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$$C(a, b) = \int d\lambda \Delta(\lambda) A(a, \lambda) B(b, \lambda) \quad A(a, \lambda) = \pm 1, \quad B(b, \lambda) = \pm 1 \quad \text{Local realism: } \left. \begin{array}{l} \\ \\ \\ \end{array} \right\} \text{local realism}$$

Consider two different configurations for A and B:  $\{a, a'\}$ ,  $\{b, b'\}$

$B(b, \lambda) = \pm 1$  Local realism: { Locality  $A(a, \lambda) \neq A$

$B(b, \lambda) = \pm 1$  Local realism:  $\left\{ \begin{array}{l} \text{Locality } A(a) \neq A(a, b), B \neq B(a) \\ \text{Realism } A(a, \lambda), B(b, \lambda) \end{array} \right.$

$$C(a, b) = \int d\lambda \Delta(\lambda) A(a, \lambda) B(b, \lambda) \quad A(a, \lambda) = \pm 1, \quad B(b, \lambda) = \pm 1 \quad \text{Lo}$$

Consider two different configurations for A and B:  $\{a, a'\}$ ,  $\{b, b'\}$

$$\begin{aligned} C(a, b) - C(a, b') &= \int d\lambda \Delta(\lambda) (A(a, \lambda) B(b, \lambda) - A(a, \lambda) B(b', \lambda)) = \\ &= \int d\lambda \Delta(\lambda) \left[ A(a, \lambda) B(b, \lambda) (1 \pm A(a', \lambda) B(b', \lambda)) \right] - \int d\lambda \Delta(\lambda) A(a, \lambda) B(b', \lambda) \end{aligned}$$

$$C(a, b) = \int d\lambda \Delta(\lambda) A(a, \lambda) B(b, \lambda) \quad A(a, \lambda) = \pm 1, \quad B(b, \lambda) = \pm 1 \quad \left. \begin{array}{l} \text{Local realism:} \\ \text{Locality} \\ \text{Realism} \end{array} \right\}$$

Consider two different configurations for A and B:  $\{a, a'\}$ ,  $\{b, b'\}$

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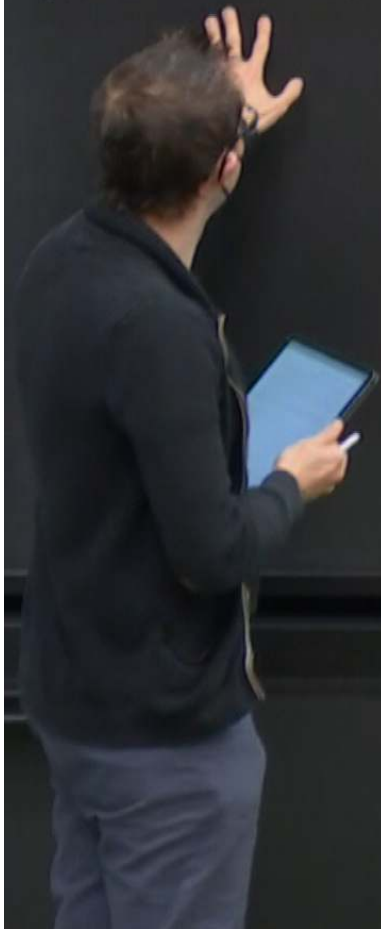
$$C(a, b) = \int d\lambda \Delta(\lambda) A(a, \lambda) B(b, \lambda) \quad A(a, \lambda) = \pm 1, \quad B(b, \lambda)$$

Consider two different configurations for A and B:  $\{a, a'\}$ ,  $\{b, b'\}$

$$\begin{aligned} C(a, b) - C(a, b') &= \int d\lambda \Delta(\lambda) (A(a, \lambda) B(b, \lambda) - A(a, \lambda) B(b', \lambda)) = \\ &= \int d\lambda \Delta(\lambda) \left[ A(a, \lambda) B(b, \lambda) (1 \pm A(a', \lambda) B(b', \lambda)) \right] - \int d\lambda \Delta(\lambda) A(a, \lambda) B(b', \lambda) \end{aligned}$$

Since  $|A(a, \lambda)| \leq 1$      $|B(b, \lambda)| \leq 1$

$$|C(a,b) - C(a',b')| \leq \int d\lambda \Lambda(\lambda) (1 \pm A(a',\lambda) B(b',\lambda)) + \int d\lambda \Lambda(\lambda) (1 \pm A(a,\lambda) B(b,\lambda))$$



$$|C(a,b) - C(a,b')| \leq \int d\lambda \Lambda(\lambda) (1 \pm A(a',\lambda) B(b,\lambda)) + \int d\lambda \Lambda(\lambda) (1 \pm A(a,\lambda) B(b',\lambda))$$

$$|C(a,b) - C(a,b')| + |C(a',b') + C(a',b)| \leq 2$$

$$\int d\lambda \Delta(\lambda) = 1$$

$$\int d\lambda \Delta(\lambda) (1 \pm A(a', \lambda) B(b, \lambda)) = 2 \pm (C(a', b') + C(a', b))$$

Bell Inequality

Consider a system of two two-level quantum systems

Let's consider a bipartite state  $|\Psi_{-}\rangle = \frac{1}{\sqrt{2}} (|0_A 1_B\rangle - |1_A 0_B\rangle)$

Since  $|A(a, \lambda)| \leq 1$   $|B(b, \lambda)| \leq 1$

$$|C(a, b) - C(a, b')| \leq \int d\lambda \Delta(\lambda) (1 \pm A(a', \lambda) B(b', \lambda)) + \int d\lambda \Delta(\lambda) (1 \pm A(a, \lambda) B(b, \lambda))$$

$$|C(a, b) - C(a, b')| + |C(a', b') + C(a', b)| \leq 2$$

Bell Inequality

Let's consider

$$A(a) = \hat{\sigma}_{2A} \otimes \mathbb{1}_B$$

$$B(b) = \frac{1}{\sqrt{2}} \mathbb{1}_A \otimes (\hat{\sigma}_{2B} + \hat{\sigma}_{yA})$$

Since  $|A(a, \lambda)| \leq 1$   $|B(b, \lambda)| \leq 1$

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$$|C(a, b) - C(a, b')| + |C(a', b') + C(a', b)| \leq 2$$

Bell Inequality

Let's consider

$$A(a) = \hat{\sigma}_{z_A} \otimes \mathbb{1}_B$$

$$B(b) = \frac{1}{\sqrt{2}} \mathbb{1}_A \otimes (\hat{\sigma}_{z_B} + \hat{\sigma}_{y_B})$$

$$|\Delta(\lambda)| \leq 1$$

$$\int d\lambda \Delta(\lambda) = 1$$

$$+ \int d\lambda \Delta(\lambda) (1 \pm A(a', \lambda) B(b, \lambda)) = 2 \pm (C(a', b') + C(a', b))$$

Bell Inequality

Consider a system of two two-level quantum systems

Let's consider a bipartite state  $|\Psi_{-}\rangle = \frac{1}{\sqrt{2}} (|0_A 1_B\rangle - |1_A 0_B\rangle)$

$$\hat{\sigma}_z |0\rangle = -|0\rangle$$

$$\hat{\sigma}_z |1\rangle = |1\rangle$$

$$\hat{\sigma}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\langle \hat{A}(a) \hat{B}(b) \rangle_{|\psi\rangle} = \langle \hat{A}(a') \hat{B}(b') \rangle_{|\psi\rangle} = \langle \hat{A}(a') \hat{B}(b) \rangle_{|\psi\rangle} = -\langle \hat{A}(a) \hat{B}(b') \rangle_{|\psi\rangle} = \frac{1}{\sqrt{2}}$$

$$\left| \underbrace{\langle \hat{A}(a) \hat{B}(b) \rangle}_{1/\sqrt{2}} - \underbrace{\langle \hat{A}(a) \hat{B}(b') \rangle}_{-1/\sqrt{2}} \right| + \left| \underbrace{\langle \hat{A}(a') \hat{B}(b) \rangle}_{1/\sqrt{2}} + \underbrace{\langle \hat{A}(a') \hat{B}(b') \rangle}_{1/\sqrt{2}} \right| = 2\sqrt{2} > 2$$



$$|D^{10}\rangle = \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} (|0\rangle_A |0\rangle_B + |1\rangle_A |1\rangle_B)$$

$$|D^{10}\rangle = \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} (|0\rangle_A |0\rangle_B + |1\rangle_A |1\rangle_B)$$

$$|0\rangle_A |1\rangle_B = |11\rangle$$

State separability and quantum entanglement



$$\hat{B}(b) = \frac{1}{\sqrt{2}} \mathbb{1}_A \otimes (\hat{\sigma}_{zA} + \hat{\sigma}_{zB})$$

$$\hat{B}(b') = \frac{1}{\sqrt{2}} \mathbb{1}_A \otimes (\hat{\sigma}_{zA} - \hat{\sigma}_{zB})$$

$$\hat{\sigma}_z |1\rangle =$$

State separability and quantum entanglement  
 for a bipartite state  $\hat{\rho}_{AB}$ , is it enough to know everything about subsystem A  
 for a pure bipartite state  $|\psi_{AB}\rangle$  is it enough to fully know  $\hat{\rho}_A = \text{tr}_B$

It's true if the pure state is a product state  $|\psi_{AB}\rangle = |\psi_A\rangle \otimes |\psi_B\rangle$   
 $|\psi_{AB}\rangle = (|\psi_{A1}\rangle + |\psi_{A2}\rangle) \otimes |\psi_B\rangle$

$$\frac{1}{\sqrt{2}} |1_A\rangle \otimes (\hat{\sigma}_z |0\rangle - \hat{\sigma}_z |1\rangle) \quad \hat{\sigma}_z |1\rangle = -|1\rangle$$

entanglement

to know everything about subsystem A and B to describe  $\hat{\rho}_{AB}$

is it enough to fully know  $\hat{\rho}_A = \text{tr}_B(\hat{\rho}_{AB})$  and  $\hat{\rho}_B = \text{tr}_A(\hat{\rho}_{AB})$ ?

direct state

$$|\psi_{AB}\rangle = |\psi_A\rangle \otimes |\psi_B\rangle$$

$$|\psi_{AB}\rangle = (|\psi_{A1}\rangle + |\psi_{A2}\rangle) \otimes (|\psi_{B1}\rangle + |\psi_{B2}\rangle)$$

$$|\psi_{AB}\rangle \in \mathcal{H}_{AB}$$

$$|\psi_{A/B}\rangle \in \mathcal{H}_{A/B}$$

$$|\psi_s\rangle = \frac{1}{2} \left( |0_A 0_B\rangle + |1_A 1_B\rangle + |0_A 1_B\rangle + |1_A 0_B\rangle \right) = \frac{1}{2} \left( |0_A \right.$$
$$|\psi_c\rangle = \frac{1}{\sqrt{2}} \left( |0_A 0_B\rangle + |1_A 1_B\rangle \right)$$

$$= \frac{1}{2} \left( |0_A\rangle + |1_A\rangle \right) \otimes \left( |0_B\rangle + |1_B\rangle \right) \quad \text{product state}$$

$|\psi_S^A\rangle$                        $|\psi_S^B\rangle$

$$|\psi_s\rangle = \frac{1}{2} \left( |0_A 0_B\rangle + |1_A 1_B\rangle + |0_A 1_B\rangle + |1_A 0_B\rangle \right) = \frac{1}{2} \underbrace{\left( |0_A\rangle + |1_A\rangle \right)}_{|\psi_A\rangle} \otimes$$

$$|\psi_c\rangle = \frac{1}{\sqrt{2}} \left( |0_A 0_B\rangle + |1_A 1_B\rangle \right) \neq \left( \alpha_A |0_A\rangle + \beta_A |1_A\rangle \right) \otimes \left( \alpha_B |0_B\rangle + \beta_B |1_B\rangle \right)$$

$$|\psi_s\rangle = \frac{1}{2} \left( |0_A 0_B\rangle + |1_A 1_B\rangle + |0_A 1_B\rangle + |1_A 0_B\rangle \right) = \frac{1}{2} \underbrace{\left( |0_A\rangle + |1_A\rangle \right)}_{|\psi_A\rangle} \otimes$$

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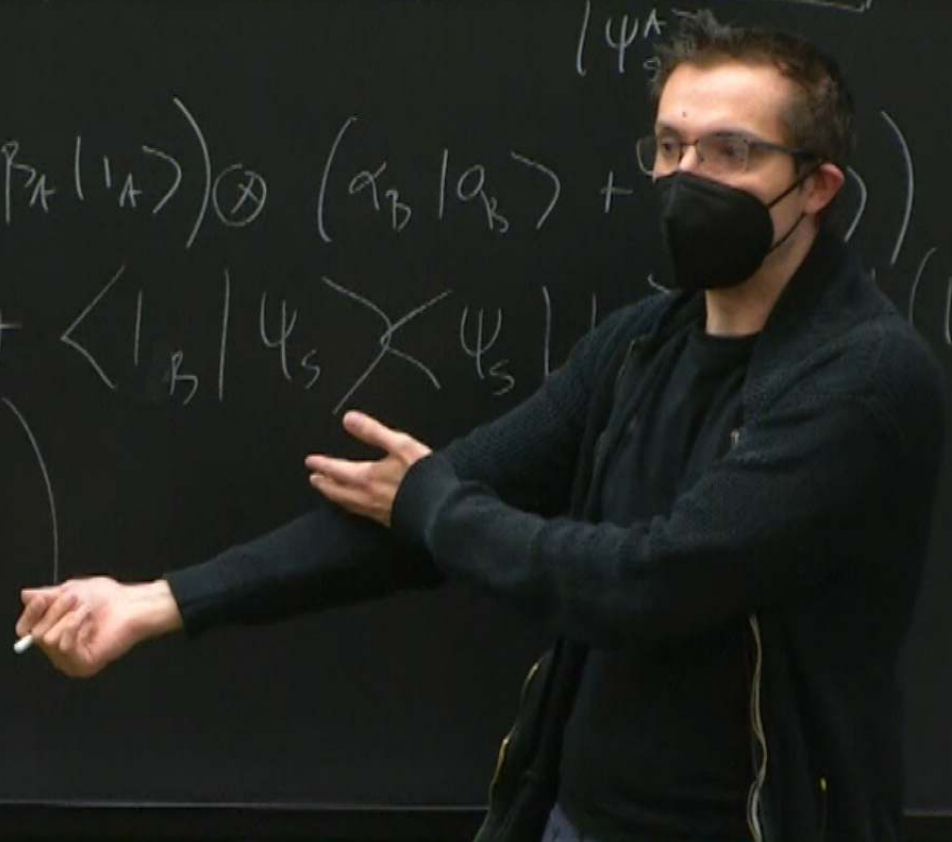
$$\hat{\rho}_s^A = \text{Tr}_B \left( |\psi_s\rangle \langle \psi_s| \right) = \langle 0_B | \psi_s \rangle \langle \psi_s | 0_B \rangle + \langle 1_B | \psi_s \rangle \langle \psi_s | 1_B \rangle$$

$$| \psi \rangle \otimes \left( \frac{|0_B\rangle + |1_B\rangle}{\sqrt{2}} \right) \quad \text{product state}$$

$|\psi_B^B\rangle$

$$\frac{1}{2} \left( |0_A\rangle\langle 0_A| + |0_A\rangle\langle 1_A| + |1_A\rangle\langle 0_A| + |1_A\rangle\langle 1_A| \right) = |\psi_S^A\rangle\langle \psi_S^A|$$

$$\begin{aligned}
 & (|0_A 0_B\rangle + |1_A 1_B\rangle + |0_A 1_B\rangle + |1_A 0_B\rangle) = \frac{1}{2} (|0_A\rangle + |1_A\rangle) \otimes (|0_B\rangle + |1_B\rangle) \\
 & (|0_A 0_B\rangle + |1_A 1_B\rangle) \neq (\alpha_A |0_A\rangle + \beta_A |1_A\rangle) \otimes (\alpha_B |0_B\rangle + \beta_B |1_B\rangle) \\
 & \langle \Psi_S | \Psi_S \rangle = \langle 0_B | \Psi_S \rangle \langle \Psi_S | 0_B \rangle + \langle 1_B | \Psi_S \rangle \langle \Psi_S | 1_B \rangle \\
 & \langle \Psi_S | \Psi_S \rangle = \frac{1}{2} (|0_A\rangle \langle 0_A| + |1_A\rangle \langle 1_A|)
 \end{aligned}$$



$$(\alpha_A |0_B\rangle + \beta_A |1_B\rangle) = \frac{1}{2} \underbrace{(|0_A\rangle + |1_A\rangle)}_{|\psi_A\rangle} \otimes \underbrace{(|0_B\rangle + |1_B\rangle)}_{|\psi_B\rangle} \quad \text{product state}$$

$$\begin{aligned}
 & (\alpha_A |0_A\rangle + \beta_A |1_A\rangle) \otimes (\alpha_B |0_B\rangle + \beta_B |1_B\rangle) \\
 & \langle \psi_A | \langle \psi_B | (\alpha_A |0_A\rangle + \beta_A |1_A\rangle) \otimes (\alpha_B |0_B\rangle + \beta_B |1_B\rangle) = \frac{1}{2} (|0_A\rangle\langle 0_A| + |0_A\rangle\langle 1_A| + |1_A\rangle\langle 0_A| + |1_A\rangle\langle 1_A|) = \\
 & (|0_A\rangle\langle 0_A| + |1_A\rangle\langle 1_A|) \quad |X\rangle\langle X| \quad |1\rangle\langle 1| \quad |1\rangle\langle 1| \\
 & \quad \quad \quad |X\rangle\langle X|
 \end{aligned}$$

$$\hat{A}(a) = \frac{1}{\sqrt{2}} \hat{\sigma}_A^x \otimes \mathbb{1}_B$$

$$\hat{B}(b) = \frac{1}{\sqrt{2}} \mathbb{1}_A \otimes (\hat{\sigma}_B^z + \hat{\sigma}_B^x)$$

$$\hat{A}(a') = \frac{1}{\sqrt{2}} \hat{\sigma}_A^x \otimes \mathbb{1}_B$$

$$\hat{B}(b') = \frac{1}{\sqrt{2}} \mathbb{1}_A \otimes (\hat{\sigma}_B^z - \hat{\sigma}_B^x)$$

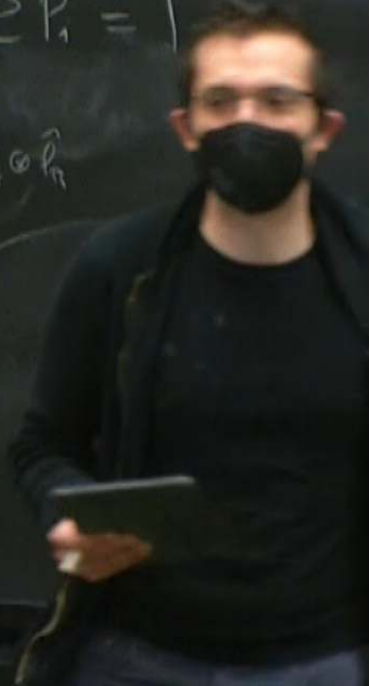
$$\hat{\sigma}_z |1\rangle = |1\rangle$$

A bipartite state is non-separable (entangled) iff it cannot be expressed as a classical mixture of product states

$$\hat{\rho}_{AB} \text{ entangled} \Leftrightarrow \hat{\rho}_{AB} \neq \sum_i P_i \hat{\rho}_A^i \otimes \hat{\rho}_B^i \quad 0 \leq P_i \leq 1 \quad \sum_i P_i = 1$$

For pure states  $|\psi_{AB}\rangle$ , separable (pure)  $\Leftrightarrow$  product (pure)  $\hat{\rho}_A \otimes \hat{\rho}_B$

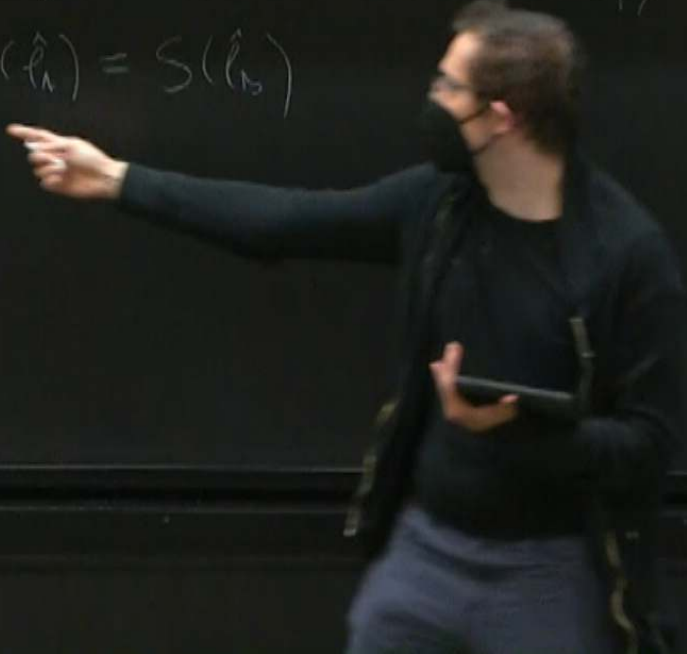
$$\hat{\rho}_{AB} = \sum_{i,j} C_{ij} \hat{\rho}_A^i \otimes \hat{\rho}_B^j$$



$$\frac{1}{\sqrt{2}} \langle A(a) B(b) \rangle - \frac{1}{\sqrt{2}} \langle A(a) B(b) \rangle + \frac{1}{\sqrt{2}} \langle A(a) B(b) \rangle + \frac{1}{\sqrt{2}} \langle A(a) B(b) \rangle$$

Entanglement entropy: The Von Neumann entropy of the partial states. Given  $\hat{\rho}_{AB}$

$$S_E(\hat{\rho}_{AB}) := S(\hat{\rho}_A) = S(\hat{\rho}_B)$$



$$\left| \frac{\langle \hat{A}(a) \hat{B}(b) \rangle - \langle \hat{A}(a) \rangle \langle \hat{B}(b) \rangle}{1/\sqrt{2}} \right| + \left| \frac{\langle \hat{A}(a) \hat{B}(b) \rangle + \langle \hat{A}(a) \rangle \langle \hat{B}(b) \rangle}{1/\sqrt{2}} \right|$$

Entanglement entropy: The von Neumann entropy of the partial states. Given  $\hat{\rho}_{AB}$

$$S_E(\hat{\rho}_{AB}) := S(\hat{\rho}_A) = S(\hat{\rho}_B)$$

Works when  $\hat{\rho}_{AB} = |\Psi_{AB}\rangle\langle\Psi_{AB}|$  if  $\hat{\rho}_A$  has "ignorance" it has to come from entanglement in  $\hat{\rho}_{AB}$ .

$$\left| \frac{\langle \hat{A}(a) \hat{B}(b) \rangle}{1/\sqrt{2}} - \frac{\langle \hat{A}(a) \rangle \langle \hat{B}(b) \rangle}{-1/\sqrt{2}} \right| + \left| \frac{\langle \hat{A}(a) \hat{B}(b) \rangle}{1/\sqrt{2}} + \frac{\langle \hat{A}(a) \rangle \langle \hat{B}(b) \rangle}{1/\sqrt{2}} \right|$$

Entanglement entropy: The Von Neumann entropy of the partial states. Given  $\hat{\rho}_{AB}$

$$S_E(\hat{\rho}_{AB}) := S(\hat{\rho}_A) = S(\hat{\rho}_B)$$

Works when  $\hat{\rho}_{AB} = |\Psi_{AB}\rangle\langle\Psi_{AB}|$  if  $\hat{\rho}_A$  has 'ignorance' to come from entanglement in  $\hat{\rho}_{AB}$

Measure of entanglement for pure states:

$$\begin{aligned}
 |\psi_+\rangle\langle\psi_+| &\Rightarrow \hat{\rho}_{AB} = \text{tr}_B(|\psi_+\rangle\langle\psi_+|) = \frac{1}{2}\mathbb{1}_A \\
 S_E(\rho_{AB}) &= S\left(\frac{1}{2}\mathbb{1}_A\right) = \log_2 2 = 1 \\
 &= \left(\frac{1}{2}\mathbb{1}_A\right) \otimes \left(\frac{1}{2}\mathbb{1}_B\right) \\
 &= S(\hat{\rho}_A) = S\left(\frac{1}{2}\mathbb{1}_A\right) = \log_2 2 = 1
 \end{aligned}$$

$$\langle \hat{B}(b) \rangle_{|\Psi_-\rangle} = - \langle \hat{A}(a) \hat{B}(b') \rangle_{|\Psi_-\rangle} = \frac{1}{\sqrt{2}}$$

$$2 + \langle \hat{A}(a) \hat{B}(b) \rangle = 2\sqrt{2} > 2$$

$\frac{1}{\sqrt{2}}$        $|\Psi_+\rangle \langle \Psi_+|$

$\frac{1}{4} \mathbb{1}_{AB}$

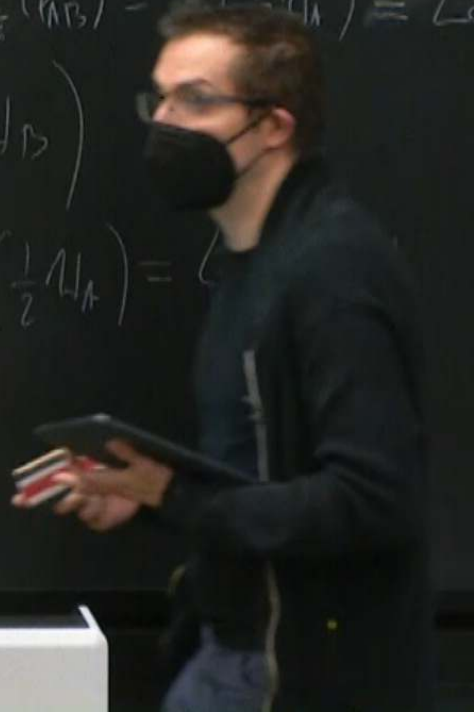
$\rho = a |\Psi_+\rangle \langle \Psi_+| + (1-a) \frac{1}{4} \mathbb{1}_{AB}$

$$\hat{\rho}_{AB} = |\Psi_+\rangle \langle \Psi_+| \Rightarrow \hat{\rho}_A = \text{tr}_B(|\Psi_+\rangle \langle \Psi_+|) = \frac{1}{2} \mathbb{1}_A$$

$$S_E(\hat{\rho}_{AB}) = S(\frac{1}{4} \mathbb{1}_{AB}) = \dots$$

$$\hat{\rho}_{AB} = \frac{1}{4} \mathbb{1}_{AB} = \left(\frac{1}{2} \mathbb{1}_A\right) \otimes \left(\frac{1}{2} \mathbb{1}_B\right)$$

$$S_E(\hat{\rho}_{AB}) = S(\hat{\rho}_A) = S\left(\frac{1}{2} \mathbb{1}_A\right) = \dots$$



$$\hat{\beta}(a) = \frac{1}{\sqrt{2}} \mathbb{1}_A \otimes (\hat{\sigma}_{z_B} + \hat{\sigma}_{x_B})$$

$$\hat{\beta}(b) = \frac{1}{\sqrt{2}} \mathbb{1}_A \otimes (\hat{\sigma}_{z_B} - \hat{\sigma}_{x_B})$$

$$\hat{\sigma}_z |1\rangle = |1\rangle$$

Entanglement measures  $\mathcal{E}(\rho_{AB}) \in \mathbb{R}$

- Must be maximum for maximally entangled states
- Must be zero for separable states
- Must be non-zero for non-separable states
- Must not increase under Local operations + classical communication (LOCC)

# Peres Criterion

Partial transpose:

$$\hat{\rho}_{AB} = \sum_{ijkl} \rho_{ijkl} |i\rangle_A |j\rangle_B \langle k|_A \langle l|_B$$

$$\rho^{PT}_B = \sum_{ijkl} \rho_{ijkl} |i\rangle_A |j\rangle_B \langle k|_A \langle l|_B$$

$$\rho^{PT}_A = \sum_{ijkl} \rho_{ijkl} |i\rangle_A |j\rangle_B \langle k|_A \langle l|_B$$

$$\rho_{AB} = \frac{1}{2} (|01\rangle\langle 00| + |00\rangle\langle 01| + |11\rangle\langle 11| + |00\rangle\langle 00|)$$

$$\rho^{PT}_{AB} = \frac{1}{2} (|00\rangle\langle 01| + |01\rangle\langle 00| + |11\rangle\langle 11| + |00\rangle\langle 00|)$$

# Peters Criterion

Partial transpose:

$$\hat{P}_{AB} = \sum_{ijkl} P_{ijkl} |i\rangle_A |j\rangle_B \langle k|_A \langle l|_B$$

$$P_{AB} = \frac{1}{2} (|01\rangle\langle 00| + |00\rangle\langle 01| + |11\rangle\langle 11| + |00\rangle\langle 00|)$$

$$P_{AB}^{PT} = \frac{1}{2} (|00\rangle\langle 01| + |01\rangle\langle 00| + |11\rangle\langle 11| + |00\rangle\langle 00|)$$

$$\frac{1}{2} \text{Tr}(\dots)$$

$$P_{AB}^{PT} = \sum_{ijkl}$$

$$P_{AB}^{PT} = \sum_{ijkl}$$

$$|k\rangle_A |l\rangle_B \langle k|_A \langle l|_B$$

$$|k\rangle_A |j\rangle_B \langle i|_A \langle l|_B$$

$$\hat{\rho}_{AB} = \sum p_n \hat{\rho}_A^{nT} \otimes \hat{\rho}_B^n$$

If  $\hat{\rho}_{AB}$  is separable  $\Rightarrow \hat{\rho}_{AB}^{PT}$  is a density operator

If  $\hat{\rho}_{AB}^{PT}$  is not a positive (semidefinite) operator then  $\hat{\rho}_{AB}$  is non-separable

operator then  $\hat{\rho}_{AB}$  is non-separable

if  $\dim(\mathcal{H}_A) \times \dim(\mathcal{H}_B)$  is  $2 \times 2$  or  $2$

then  $\hat{\rho}_{AB}^{PT}$  non positive  $\Leftrightarrow \hat{\rho}_{AB}$  entangled

	$ 00\rangle$	$ 01\rangle$	$ 10\rangle$	$ 11\rangle$
$ 00\rangle$	1			
$ 01\rangle$		0	1	
$ 10\rangle$		1	0	
$ 11\rangle$				1

$$\lambda^2 - 1 = 0$$

$$\lambda = \pm 1$$