Title: Classical and Quantum Chaos 2021/2022 - Lecture 1

Speakers: Meenu Kumari

Collection: Classical and Quantum Chaos 2021/2022

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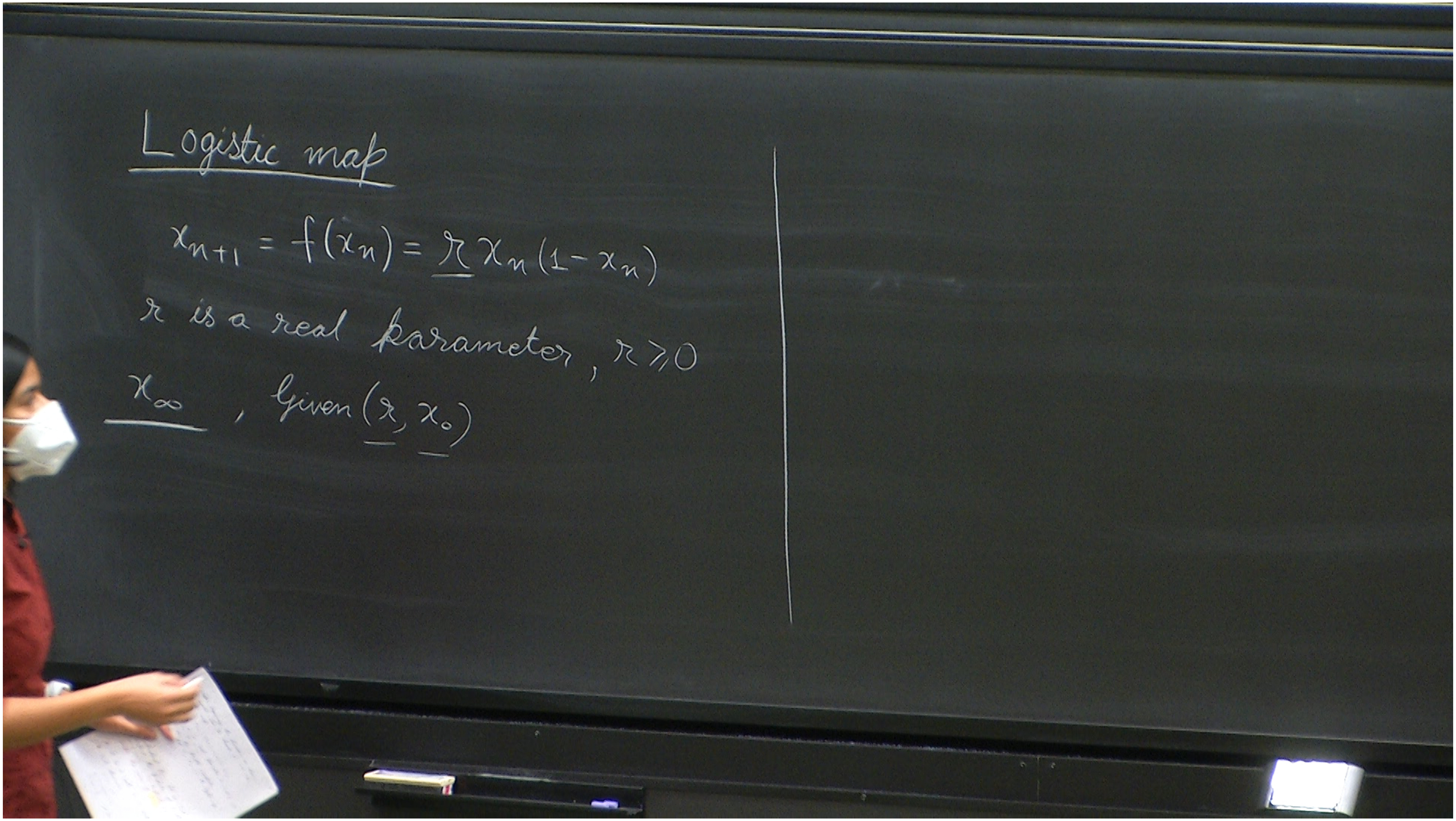
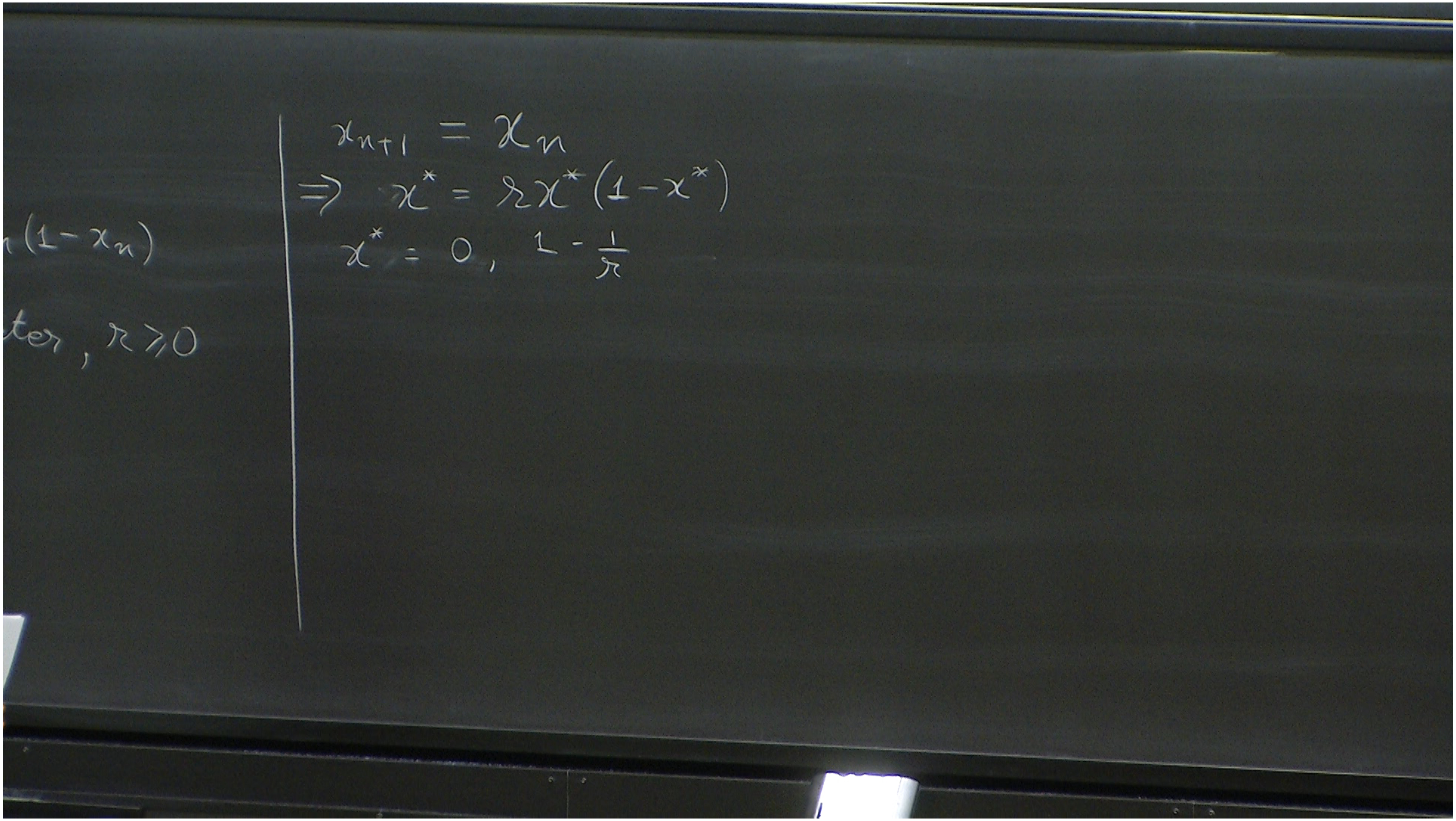
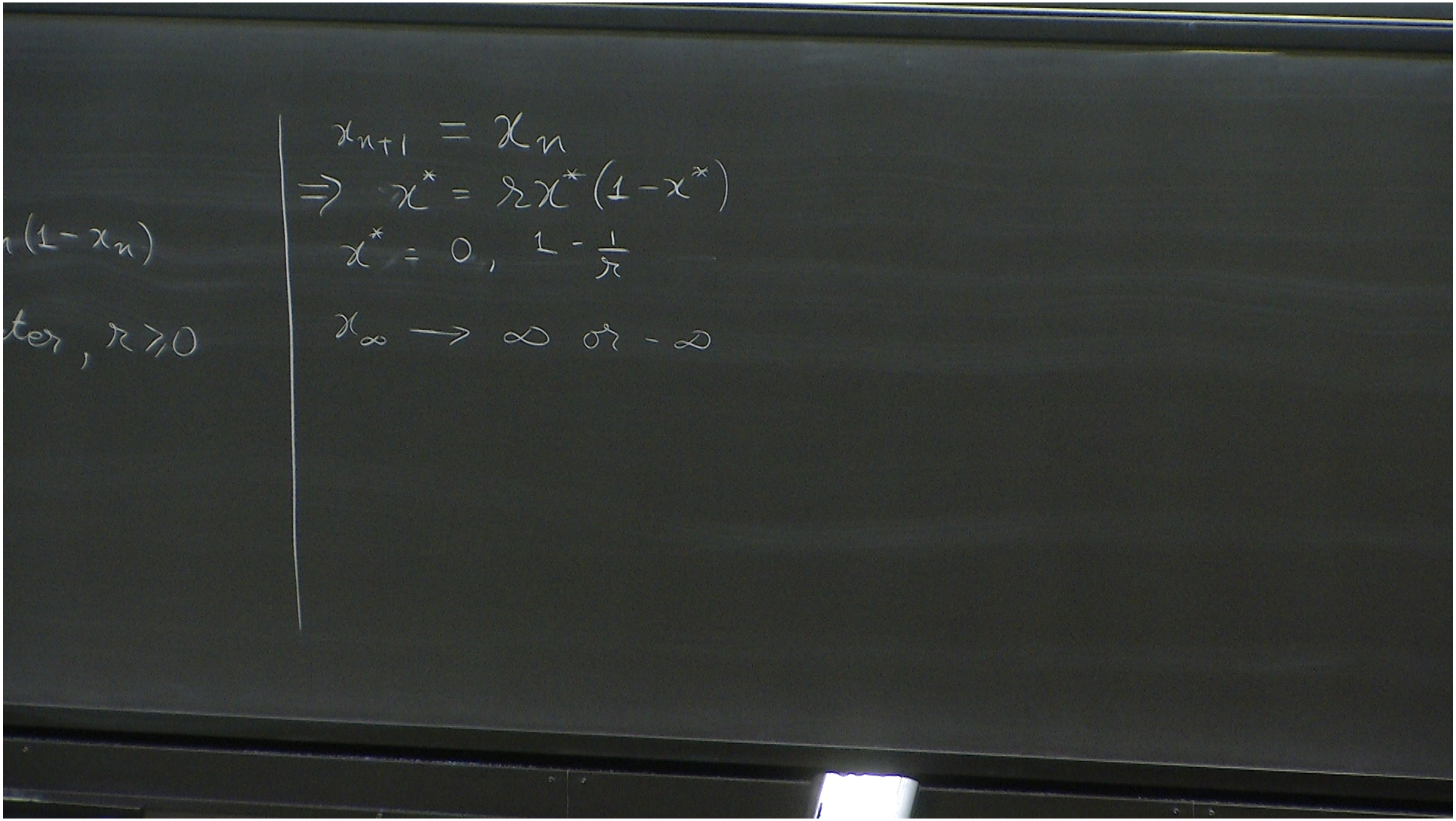
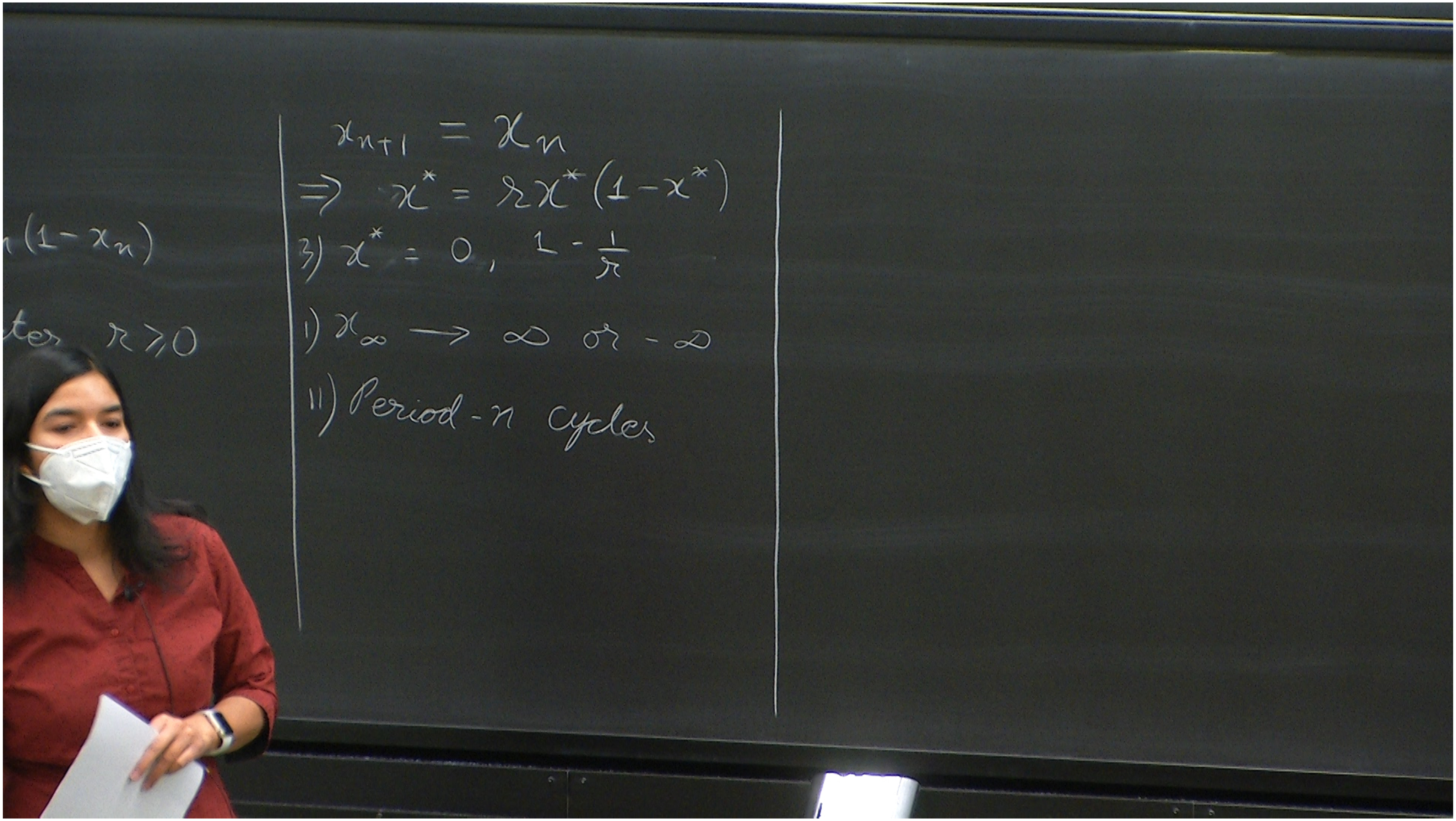
URL: https://pirsa.org/22030046


Logistic map
$x_{n+1} = f(x_n) = r x_n (1 - x_n)$
r is a real parameter, $r \geq 0$
x_∞, given (r, x_0)


$x_{n+1} = x_n$
$\Rightarrow x^* = r x^* (1 - x^*)$
$x^* = 0, \ 1 - \frac{1}{r}$
$(1 - x_n)$
ter, $r \geq 0$


$x_{n+1} = x_n$
$\Rightarrow x^* = r x^* (1 - x^*)$
$x^* = 0, \ 1 - \frac{1}{r}$
$x_\infty \longrightarrow \infty$ or $-\infty$
$n(1 - x_n)$
ter, $r \geq 0$



n (1 - x_n)
ter r ≥ 0
x_{n+1} = x_n
⇒ x* = r x* (1 - x*)
3) x* = 0, 1 - 1/r
1) x_∞ → ∞ or -∞
11) Period-n cycles

$^*(1-x^*)$

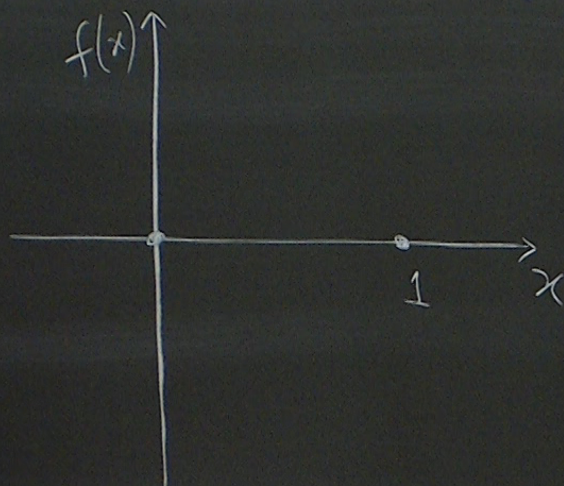
$= \frac{1}{r}$

or $-\infty$

cycles

$\underline{f(x) = rx(1-x)}$, $r \geq 0$

2 roots: $x = 0, 1$



$f'(x) = 0$

$\Rightarrow r - 2xr = 0$

$\Rightarrow x = \frac{1}{2}$ is an extremum

$f(x = \frac{1}{2}) = \frac{r}{2}\left(1 - \frac{1}{2}\right) = \frac{r}{4}$

$f''(x) < 0 \Rightarrow$ maximum

$-2r$

$^*(1-x^*)$

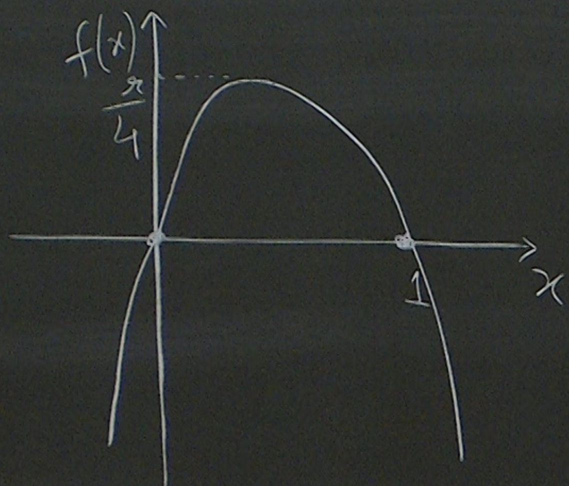
$-\frac{1}{r}$

or $-\infty$

ycles

$\underline{f(x) = r x(1-x)}$, $r \geq 0$

2 roots : $x = 0, 1$



$f'(x) = 0$

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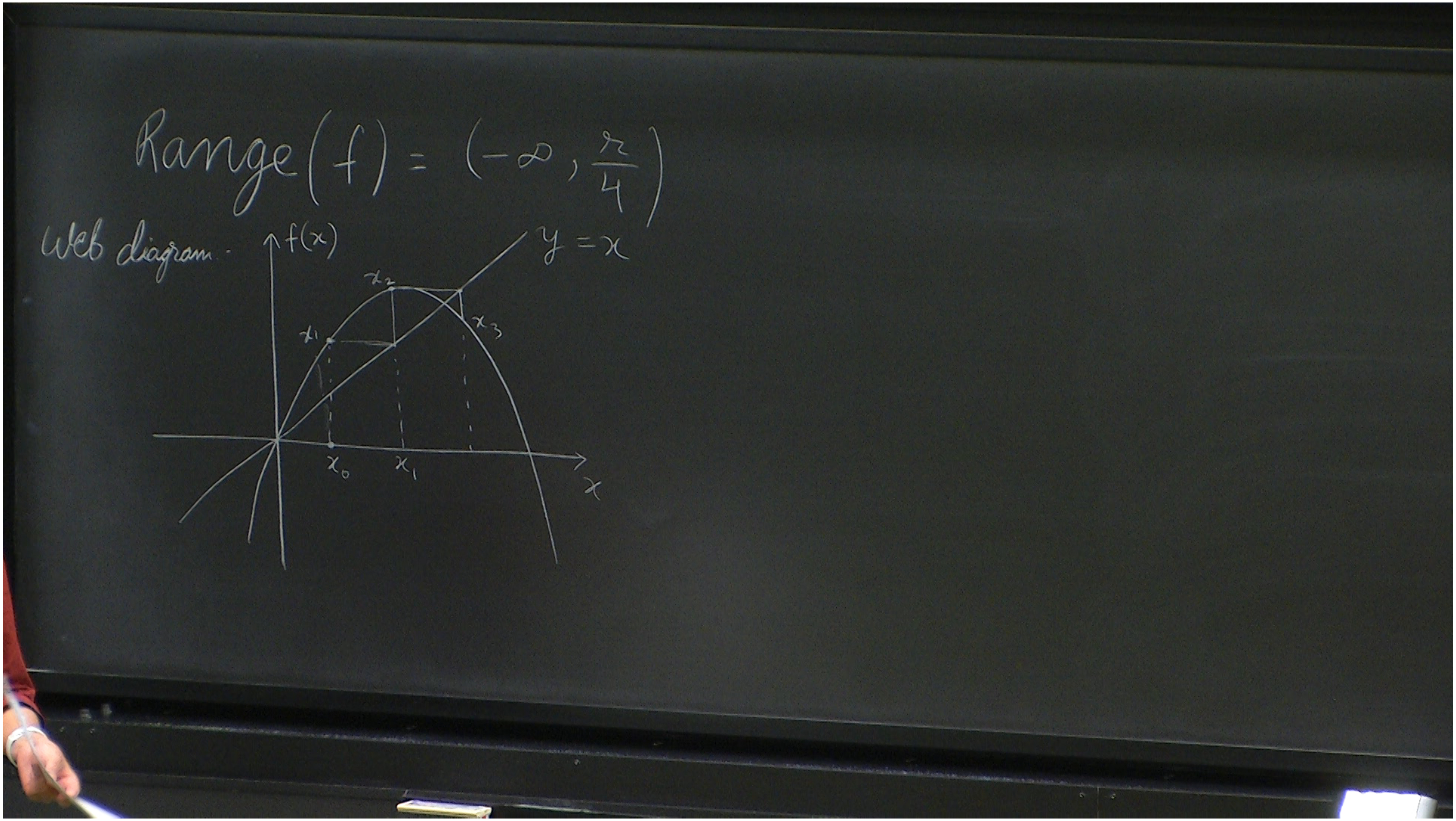
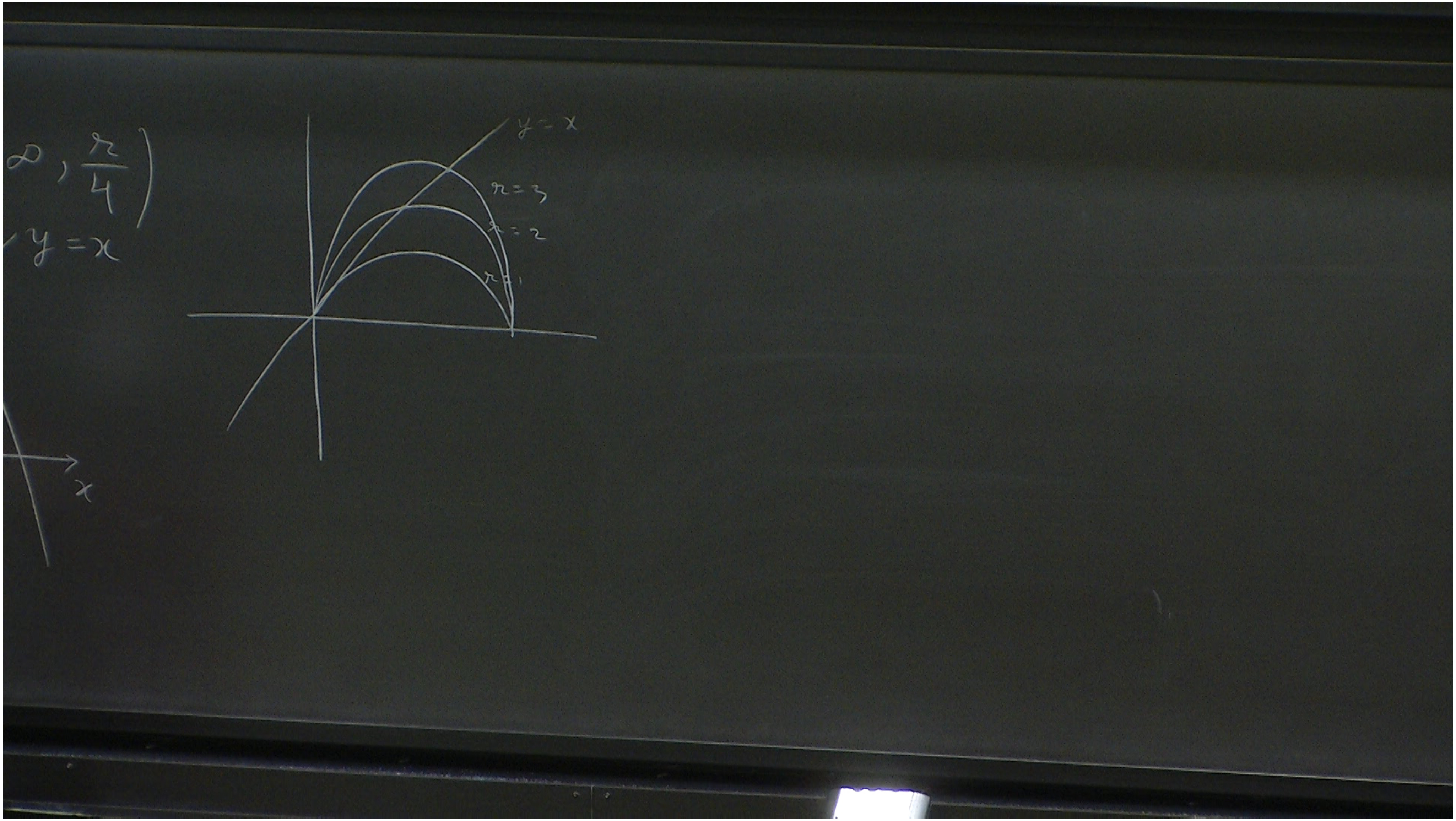
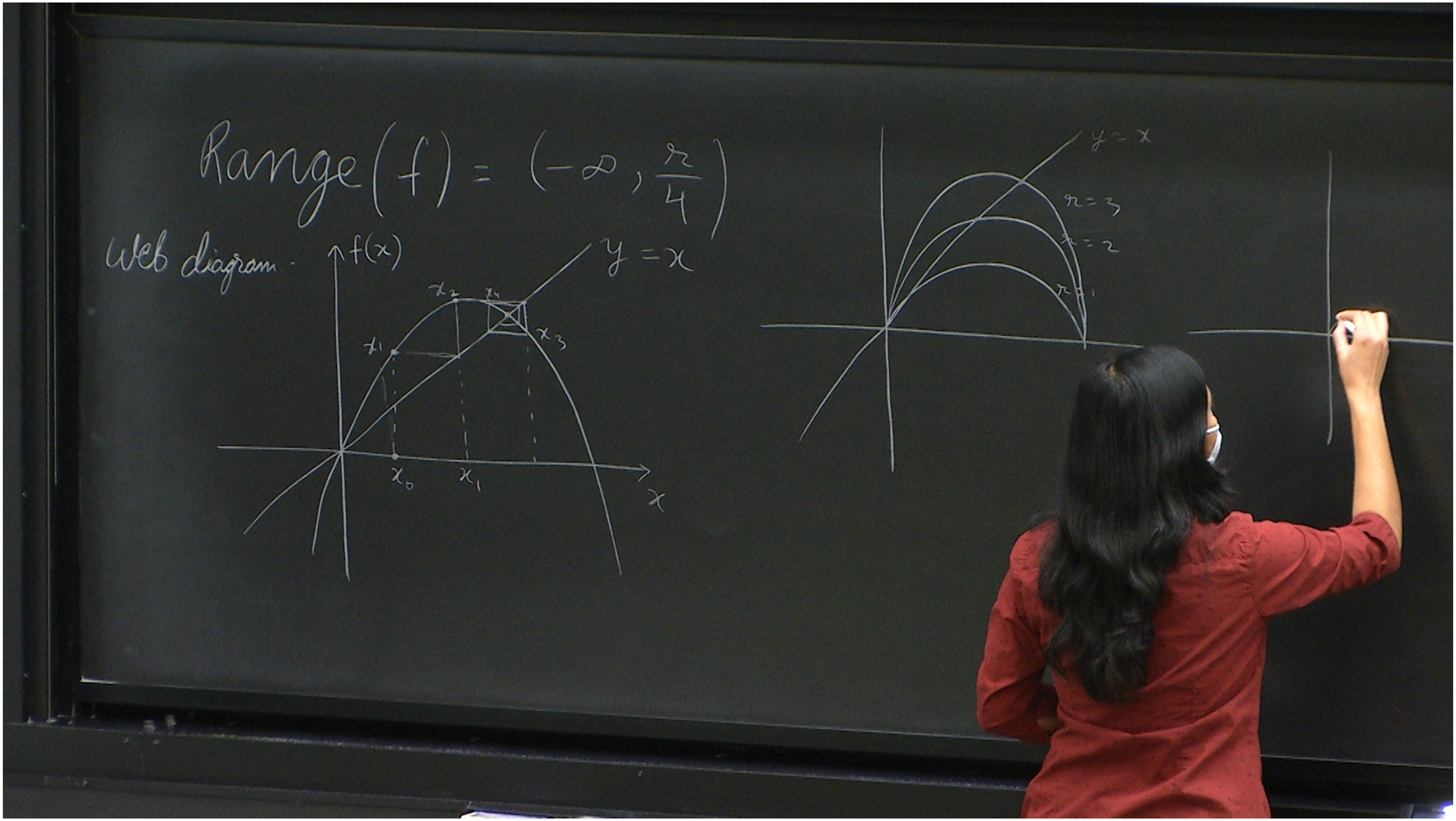
$f\left(x = \frac{1}{2}\right) = \frac{r}{2}\left(1 - \frac{1}{2}\right) = \frac{r}{4}$

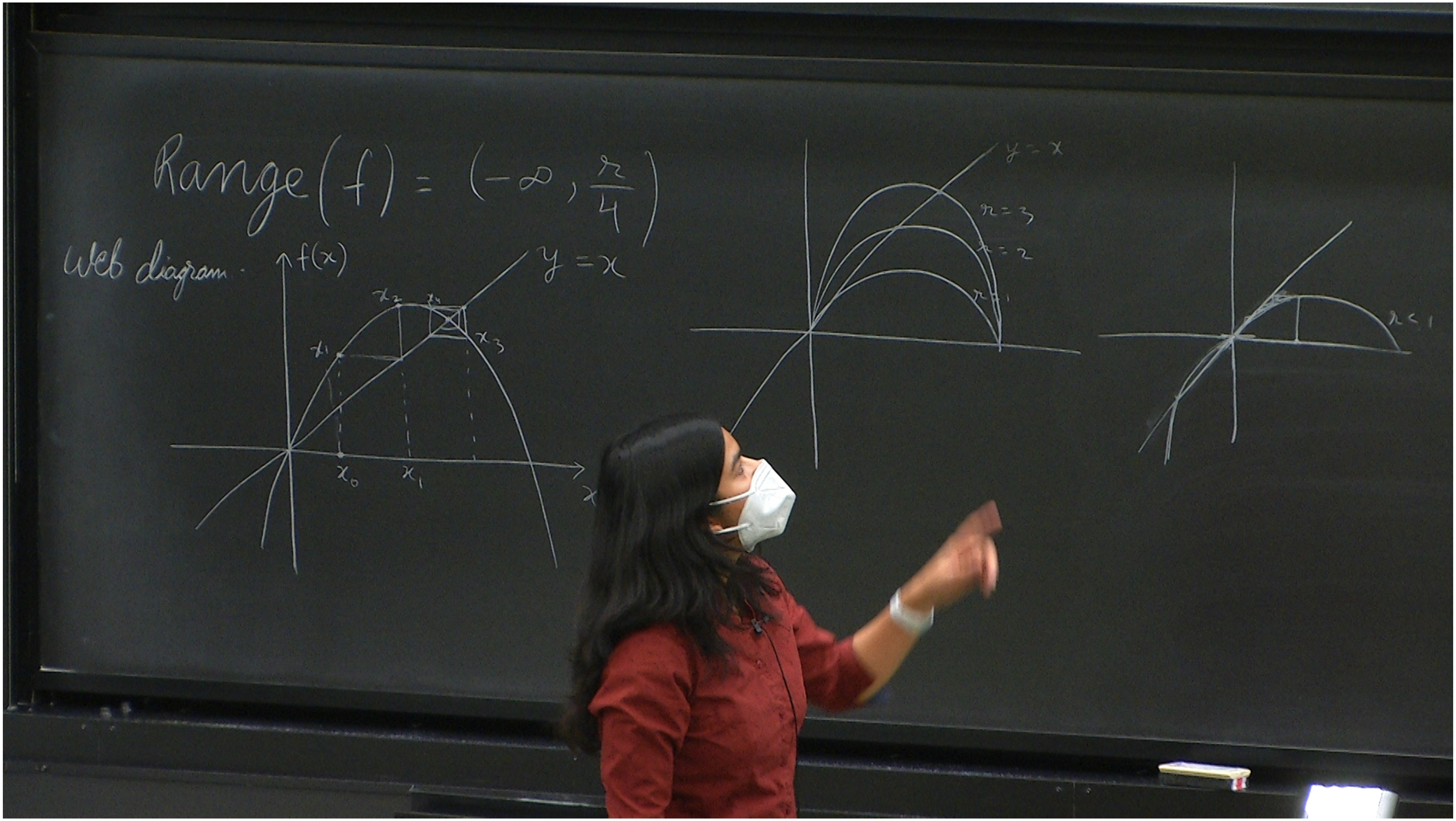
$f''(x) < 0 \Rightarrow$ maximum

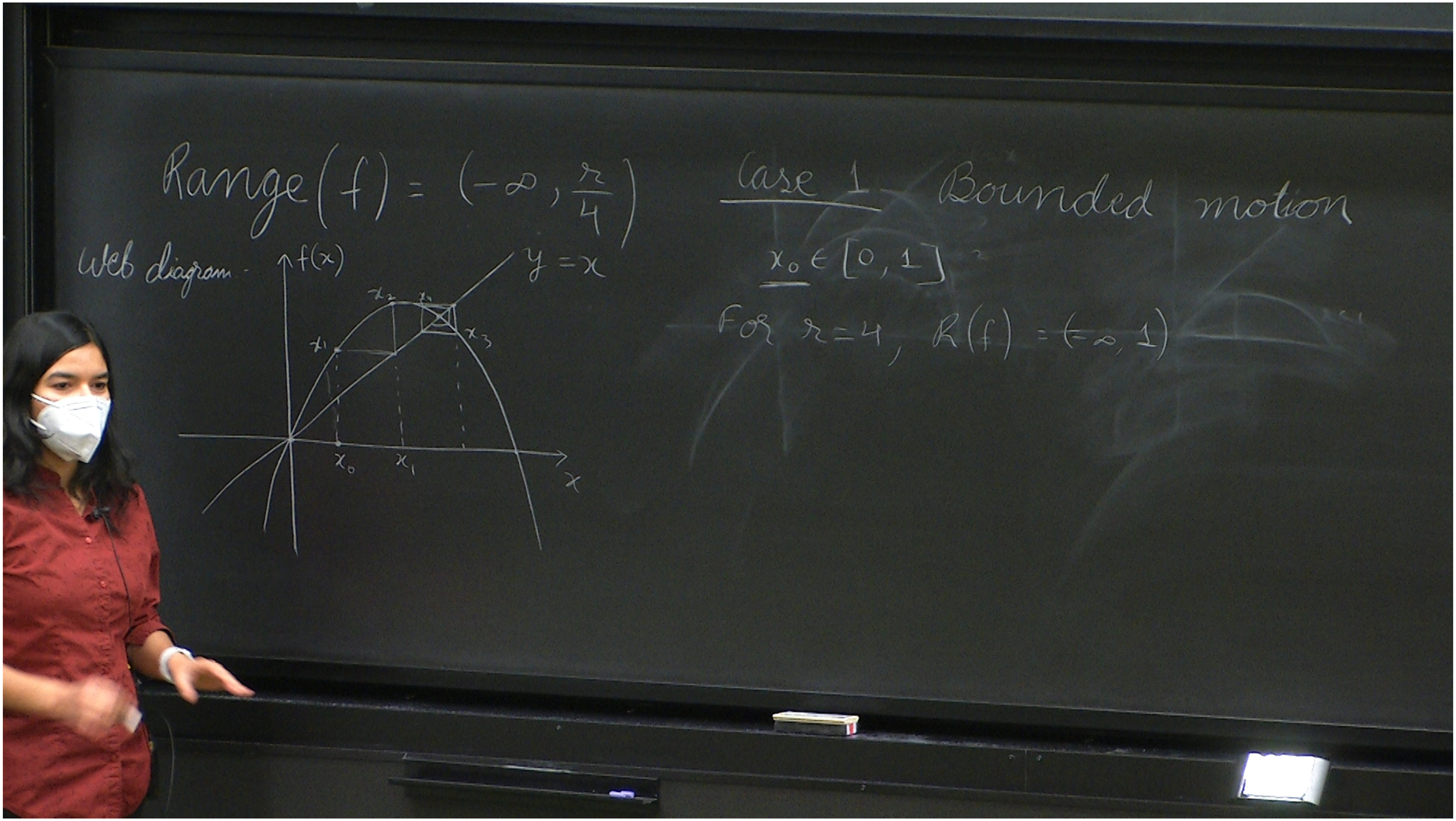
$-2r$

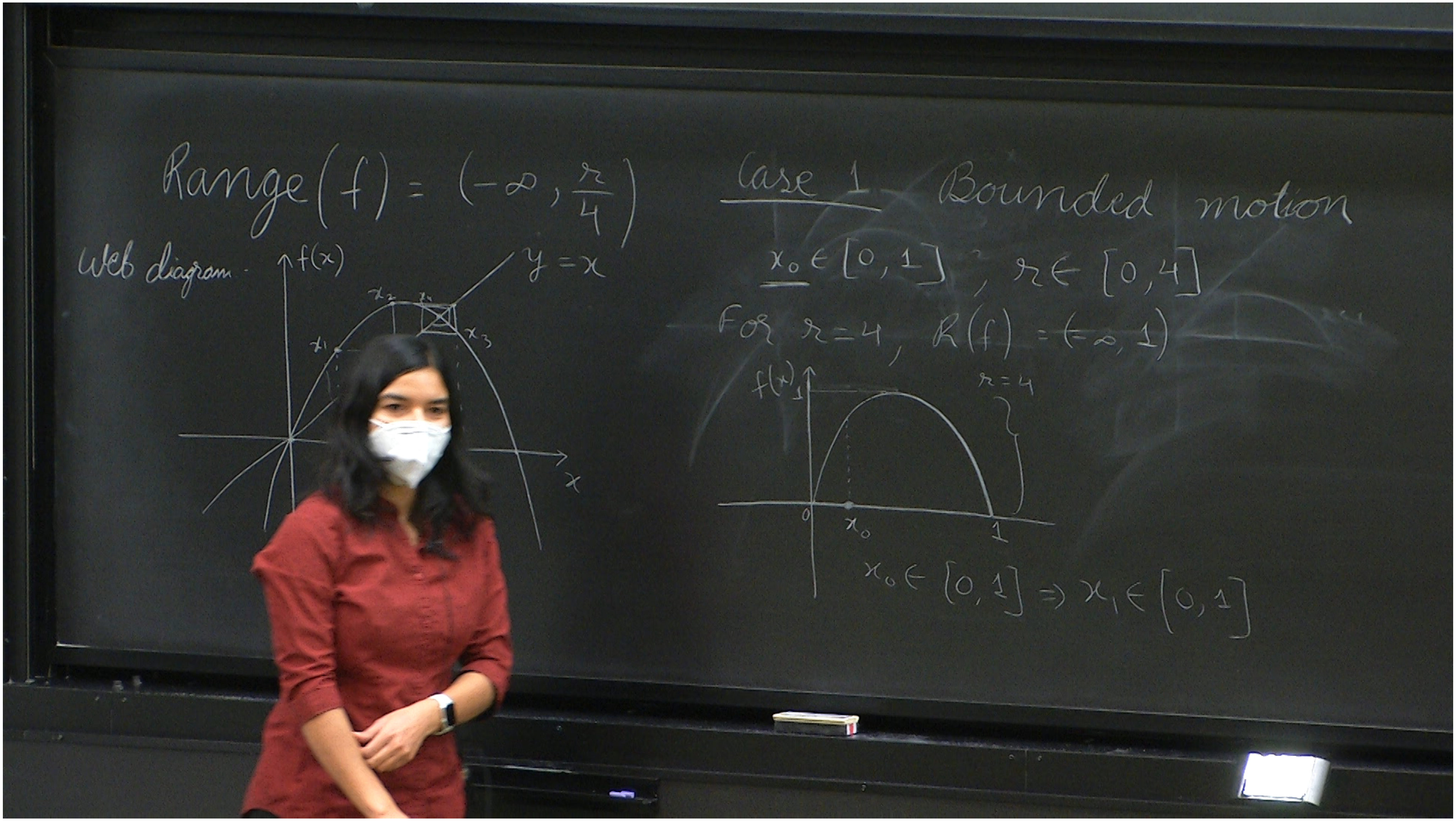

Range(f) = (−∞, r/4)
Web diagram.
f(x)
y = x
x₂
x₁
x₃
x₀
x₁
x


y = x
y = x
r = 3
r = 2
x

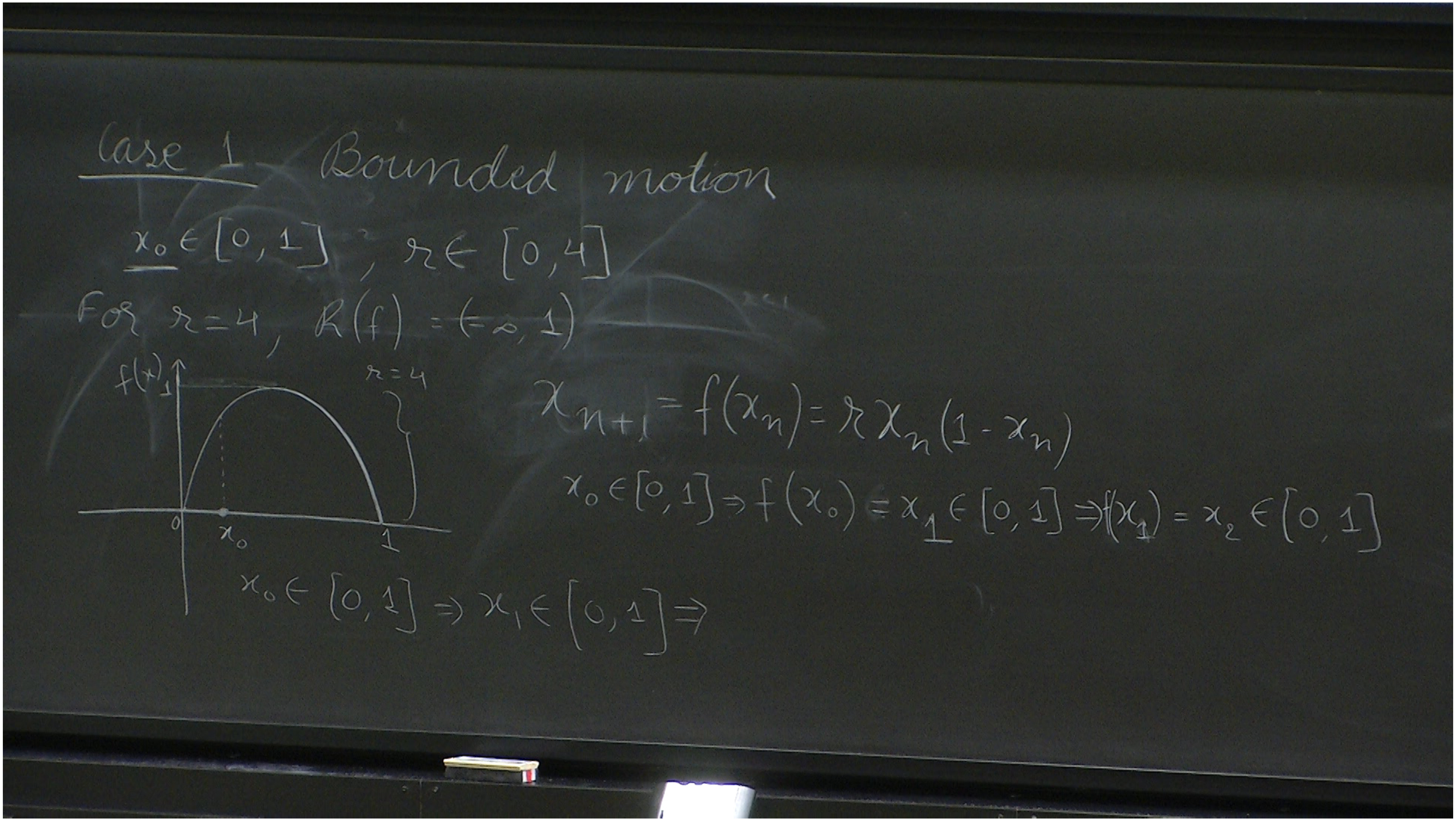

Range(f) = (−∞, r/4)
Web diagram.
f(x)
y = x
x₀
x₁
x₂
x₃
x
r = 3
r = 2
y = x


Range(f) = (−∞, r/4)
Web diagram.
f(x)
y = x
x₀
x₁
x₂
x₃
r = 3
r = 2
r < 1


Range(f) = (−∞, r/4)
Case 1 Bounded motion
Web diagram.
f(x)
y = x
x2
x1
x3
x0
x1
x
x0 ∈ [0, 1]
For r=4, R(f) = (−∞, 1)



Range(f) = (−∞, r/4)
Web diagram.
f(x)
y = x
x
Case 1 Bounded motion
x₀ ∈ [0, 1] ; r ∈ [0, 4]
For r = 4, R(f) = (−∞, 1)
f(x)
r = 4
0
1
x₀ ∈ [0, 1] ⇒ x₁ ∈ [0, 1]

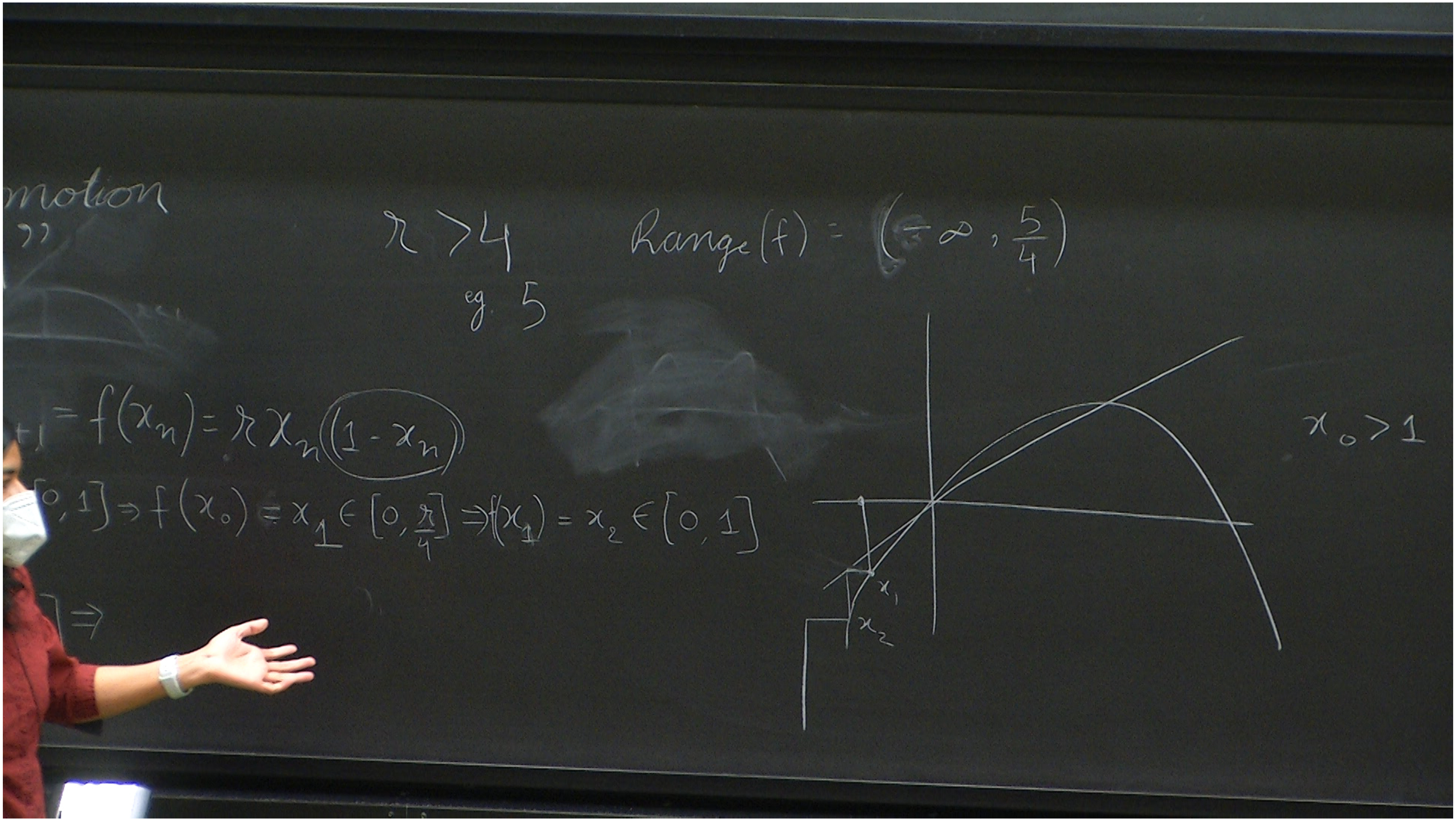

Case 1 Bounded motion
$x_0 \in [0,1]$; $r \in [0,4]$
For $r=4$, $R(f) = (-\infty, 1)$
$f(x)$
$r=4$
$x_{n+1} = f(x_n) = r x_n (1 - x_n)$
$x_0 \in [0,1] \Rightarrow f(x_0) = x_1 \in [0,1] \Rightarrow f(x_1) = x_2 \in [0,1]$
$x_0 \in [0,1] \Rightarrow x_1 \in [0,1] \Rightarrow$

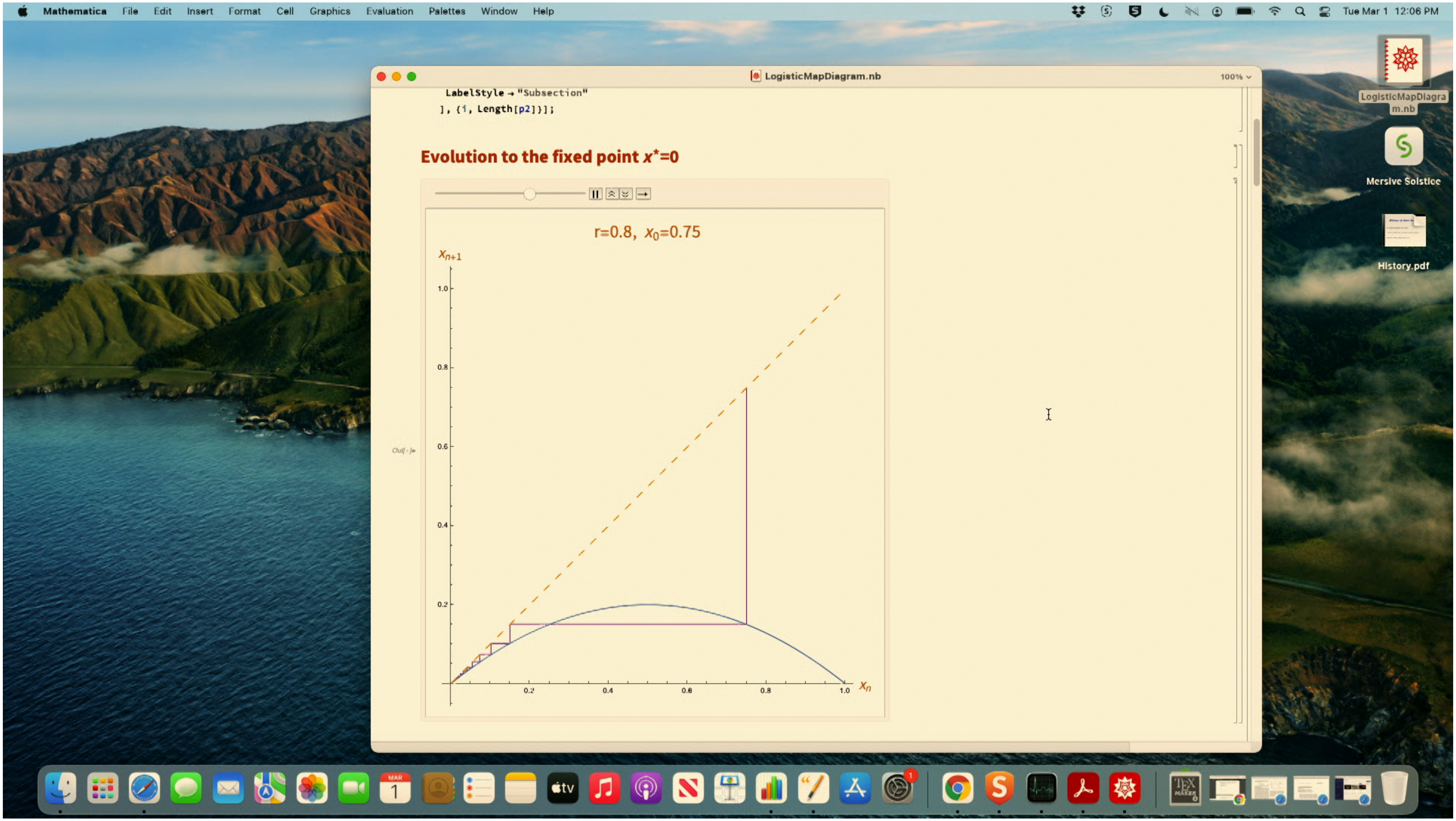
$r > 4$ Range$(f) = \left(-\infty, \frac{5}{4}\right)$

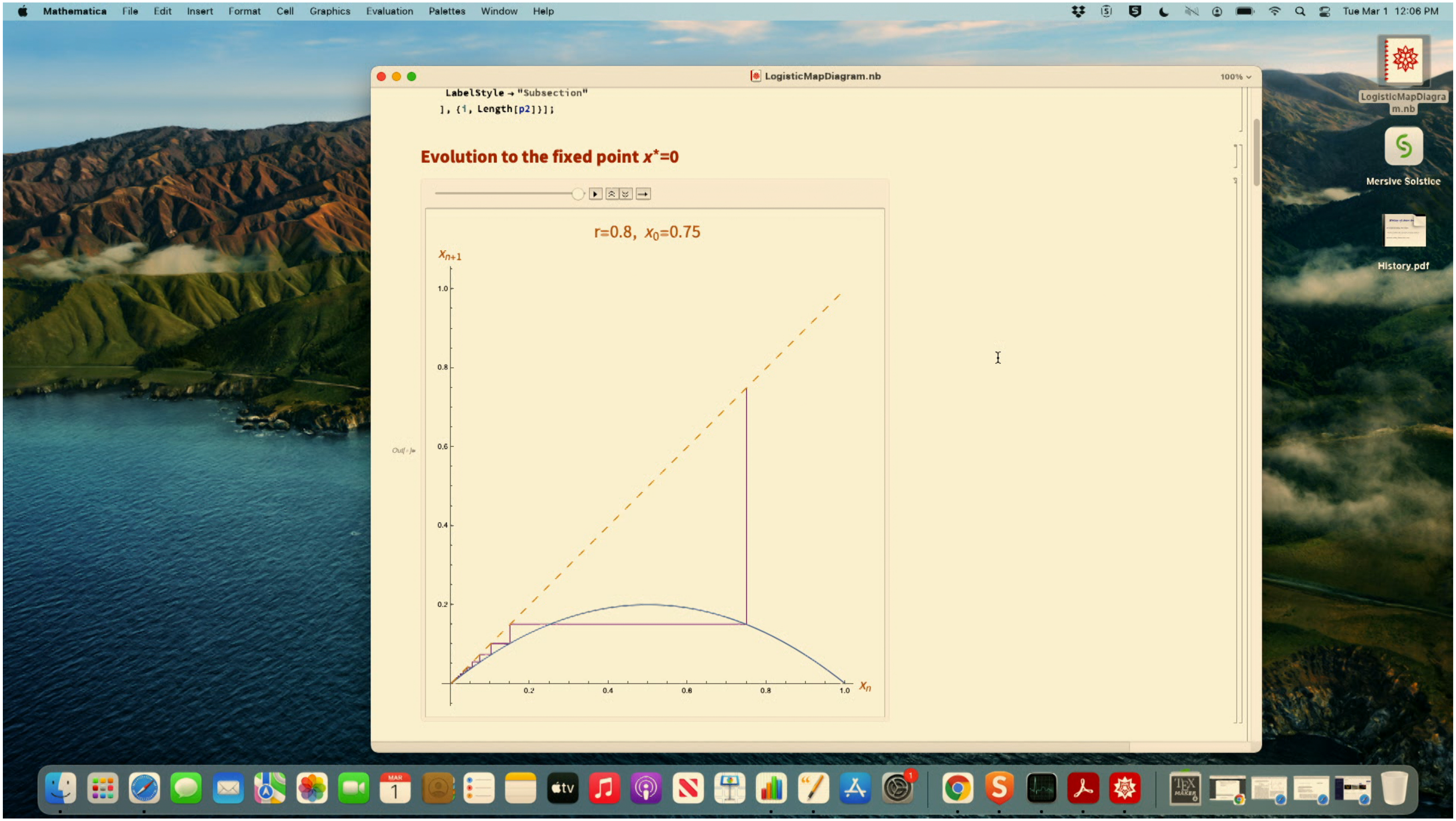
eg. 5

$r x_n (1 - x_n)$

$) = x_1 \in \left[0, \frac{r}{4}\right] \Rightarrow f(x_1) = x_2 \in [0, 1]$


motion
r > 4
eg. 5
Range(f) = (−∞, 5/4)
= f(x_n) = r x_n (1 - x_n)
[0, 1] ⇒ f(x_0) = x_1 ∈ [0, r/4] ⇒ f(x_1) = x_2 ∈ [0, 1]
x_1
x_2
x_0 > 1


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
Tue Mar 1 12:06 PM
LogisticMapDiagram.nb
LabelStyle → "Subsection"
], {i, Length[p2]}];
Evolution to the fixed point x*=0
r=0.8, x0=0.75
xn+1
xn
LogisticMapDiagram.nb
Mersive Solstice
History.pdf


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
Tue Mar 1 12:06 PM
LogisticMapDiagram.nb
100%
LabelStyle → "Subsection"
], {i, Length[p2]}];
Evolution to the fixed point x*=0
r=0.8, x0=0.75
xn+1
xn
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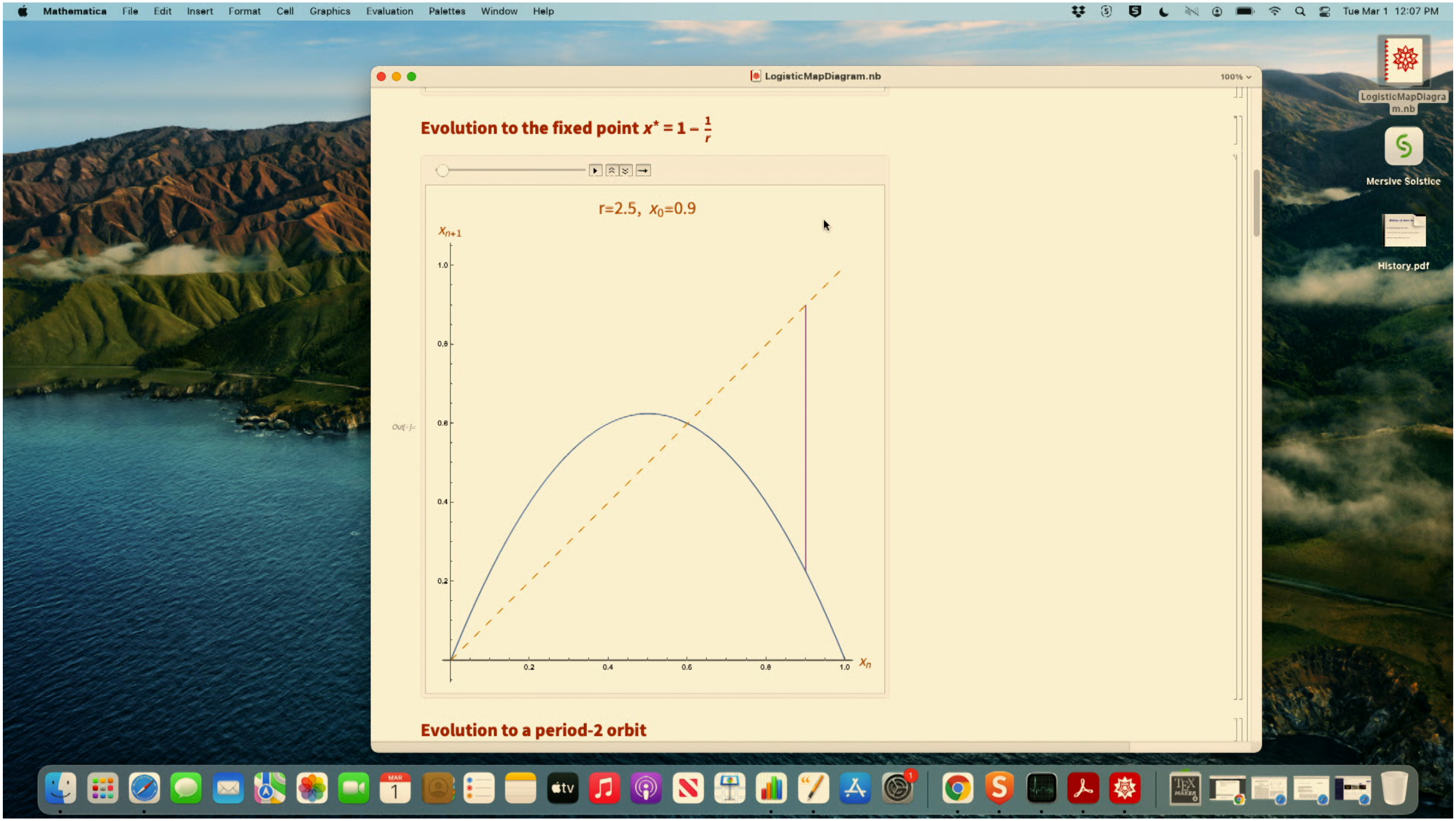

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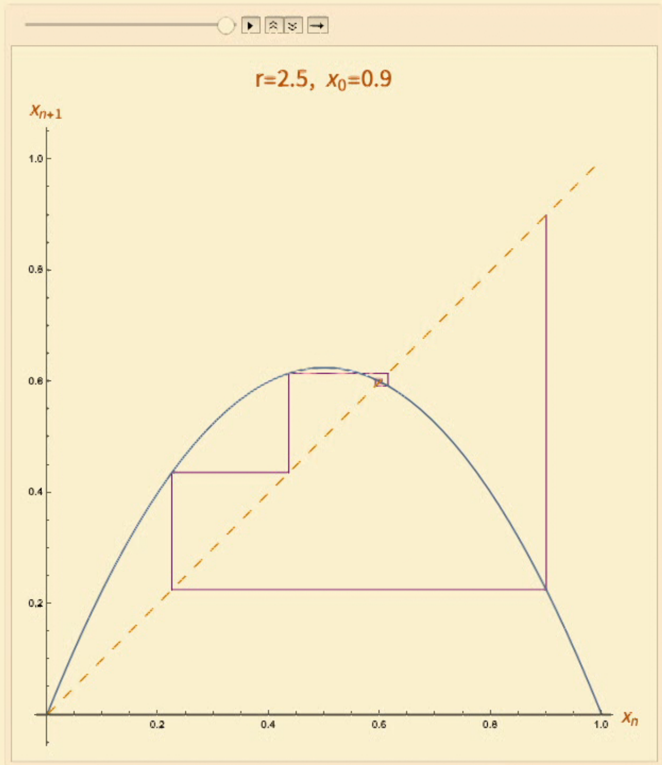
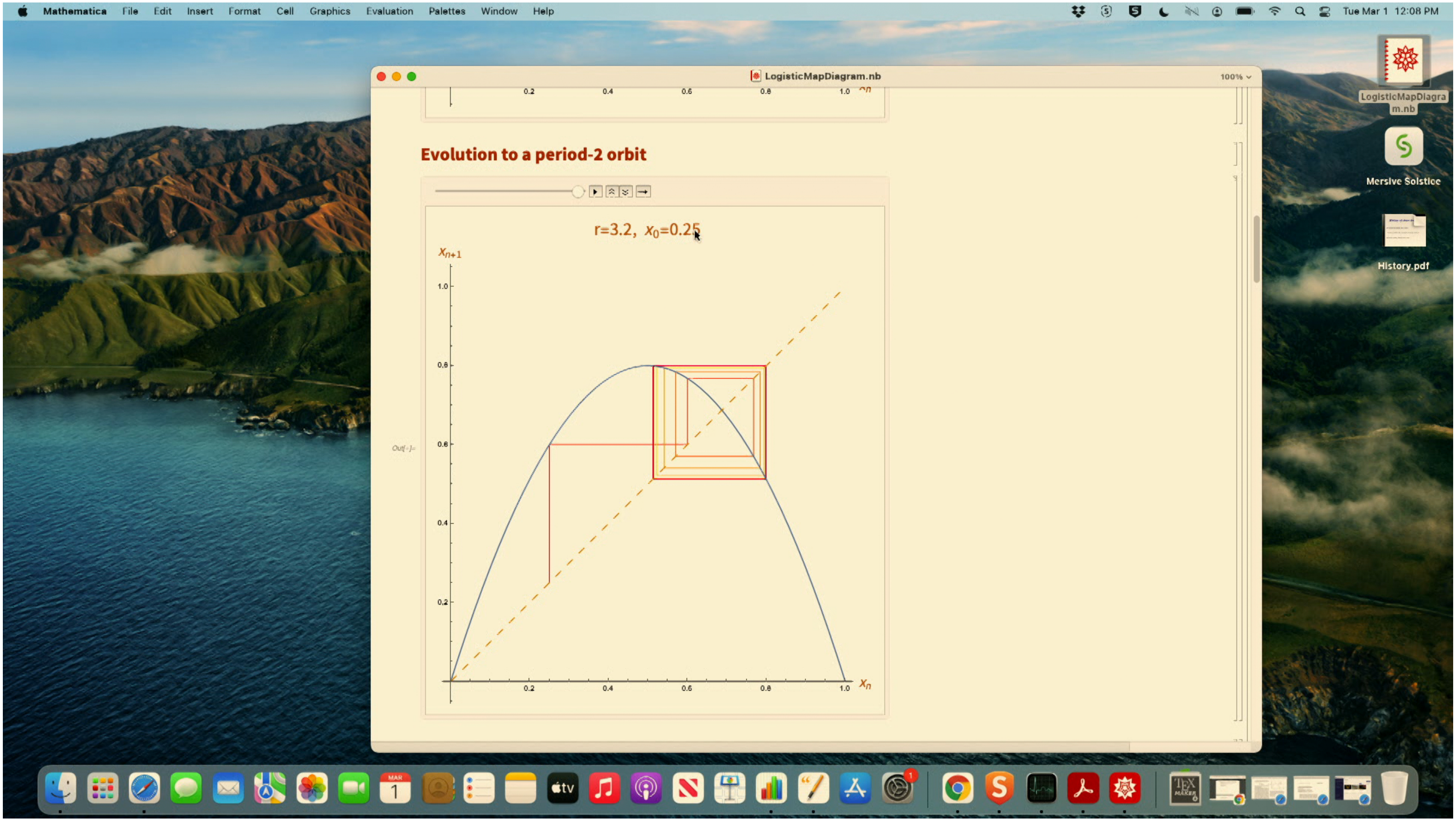
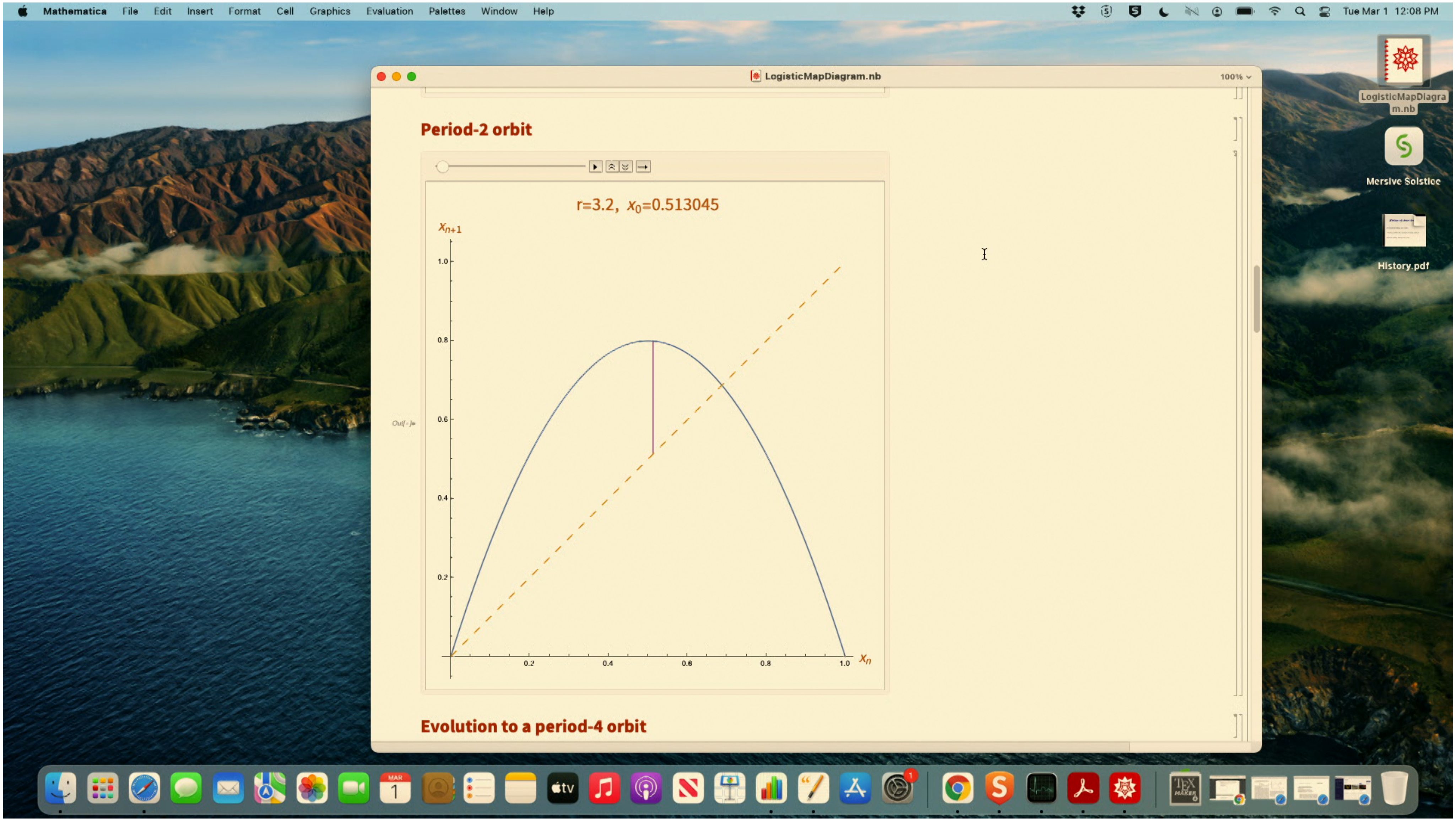
LogisticMapDiagram.nb
Evolution to the fixed point $x^* = 1 - \frac{1}{r}$
r=2.5, x_0=0.9
x_{n+1}
x_n
Evolution to a period-2 orbit
LogisticMapDiagram.nb
Mersive Solstice
History.pdf

Evolution to the fixed point $x^* = 1 - \frac{1}{r}$



Evolution to a period-2 orbit


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
Tue Mar 1 12:08 PM
LogisticMapDiagram.nb
100%
Evolution to a period-2 orbit
r=3.2, x0=0.25
xn+1
1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0
xn
LogisticMapDiagram.nb
Mersive Solstice
History.pdf



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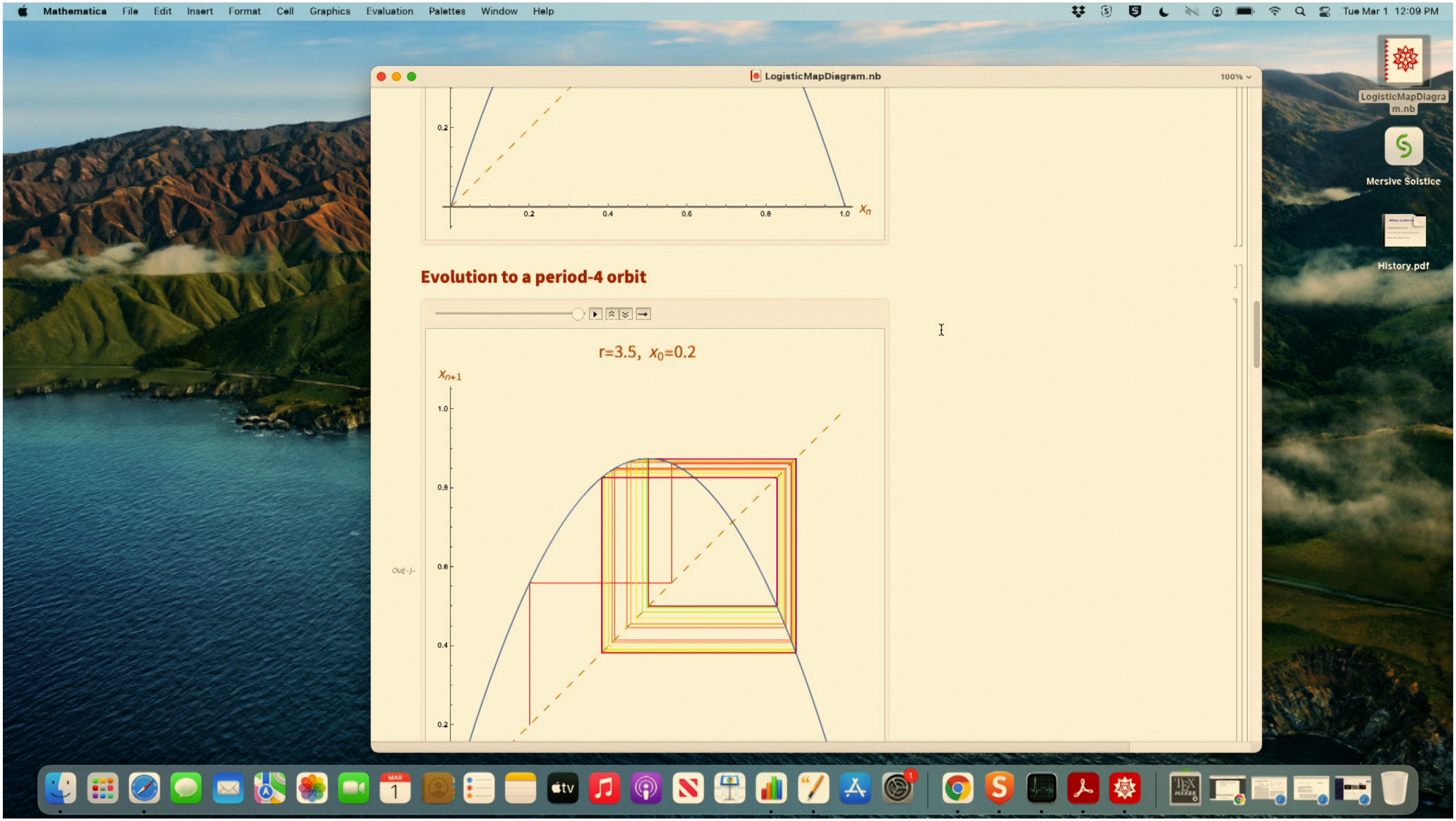
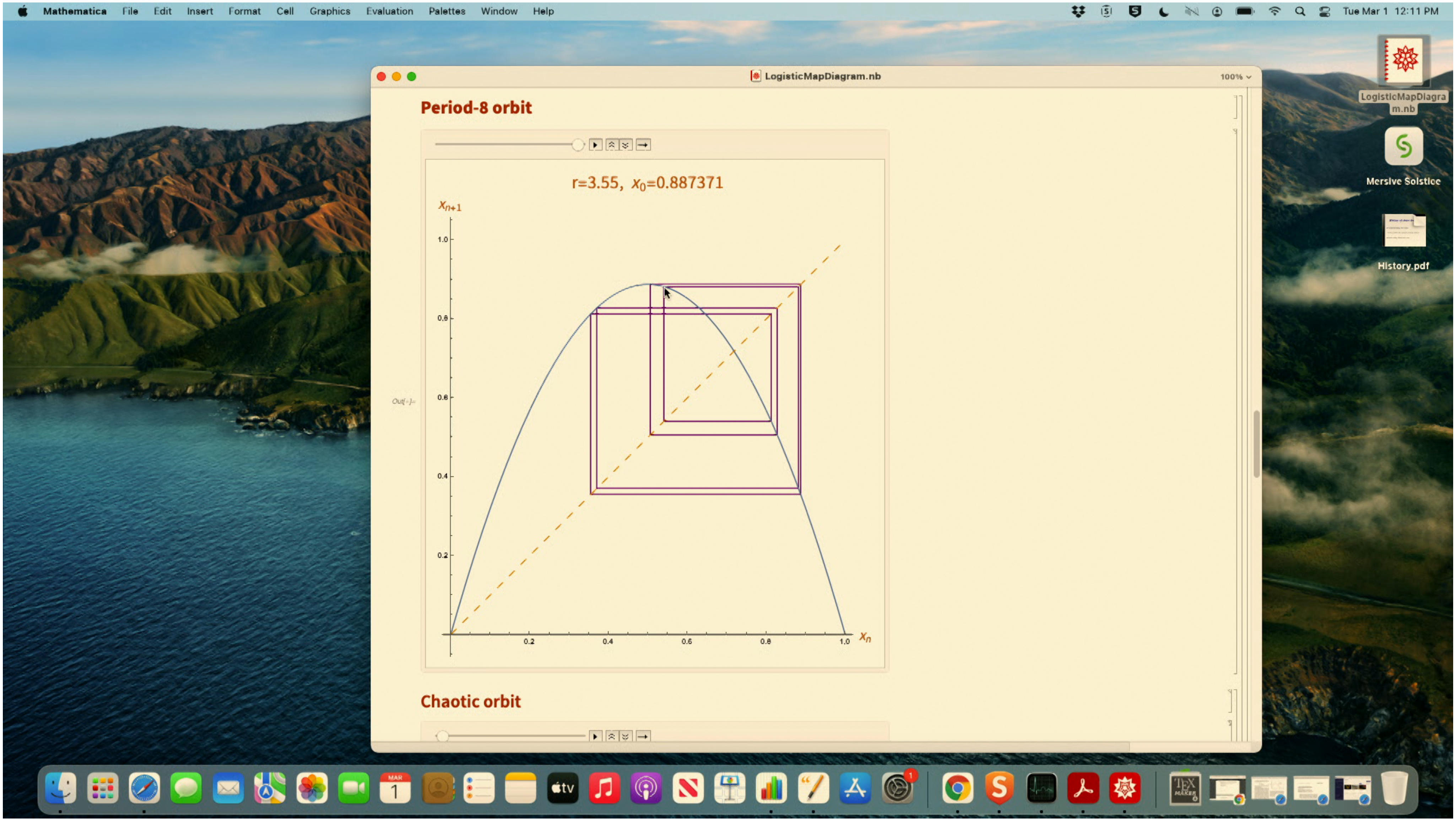
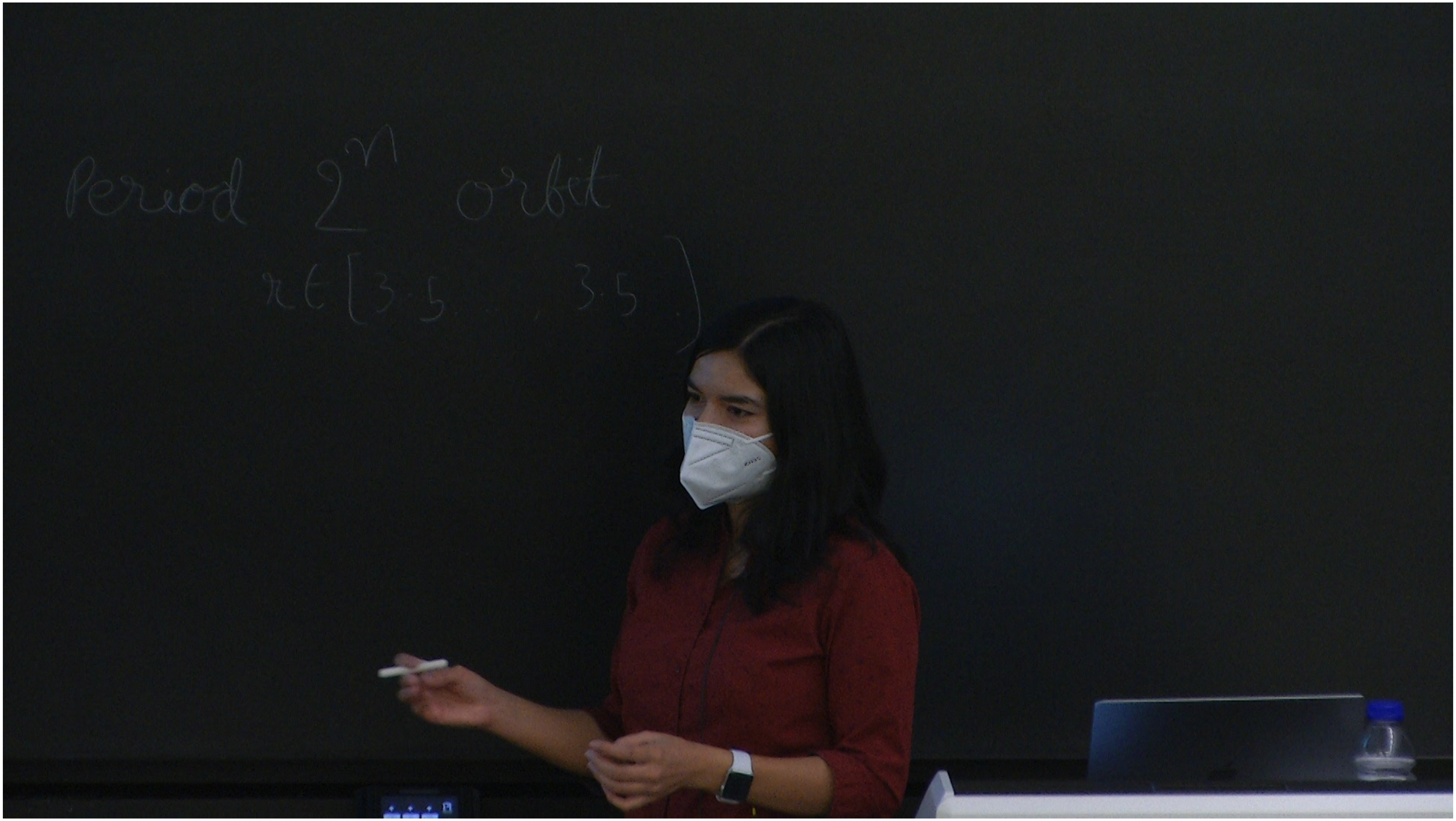
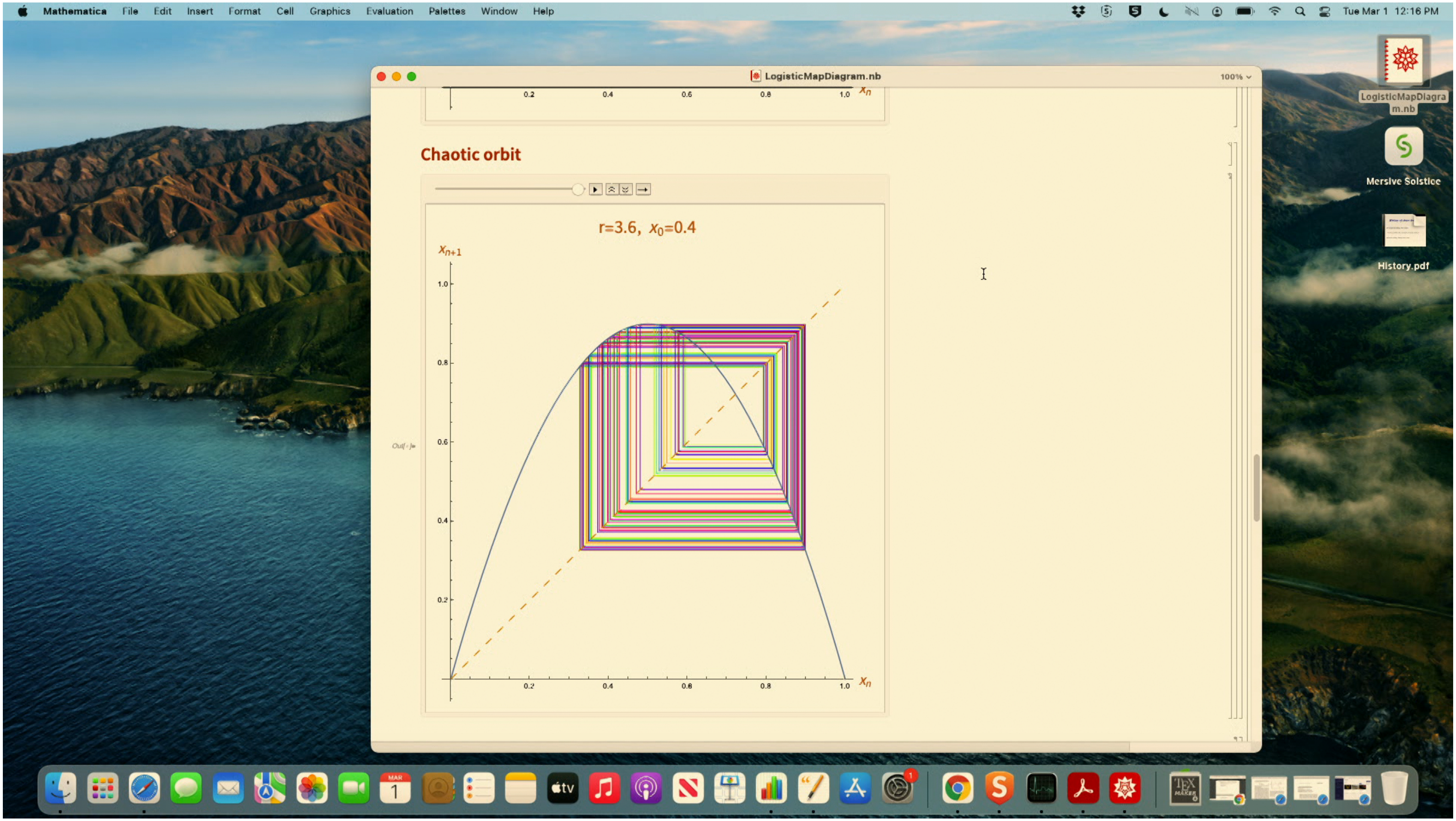
Tue Mar 1 12:08 PM
LogisticMapDiagram.nb
100%
Period-2 orbit
r=3.2, x_0=0.513045
x_{n+1}
1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0
x_n
Evolution to a period-4 orbit
LogisticMapDiagram.nb
Mersive Solstice
History.pdf

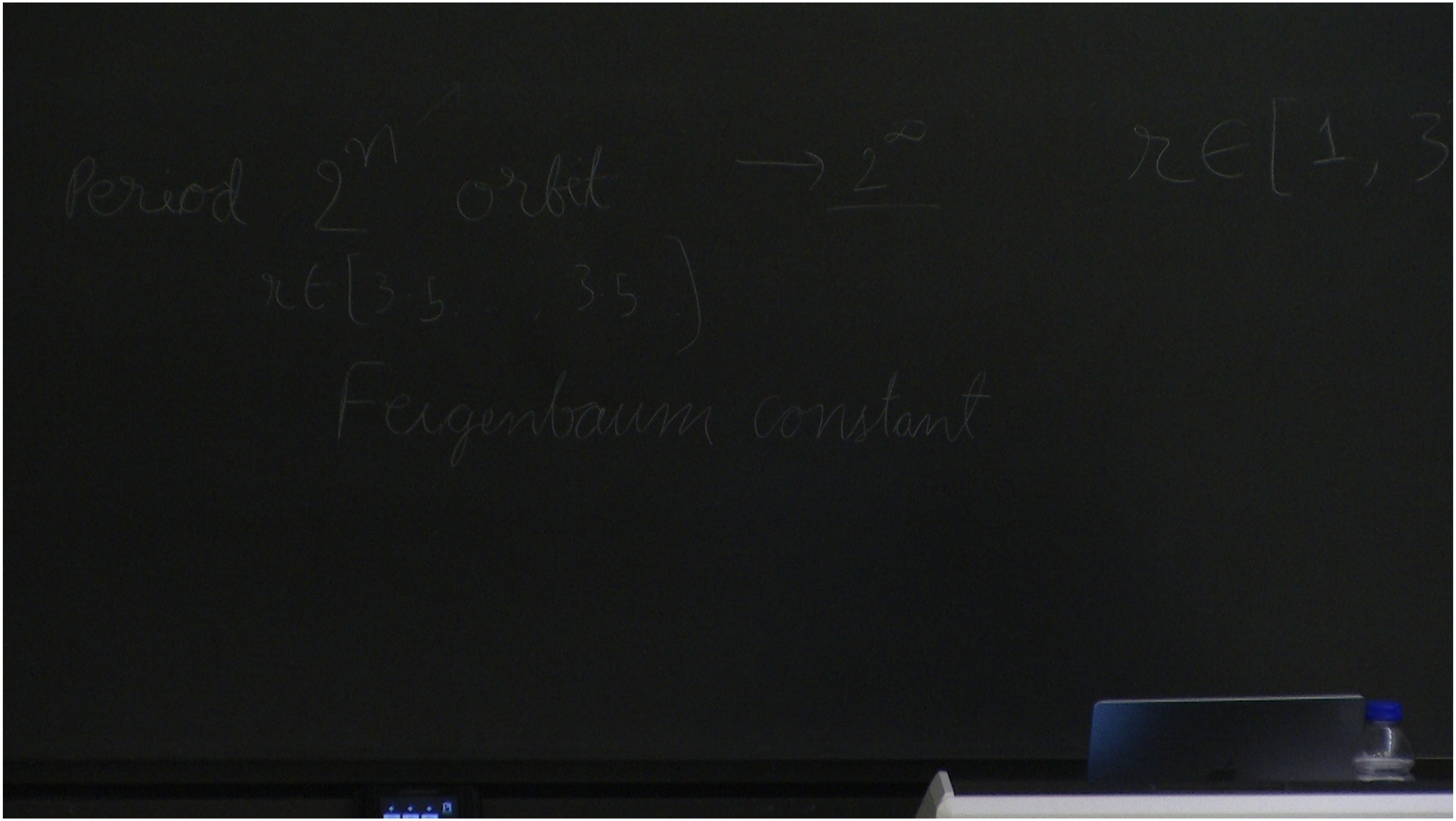

LogisticMapDiagram.nb
Evolution to a period-4 orbit
r=3.5, x_0=0.2
x_{n+1}
x_n
1.0
0.8
0.6
0.4
0.2

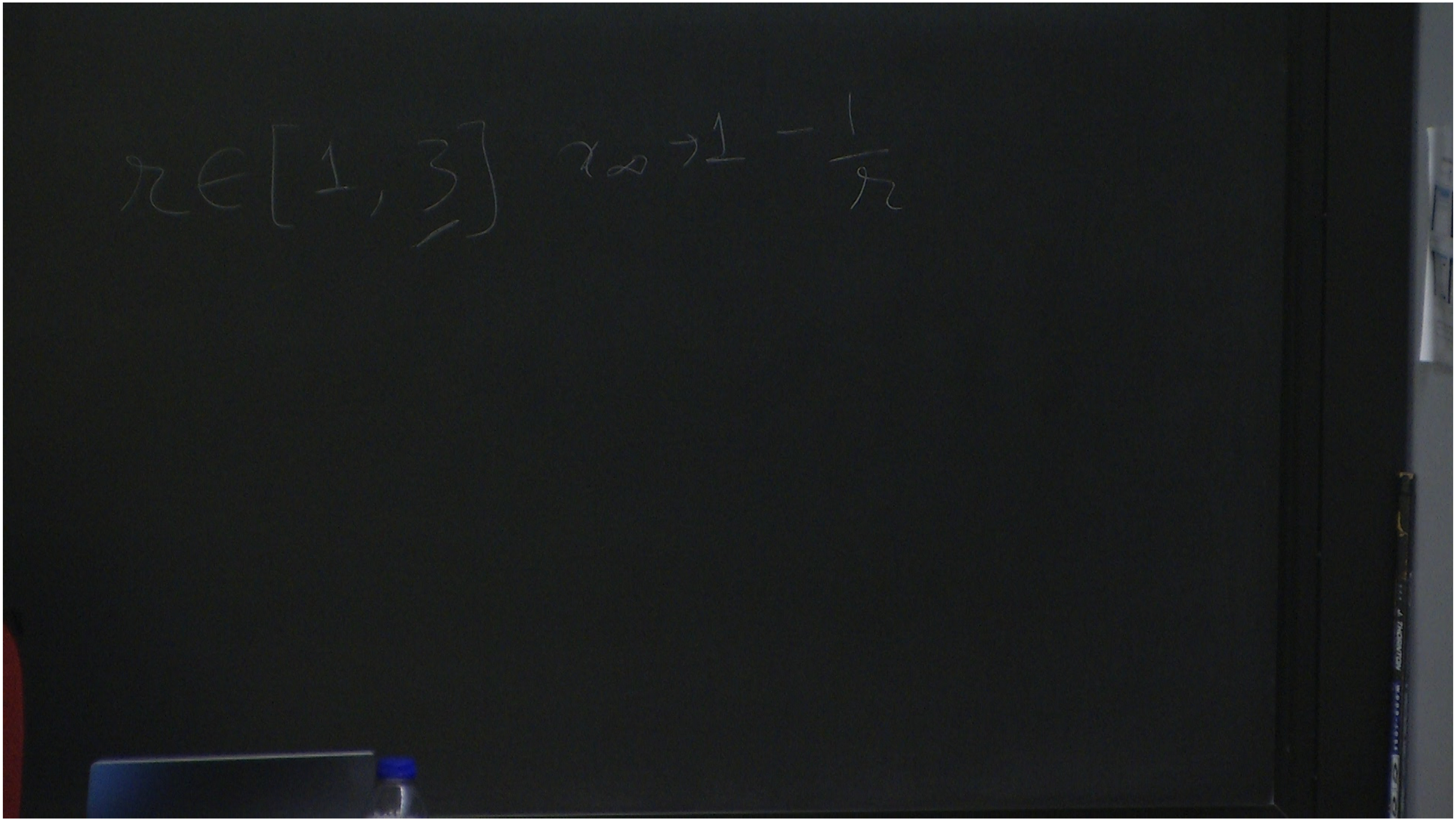


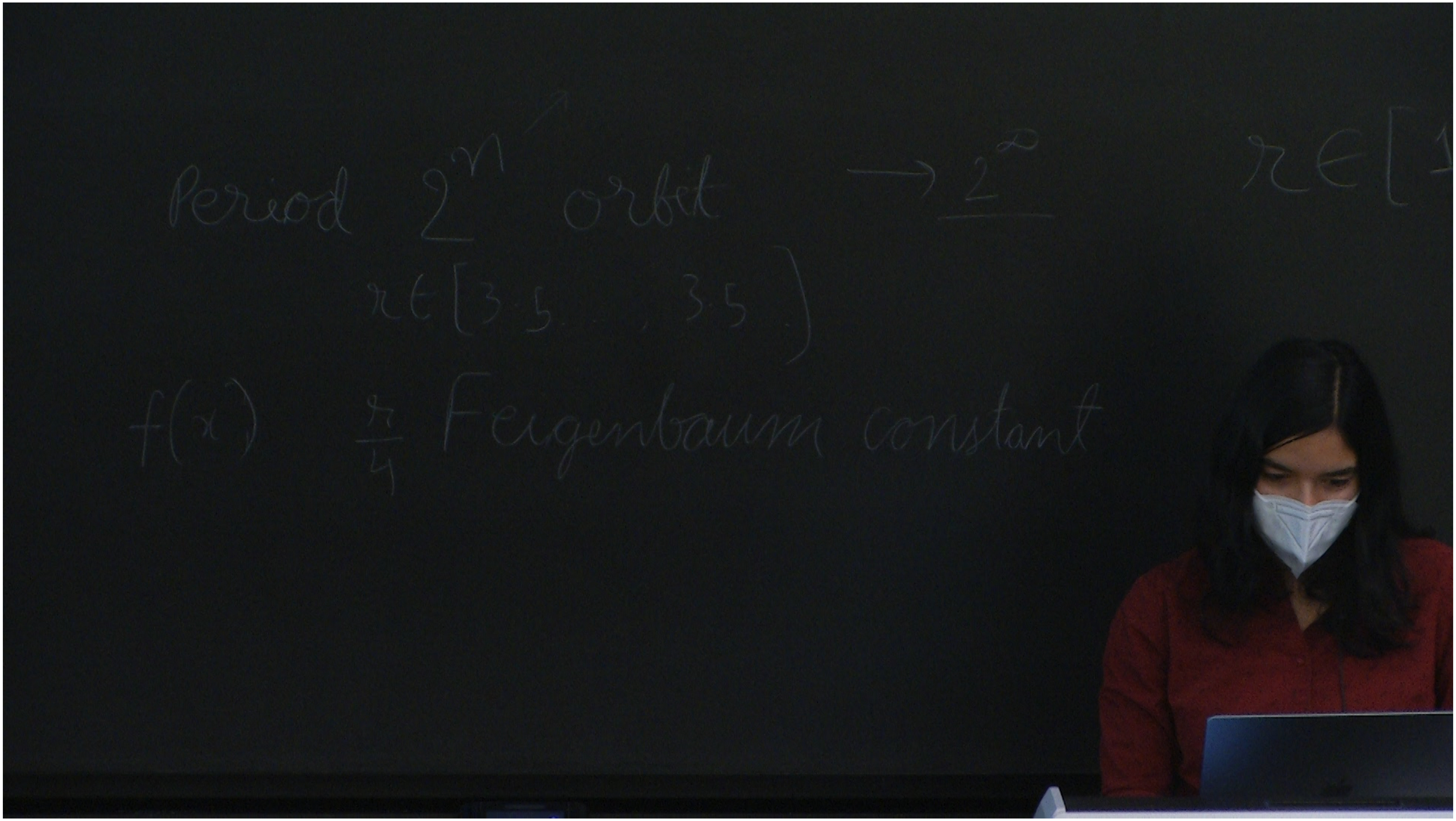
LogisticMapDiagram.nb
Period-8 orbit
r=3.55, x_0=0.887371
x_{n+1}
x_n
Chaotic orbit


Period 2^n orbit
$r \in [3.5\ldots, 3.5\ldots)$

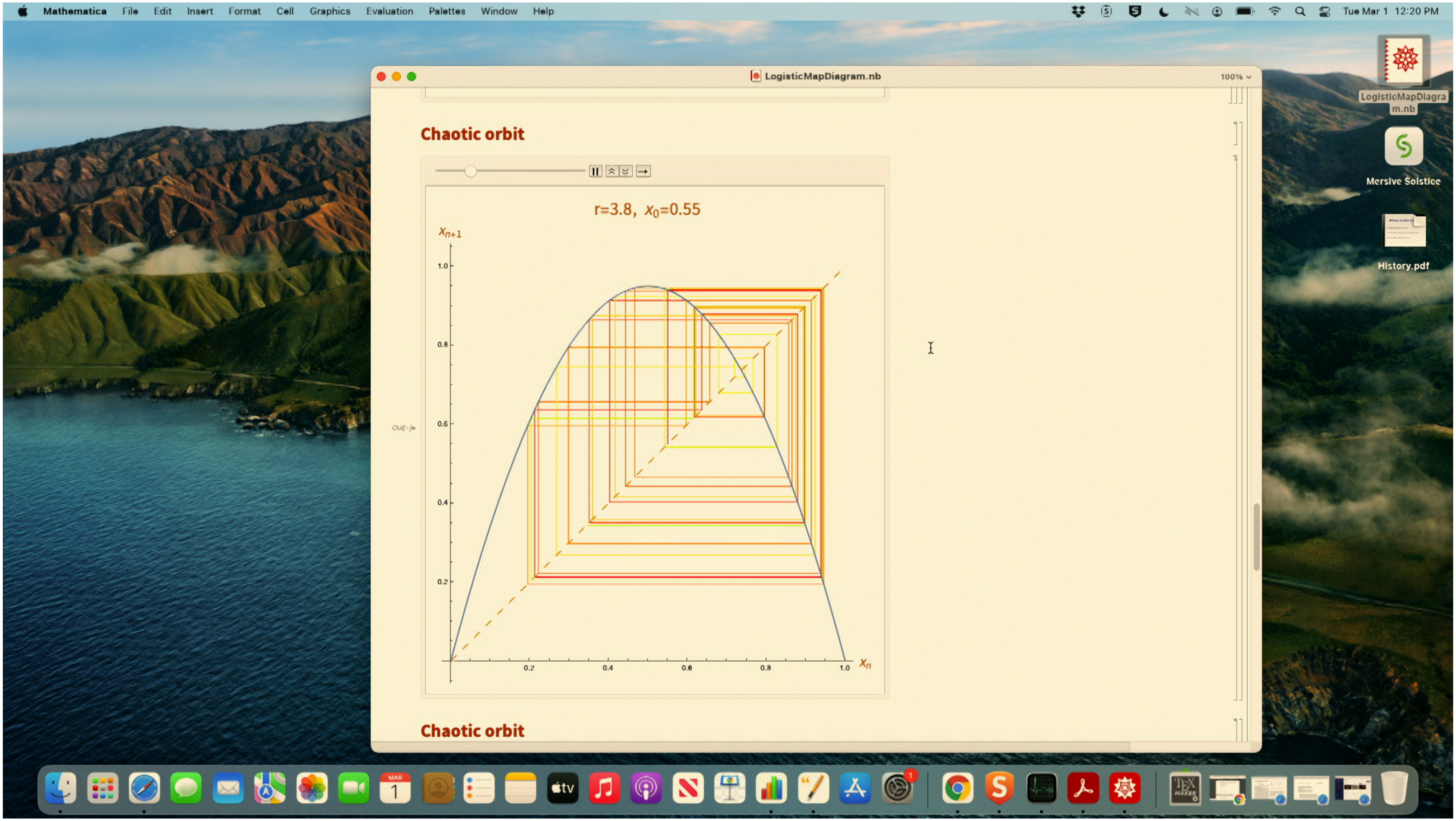

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
Tue Mar 1 12:16 PM
LogisticMapDiagram.nb
100%
Chaotic orbit
r=3.6, x0=0.4
xn+1
xn
0.2
0.4
0.6
0.8
1.0
LogisticMapDiagram.nb
Mersive Solstice
History.pdf


Period 2^n orbit → 2^∞
r ∈ [1, 3
r ∈ [3.5..., 3.5...)
Feigenbaum constant


r ∈ [1, 3]
x∞ → 1 − 1/r



Period 2^n orbit → 2^∞
r∈[3.5..., 3.5...]
f(x)
r/4
Feigenbaum constant



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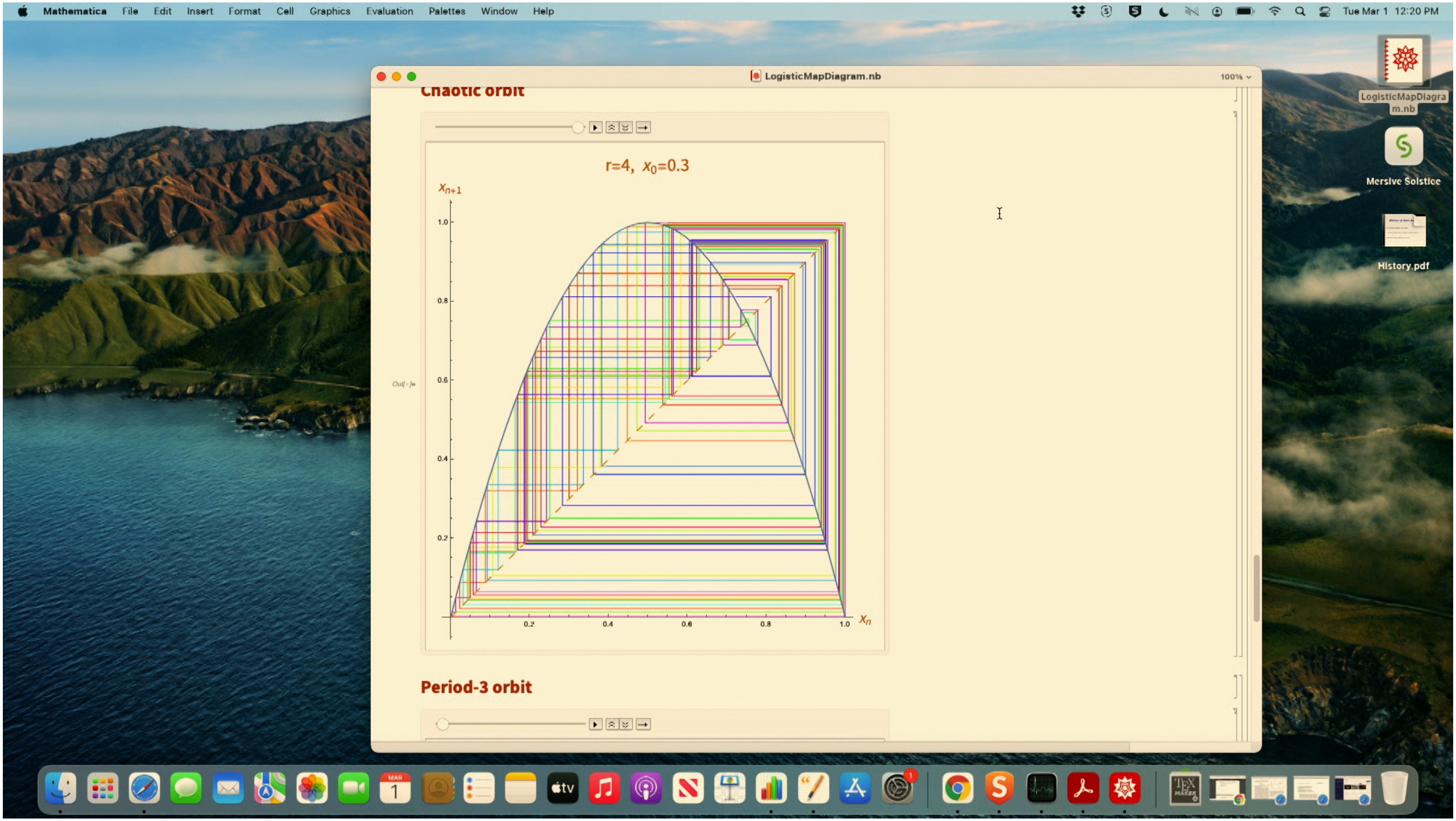
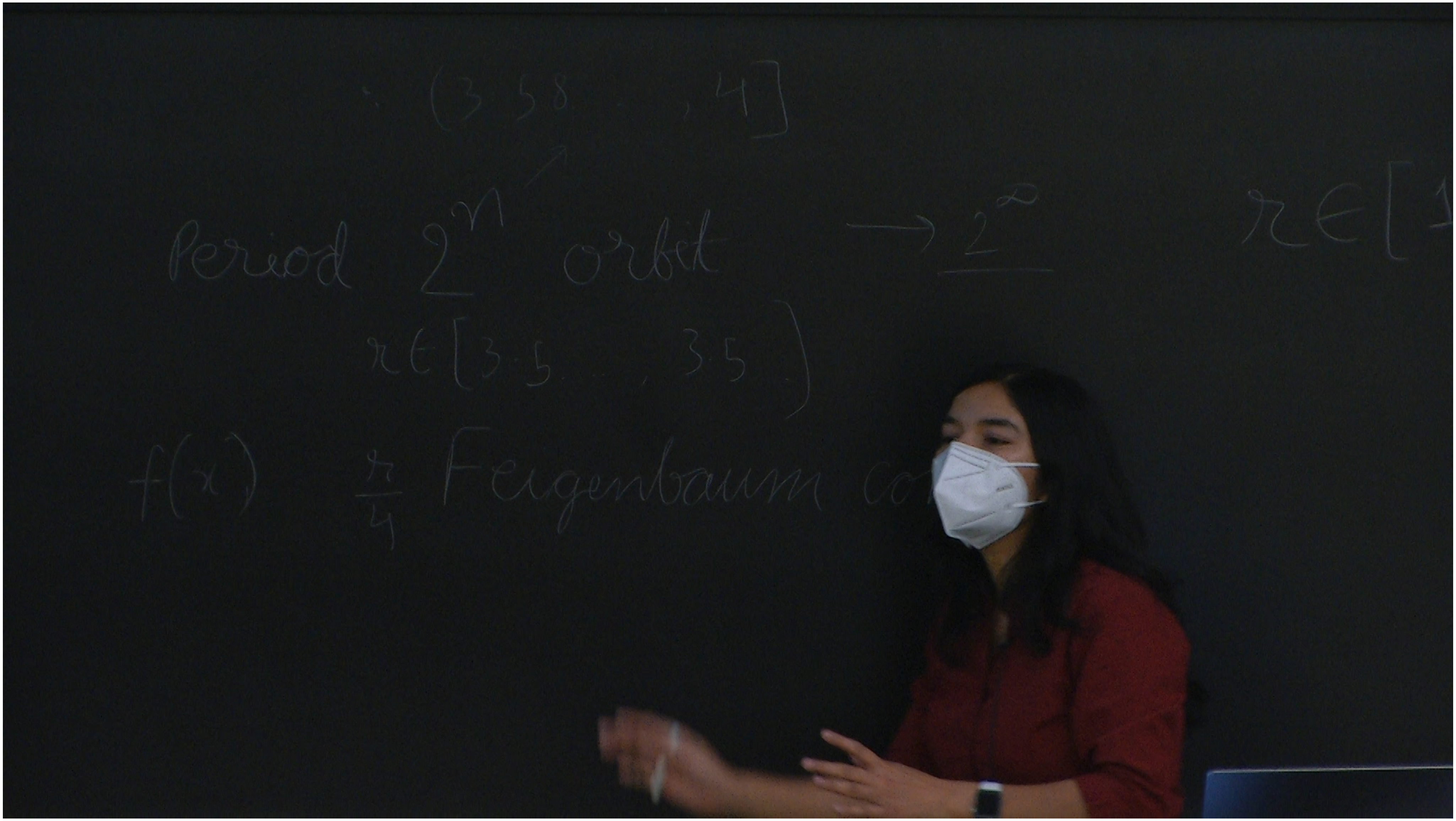
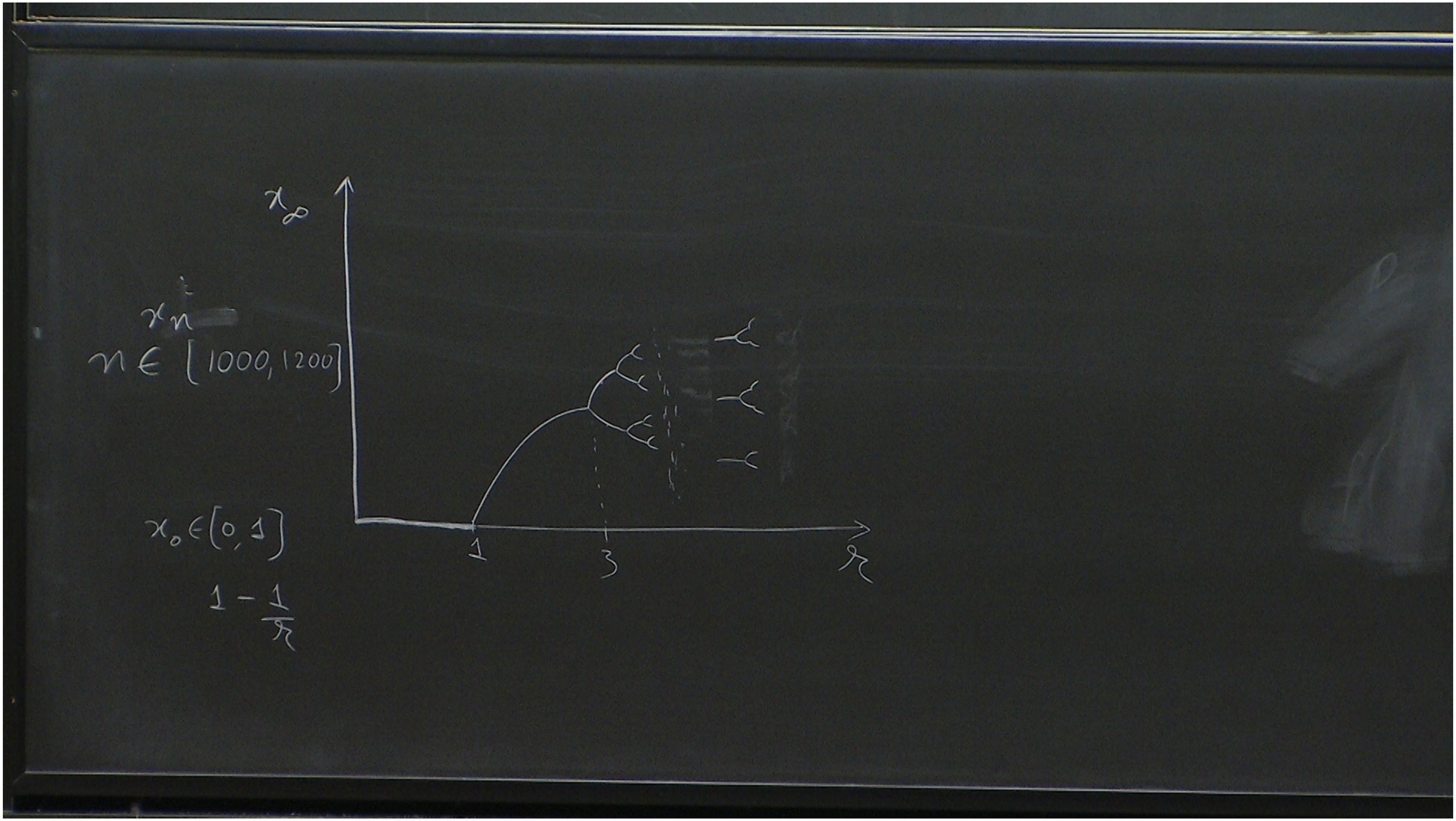
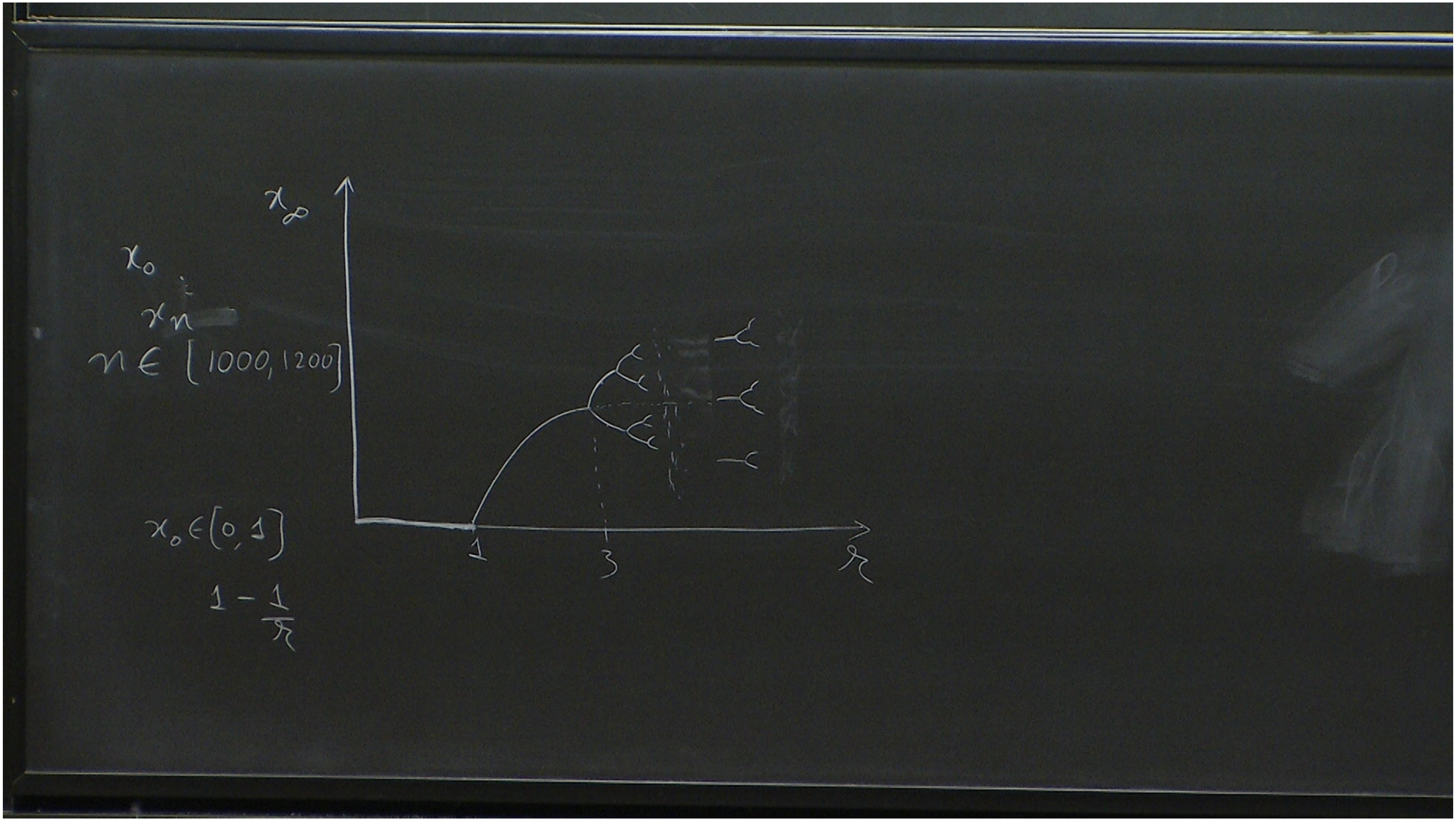
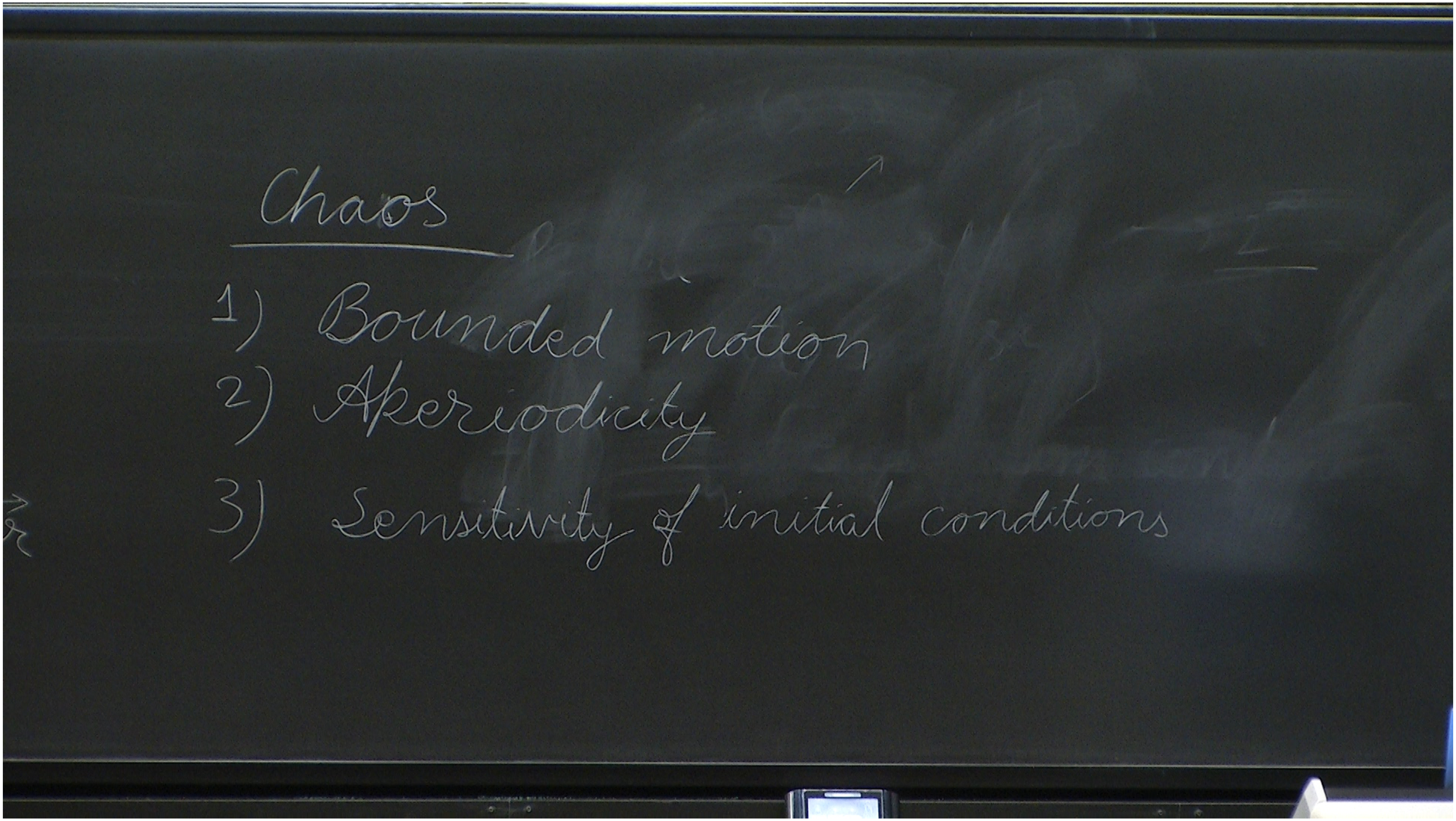
Tue Mar 1 12:20 PM
LogisticMapDiagram.nb
Chaotic orbit
r=3.8, x_0=0.55
x_{n+1}
x_n
Chaotic orbit


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
Tue Mar 1 12:20 PM
LogisticMapDiagram.nb
Chaotic orbit
r=4, x0=0.3
xn+1
xn
1.0
0.8
0.6
0.4
0.2
Period-3 orbit
LogisticMapDiagram.nb
Mersive Solstice
History.pdf


Period 2^n orbit
Feigenbaum


x∞
xn
n ∈ [1000, 1200]
x0 ∈ [0, 1]
1 − 1/r
1
3
r


x∞
x0
xn
n ∈ [1000, 1200]
x0 ∈ [0, 1]
1 − 1/r
1
3
r


Chaos
1) Bounded motion
2) Aperiodicity
3) Sensitivity of initial conditions