Title: AdS/CFT 2021/2022

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Collection: AdS/CFT 2021/2022

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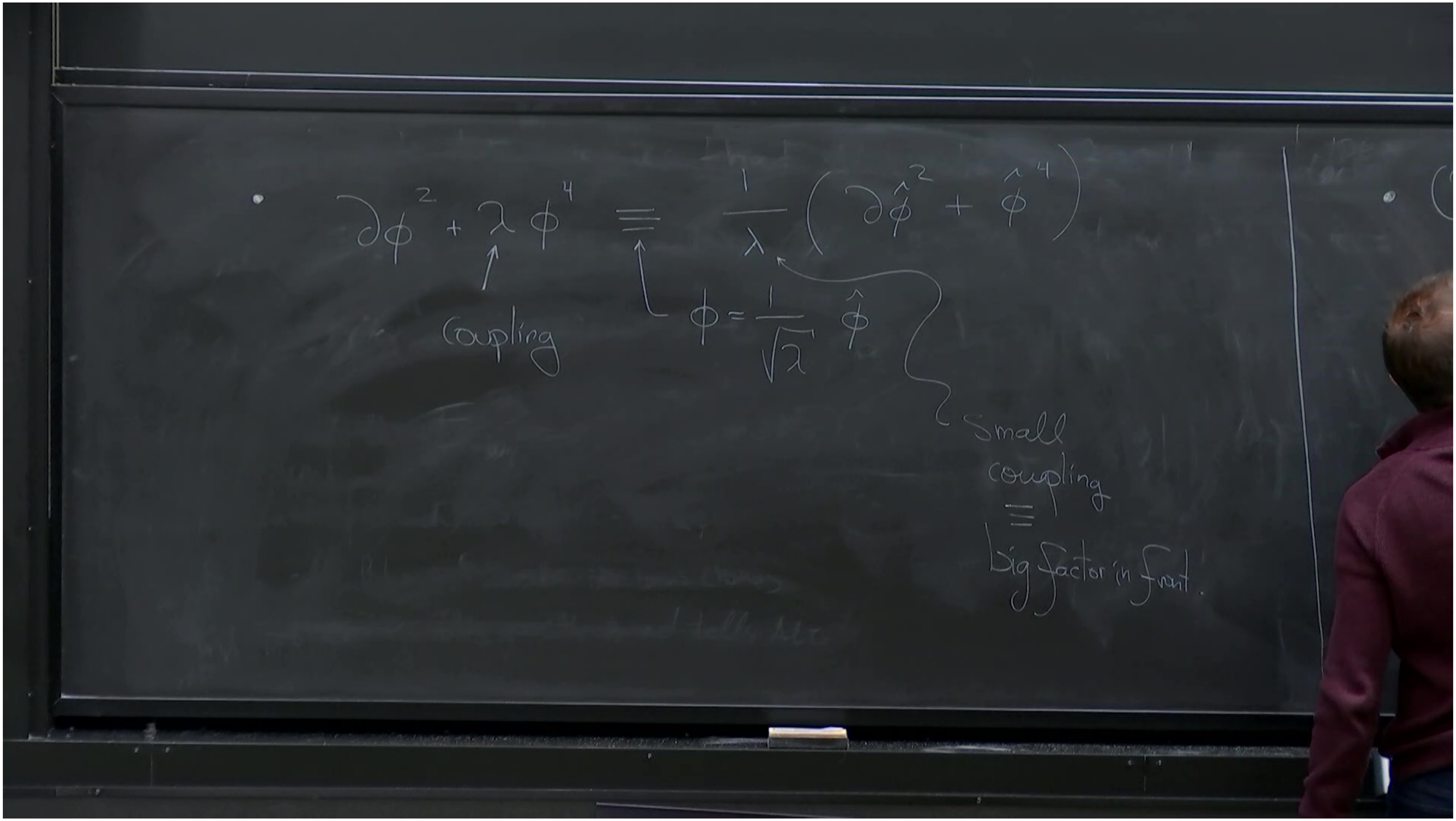

$\partial\phi^2 + \lambda\phi^4 \equiv \frac{1}{\lambda}\left(\partial\hat{\phi}^2 + \hat{\phi}^4\right)$
coupling
$\phi = \frac{1}{\sqrt{\lambda}}\hat{\phi}$
Small coupling $\equiv$ big factor in front.

front.

• $\mathcal{O}(N)$ non-linear σ-model.

$$S = \underbrace{\frac{1}{2e_0^2}}_{\text{coupling}} \int d^2x \left(\partial_\mu \vec{n} \cdot \partial^\mu \vec{n} \right) \qquad \text{with } \vec{n}^2 = 1$$

$(n_1 \ldots\ldots n_N)$

N Scalars

front.

• $O(N)$ non-linear σ-model.

Classically conformal.

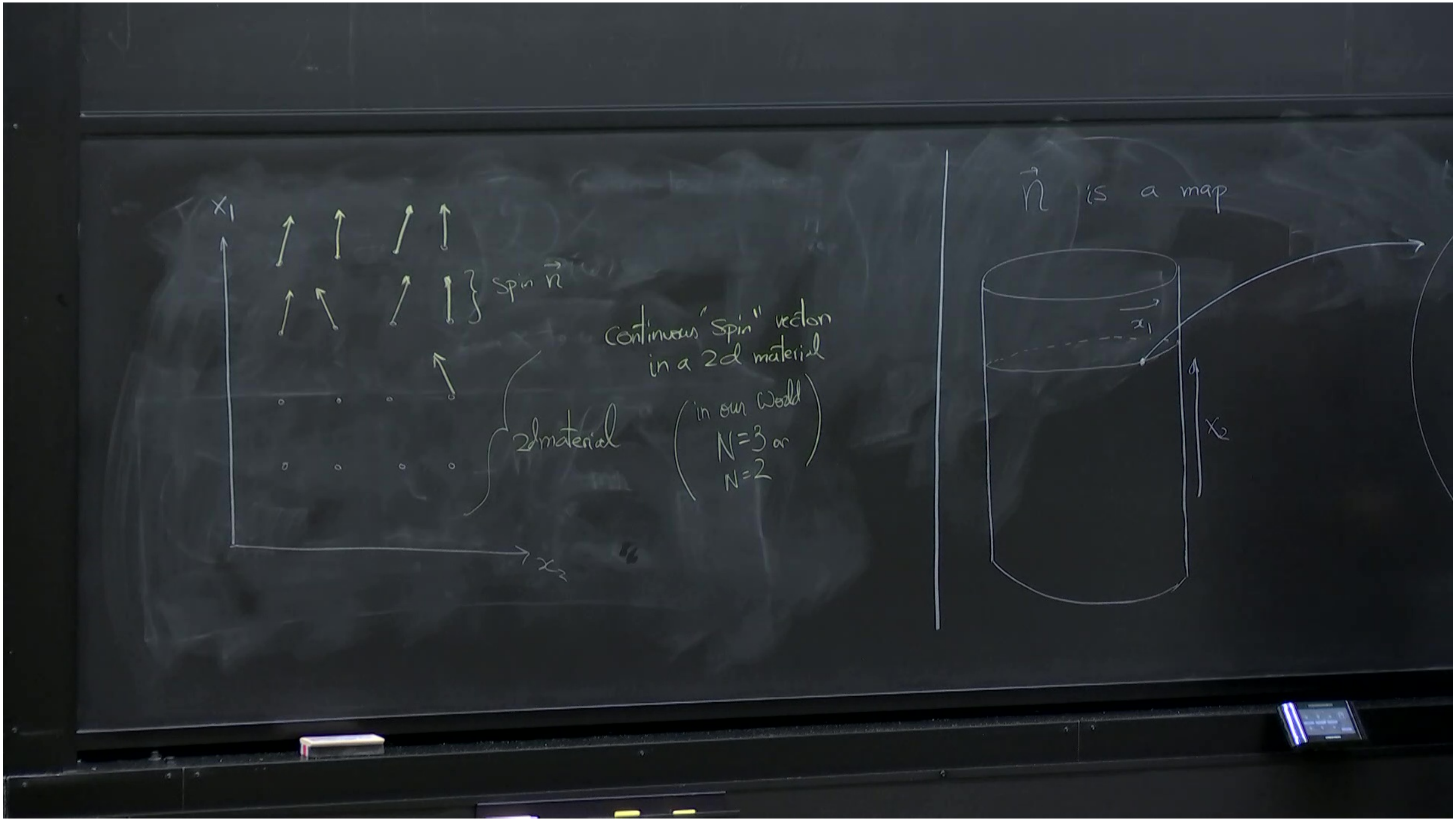
$$S = \frac{1}{2e_0^2}\int d^2x \left(\partial_\mu \vec{n}\cdot\partial^\mu \vec{n}\right) \quad \text{with } \vec{n}^2 = 1$$

coupling

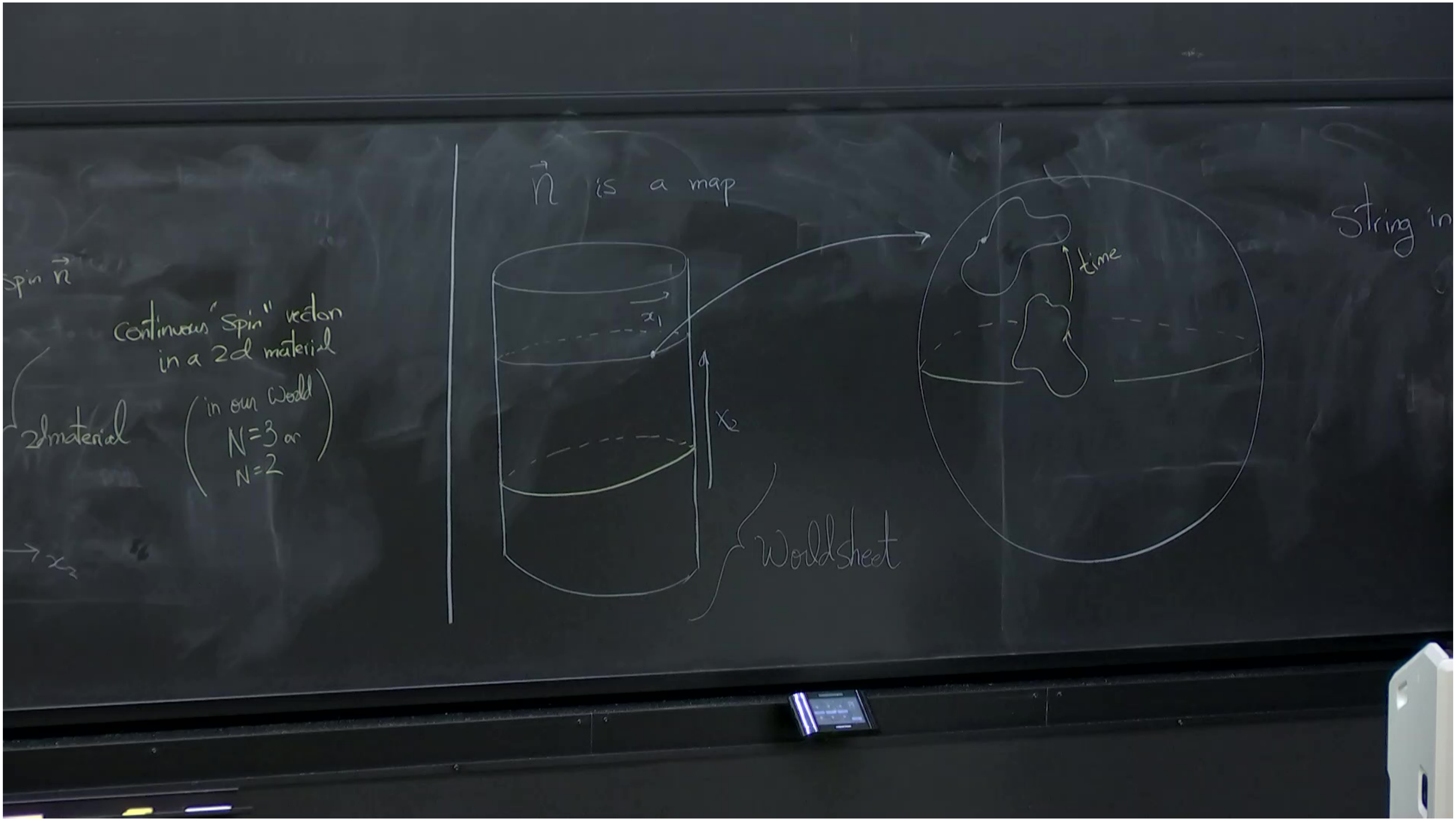
$(n_1 \ldots\ldots n_N)$

N Scalars

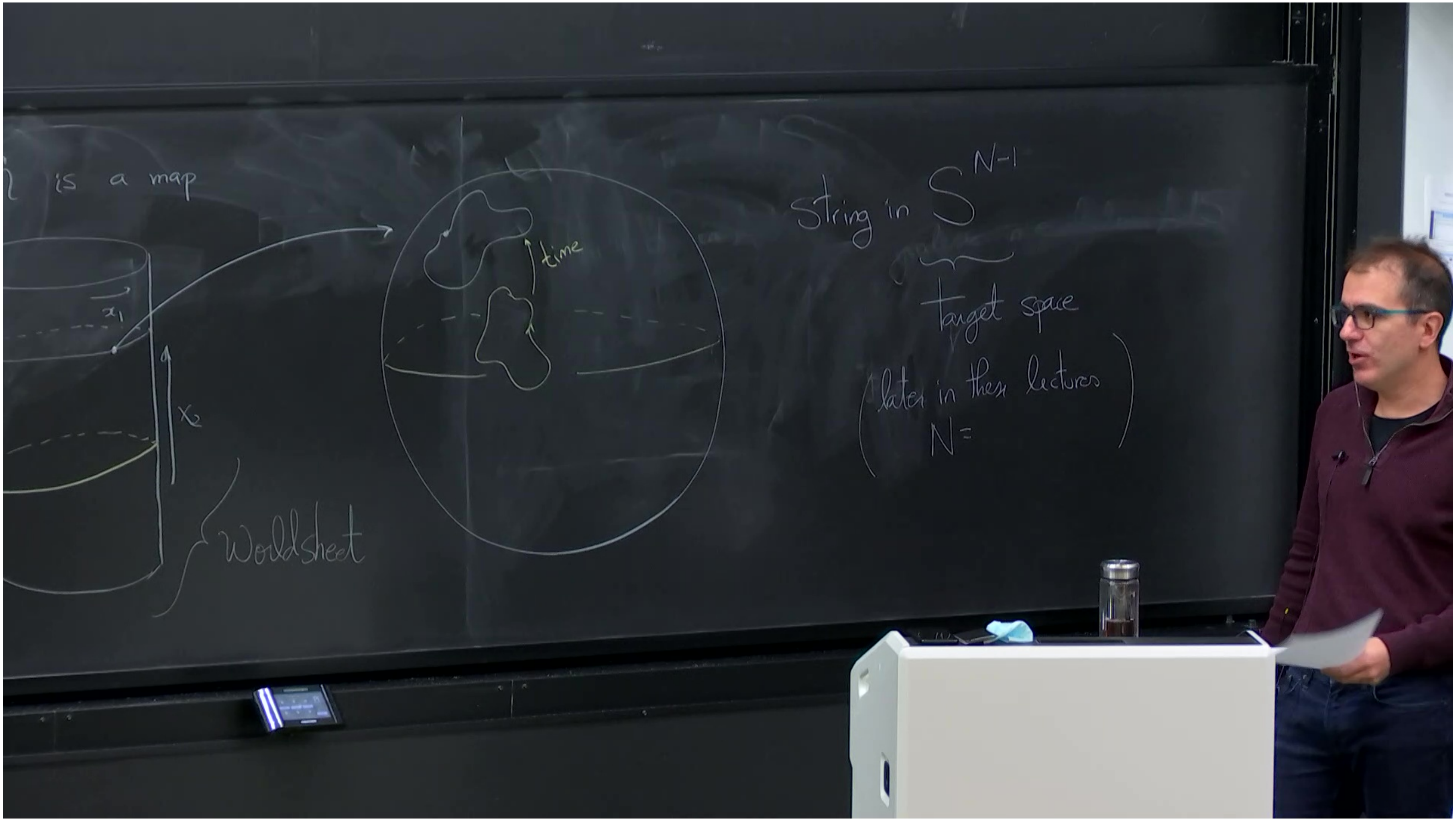
$x^\mu = (x^1, x^2)$

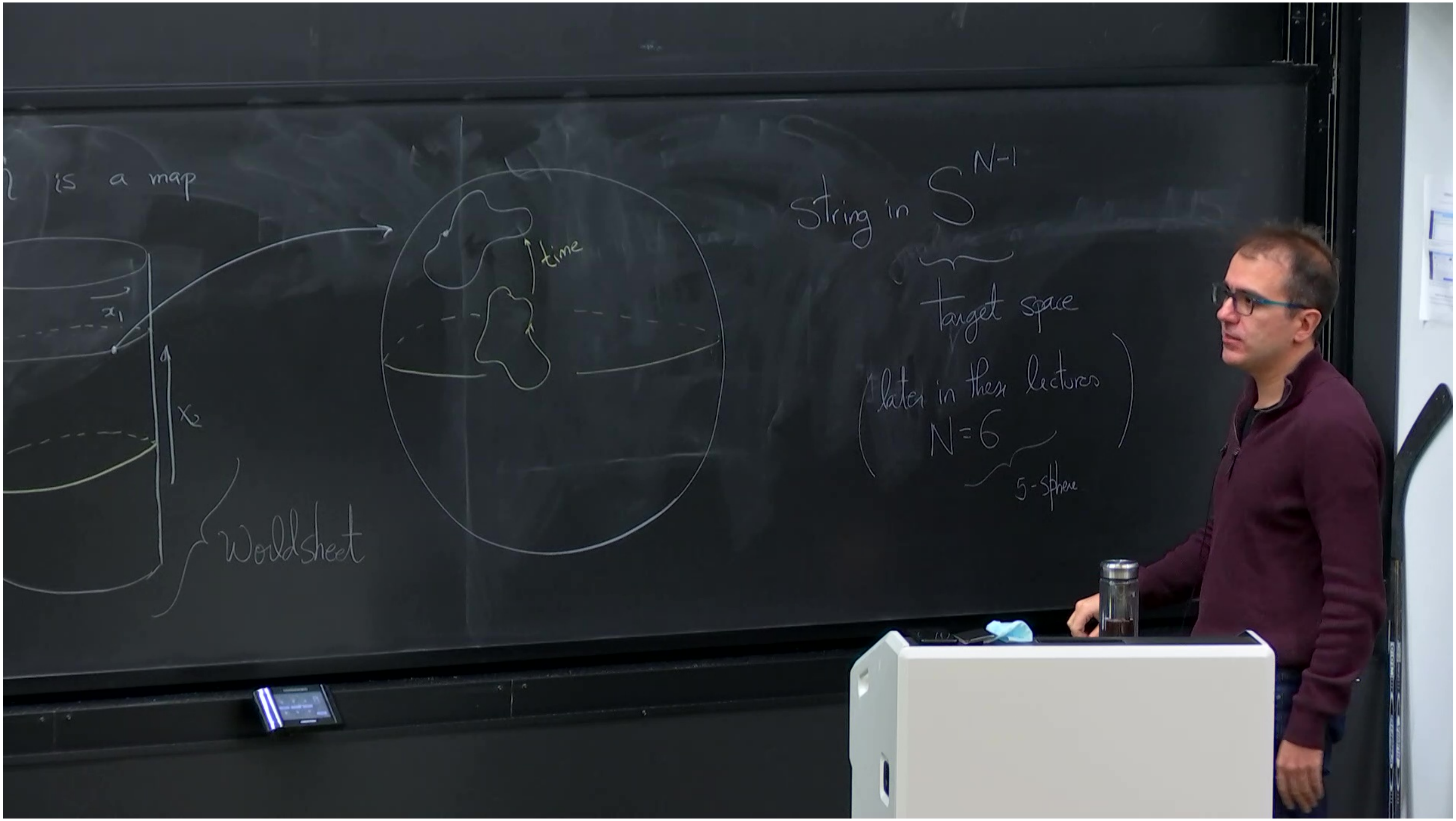


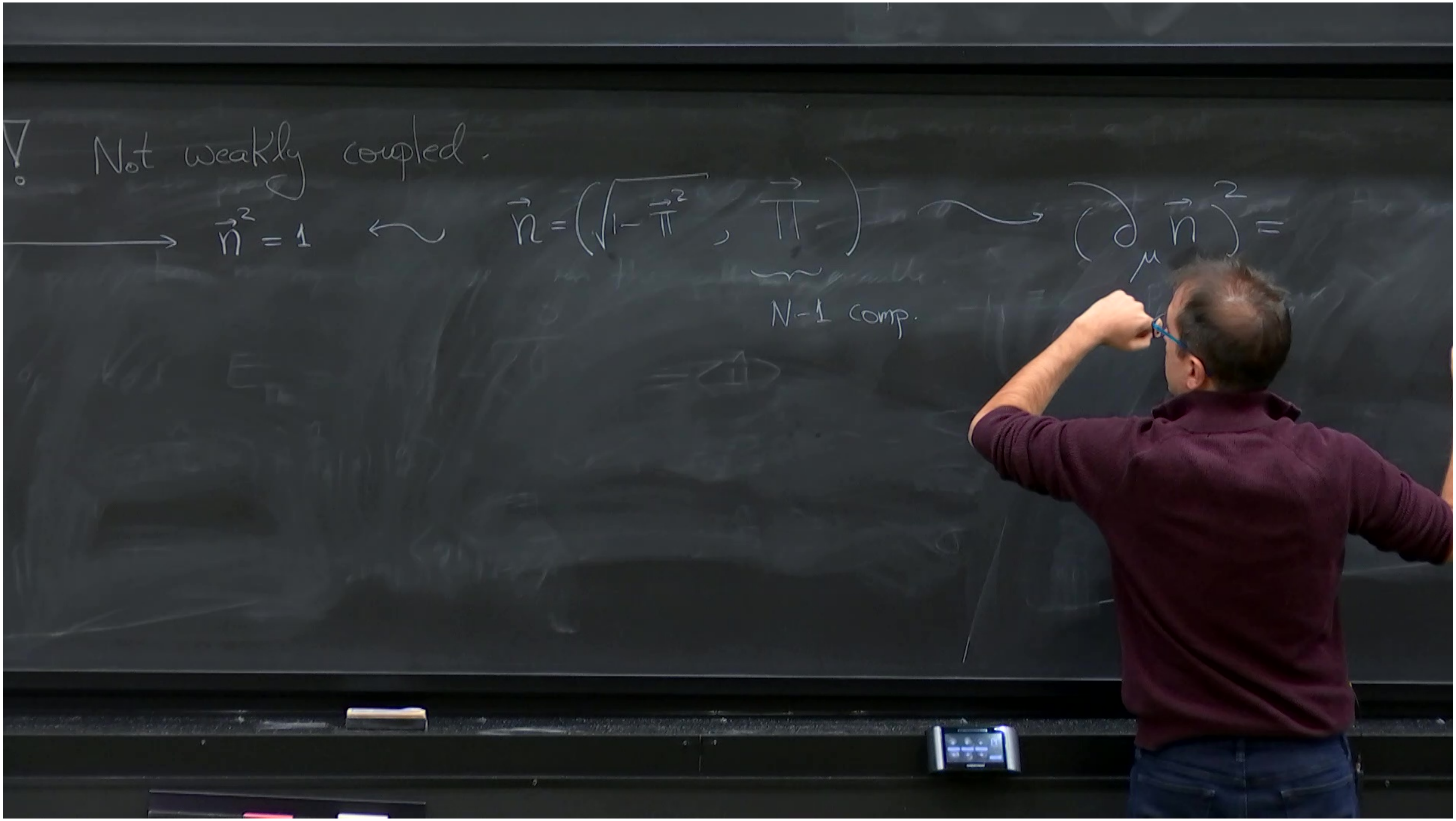
x_1
} spin $\vec{n}$
Continuous "spin" vector in a 2d material
2d material
(in our World N=3 or N=2)
x_2
$\vec{n}$ is a map
x_1
X_2

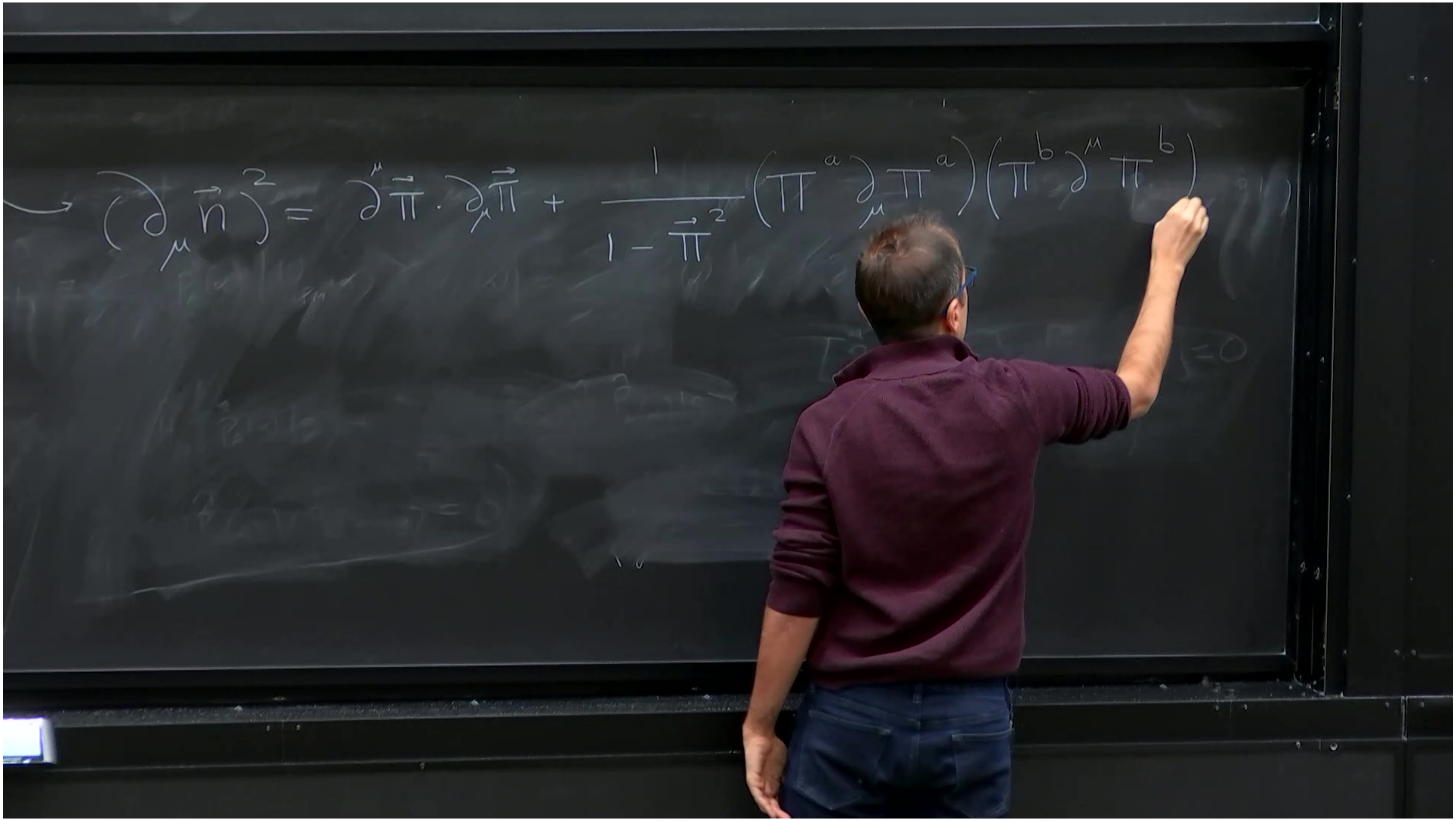


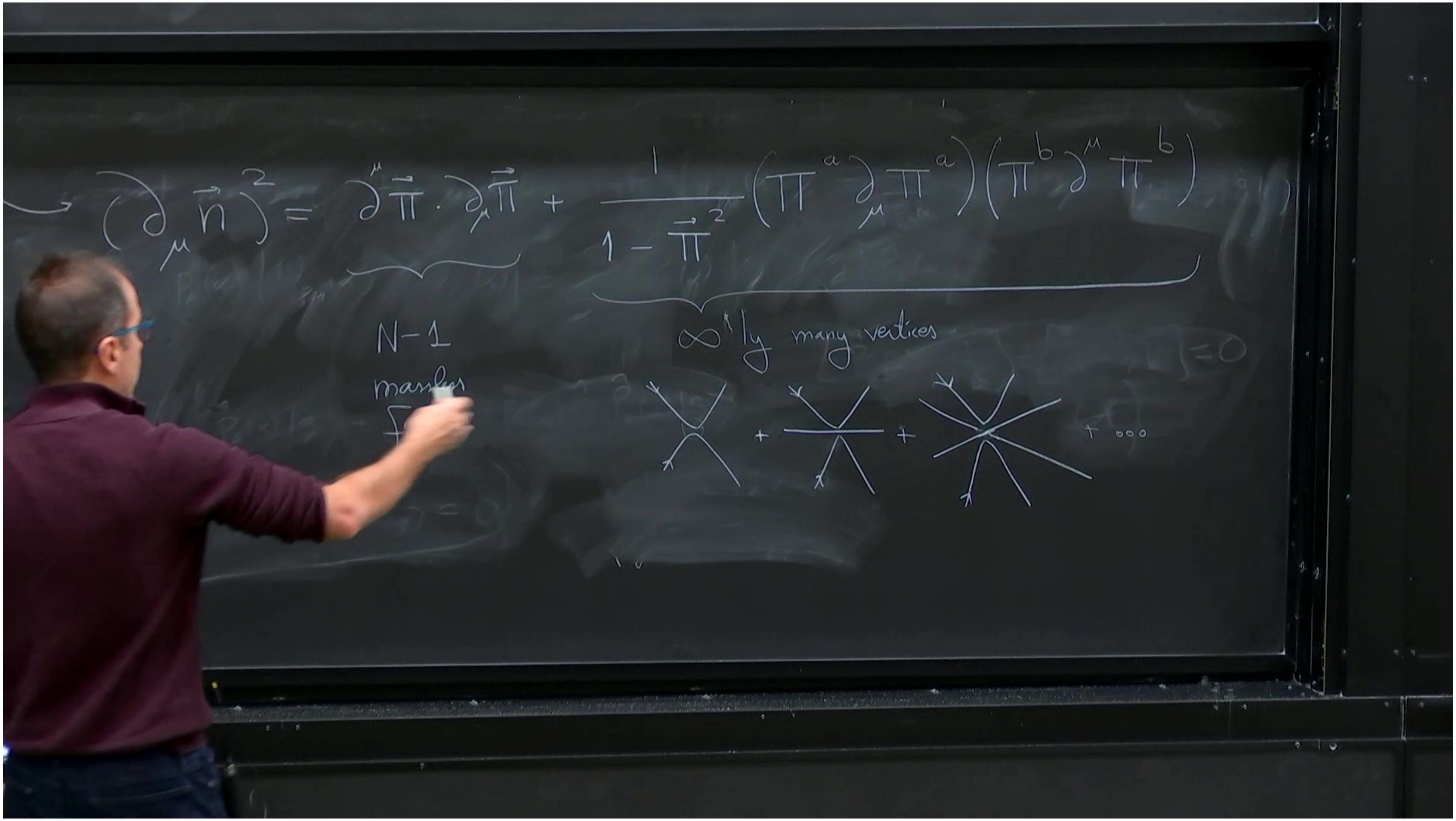
spin $\vec{n}$
Continuous "spin" vector
in a 2d material
2dmaterial
in our World
N=3 or
N=2
$\vec{n}$ is a map
x_1
x_2
Worldsheet
time
String in


is a map
x1
x2
Worldsheet
time
string in S N-1
target space
later in these lectures
N=


is a map
x_1
x_2
Worldsheet
time
string in S^{N-1}
target space
later in these lectures
$N=6$
5-sphere

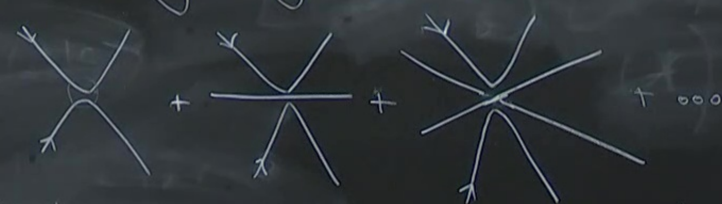

Not weakly coupled.
$\vec{n}^2 = 1$
$\vec{n} = \left(\sqrt{1-\vec{\pi}^2}, \vec{\pi}\right)$
N-1 comp.
$(\partial_\mu \vec{n})^2 =$


$(\partial_\mu \vec{n})^2 = \partial^\mu \vec{\pi} \cdot \partial_\mu \vec{\pi} + \frac{1}{1-\vec{\pi}^2}(\pi^a \partial_\mu \pi^a)(\pi^b \partial^\mu \pi^b)$



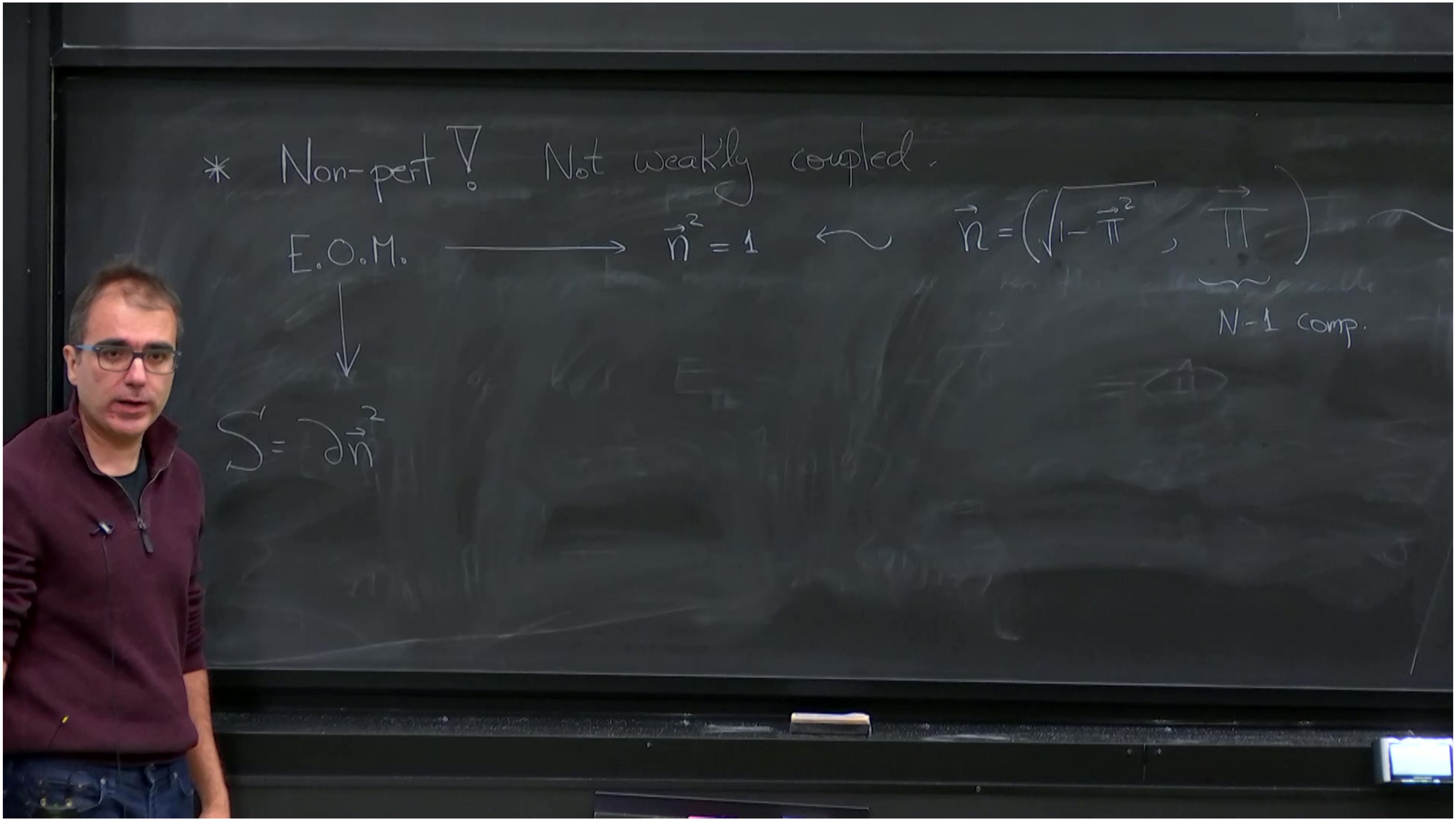
$(\partial_\mu \vec{n})^2 = \partial^\mu \vec{\pi} \cdot \partial_\mu \vec{\pi} + \frac{1}{1-\vec{\pi}^2}(\pi^a \partial_\mu \pi^a)(\pi^b \partial^\mu \pi^b)$
N−1
massless
∞^{ly} many vertices
+ ...

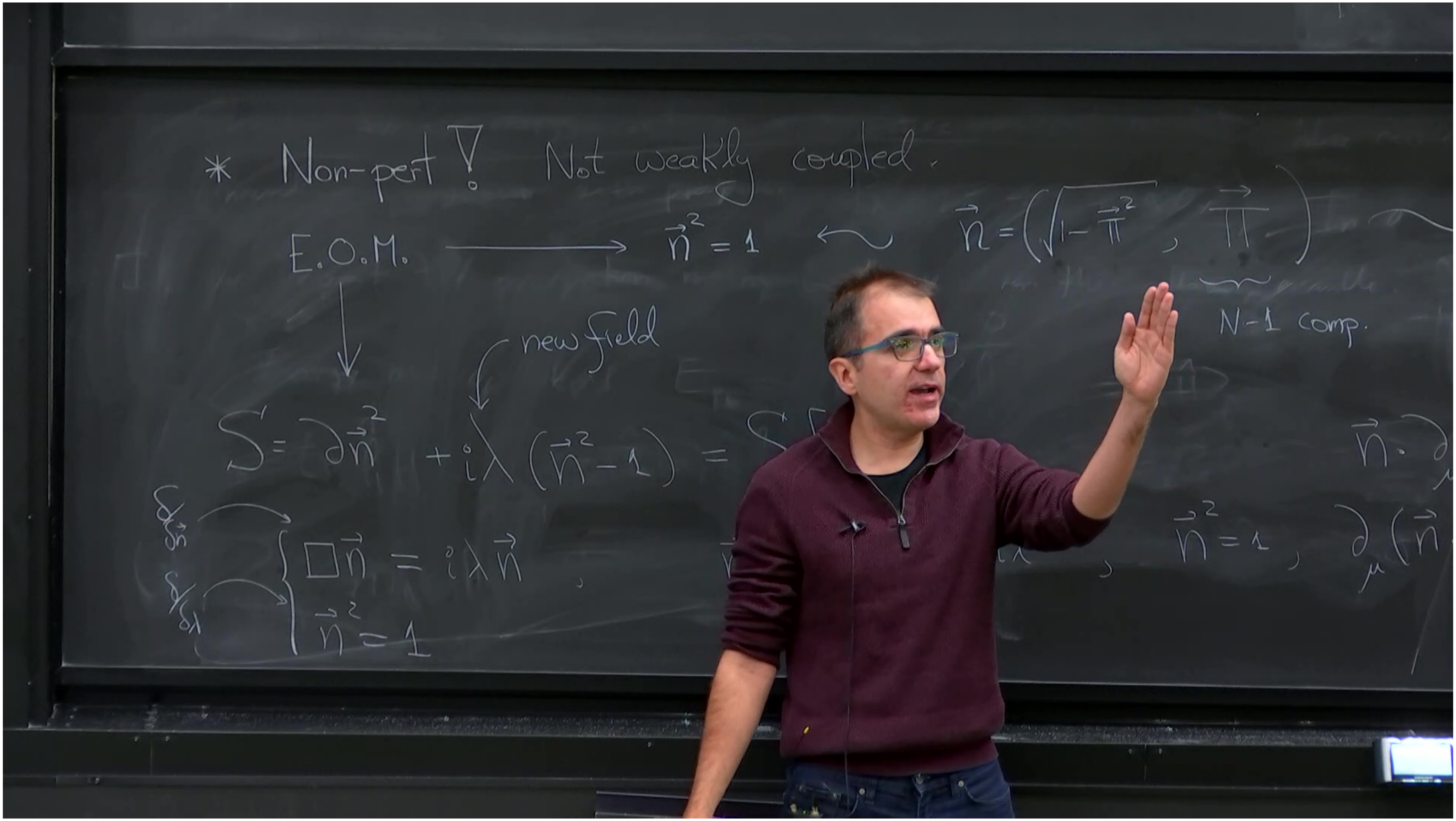
$$\to (\partial_\mu \vec{n})^2 = \underbrace{\partial^\mu \vec{\pi} \cdot \partial_\mu \vec{\pi}}_{\substack{N-1 \\ \text{massless} \\ \text{fields}}} + \underbrace{\frac{1}{1-\vec{\pi}^2}(\pi^a \partial_\mu \pi^a)(\pi^b \partial^\mu \pi^b)}_{\infty\text{'ly many vertices}}$$



$$\Box \vec{\pi} = \infty \text{ set of vectors}$$

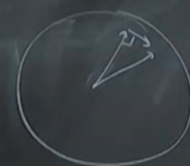
useful in the UV


* Non-pert ! Not weakly coupled.
E.O.M.
$\vec{n}^2 = 1$
$\vec{n} = \left(\sqrt{1-\vec{\pi}^2}, \vec{\pi}\right)$
N-1 comp.
$S = \partial \vec{n}^2$


* Non-pert ! Not weakly coupled.
E.O.M. ⟶ $\vec{n}^2 = 1$
$\vec{n} = (\sqrt{1-\vec{\pi}^2}, \vec{\pi})$
N-1 comp.
new field
$S = \partial\vec{n}^2 + i\lambda(\vec{n}^2 - 1) =$
$\Box\vec{n} = i\lambda\vec{n}$
$\vec{n}^2 = 1$

...akly coupled.

$$\vec{n}^2 = 1 \quad \leftsquigarrow \quad \vec{n} = \Big(\sqrt{1-\vec{\pi}^2}\,,\ \underbrace{\vec{\pi}}_{N-1\ \text{comp.}}\Big) \rightsquigarrow (\partial_\mu \vec{n})^2 = \underbrace{\partial^\mu \vec{\pi}\cdot\partial_\mu\vec{\pi}}_{\substack{N-1\\ \text{massless}\\ \text{fields}}} + \frac{1}{1-\vec{\pi}^2} \ldots$$

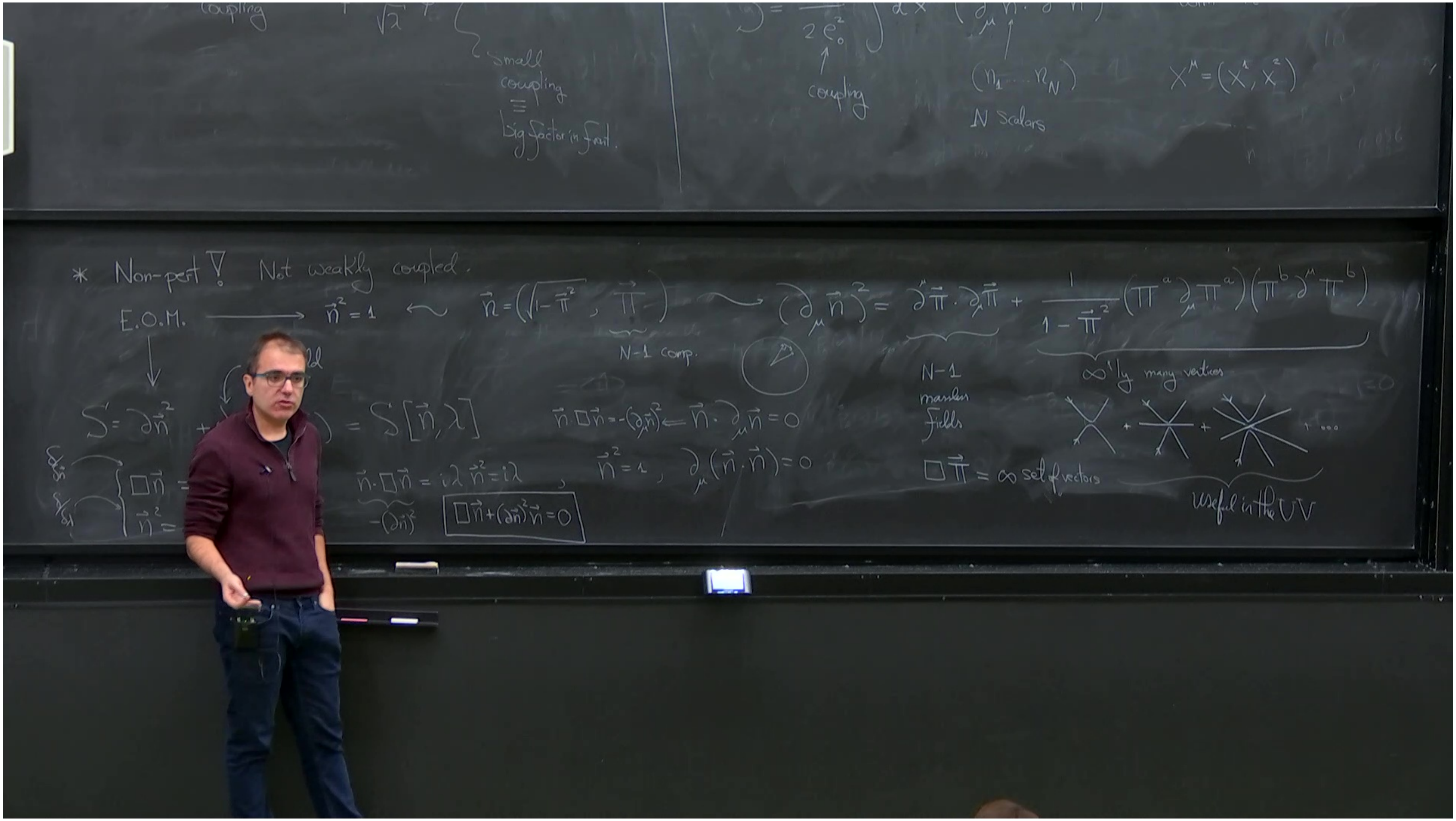


$$= S[\vec{n}, \lambda]$$

$$\vec{n}\cdot\Box\vec{n} = -(\partial_\mu\vec{n})^2 \Leftarrow \vec{n}\cdot\partial_\mu\vec{n} = 0$$

$$\vec{n}\cdot\Box\vec{n} = i\lambda\,\vec{n}^2 = i\lambda\,, \qquad \vec{n}^2 = 1\,, \quad \partial_\mu(\vec{n}\cdot\vec{n}) = 0$$

$\Box\vec{\pi} = \infty$ set of vectors


small coupling
big factor in front
coupling
N Scalars
* Non-pert ! Not weakly coupled.
E.O.M.
N-1 comp.
N-1 massless fields
many vertices
Useful in the UV

coupling $\sqrt{\lambda}$

small coupling $=$ big factor in front.

$2e_0^2$

coupling

$(n_1 \dots n_N)$

N Scalars

$x^\mu = (x^1, x^2)$

* Non-pert ! Not weakly coupled.

E.O.M. $\longrightarrow$ $\vec{n}^2 = 1$ $\sim$ $\vec{n} = (\sqrt{1-\vec{\pi}^2}, \underbrace{\vec{\pi}}_{N-1 \text{ comp.}})$ $\sim$ $(\partial_\mu \vec{n})^2 = \underbrace{\partial^\mu \vec{\pi} \cdot \partial_\mu \vec{\pi}}_{\substack{N-1 \\ \text{massless} \\ \text{fields}}} + \underbrace{\frac{1}{1-\vec{\pi}^2} (\pi^a \partial_\mu \pi^a)(\pi^b \partial^\mu \pi^b)}_{\infty\text{'ly many vertices}}$

new field

$S = \partial \vec{n}^2 + i\lambda (\vec{n}^2 - 1) = S[\vec{n}, \lambda]$

$\vec{n} \cdot \Box \vec{n} = -(\partial_\mu \vec{n})^2 \Leftarrow \vec{n} \cdot \partial_\mu \vec{n} = 0$

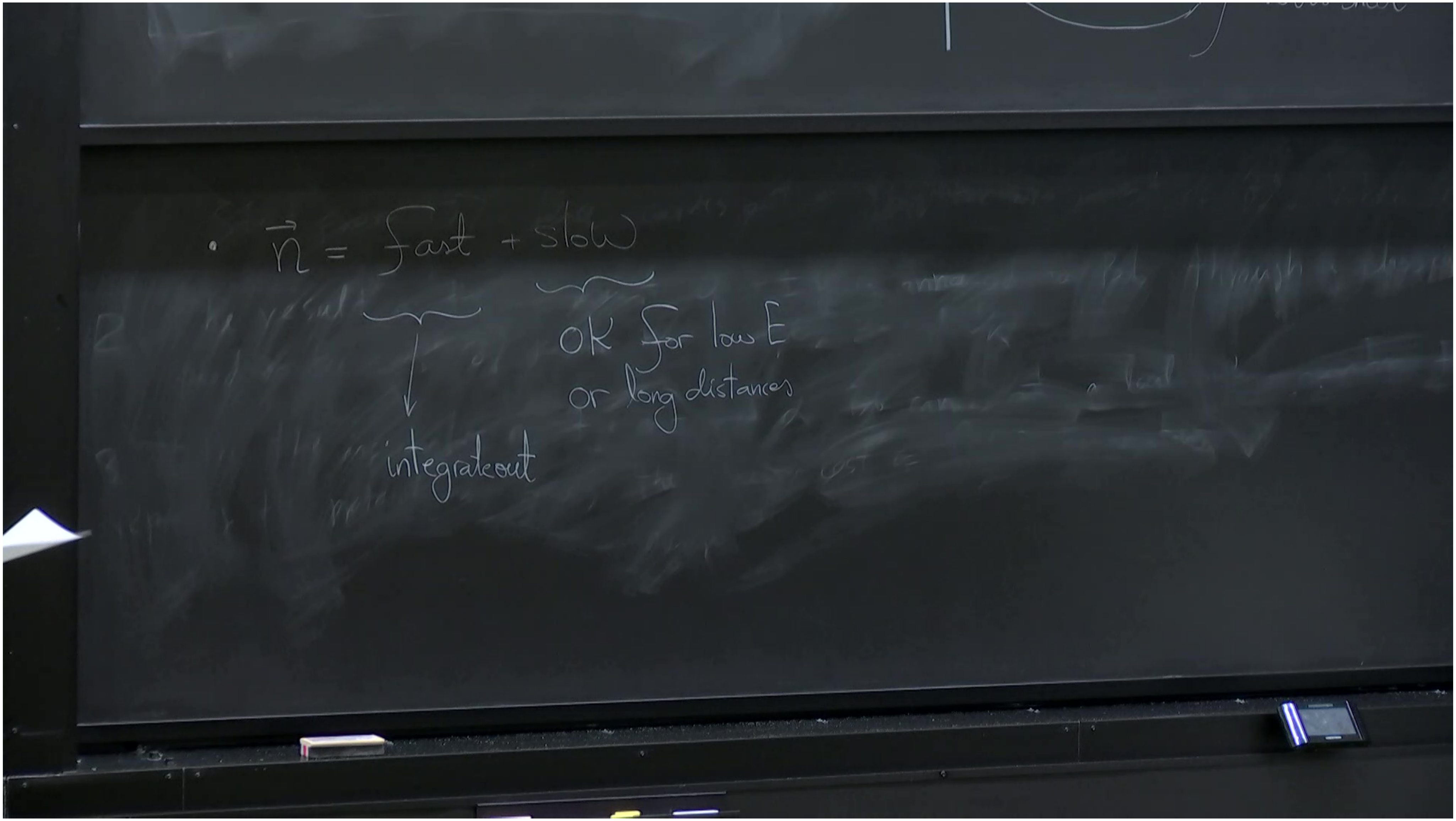
$\frac{\delta}{\delta \vec{n}}$, $\frac{\delta}{\delta \lambda}$: $\begin{cases} \Box \vec{n} = i\lambda \vec{n} \\ \vec{n}^2 = 1 \end{cases}$

$\underbrace{\vec{n} \cdot \Box \vec{n}}_{-(\partial \vec{n})^2} = i\lambda \vec{n}^2 = i\lambda$, $\vec{n}^2 = 1$, $\partial_\mu (\vec{n} \cdot \vec{n}) = 0$

$\boxed{\Box \vec{n} + (\partial \vec{n})^2 \vec{n} = 0}$ $\sim$ non-trivial dynamics

$\Box \vec{\pi} = \infty$ set of vectors

useful in the UV



$\vec{n}$ = fast + slow
integrate out
OK for low E
or long distances

- $\vec{n} = \underbrace{\text{fast}}_{} + \underbrace{\text{slow}}_{}$

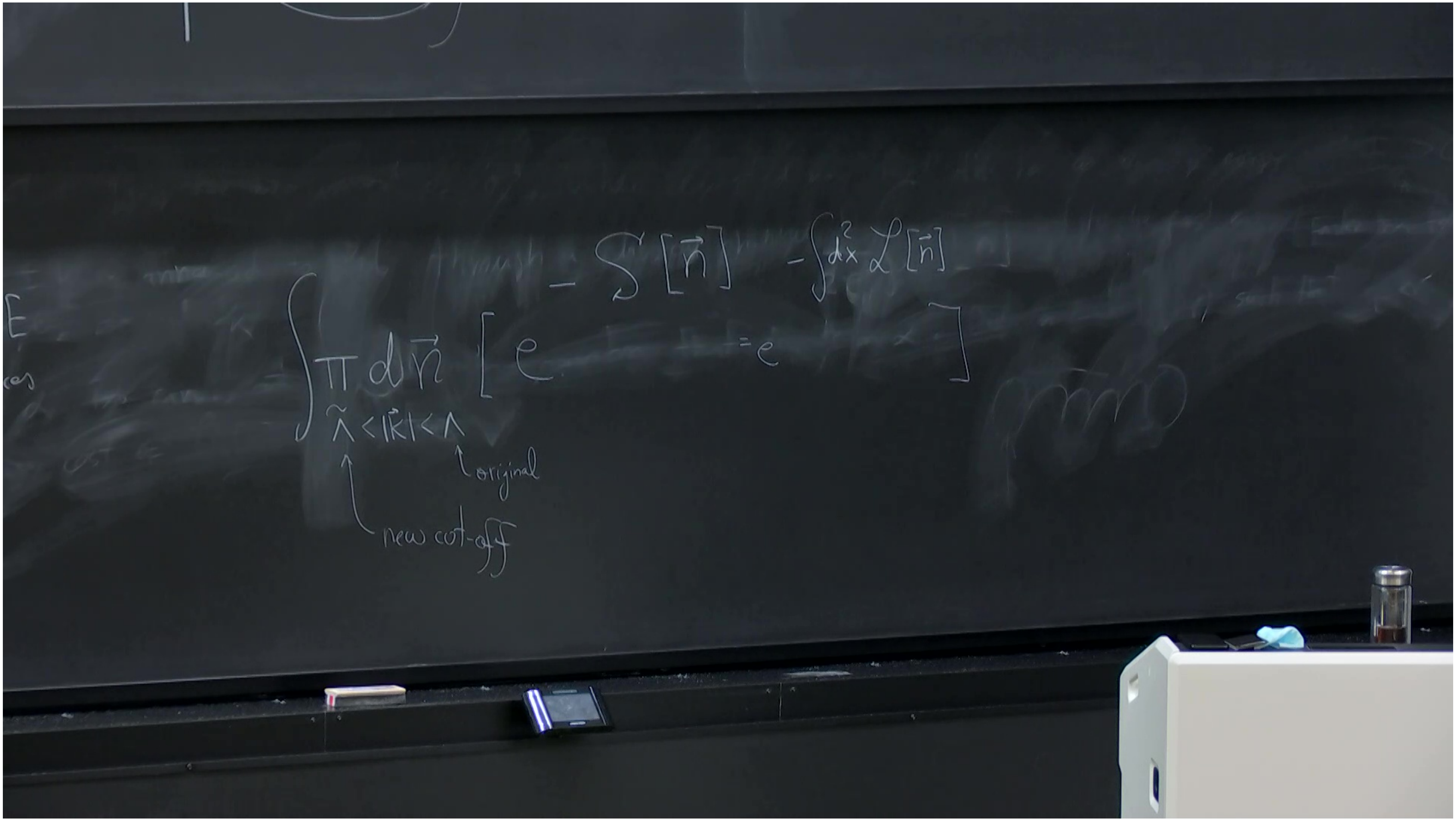
OK for low E
or long distances

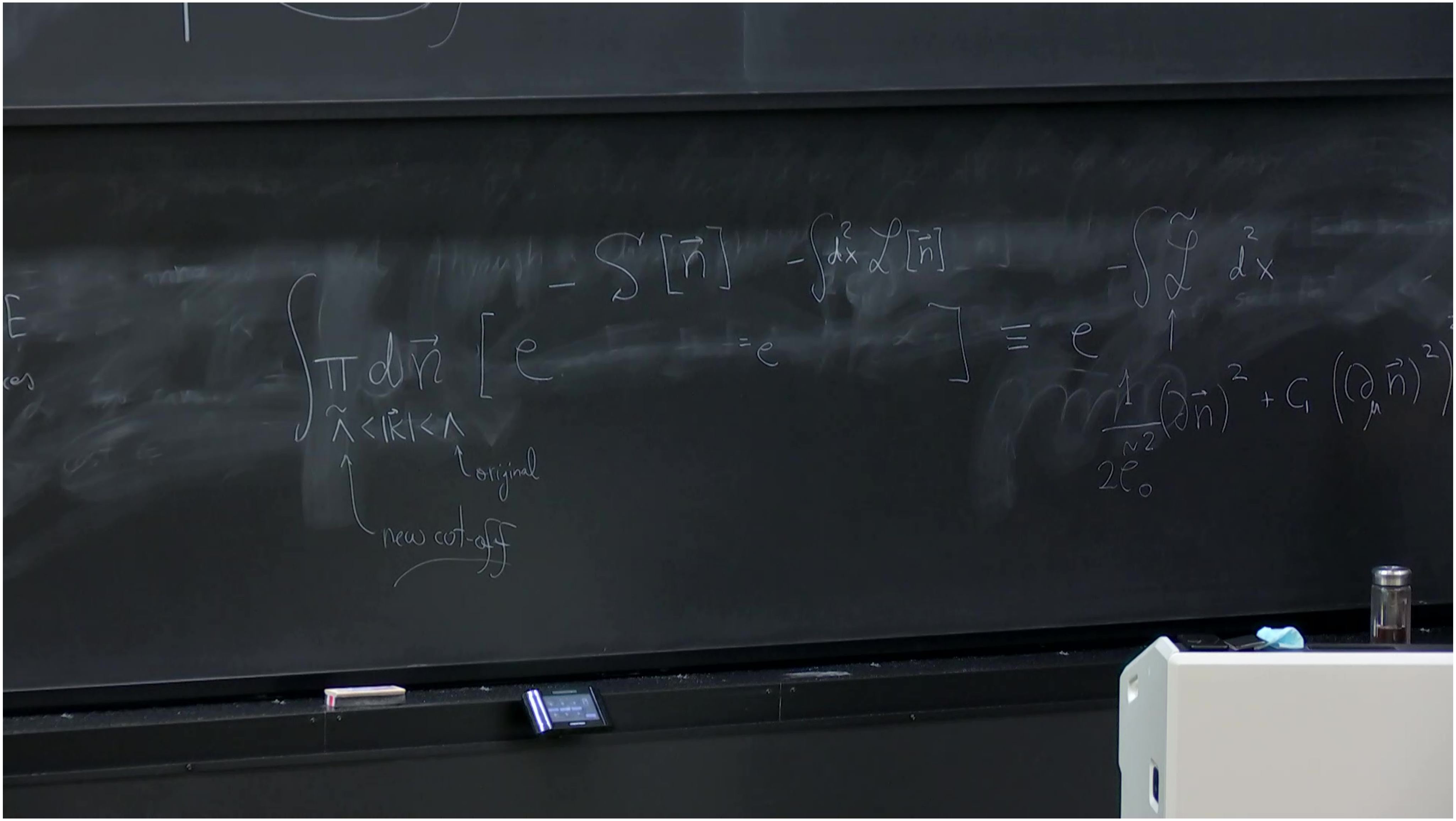
$\downarrow$

can integrate out
if we care about low E
or long dist. questions.

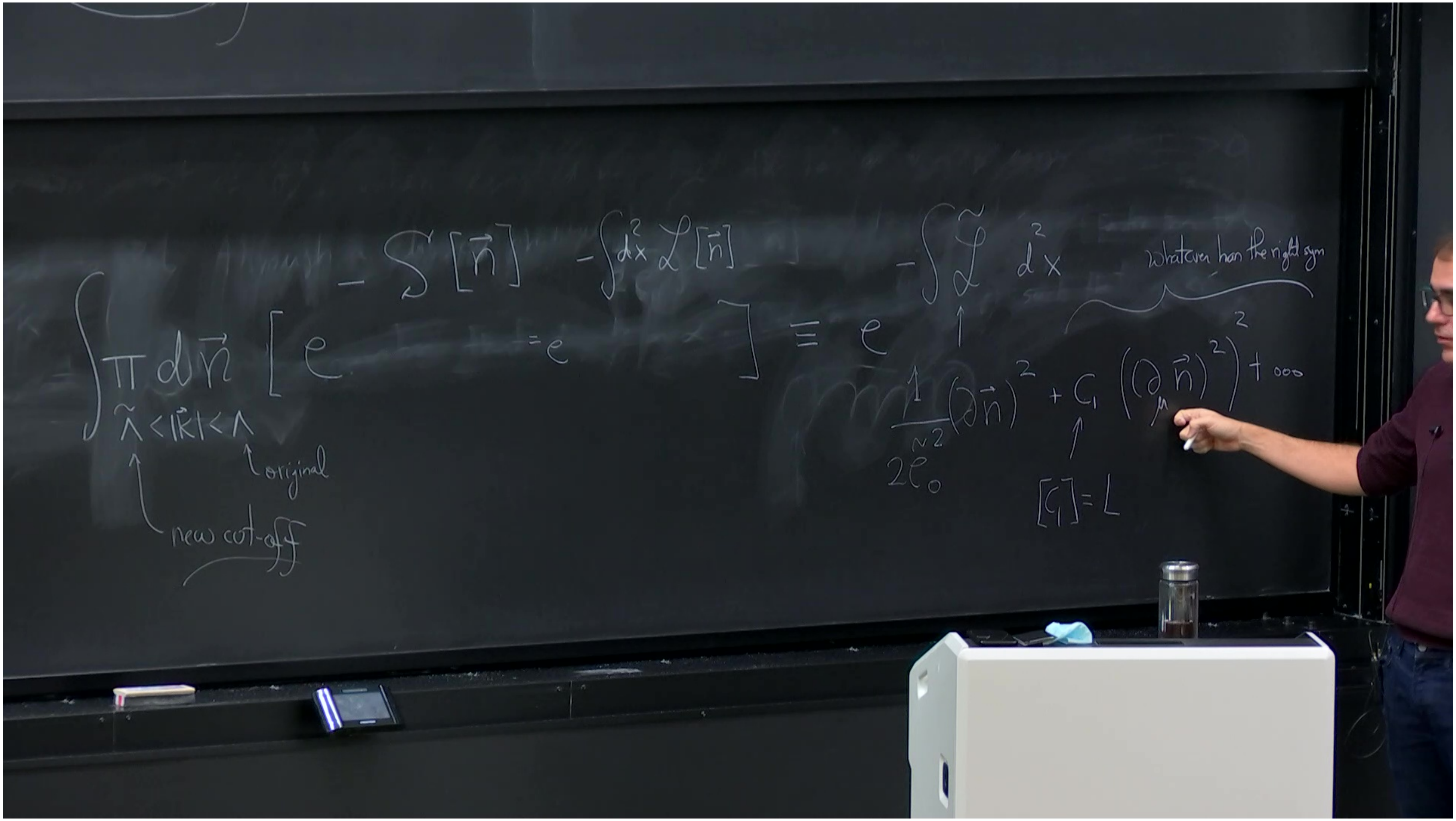
$- S[\vec{n}]$

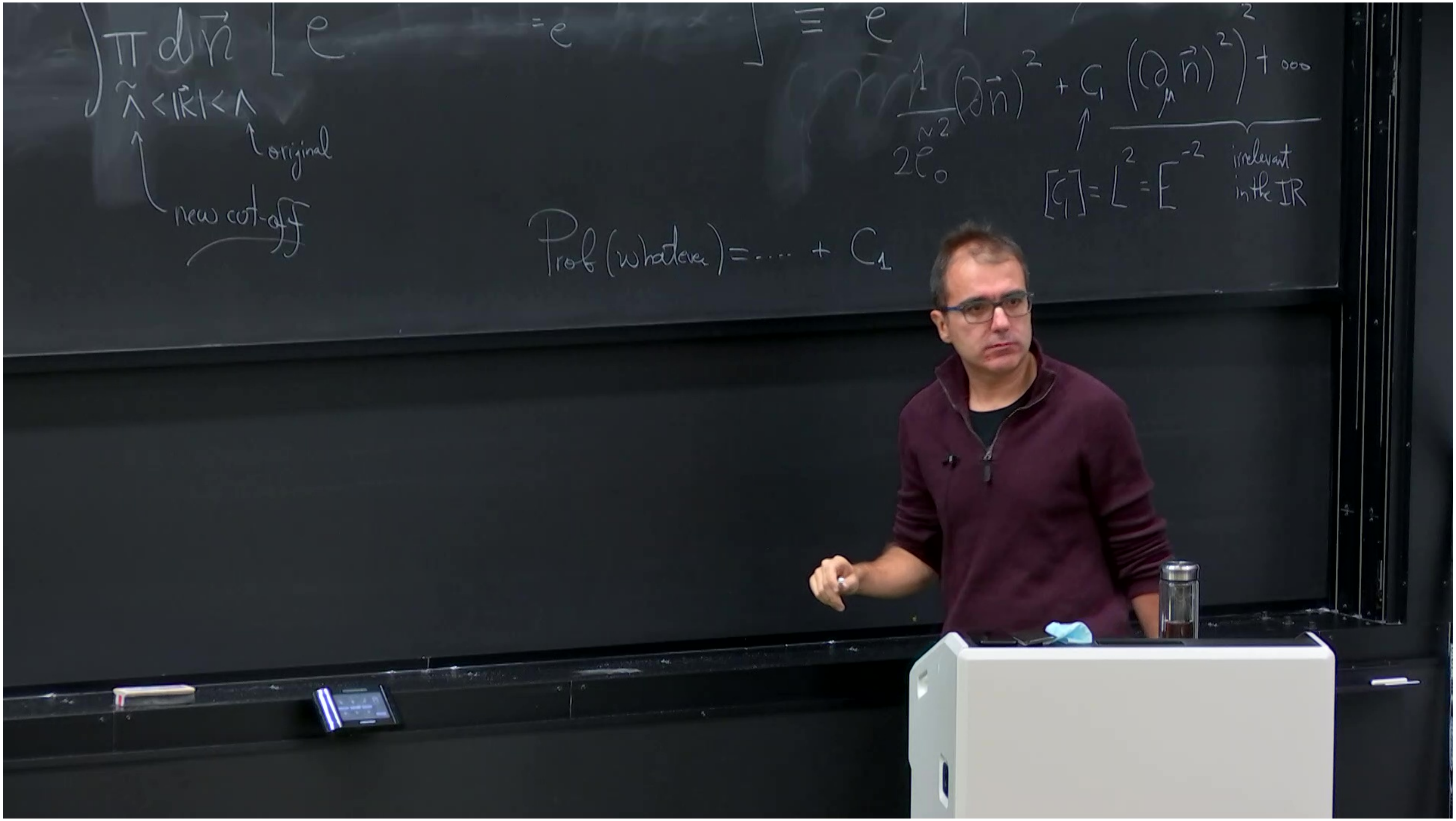
e

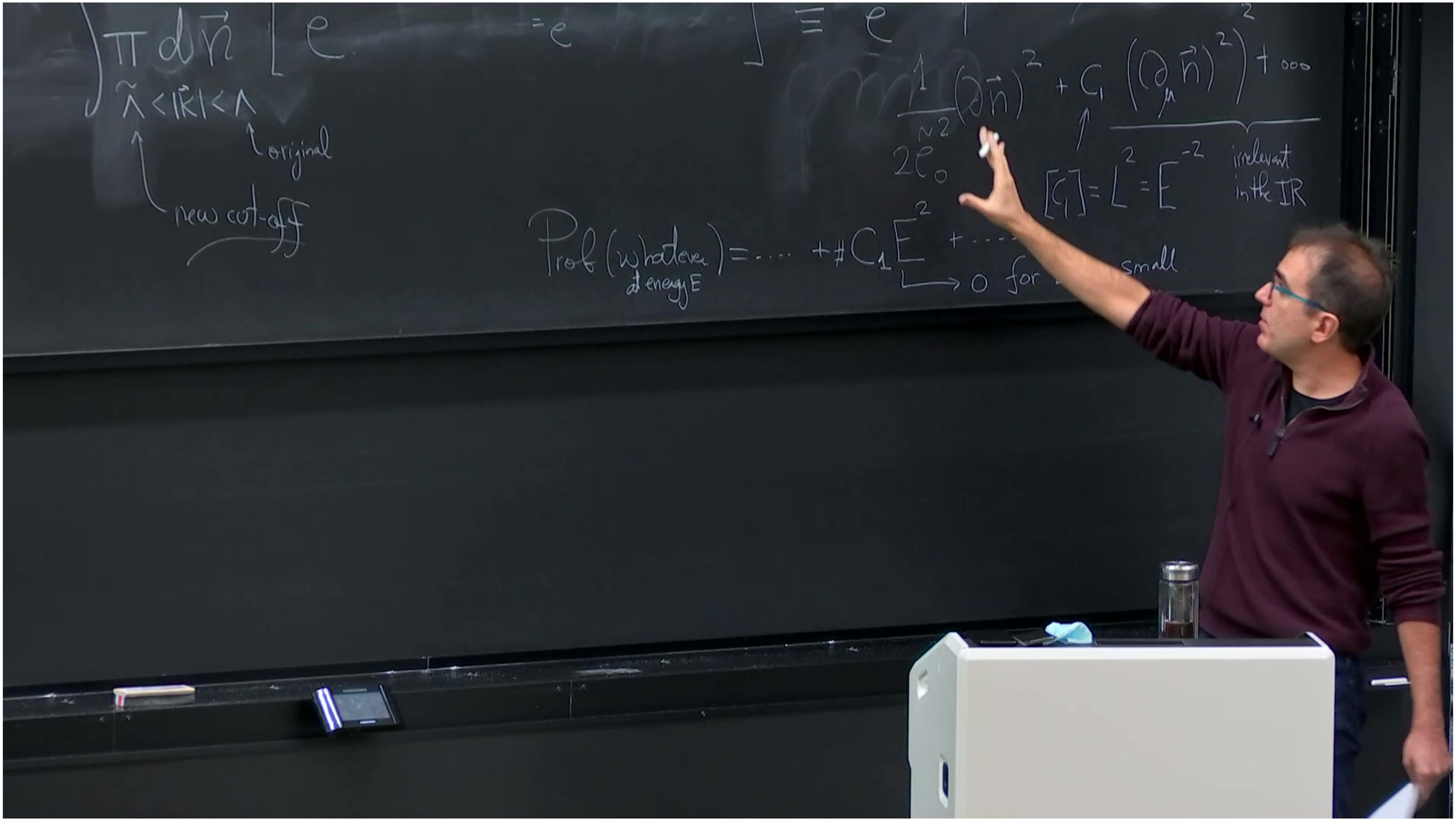

$\int \prod_{\tilde{\Lambda} < |\vec{k}| < \Lambda} d\vec{n} \left[e^{-S[\vec{n}]} = e^{-\int d^2x \, \mathcal{L}[\vec{n}]} \right]$
original
new cut-off

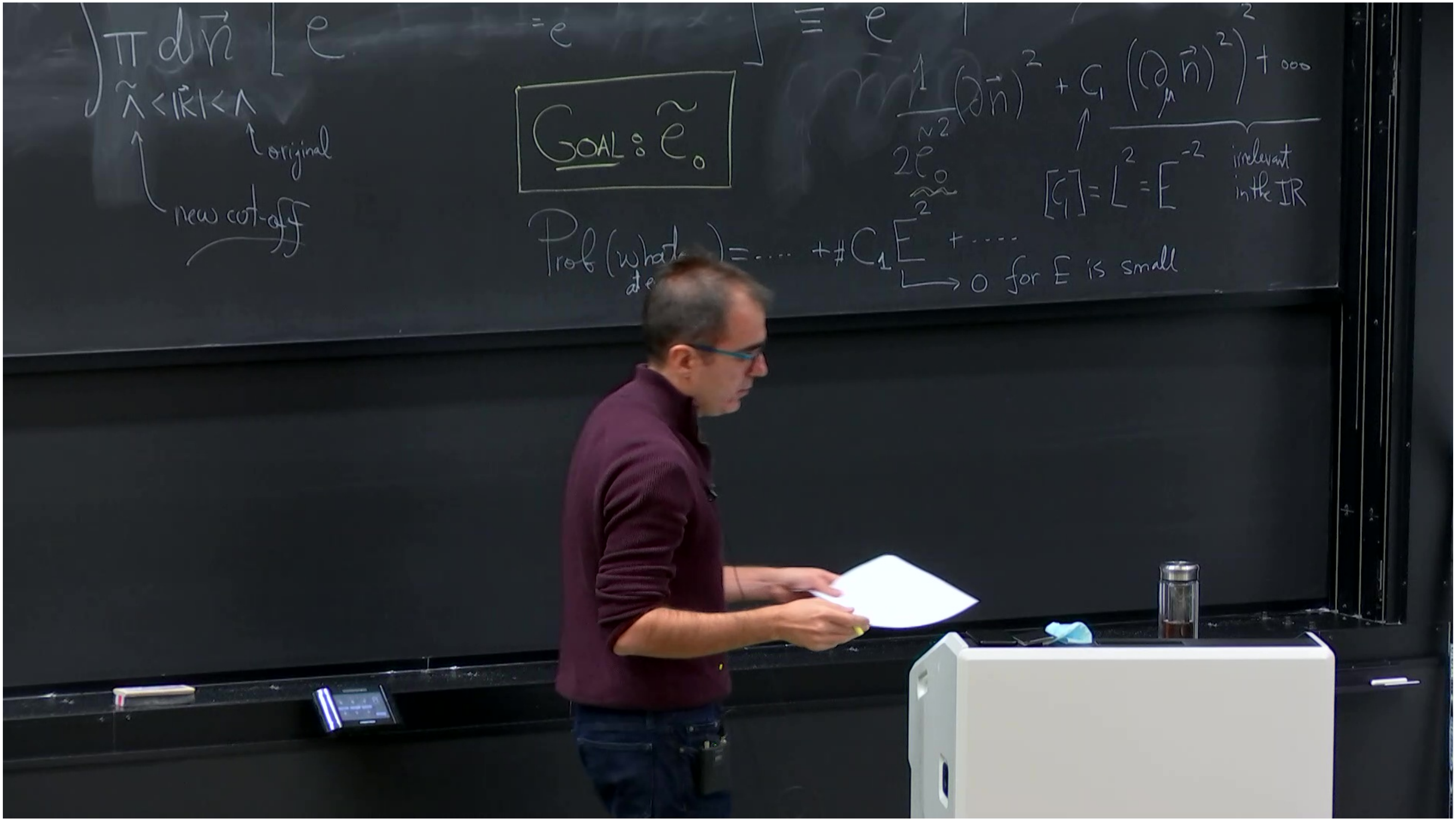


$\int \prod_{\tilde{\Lambda} < |\vec{k}| < \Lambda} d\vec{n} \left[e^{-S[\vec{n}]} = e^{-\int d^2x \, \mathcal{L}[\vec{n}]} \right] \equiv e^{-\int \tilde{\mathcal{L}} \, d^2x}$
$\frac{1}{2\tilde{e}_0^2}(\partial\vec{n})^2 + c_1((\partial_\mu \vec{n})^2)$
original
new cut-off

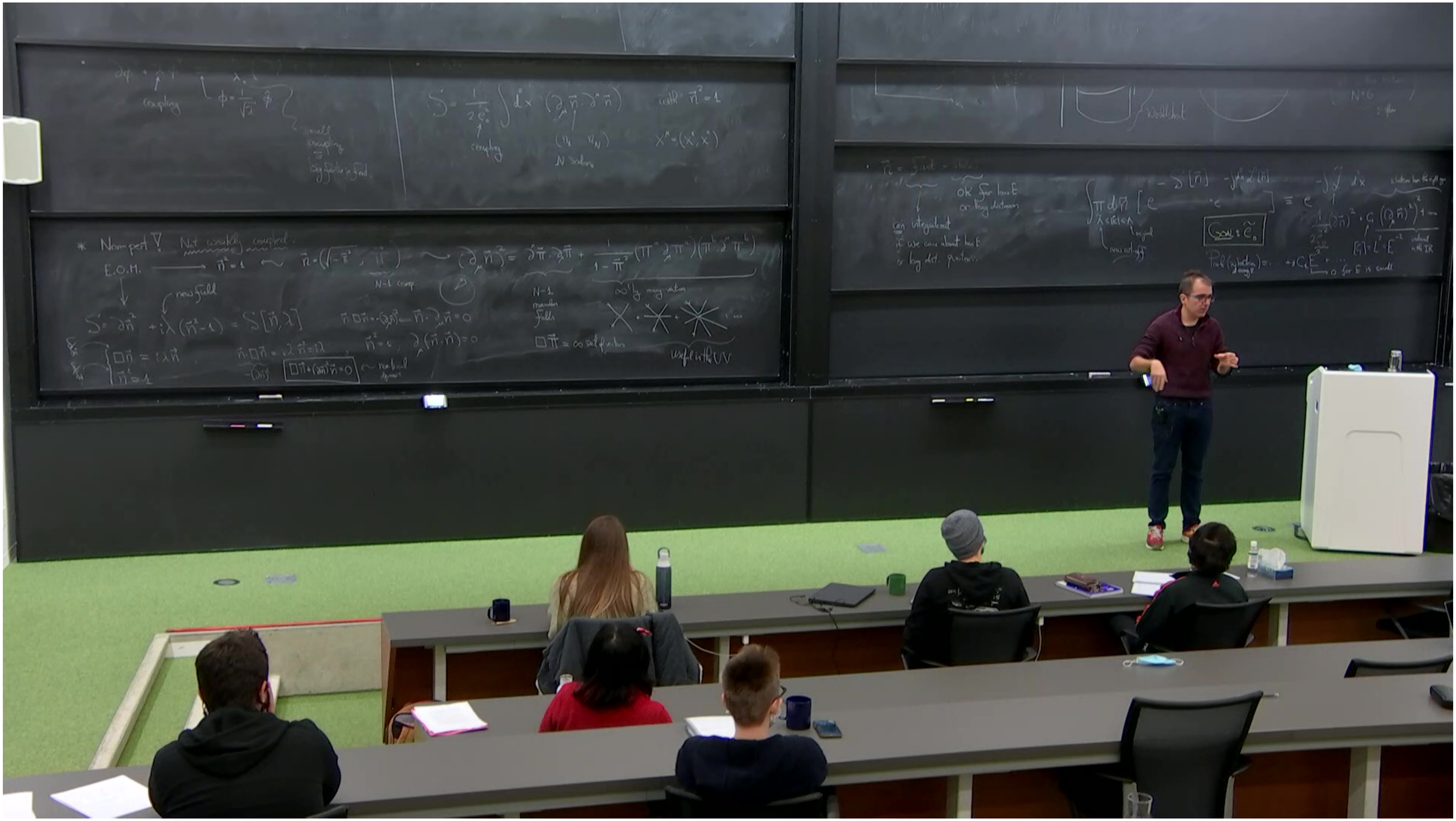

∫ ∏ dn⃗ [e^{−S[n⃗]} = e^{−∫d²x 𝓛[n⃗]}] ≡ e^{−∫ 𝓛̃ d²x}
Λ̃ < |k⃗| < Λ
original
new cut-off
whatever has the right sym
1/(2ẽ₀²) (∂n⃗)² + c₁ ((∂_μ n⃗)²)² + ...
[c₁] = L

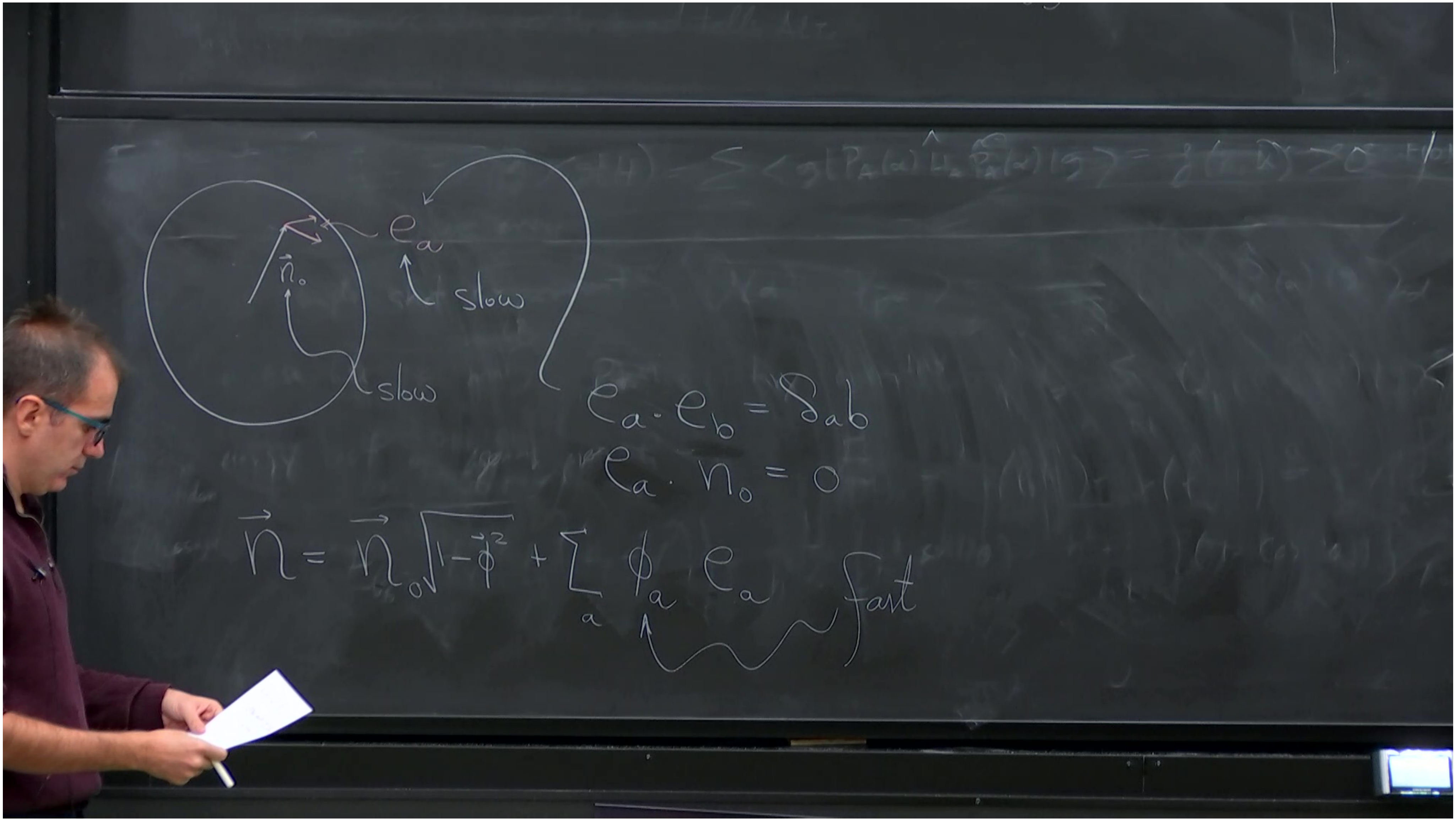

original
new cut-off
Prob (whatever) = ... + C1
[C1] = L^2 = E^-2
irrelevant
in the IR

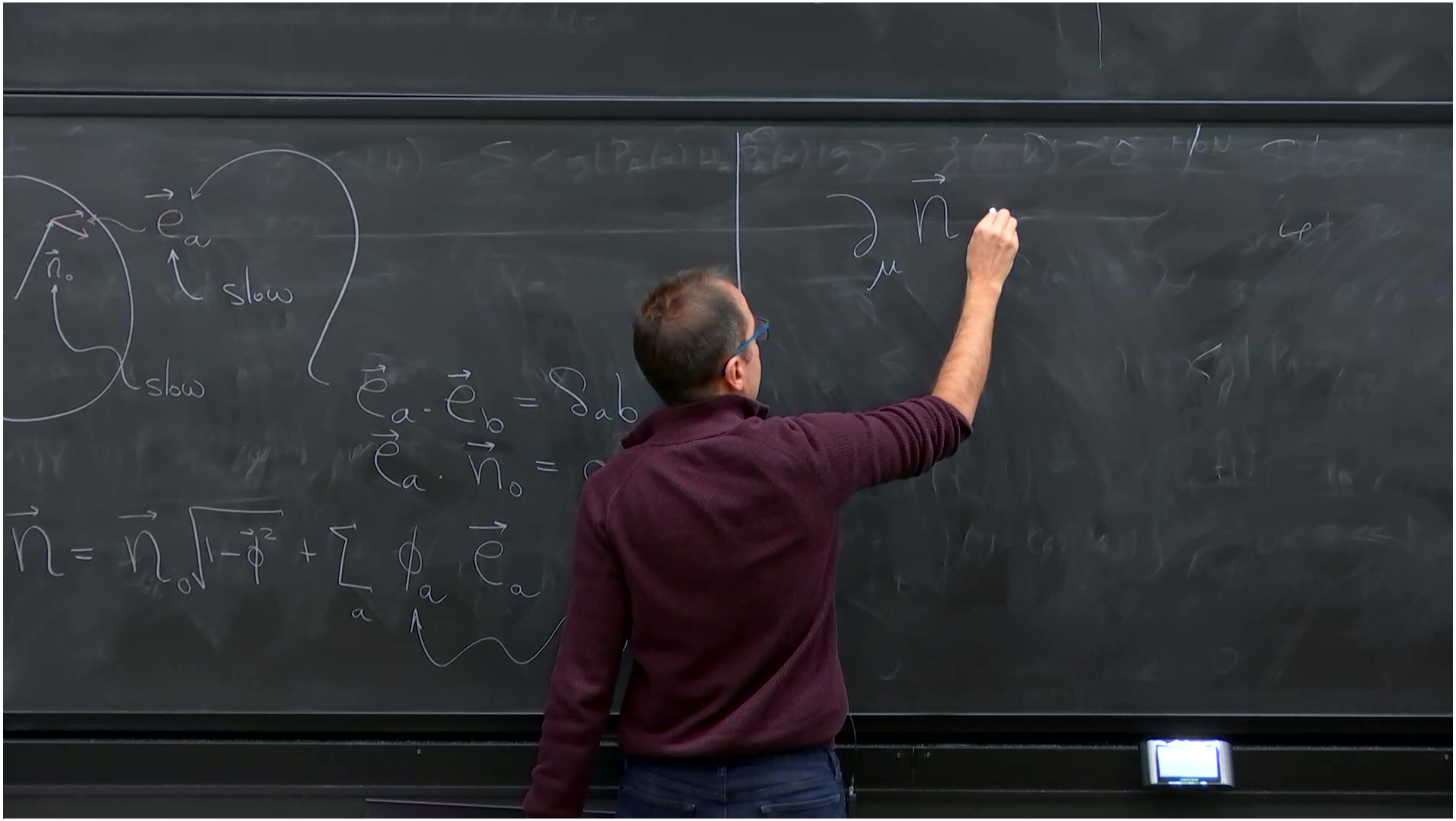

original
new cut-off
irrelevant in the IR
Prob (whatever at energy E) = ... + # C1 E^2 + ...
→ 0 for ... small
[C1] = L^2 = E^-2

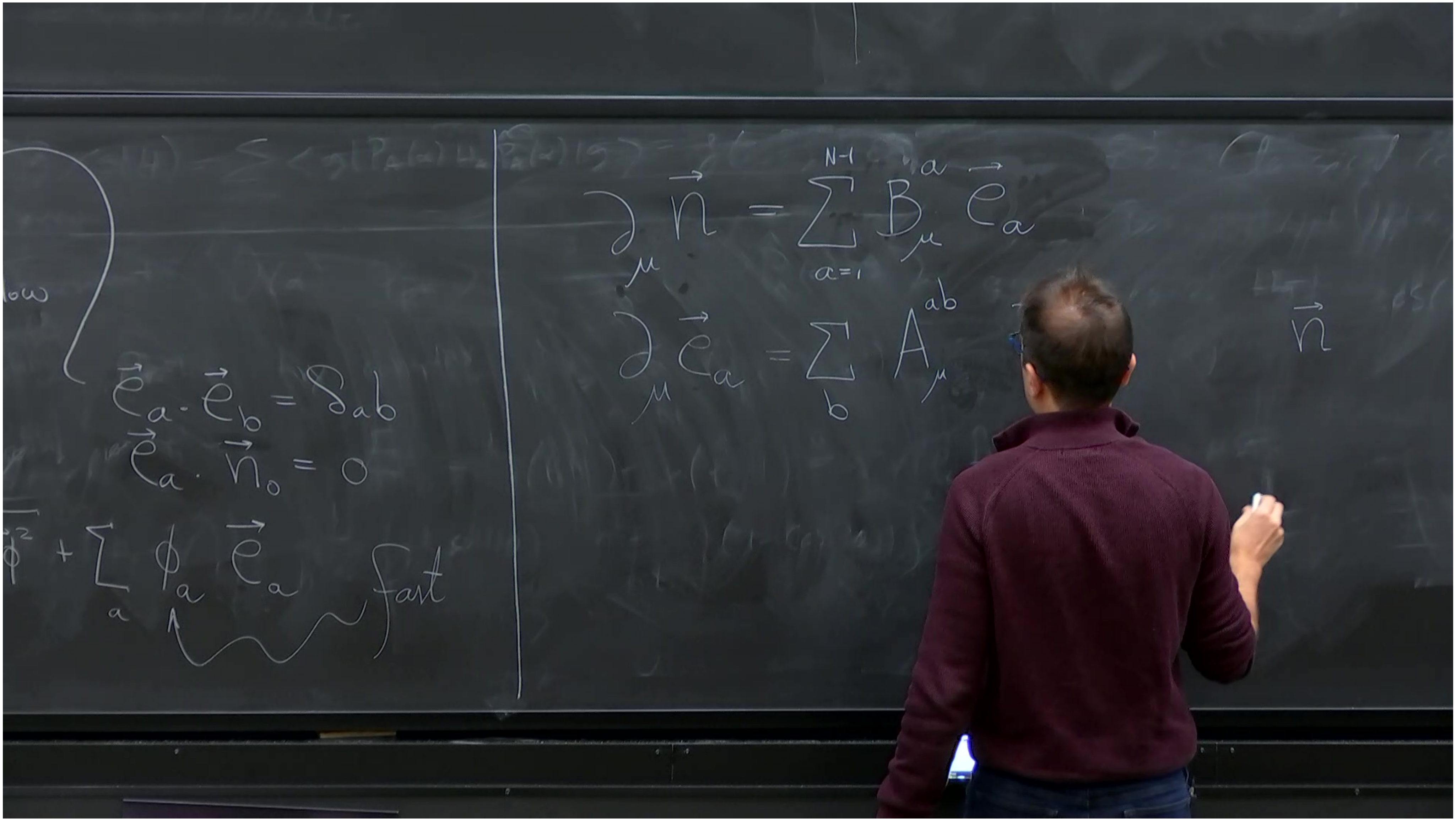


GOAL: $\tilde{e}_0$
original
new cut-off
irrelevant in the IR
→ 0 for E is small




e_a
slow
$\vec{n}_0$
slow
$e_a \cdot e_b = \delta_{ab}$
$e_a \cdot n_0 = 0$
$\vec{n} = \vec{n}_0 \sqrt{1-\vec{\phi}^2} + \sum_a \phi_a e_a$
fast


$\vec{e}_a$
$\vec{n}_0$
slow
slow
$\vec{e}_a \cdot \vec{e}_b = \delta_{ab}$
$\vec{e}_a \cdot \vec{n}_0 = 0$
$\vec{n} = \vec{n}_0\sqrt{1-\vec{\phi}^2} + \sum_a \phi_a \vec{e}_a$
$\partial_\mu \vec{n}$



$\partial_\mu \vec{n} = \sum_{a=1}^{N-1} B_\mu^a \vec{e}_a$
$\partial_\mu \vec{e}_a = \sum_b A_\mu^{ab}$
$\vec{n}$
$\vec{e}_a \cdot \vec{e}_b = \delta_{ab}$
$\vec{e}_a \cdot \vec{n}_0 = 0$
$+ \sum_a \phi_a \vec{e}_a$
fast

low

$$\vec{e}_a \cdot \vec{e}_b = \delta_{ab}$$

$$\vec{e}_a \cdot \vec{n}_0 = 0$$

$$\phi^2 + \sum_a \phi_a \vec{e}_a$$

fast

$$\partial_\mu \vec{n}_0 = \sum_{a=1}^{N-1} B_\mu^a \vec{e}_a$$

$$n_0 \cdot e_a = 0 \Rightarrow$$

$$\partial_\mu \vec{e}_a = \sum_b A_\mu^{ab} \vec{e}_b - B_\mu^a \vec{n}_0$$

$$\partial_\mu \vec{n} = \Big[\qquad \Big] \vec{n}_0$$

$$+ \sum_a \Big[$$

low

$$\vec{e}_a \cdot \vec{e}_b = \delta_{ab}$$

$$\vec{e}_a \cdot \vec{n}_0 = 0$$

$$\ldots\phi^2 + \sum_a \phi_a \vec{e}_a$$

fast

$$\partial_\mu \vec{n}_0 = \sum_{a=1}^{N-1} B^a_\mu \vec{e}_a$$

$$n_0 \cdot e_a = 0 \Rightarrow$$

$$\partial_\mu \vec{e}_a = \sum_b A^{ab}_\mu \vec{e}_b - B^a_\mu \vec{n}_0$$

$$\partial_\mu \vec{n} = \left[\partial_\mu \sqrt{1-\vec{\phi}^2} - B^a_\mu \phi_a\right] \vec{n}_0$$

$$+ \sum_a \left[\sqrt{1-\phi^2}\, B^a_\mu + \partial_\mu \phi_a + A^{ab}_\mu \phi_b\right.$$

$$\sum_{a=1}^{N-1} B^a_\mu \vec{e}_a$$

$$n_0 \cdot e_a = 0 \Rightarrow \partial n_0 \cdot e_a = - n_0 \cdot \partial e_a$$

$$= \sum_b A^{ab}_\mu \vec{e}_b - B^a_\mu \vec{n}_0$$

$$\left[\partial_\mu \sqrt{1-\vec{\phi}^2} - B^a_\mu \phi_a\right]\vec{n}_0$$

$$\mathcal{L} = \frac{1}{2e_0^2}\left([\ldots]^2 + \sum_a [\ldots]^2 \right)$$

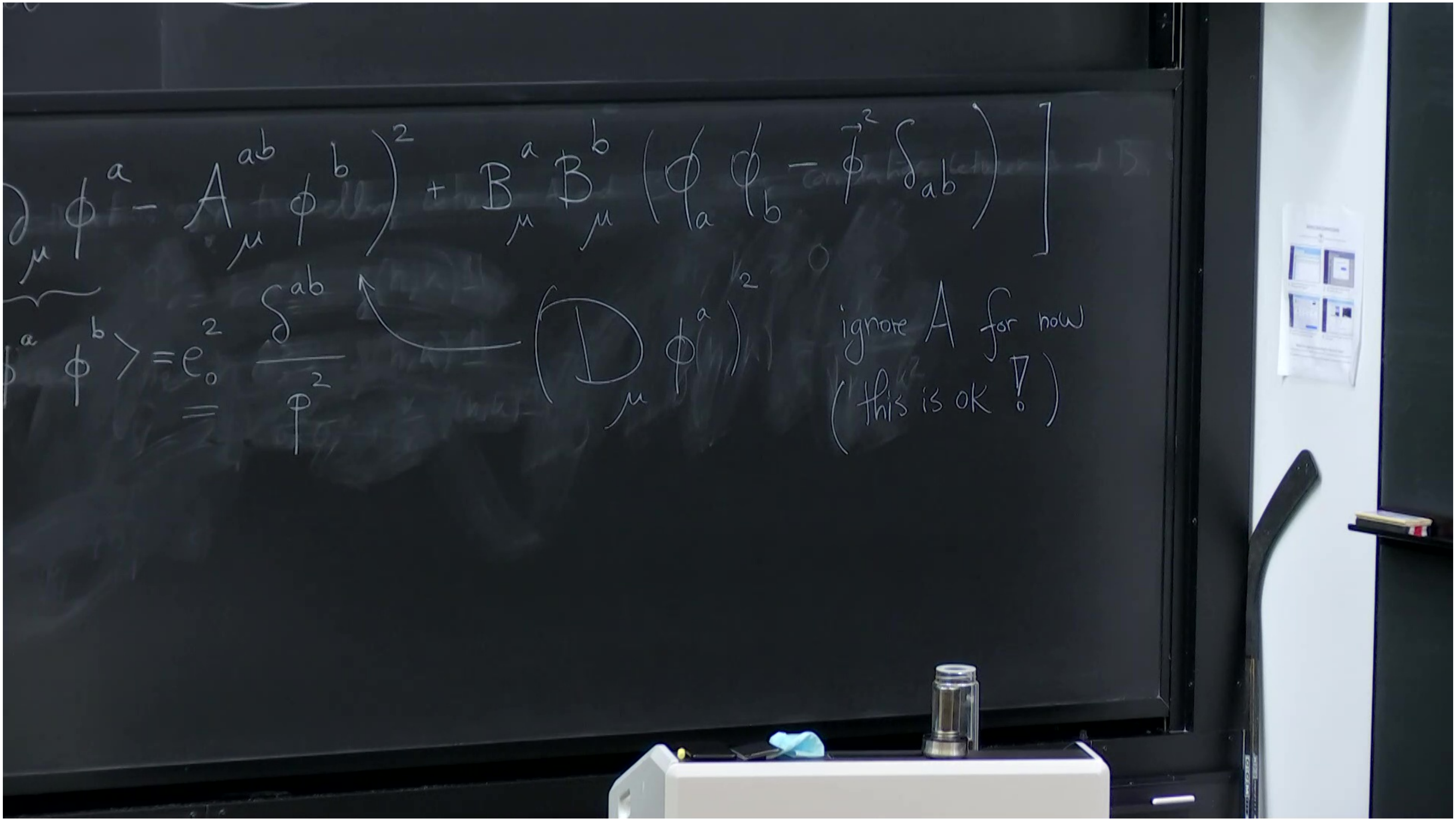
$$\sum_a \left[\sqrt{1-\phi^2}\, B^a_\mu + \partial_\mu \phi_a + A^{ab}_\mu \phi_b\right]\vec{e}_a$$

Worldsheet

$$\mathcal{L}\Big|_{\substack{\text{up to quadractic}\\ \text{terms in } \phi}} \doteq \frac{1}{2e_0^2}\int d^2x \, (B^a_\mu)^2 + \frac{1}{2e_0^2}\int d^2x \, (\partial_\mu \phi$$

$$)^2 + \frac{1}{2e_0^2}\int d^2x\left[\left(\underbrace{\partial_\mu\phi^a}_{} - A_\mu^{ab}\phi^b\right)^2 + B_\mu^a B_\mu^b\left(\phi_a\phi_b - \vec{\phi}^2\delta\right.\right.$$

$$\langle\phi^a\phi^b\rangle = e_0^2\,\frac{\delta^{ab}}{\varphi^2} \qquad \left(D_\mu\phi^a\right)^2$$


$\partial_\mu \phi^a - A_\mu^{ab} \phi^b)^2 + B_\mu^a B_\mu^b (\phi_a \phi_b - \vec{\phi}^2 \delta_{ab})]$
$\phi^a \phi^b \rangle = e_0^2 \frac{\delta^{ab}}{\varphi^2}$
$(D_\mu \phi^a)^2$
ignore A for now
(this is OK!)

$$\mathcal{L}\Big|_{\text{up to quadractic terms in } \phi} = \frac{1}{2e_0^2}\int d^2x\, (B^a_\mu)^2 + \frac{1}{2e_0^2}\int d^2x \Big[\underbrace{(\partial_\mu \phi}_{} \ldots$$

$$\langle \phi^a \phi \ldots$$

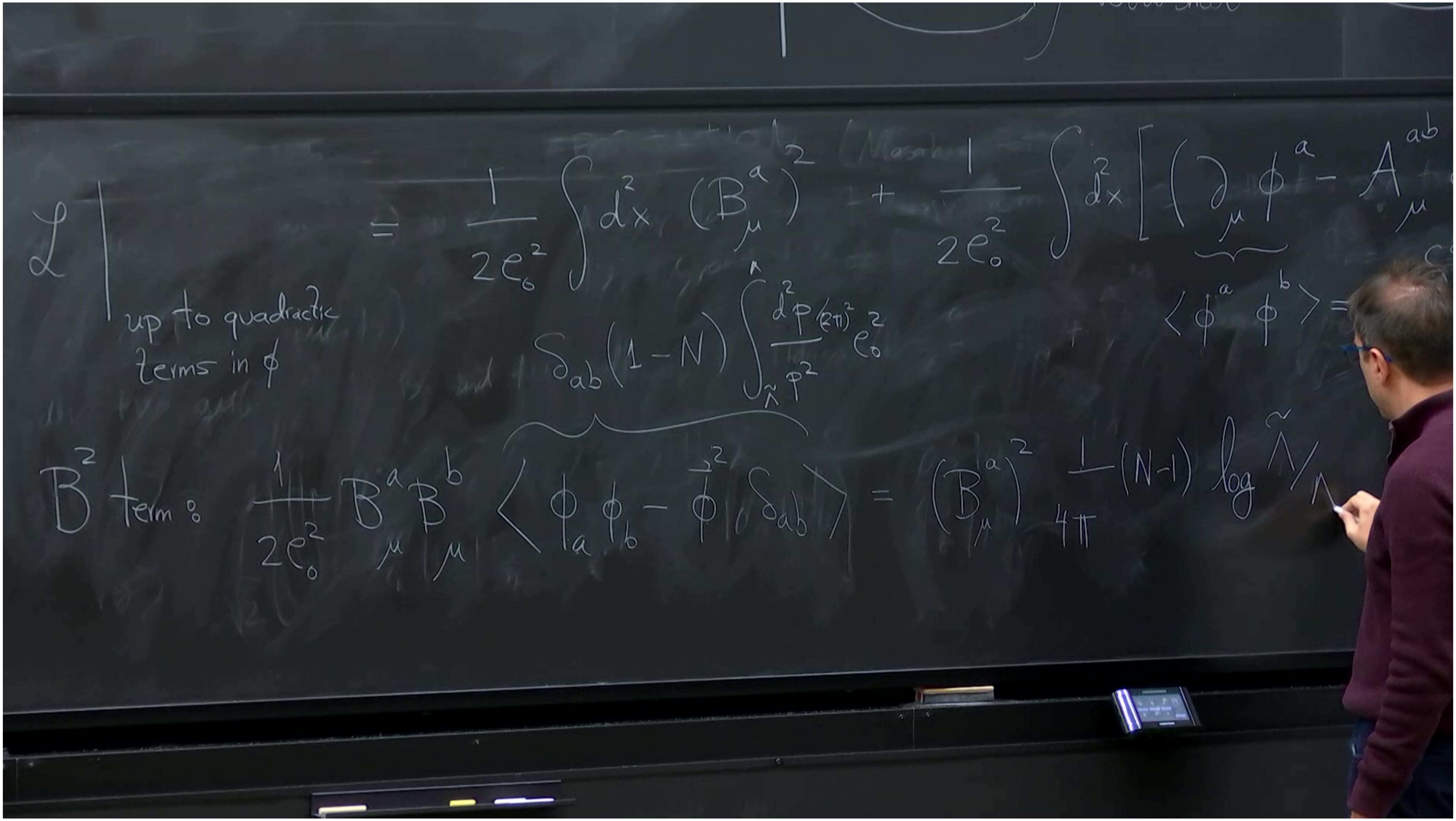
B^2 term : $\frac{1}{2e_0^2} B^a_\mu B^b_\mu \langle \phi_a \phi_b - \vec{\phi}^2 \delta_{ab} \rangle$

$$\mathcal{L}\Big|_{\text{up to quadratic terms in } \phi} = \frac{1}{2e_0^2}\int d^2x \, (B^a_\mu)^2 + \frac{1}{2e_0^2}\int d^2x \Big[\big(\underbrace{\partial_\mu \phi^a} - A^{ab}_\mu$$

$$\langle \phi^a \phi^b \rangle = e_0^2$$

$$\underbrace{\delta_{ab}(1-N)\int_{\tilde\Lambda}^{\Lambda} \frac{d^2p}{p^2 (2\pi)^2} e_0^2}$$

$$B^2 \text{ term}: \quad \frac{1}{2e_0^2} B^a_\mu B^b_\mu \langle \phi_a \phi_b - \vec{\phi}^2 \delta_{ab} \rangle$$



$\mathcal{L}\Big|_{\text{up to quadratic terms in } \phi} = \frac{1}{2e_0^2}\int d^2x\, (B^a_\mu)^2 + \frac{1}{2e_0^2}\int d^2x \left[(\partial_\mu \phi^a - A^{ab}_\mu$
$\delta_{ab}(1-N)\int_{\tilde{\Lambda}}^{\Lambda} \frac{d^2p}{(2\pi)^2}\frac{e_0^2}{p^2}$
$\langle \phi^a \phi^b \rangle =$
B^2 term: $\frac{1}{2e_0^2} B^a_\mu B^b_\mu \langle \phi_a \phi_b - \vec{\phi}^2 \delta_{ab} \rangle = (B^a_\mu)^2 \frac{1}{4\pi}(N-1)\log \tilde{\Lambda}/\Lambda$

$$\mathcal{L}\Big|_{\substack{\text{up to quadractic}\\ \text{terms in } \phi}} = \frac{1}{2e_0^2}\int d^2x\,(B^a_\mu)^2 + \frac{1}{2e_0^2}\int d^2x\Big[\big(\underbrace{\partial_\mu\phi^a}\ - A^{ab}_\mu$$

$$\langle \phi^a \phi^b \rangle = e_0^2$$

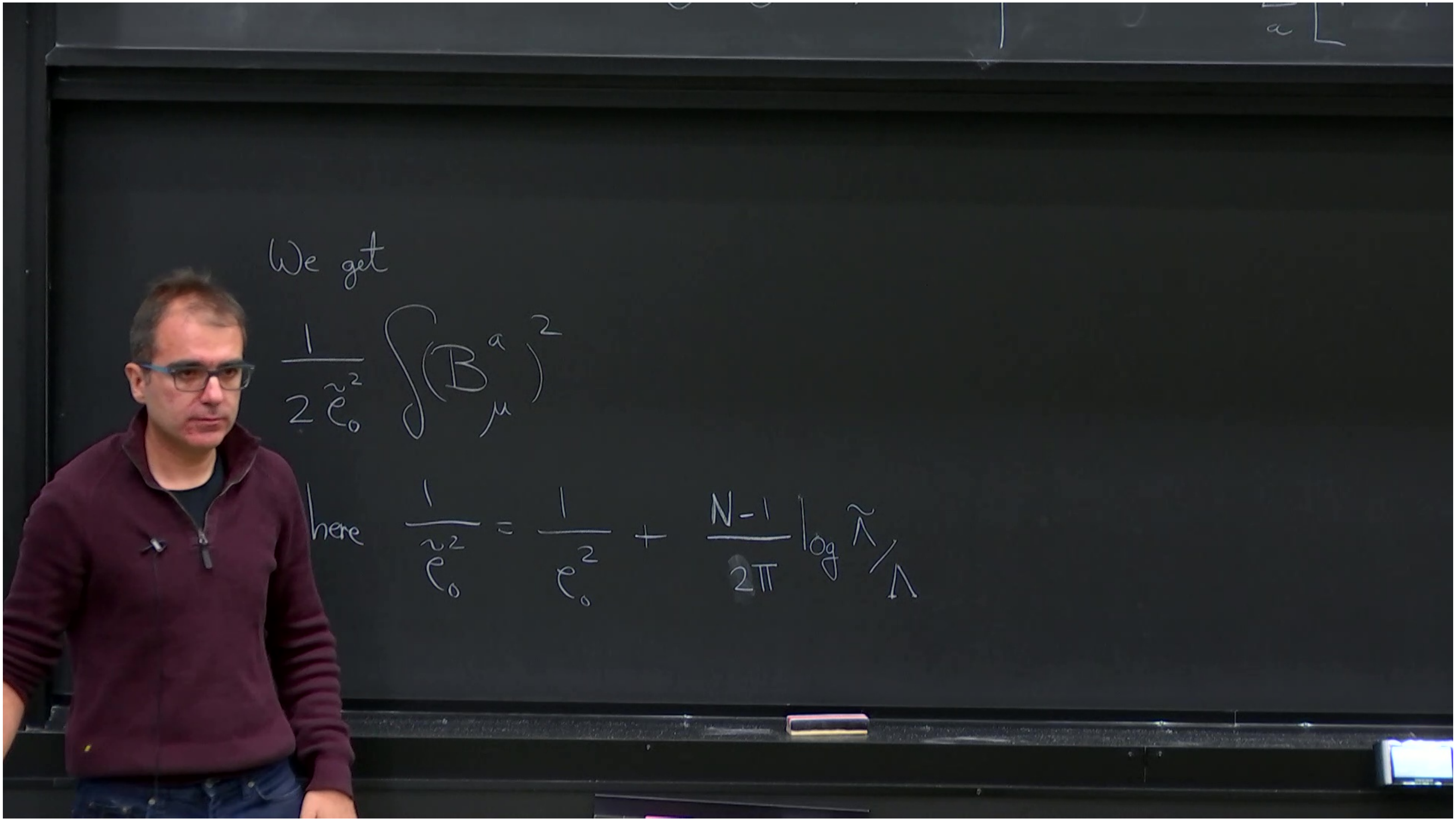
$$\delta_{ab}(1-N)\int_{\tilde\Lambda}^{\Lambda}\frac{d^2p}{p^2 (2\pi)^2}\, e_0^2$$

$$B^2 \text{ term: } \frac{1}{2e_0^2} B^a_\mu B^b_\mu \overbrace{\langle \phi_a\phi_b - \vec\phi^{\,2}\delta_{ab}\rangle} = \boxed{(B^a_\mu)^2 \frac{1}{4\pi}(N-1)\log\tilde\Lambda/\Lambda}$$

We get

$$\frac{1}{2\tilde{e}_0^2}\int (B^a_\mu)^2$$

Where $\frac{1}{\tilde{e}_0^2} = \frac{1}{e_0^2} + \frac{N-1}{4\pi}\log \tilde{\Lambda}/\Lambda$


We get
$\frac{1}{2\tilde{e}_0^2}\int(B^a_\mu)^2$
here $\frac{1}{\tilde{e}_0^2} = \frac{1}{e_0^2} + \frac{N-1}{2\pi}\log \tilde{\Lambda}/\Lambda$

We get

$$\frac{1}{2\tilde{e}_0^2}\int (B_\mu^a)^2$$

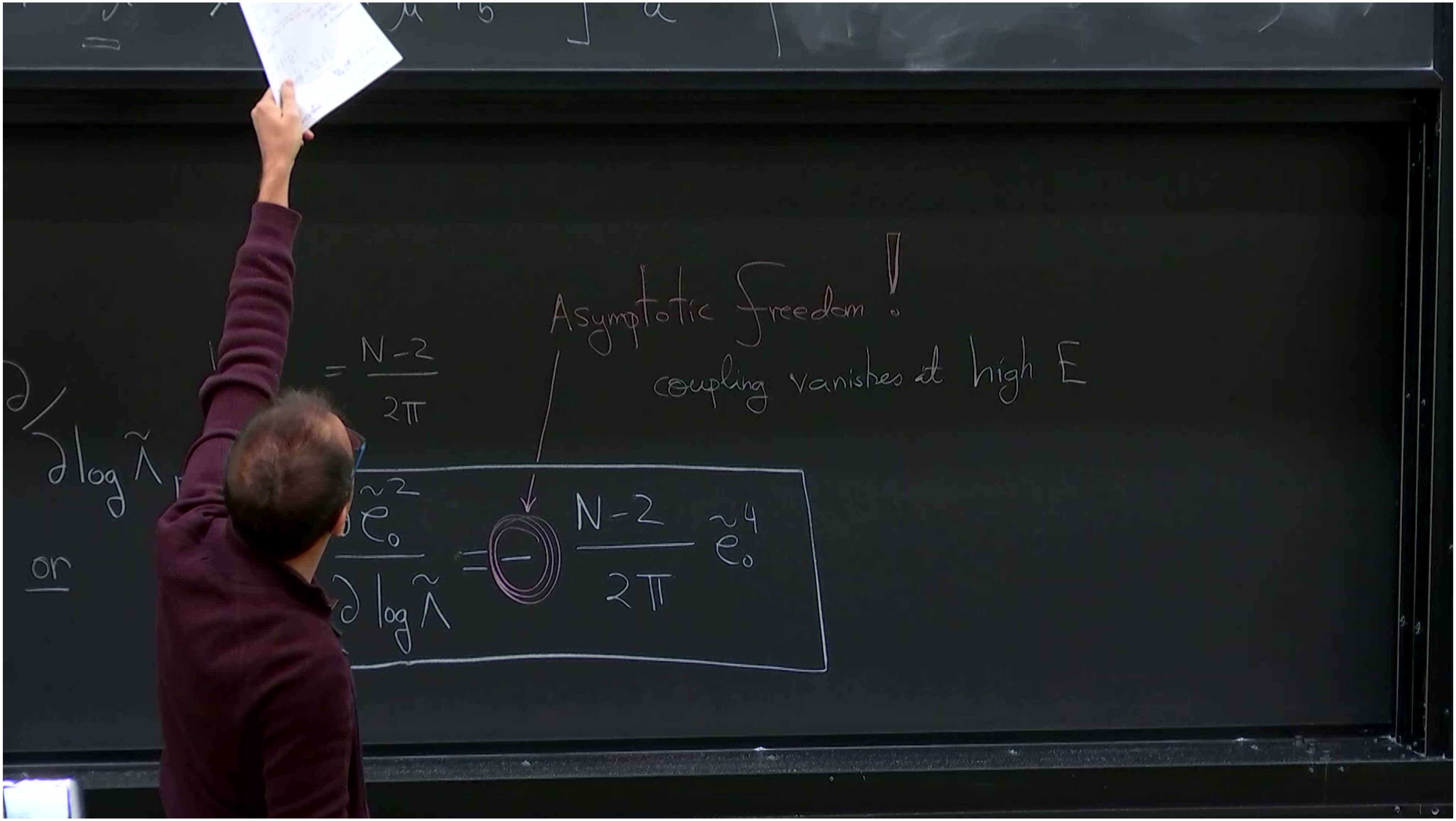
Where $\displaystyle \frac{1}{\tilde{e}_0^2} = \frac{1}{e_0^2} + \frac{N-1}{2\pi}\log \tilde{\Lambda}/\Lambda$ $\leftsquigarrow$ new coupling

$$\frac{\partial}{\partial \log \tilde{\Lambda}} \frac{1}{\tilde{e}_0^2} = \frac{N-2}{2\pi}$$

~ new coupling

or

$$\boxed{\beta \equiv \frac{\partial \tilde{e}_0^2}{\partial \log \tilde{\Lambda}} = -\frac{N-2}{2\pi} \tilde{e}_0^4}$$


Asymptotic freedom!
coupling vanishes at high E
or
$\frac{\partial \tilde{e}_0^2}{\partial \log \tilde{\Lambda}} = - \frac{N-2}{2\pi} \tilde{e}_0^4$

$$\frac{\partial}{\partial \log \tilde{\Lambda}} \frac{1}{\tilde{e}_0^2} = \frac{N-2}{2\pi}$$

or

$$\boxed{\beta \equiv \frac{\partial \tilde{e}_0^2}{\partial \log \tilde{\Lambda}} = - \frac{N-2}{2\pi} \tilde{e}_0^4}$$

Asymptotic freedom!

coupling vanishes at high E $\rightsquigarrow$ N−1 free scalars

coupling grows at low E $\rightarrow$ strong dynamics in the IR.

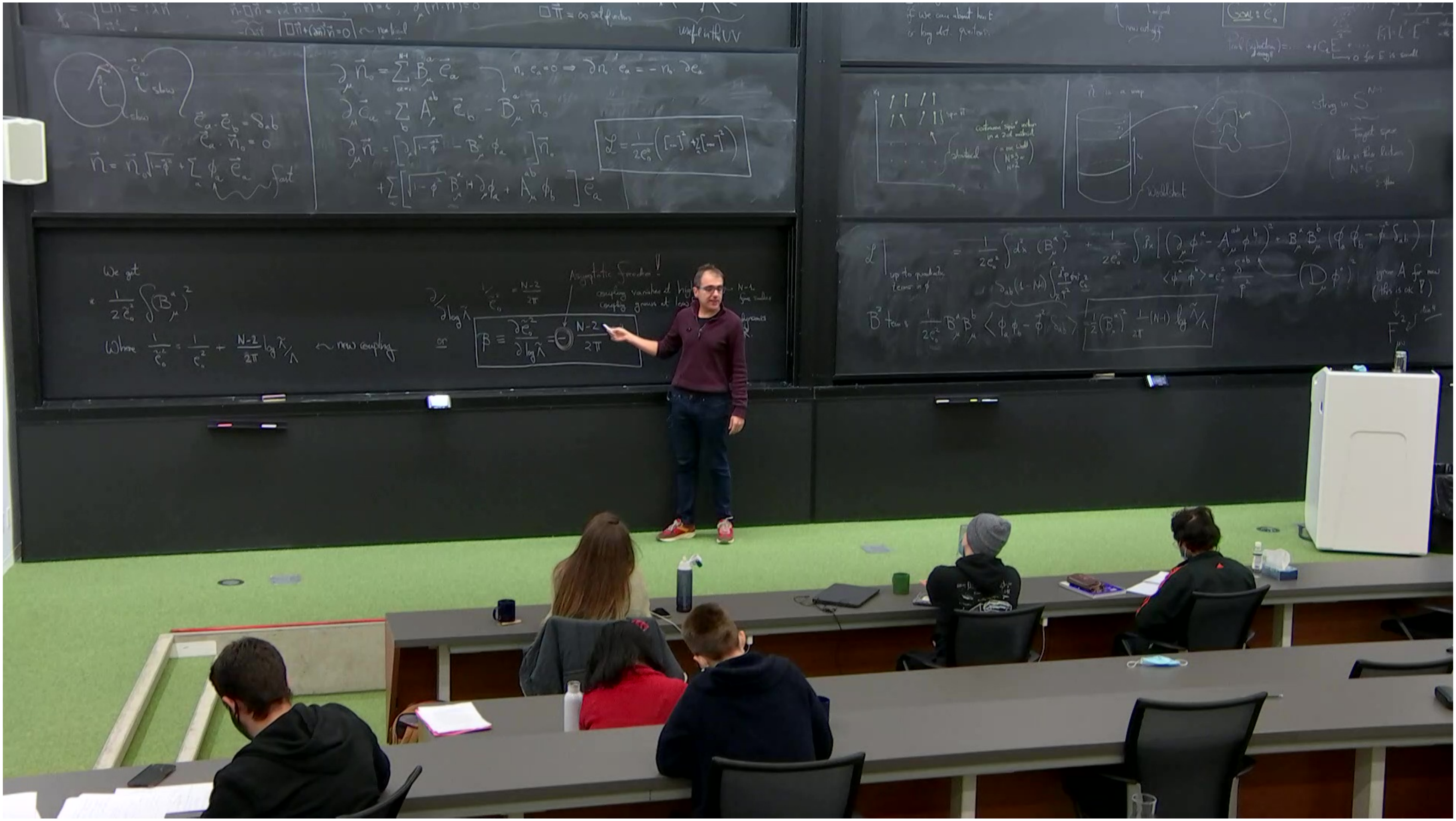
$$\int d^2x \left[\underbrace{(\partial_\mu \phi^a - A_\mu^{ab}\phi^b)^2}_{} + B_\mu^a B_\mu^b \left(\phi_a \phi_b - \vec{\phi}^2 \delta_{ab}\right) \right]$$

$$\langle \phi^a \phi^b \rangle = e_0^2 \frac{\delta^{ab}}{\rho^2} \qquad (D_\mu \phi^a)^2$$

ignore A for now

(this is ok !)

$$\boxed{\frac{1}{2\pi}(N-1)\log \tilde{\Lambda}/\Lambda}$$


We got
Where
new coupling
Asymptotic freedom!
worldsheet