Title: Fold-Transversal Clifford Gates for Quantum Codes

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Series: Perimeter Institute Quantum Discussions

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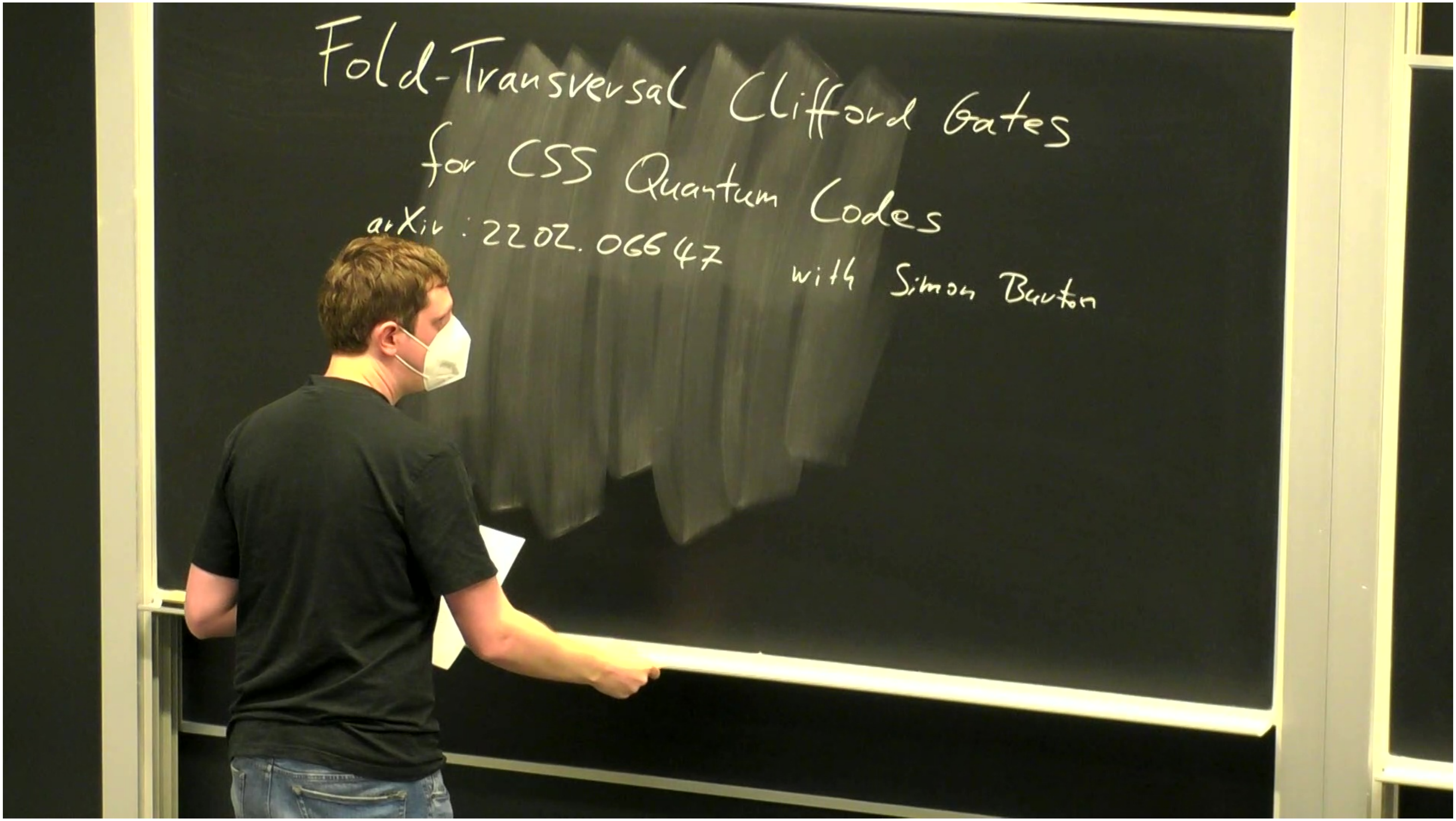
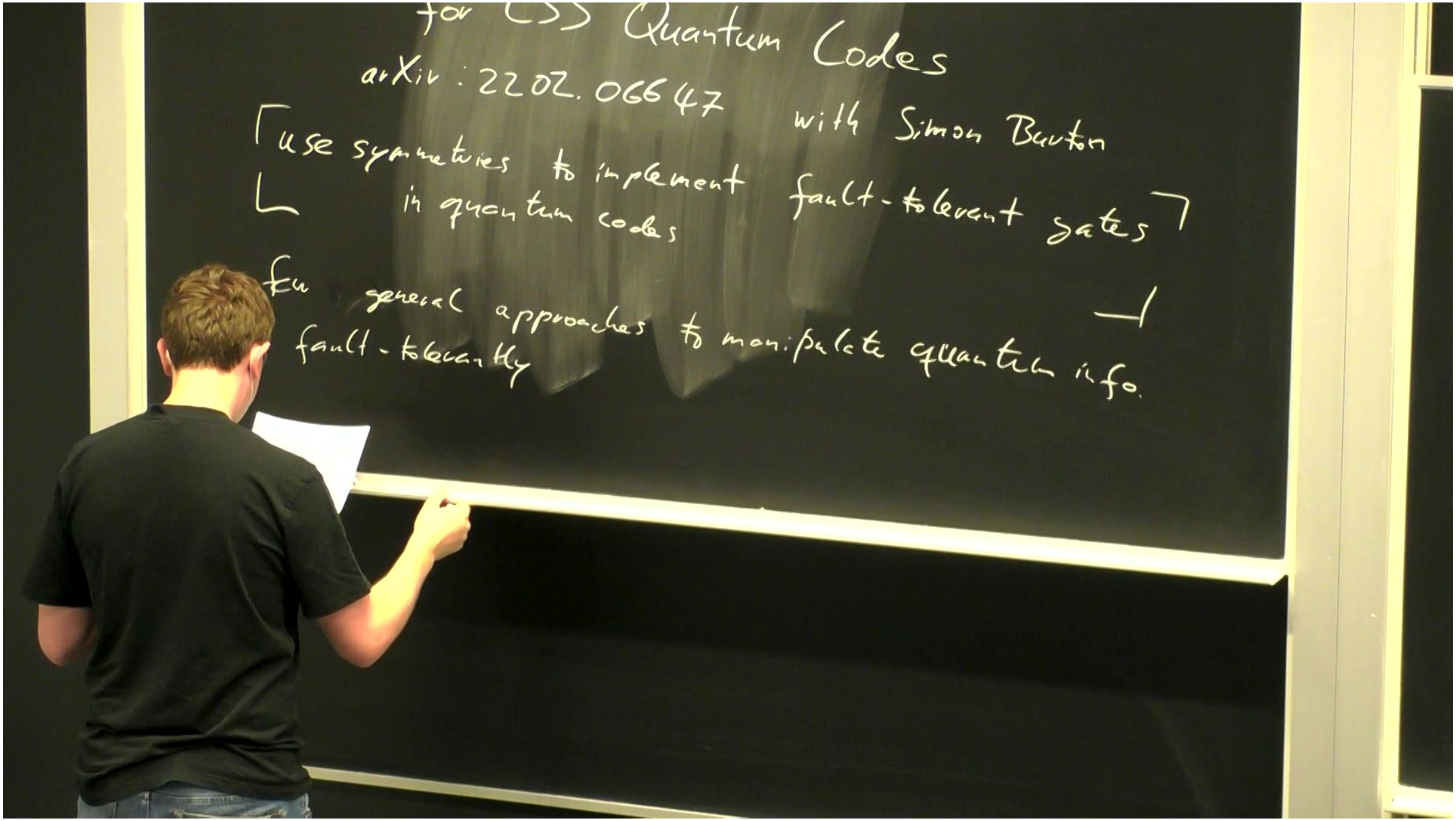
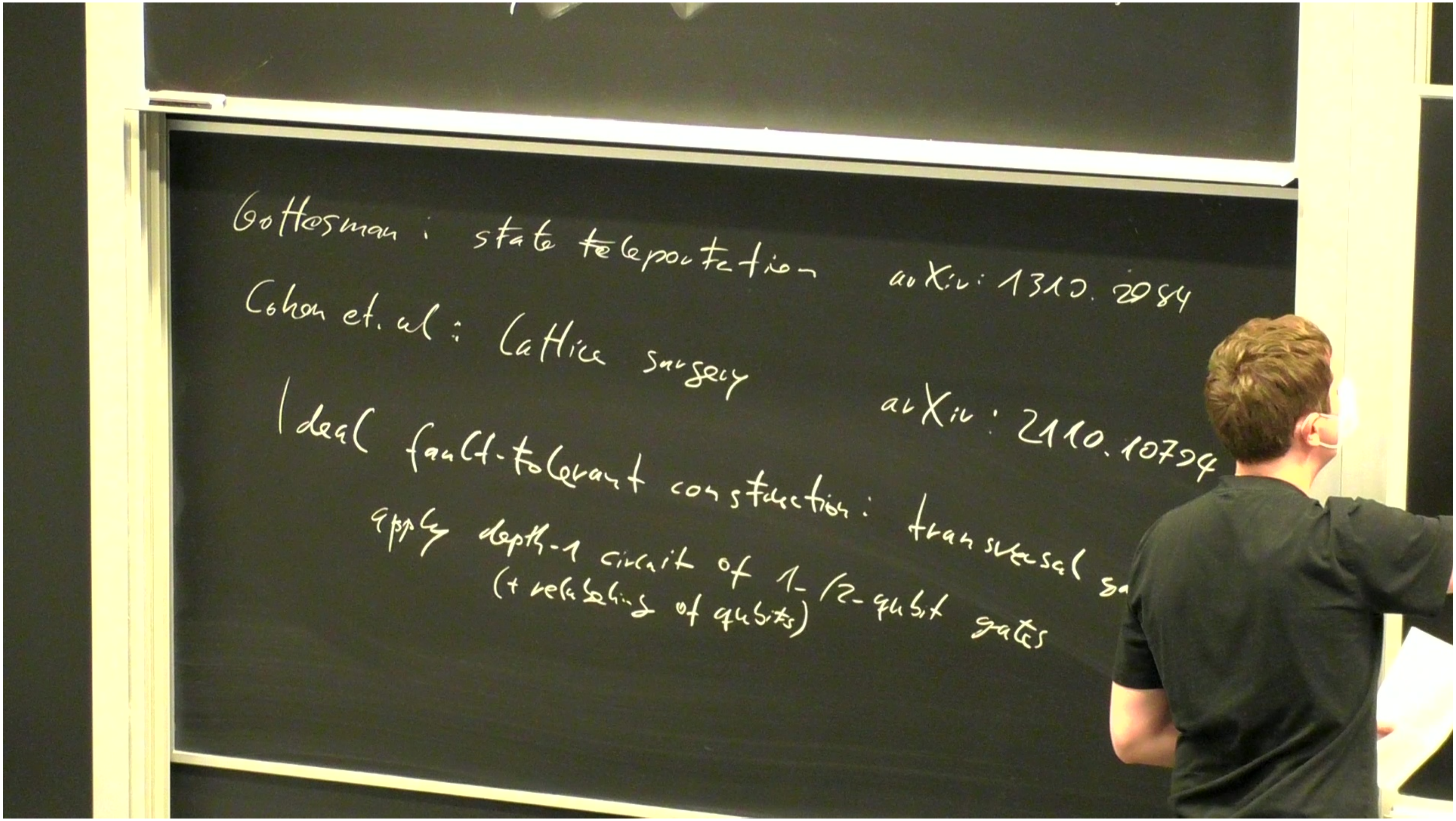
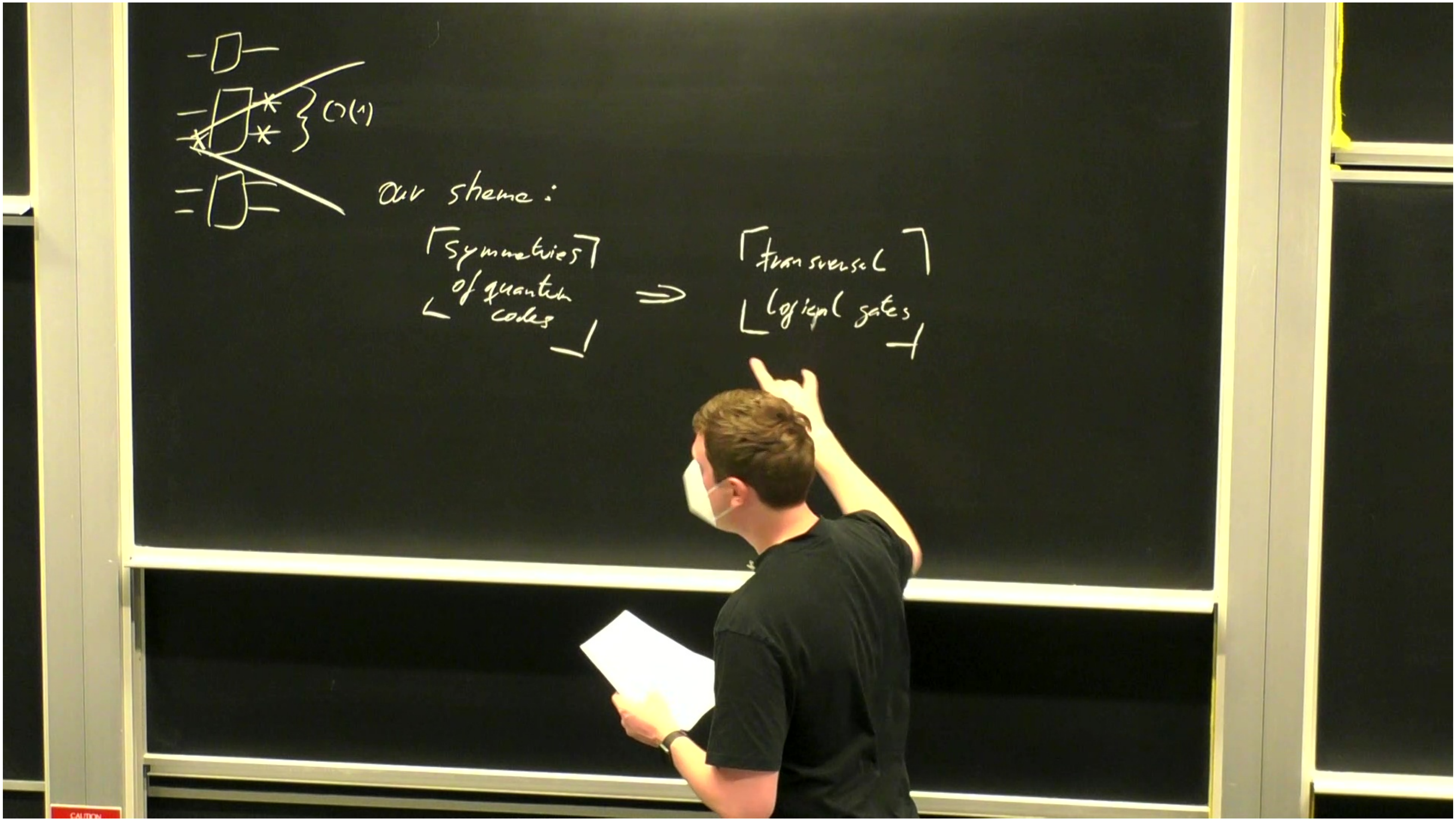
Abstract: We generalize the concept of folding from surface codes to CSS codes by considering certain dualities within them. In particular, this gives a general method to implement logical operations in suitable LDPC quantum codes using transversal gates and qubit permutations only. To demonstrate our approach, we specifically consider a [[30, 8, 3]] hyperbolic quantum code called Bring's code. Further, we show that by restricting the logical subspace of Bring's code to four qubits, we can obtain the full Clifford group on that subspace.

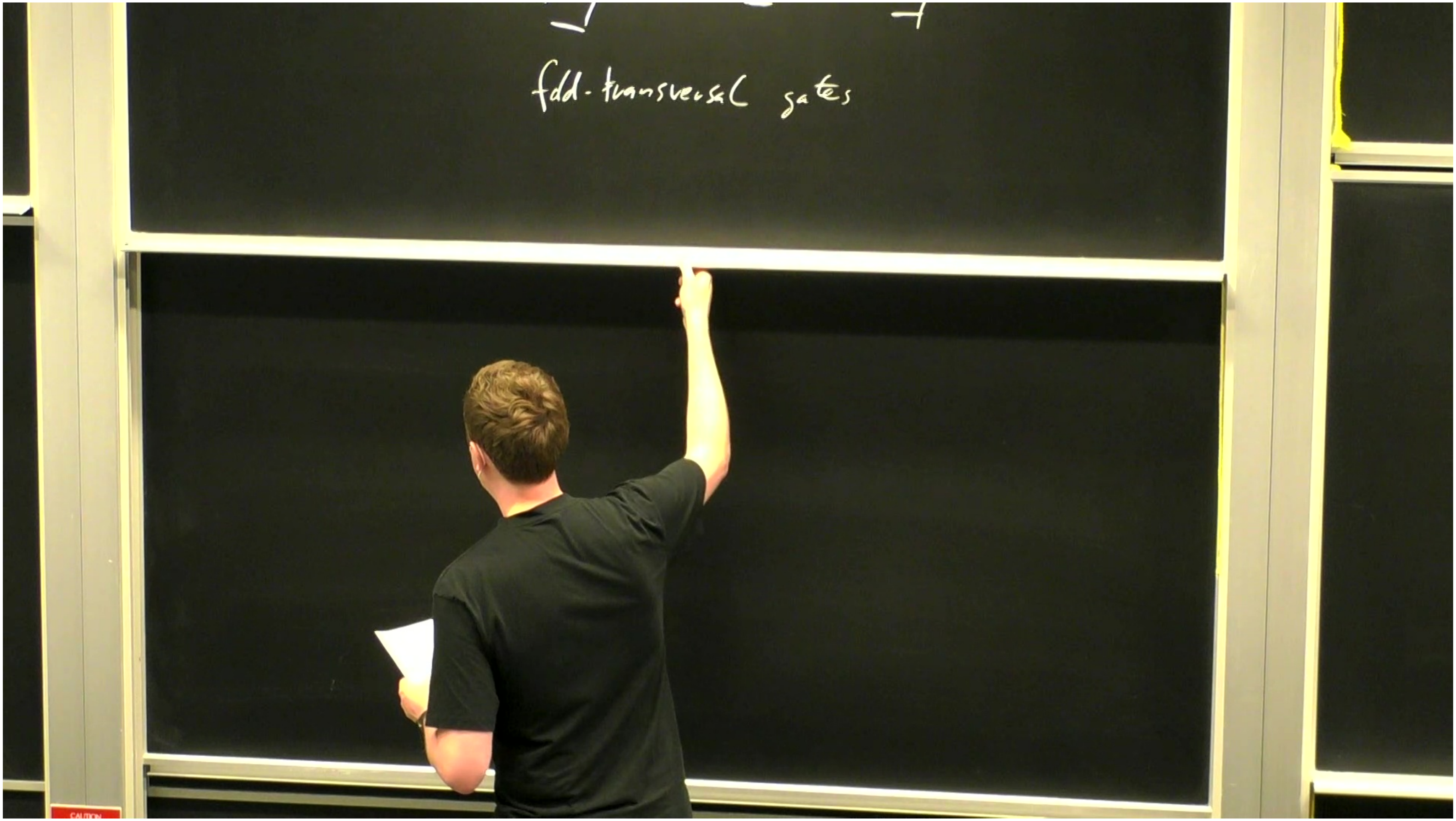
Zoom Link: https://pitp.zoom.us/j/94852018243?pwd=RmFHM1QyNS9CK0RMRm5yUEt0MzdSZZz09

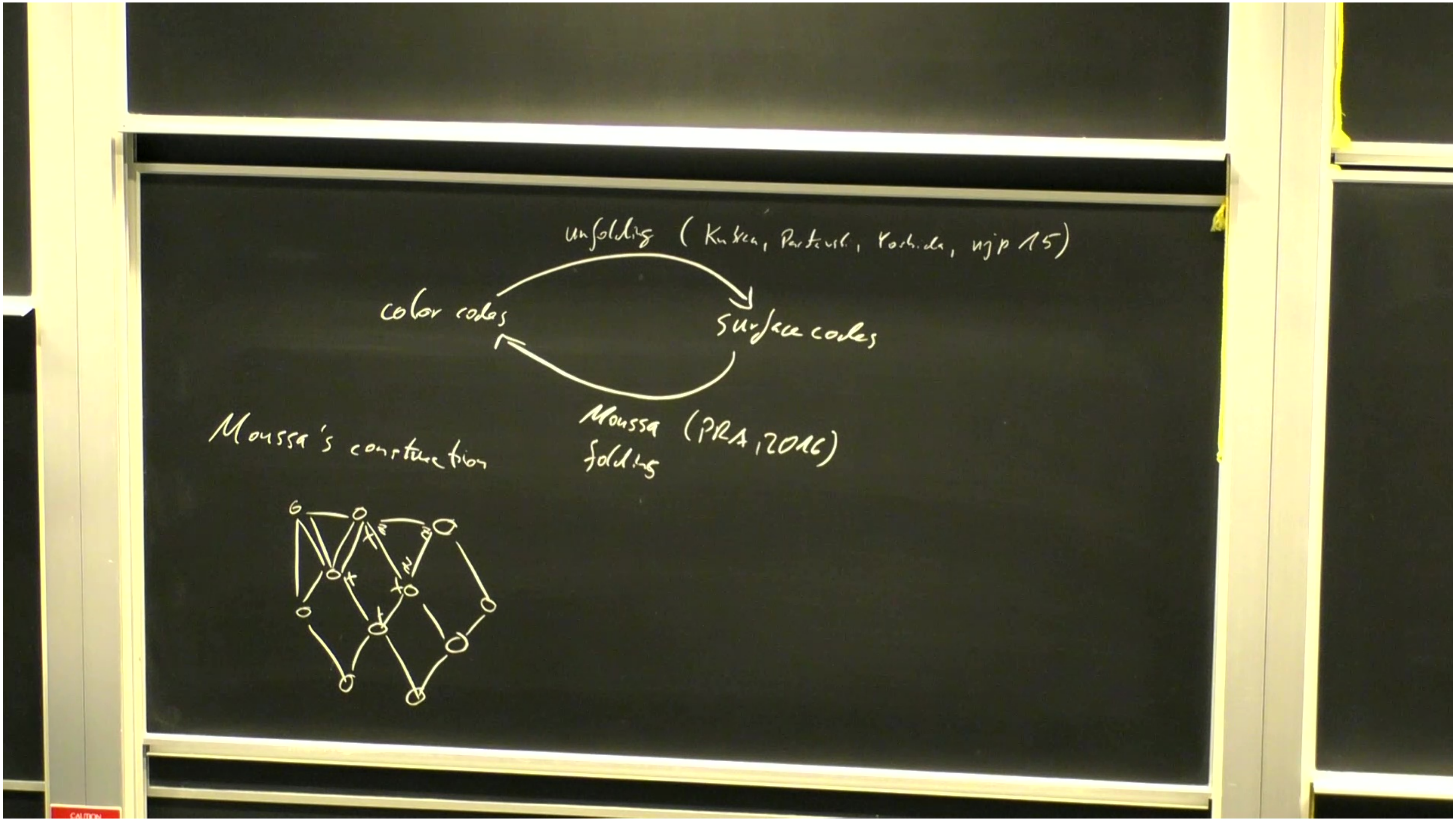

Fold-Transversal Clifford Gates
for CSS Quantum Codes
arXiv: 2202.06647
with Simon Burton


for CSS Quantum Codes
arXiv: 2202.06647 with Simon Burton
use symmetries to implement fault-tolerant gates
in quantum codes
for general approaches to manipulate quantum info.
fault-tolerantly


Gottesman: state teleportation
arXiv: 1310.2984
Cohen et. al: Lattice surgery
arXiv: 2110.10794
Ideal fault-tolerant construction: transversal ga
apply depth-1 circuit of 1-/2-qubit gates
(+ relabeling of qubits)

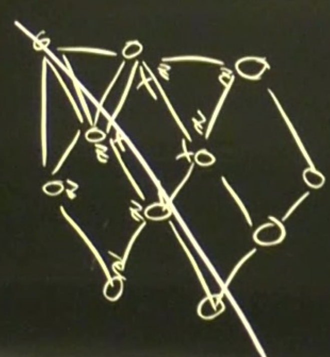

our sheme:
symmetries of quantum codes
⇒
transversal logical gates

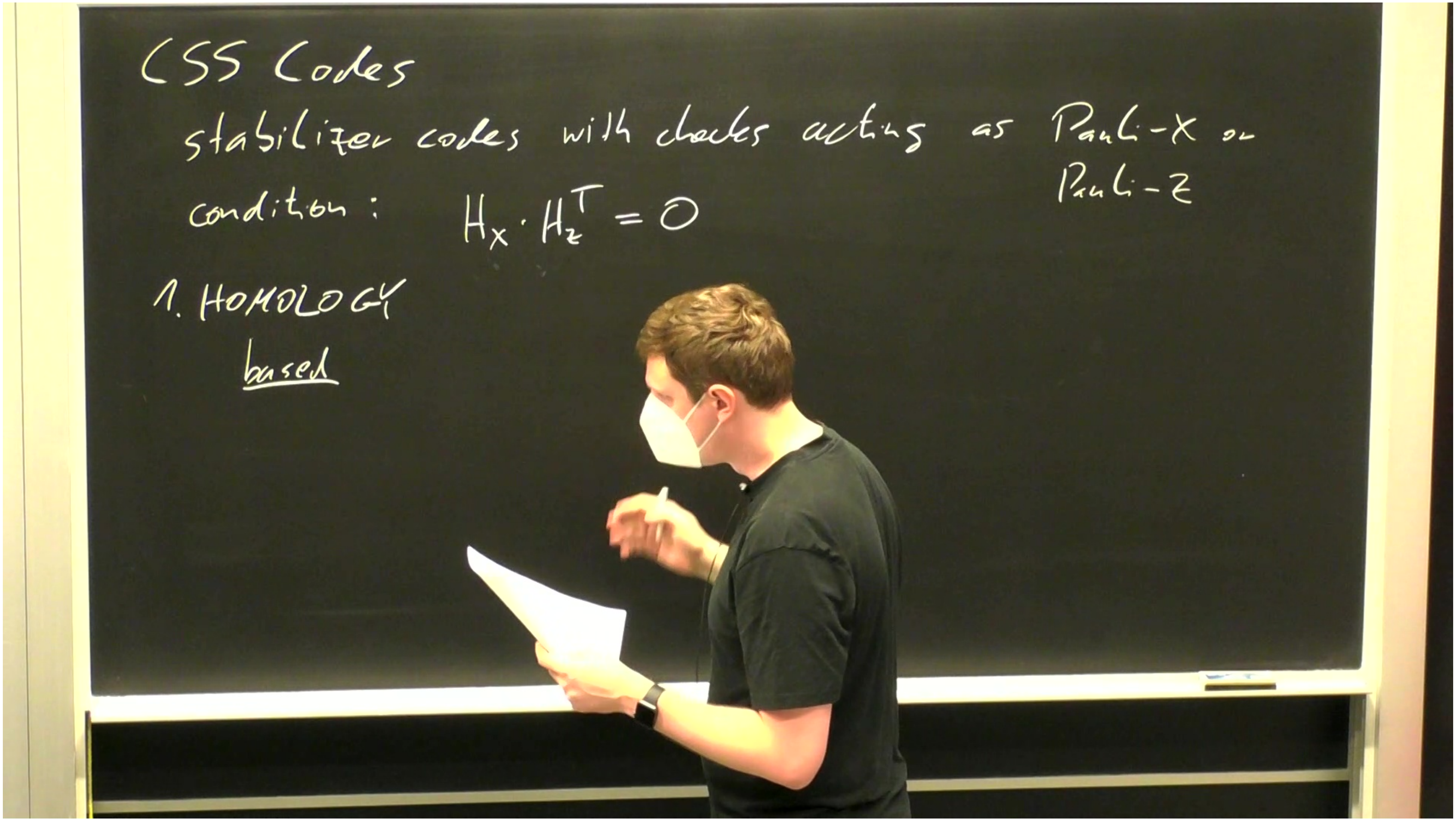

fold-transversal gates

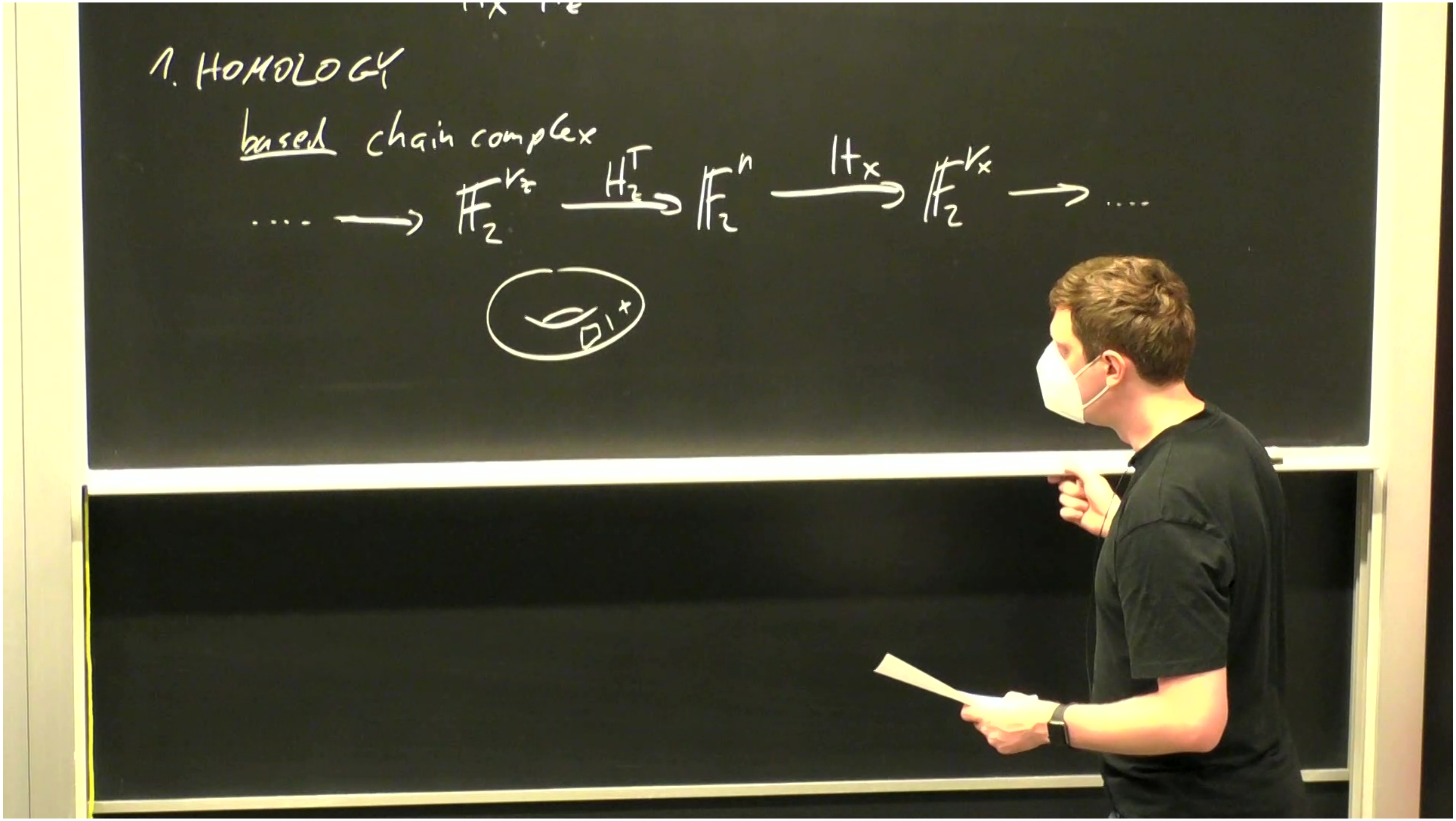

unfolding (Kubica, Pastawski, Yoshida, njp 15)
color codes
surface codes
Moussa (PRA, 2016)
folding
Moussa's construction

$$\begin{tikzcd} \text{color codes} \arrow[r, bend left, "\text{unfolding (Kubica, Pastawski, Yoshida, njp 15)}"] & \text{surface codes} \arrow[l, bend left, "\text{Moussa (PRA, 2016) folding}"] \end{tikzcd}$$

Moussa's construction




CSS Codes
stabilizer codes with checks acting as Pauli-X or
Pauli-Z
condition: $H_X \cdot H_Z^T = 0$
1. HOMOLOGY
based

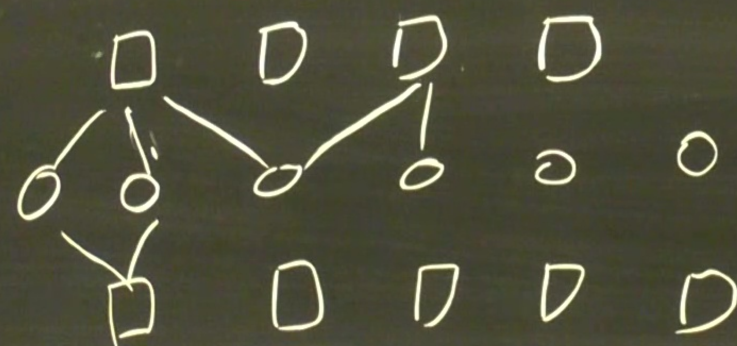

1. HOMOLOGY
based chain complex
$\dots \longrightarrow \mathbb{F}_2^{r_z} \xrightarrow{H_z^T} \mathbb{F}_2^{n} \xrightarrow{H_x} \mathbb{F}_2^{r_x} \longrightarrow \dots$

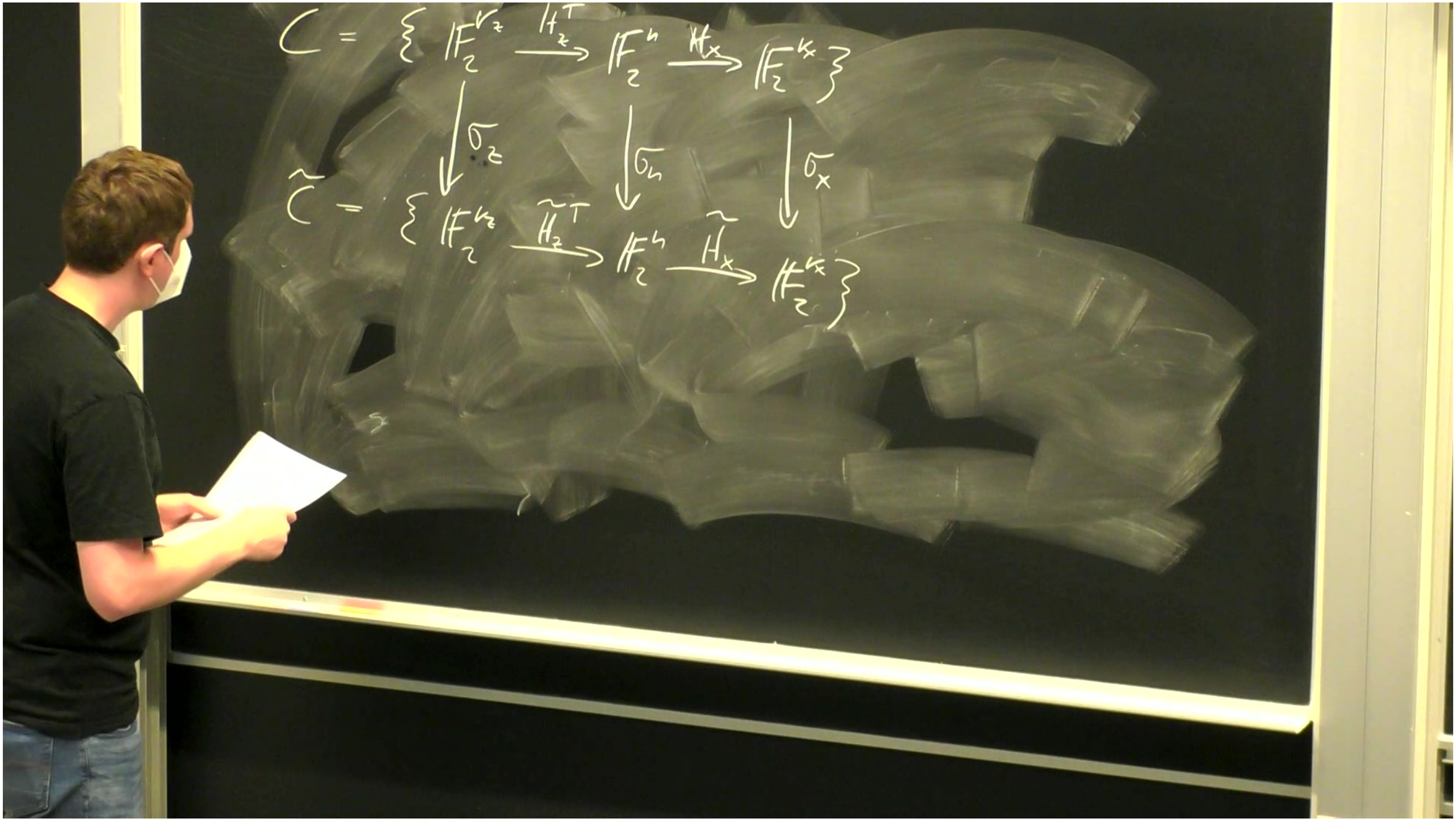
DUAL CODE

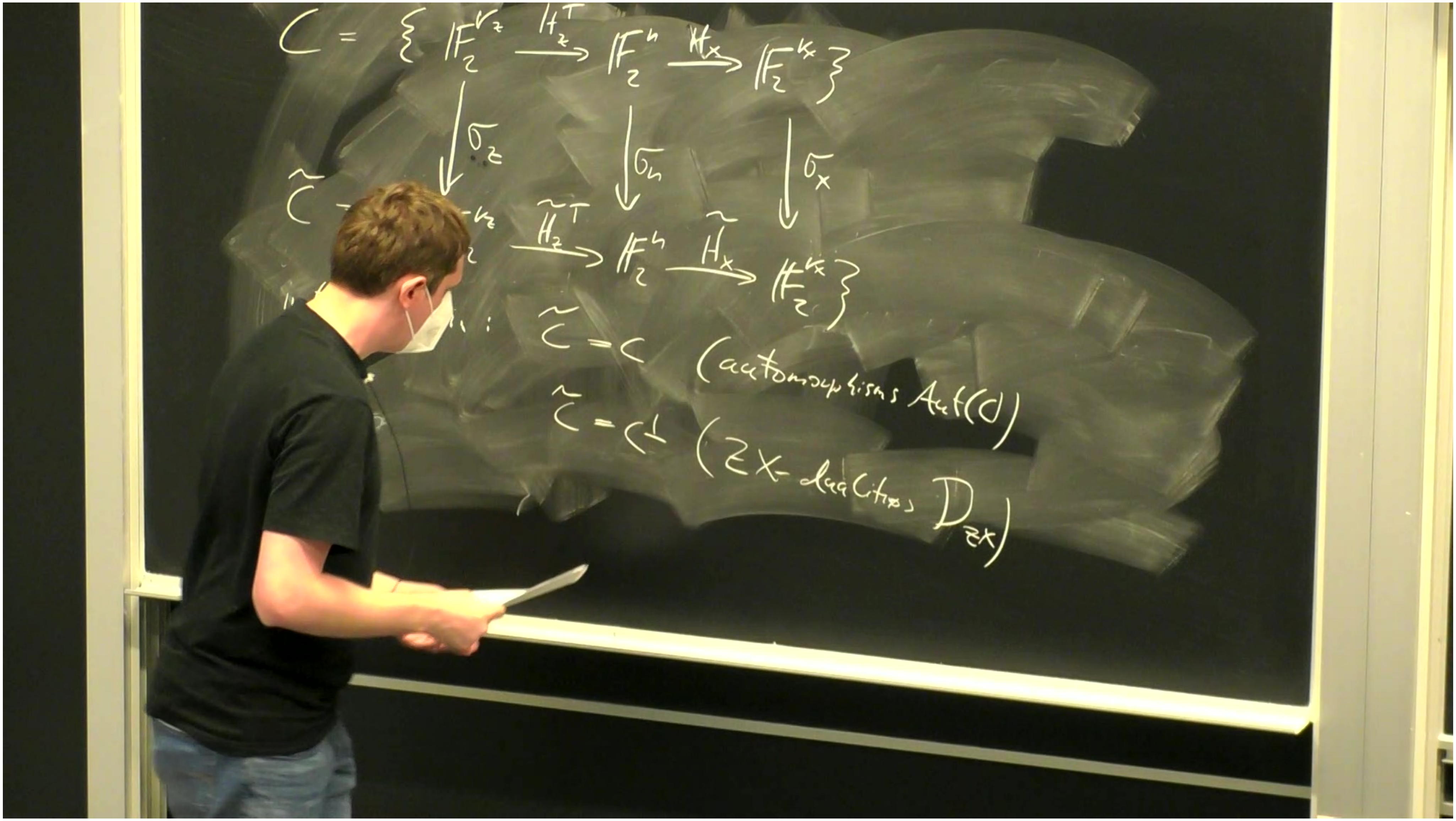
$$C^T = \{ \mathbb{F}_2^{r_x} \xrightarrow{H_x^T} \mathbb{F}_2^{n} \xrightarrow{H_z} \mathbb{F}_2^{r_z} \}$$

self-dual CSS code $\quad C = C^\perp$

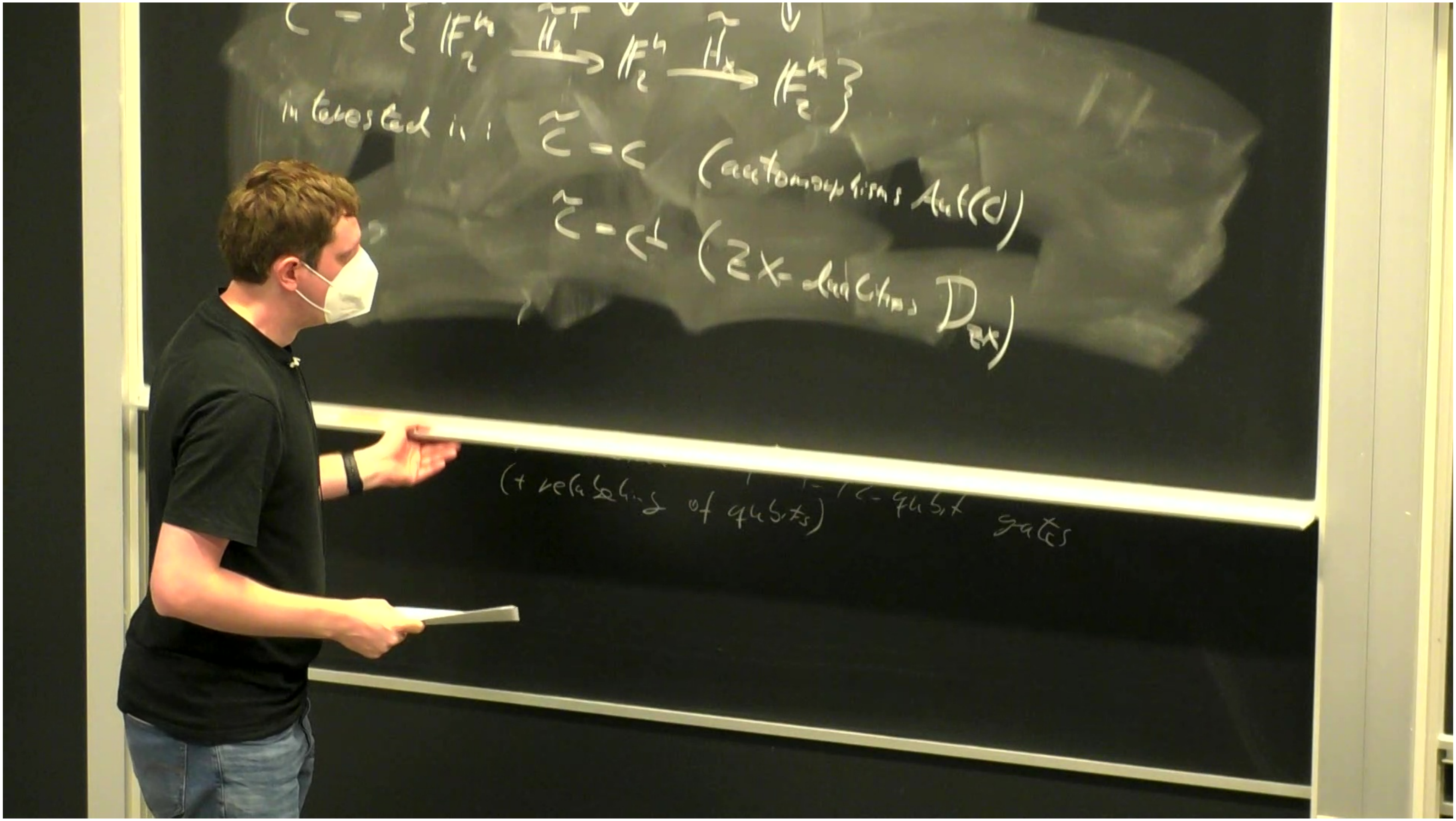
2. TANNER GRAPH / POSET DIAGRAM

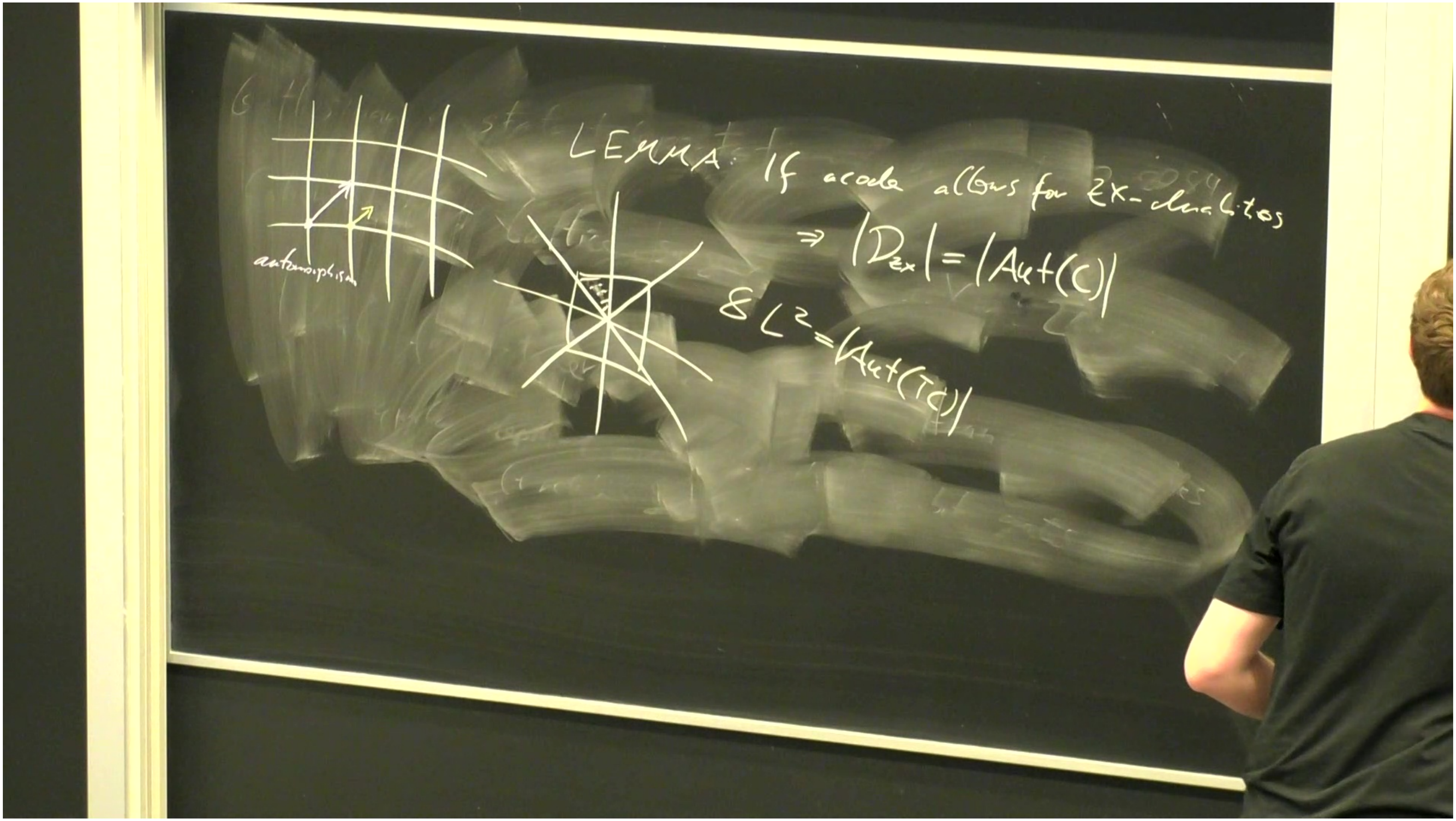


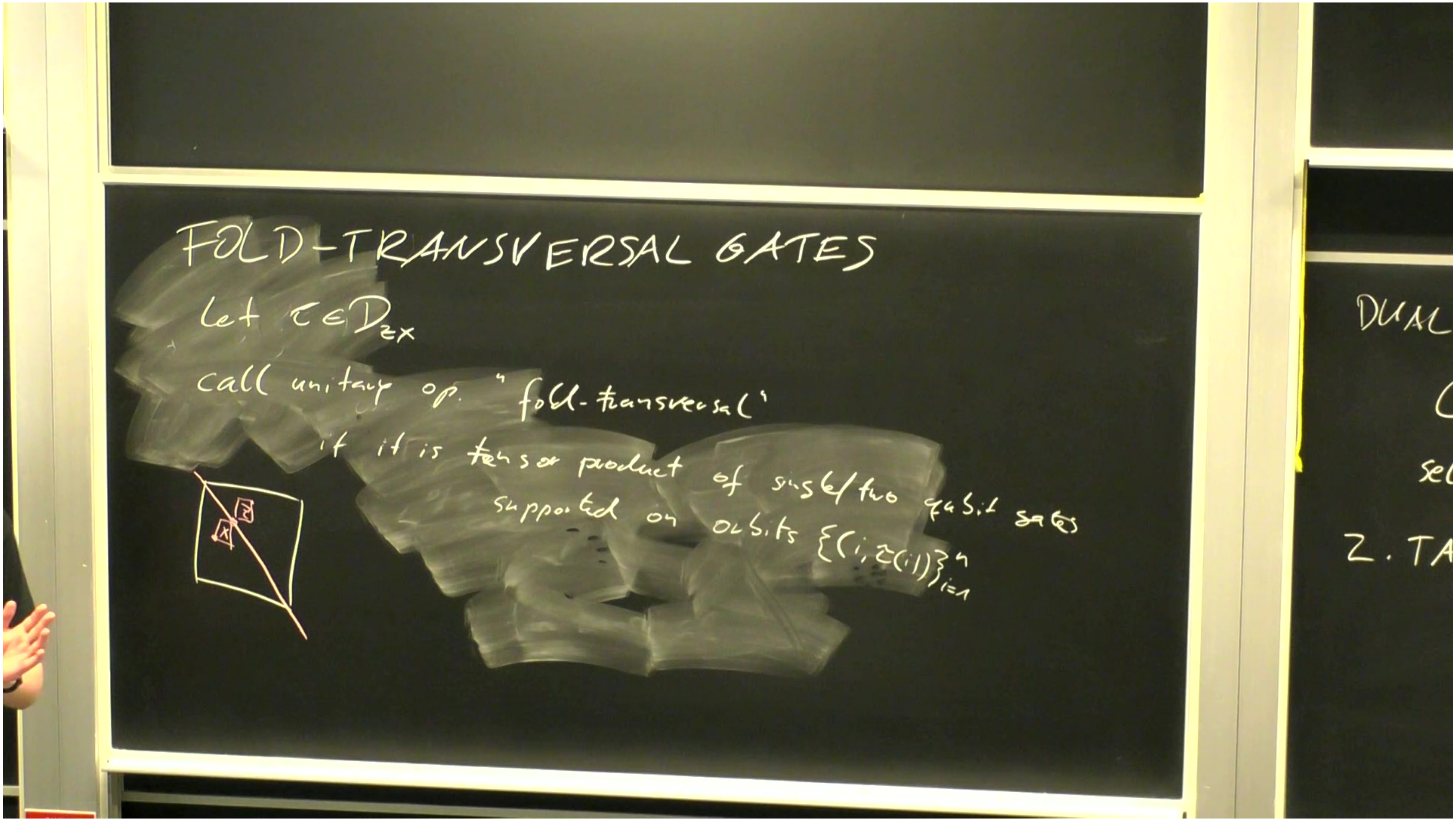

$C = \{ \mathbb{F}_2^{r_z} \xrightarrow{H_z^T} \mathbb{F}_2^{n} \xrightarrow{H_x} \mathbb{F}_2^{k_x} \}$
σ_z
σ_n
σ_x
$\tilde{C} = \{ \mathbb{F}_2^{k_z} \xrightarrow{\tilde{H}_z^T} \mathbb{F}_2^{n} \xrightarrow{\tilde{H}_x} \mathbb{F}_2^{k_x} \}$

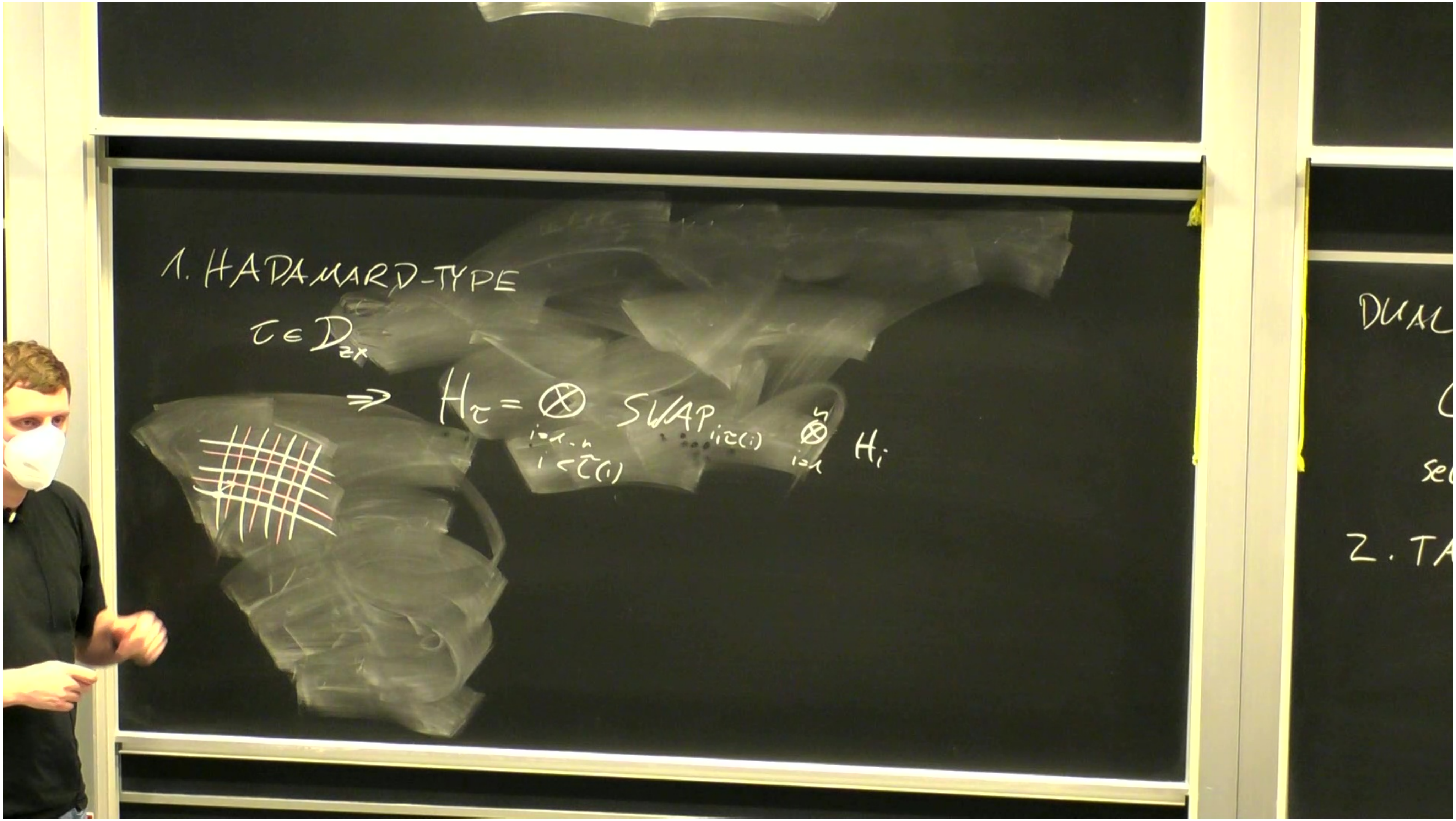


$C = \{ \mathbb{F}_2^{r_z} \xrightarrow{H_z^T} \mathbb{F}_2^n \xrightarrow{H_x} \mathbb{F}_2^{r_x} \}$
σ_z
σ_n
σ_x
$\tilde{C} = \{ \tilde{H}_z^T \; \mathbb{F}_2^n \xrightarrow{\tilde{H}_x} \mathbb{F}_2^{r_x} \}$
$\tilde{C} = C$ (automorphisms $\mathrm{Aut}(C)$)
$\tilde{C} = C^\perp$ (ZX-dualities $\mathbb{D}_{ZX}$)


interested in:
(automorphisms Aut(C))
(ZX-dualities
(+ relabeling of qubits)
qubit gates


LEMMA: If a code allows for ZX-dualities
⇒ |D_ZX| = |Aut(C)|
& L^2 = |Aut(TC)|
automorphism


FOLD-TRANSVERSAL GATES
Let τ ∈ D_ZX
call unitary op. "fold-transversal"
if it is tensor product of single/two qubit gates
supported on orbits {(i, τ(i))}_{i=1}^n
Z
X
DUAL
2. TA


1. HADAMARD-TYPE
$\tau \in D_{ZX}$
$\Rightarrow H_\tau = \bigotimes_{\substack{i=1..n \\ i<\tau(i)}} SWAP_{i,\tau(i)} \bigotimes_{i=1}^{n} H_i$
DUAL
2. TA

1. HADAMARD-TYPE

$\tau \in D_{2x}$

$$\Rightarrow \quad H_\tau = \bigotimes_{\substack{i=1..n \\ i<\tau(i)}} SWAP_{i\tau(i)} \bigotimes_{i=1}^{n} H_i$$

In general:

- does <u>not</u> map $X_i \longleftrightarrow Z_i$
- $H_\tau^2 \neq I$

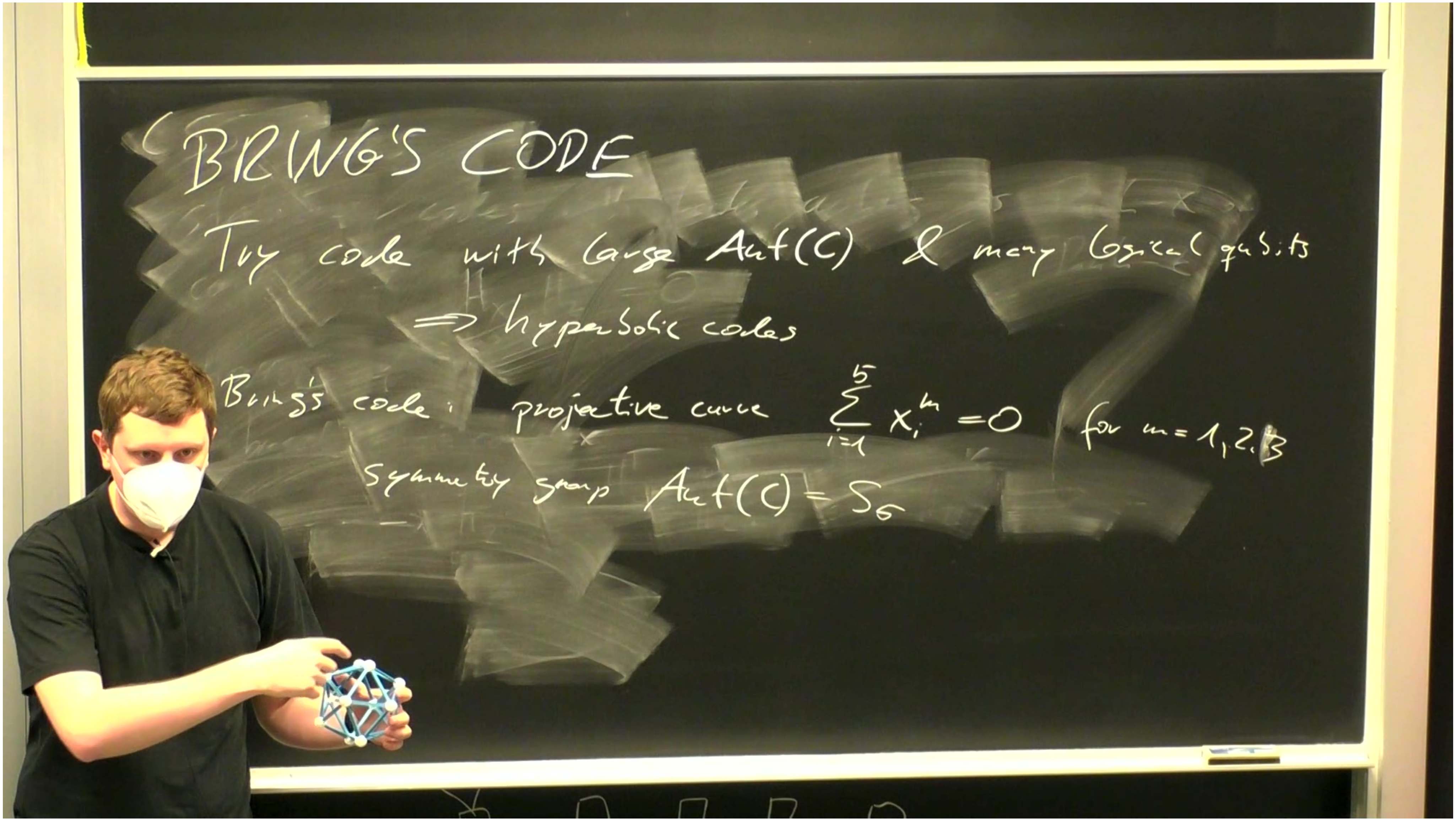
DUAL

2. TA

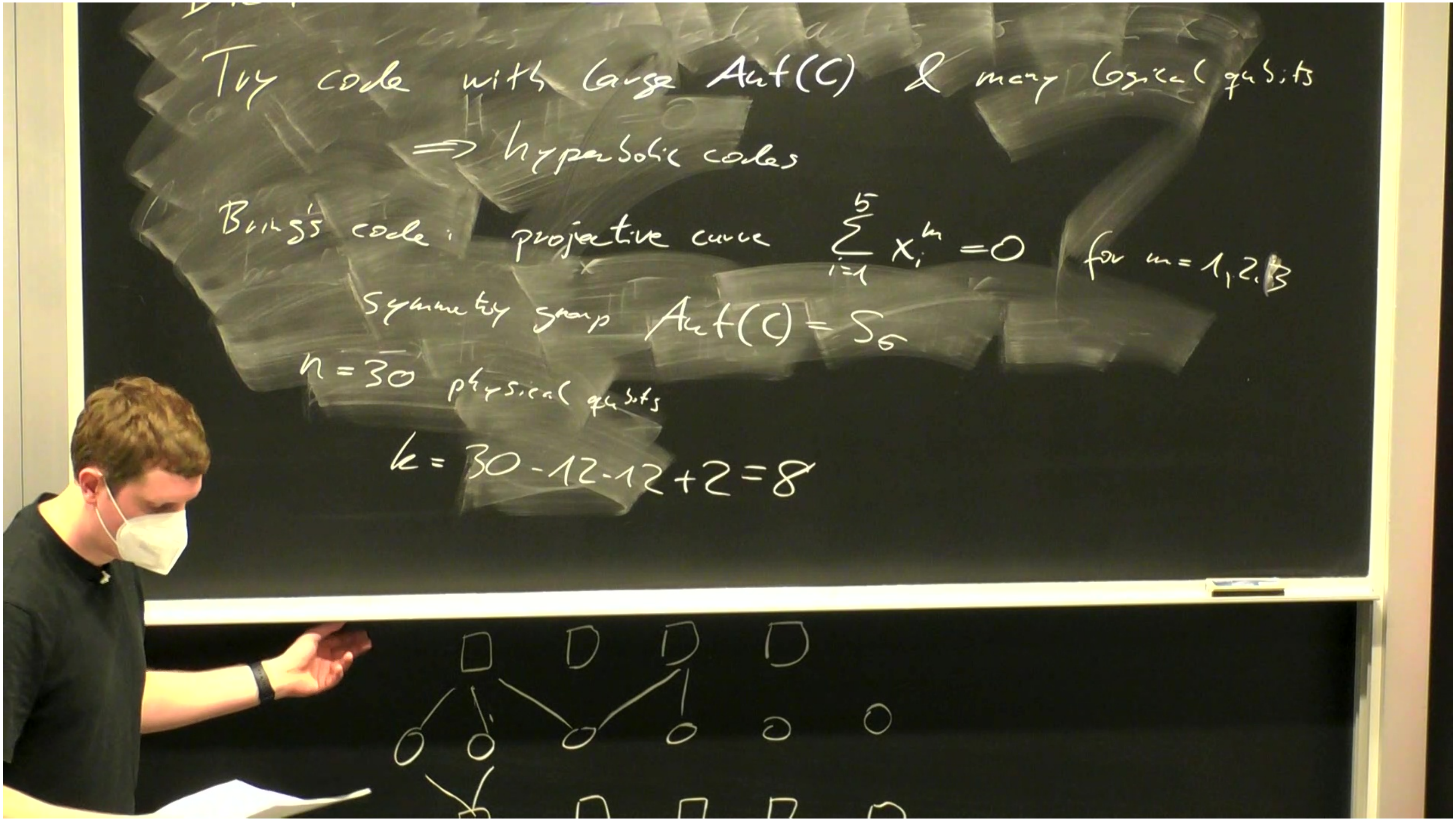
2. PHASE-TYPE GATE

given suitable $\tau \in D_{ex}$

$$\Rightarrow \quad S_\tau = \bigotimes_{\substack{i=1 \\ i=\tau(i)}}^{n} S_i^{(+)} \; \bigotimes_{\substack{i=1 \\ i<\tau(i)}}^{n} CZ_{i,\tau(i)}$$



BRING'S CODE
Try code with large Aut(C) & many logical qubits
⇒ hyperbolic codes
Bring's code: projective curve $\sum_{i=1}^{5} x_i^m = 0$ for m = 1, 2, 3
symmetry group Aut(C) = S_5


Try code with large Aut(C) & many logical qubits
⇒ hyperbolic codes
Bring's code: projective curve $\sum_{i=1}^{5} x_i^m = 0$ for m = 1, 2, 3
Symmetry group $Aut(C) = S_6$
n = 30 physical qubits
k = 30 - 12 - 12 + 2 = 8

physical qubits

$k = 30 - 12 - 12 + 2 = 8$

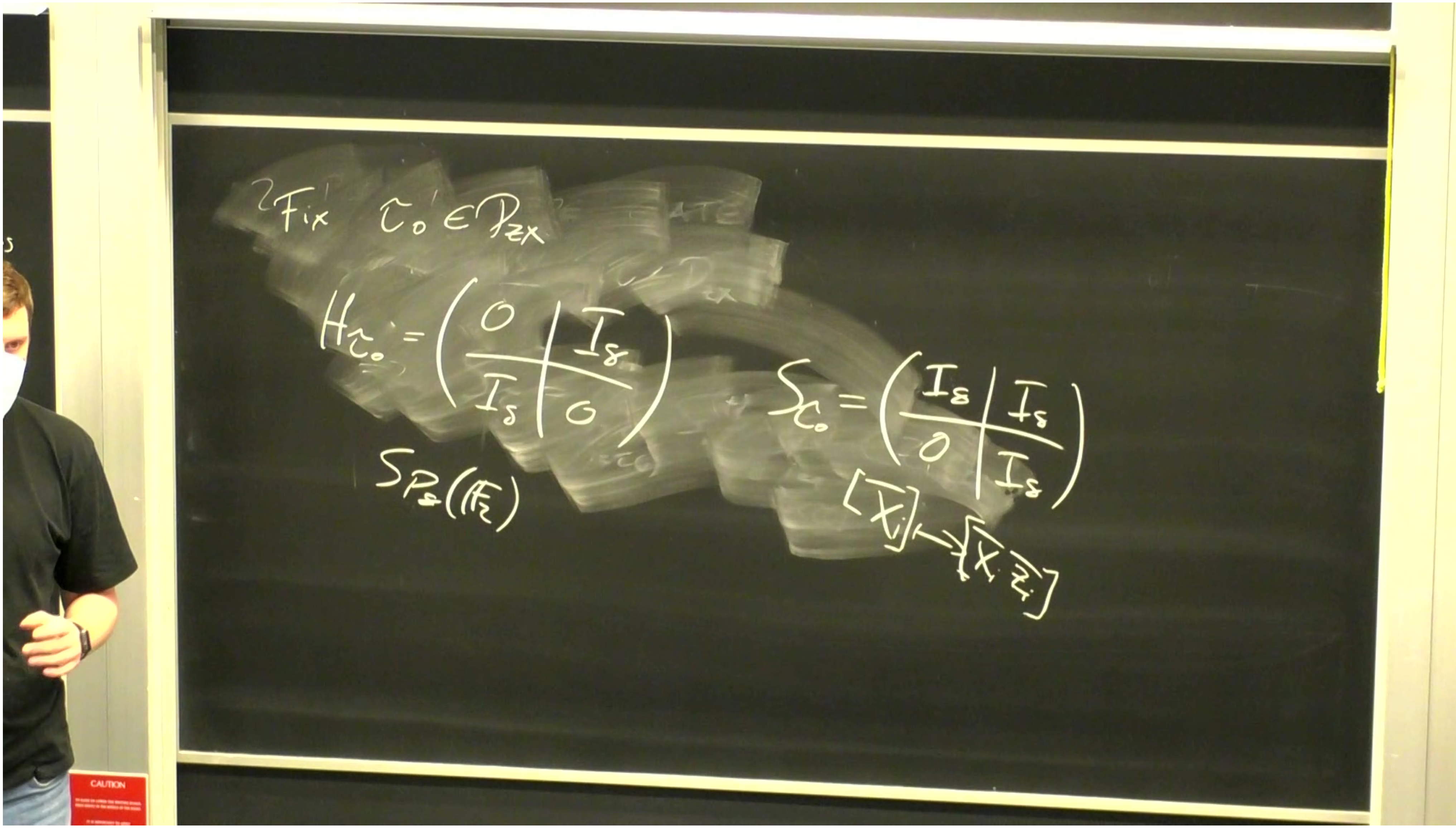
PERMUTATION GATES

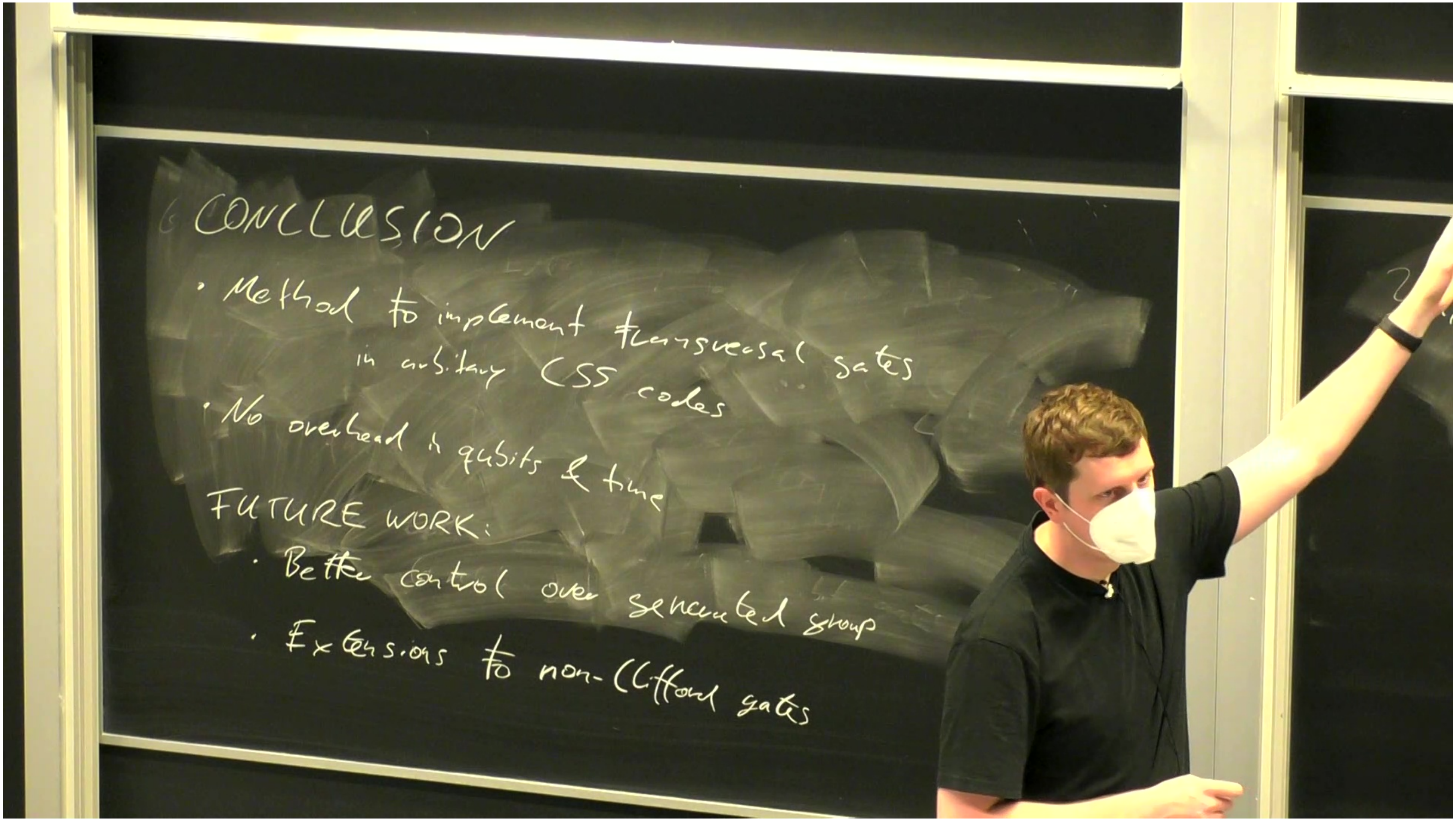
use that $\overline{\mathcal{P}_k}/\langle i \rangle \cong H_1 \oplus H^1$

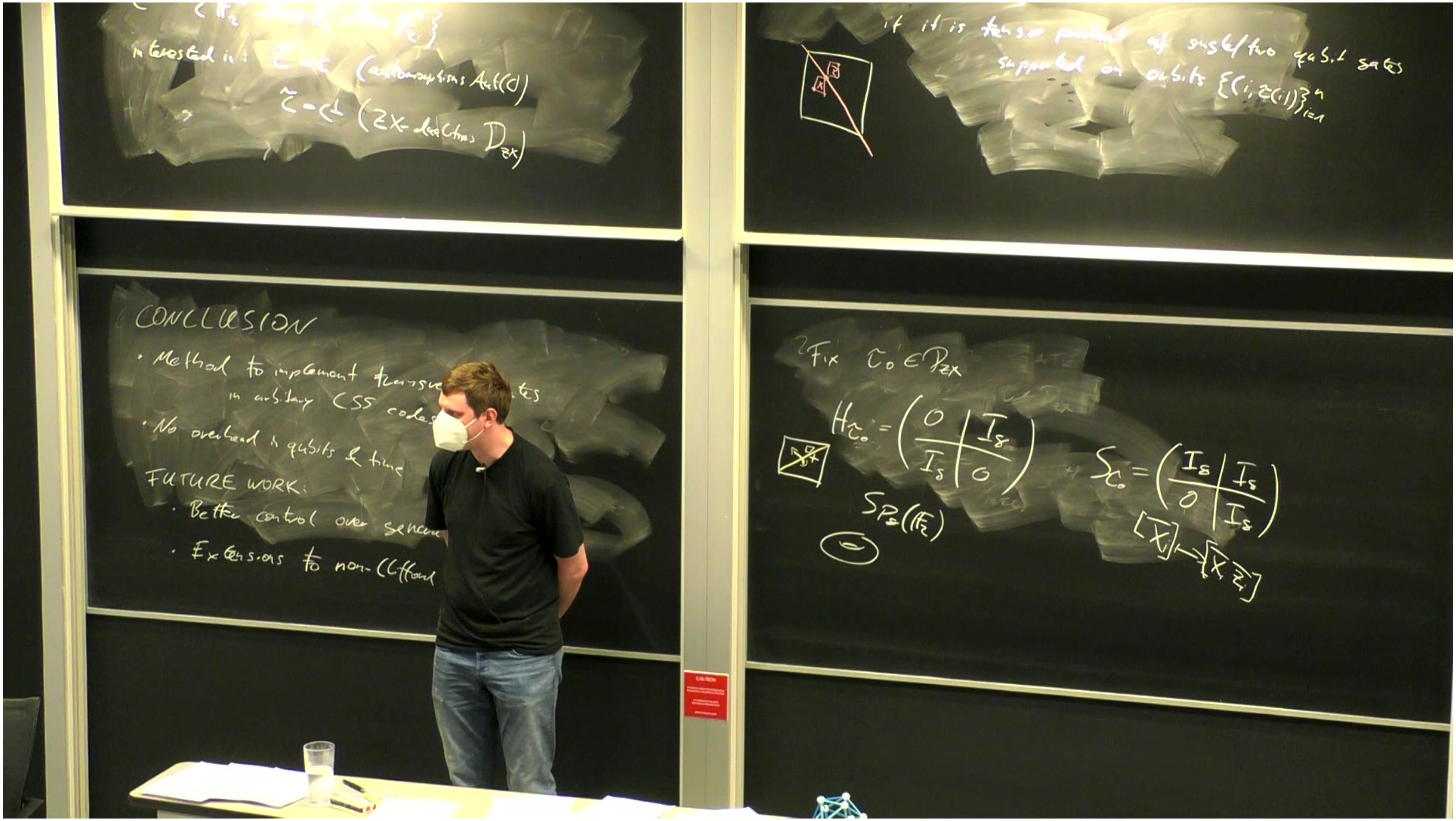
& $\overline{\mathcal{C}_k}/\overline{\mathcal{P}_k} = Sp_{2k}(\mathbb{F}_2)$

$S_5 \longrightarrow GL(H_1 \oplus H^1)$ faithful

$S_5 < Sp_{16}(\mathbb{F}_2)$


Fix $\tau_0 \in P_{2X}$
$H_{\tau_0} = \left(\begin{array}{c|c} 0 & I_8 \\ \hline I_8 & 0 \end{array}\right)$
$Sp_8(\mathbb{F}_2)$
$S_{\tau_0} = \left(\begin{array}{c|c} I_8 & I_8 \\ \hline 0 & I_8 \end{array}\right)$
$[X_i] \longmapsto [X_i Z_i]$


CONCLUSION
• Method to implement transversal gates
in arbitrary CSS codes
• No overhead in qubits & time
FUTURE WORK:
• Better control over generated group
• Extensions to non-Clifford gates


CONCLUSION
· No overhead in qubits & time
FUTURE WORK:
· Extensions to non-Clifford