

Title: Quantum Information 2021/2022

Speakers: Eduardo Martin-Martinez, Philippe Allard Guerin

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Bipartite systems: Composite systems made of two subsystems A and B. If $\mathcal{H}_{A/B}$ is the Hilbert space of the composite system is $\mathcal{H}_{AB} = \mathcal{H}_A \otimes \mathcal{H}_B$

If $\{|a_i\rangle\}$ is an orthonormal basis of $\mathcal{H}_A \Rightarrow \{|a_i\rangle \otimes |b_j\rangle \equiv |a_i b_j\rangle\}$ is an orthonormal basis of $\mathcal{H}_{AB}$
 If $\{|b_j\rangle\}$ is an orthonormal basis of $\mathcal{H}_B$

systems made of two subsystems A and B. If $\mathcal{H}_{A/B}$ is the Hilbert space associated with subsystem A/B the system is $\mathcal{H}_{AB} = \mathcal{H}_A \otimes \mathcal{H}_B$

basis of $\mathcal{H}_A \Rightarrow \{|a_i\rangle \otimes |b_j\rangle \equiv |a_i b_j\rangle\}$ is an orthonormal basis of $\mathcal{H}_{AB}$
 basis of $\mathcal{H}_B$

determine the stability - we consider small perturbations and try to understand the nature of the perturbed initial conditions

A bipartite system is in a state $\hat{P}_{AB}$.

What is the state $\hat{P}_A$ and $\hat{P}_B$ of the

Dynamical Systems

any system that evolves in time

where X is a state space/phase space

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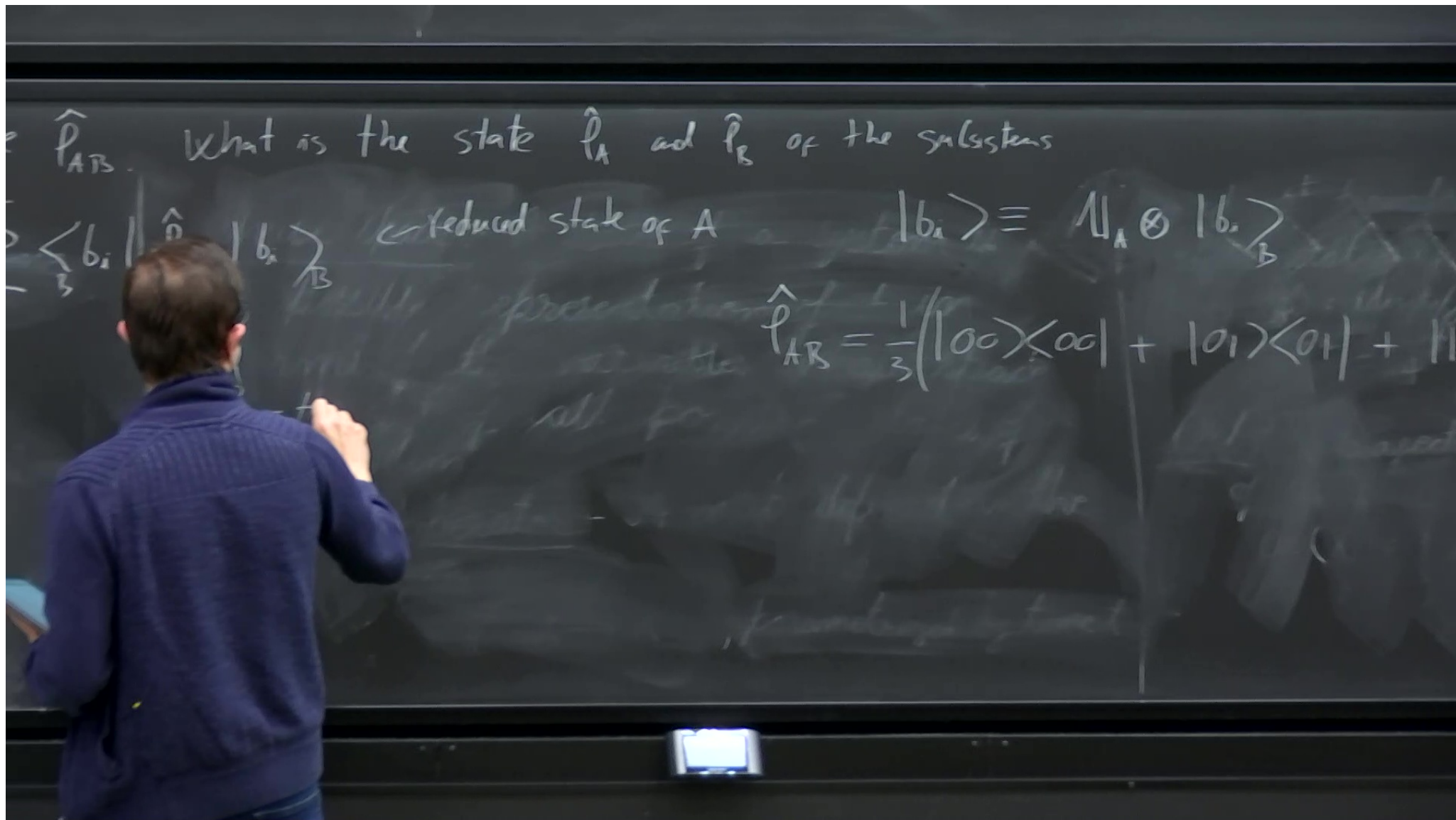
where X is a state space/phase space

phase space: a state of a system

a possible representation of a system

the system

operator



What is the state $\hat{\rho}_A$ and $\hat{\rho}_B$ of the subsystems

$|b_A\rangle \hat{\rho}_{AB} |b_A\rangle$ ← reduced state of A $|b_A\rangle \equiv |1\rangle_A \otimes |b_A\rangle_B$

$$\hat{\rho}_{AB} = \frac{1}{3} (|00\rangle\langle 00| + |01\rangle\langle 01| + |10\rangle\langle 10|)$$

$$\hat{\rho}_A = \text{tr}_B \hat{\rho}_{AB} = \sum_{n_B=0}^1 \langle n_B | \hat{\rho}_{AB} | n_B \rangle = \frac{1}{3} (\langle 0_B | \hat{\rho}_{AB} | 0_B \rangle + \langle 1_B | \hat{\rho}_{AB} | 1_B \rangle)$$

What is the state $\hat{\rho}_A$ and $\hat{\rho}_B$ of the subsystems

$\hat{\rho}_{AB} |b_A b_B\rangle$ reduced state of A $|b_A\rangle \equiv \mathbb{1}_A \otimes |b_A\rangle_B$

$$\hat{\rho}_{AB} = \frac{1}{3} (|00\rangle\langle 00| + |01\rangle\langle 01| + |11\rangle\langle 11|)$$

$$\hat{\rho}_A = \text{tr}_B \hat{\rho}_{AB} = \sum_{n_B=0}^1 \langle n_B | \hat{\rho}_{AB} | n_B \rangle = \frac{1}{3} (\langle 0_B | \hat{\rho}_{AB} | 0_B \rangle + \langle 1_B | \hat{\rho}_{AB} | 1_B \rangle)$$

$$= \frac{1}{3} ((\mathbb{1}_A \otimes \langle 0_B |) |0_A\rangle \otimes |0_B\rangle + (\mathbb{1}_A \otimes \langle 1_B |) |1_A\rangle \otimes |1_B\rangle) = \frac{1}{3} (|0_A\rangle\langle 0_A| + |1_A\rangle\langle 1_A|)$$

A bipartite system is in a state $\hat{\rho}_{AB}$. What is the state $\hat{\rho}_A$ and $\hat{\rho}_B$ of the

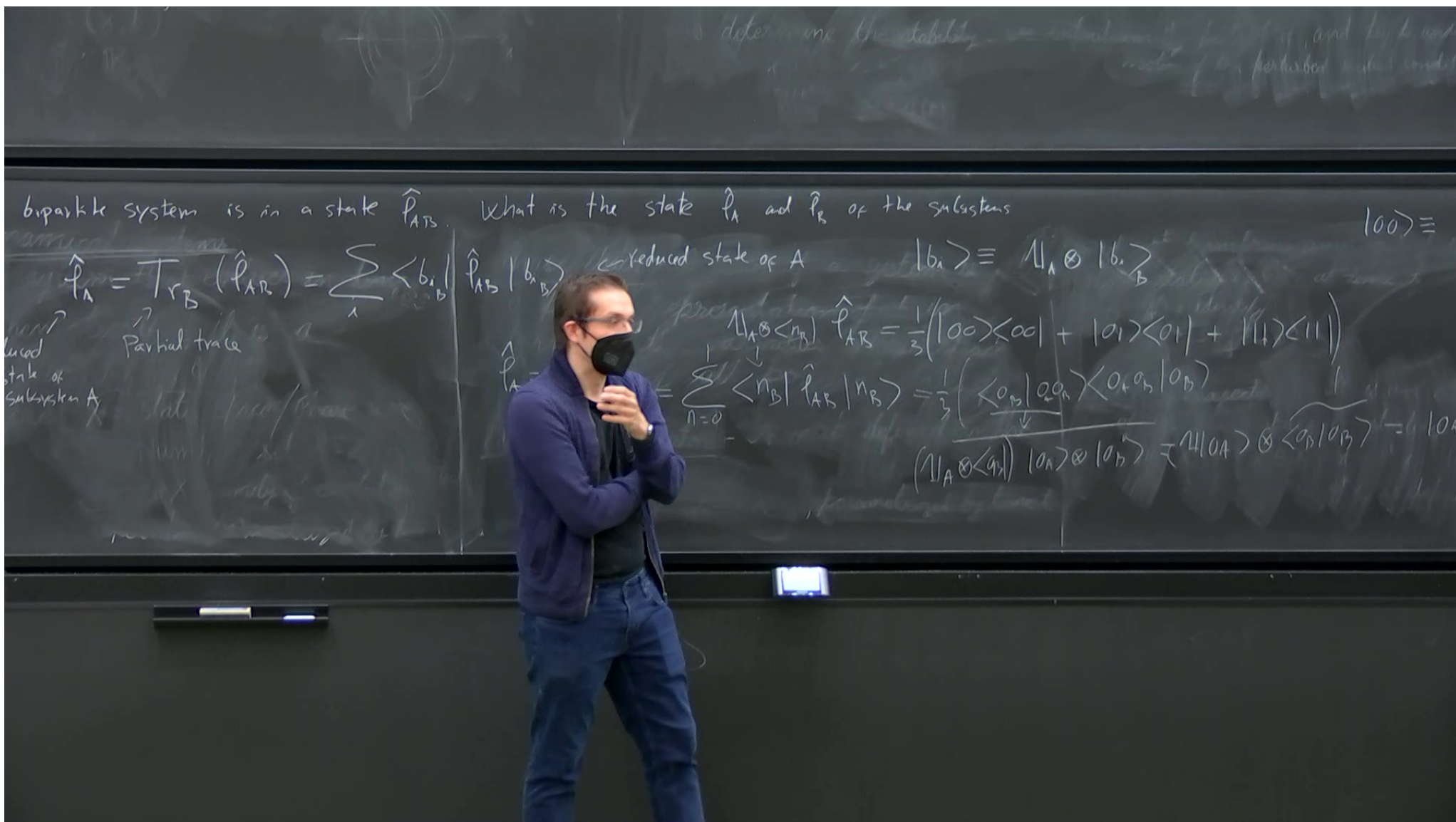
$$\hat{\rho}_A = \text{Tr}_B (\hat{\rho}_{AB}) = \sum_i \langle b_i | \hat{\rho}_{AB} | b_i \rangle$$

← reduced state of A

reduced
state of
subsystem A

Partial trace

$$\hat{\rho}_A = \text{Tr}_B \hat{\rho}_{AB} = \sum_{n=0}^1 \langle n_B | \hat{\rho}_{AB} | n_B \rangle = \frac{1}{3} \mathbb{1}_A \otimes \langle n_B | \hat{\rho}_{AB} = \frac{1}{3}$$



EPR argument and Bell inequalities

$$|\Phi^{\pm}\rangle = \frac{1}{\sqrt{2}} (|0_A 0_B\rangle \pm |1_A 1_B\rangle) \quad |\Psi^{\pm}\rangle = \frac{1}{\sqrt{2}} (|0_A 1_B\rangle \pm |1_A 0_B\rangle)$$

$$|0\rangle_A \otimes |0\rangle_B$$

EPR argument and Bell inequalities

$$|\Phi^\pm\rangle = \frac{1}{\sqrt{2}} (|0_A 0_B\rangle \pm |1_A 1_B\rangle) \quad |\Psi^\pm\rangle = \frac{1}{\sqrt{2}} (|0_A 1_B\rangle \pm |1_A 0_B\rangle)$$

Measure A, obtain a 0 $\Rightarrow |\Phi^\pm\rangle \xrightarrow{\text{PVM}}$

$$\frac{\hat{P}_{|0\rangle \otimes 0} |\Phi^\pm\rangle}{|\hat{P}_{|0\rangle \otimes 0} |\Phi^\pm\rangle|} = |0_A 0_B\rangle$$

Consider two systems A and B and sets of measurements that yield the outcomes ± 1
The result of a measurement on A

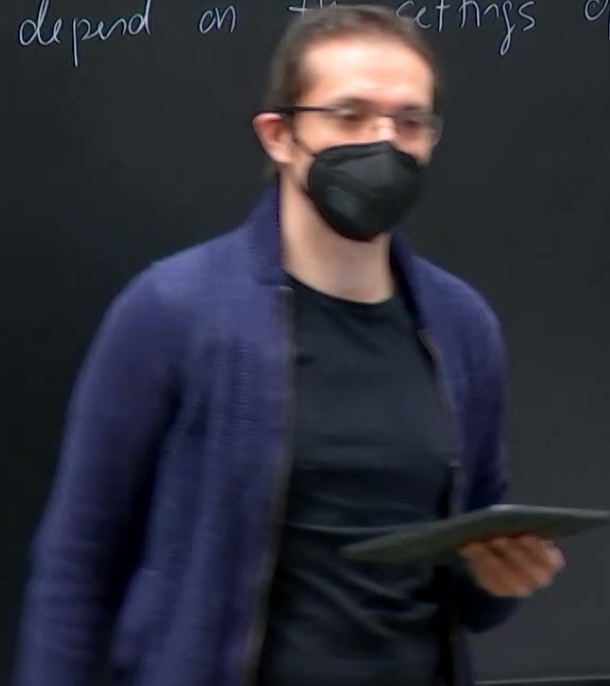
Consider two systems A and B and sets of measurements that yield the outcome

The result of a measurement on A will depend on the settings of the experiment

$$A(a, \lambda) \in \{-1, 1\}$$

likewise for B

$$B(b, \lambda)$$



systems A and B and sets of measurements that yield the outcomes ± 1 when measured
of a measurement on A will depend on the settings of the experiment a and possibly a
 $\in \{-1, 1\}$
Locality: $A(a, \lambda) \neq A(a, b, \lambda)$
Realism: A, B are functions of λ

C

ts of measurements that yield the outcomes ± 1 when measured locally

depend on the settings of the experiment a and possibly a set of hidden variables λ

$$A(a, \lambda) \neq A(a, b, \lambda)$$

A, B are functions of λ

$$C(a, b) = \int d\lambda A(a, \lambda) B(b, \lambda) \Lambda(\lambda) \quad \int d\lambda \Lambda(\lambda)$$

