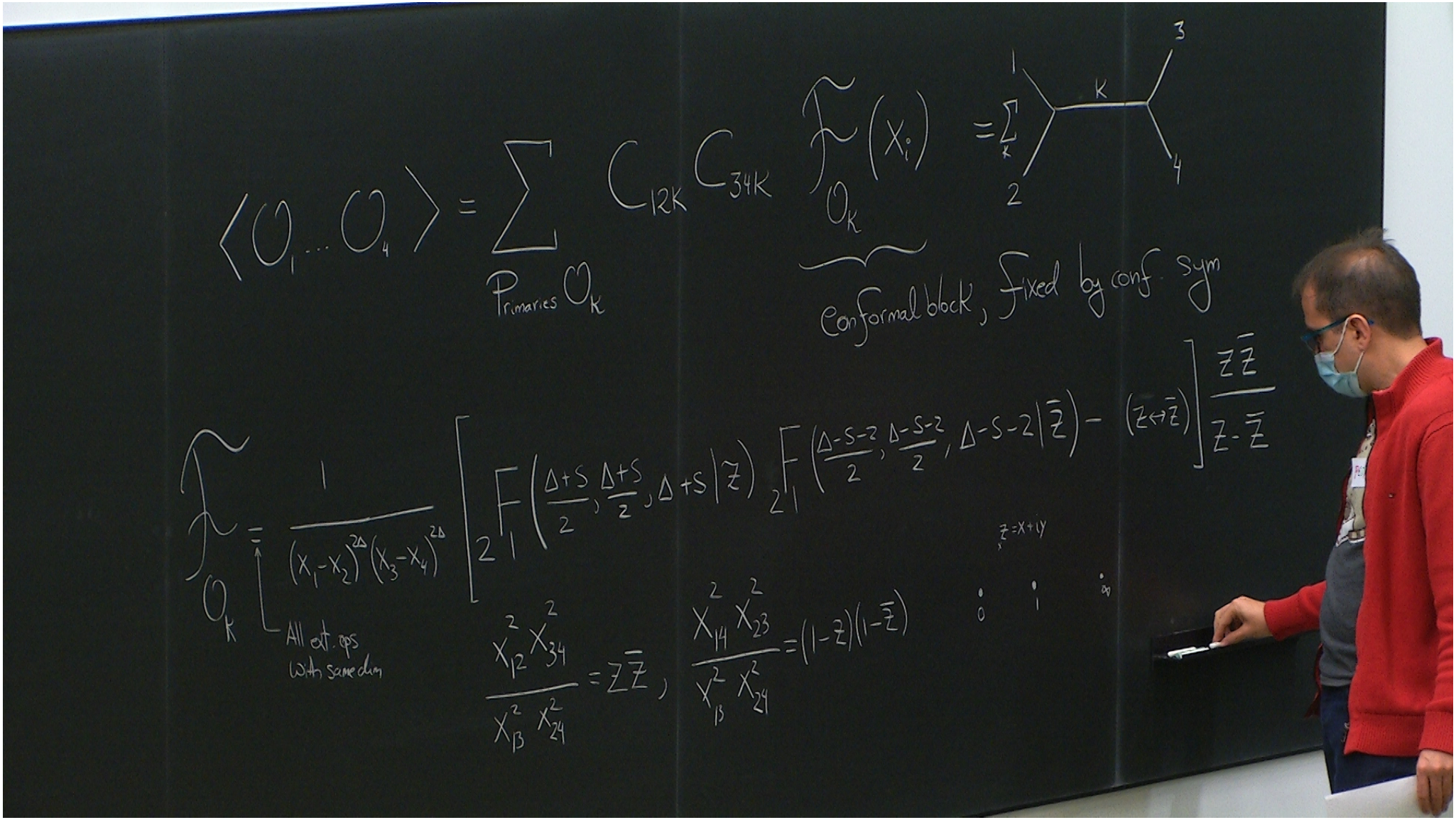
Title: AdS/CFT 2021/2022

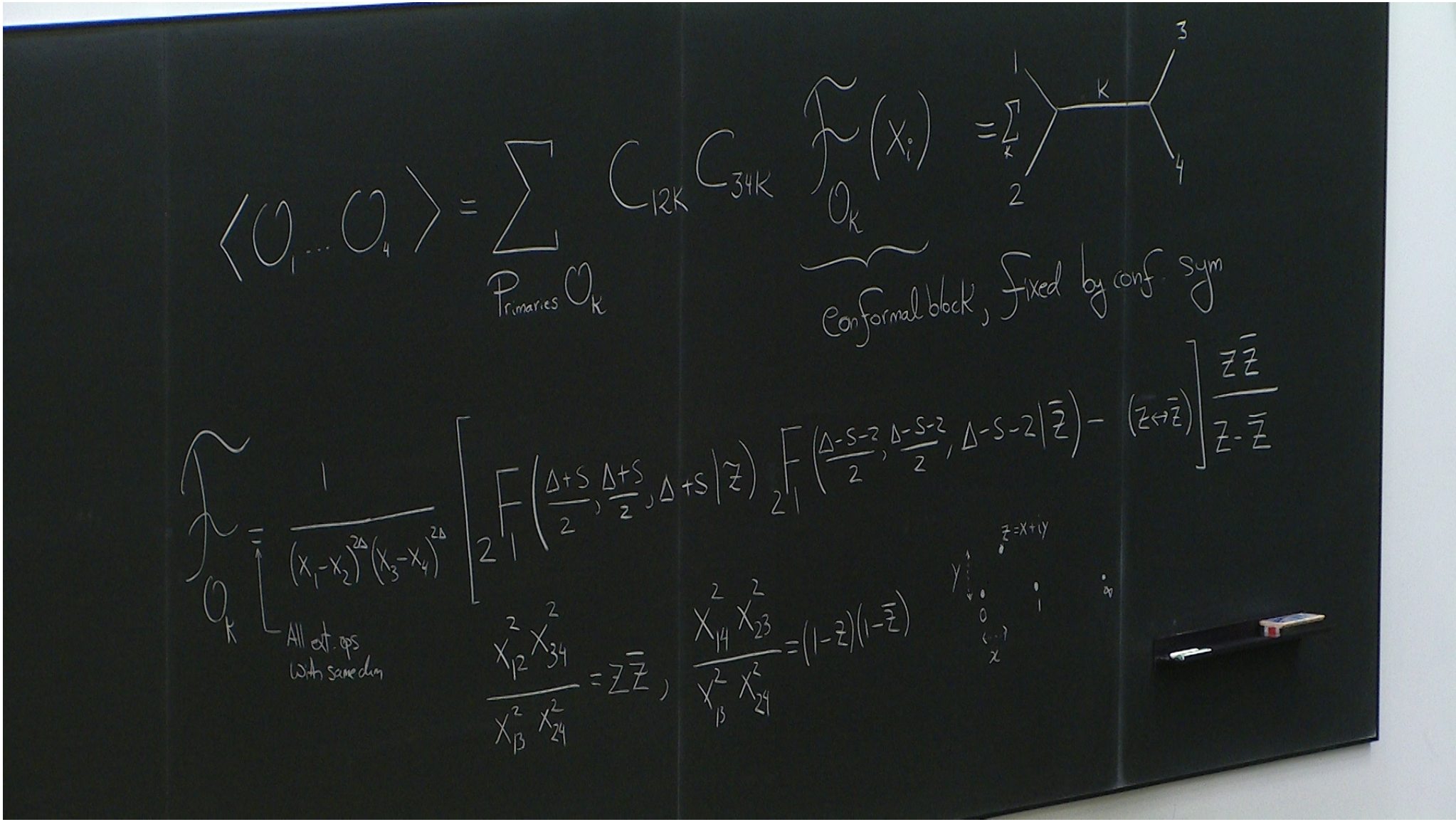
Speakers: Pedro Vieira, Gang Xu

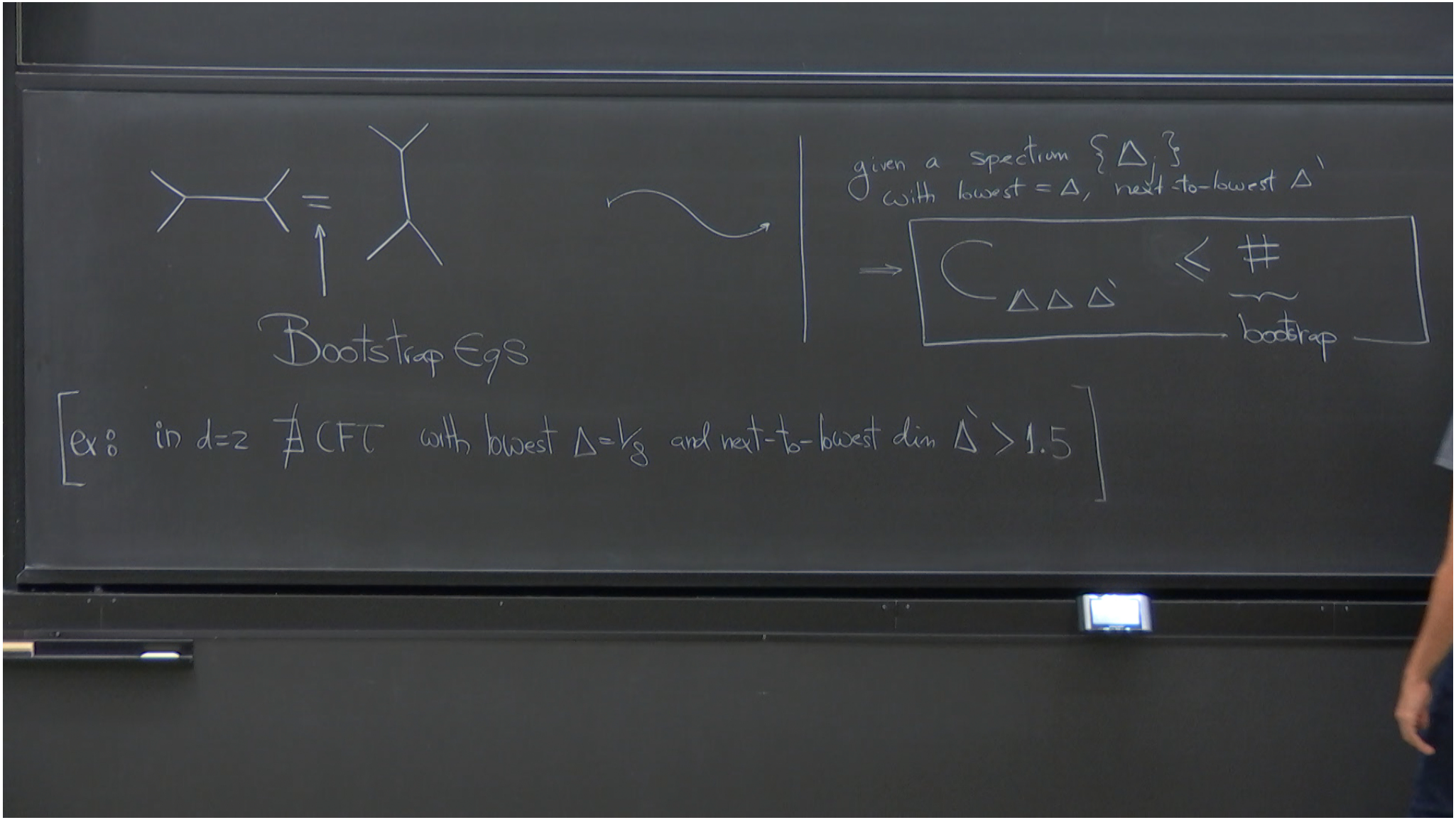
Collection: AdS/CFT 2021/2022

Date: March 24, 2022 - 1:00 PM

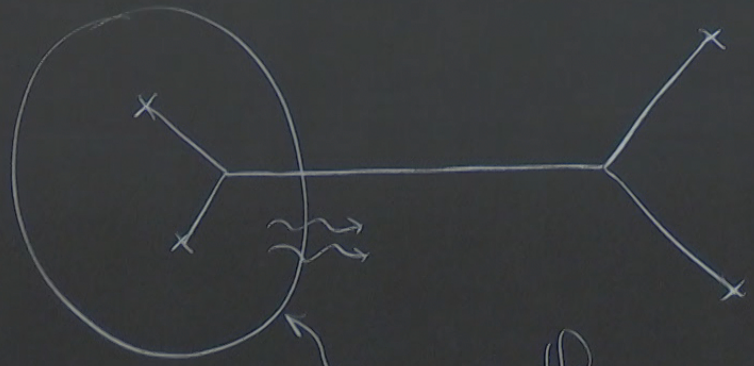
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$\langle \mathcal{O}_1 \dots \mathcal{O}_4 \rangle = \sum_{\text{Primaries } \mathcal{O}_k} C_{12k} C_{34k} \underbrace{\mathcal{F}_{\mathcal{O}_k}(x_i)}_{\text{conformal block, fixed by conf. sym}}$
1
2
3
4
k
$\mathcal{F} = \frac{1}{(x_1-x_2)^{2\Delta}(x_3-x_4)^{2\Delta}} \left[{}_2F_1\left(\frac{\Delta+s}{2}, \frac{\Delta+s}{2}, \Delta+s \,|\, z\right) {}_2F_1\left(\frac{\Delta-s-2}{2}, \frac{\Delta-s-2}{2}, \Delta-s-2 \,|\, \bar{z}\right) - (z \leftrightarrow \bar{z}) \right] \frac{z\bar{z}}{z-\bar{z}}$
$\mathcal{O}_k$
All ext. ops with same dim
$\frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2} = z\bar{z}, \quad \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2} = (1-z)(1-\bar{z})$
$z = x + iy$
0
1
∞

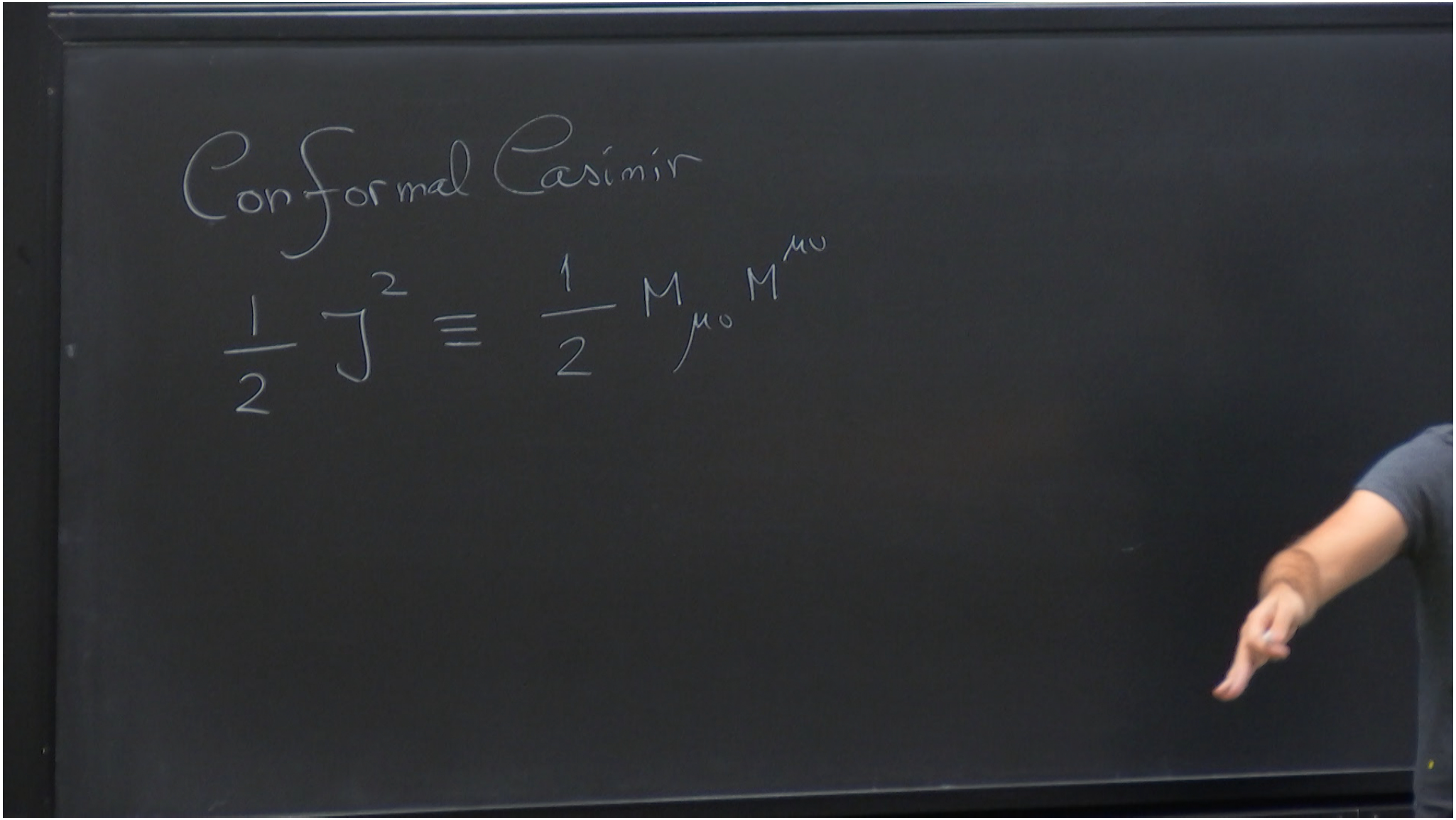

$\langle \mathcal{O}_1 \dots \mathcal{O}_4 \rangle = \sum_{\text{Primaries } \mathcal{O}_k} C_{12k} C_{34k} \mathcal{F}_{\mathcal{O}_k}(x_i)$
conformal block, fixed by conf. sym
$= \sum_k$
1
2
3
4
k
$\mathcal{F} = \frac{1}{(x_1-x_2)^{2\Delta}(x_3-x_4)^{2\Delta}} \left[{}_2F_1\left(\frac{\Delta+s}{2}, \frac{\Delta+s}{2}, \Delta+s \,\middle|\, z\right) {}_2F_1\left(\frac{\Delta-s-2}{2}, \frac{\Delta-s-2}{2}, \Delta-s-2 \,\middle|\, \bar{z}\right) - (z \leftrightarrow \bar{z}) \right] \frac{z\bar{z}}{z-\bar{z}}$
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$z = x+iy$
y
0
1
∞
x

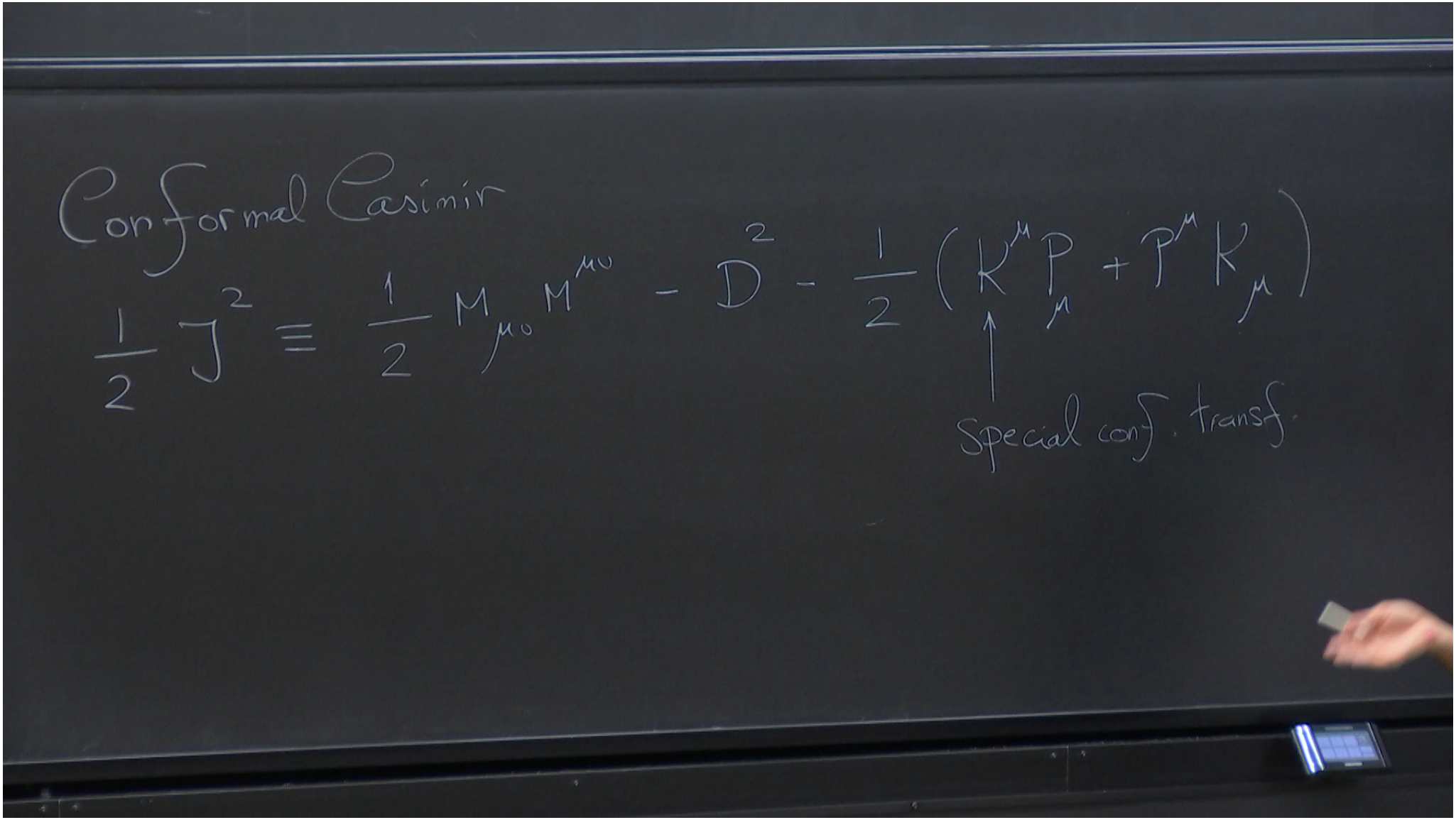

Bootstrap Eqs
given a spectrum $\{\Delta_i\}$
with lowest = Δ, next-to-lowest Δ'
$C_{\Delta\Delta\Delta'} \leq \#$
bootstrap
[ex: in d=2 $\nexists$ CFT with lowest $\Delta = 1/8$ and next-to-lowest dim $\Delta' > 1.5$]

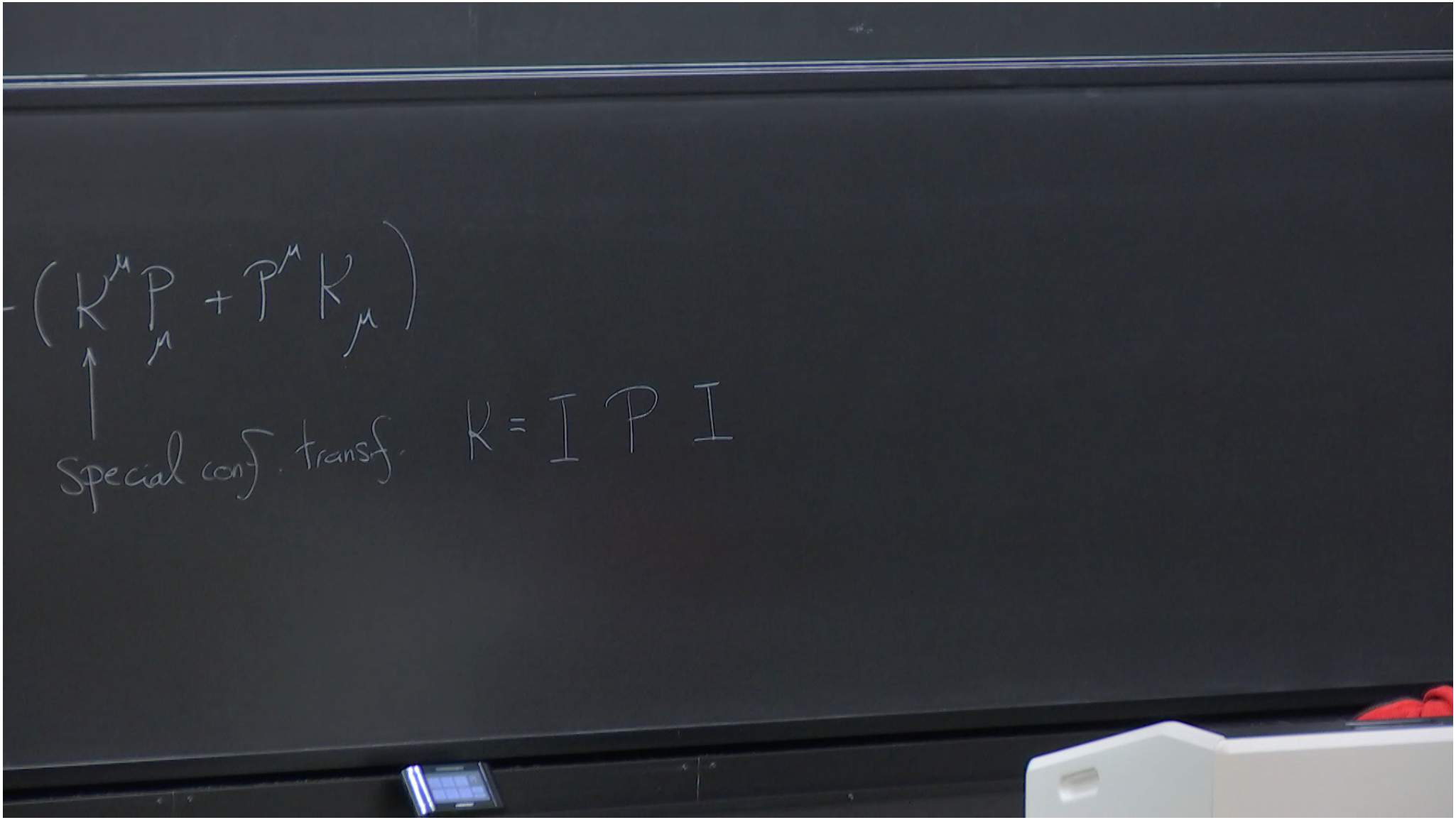
Alternative derivation of the blocks $\mathcal{F}_{OR}$

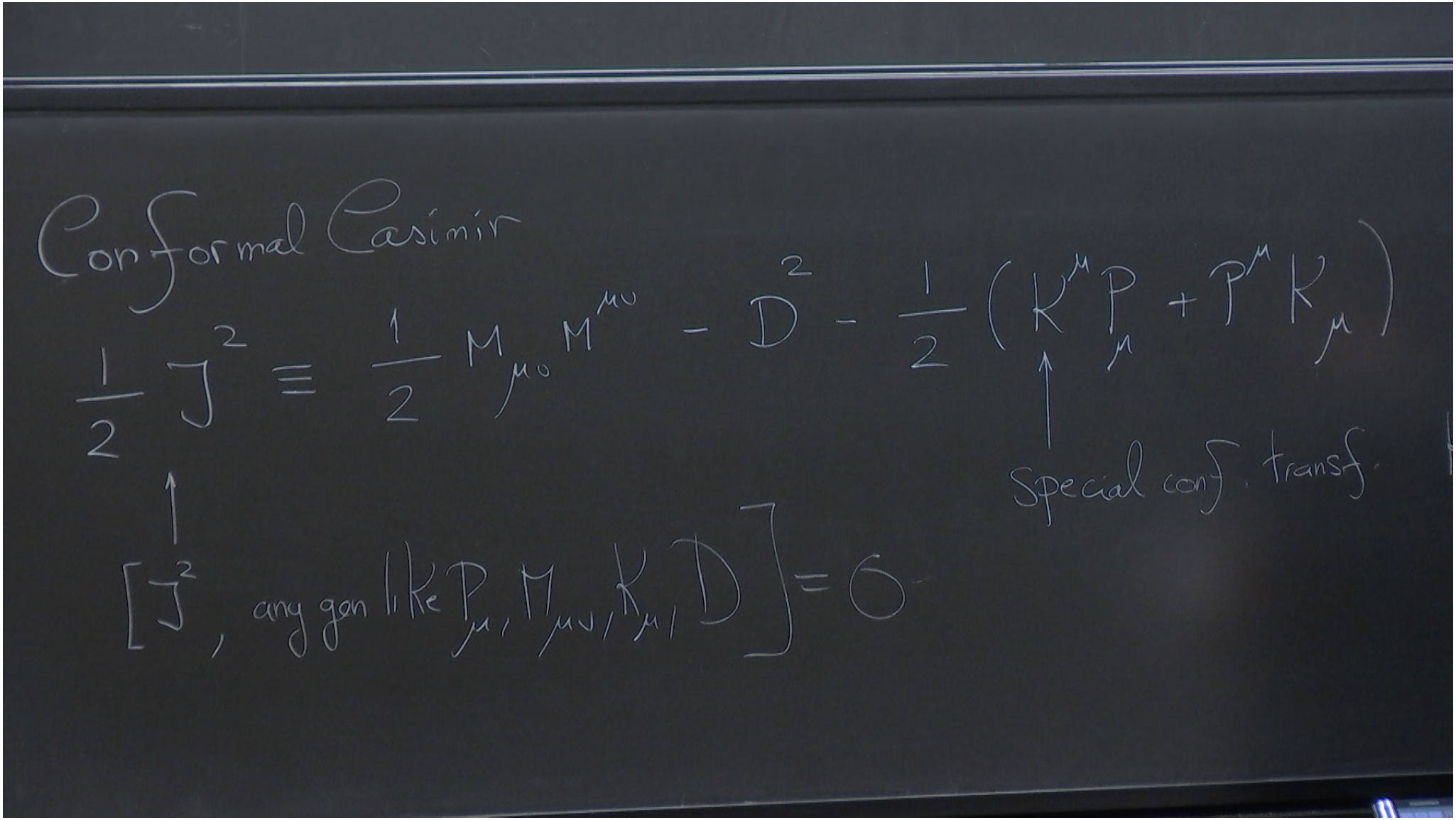


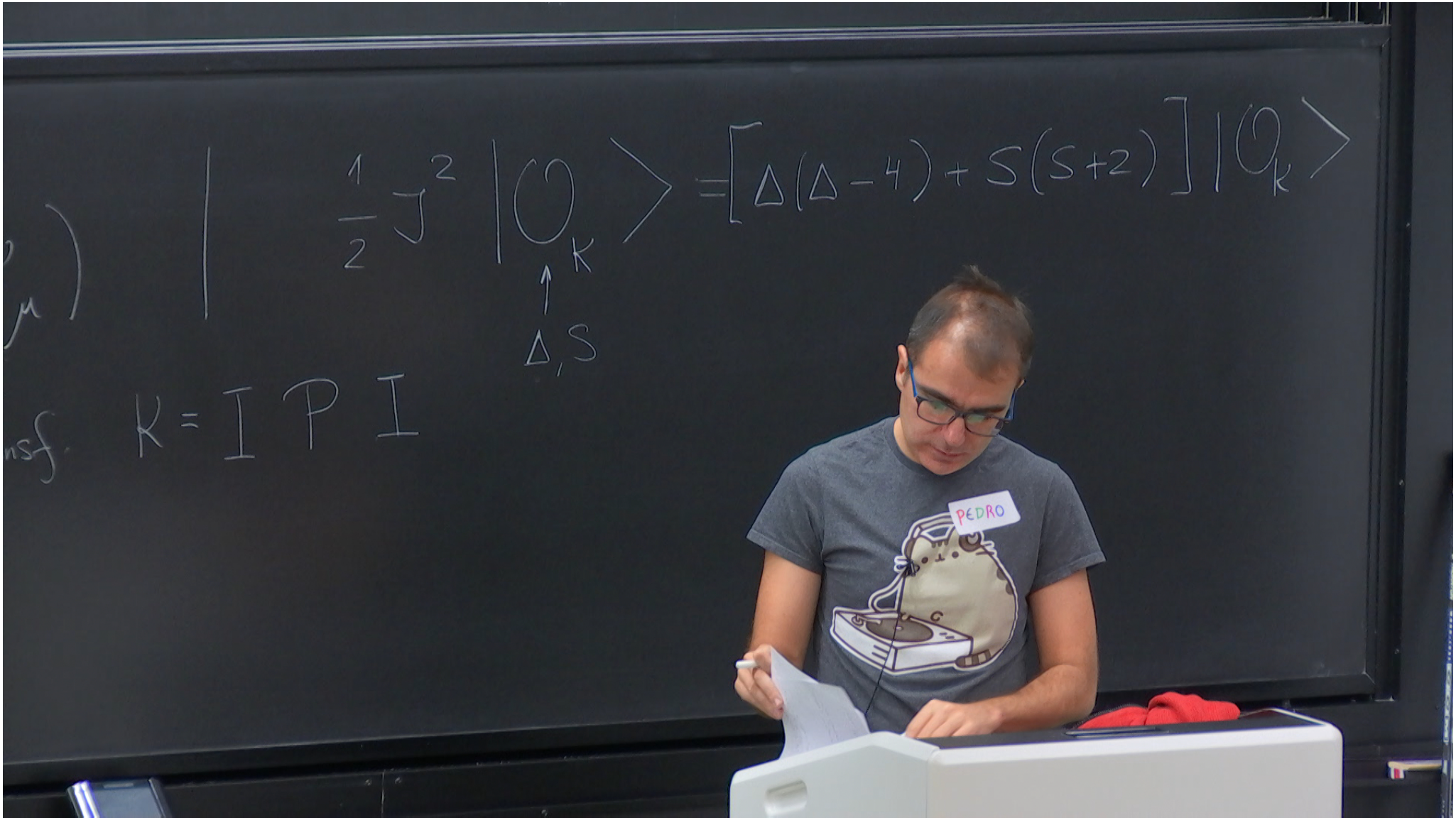
measure the
qnt numbers of what is flowing

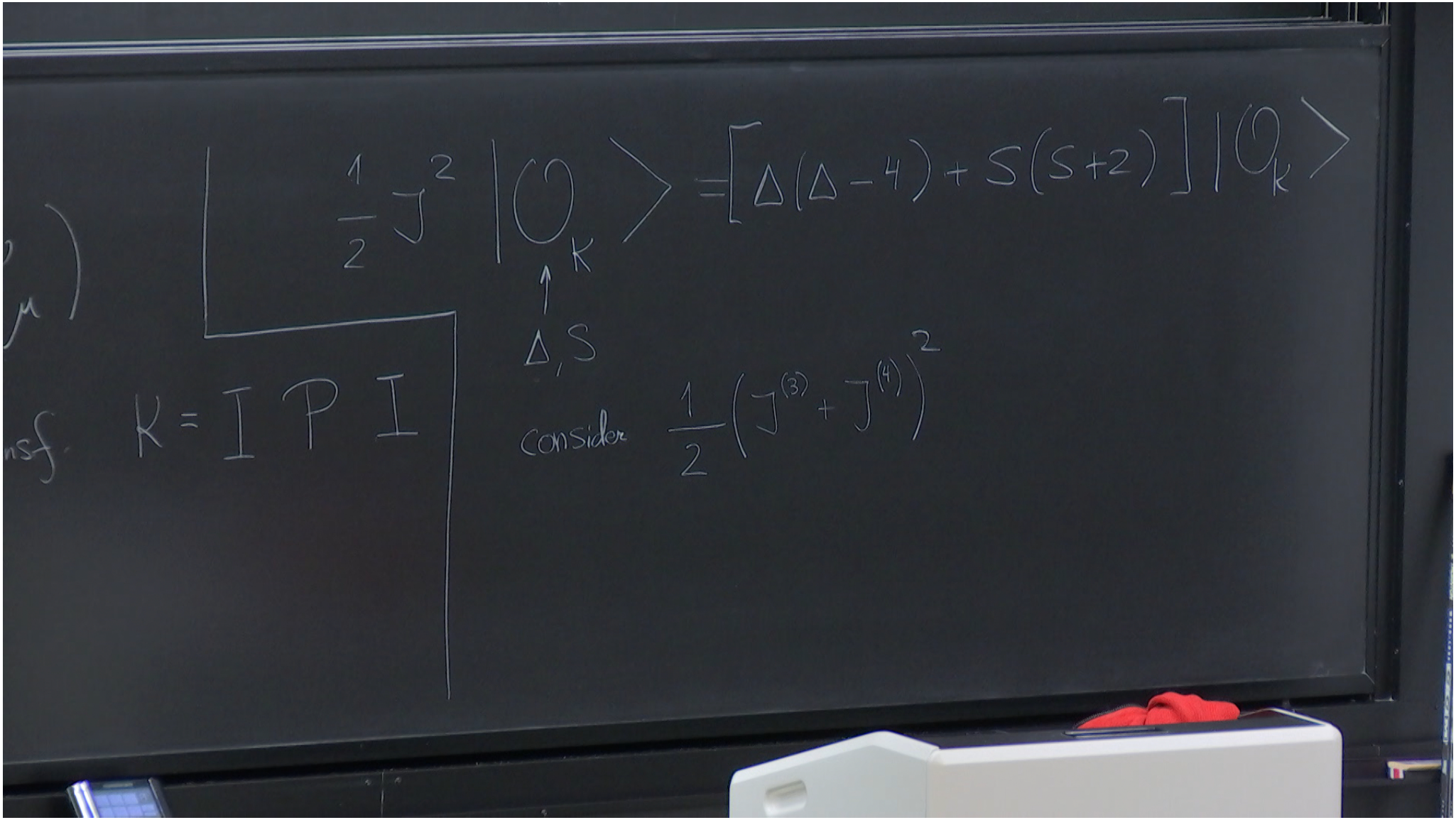

Conformal Casimir
$\frac{1}{2} J^2 \equiv \frac{1}{2} M_{\mu_0} M^{\mu_0}$

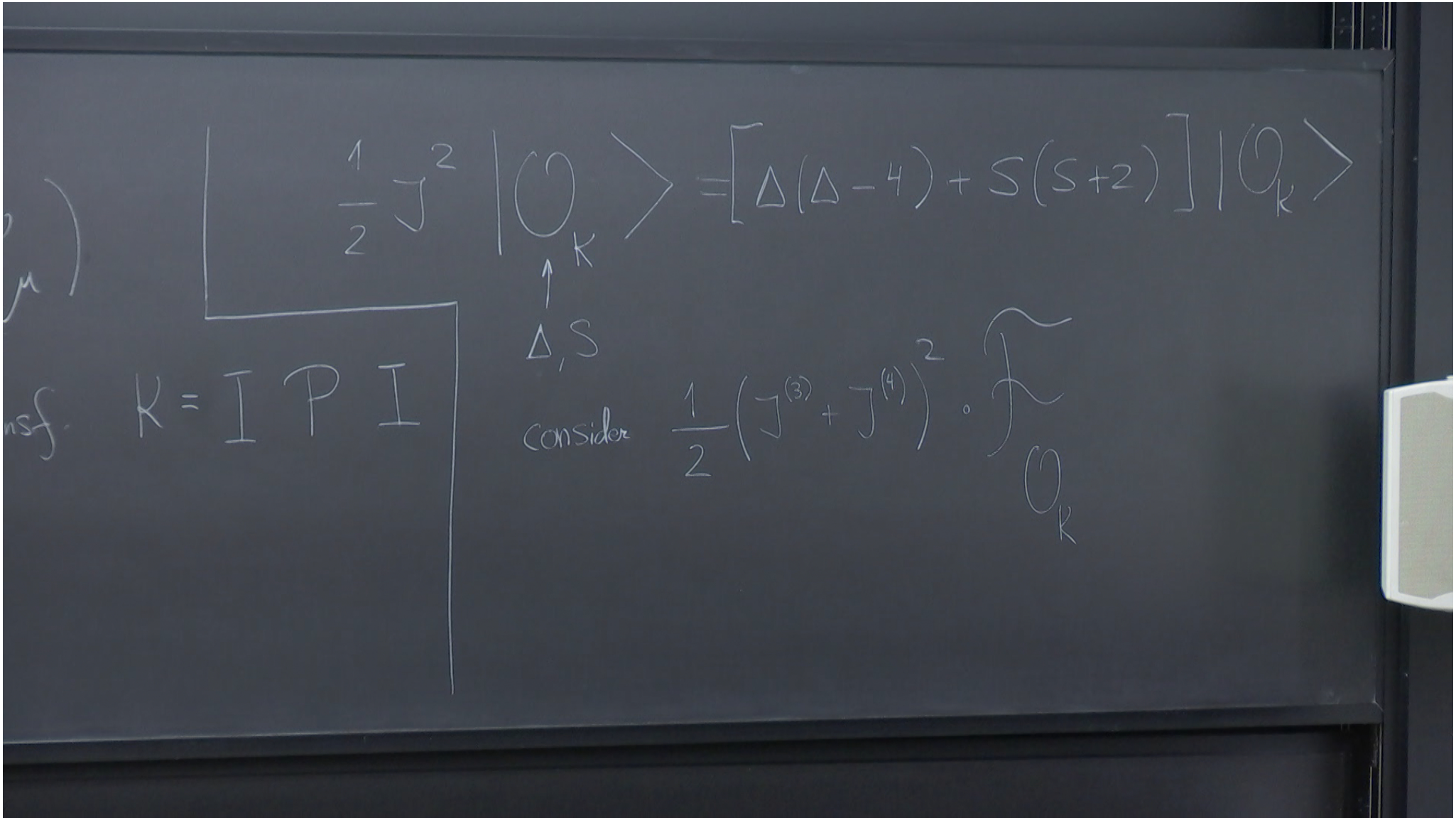

Conformal Casimir
$\frac{1}{2}J^2 \equiv \frac{1}{2}M_{\mu\nu}M^{\mu\nu} - D^2 - \frac{1}{2}(K^\mu P_\mu + P^\mu K_\mu)$
Special conf. transf.

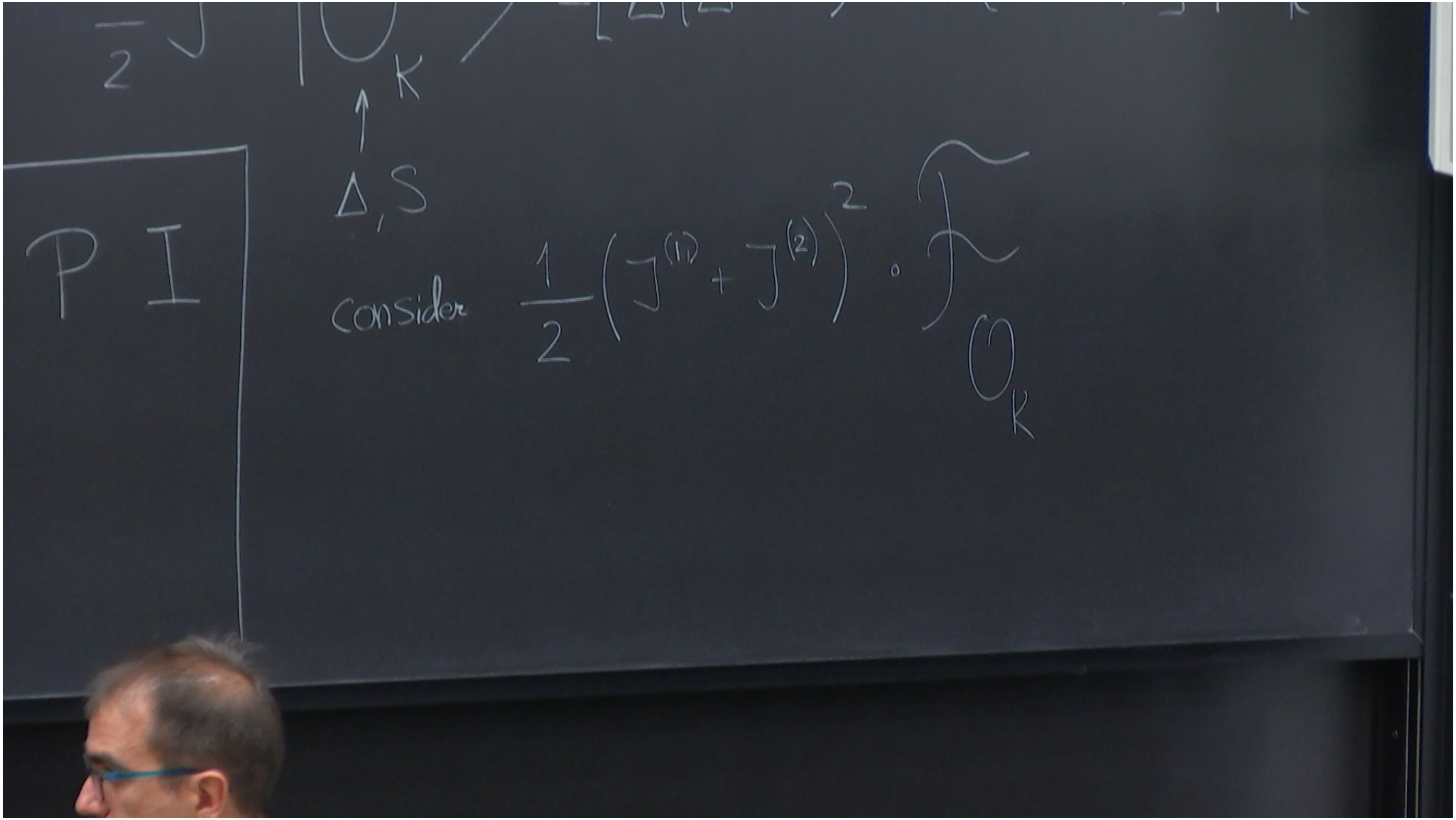

$-(K^\mu P_\mu + P^\mu K_\mu)$
Special conf. transf.
$K = I\,P\,I$

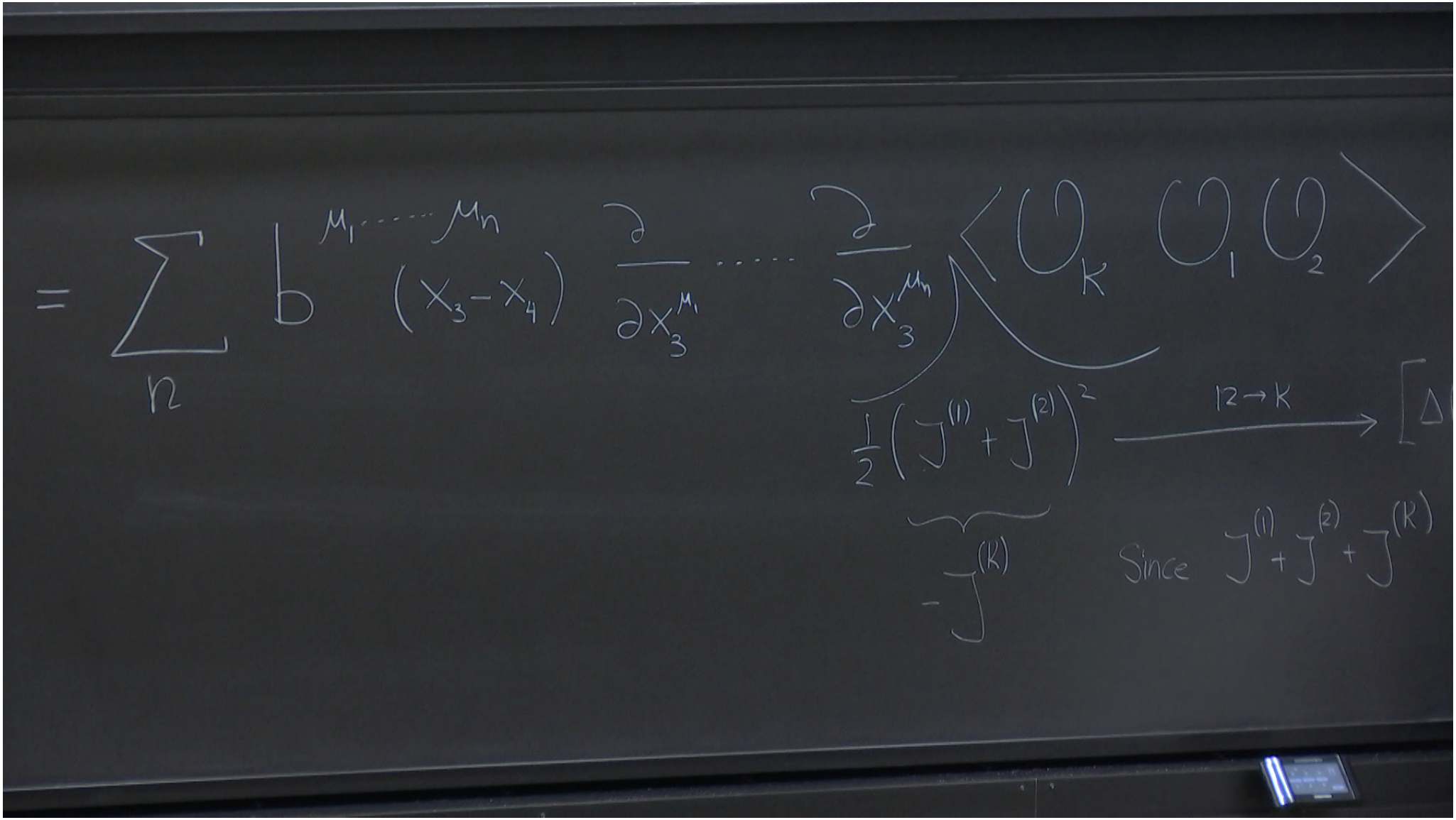

Conformal Casimir
$\frac{1}{2}J^2 \equiv \frac{1}{2}M_{\mu\nu}M^{\mu\nu} - D^2 - \frac{1}{2}(K^\mu P_\mu + P^\mu K_\mu)$
Special conf. transf.
$[J^2, \text{any gen. like } P_\mu, M_{\mu\nu}, K_\mu, D] = 0$

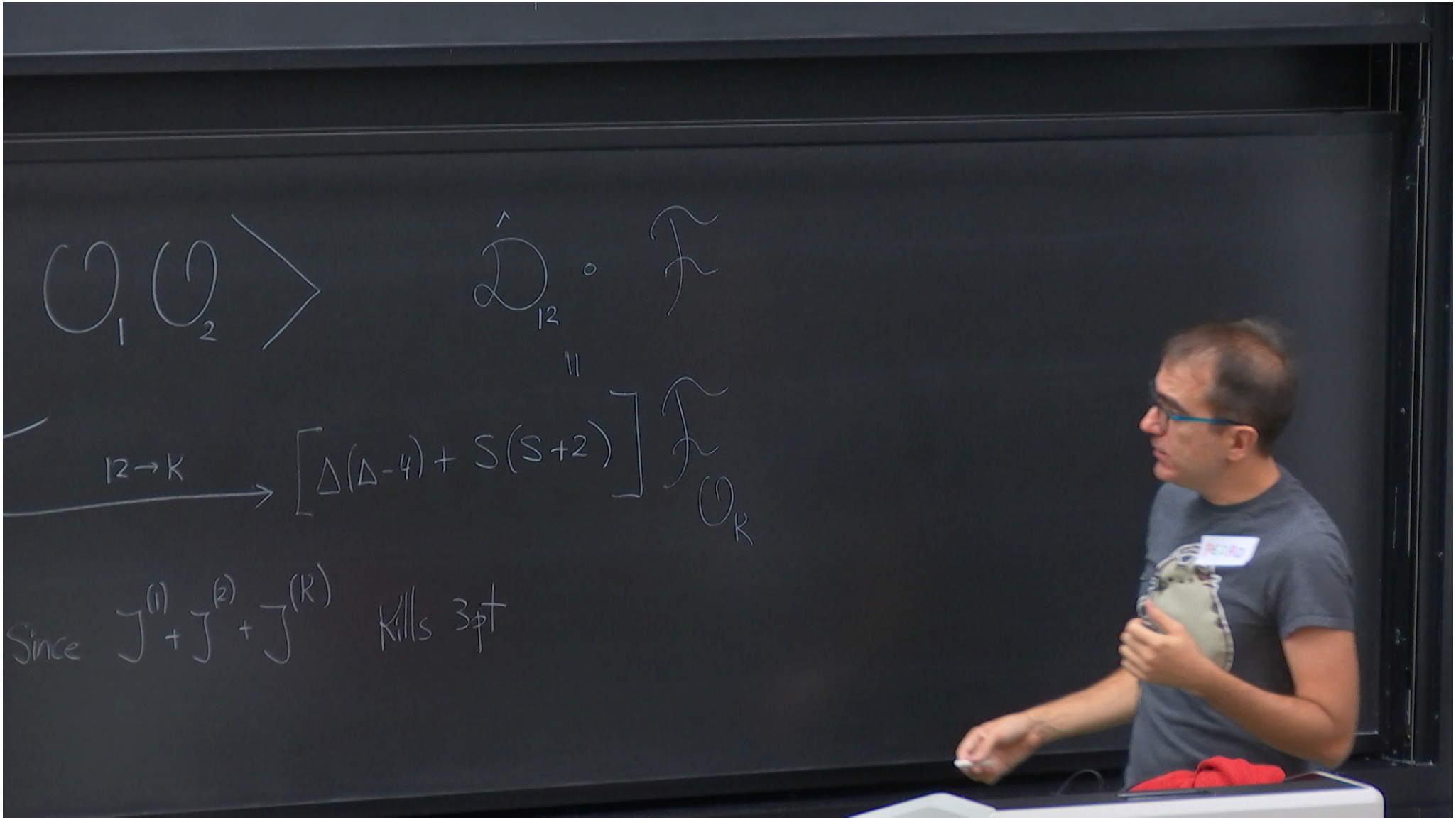

$\frac{1}{2}J^2|\mathcal{O}_k\rangle = [\Delta(\Delta-4)+S(S+2)]|\mathcal{O}_k\rangle$
Δ, S
$K = I P I$
PEDRO

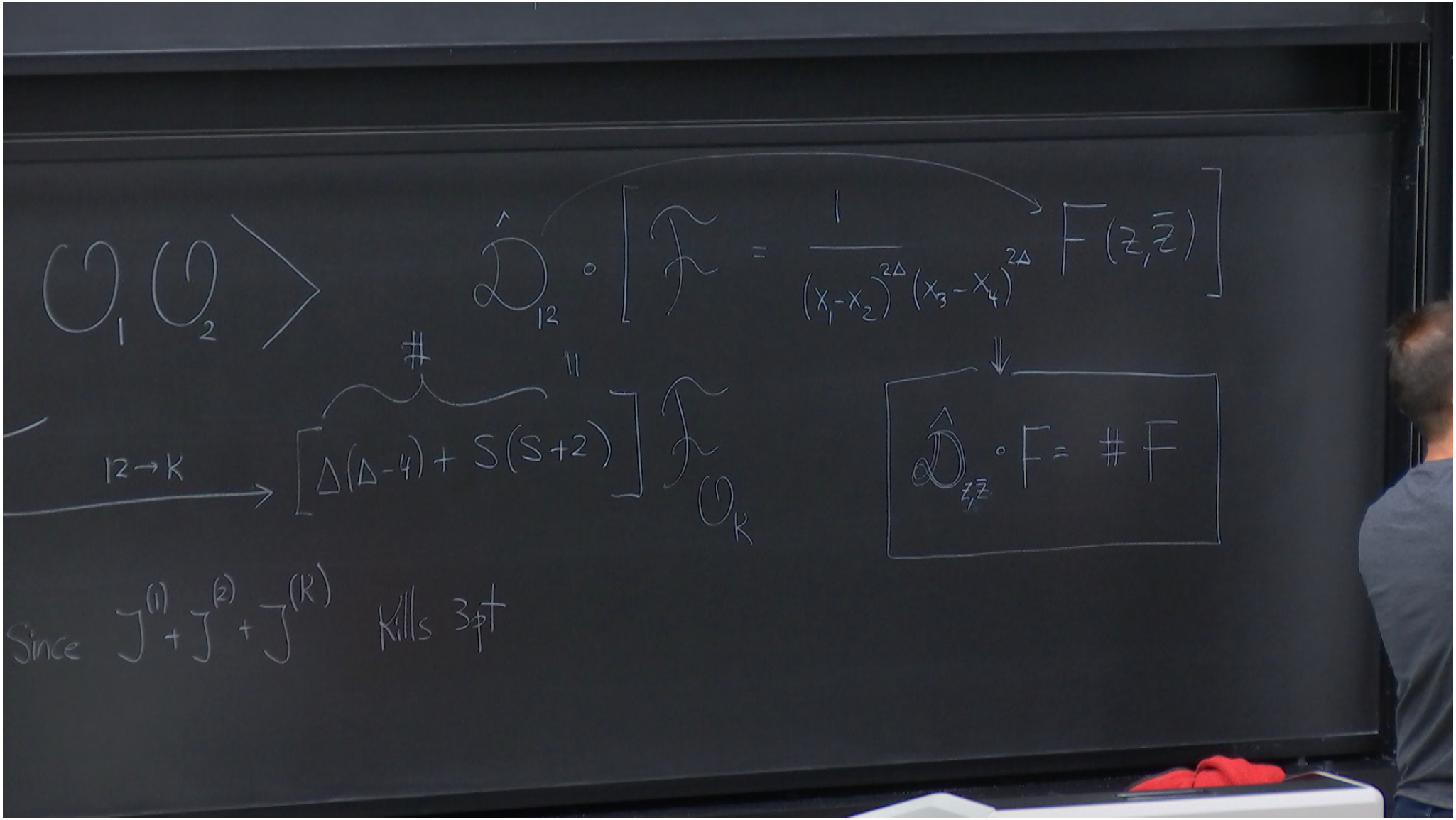

$\frac{1}{2}J^2 |O_K\rangle = [\Delta(\Delta-4)+S(S+2)]|O_K\rangle$
Δ, S
$K = I P I$
consider $\frac{1}{2}\left(J^{(3)}+J^{(4)}\right)^2$

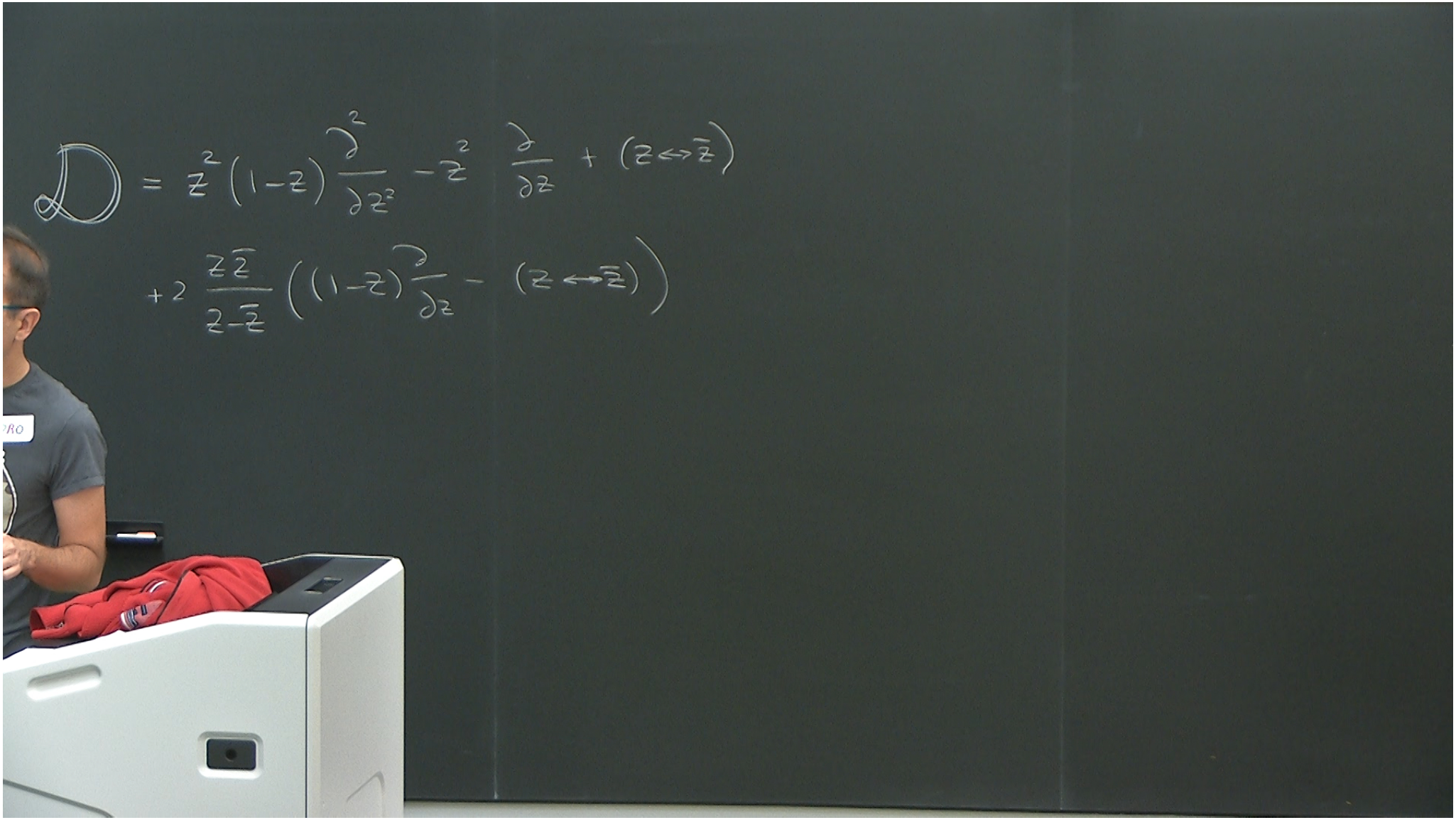

$\frac{1}{2}J^2 |O_k\rangle = [\Delta(\Delta-4) + S(S+2)]\,|O_k\rangle$
Δ, S
K = I P I
consider $\frac{1}{2}\left(J^{(3)} + J^{(4)}\right)^2 \cdot \mathcal{F}\, O_k$

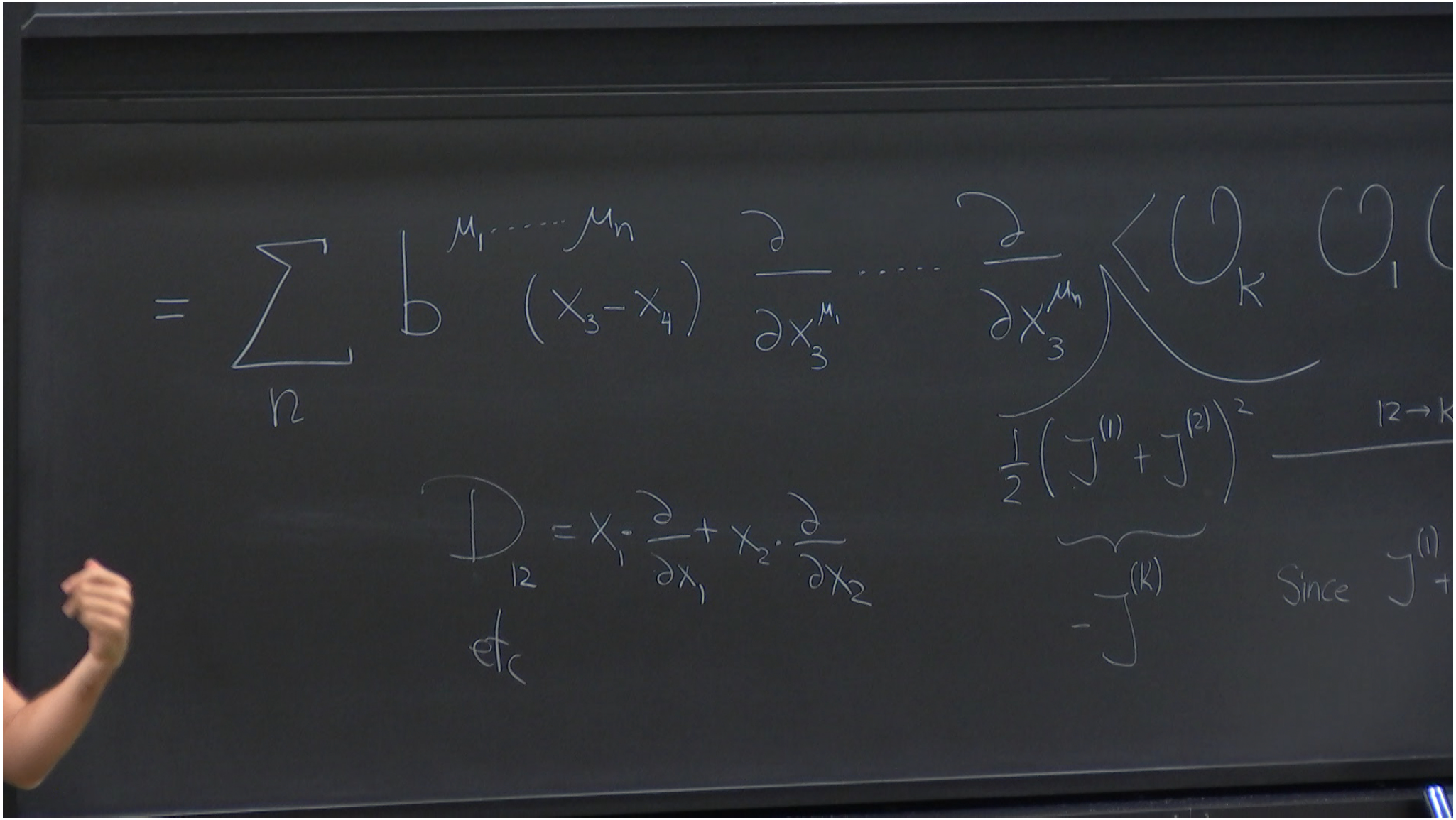

PI
consider

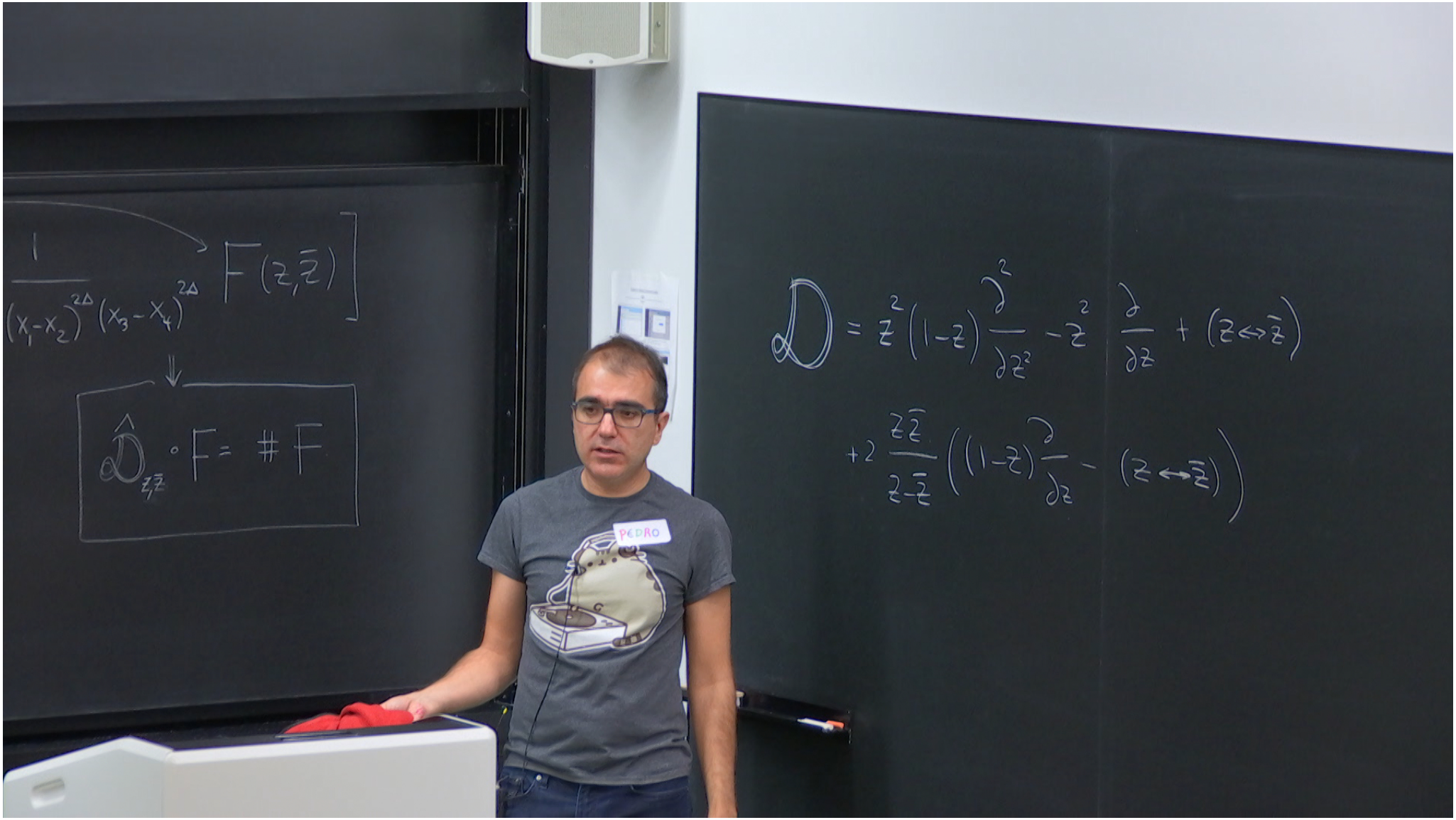

$= \sum_n b^{\mu_1 \cdots \mu_n}(x_3 - x_4) \frac{\partial}{\partial x_3^{\mu_1}} \cdots \frac{\partial}{\partial x_3^{\mu_n}} \langle O_k O_1 O_2 \rangle$
$\frac{1}{2}(J^{(1)} + J^{(2)})^2$
$12 \to k$
$-J^{(k)}$
Since $J^{(1)} + J^{(2)} + J^{(k)}$

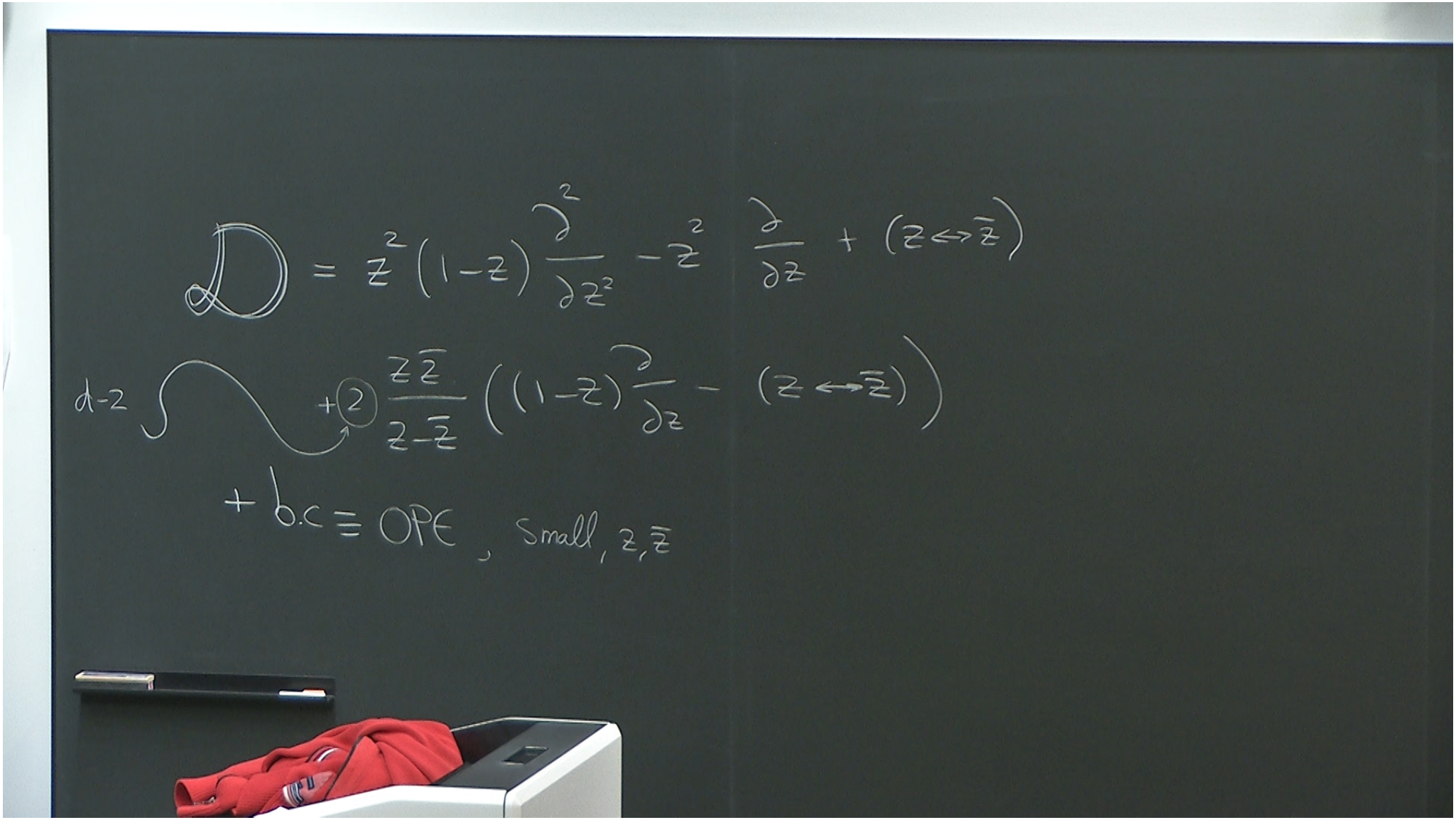

$\mathcal{O}_1 \mathcal{O}_2\rangle$
$\hat{\mathcal{D}}_{12} \cdot \mathcal{F}$
$=$
$12 \to K$
$[\Delta(\Delta-4) + S(S+2)]\,\mathcal{F}_{\mathcal{O}_K}$
Since $J^{(1)} + J^{(2)} + J^{(K)}$ Kills 3pt


$\mathcal{O}_1\mathcal{O}_2\rangle$
$\hat{\mathcal{D}}_{12} \circ \left[\mathcal{F} = \frac{1}{(x_1-x_2)^{2\Delta}(x_3-x_4)^{2\Delta}} F(z,\bar{z})\right]$
$12 \to K$
$\left[\Delta(\Delta-4) + S(S+2)\right] \mathcal{F}_{\mathcal{O}_K}$
#
=
$\hat{\mathcal{D}}_{z\bar{z}} \circ F = \# F$
Since $J^{(1)} + J^{(2)} + J^{(K)}$ Kills 3pt


$\mathcal{D} = z^2(1-z)\frac{\partial^2}{\partial z^2} - z^2\frac{\partial}{\partial z} + (z \leftrightarrow \bar{z})$
$+2\frac{z\bar{z}}{z-\bar{z}}\left((1-z)\frac{\partial}{\partial z} - (z \leftrightarrow \bar{z})\right)$


$= \sum_n b^{\mu_1 \cdots \mu_n} (x_3 - x_4) \frac{\partial}{\partial x_3^{\mu_1}} \cdots \frac{\partial}{\partial x_3^{\mu_n}}$
$\frac{1}{2}(J^{(1)} + J^{(2)})^2$
$12 \to k$
$J^{(k)}$
Since
$D_{12} = x_1 \cdot \frac{\partial}{\partial x_1} + x_2 \cdot \frac{\partial}{\partial x_2}$
etc


$\frac{1}{(x_1-x_2)^{2\Delta}(x_3-x_4)^{2\Delta}} F(z,\bar{z})$
$\hat{\mathcal{D}}_{z,\bar{z}} \cdot F = \# F$
$\mathcal{D} = z^2(1-z)\frac{\partial^2}{\partial z^2} - z^2\frac{\partial}{\partial z} + (z \leftrightarrow \bar{z})$
$+2\frac{z\bar{z}}{z-\bar{z}}\left((1-z)\frac{\partial}{\partial z} - (z \leftrightarrow \bar{z})\right)$
PEDRO


$\mathcal{D} = z^2(1-z)\frac{\partial^2}{\partial z^2} - z^2\frac{\partial}{\partial z} + (z \leftrightarrow \bar{z})$
$d-2$
$+2\frac{z\bar{z}}{z-\bar{z}}\left((1-z)\frac{\partial}{\partial z} - (z \leftrightarrow \bar{z})\right)$
$+ b.c \equiv OPE$, small $z, \bar{z}$

So far we have played with:

- $\mathbb{R}^d$: $ds^2 = dx_\mu dx^\mu$
- spheres : $\{\vec{Y} \mid Y_1^2 + Y_2^2 + \cdots + Y_d^2 = 1\}$
- cylinders : $ds^2 = dr^2 + d\Omega^2_{S^{d-1}}$

- $\mathbb{R}^d$ is <u>not</u> inv. under conf. transf.

$x^\mu \to \tilde{x}^\mu$, $d\tilde{x}_\mu d\tilde{x}^\mu = \Omega^2(x)\, \underbrace{dx_\mu dx^\mu}_{\mathbb{R}^d}$

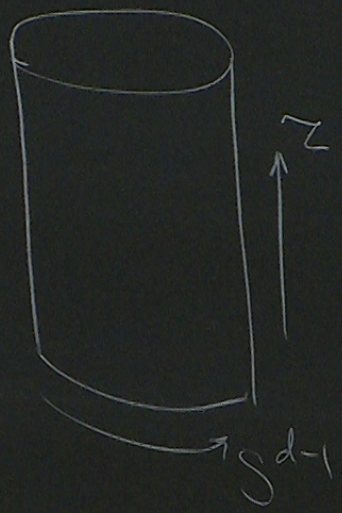
[for inversion $\tilde{x} = \frac{x}{x^2}$, $\Omega = \frac{1}{x^2}$]

for a finite K

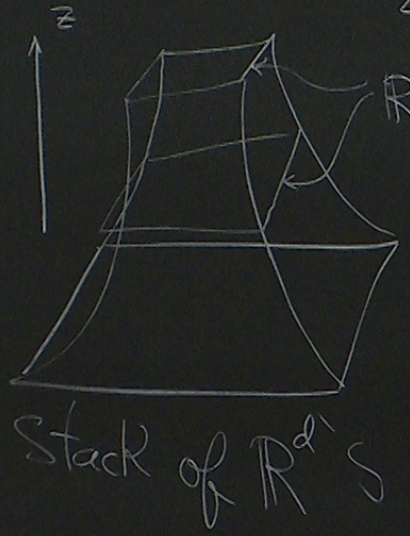
$$\vec{\tilde{x}} = \frac{\vec{x} + \vec{b}\, x^2}{1 + 2\vec{b}\cdot\vec{x} + b^2 x^2}, \quad ds^2 \to \Omega^2 ds^2$$

- spheres with a few sign flips ⇝ hyperboloids, ...
- cylinder has no interior

just the cover!



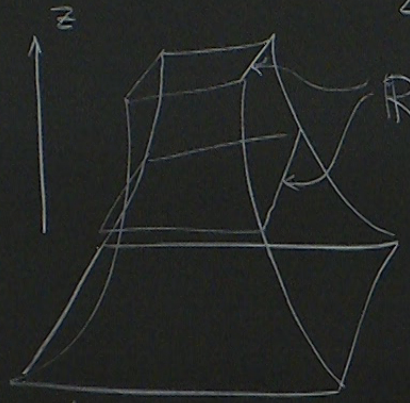
$$\boxed{A} = ds^2 \equiv \frac{dz^2 + dx_\mu dx^\mu}{z^2} \quad , z \geq 0$$



CLAIM : Each conf. transf $X \to \tilde{X}$ can be promoted to an isommetry of this Space-time!

I.e. Sym group of this space-time = Conformal group

$$\boxed{A} = ds^2 \equiv \frac{dz^2 + dx_\mu dx^\mu}{z^2}, \quad z \geq 0$$



Stack of R^d's

CLAIM: Each conf. transf $X \to \tilde{X}$ can be promoted to an isommetry of this Space-time!

I.e. Sym group of this space-time = Conformal group

Example: $\tilde{X} = \lambda X$

Example:

$$x \longmapsto \tilde{x} = \frac{x + b\langle x^2}{1 + 2b \cdot x + b^2\langle x^2}$$

Example:

$$x \longrightarrow \tilde{x} = \frac{x + b(x^2 + z^2)}{1 + 2b \cdot x + b^2(x^2 + z^2)}$$

$$z \longrightarrow \tilde{z} = \frac{z}{\text{Same den}}$$

is an isometry?

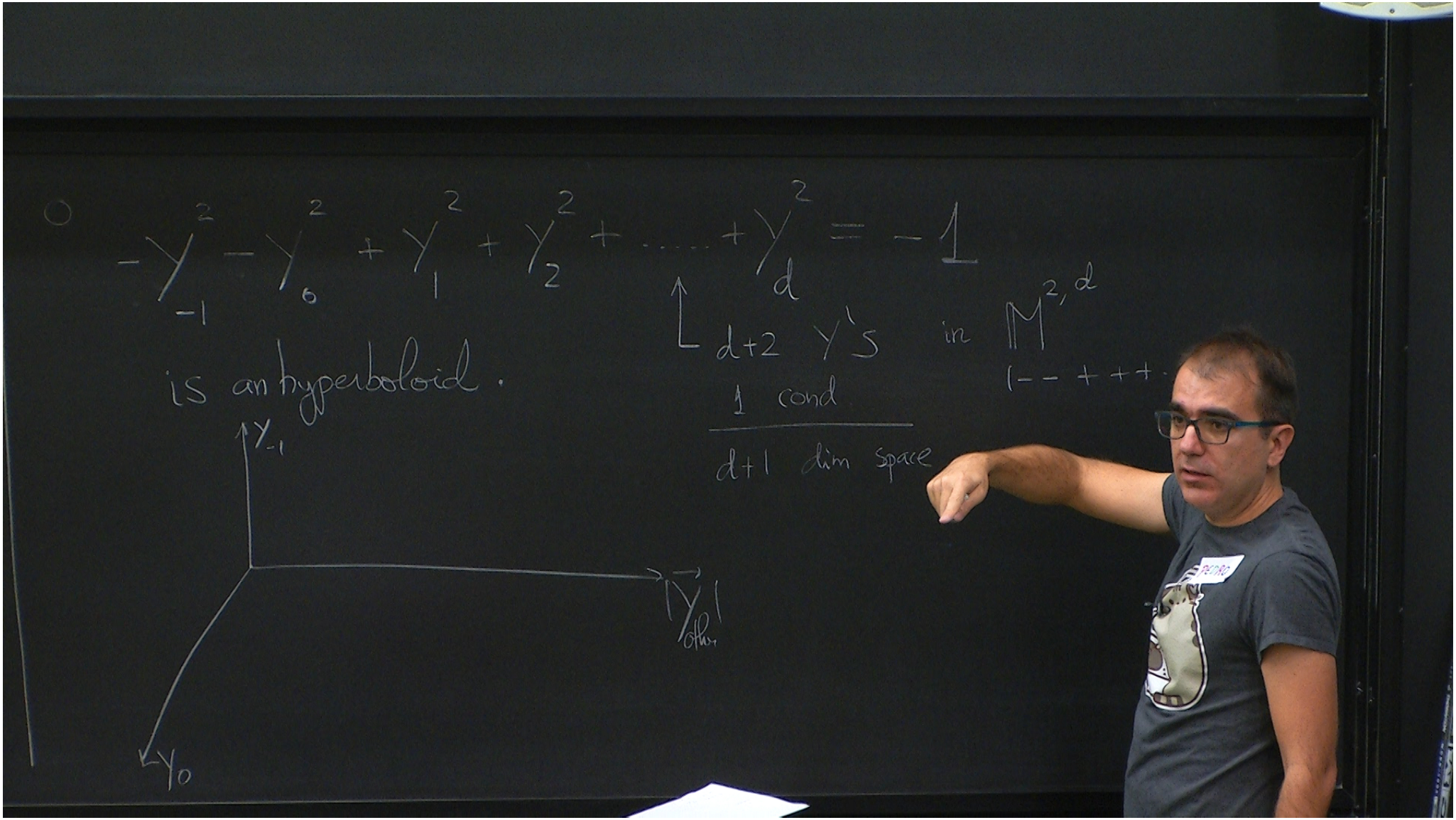
Example 1:

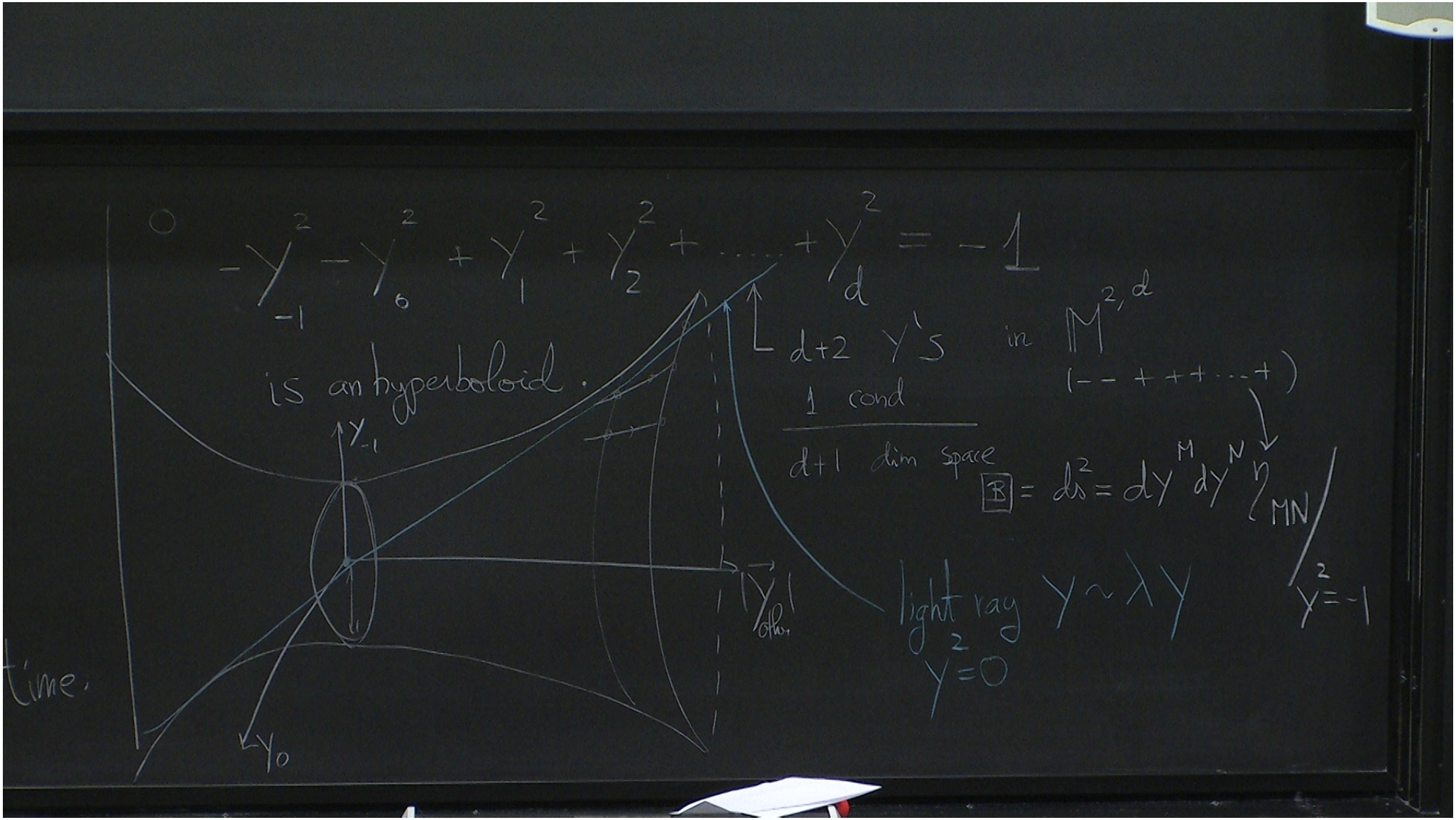
$$X \longrightarrow \tilde{X} = \frac{X + b(X^2 + Z^2)}{1 + 2b \cdot X + b^2(X^2 + Z^2)}$$

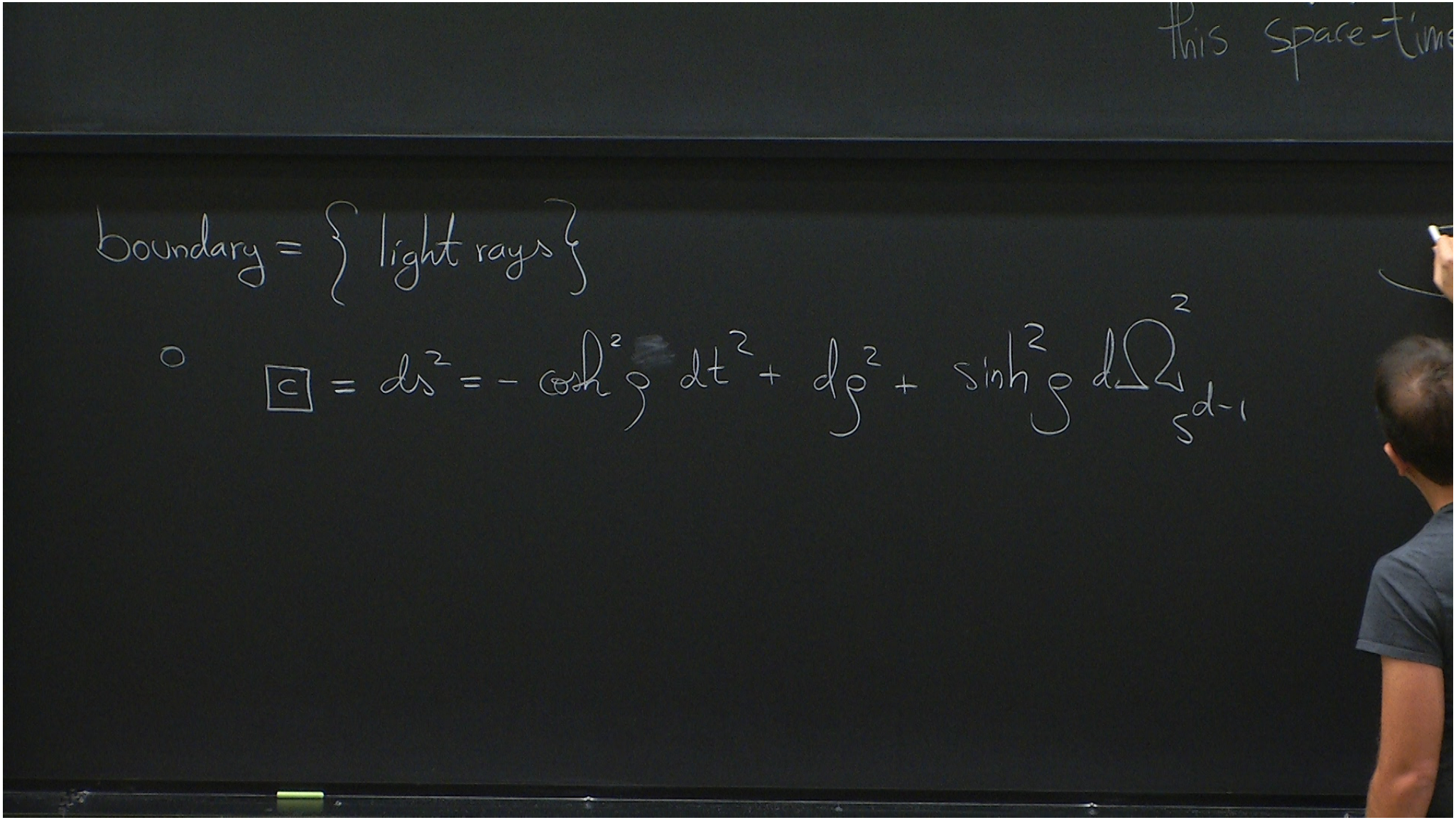
← usual CT as $Z \to 0$

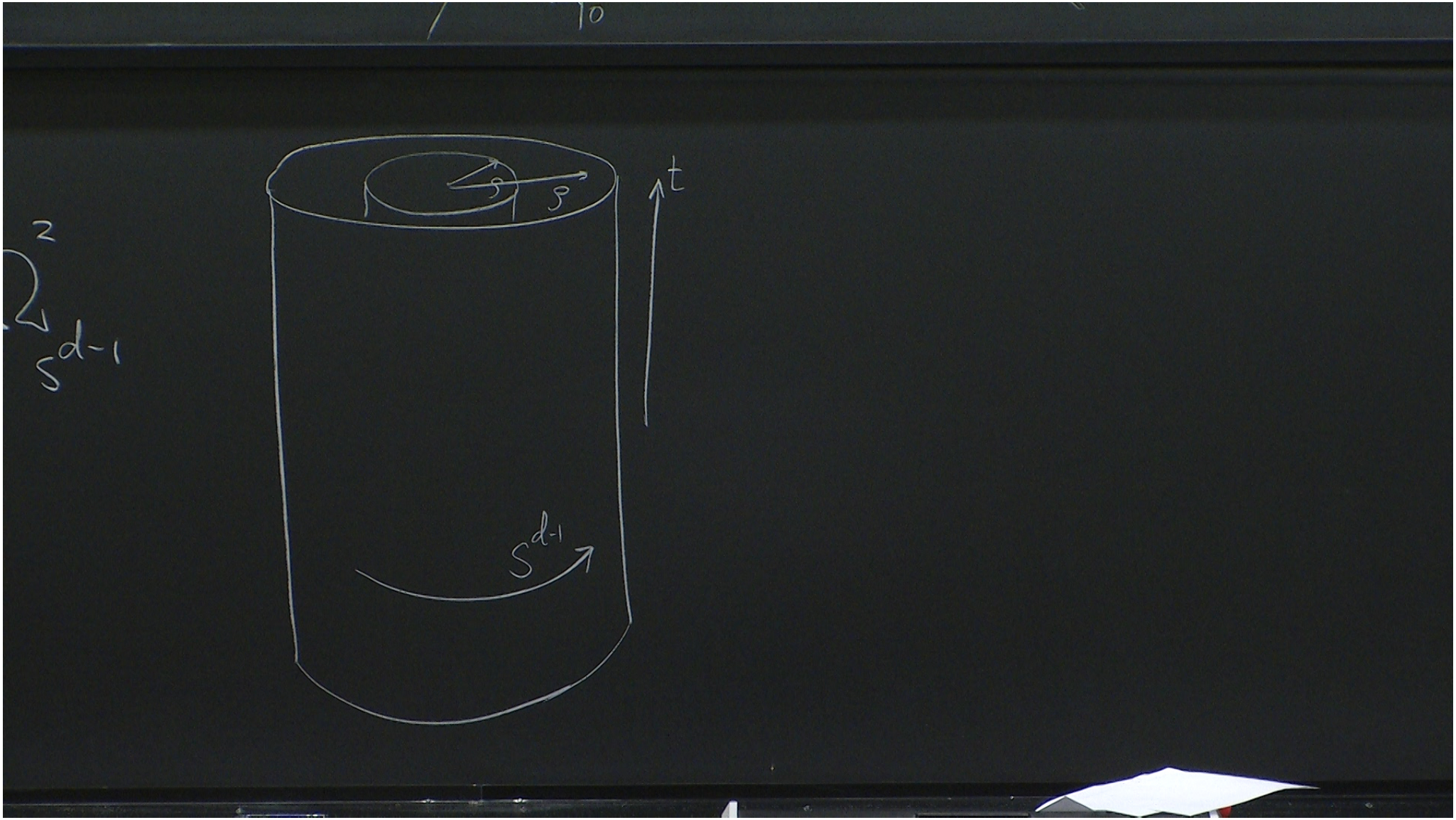
$$Z \longrightarrow \tilde{Z} = \frac{Z}{\text{Same den}}$$

is an isometry? $ds^2 \to ds^2$


$-Y_{-1}^2 - Y_0^2 + Y_1^2 + Y_2^2 + \ldots + Y_d^2 = -1$
is an hyperboloid.
$d+2$ Y's in $\mathbb{M}^{2,d}$
(- - + + + ...)
1 cond
$d+1$ dim space
Y_{-1}
Y_0
$\vec{Y}_{\text{other}}$


$-Y_{-1}^2 - Y_0^2 + Y_1^2 + Y_2^2 + \ldots + Y_d^2 = -1$
is an hyperboloid.
Y_{-1}
Y_0
time.
$d+2$ Y^Is in $\mathbb{M}^{2,d}$
$(--+++\ldots+)$
1 cond
$d+1$ dim space
$\boxed{B} = ds^2 = dY^M dY^N \eta_{MN}$
$\vec{Y}_{\text{othr.}}$
light ray $Y \sim \lambda Y$
$Y^2 = 0$
$Y^2 = -1$


This space-time
boundary = { light rays }
$\boxed{c} = ds^2 = -\cosh^2\rho\, dt^2 + d\rho^2 + \sinh^2\rho\, d\Omega^2_{S^{d-1}}$


10
z
S^{d-1}
s
s
t
S^{d-1}

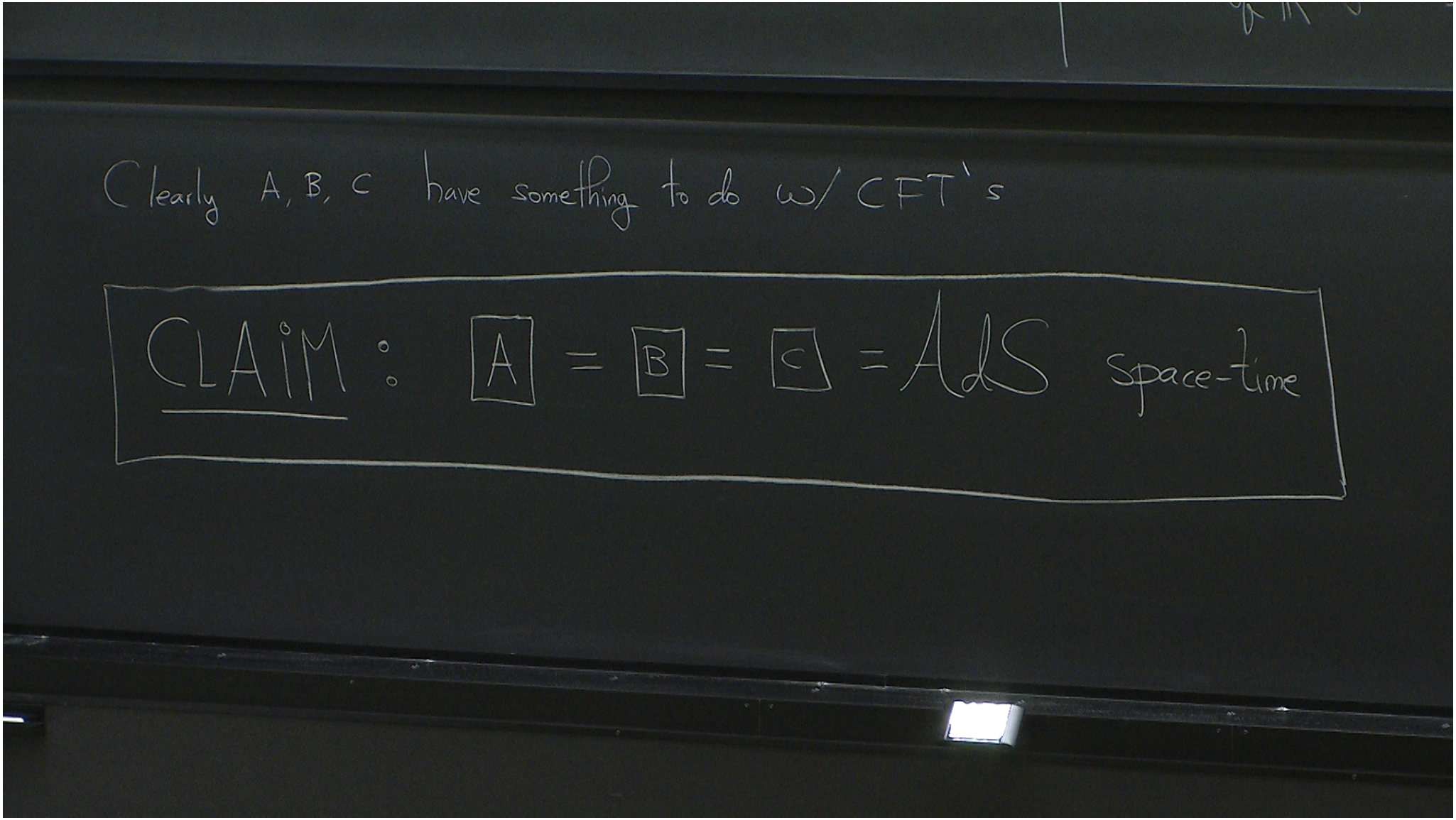
boundary = { light rays }

$$\boxed{c} = ds^2 = -\cosh^2\rho\, dt^2 + d\rho^2 + \sinh^2\rho\, d\Omega^2_{S^{d-1}}$$

$\rho \to 0$

$$\sim \frac{e^{2\rho}}{4}\underbrace{\left[-dt^2 + d\Omega^2_{S^{d-1}}\right]}_{\text{cylinder}}$$

boundary = cylinder


Clearly A, B, C have something to do w/ CFT's
CLAIM: A = B = C = AdS space-time