Title: Classical and Quantum Chaos 2021/2022 - Lecture 11

Speakers: Meenu Kumari

Collection: Classical and Quantum Chaos 2021/2022

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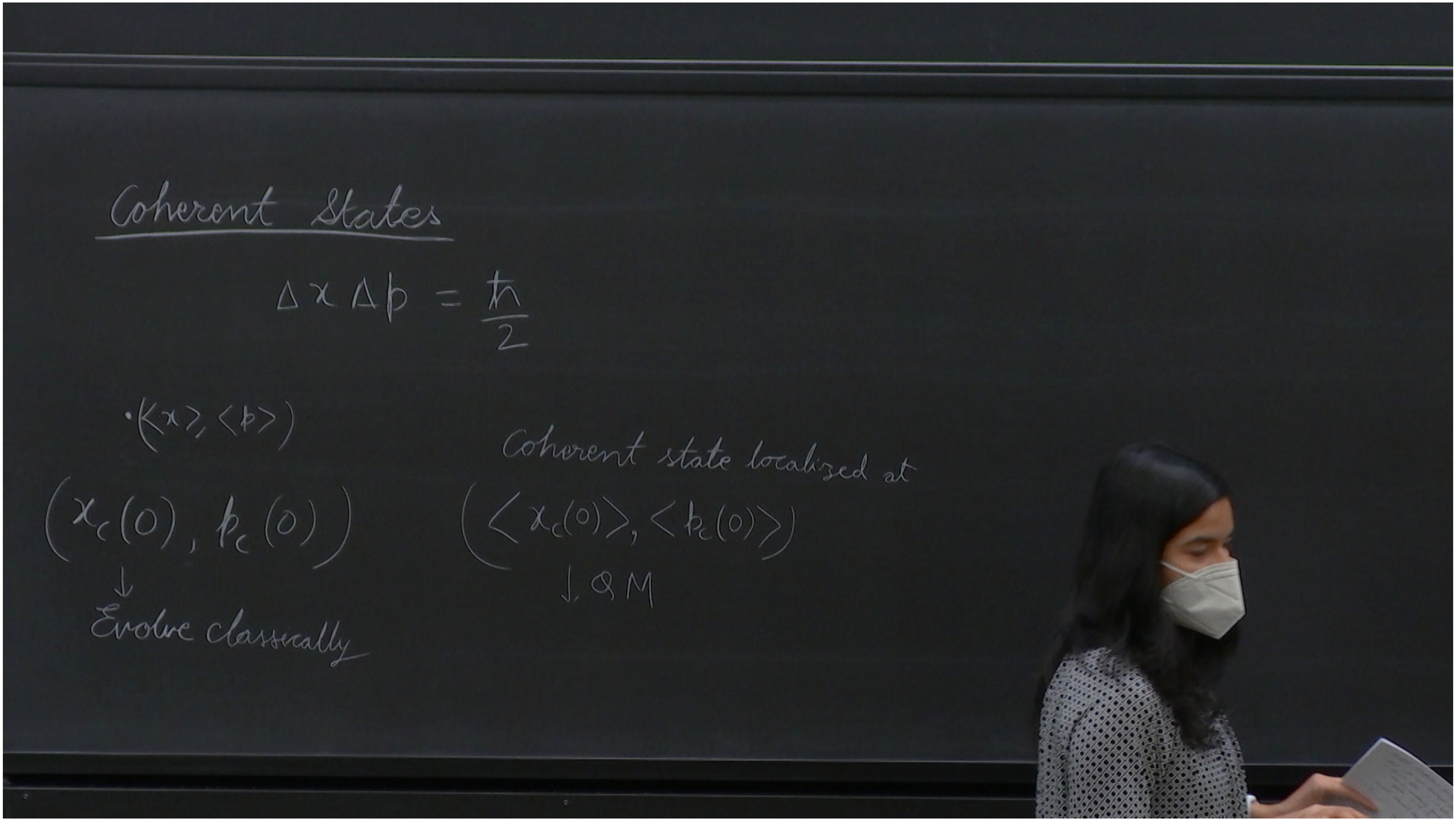
Coherent States
$\Delta x \Delta p = \frac{\hbar}{2}$
$\cdot(\langle x \rangle, \langle p \rangle)$
$(x_c(0), p_c(0))$
↓
Evolve classically
Coherent state localized at
$(\langle x_c(0) \rangle, \langle p_c(0) \rangle)$
↓ QM

Coherent States

$$\Delta x \Delta p = \frac{\hbar}{2}$$

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$(x_c(0), p_c(0))$

$\downarrow$

Evolve classically

Coherent state localized at

$(\langle x_c(0) \rangle, \langle p_c(0) \rangle)$

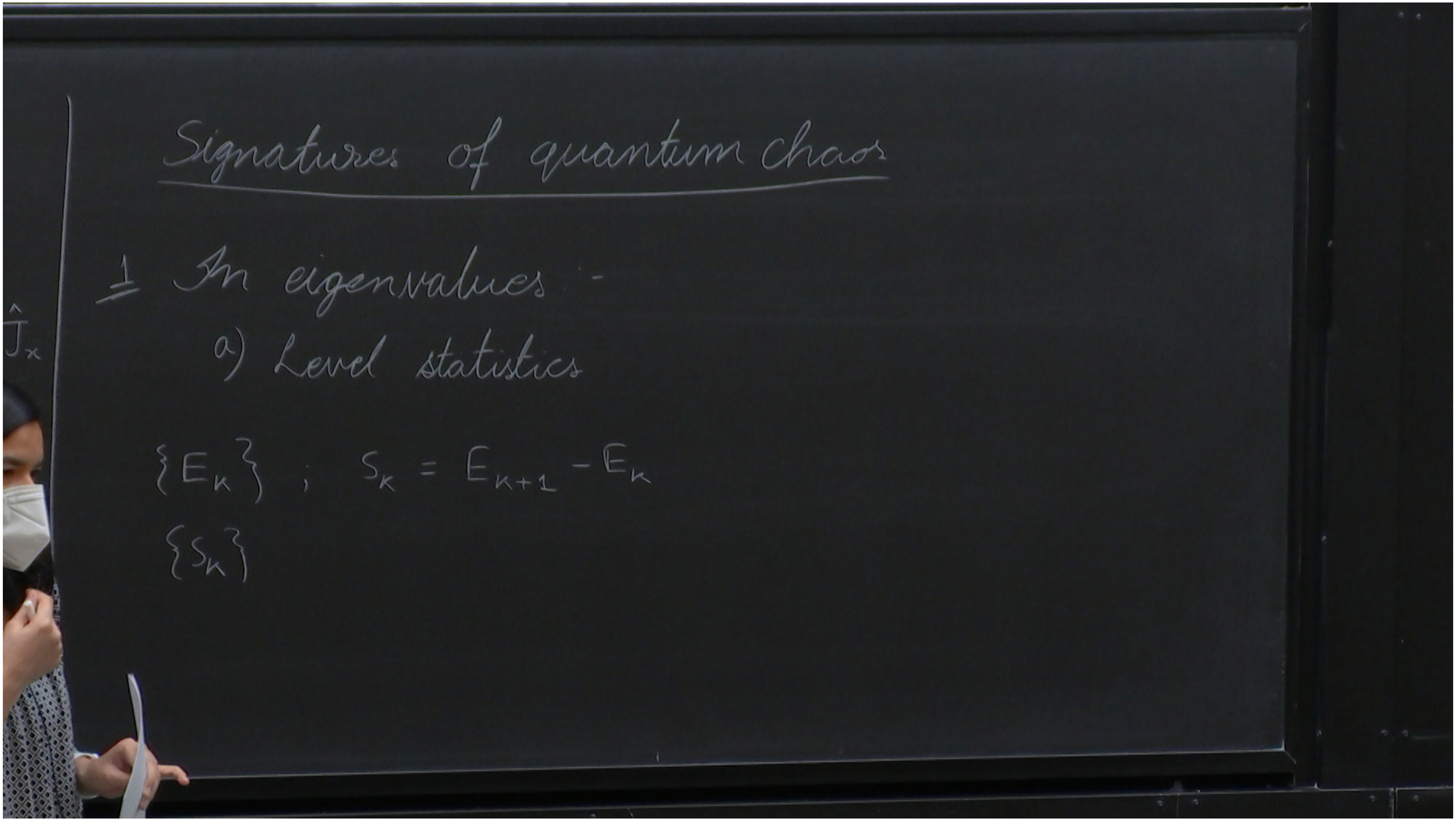
$\downarrow$ QM

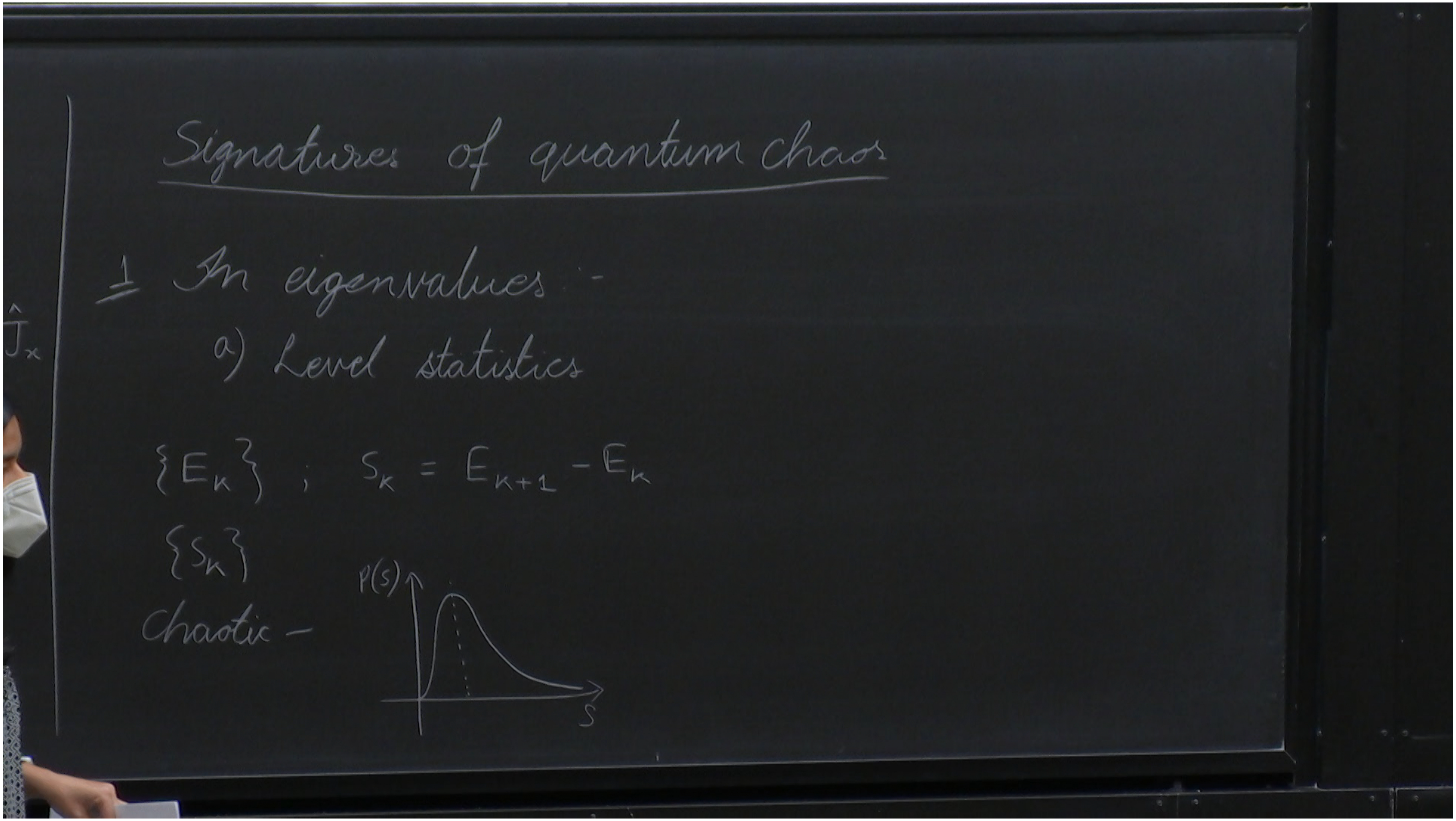
t_{Eh}

Measurement and quantum chaos

PRA 69, 052116 (2004)

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2} m\omega^2 \hat{z}^2 + b\hat{z}\hat{J}_z + c\hat{J}_x$$


Signatures of quantum chaos
1 In eigenvalues :-
a) Level statistics
$\{E_k\}$; $S_k = E_{k+1} - E_k$
$\{S_k\}$
$\hat{J}_x$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Chaotic -
$P(S)$
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Signatures of quantum chaos

1 In eigenvalues :-

a) Level statistics

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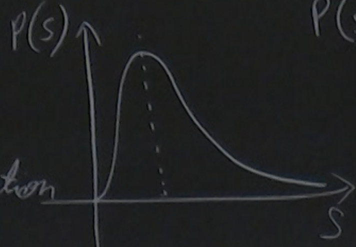
$\{S_k\}$

Chaotic -

Wigner Dyson distribution



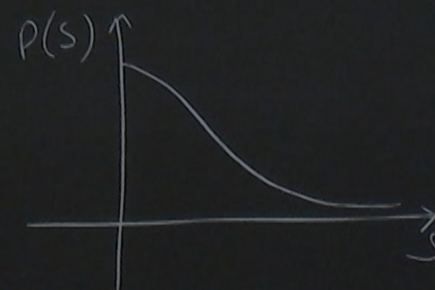
$P(s) \sim s^{\beta}, s \to 0$

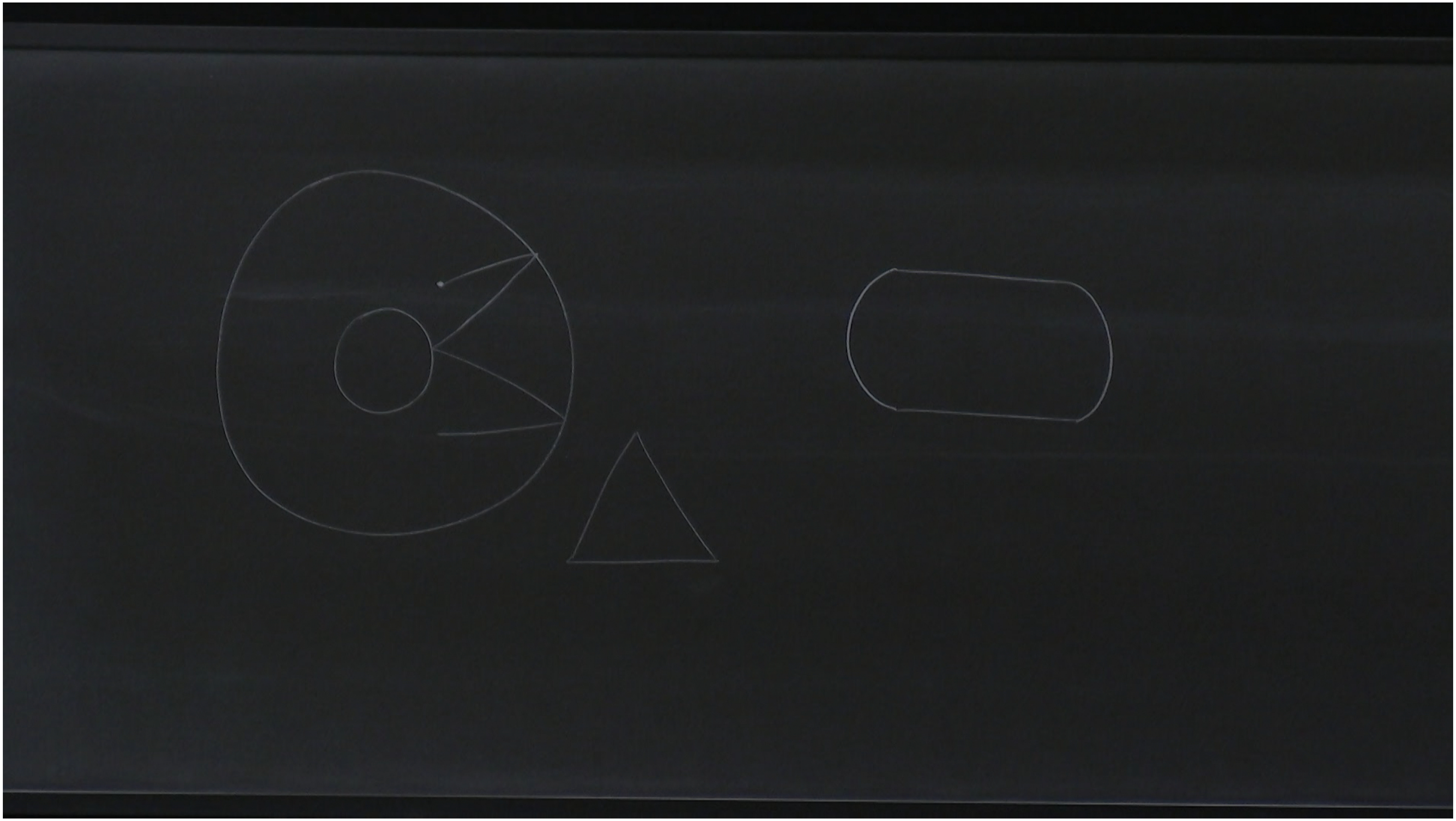
$\beta = 1, 2, 4$

Regular :-

$\{S_k\}$ $P(s) \sim \exp(-s)$

Poissonian dist





b) Spectral Form Factor

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2. Dynamical properties

a) Fidelity decay :-

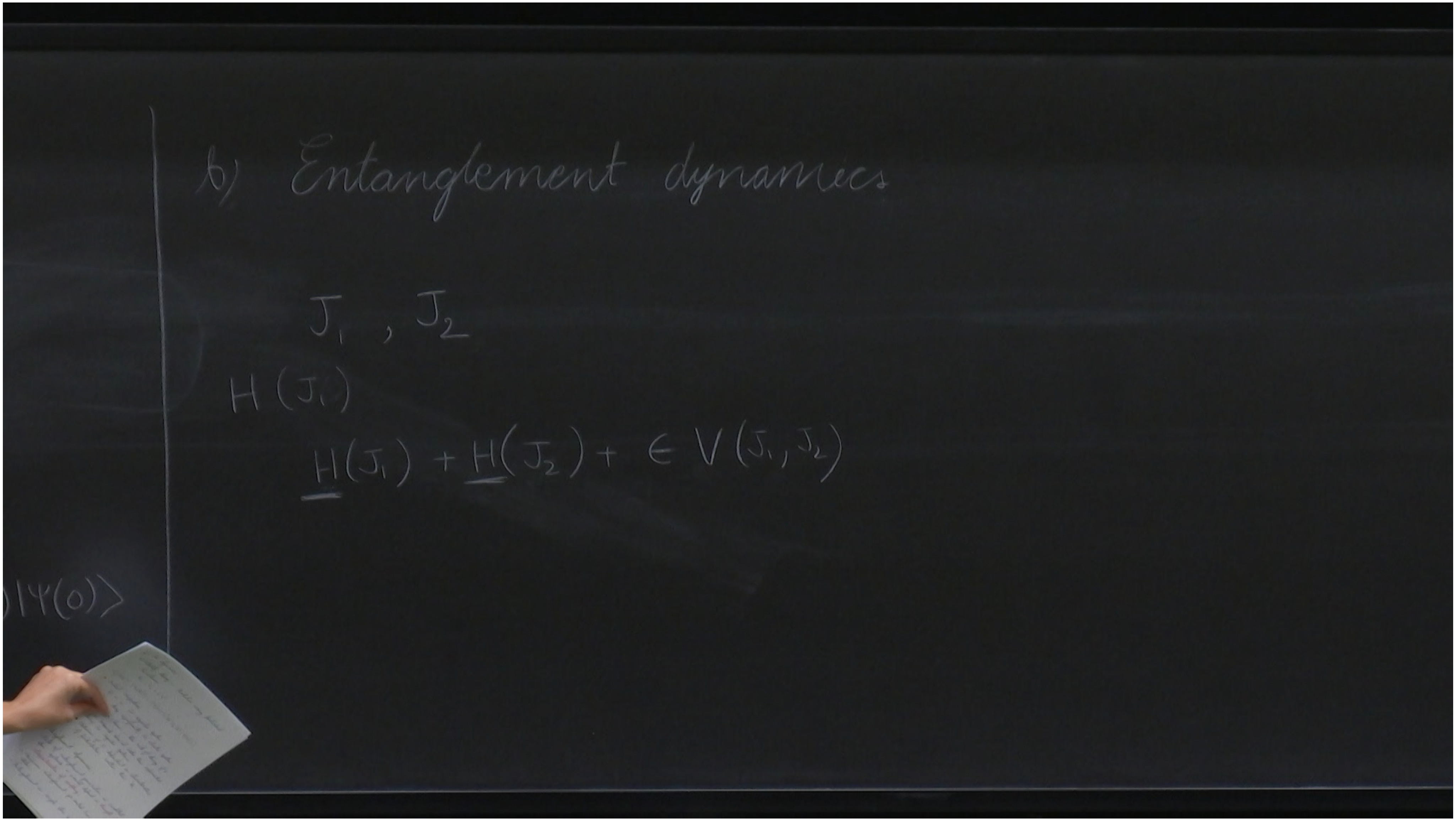
b) Spectral Form Factor

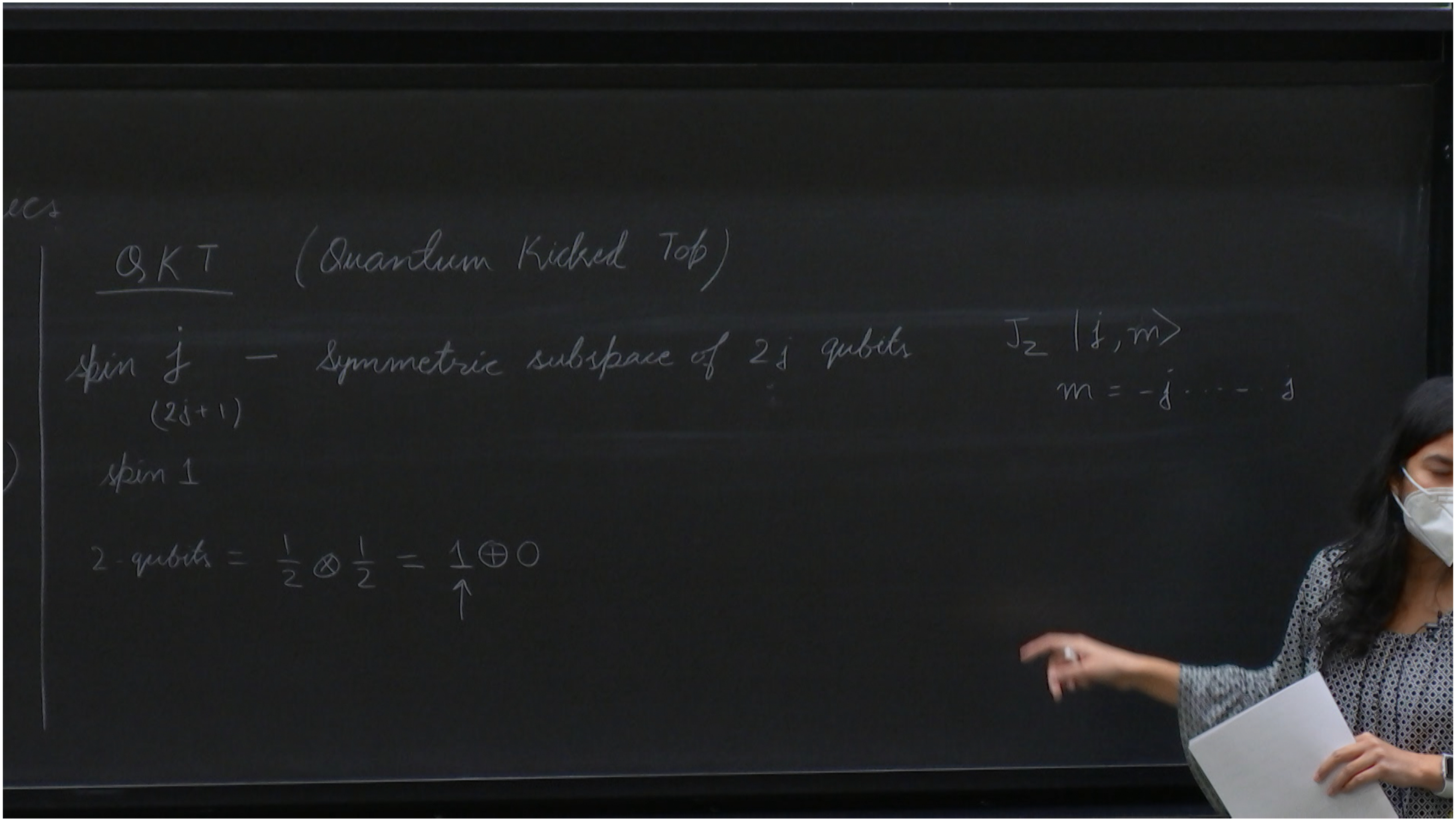
2 Dynamical properties

a) Fidelity decay :-

$|\Psi(0)\rangle : H_0$, $H_\epsilon = H_0 + \epsilon V$

$$F\left(|\Psi(t)\rangle, |\Psi_\epsilon(t)\rangle\right) = |\langle \Psi(0)| U^\dagger(t) U_\epsilon(t) |\Psi(0)\rangle$$


b) Entanglement dynamics
J_1 , J_2
$H(J_1)$
$\underline{H}(J_1) + \underline{H}(J_2) + \epsilon V(J_1, J_2)$
$|\Psi(0)\rangle$


QKT (Quantum Kicked Top)
spin j — Symmetric subspace of 2j qubits
(2j+1)
$J_z |j, m\rangle$
m = -j ..... j
spin 1
2-qubits = 1/2 ⊗ 1/2 = 1 ⊕ 0

Model: Kicked Top

$$H = pJ_y \sum_{n=0}^{\infty} \delta(t - n\tau) + \frac{\kappa}{2j\tau} J_z^2$$

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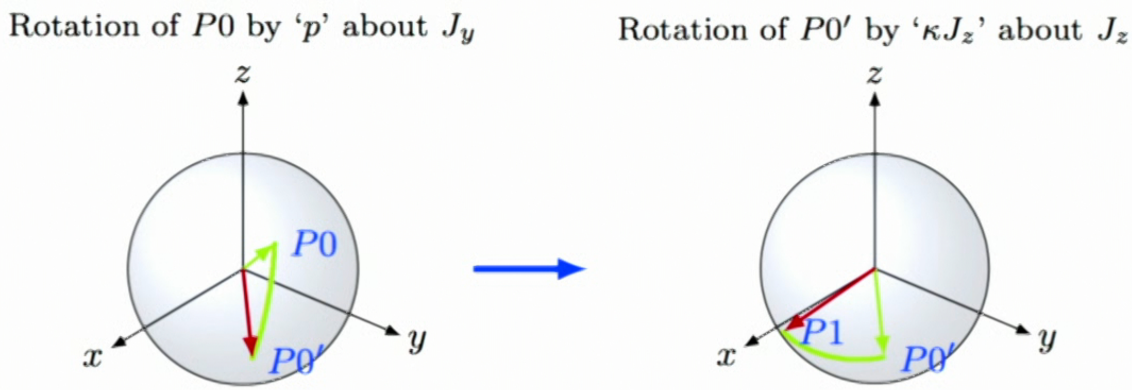

Chaotic Model: Kicked Top

$$H = \frac{\kappa}{2j\tau} J_z^2 + pJ_y \sum_{n=0}^{\infty} \delta(t - n\tau)$$

$X = \frac{J_x}{j}$, $Y = \frac{J_y}{j}$, $Z = \frac{J_z}{j}$

Classical equations of motion

$$X' = Z \cos \kappa X + Y \sin \kappa X$$

$$Y' = -Z \sin \kappa X + Y \cos \kappa X$$

$$Z' = -X$$

$X' \equiv X(n\tau + 1), X \equiv X(n\tau)$

$X^2 + Y^2 + Z^2 = 1.$

$\therefore$ Polar co-ordinates

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∴ Polar co-ordinates

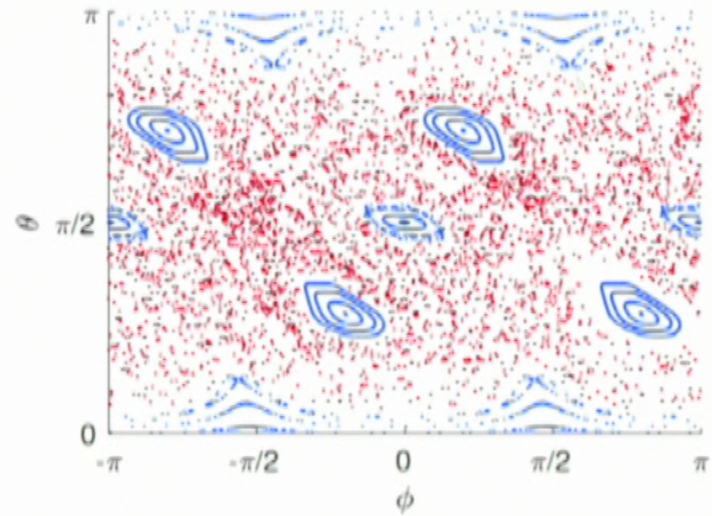


Figure: Classical stroboscopic phase space. $\kappa = 3.0$ and $p = \pi/2$

Kicked top: Periodic orbits

$$H = \frac{\kappa}{2j\tau} J_z^2 + pJ_y \sum_{n=0}^{\infty} \delta(t - n\tau)$$

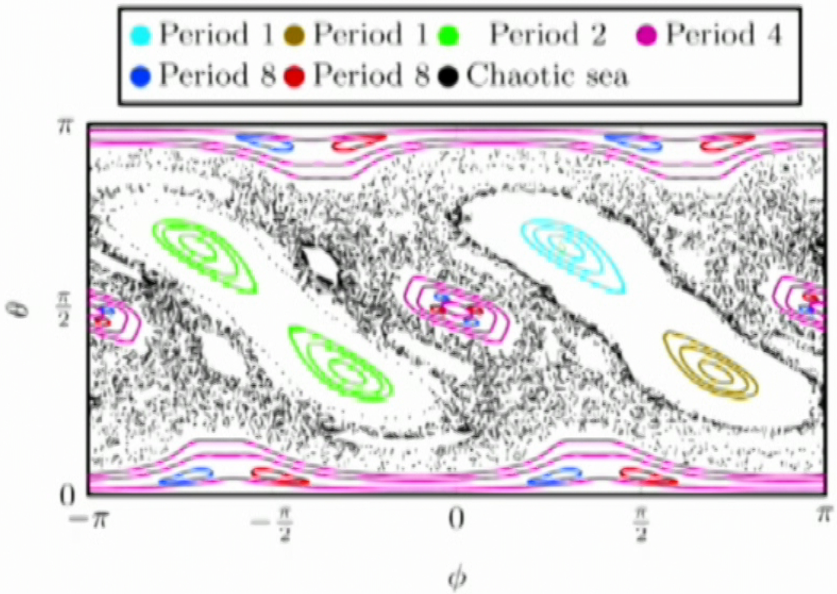


Figure: Classical stroboscopic phase space,[1] $\kappa = 2.5$

[1] **Ruebeck et. al, PRE 95, 062222 (2017).**

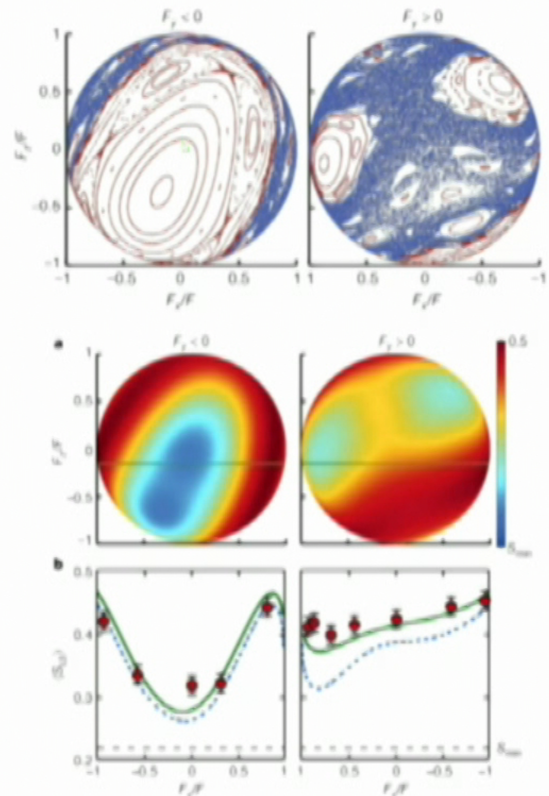
Motivation: Entanglement vs classical dynamics?

- **Chaudhury *et. al*, Nature 461, 768 (2009):**
  "*...purely quantum property of entanglement is a good signature of classical chaos*" in a 6-qubit kicked top.

Classical stroboscopic phase space

Theoretical time-averaged entanglement (as a function of initial conditions)

Experimental time-averaged entanglement (as a function of initial conditions lying on Horizontal green line in middle figure)



Motivation: Entanglement vs classical dynamics?

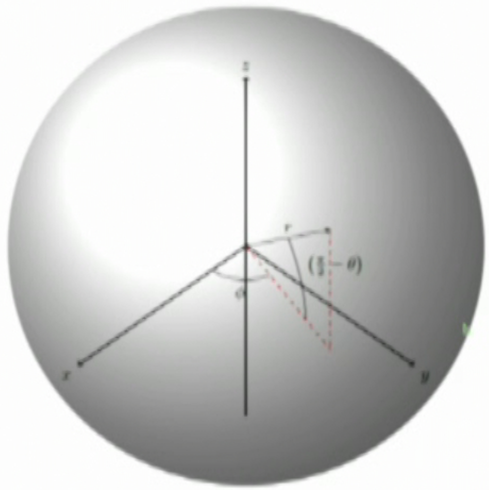
Experimental investigations:

- "Quantum signatures of chaos in a kicked top", **Chaudhury *et. al*, Nature 461, 768 (2009):**
  "*...purely quantum property of entanglement is a good signature of classical chaos*" in a 6-qubit kicked top
- "Ergodic dynamics and thermalization in an isolated quantum system", **Neill *et. al*, Nature Physics 12, 1037 (2016):**
  In a 3-qubit kicked top,
  Classically chaotic dynamics - high entanglement
  Regular dynamics - low entanglement.

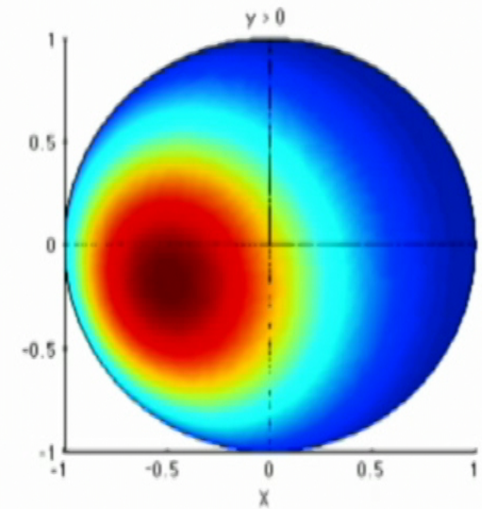
Theoretical and numerical investigations:

- **Ruebeck *et. al*, PRE 95, 062222 (2017)**:
  In a 2-qubit kicked top,
  classically chaotic dynamics - medium level entanglement
  regular dynamics - either high or low entanglement.

SCS: Quantum analog of classical states



Classical state



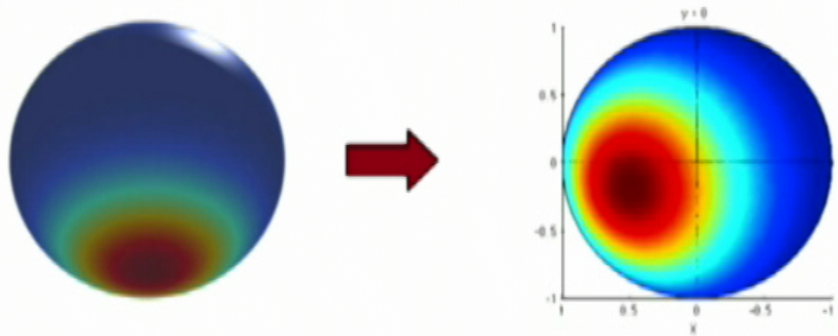
Quantum state

SCS: Quantum analog of classical states

Spin coherent state (SCS)

$$|\theta, \phi\rangle = R(\theta, \phi)|j, j\rangle$$

where $R(\theta, \phi) = \exp\left(i\theta(J_x \sin\phi - J_y \cos\phi)\right)$



Quantum dynamics using Husimi function

Husimi phase space distribution

$$Q(\theta, \phi) = \frac{1}{4\pi}(2J+1)\langle\theta, \phi|\rho|\theta, \phi\rangle$$

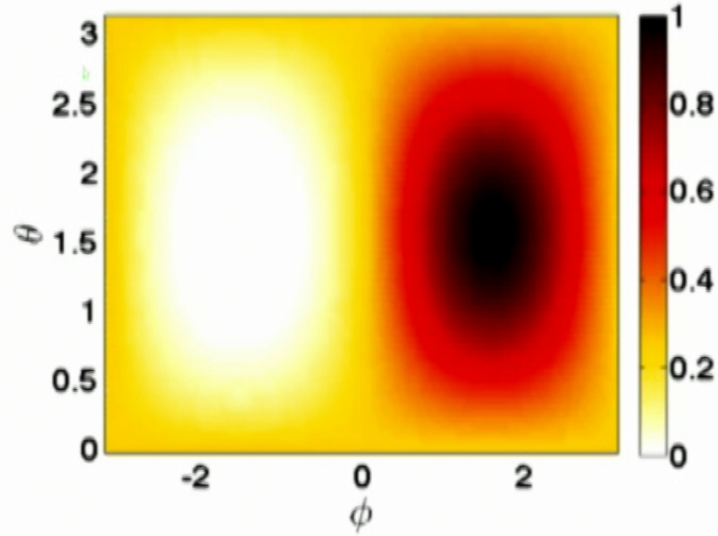
Figure: Evolution of SCS with Hamiltonian

Quantum dynamics using Husimi function

Husimi phase space distribution

$$Q(\theta,\phi) = \frac{1}{4\pi}(2J+1)\langle\theta,\phi|\rho|\theta,\phi\rangle$$

Evolution on sphere = Evolution in (θ,ϕ)



QKT: Quantum dynamics for a stable period-2 orbit

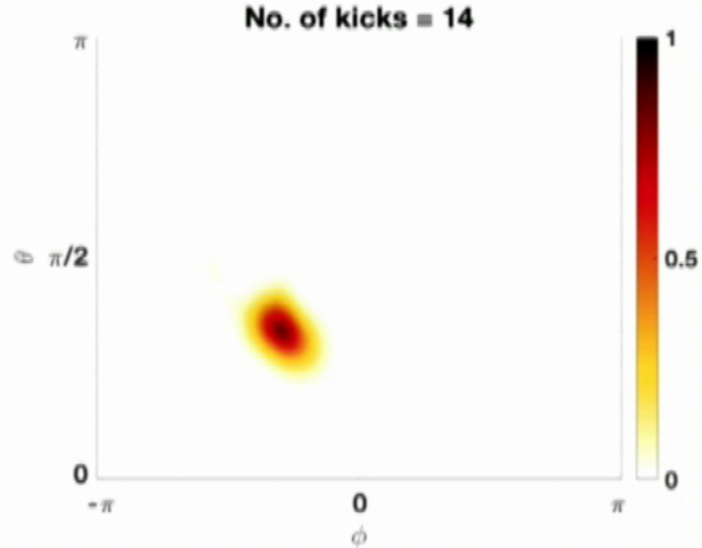


Figure: $\kappa = 2.5$, $j = 40$

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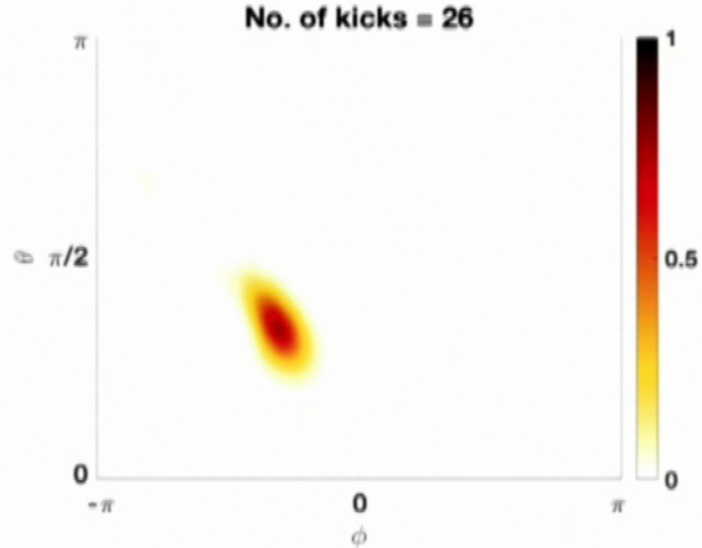

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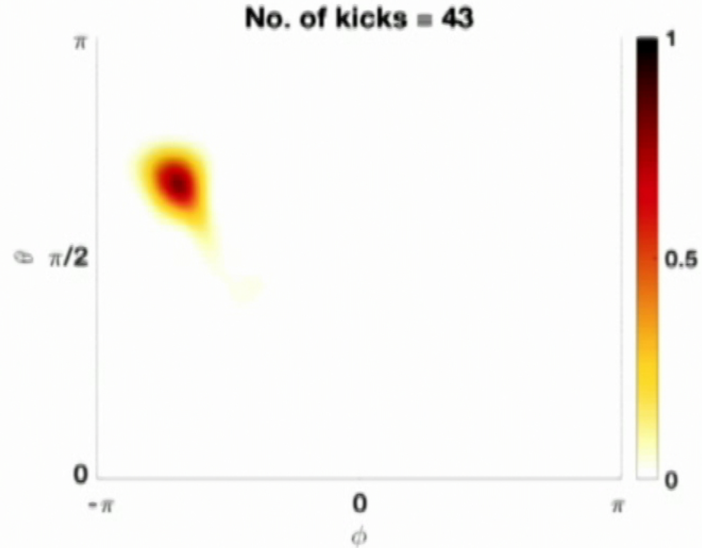

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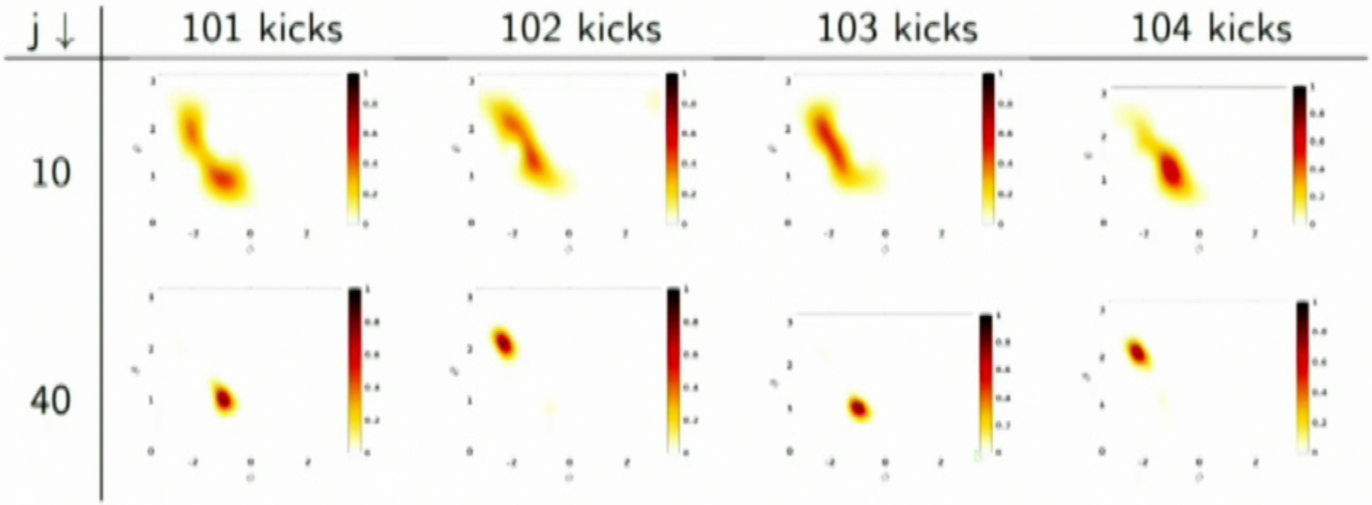

QKT: Quantum dynamics in the chaotic regime

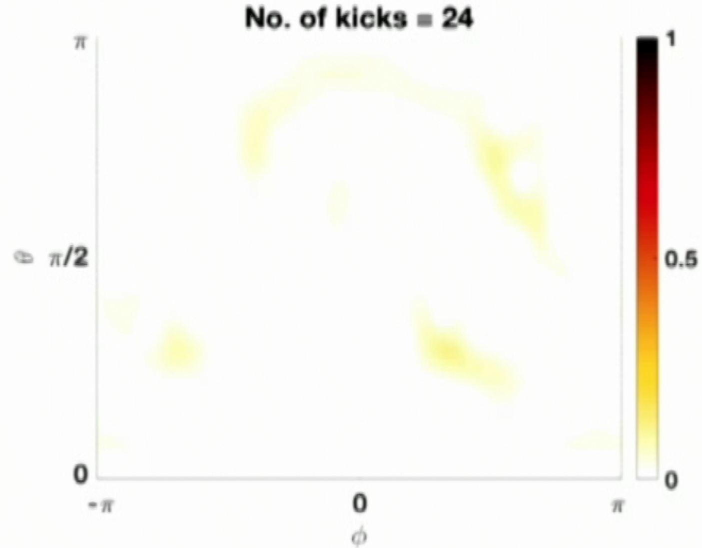

Figure: $\kappa = 3$, $j = 40$

QKT: Quantum dynamics in the chaotic regime

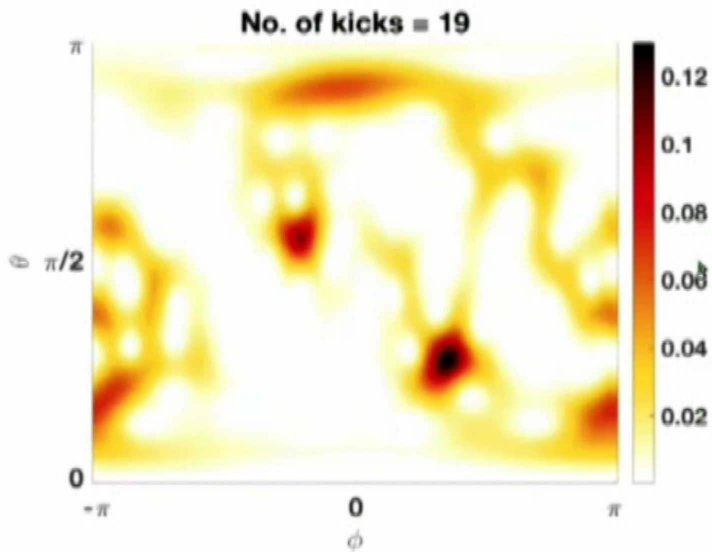

Figure: $\kappa = 3$, $j = 40$ (different range for colour axis)

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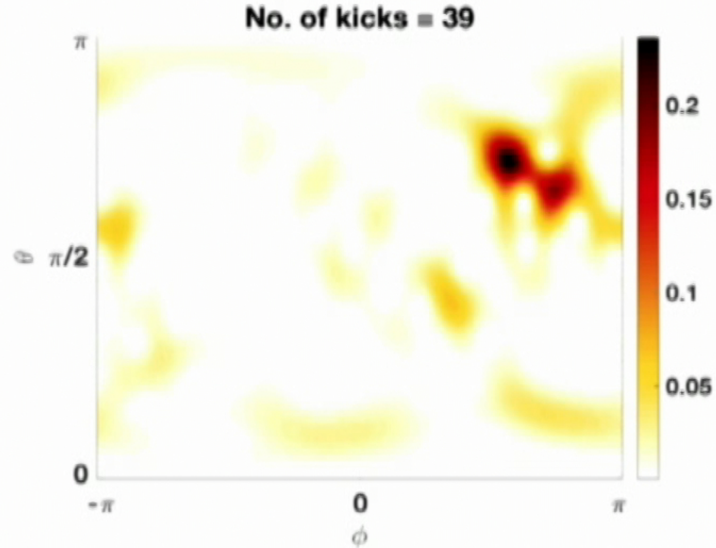

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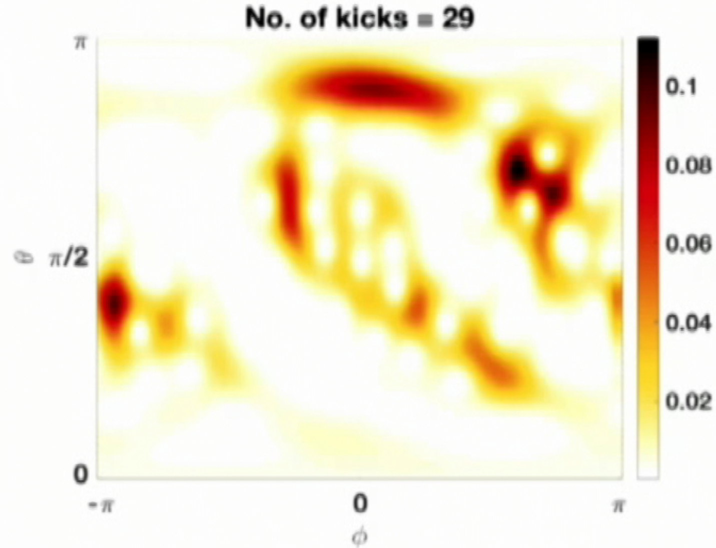

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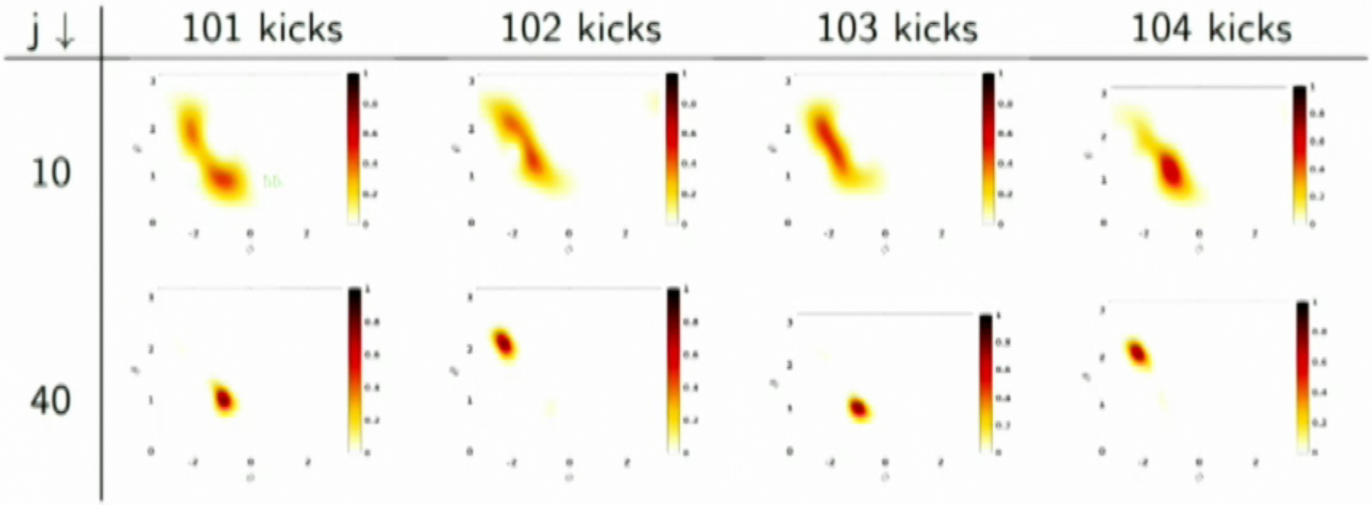


QKT: Husimi distribution of eigenfunctions

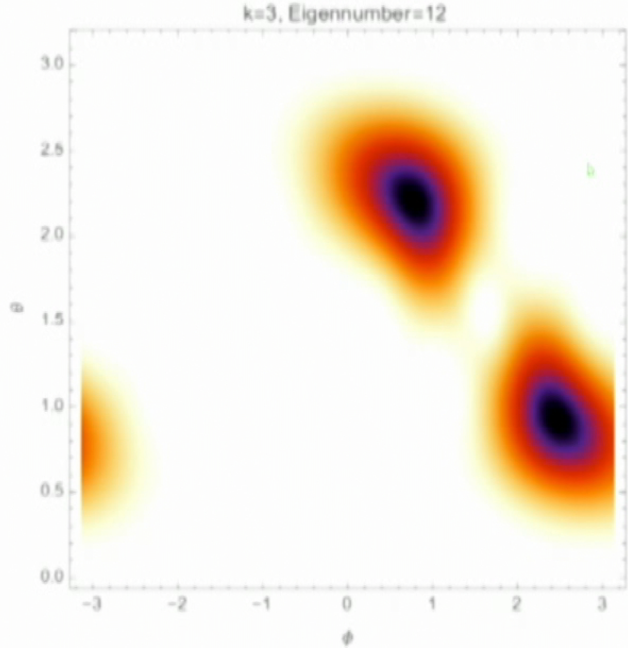
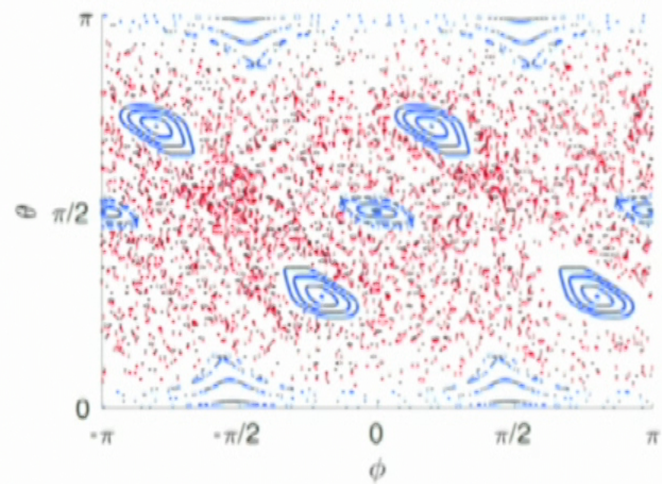


Figure: Husimi distribution for eigenfunctions. $\kappa = 3.0$ and $p = \pi/2$



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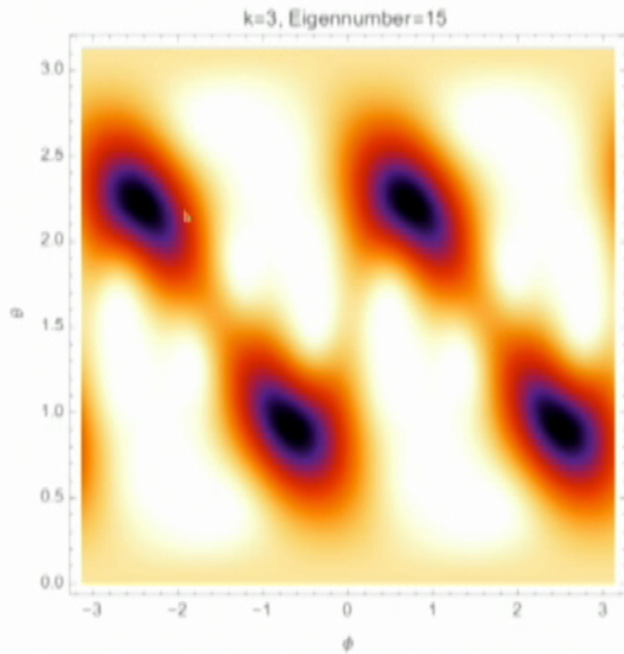

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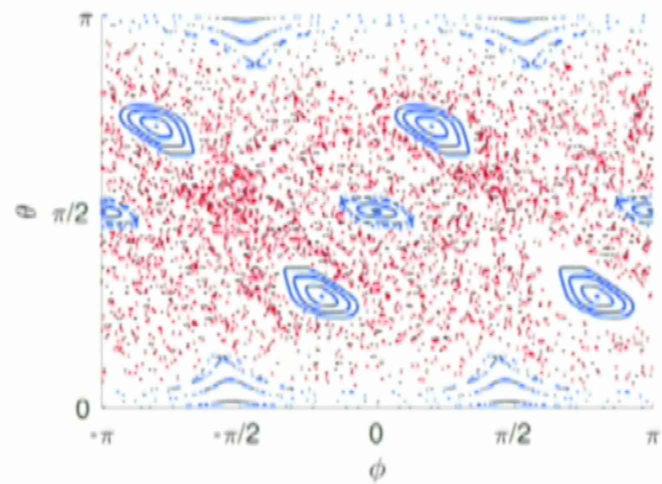

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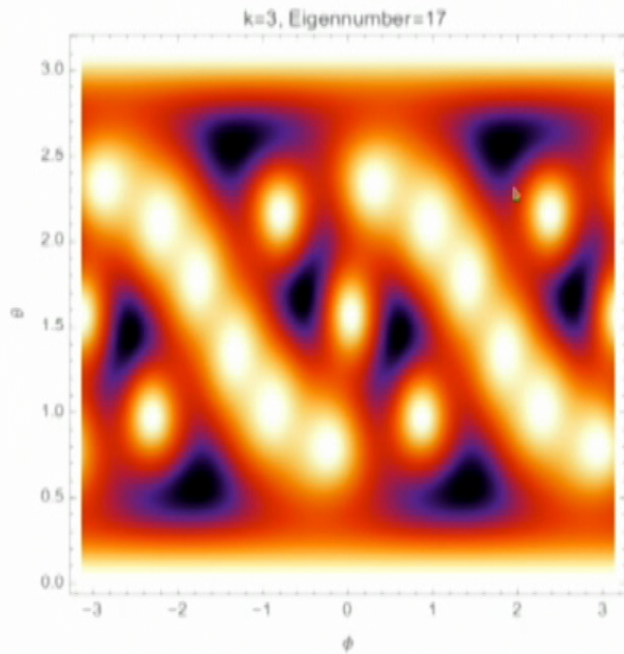

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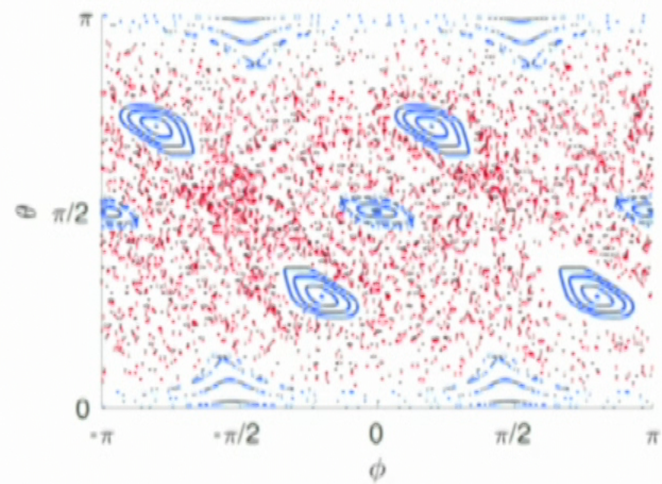

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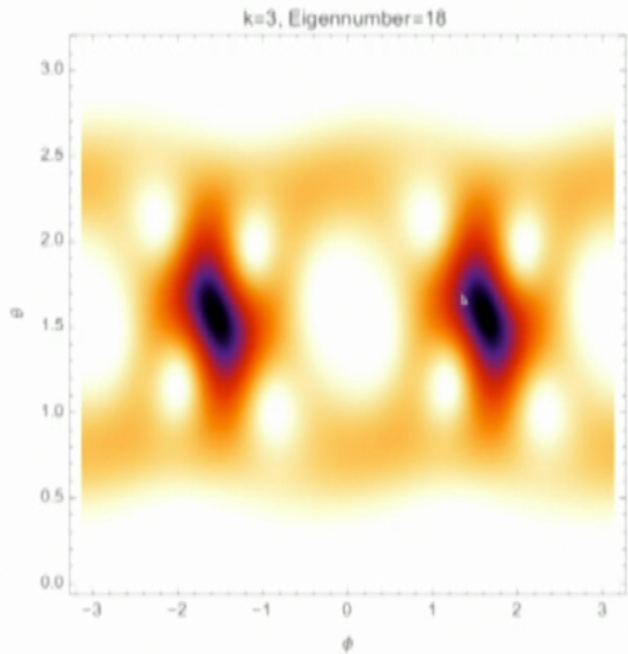

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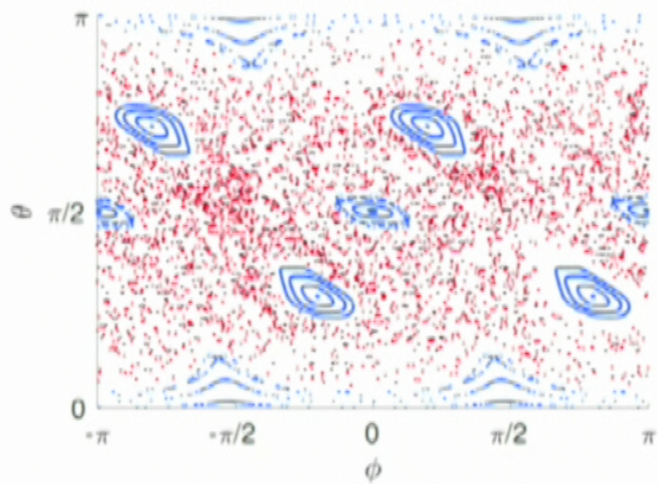

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