Title: AdS/CFT 2021/2022

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Collection: AdS/CFT 2021/2022

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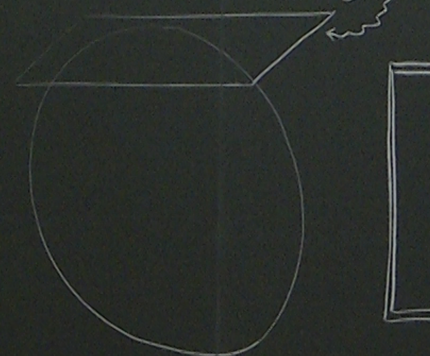
$$S = \underbrace{\frac{1}{2e_0^2}}_{\text{coupling}}\int d^2x\,(\partial\vec{n})^2 + \underbrace{\lambda(\vec{n}^2-1)}_{\vec{n}\text{ is in a }N-1\text{ sphere}} = \int d^2x\left[\underbrace{\frac{\partial\vec{\pi}}{2}^2}_{N-1\text{ massless fields}} + e_0^2\,(\vec{\pi}\cdot\partial^\mu\vec{\pi})^2 + e_0^4\;\vec{\pi}^2\,(\vec{\pi}\cdot\partial\vec{\pi})^2 + \dots\right]$$

$$\vec{n} = \left(\sqrt{1-\vec{\pi}^2},\ \vec{\pi}\right)$$

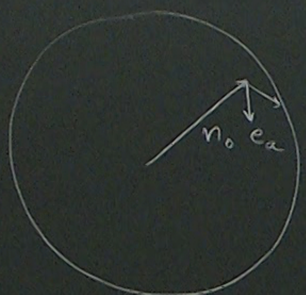
(plus simple rescaling of $\vec{\pi}$)



$$\boxed{\beta \equiv \frac{\partial\tilde{e}_0^2}{\partial\log\tilde{\lambda}} = -\frac{N-2}{2\pi}\,\tilde{e}_0^4}$$

RG

$$\vec{n} = \vec{n}_0 \sqrt{1-\vec{\phi}^2} + \sum_a \phi_a \vec{e}_a \sim S = S[n_0] + \text{Quadractic equation for fast fields } \vec{\phi}$$



Ⓐ $\quad$ $\tilde{\text{Ⓐ}}$

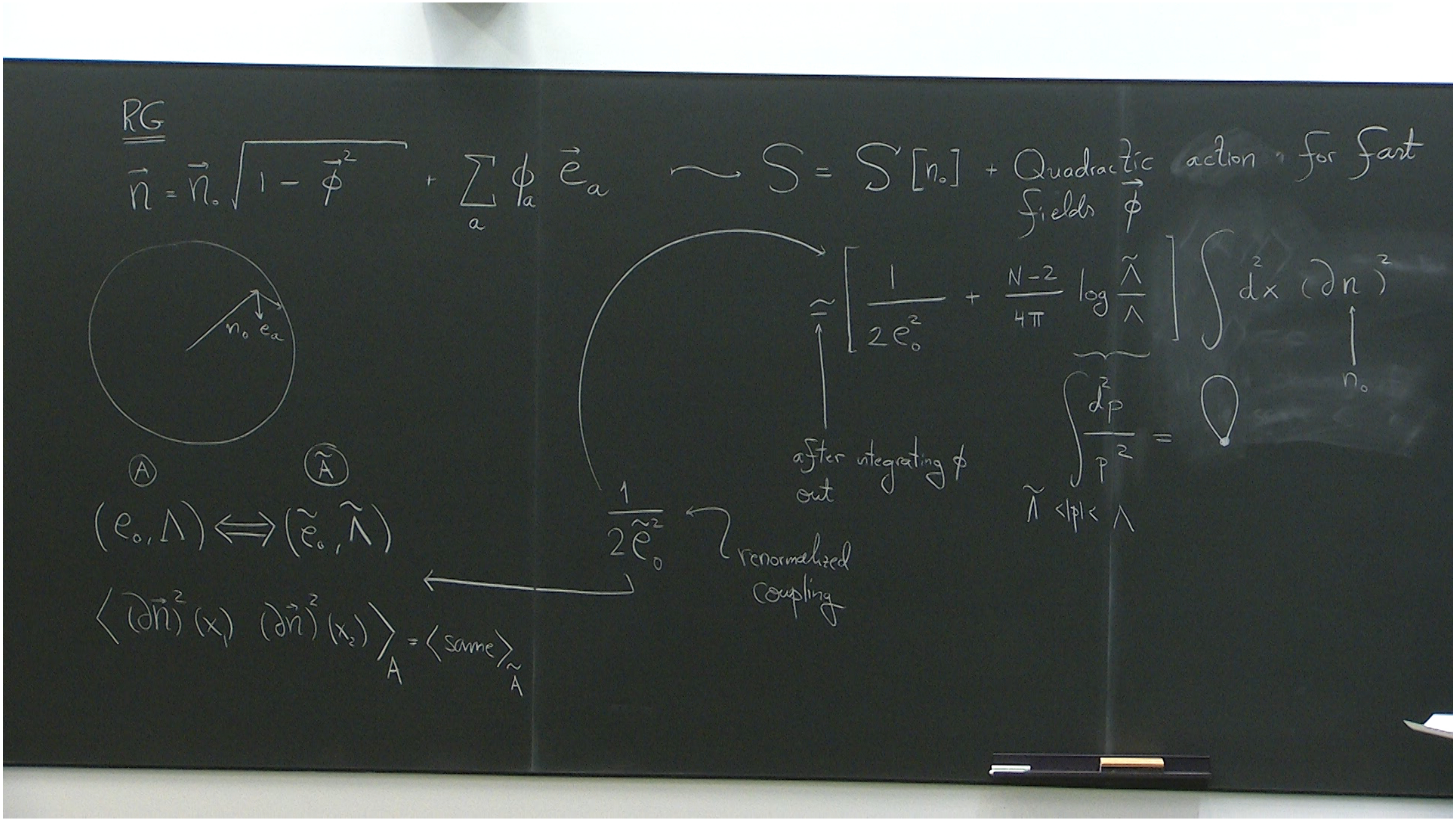
$$(e_0, \Lambda) \Longleftrightarrow (\tilde{e}_0, \tilde{\Lambda})$$

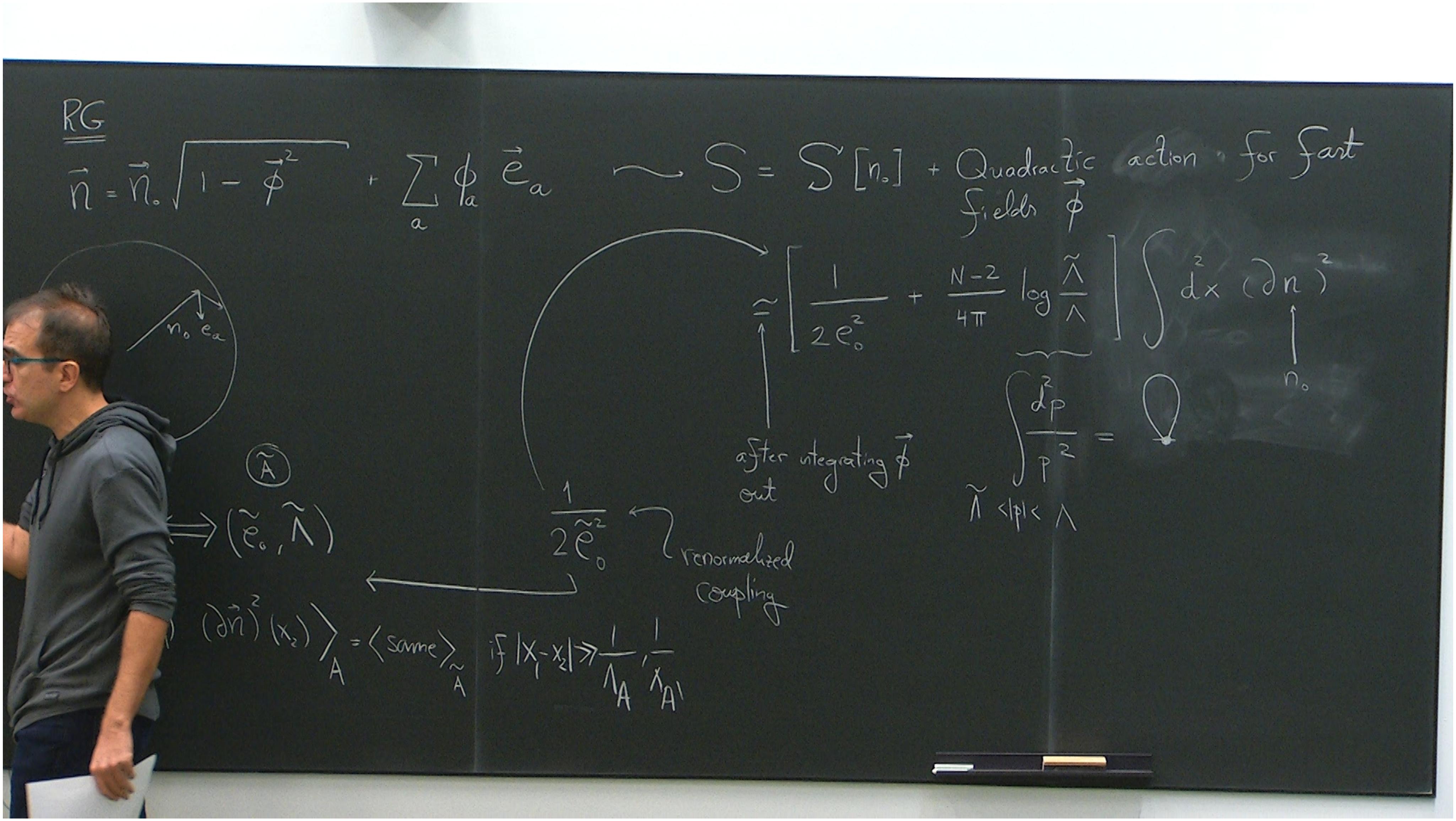
$$\left\langle (\partial\vec{n})^2(x_1)\ (\partial\vec{n})^2(x_2) \right\rangle_{\Lambda} = \left\langle \text{same} \right\rangle_{\tilde{\Lambda}}$$

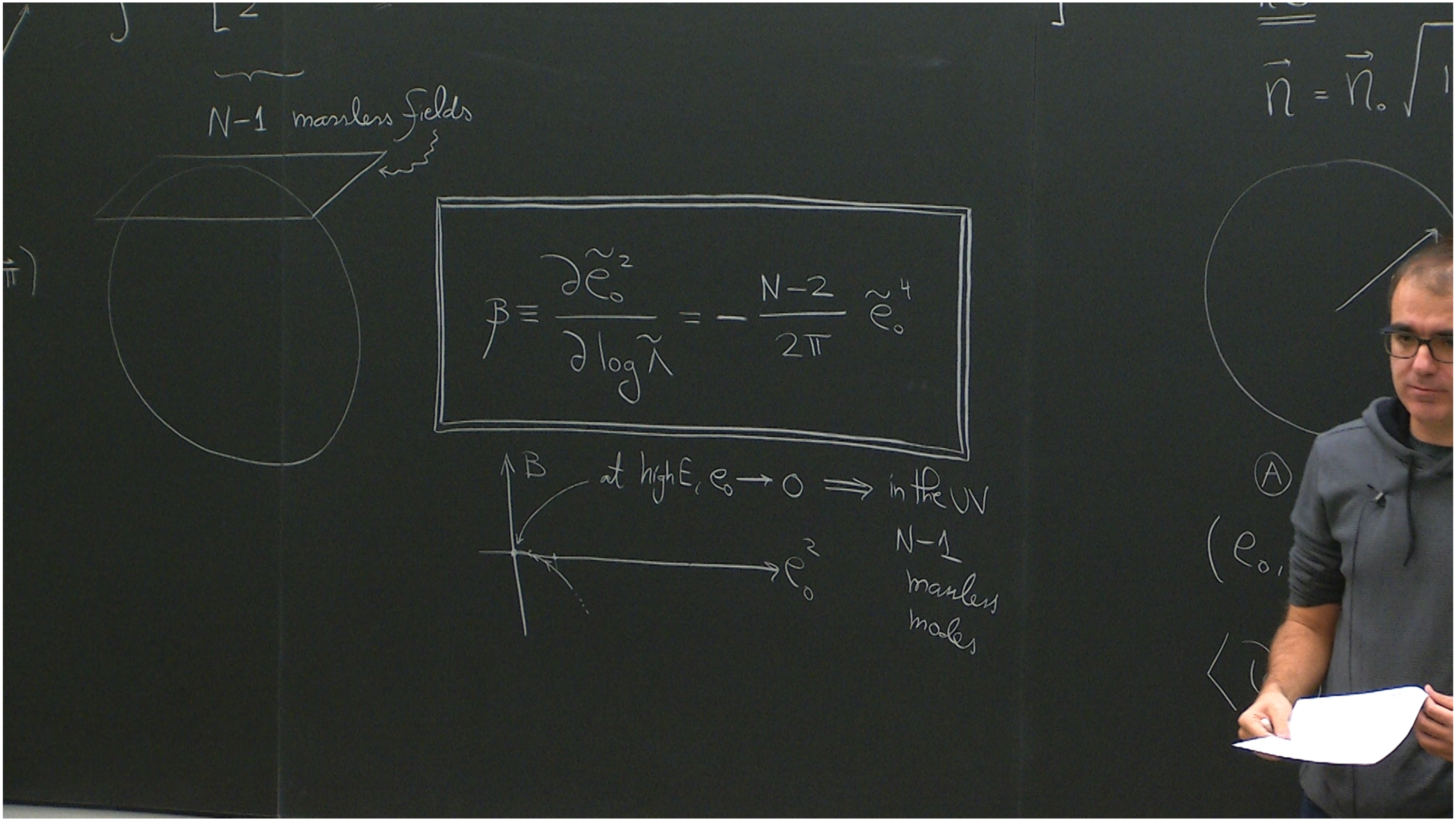
$$\simeq \left[\frac{1}{2e_0^2} + \underbrace{\frac{N-2}{4\pi} \log \frac{\tilde{\Lambda}}{\Lambda}}_{\int_{\tilde{\Lambda}<|p|<\Lambda} \frac{d^2p}{p^2}} \right] \int d^2x\, (\partial \underset{n_0}{n})^2$$

after integrating ϕ out

$$\frac{1}{2\tilde{e}_0^2}$$ renormalized coupling


RG
$\vec{n} = \vec{n}_0 \sqrt{1 - \vec{\phi}^2} + \sum_a \phi_a \vec{e}_a \sim S = S[n_0]$ + Quadractic action for fast fields $\vec{\phi}$
n_0 e_a
Ⓐ Ⓐ̃
$(e_0, \Lambda) \Longleftrightarrow (\tilde{e}_0, \tilde{\Lambda})$
$\left\langle (\partial\vec{n})^2(x_1)\ (\partial\vec{n})^2(x_2) \right\rangle_A = \langle \text{same} \rangle_{\tilde{A}}$
$\frac{1}{2\tilde{e}_0^2}$ renormalized coupling
$\simeq \left[\frac{1}{2e_0^2} + \frac{N-2}{4\pi} \log \frac{\tilde{\Lambda}}{\Lambda} \right] \int d^2x\, (\partial n)^2$
after integrating ϕ out
n_0
$\int_{\tilde{\Lambda} < |p| < \Lambda} \frac{d^2p}{p^2} =$


RG
n = n₀ √(1 − φ²) + Σ_a φ_a e_a ∼ S = S[n₀] + Quadractic action for fast fields φ
n₀ e_a
≃ [1/(2e₀²) + (N−2)/(4π) log (Λ̃/Λ)] ∫ d²x (∂n)²
n₀
∫ d²p/p² =
Λ̃ <|p|< Λ
after integrating φ out
Ã
⟹ (ẽ₀, Λ̃)
1/(2ẽ₀²)
renormalized coupling
(∂n)² (x₂) ⟩_Λ = ⟨ same ⟩_Λ̃ if |x₁ − x₂| ≫ 1/Λ, 1/Λ'


N−1 massless fields
$\beta \equiv \frac{\partial \tilde{e}_0^2}{\partial \log \tilde{\lambda}} = -\frac{N-2}{2\pi}\tilde{e}_0^4$
B
at high E, $e_0 \to 0 \Rightarrow$ in the UV
N−1
massless
modes
e_0^2
A

$$+\sum_a \phi_a \vec{e}_a \rightsquigarrow S = S[n_0] + \text{Quadractic action for fast fields } \vec{\phi}$$

$$\simeq \left[\frac{1}{2e_0^2} + \frac{N-2}{4\pi} \log \frac{\tilde{\Lambda}}{\Lambda} \right] \int d^2x \, (\partial n_0)^2$$

after integrating $\vec{\phi}$ out

$$\int_{\tilde{\Lambda} < |p| < \Lambda} \frac{d^2p}{p^2} =$$ 

$$\frac{1}{2\tilde{e}_0^2}$$ renormalized coupling

$\int$ out fast modes $\Longrightarrow e_0 \nearrow$

$S \Longleftrightarrow$ decreasing magnitude of $\vec{n}$

$= \langle \text{same} \rangle_{\tilde{A}}$ if $|x_1 - x_2| \gg \frac{1}{\Lambda_A}, \frac{1}{\tilde{\Lambda}_{A'}}$

$$\sum_a \phi_a \vec{e}_a \rightsquigarrow S = S'[n_0] + \text{Quadratic action for fast fields } \vec{\phi}$$

$$\simeq \left[\frac{1}{2e_0^2} + \frac{N-2}{4\pi} \underbrace{\log \frac{\tilde{\Lambda}}{\Lambda}}_{\int \frac{d^2p}{p^2} = \bigcirc} \right] \int d^2x \, (\partial n)^2$$

n_0

$\tilde{\Lambda} < |p| < \Lambda$

after integrating $\vec{\phi}$ out

$$\frac{1}{2\tilde{e}_0^2}$$

renormalized coupling

$\int$ out Fast modes

$\Longrightarrow e_0 \nearrow$

$S \Longleftrightarrow$ ~~decreasing~~ magnitude of $\vec{n}$

$\sim$ smaller

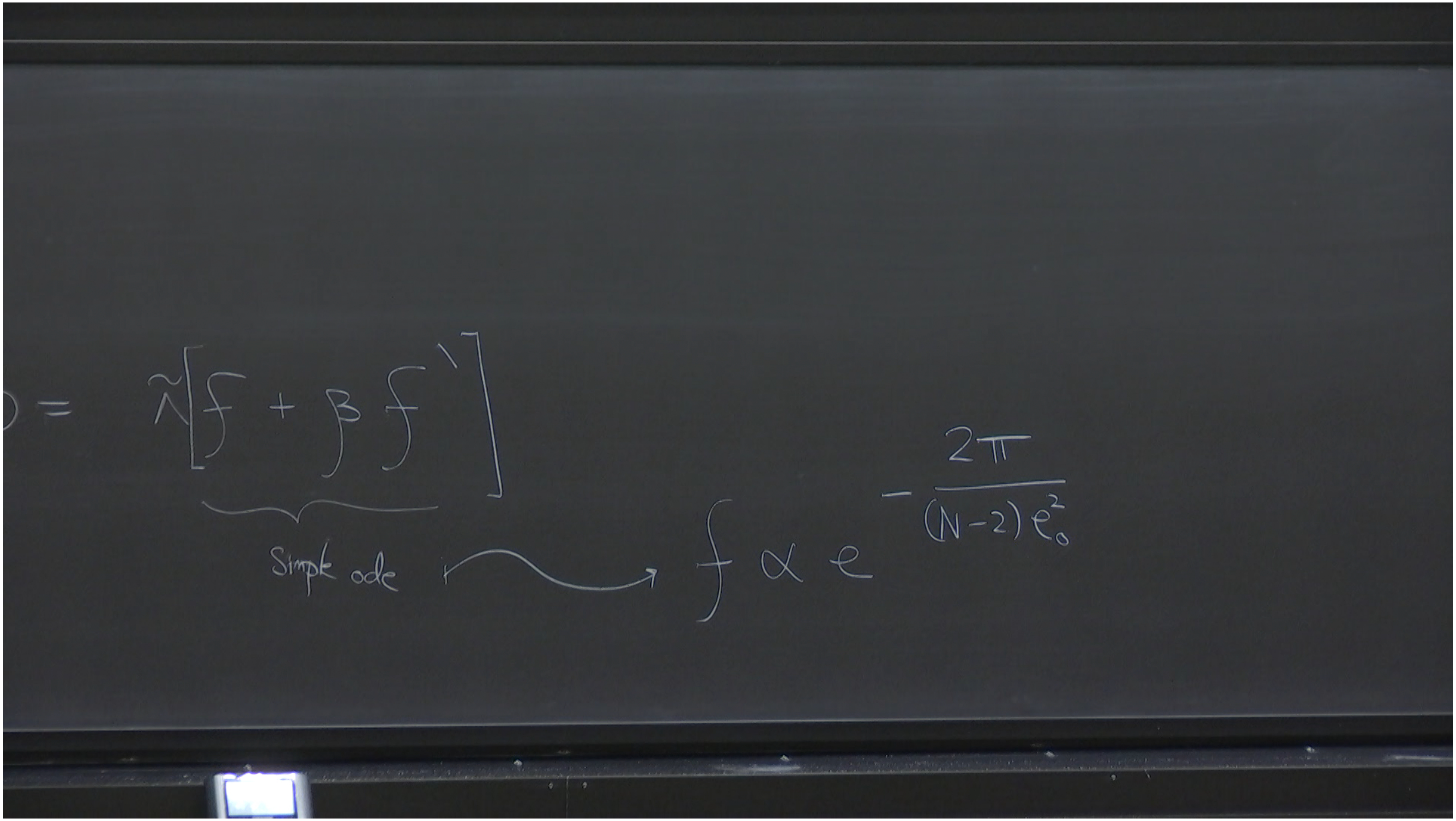
$me\rangle_{\tilde{A}}$ if $|x_1 - x_2| \gg \frac{1}{\Lambda_A}, \frac{1}{\Lambda_{A'}}$

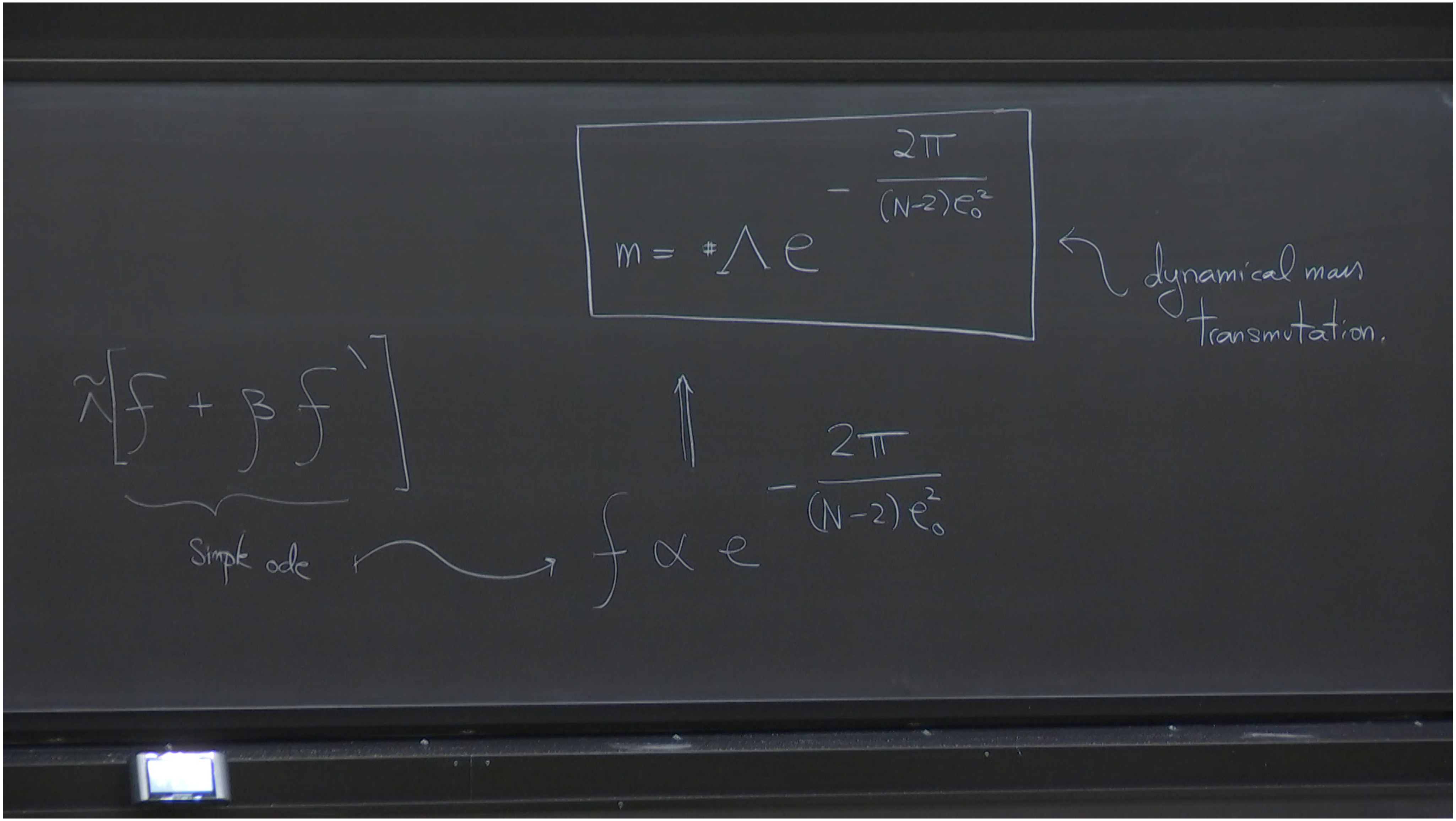
@ low E we have N particles which develop mass m.

$$m = ?$$

$$m = \Lambda f(e_0^2)$$
$$= \tilde{\Lambda} f(\tilde{e}_0^2)$$

$$\frac{d}{d\log\tilde{\Lambda}}\left(\tilde{\Lambda} f(\tilde{e}_0^2)\right) = 0 = \tilde{\Lambda}\Big[\underbrace{f + \beta f'}_{\text{Simple ode}}\Big]$$


$= \tilde{\lambda}[f + \beta f']$
Simple ode
$f \propto e^{-\frac{2\pi}{(N-2)e_0^2}}$



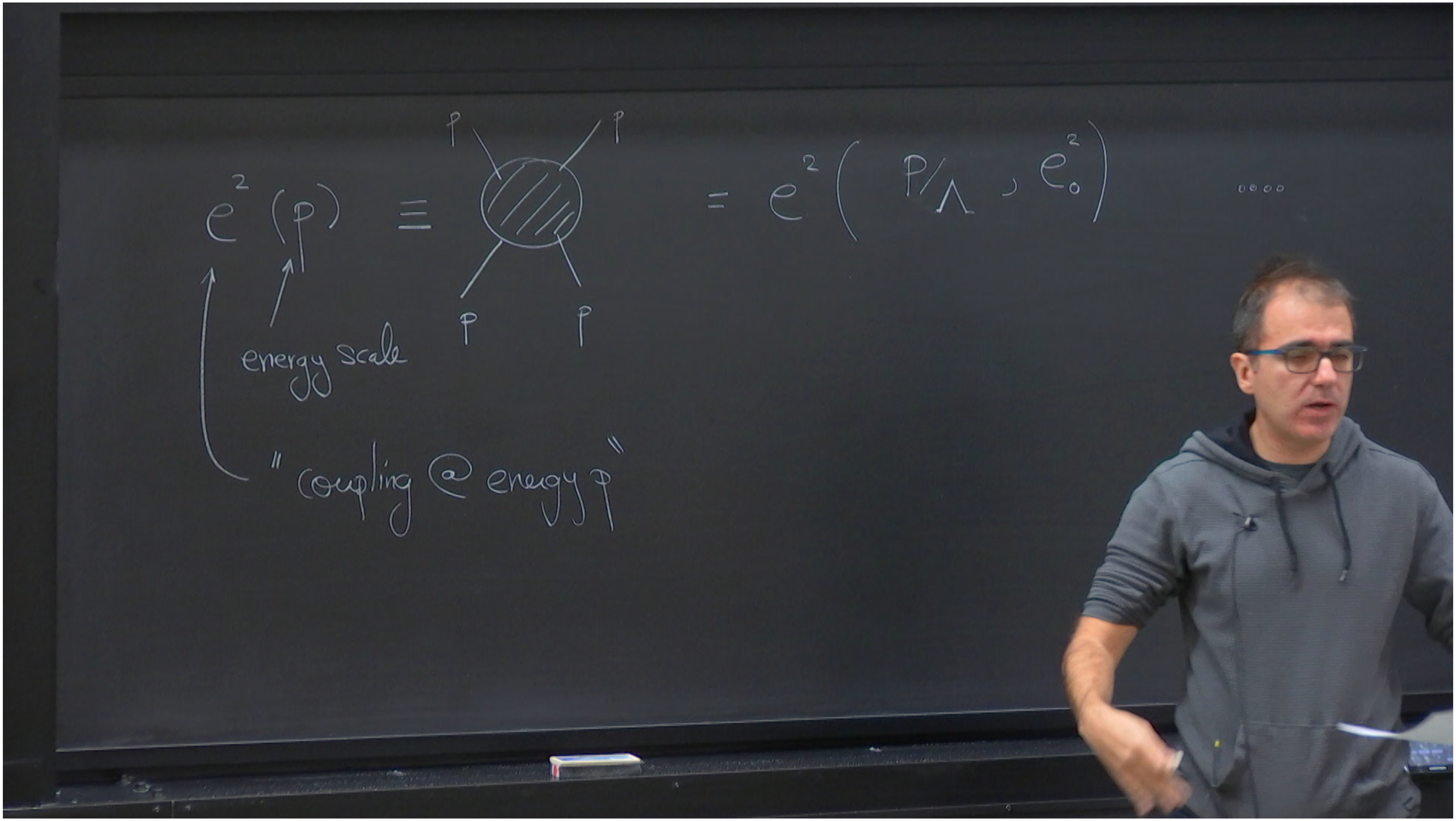
$m = \# \Lambda e^{-\frac{2\pi}{(N-2)e_0^2}}$
dynamical mass transmutation.
$\tilde{\lambda}[f + \beta f']$
Simple ode
$f \propto e^{-\frac{2\pi}{(N-2)e_0^2}}$

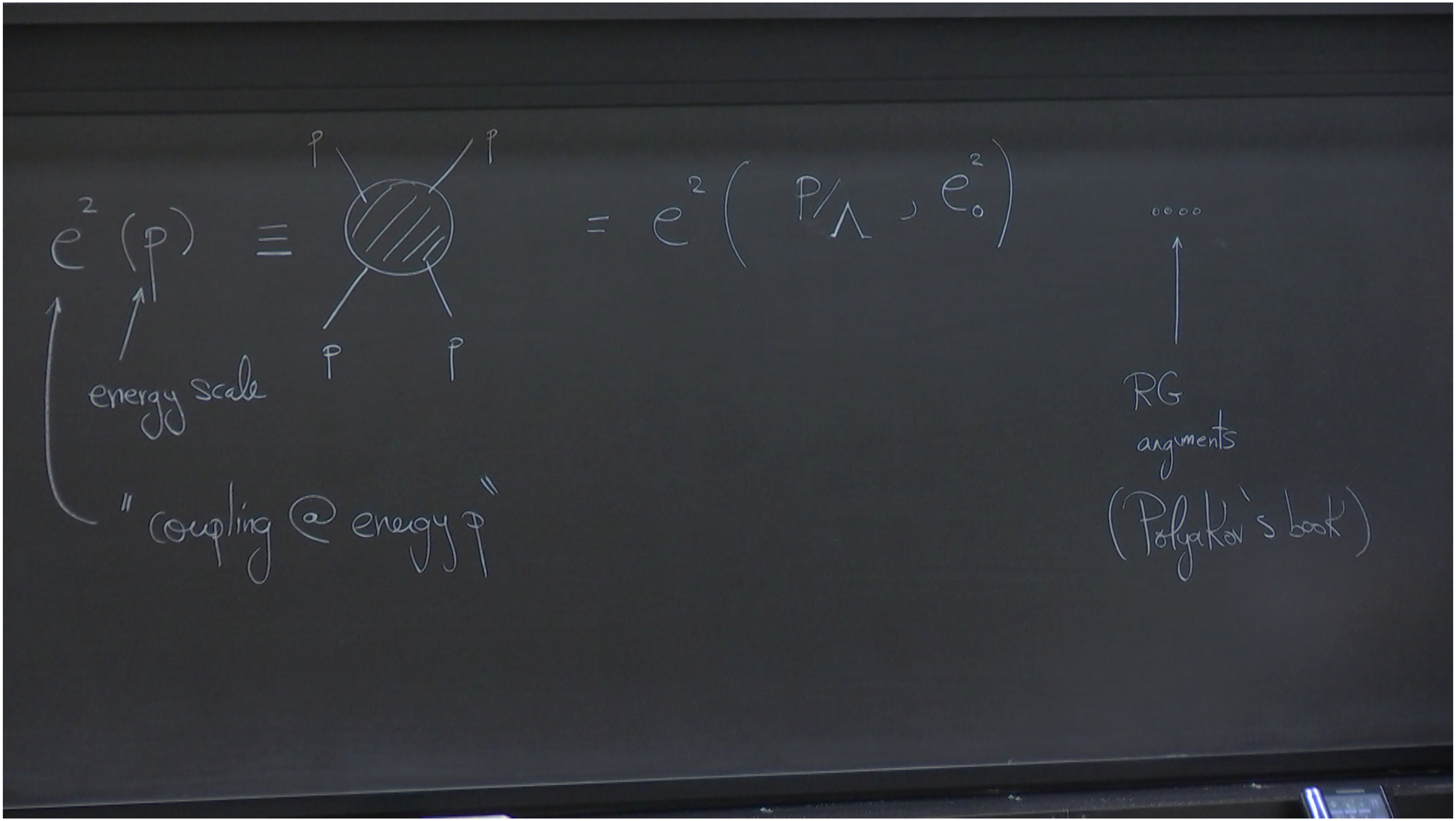
$$\boxed{m = \# \Lambda e^{-\frac{2\pi}{(N-2)e_0^2}}}$$

very small if e_0 is small

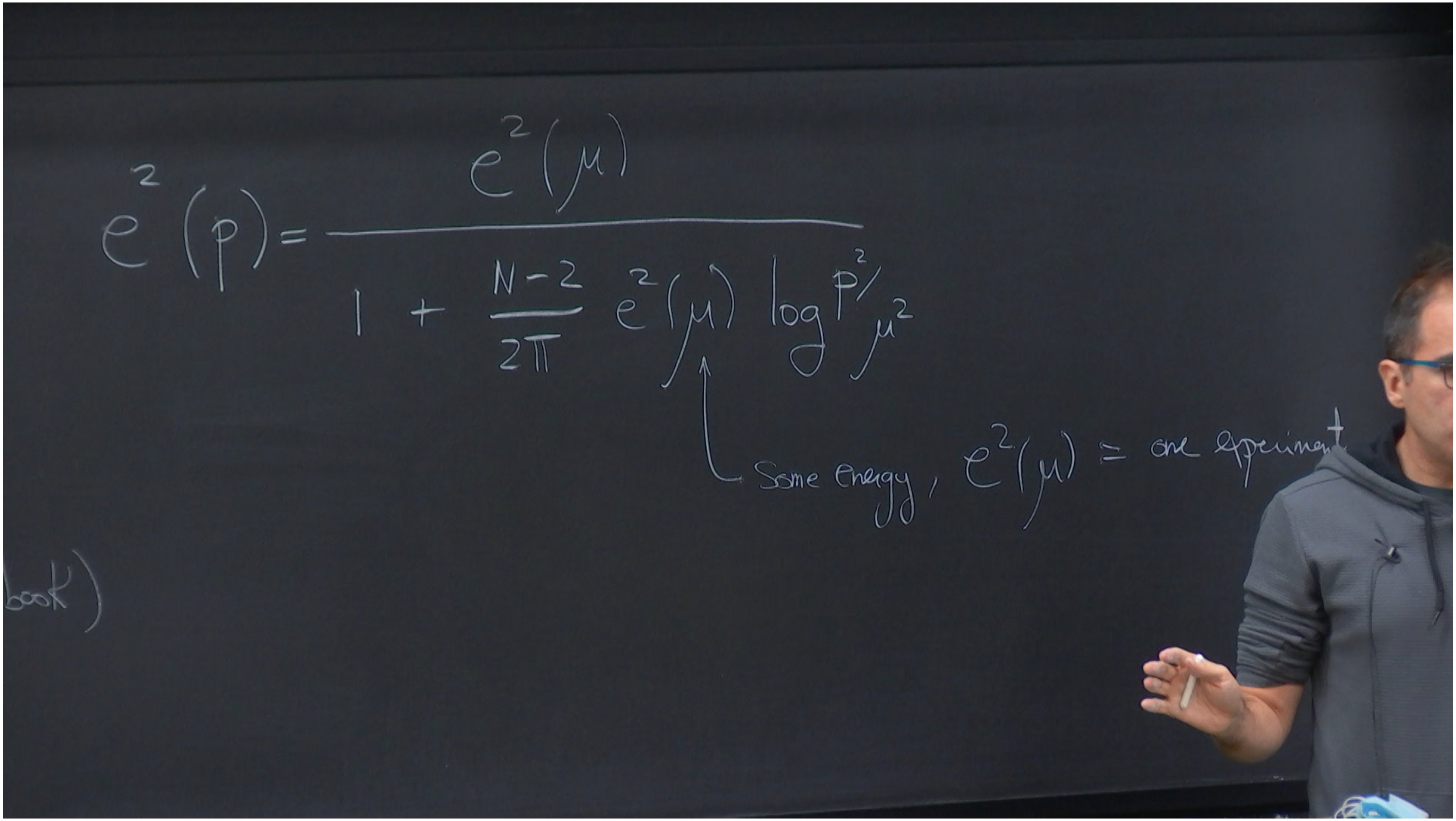
dynamical mass transmutation.

$$\underbrace{\tilde{\Lambda}\left[f + \beta f'\right]}_{\text{Simple ode}} \rightsquigarrow f \propto e^{-\frac{2\pi}{(N-2)e_0^2}}$$


$e^2(p) \equiv$
p p p p
$= e^2(p/\Lambda, e_0^2)$ ....
energy scale
"coupling @ energy p"



$e^2(p) \equiv$
$= e^2(p/\Lambda, e_0^2)$ ....
p
energy scale
"coupling @ energy p"
RG arguments
(Polyakov's book)



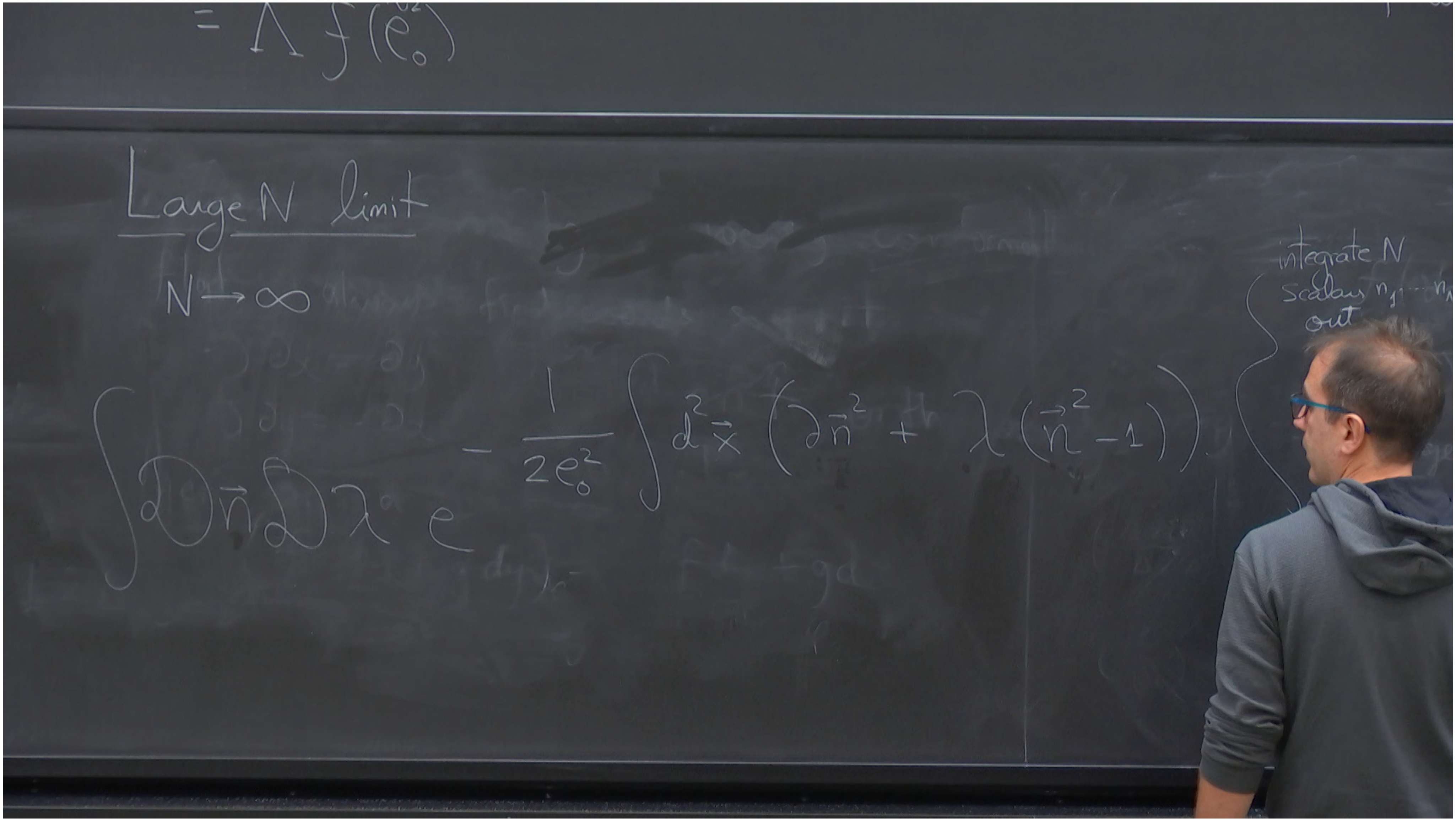
$e^2(p) = \frac{e^2(\mu)}{1 + \frac{N-2}{2\pi} e^2(\mu) \log p^2/\mu^2}$
Some energy, $e^2(\mu)$ = one experiment
book)

$$\boxed{e^2(p) = \frac{e^2(\mu)}{1 + \frac{N-2}{2\pi} e^2(\mu) \log p^2/\mu^2}}$$

(ok for small coupling)

some energy, $e^2(\mu)$ = one experiment

book)


$= \Lambda f(e_0^2)$
Large N limit
$N \to \infty$
$\int \mathcal{D}\vec{n}\,\mathcal{D}\lambda\, e^{-\frac{1}{2e_0^2}\int d^2\vec{x}\left((\partial\vec{n})^2 + \lambda(\vec{n}^2 - 1)\right)}$
integrate N scalars $n_1 \cdots n_N$ out

integrate N scalars $n_1, \dots, n_N$ out

$$\left(\vec{n}^2 - 1\right)\Big)$$

$$= \int \mathcal{D}\lambda \; e^{+\frac{1}{2e_0^2}\int d^2x \, \lambda}$$

integrate N scalars $n_1 \cdots n_N$ out

If N is large we can use Saddle point!

$$= \int \mathcal{D}\lambda \; e^{+\frac{1}{2e_0^2}\int d^2x\, \lambda \;+\; \frac{N}{2}\log\det\left(-\Box + \lambda_f\right)}$$

integrate N scalars $n_1 \cdots n_N$ out

If N is large we can use Saddle point !

$$= \int \mathcal{D}\lambda \; e^{+\frac{1}{2e_0^2}\int d^2x\,\lambda \; + \; \frac{N}{2}\underbrace{\log\det\left(-\Box+\lambda\right)}_{\mathrm{tr}\,\log\left(-\Box+\lambda\right)}}$$

$$0 = \frac{\delta}{\delta\lambda} S_{\mathrm{eff}}[\lambda]$$

$$0 = \frac{1}{2e_0^2} + \frac{N}{2}\underbrace{\mathrm{tr}\,\frac{1}{\lambda-\Box}}_{\int\frac{d^2p}{(2\pi)^2}\frac{1}{p^2+\lambda}}$$

$$0 = \frac{1}{2e_0^2} + \frac{N}{2}\underbrace{\int \frac{d^2p}{(2\pi)^2}\,\frac{1}{p^2+\lambda}}_{\frac{1}{2}\frac{1}{2\pi}\underbrace{\int_0^{\Lambda} d\Omega\,\frac{2\Omega}{\Omega^2+\lambda}}_{\simeq \log\left[\Lambda^2/\lambda\right]}}$$

, λ is given by

$$\frac{1}{2e_0^2} + \frac{N}{8\pi}\log \Lambda^2/\lambda = 0$$

$$\Rightarrow \boxed{\lambda^* = \Lambda^2 e^{-\frac{4\pi}{N e_0^2}}}$$

@ large N

the Lagrange mult. field

$$\lambda = \underbrace{\langle \lambda \rangle}_{\lambda^*} + O$$

suppressed by $N \to \infty$

@ large N

the Lagrange mult. field

$$\underbrace{\lambda = \langle\lambda\rangle}_{\lambda^*} + O$$

suppressed by $N \to \infty$

$$S \simeq \int (\partial \vec{n})^2 + m^2 \vec{n}^2 \quad \text{where } m = \sqrt{\lambda_*} = e^{-\frac{2\pi}{N e_0^2}} \Lambda$$

@ large N

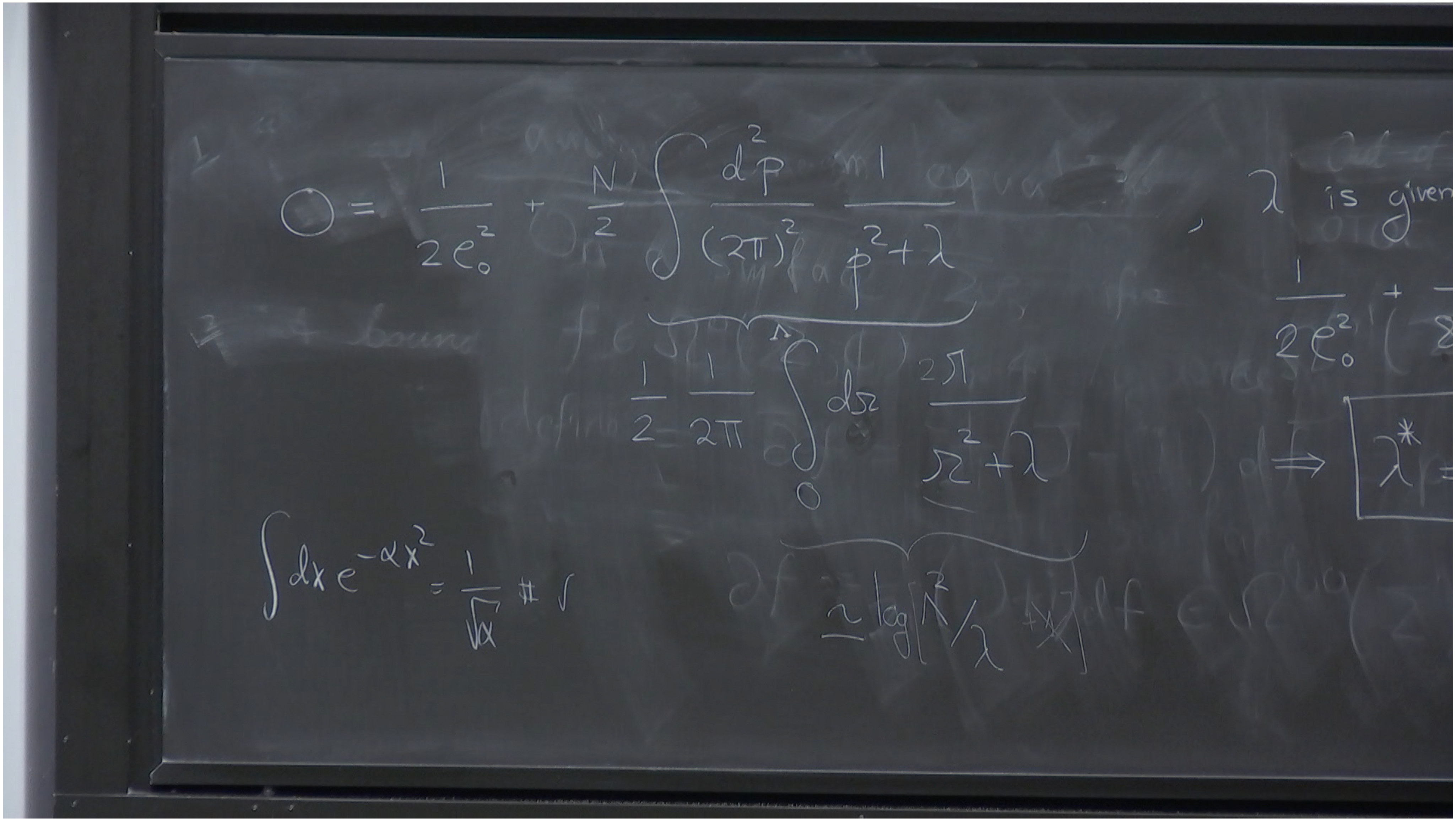
the Lagrange mult. field

$$\lambda = \underbrace{\langle \lambda \rangle}_{\lambda^*} + O$$

suppressed by $N \to \infty$

N massive fields with m

$$S \simeq \int \partial \vec{n}^2 + m^2 \vec{n}^2 \quad \text{where } m = \sqrt{\lambda_*} = e^{-\frac{2\pi}{N e_0^2}} \Lambda$$



$\bigcirc = \frac{1}{2e_0^2} + \frac{N}{2}\int \frac{d^2p}{(2\pi)^2}\frac{1}{p^2+\lambda}$
λ is given
$\frac{1}{2}\frac{1}{2\pi}\int_0^\Lambda d\Omega \frac{2\pi}{\Omega^2+\lambda}$
$\frac{1}{2e_0^2} +$
$\Rightarrow \boxed{\lambda^*}$
$\int dx\, e^{-\alpha x^2} = \frac{1}{\sqrt{\alpha}}$
$\sim \log[\Lambda^2/\lambda]$

$$\frac{1}{2e_0^2}\int d^2\vec{x}\left(\partial\vec{n}^2 + \lambda(\vec{n}^2-1)\right)$$

we can use Saddle

$$= \int \mathcal{D}\lambda\; e^{+\frac{1}{2e_0^2}\int d^2x\,\lambda \;-\; \frac{N}{2}\underbrace{\log\det\left(-\Box+\lambda\right)}_{\mathrm{tr}\log(-\Box+\lambda)}}$$

$$\frac{1}{p^2+\lambda}$$

$$d\Omega\,\frac{2\pi}{\Omega^2+\lambda}$$

$$\sim \log \Lambda^2/\lambda$$

λ is given by

$$\frac{1}{2e_0^2} - \frac{N}{8\pi}\log \Lambda^2/\lambda = 0$$

$$\Rightarrow \boxed{\lambda^* = \Lambda^2 e^{-\frac{4\pi}{Ne_0^2}}}$$

@ large N

the Lagrange mult. field is not

$$\lambda = \underbrace{\langle\lambda\rangle}_{\lambda^*} + \bigcirc$$

suppressed by $N\to\infty$

$$S \simeq \partial\vec{n}^2 + m^2\,\vec{n}^2 \quad \text{where } m =$$

UV

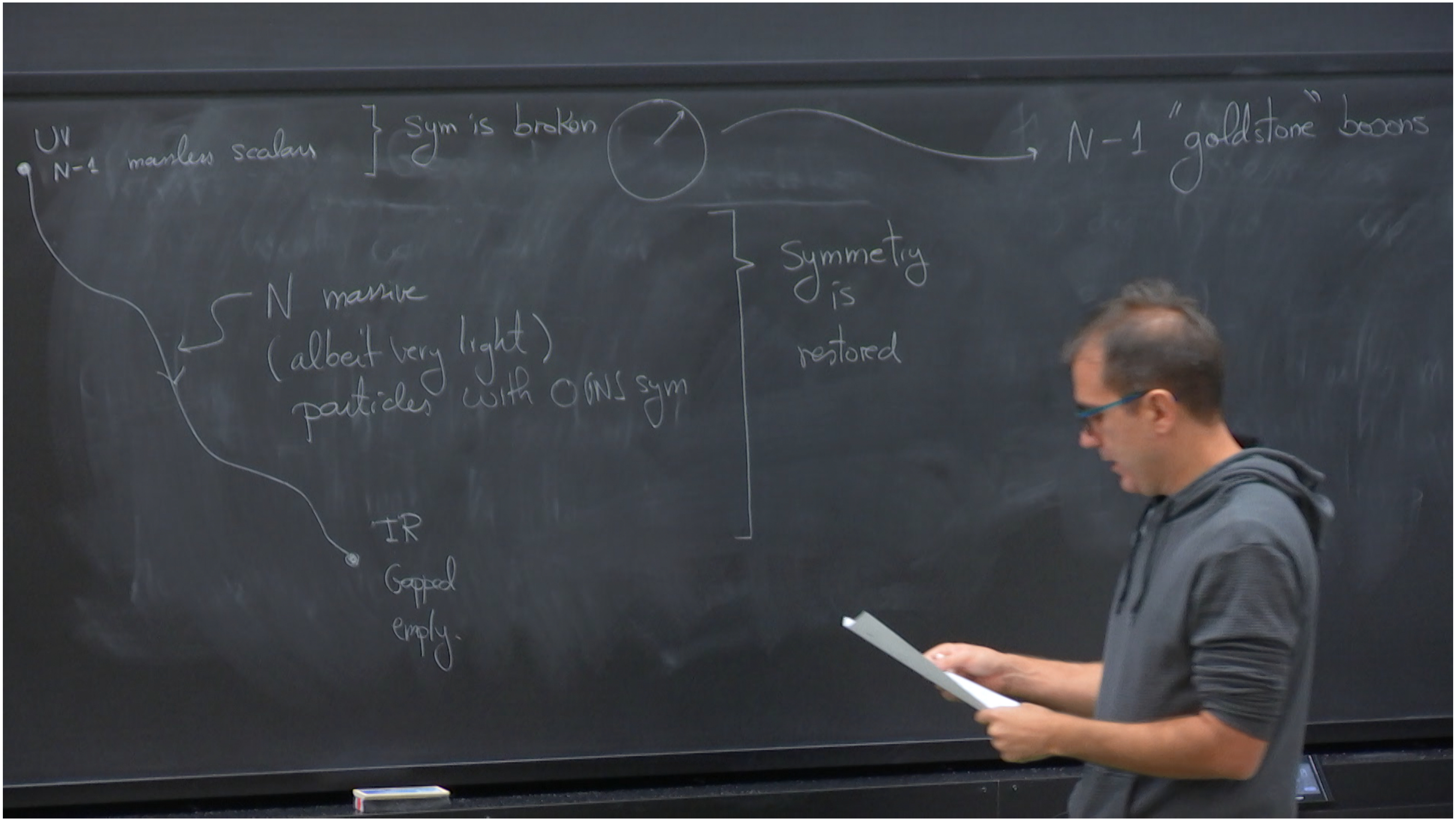
N−1 massless scalars } Sym is broken

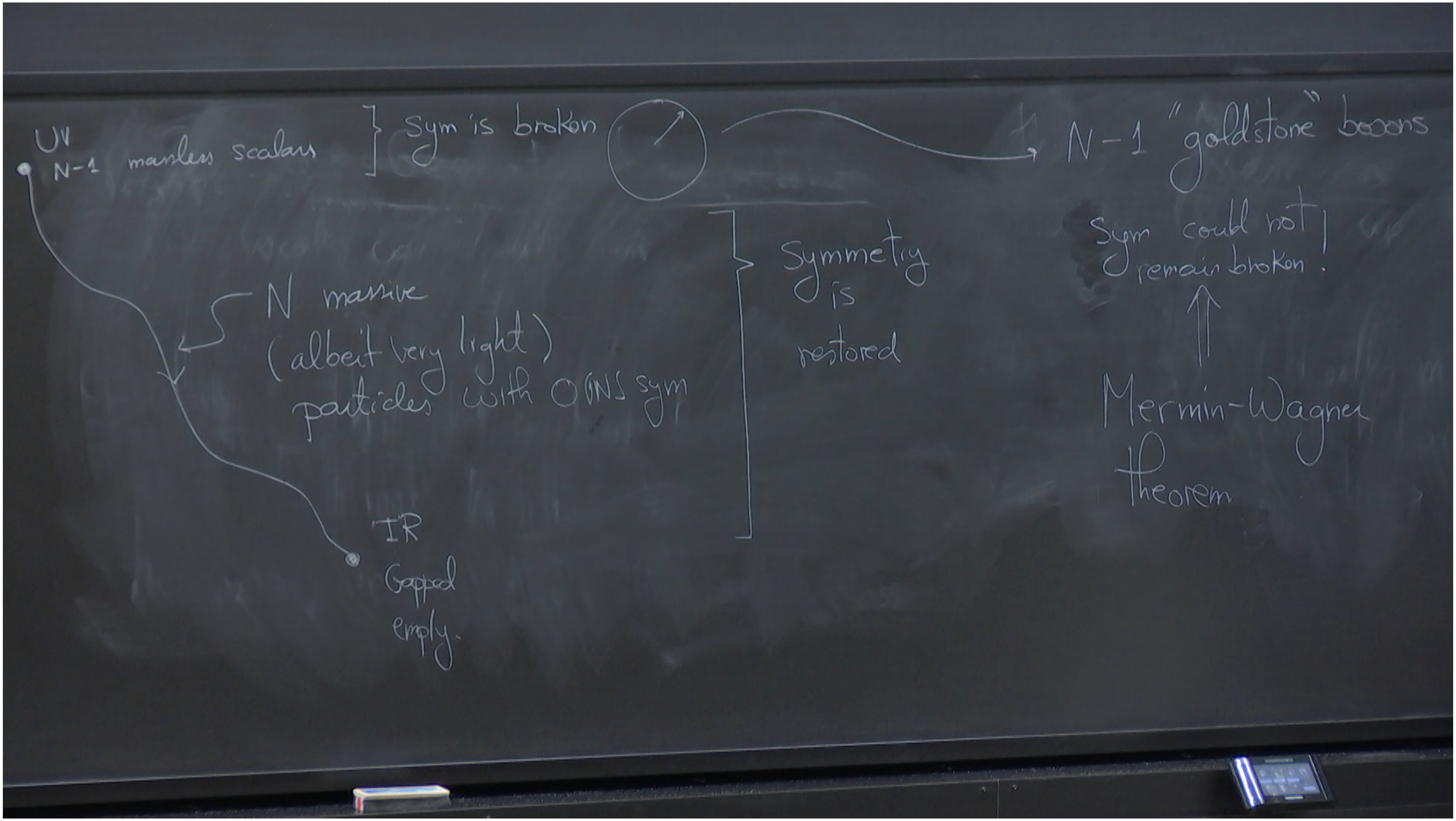
N massive
(albeit very light)
particles with $O(N)$ sym

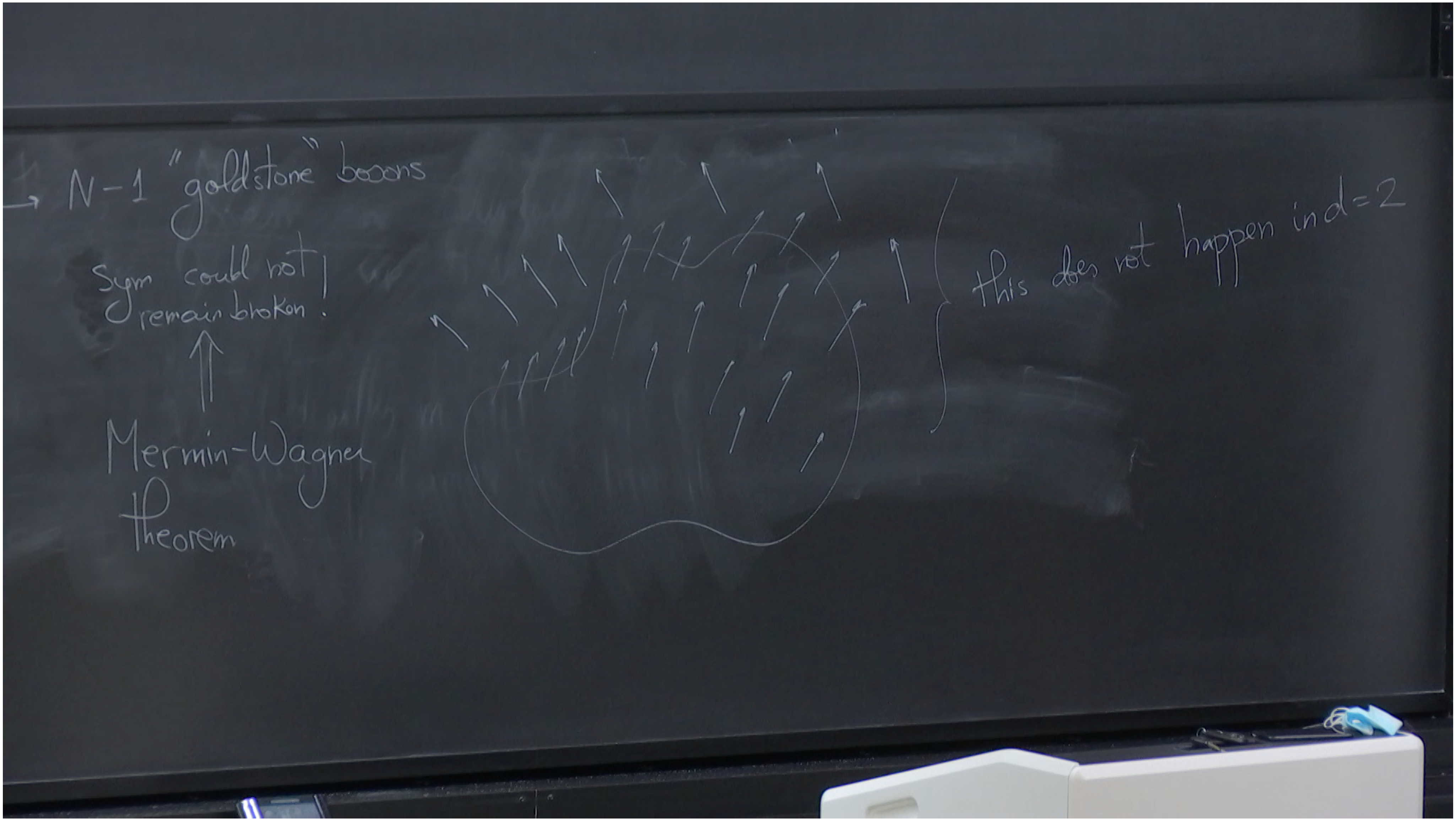
} Symmetry is restored

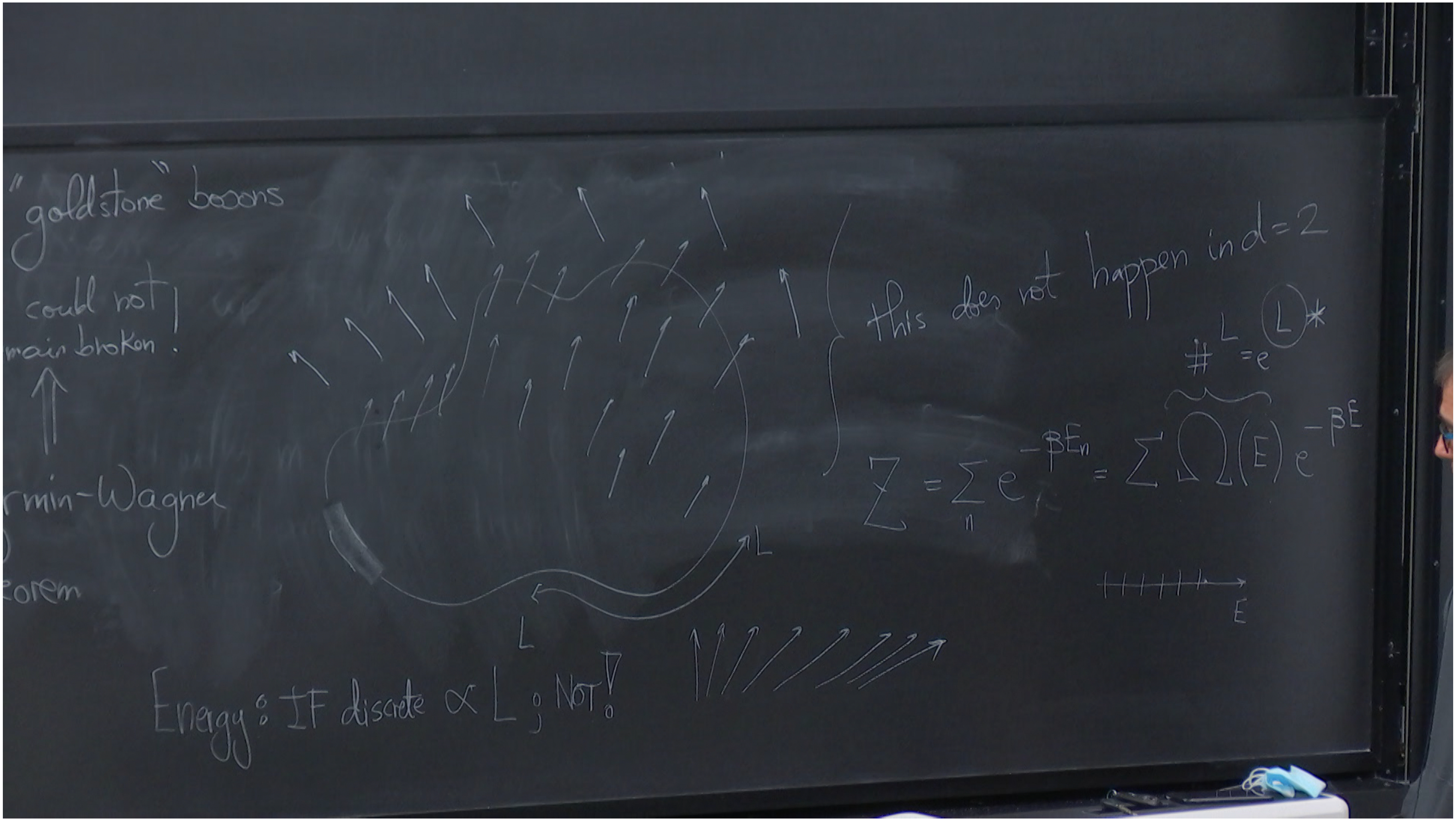
IR

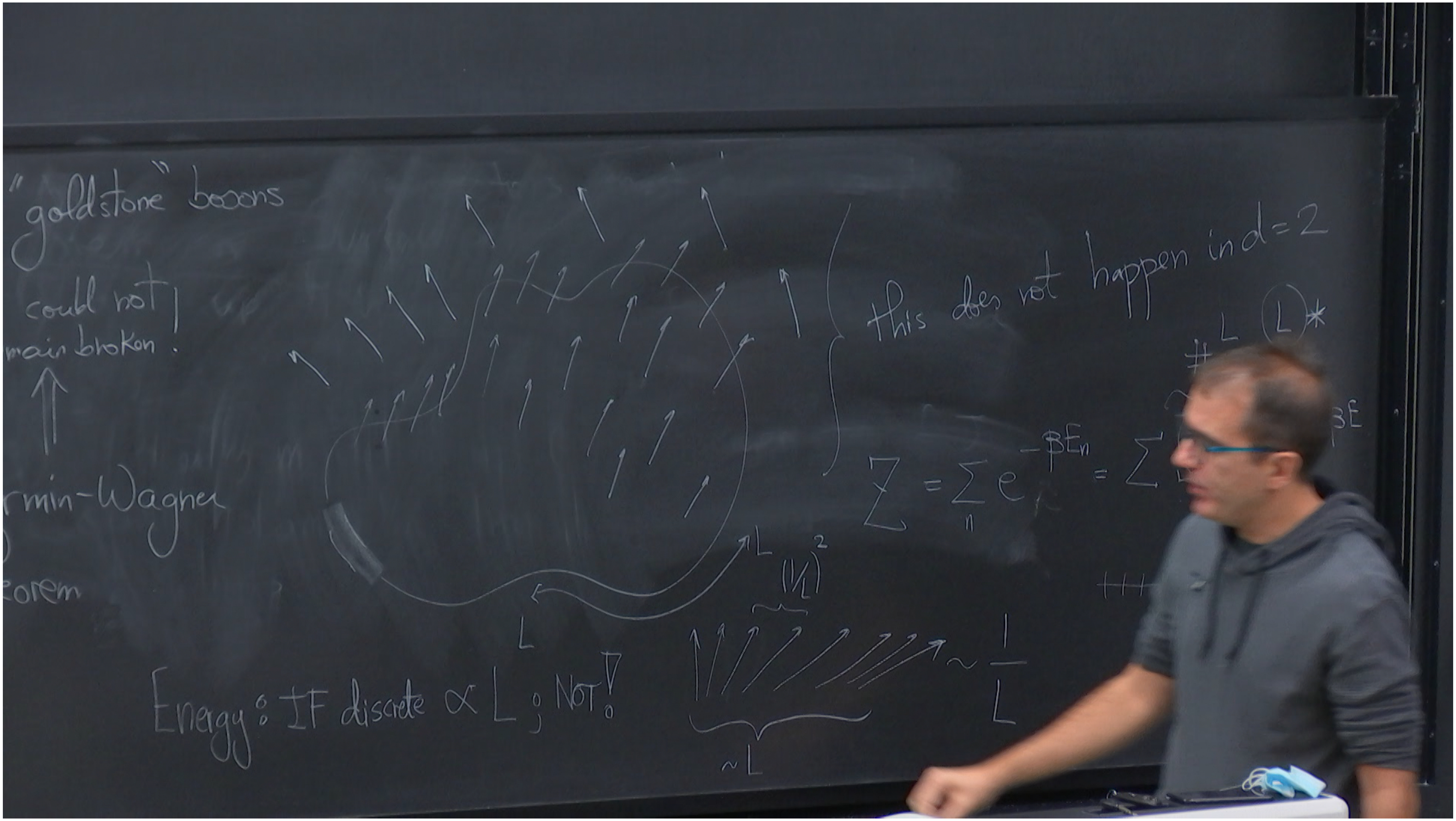
Gapped
empty.

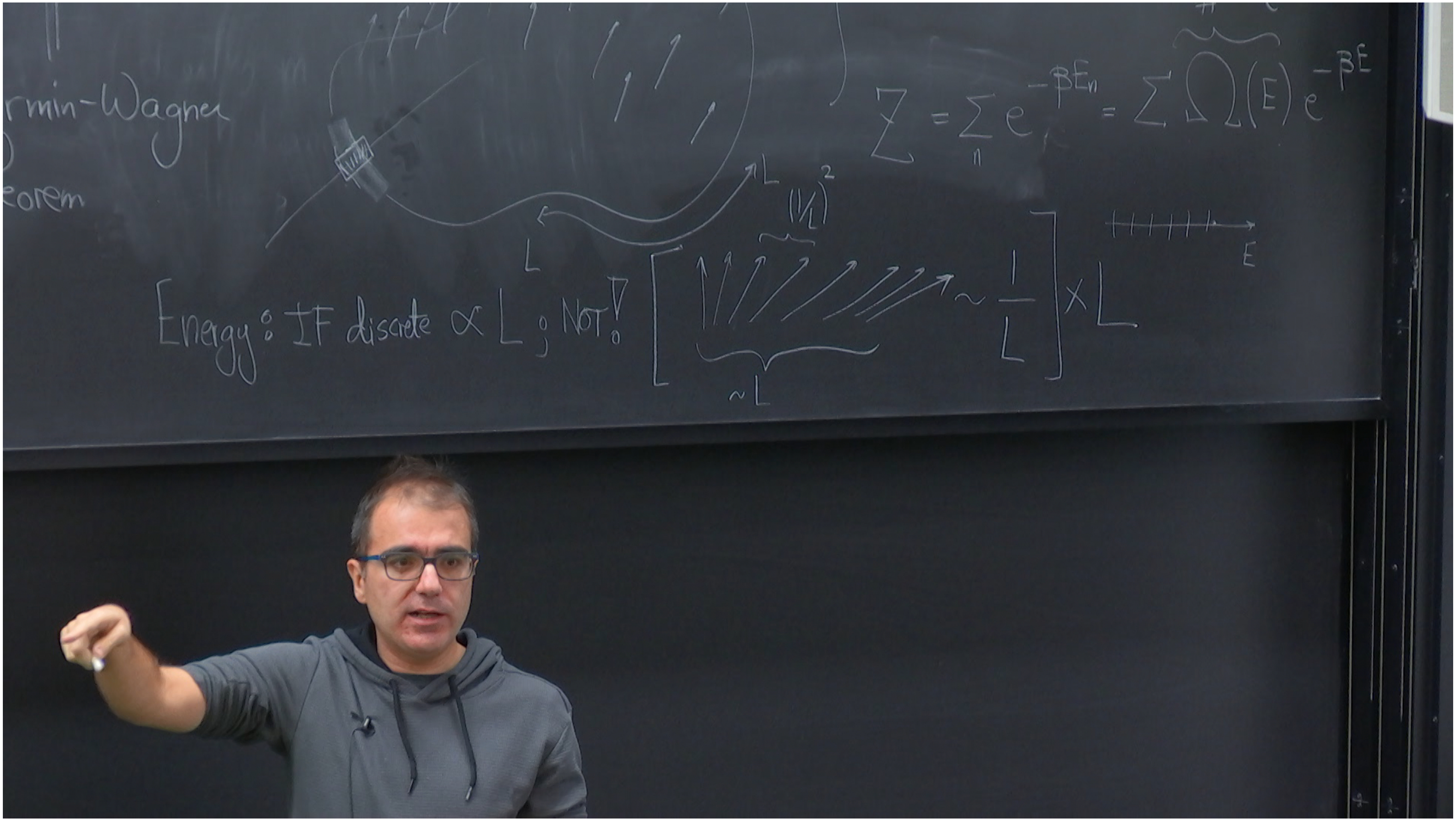

UV
N-1 massless scalars
Sym is broken
N-1 "goldstone" bosons
N massive
(albeit very light)
particles with O(N) sym
Symmetry
is
restored
IR
Gapped
empty.


UV
N-1 massless scalars
Sym is broken
N-1 "goldstone" bosons
Sym could not remain broken!
Mermin-Wagner theorem
N massive
(albeit very light)
particles with O(N) sym
Symmetry is restored
IR
Gapped
empty.

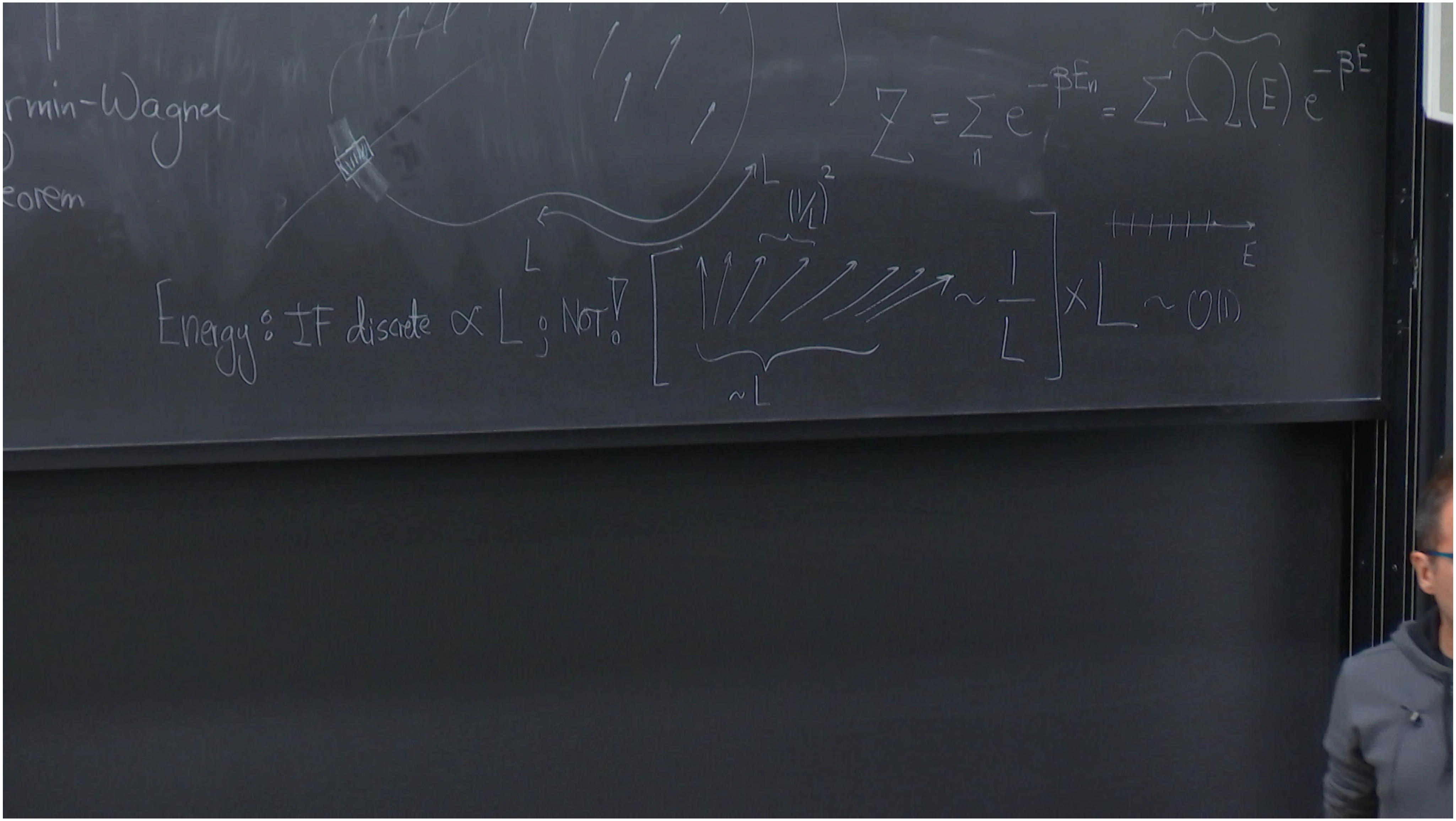

→ N−1 "goldstone" bosons
Sym could not remain broken!
Mermin-Wagner theorem
this does not happen in d=2

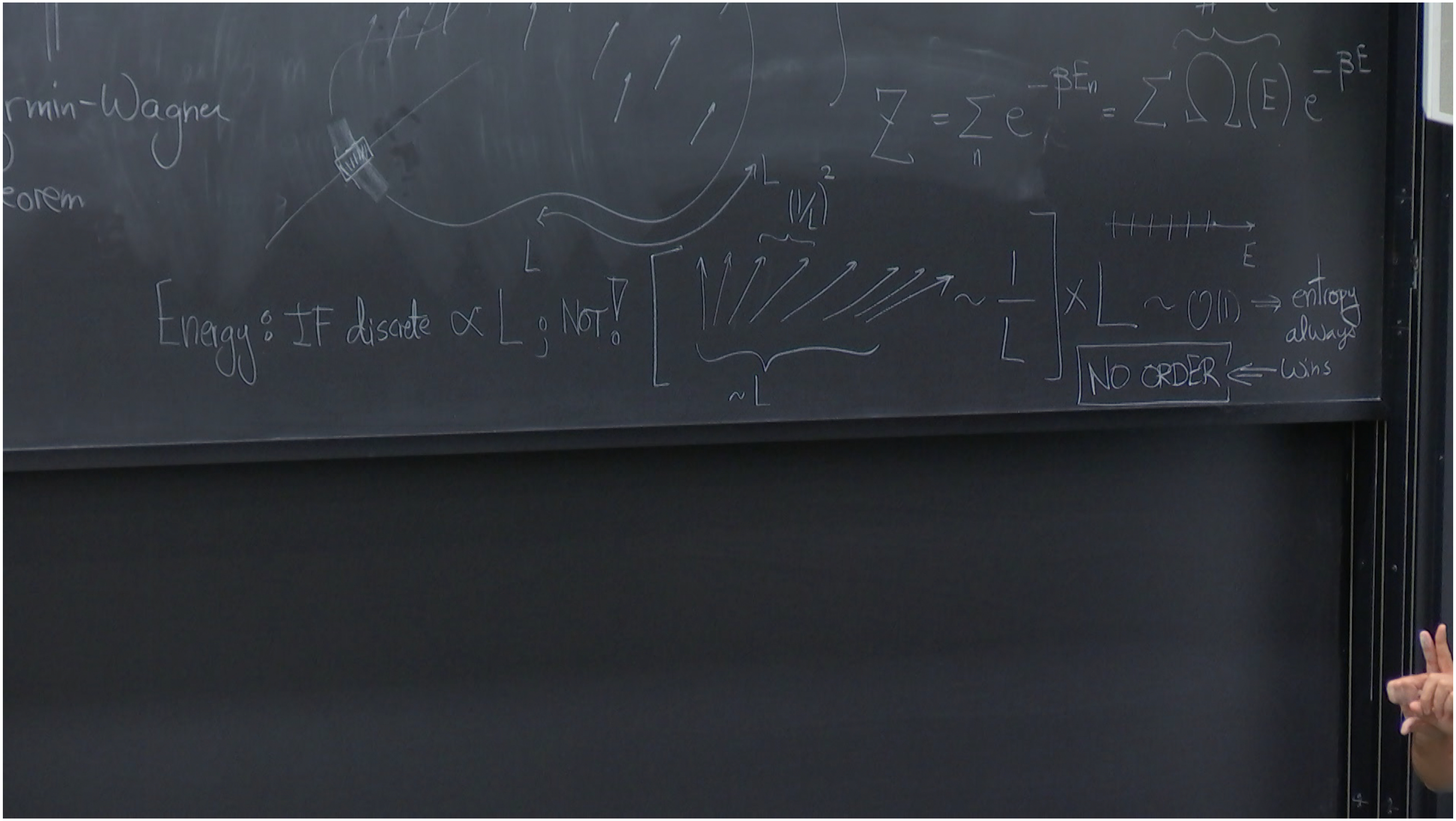

"goldstone" bosons
could not
main broken!
rmin-Wagner
eorem
this does not happen in d=2
$\# ^L = e^{L}$ *
$Z = \sum_n e^{-\beta E_n} = \sum \Omega(E) e^{-\beta E}$
E
L
L
Energy: IF discrete ∝ L; NOT!

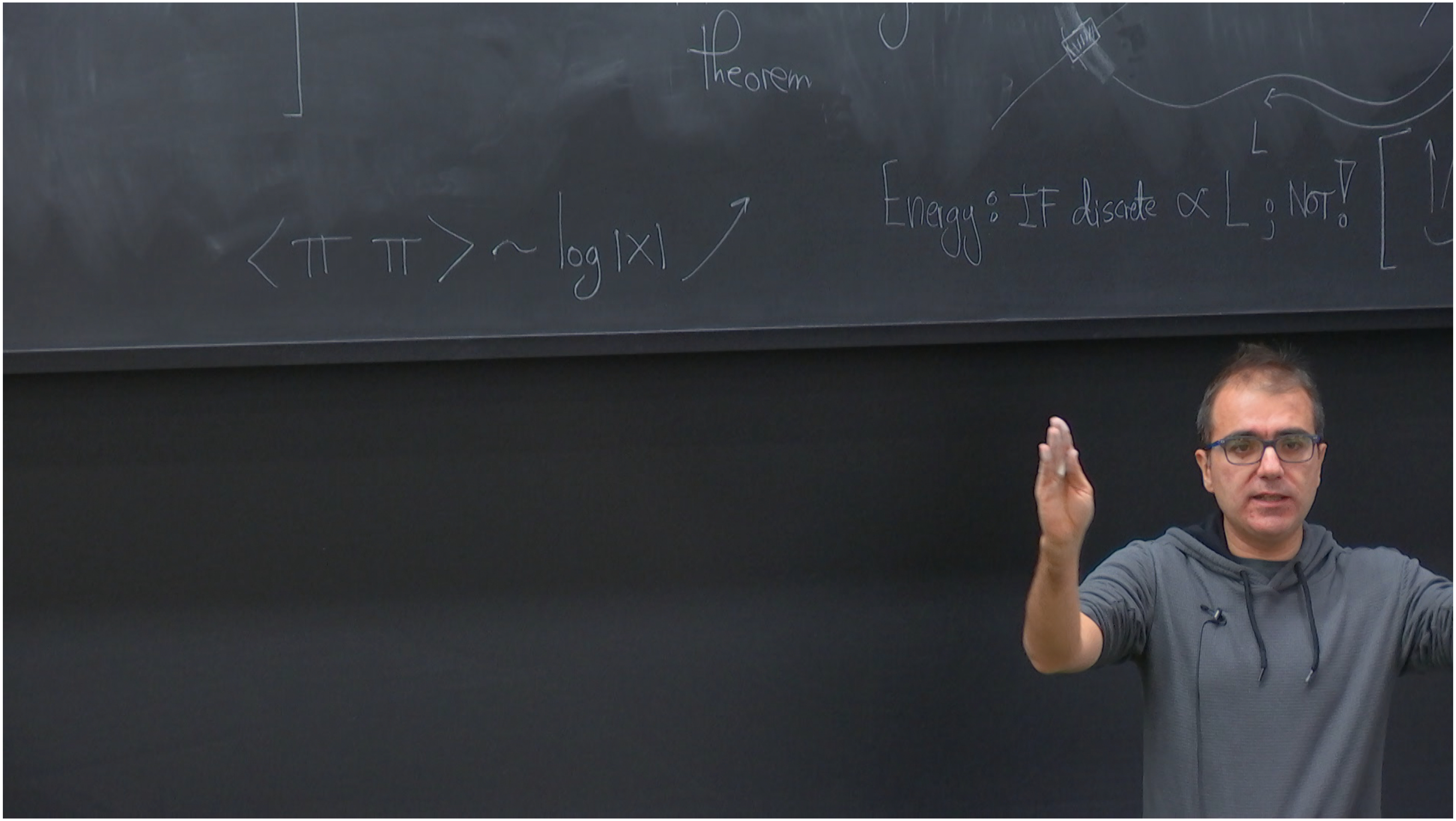

"goldstone" bosons
could not
main broken!
rmin-Wagner
eorem
this does not happen in d=2
Z = ∑ e^{-βE_n}
Energy: IF discrete ∝ L; NOT!
~ 1/L
~L
(1/L)^2

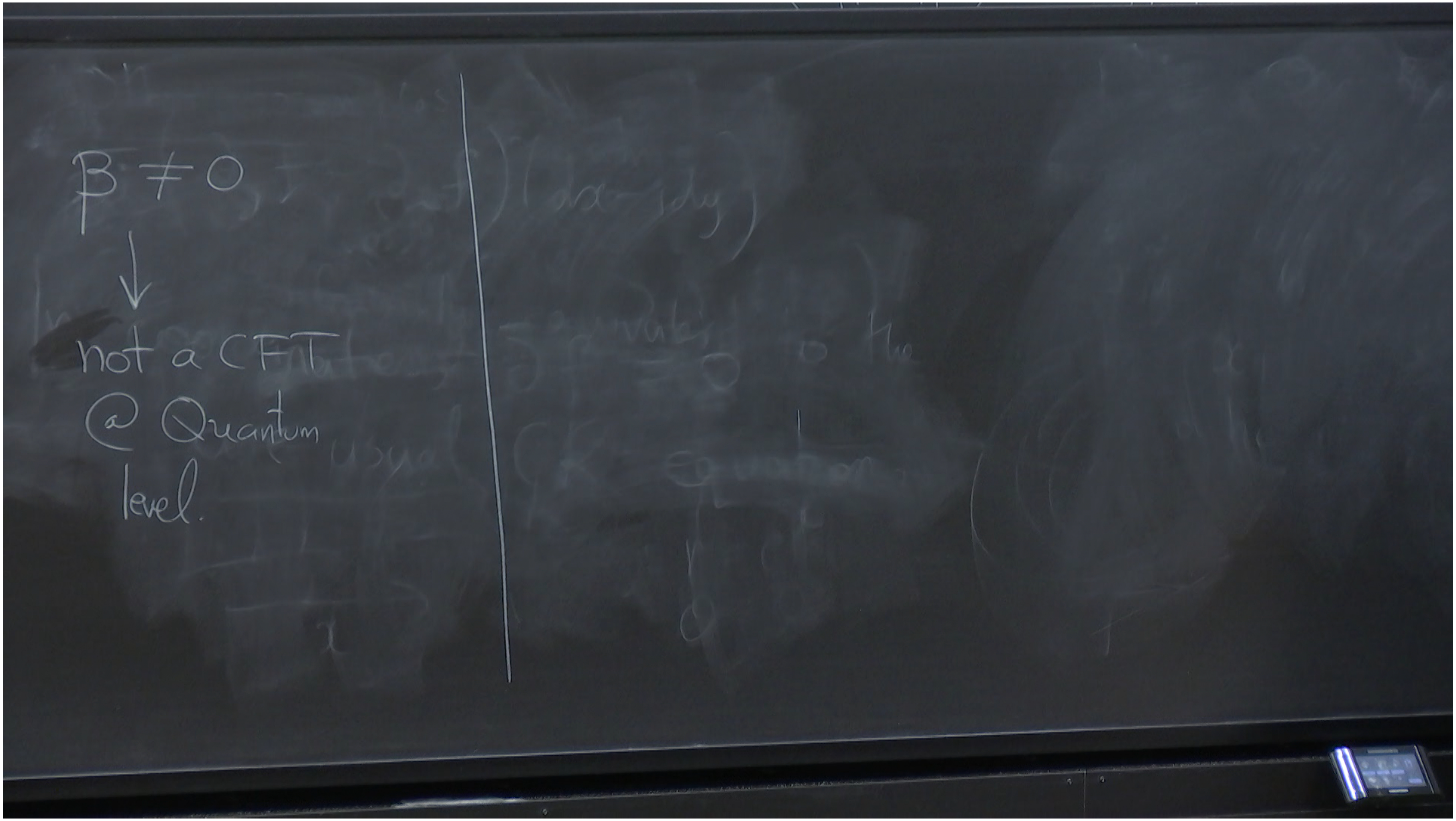


rmin-Wagner
eorem
Z = Σ e^{-βE_n} = Σ Ω(E) e^{-βE}
Energy: IF discrete ∝ L ; NOT!
~ 1/L
× L
~L
(1/L)^2
E


rmin-Wagner
eorem
Z = Σ e^{-βE_n} = Σ Ω(E) e^{-βE}
L
(1/L)^2
E
Energy: IF discrete ∝ L ; NOT!
~ 1/L
× L ~ O(1)
~ L


rmin-Wagner
eorem
Z = Σ e^{-βE_n} = Σ Ω(E) e^{-βE}
(1/L)²
E
Energy: IF discrete ∝ L; NOT!
~ 1/L × L ~ O(1) ⇒ entropy always wins
NO ORDER
~L


theorem
L
Energy: IF discrete ∝ L; NOT!
⟨π π⟩ ~ log|x|


β ≠ 0
not a CFT
@ Quantum
level.

$\langle \pi \, \pi \rangle \sim \log|X|$

Energy: If discrete $\propto L$; Not:

$$S_{YM} = \frac{1}{2g^2}\int d^4x \, \mathrm{tr}\, F^2$$

coupling

Nobel 2004

Asymptotic freedom

$$\beta = -\frac{11}{3} N \, g^3/16\pi^2 = \frac{\partial g}{\partial \log \Lambda}$$

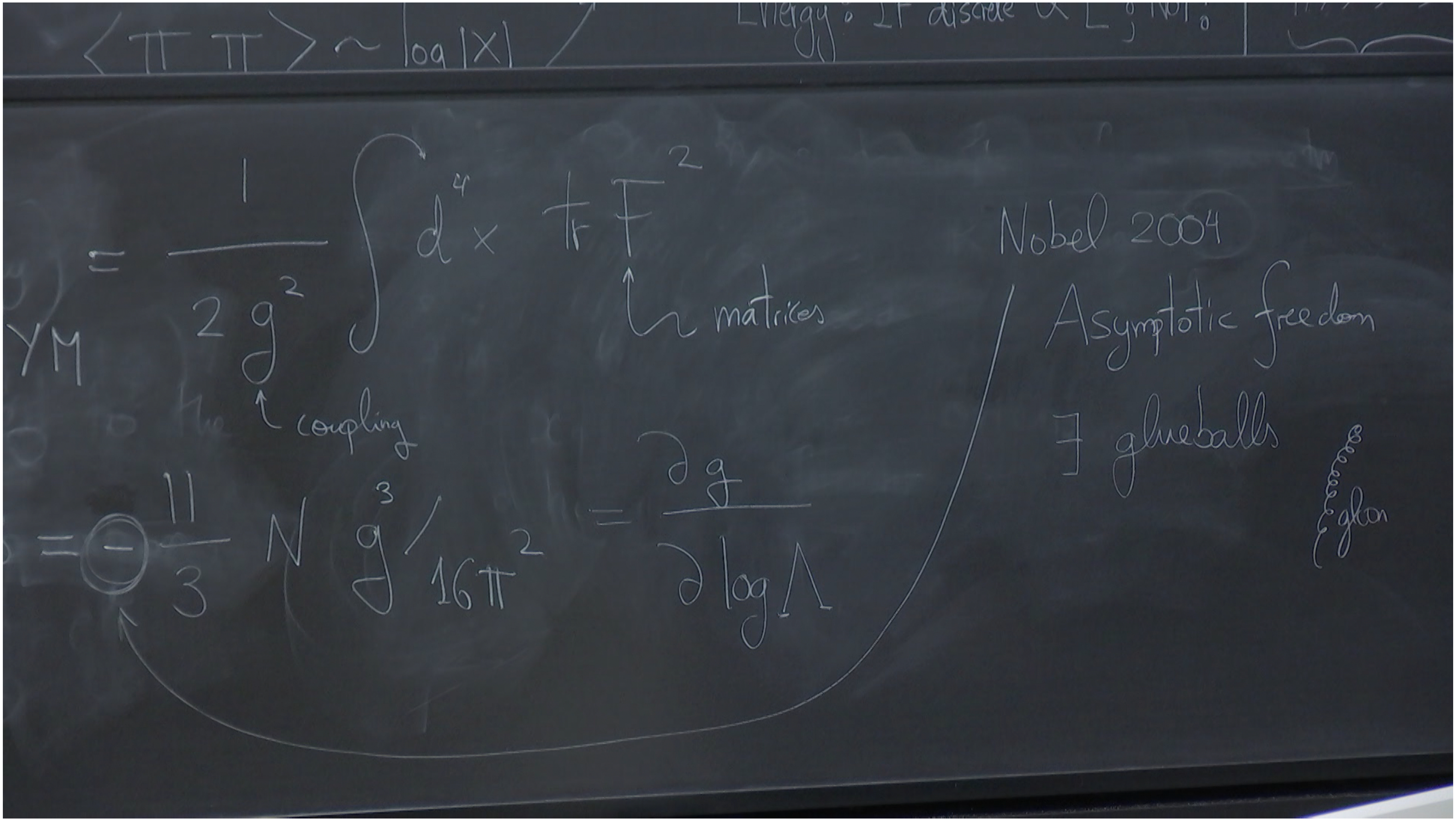
Nobel 2004

Asymptotic freedom

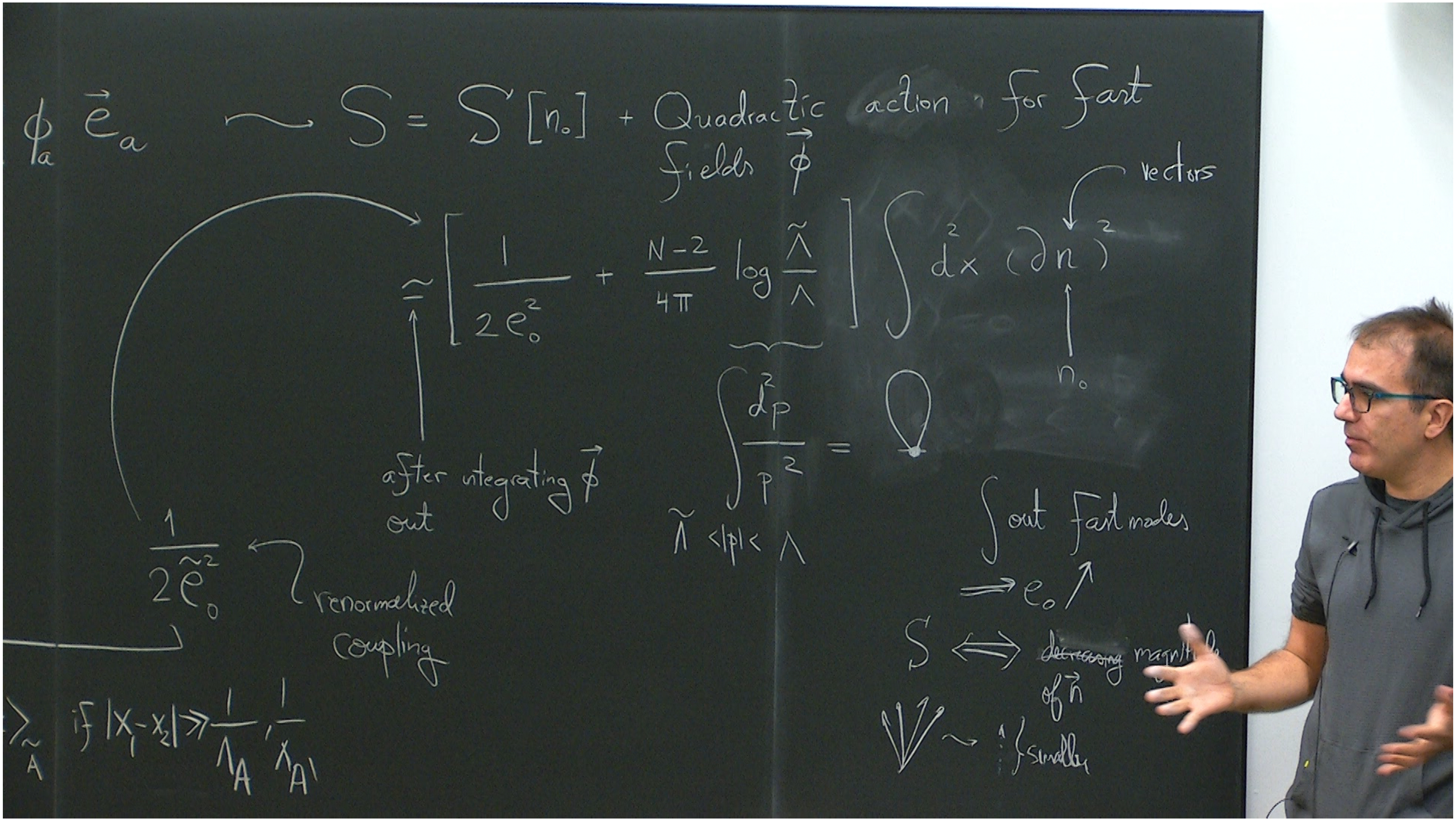
$\exists$ glueballs

gluon

, $m \sim$ dym generated.



⟨π π⟩ ~ log|X|
YM = 1/2g² ∫ d⁴x tr F²
matrices
coupling
= − 11/3 N g³/16π² = ∂g/∂log Λ
Nobel 2004
Asymptotic freedom
∃ glueballs
gluon


$\phi_a \vec{e}_a \sim S = S[n_0]$ + Quadractic action for fast fields $\vec{\phi}$
vectors
$\simeq \left[\frac{1}{2e_0^2} + \frac{N-2}{4\pi} \log \frac{\tilde{\Lambda}}{\Lambda} \right] \int d^2x \, (\partial n)^2$
n_0
$\int_{\tilde{\Lambda} < |p| < \Lambda} \frac{d^2p}{p^2} =$
after integrating $\vec{\phi}$ out
$\frac{1}{2\tilde{e}_0^2}$ renormalized coupling
$\int$ out fast modes
$\Rightarrow e_0 \nearrow$
$S \Longleftrightarrow$ ~~decreasing~~ magnitude of $\vec{n}$
smaller
if $|x_1 - x_2| \gg \frac{1}{\Lambda_A}, \frac{1}{\Lambda_{A'}}$