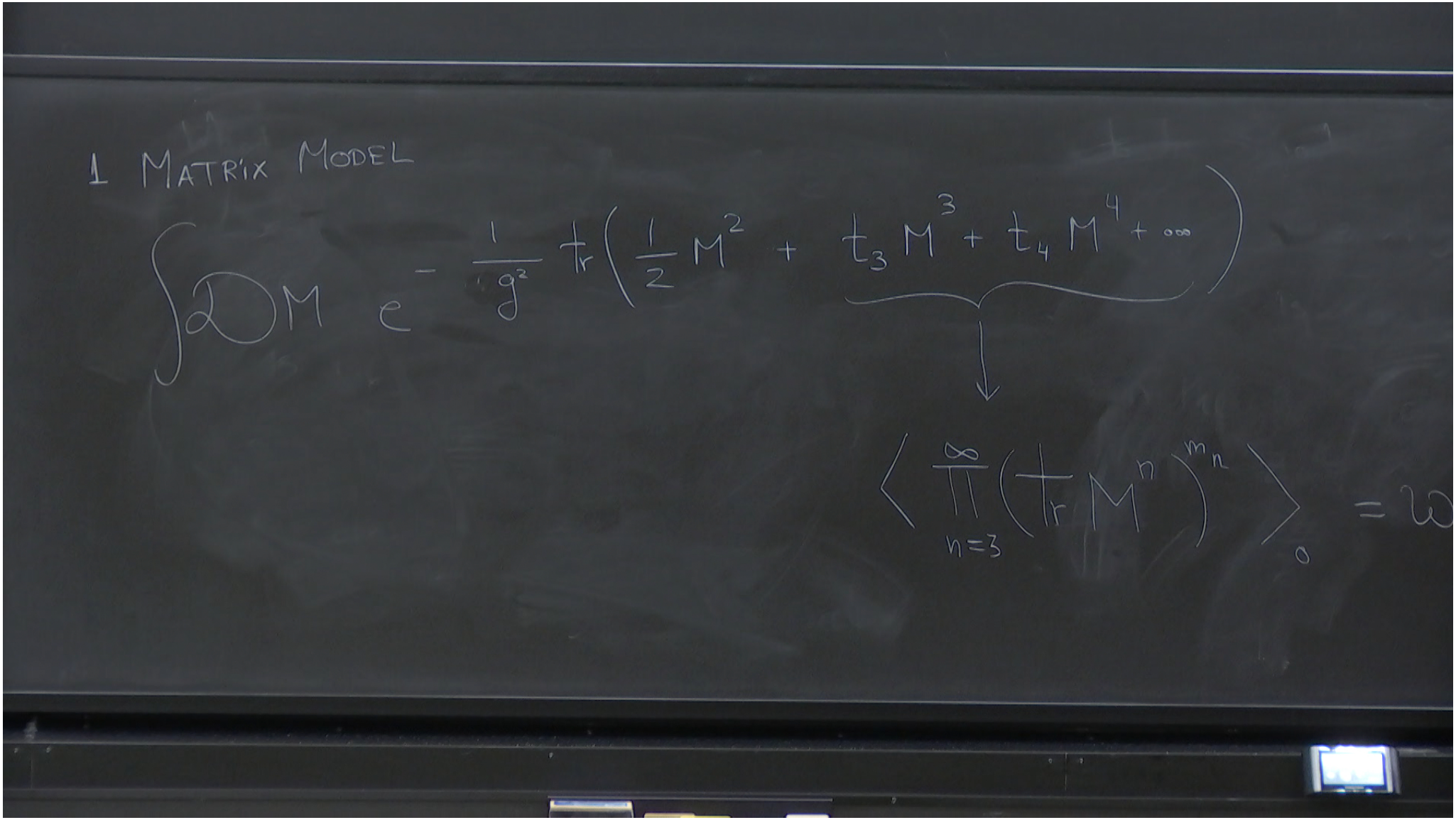
Title: AdS/CFT 2021/2022

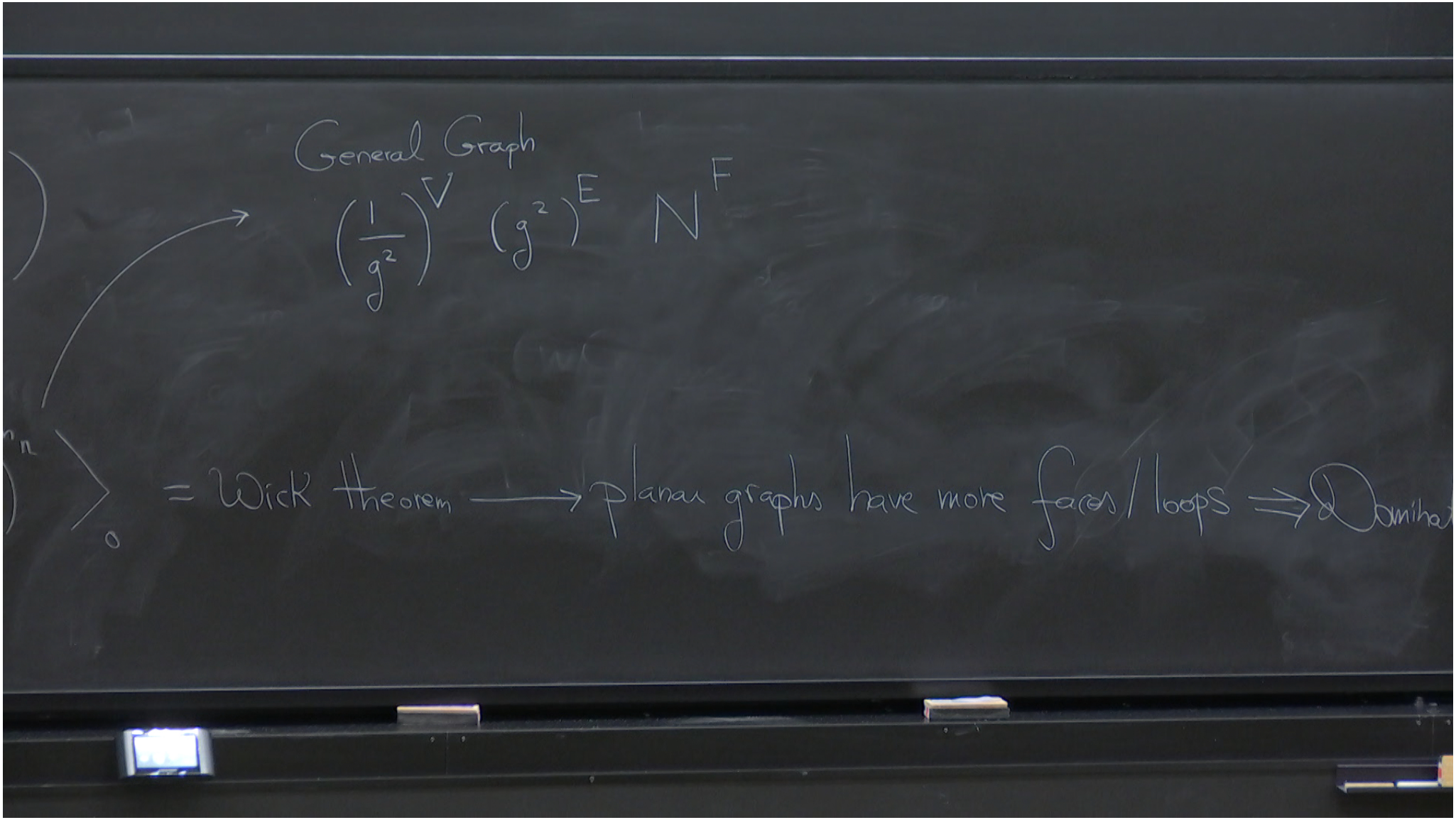
Speakers: Pedro Vieira, Gang Xu

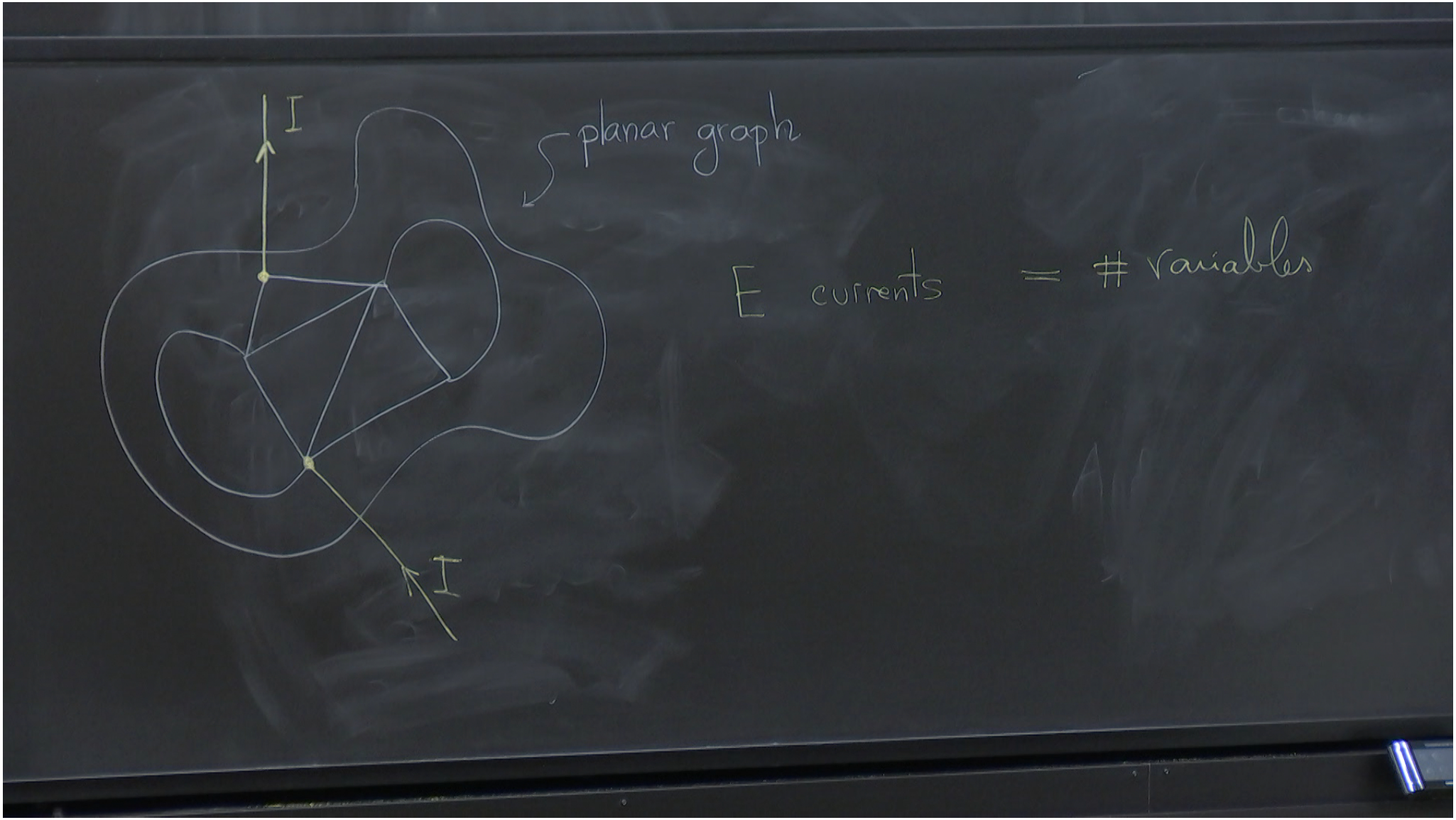
Collection: AdS/CFT 2021/2022

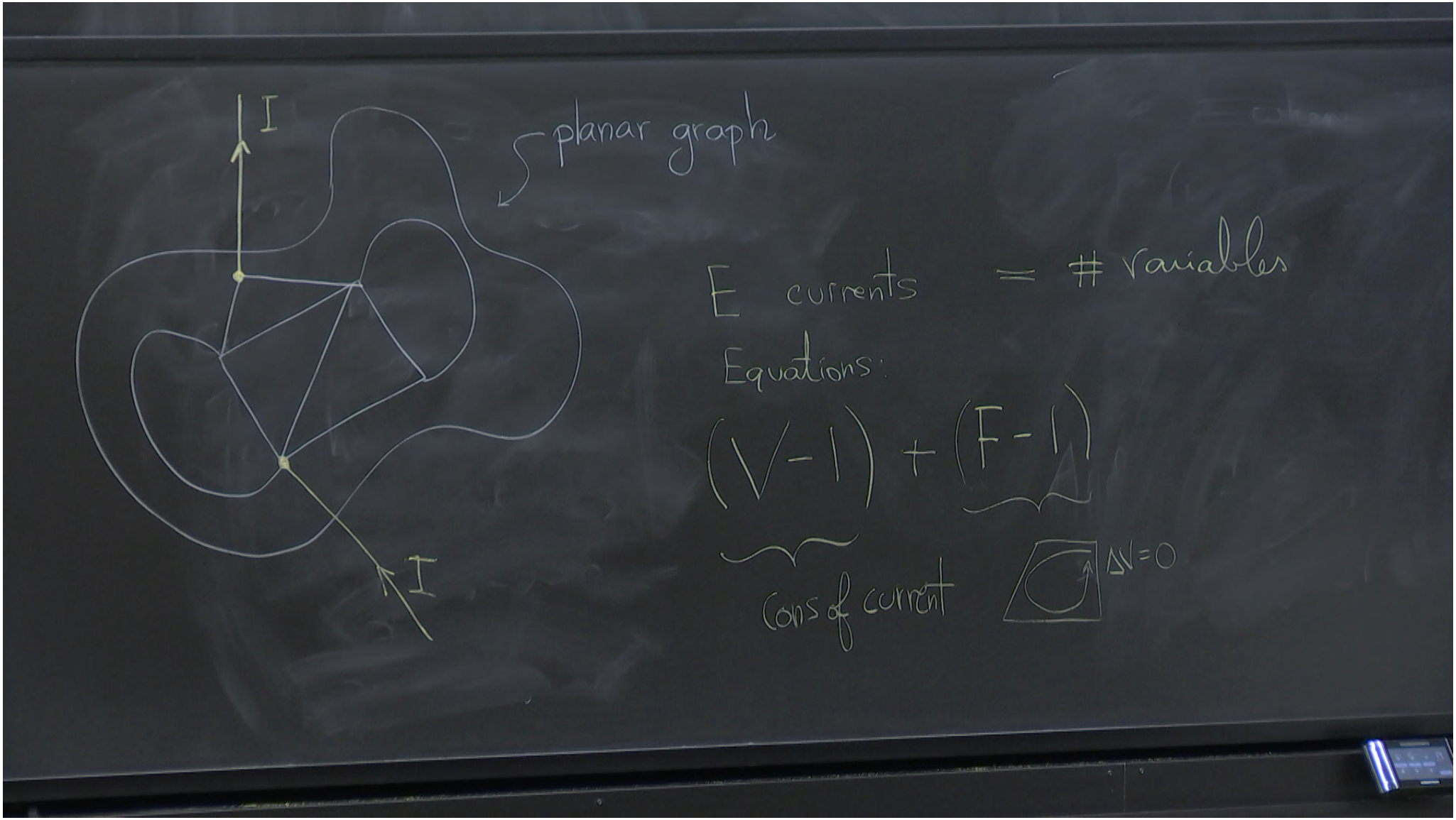
Date: March 31, 2022 - 11:30 AM

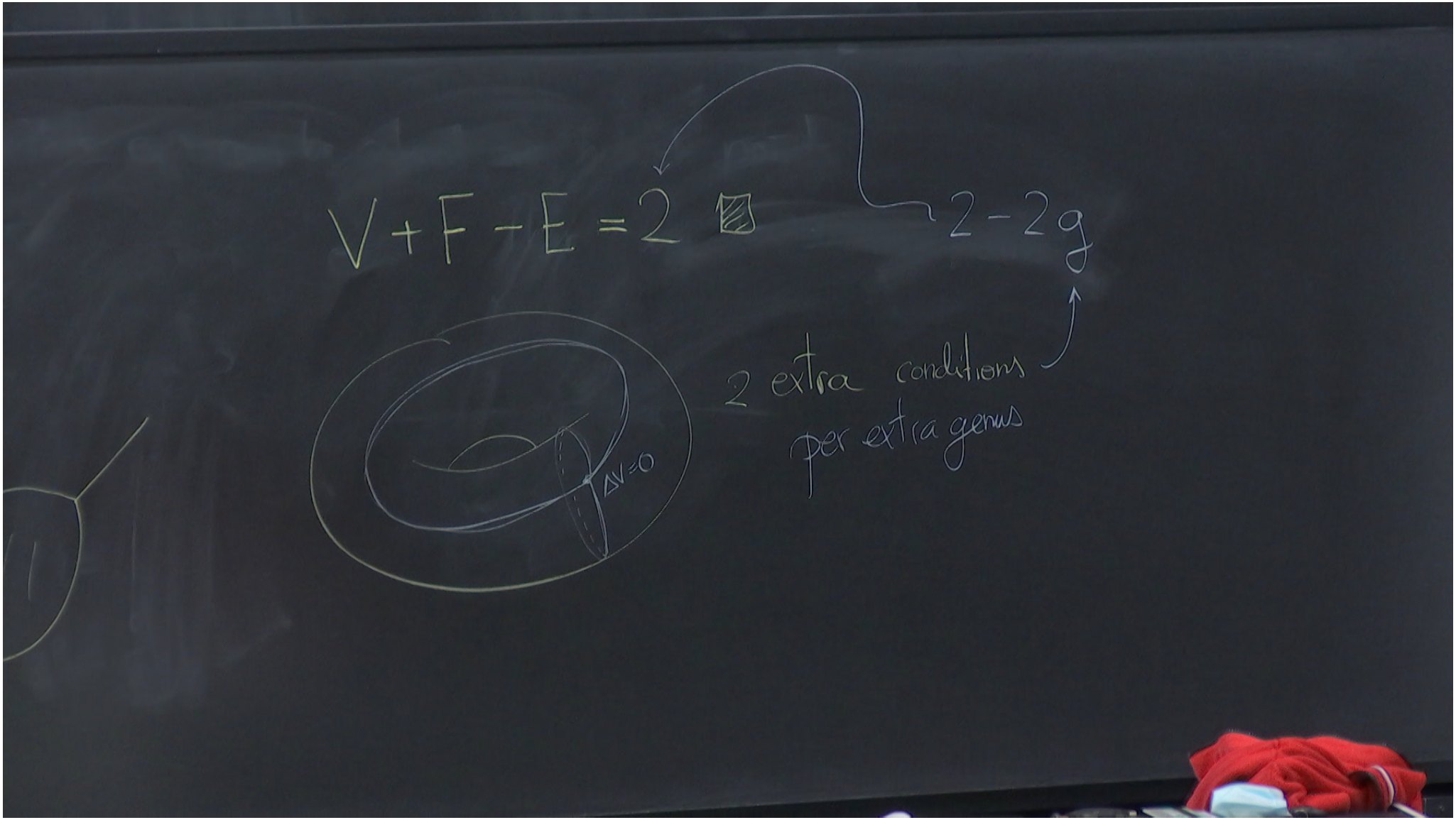
URL: https://pirsa.org/22030045


1 Matrix Model
$\int \mathcal{D}M \; e^{-\frac{1}{g^2}\mathrm{tr}\left(\frac{1}{2}M^2 + t_3 M^3 + t_4 M^4 + \cdots\right)}$
$\left\langle \prod_{n=3}^{\infty} \left(\mathrm{tr}\, M^n\right)^{m_n} \right\rangle_0 = \omega$


General Graph
$\left(\frac{1}{g^2}\right)^V (g^2)^E N^F$
$= \langle \rangle_0$
= Wick theorem ⟶ planar graphs have more faces/loops ⇒


I
planar graph
E currents = # variables
I

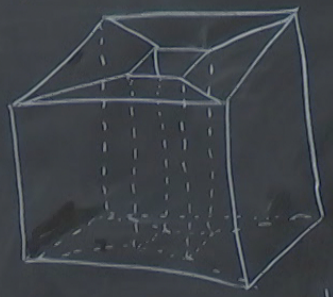

I
planar graph
E currents = # variables
Equations:
(V-1) + (F-1)
I
Cons of current
ΔV=0


V+F-E=2
2-2g
2 extra conditions per extra genus
ΔV=0

neral Graph

$\left(\frac{1}{g^2}\right)^V \quad (g^2)^E \quad N^F$

now $V + F - E = \overbrace{2 - 2g}^{\text{Euler Char. } \chi}$



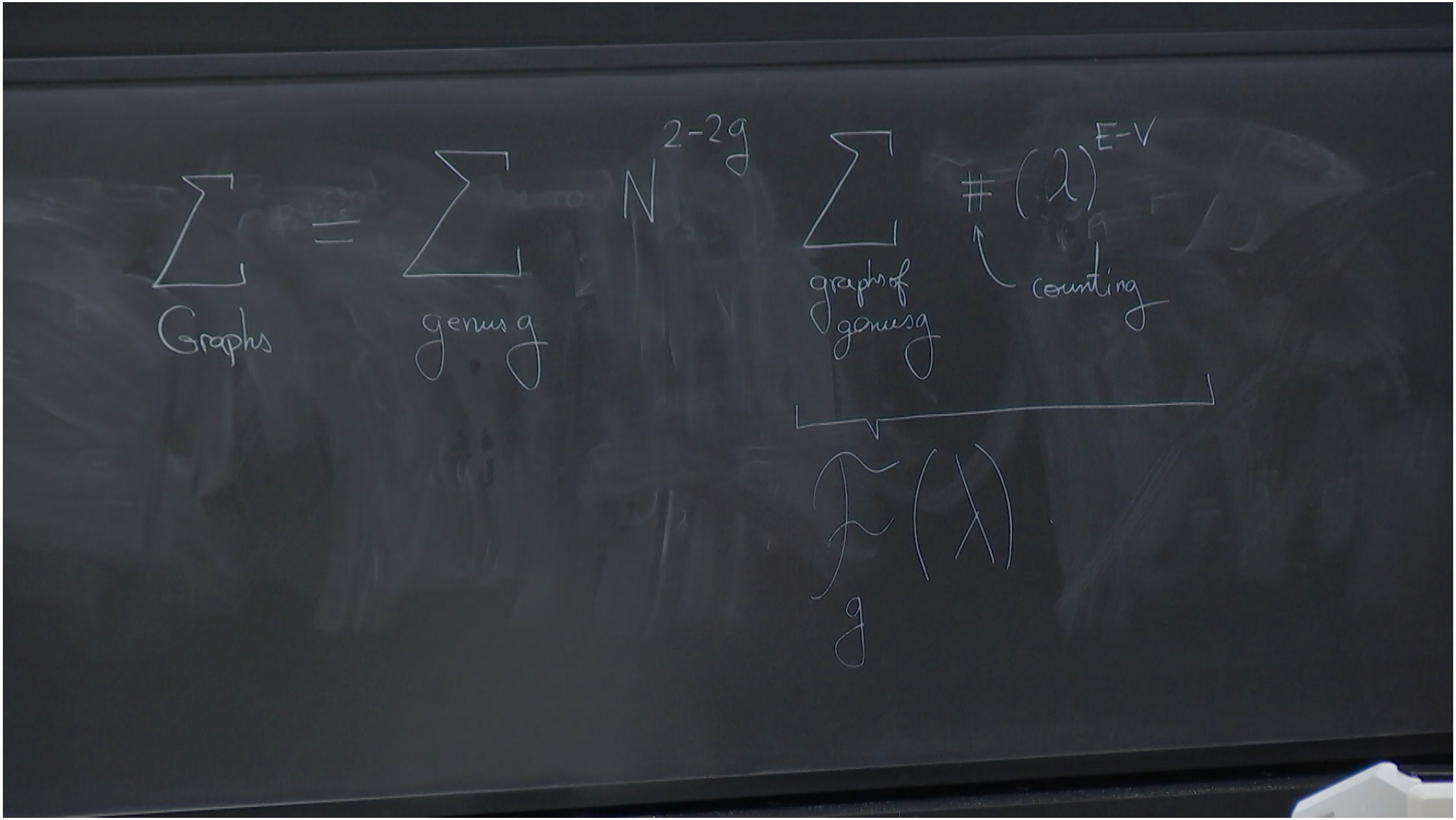
$8+6-12=2$ ✓

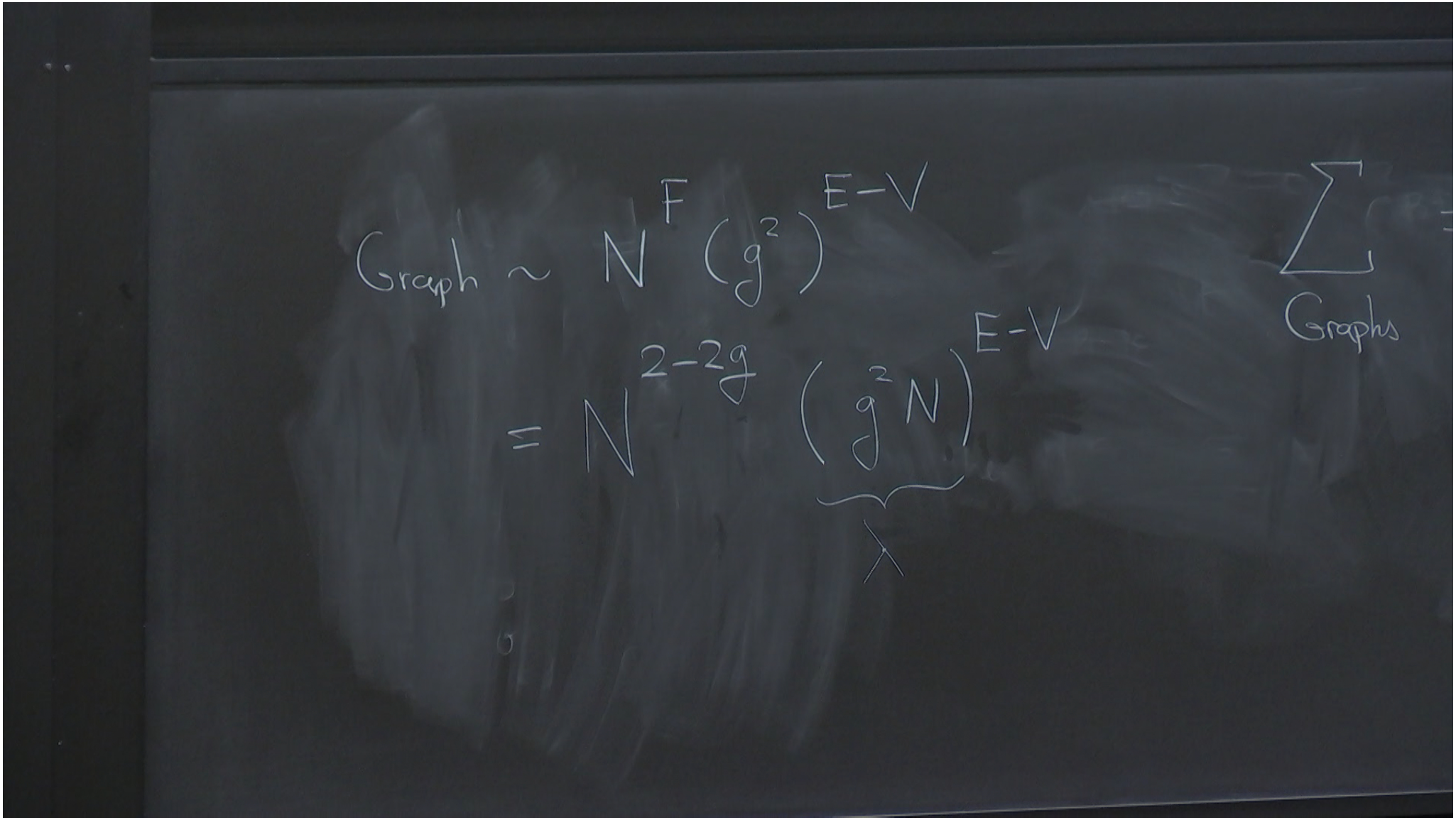
minimum genus of a surface Σ where we can draw graph.

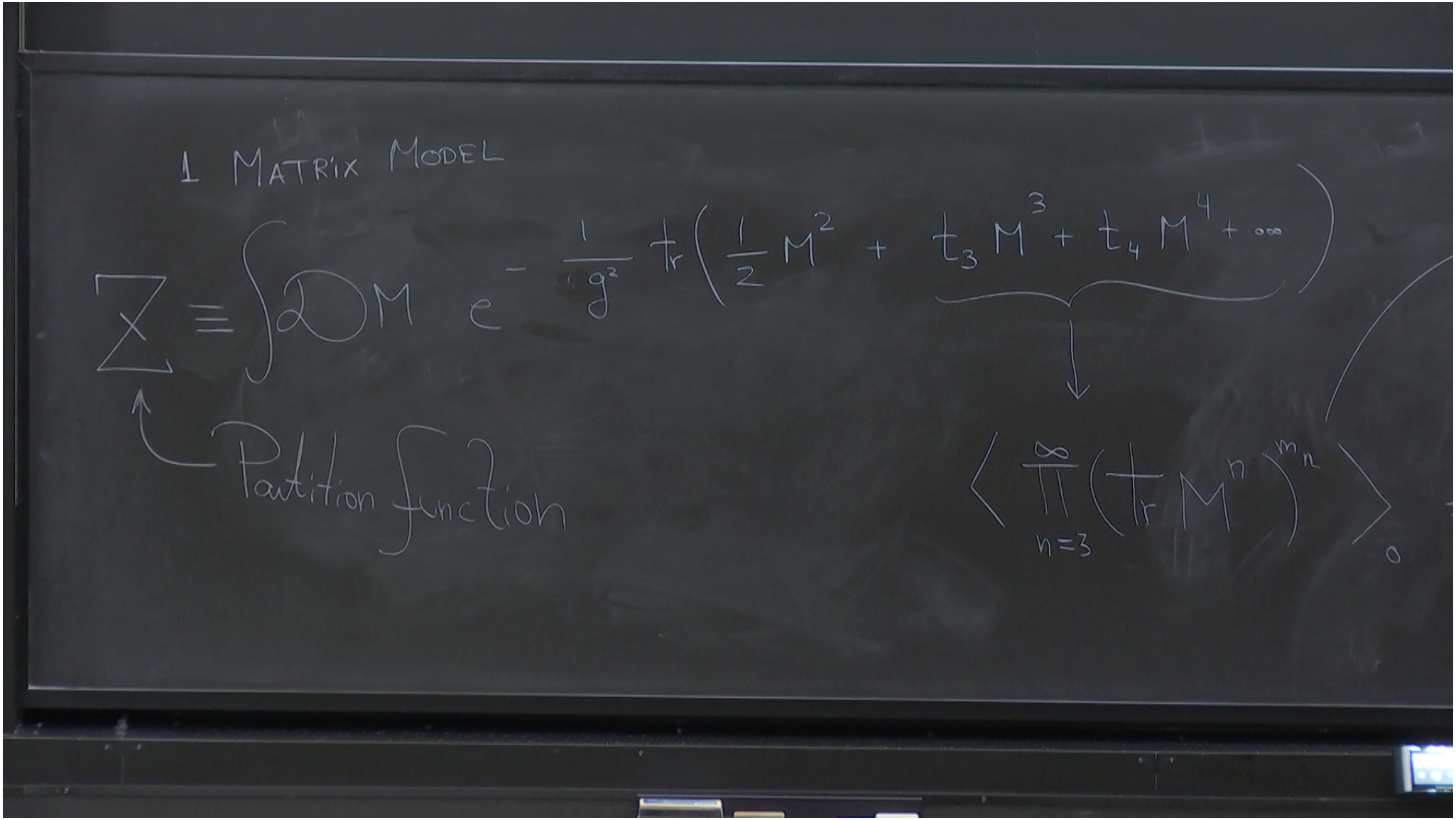
~~$0-24=2$~~ ←

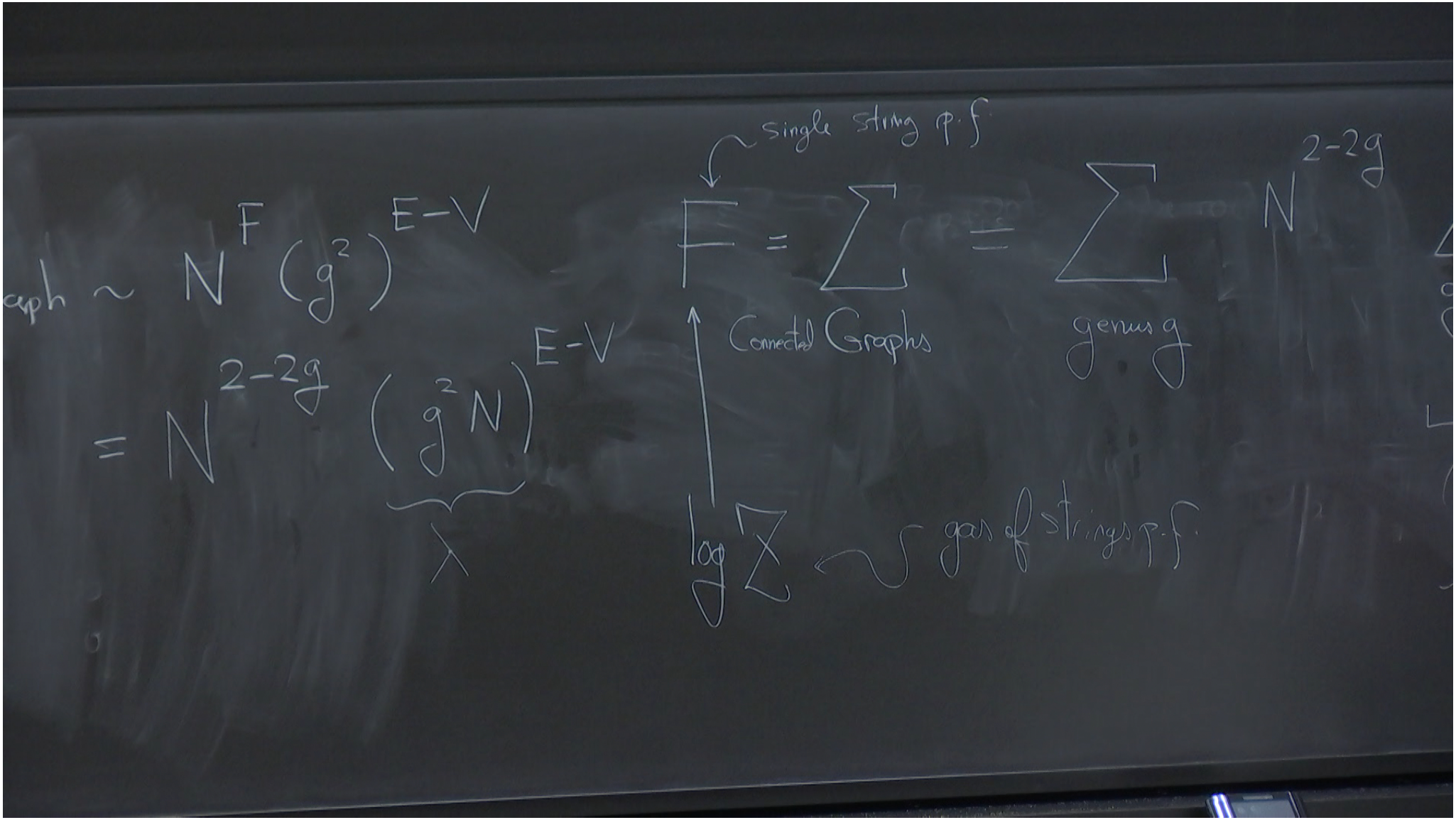
Exercise $=0$ ✓

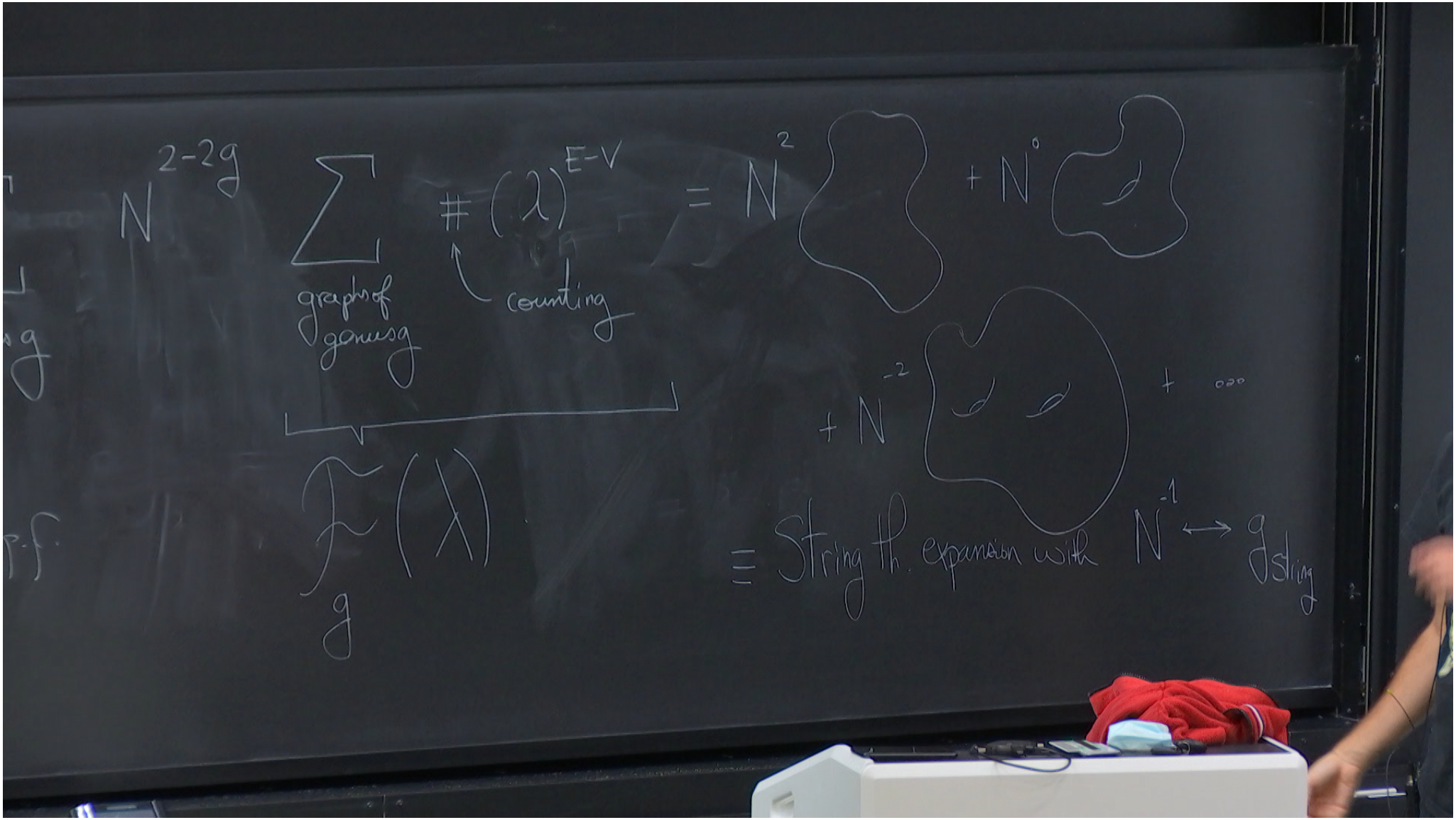
Theorem ⟶ planar graphs have more faces/loops ⟹ Dominate.

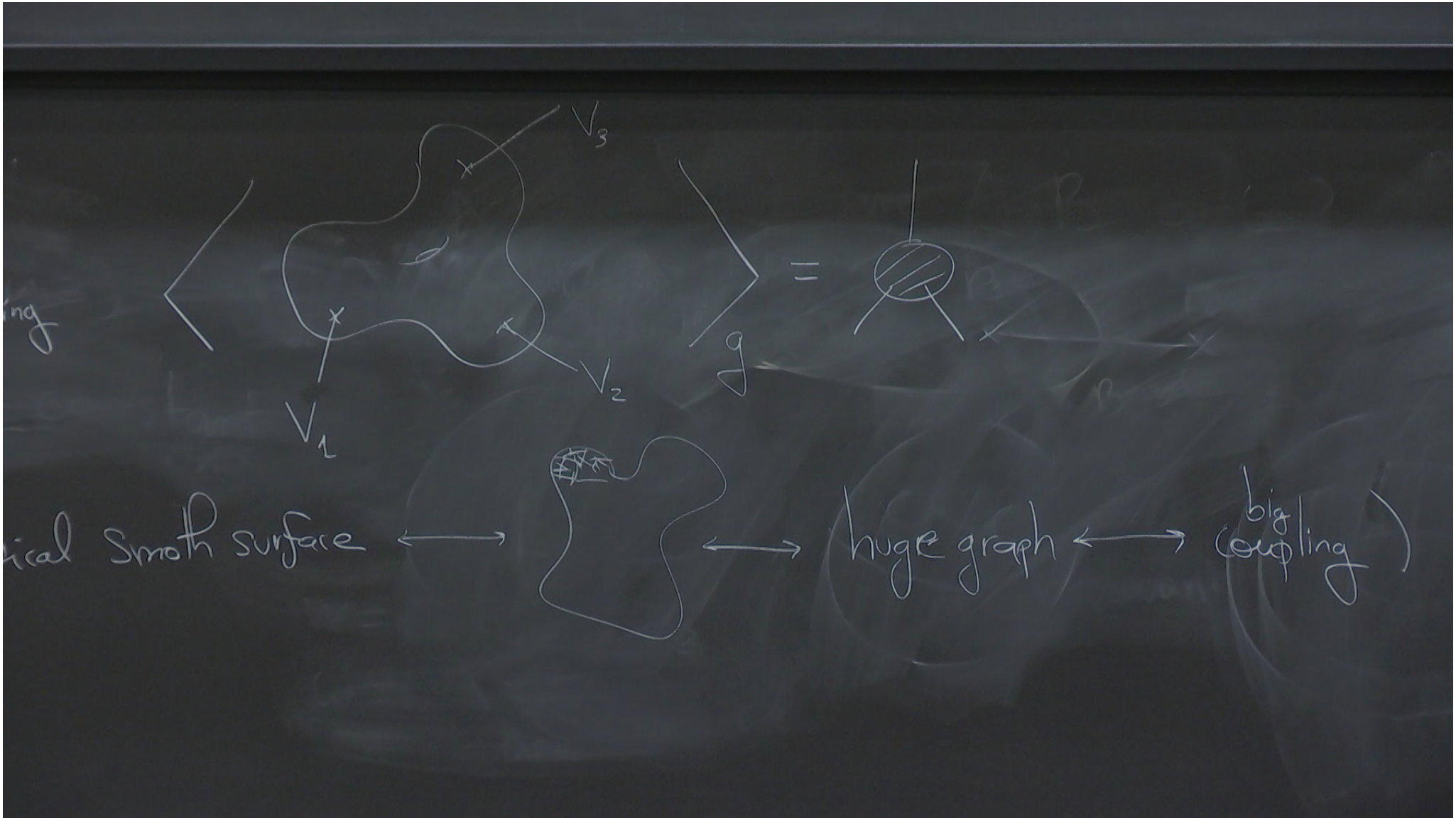

$\sum_{\text{Graphs}} = \sum_{\text{genus } g} N^{2-2g} \sum_{\text{graphs of genus } g} \#(\lambda)^{E-V}$
counting
$\tilde{\mathcal{F}}_g(\lambda)$


Graph ~ $N^{F}(g^2)^{E-V}$
$\sum$
Graphs
$= N^{2-2g}(g^2N)^{E-V}$
λ


1 MATRIX MODEL
$Z \equiv \int \mathcal{D}M \; e^{-\frac{1}{g^2}\mathrm{tr}\left(\frac{1}{2}M^2 + t_3 M^3 + t_4 M^4 + \cdots\right)}$
Partition function
$\left\langle \prod_{n=3}^{\infty} \left(\mathrm{tr}\, M^n\right)^{m_n} \right\rangle_0$

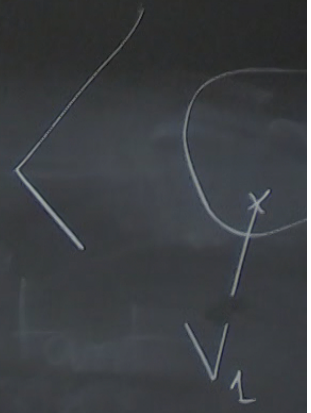

single string p.f.
$F = \sum = \sum N^{2-2g}$
Connected Graphs
genus g
$\log Z$ ~ gas of strings p.f.
$\sim N^{F} (g^2)^{E-V}$
$= N^{2-2g} (g^2 N)^{E-V}$
χ


N^{2-2g}
$\sum$
graphs of genus g
$\#(\lambda)^{E-V}$
counting
$= N^2$
$+ N^0$
$+ N^{-2}$
$+ \cdots$
$\mathcal{F}_g(\lambda)$
$\equiv$ String th. expansion with $N^{-1} \leftrightarrow g_{string}$


V_3
V_2
V_1
=
g
ical Smoth surface
huge graph
big
coupling

Tentative dictionary:

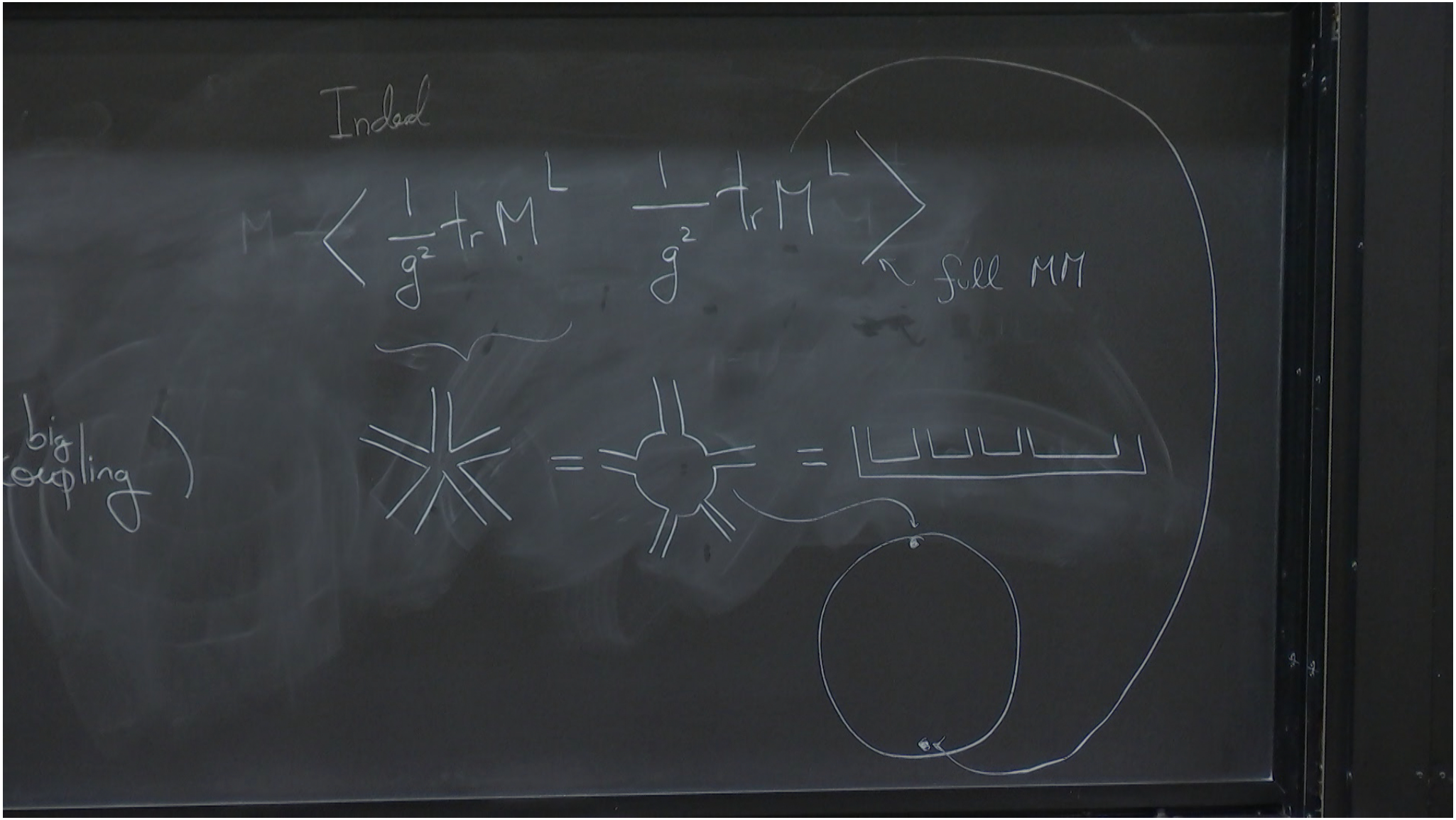
$\frac{1}{N}$ $\longleftrightarrow$ g_s , string coupling

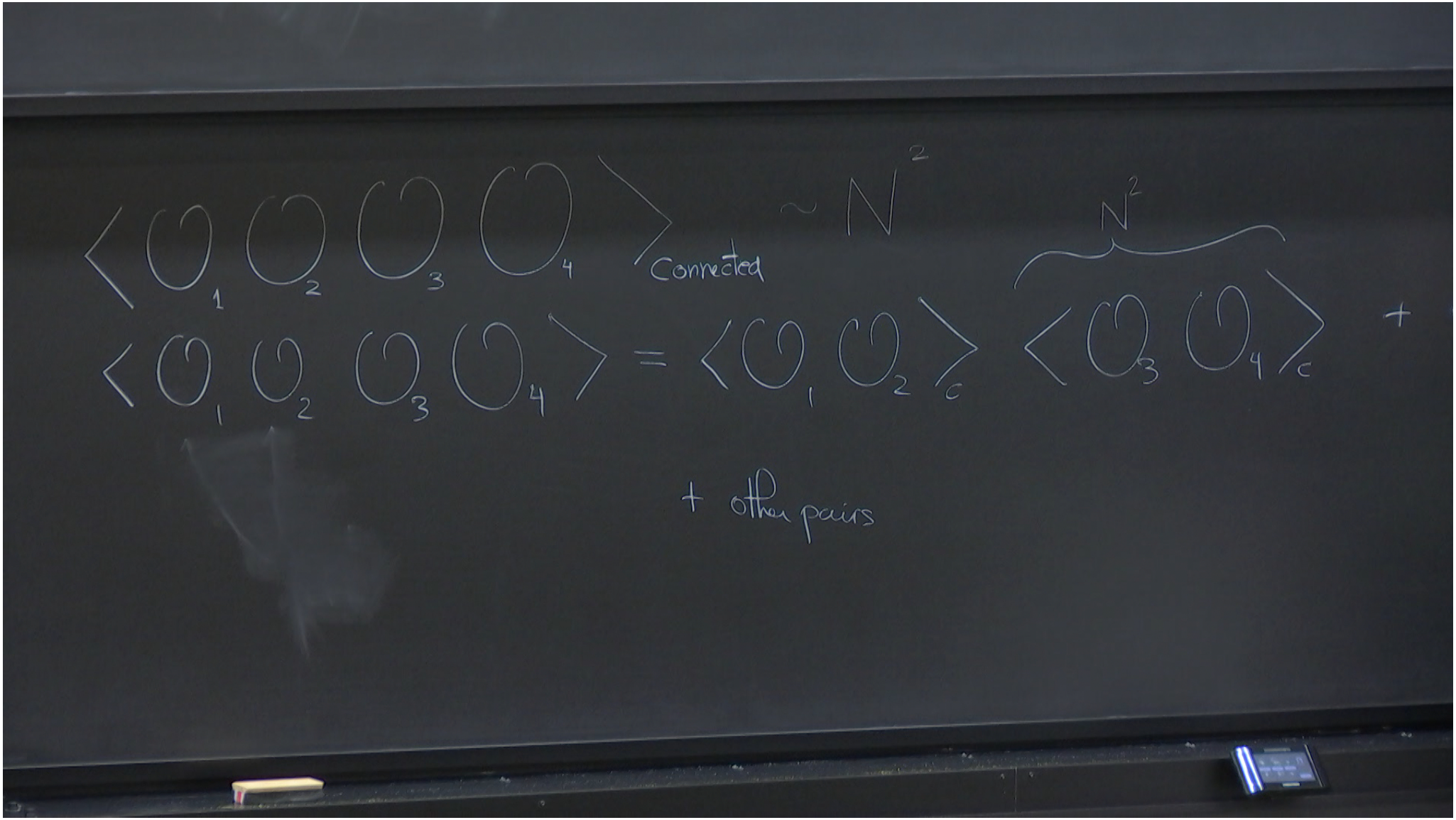


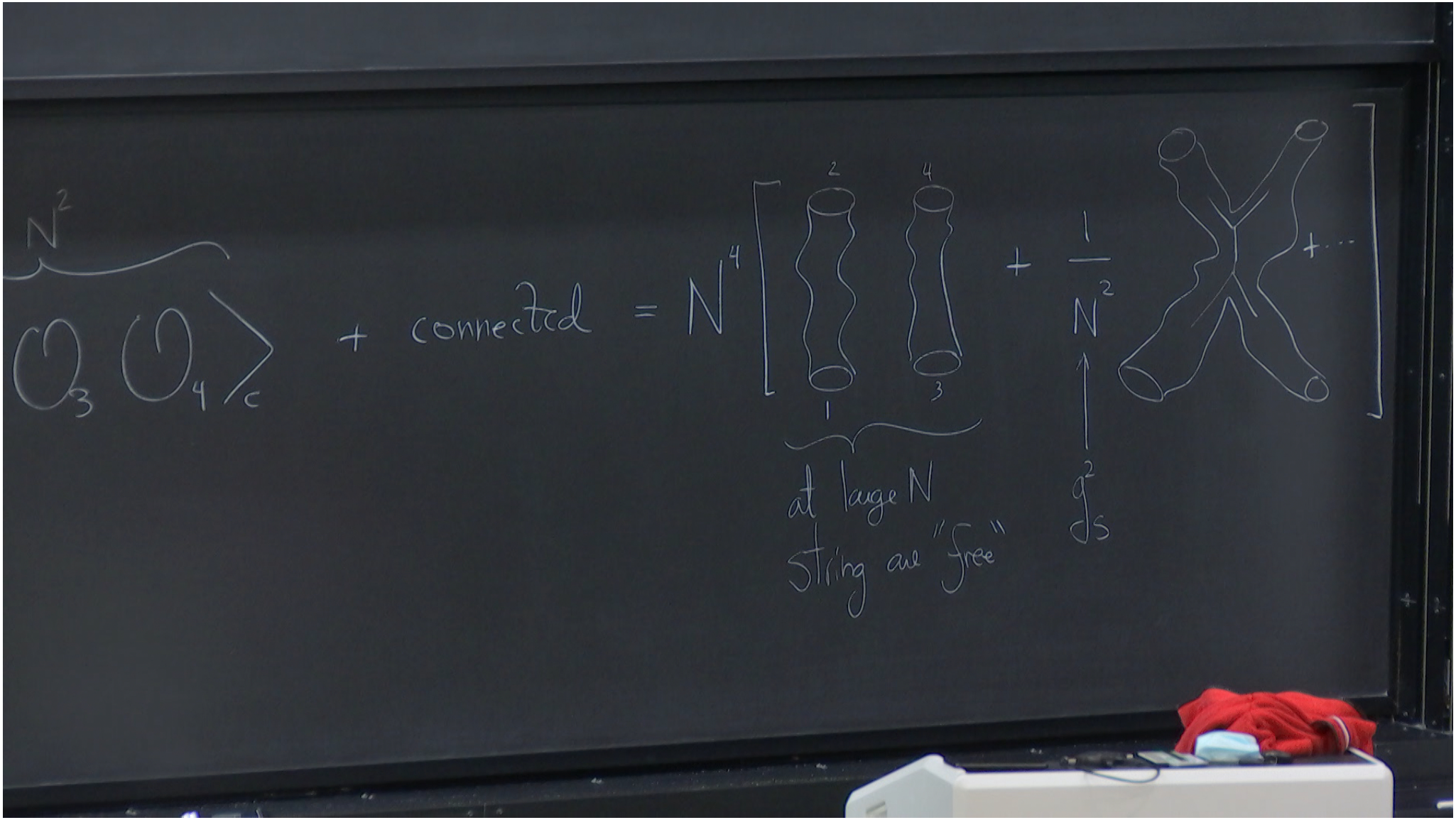
$(\lambda = g^2 N)^{\#}$ $\longleftrightarrow$ string tension T

(large $T \rightarrow$ classical smoth surface

Local op like $\mathrm{tr}\, M^L$ $\longleftrightarrow$ V , vertex op creating string

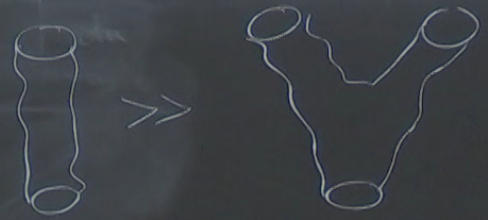

Indeed
$M \sim \left\langle \frac{1}{g^2} \mathrm{tr} M^L \quad \frac{1}{g^2} \mathrm{tr} M^L \right\rangle$
full MM
big coupling


$\langle \mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3 \mathcal{O}_4 \rangle_{\text{connected}} \sim N^2$
N^2
$\langle \mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3 \mathcal{O}_4 \rangle = \langle \mathcal{O}_1 \mathcal{O}_2 \rangle_c \langle \mathcal{O}_3 \mathcal{O}_4 \rangle_c +$
+ other pairs

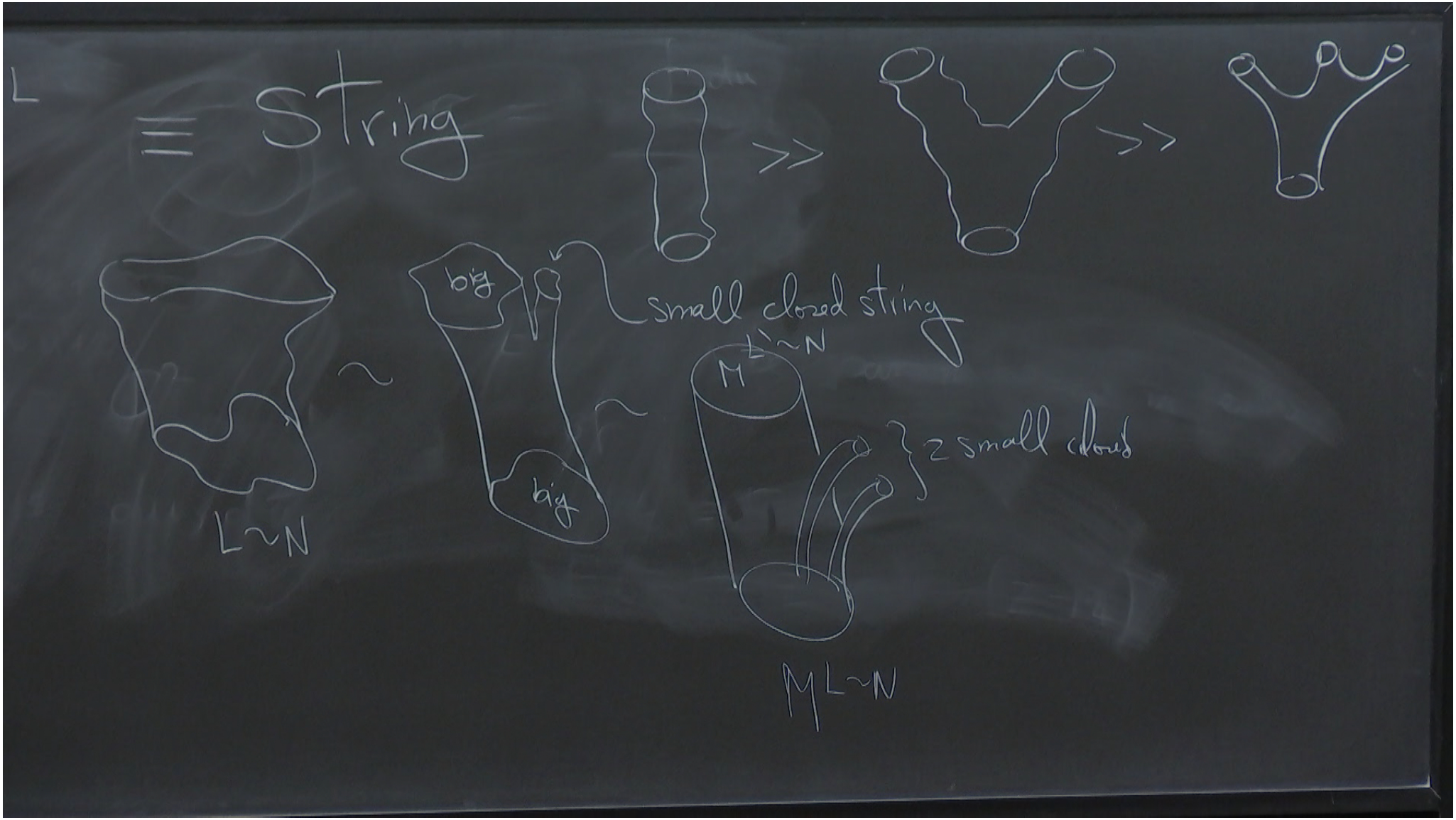

N^2
$\langle O_3 O_4 \rangle_c$ + connected = N^4
2
4
1
3
at large N
string are "free"
+ $\frac{1}{N^2}$
g_s^2
+ ...

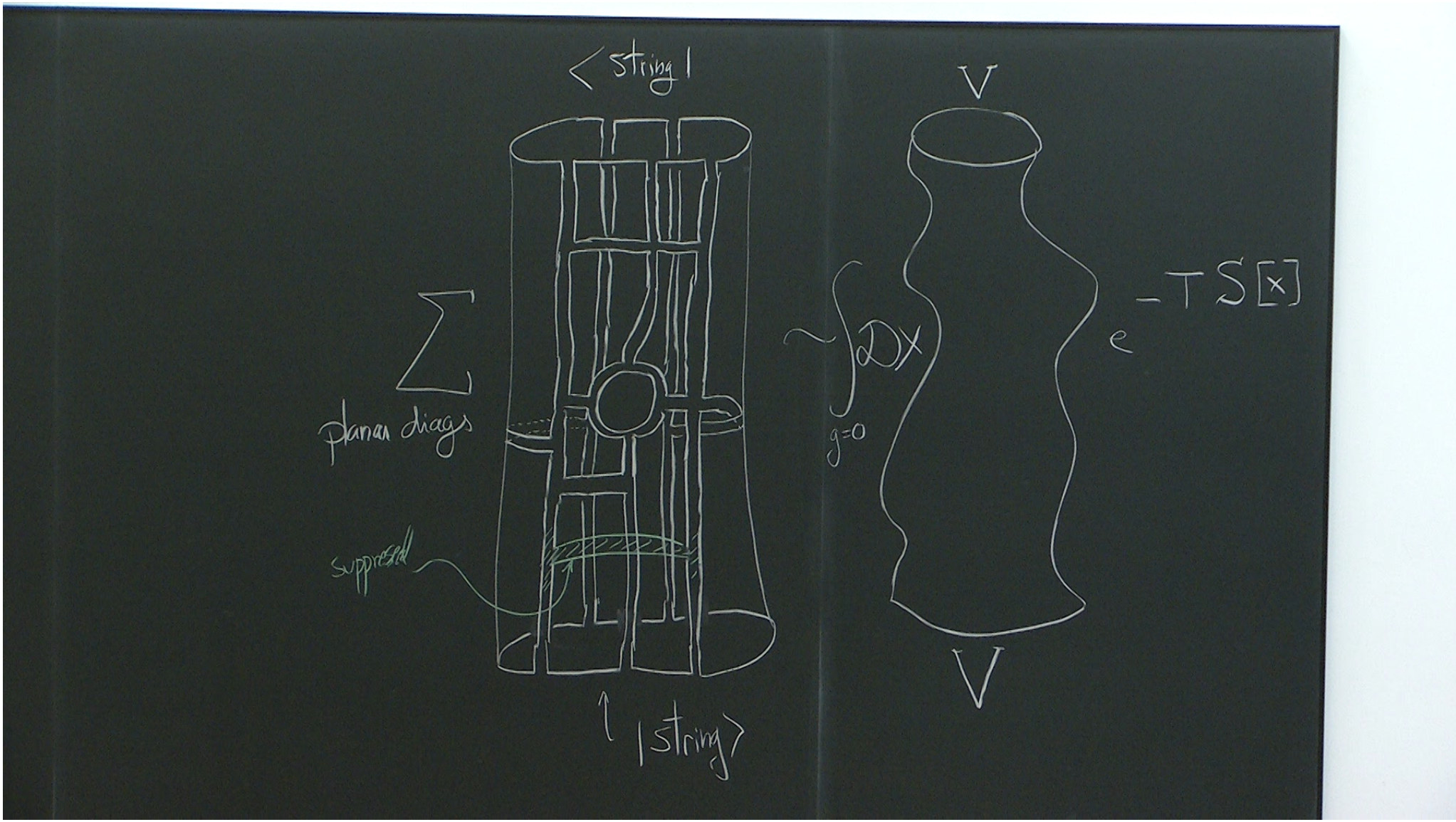
$$\frac{\langle OOO \rangle_c}{\langle OO \rangle_c^{3/2}} \sim \frac{N^2}{(N^2)^{3/2}} \sim \frac{1}{N}$$



$L = O(1)$ ⇝ $\mathrm{tr}\, M^L \equiv$ string 

$L = O(N)$ ⇝ higher genus matter


L
≡ string
big
big
small closed string
$L' \sim N$
$M^{L' \sim N}$
2 small clouds
$L \sim N$
$M^{L \sim N}$


< string |
V
Σ
planar diags
suppressed
$\sim \int_{g=0} \mathcal{D}x$
$e^{-TS[x]}$
| string >
V