(\hat\rho_B) = S_E(|\psi_i\rangle_{AB}\langle\psi_i|)$

$$EoF(\hat\rho_{AB}) = h\left(\frac{1+\sqrt{1-C(\hat\rho_{AB})^2}}{2}\right)$$

$$h(x) = -x\log_2 x - (1-x)\log_2(1-x)$$

EoF is a functional

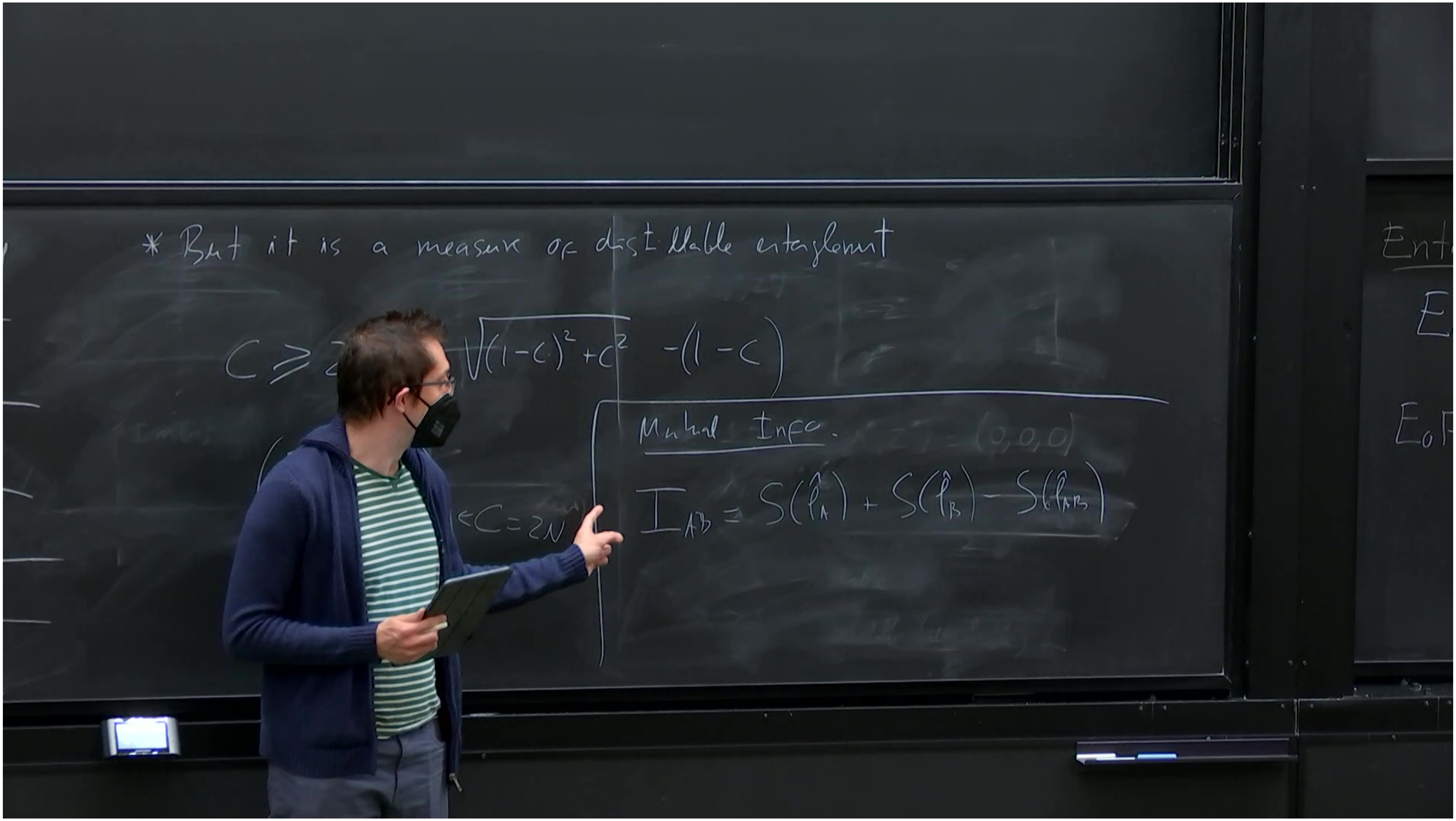

conjugation is taken on the eigenbasis of σ_z
pairs needed to prepare a target state $\hat{\rho}_{AB}$ with LOCC
$EoF(|\psi\rangle_{AB}) = S(\hat{\rho}_A) = S(\hat{\rho}_B) = S_E(|\psi\rangle_{AB}\langle\psi|)$
$-x \log_2 x - (1-x)\log_2(1-x)$
- EoF is a faithful measure of entanglement
- For pure states it's the entanglement entropy


Entanglement Entropy
Negativity
Concurrence
EoF
Mutual Info
* But it is a
Is it limited to small dim?
No
No
No (General dim)
Yes (two-qubit)
No
Is it Easy to compute?
Yes
Yes!
No! (General dim)
Yes (two-qubit)
No!
Does it have a nice interp
Does it reduce to S_E for pure states
Yes!
No
No
(Yes ish)
Yes!
Is it a measure of entanglement
NO
No*
(Yes for (2x2, 3x2))
Yes
Yes
Does it work for non-pure states
No
Yes
Yes
Yes

* But it is a measure of distillable entanglement

$$C \geq 2N \geq \sqrt{(1-c)^2 + c^2} - (1-c)$$

$$\begin{pmatrix} \cdot & & & \cdot \\ & \cdot & \cdot & \\ & \cdot & \cdot & \\ \cdot & & & \cdot \end{pmatrix} \leftarrow C = 2N$$


* But it is a measure of distillable entanglement
Mutual Info.
$I_{AB} = S(\hat{\rho}_A) + S(\hat{\rho}_B) - S(\hat{\rho}_{AB})$