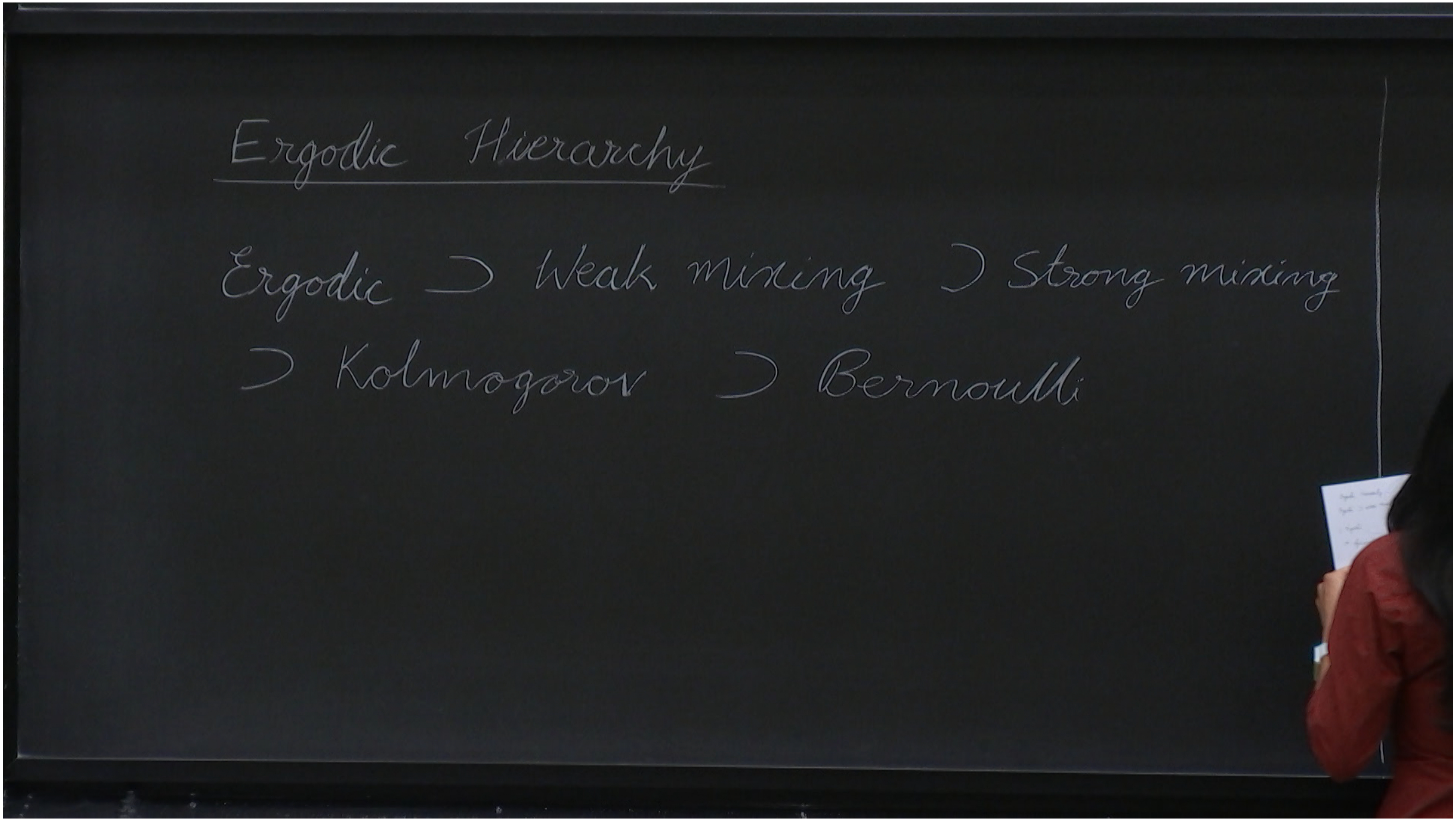
Title: Classical and Quantum Chaos 2021/2022 - Lecture 10

Speakers: Meenu Kumari

Collection: Classical and Quantum Chaos 2021/2022

Date: March 22, 2022 - 11:30 AM

URL: https://pirsa.org/22030054


Ergodic Hierarchy
Ergodic $\supset$ Weak mixing $\supset$ Strong mixing
$\supset$ Kolmogorov $\supset$ Bernoulli

Ergodic Hierarchy

Ergodic $\supset$ Weak mixing $\supset$ Strong mixing $\supset$ Kolmogorov $\supset$ Bernoulli

Ergodicity: $f^* = \bar{f}$ almost everywhere in phase space

f^*: infinite-time average

$\bar{f}$: space average

St

K-mixing

here in phase space

Strong mixing — necessary for chaos
sufficient ?

K-systems (K-mixing) — sufficient for chaos
Necessary ?

Strong mixing :-

$$\lim_{n\to\infty} \mu\left(T_n B \cap A\right) = \mu(B)\mu(A)$$

A & B are subsets of A

K-syste

Quantum Chaos :-

1. Linearity of S. equ^n

$$i\hbar \frac{\partial |\Psi(t)\rangle}{\partial t} = \hat{H}|\Psi(t)\rangle$$

$$|\Psi(0)\rangle, \ |\Psi'(0)\rangle$$

$$|\langle \Psi'(t)|\Psi(t)\rangle| = |\langle \Psi'(0)|U^{+}U|\Psi(0)\rangle$$

$$= |\langle \Psi'(0)|\Psi(0)\rangle|$$

2.

2 Heisenberg's uncertainty principle

$$\Delta x \, \Delta p \geq \frac{\hbar}{2}$$

Quantum - Classical Correspondence

Ehrenfest

Single particle in 1-D

$$H = \frac{\hat{p}^2}{2m} + V(\hat{q})$$

$$\frac{d\hat{q}}{dt} = i[H, \hat{q}] = \frac{\hat{p}}{m}$$

$$\frac{d\hat{p}}{dt} = i[H, \hat{p}] = \frac{i}{\hbar}[V(q), \hat{p}]$$

f - test fuction :-

$$[V(\hat{q}), \hat{p}]f = -i\hbar\left(V(\hat{q})\frac{\partial f}{\partial \hat{q}} - \underbrace{\frac{\partial}{\partial \hat{q}}(V(\hat{q})f)}\right)$$

$$= -i\hbar\left(V(\hat{q})\frac{\partial f}{\partial \hat{q}} - V(\hat{q})\frac{\partial f}{\partial \hat{q}} - f\frac{\partial V(\hat{q})}{\partial \hat{q}}\right)$$

$$\Rightarrow [V(\hat{q}), \hat{p}] = i\hbar \frac{\partial V(\hat{q})}{\partial \hat{q}}$$

$$\Rightarrow \frac{d\hat{p}}{dt} = -\frac{\partial V(\hat{q})}{\partial \hat{q}} = F(\hat{q}) = \text{Force operator}$$

$|\Psi\rangle$

$$\frac{d\langle\hat{q}\rangle}{dt} = \frac{\langle\hat{p}\rangle}{m}$$

$$\frac{d\langle\hat{p}\rangle}{dt} = \langle F(\hat{q})\rangle$$

<u>If</u> we can approximate

$$\langle F(\hat{q})\rangle \simeq F(\langle\hat{q}\rangle)$$

$$\frac{d\hat{p}}{dt} = F(\langle\hat{q}\rangle)$$

Let $\delta\hat{q} = \hat{q} - \langle \hat{q} \rangle$

$\delta\hat{p} = \hat{p} - \langle \hat{p} \rangle$

Apply Taylor expansion to $\langle F(q) \rangle$ about $\langle \hat{q} \rangle$

$$\frac{d\hat{p}}{dt} = \langle F(\hat{q}) \rangle = F(\langle \hat{q} \rangle) + \frac{1}{2} \langle (\delta\hat{q})^2 \rangle \frac{d^2F}{dq^2}\Big|_{q=\langle q \rangle} +$$

$\langle (\delta\hat{q})^2 \rangle$ — measure of the position prob distribution

Ehrenfest regime :- The dynamical regime where $\langle F(\hat{q}) \rangle \simeq F(\langle \hat{q} \rangle)$

$$\epsilon(t) = \sqrt{(\langle \hat{q}(t) \rangle - q_c(t))^2 + (\langle \hat{p}(t) \rangle - p_c(t))^2}$$

$(q_c(0), p_c(0))$

Let $\tilde{\epsilon}$ be the threshold for correspondent

$$\epsilon(t) \leq \tilde{\epsilon} \quad \text{for } t < t_{Eh}$$

Let $\delta\hat{q} = \hat{q} - \langle\hat{q}\rangle$

$\delta\hat{p} = \hat{p} - \langle\hat{p}\rangle$

Apply Taylor expansion to $\langle F(q)\rangle$ about $\langle\hat{q}\rangle$

$$\frac{d\langle\hat{p}\rangle}{dt} = \langle F(\hat{q})\rangle = F(\langle\hat{q}(t)\rangle) + \frac{1}{2}\langle(\delta\hat{q}(t))^2\rangle \frac{d^2F}{dq^2}\bigg|_{q=\langle q\rangle} + $$

$\langle(\delta\hat{q})^2\rangle$ — measure of the position prob distribution

1 , A_0 - characteristic system $\qquad \frac{\hbar}{A_0} \to 0$

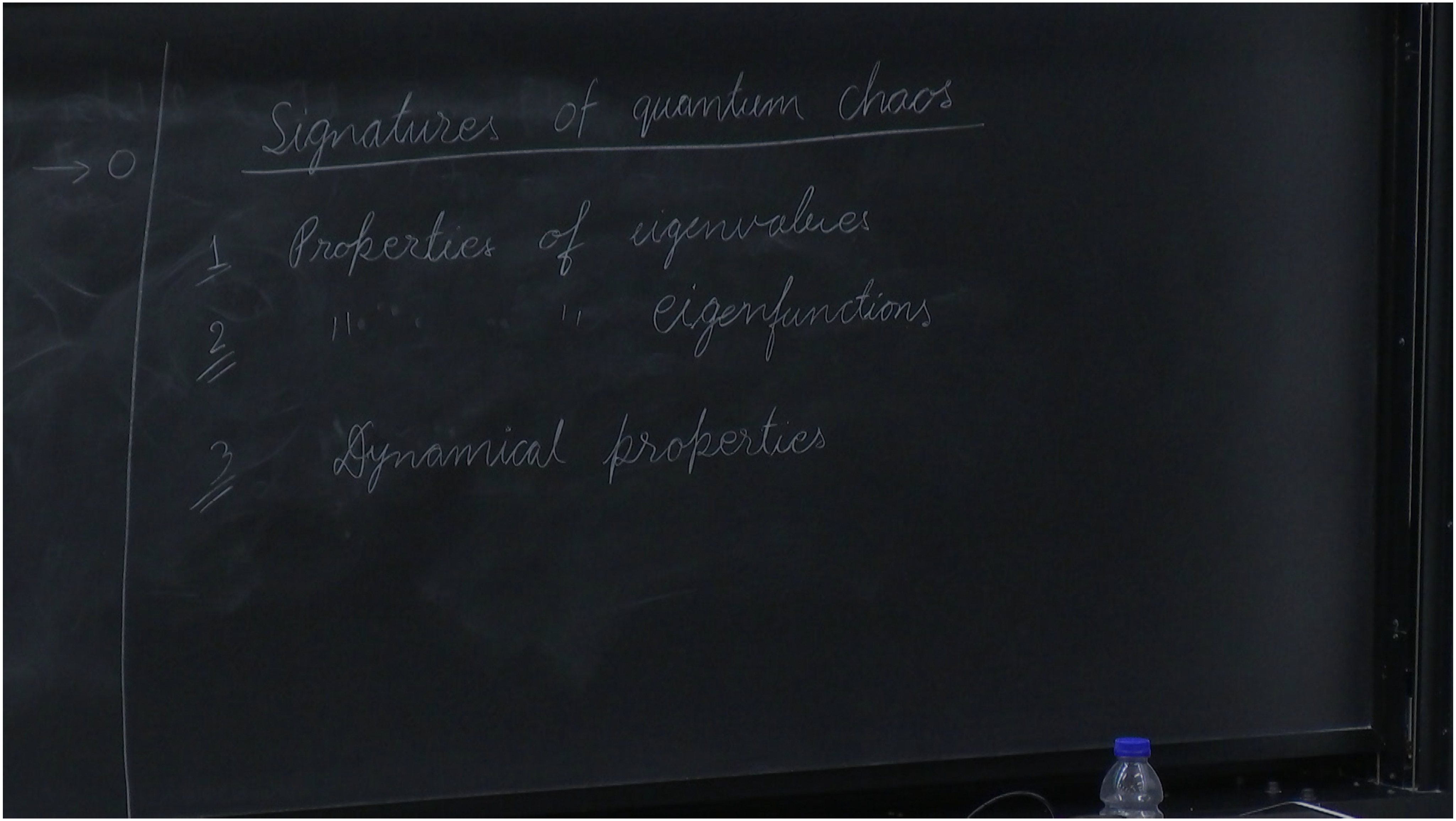
Hyperion - moon of saturn

$t_{Eh} \sim 20$ years

— Quantum Kicked top , j

$A_0 \sim j\hbar$

$t_{Eh} \sim \frac{1}{\lambda} \log j$


→ 0
Signatures of quantum chaos
1 Properties of eigenvalues
2 " " eigenfunctions
3 Dynamical properties