Title: Emergent times in holography

Speakers: Hong Liu

Series: Quantum Fields and Strings

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Abstract: In holographic duality an eternal AdS black hole is described by two copies of the boundary CFT in the thermofield double state. We provide explicit constructions in the boundary theory of infalling time evolutions which can take bulk observers behind the horizon. The constructions also help to illuminate the boundary emergence of the black hole horizons, the interiors, and the associated causal structure. A key element is the emergence, in the large N limit of the boundary theory, of a type III1 von Neumann algebraic structure and the half-sided modular translation structure associated with it.

Zoom Link: https://pitp.zoom.us/j/99458239757?pwd=YTBRdXdwTjltMkdZc0Q4WW1jQjA3QT09

Emergent times in holography

Hong Liu

Perimeter Institute, seminar

Mar. 22, 2022

based on work with Samuel Leutheusser

arXiv: 2110.05497 and arXiv: 2112.12156

Problem of time in quantum gravity

QM: time is absolute.

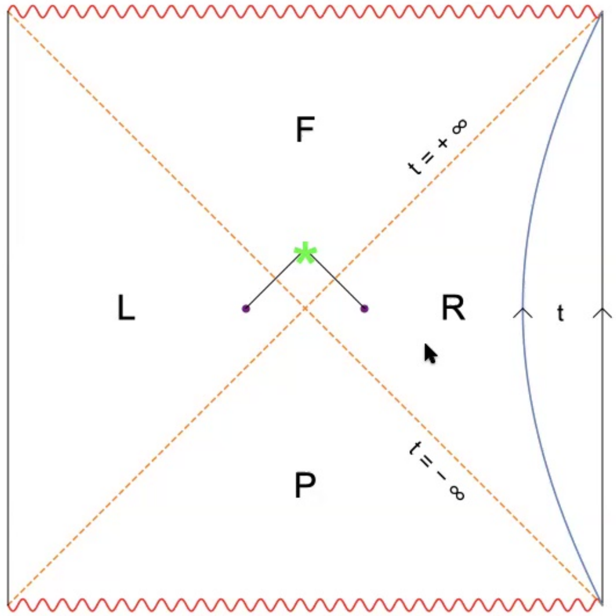
Gravity: most of the time time is meaningless as it can be changed by arbitrary gauge diffeomorphism transformations.

AdS: there is an absolute asymptotic time, which underlies AdS/CFT

QG in spacetimes of other asymptotics appears much more difficult.

In AdS/CFT: can the asymptotic time be sensibly extended to the interior?

When the bulk spacetime is time-translation invariant: yes.



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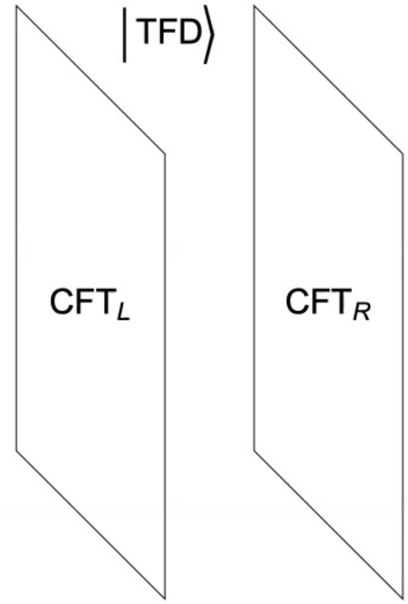


Maldacena, 2001

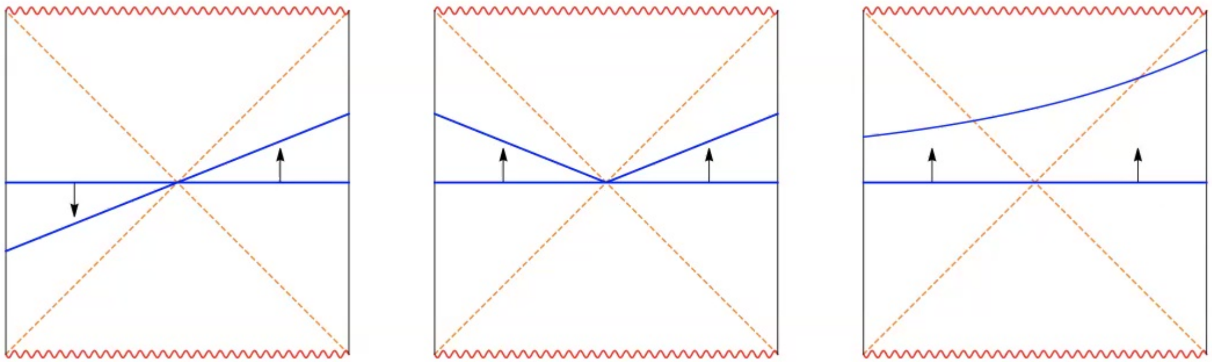
Time-like Killing vector outside the horizon

Many mysteries: F and P regions? Kruskal-like time?

horizons and associated causal structure ?

Interactions between R and L observers in the interior?

Boundary descriptions of bulk evolutions



H_R-H_L | $H_R + H_L$ (not well defined) | Emergent?

Goals:
- Boundary constructions of Kruskal-like evolutions
- Boundary emergence of horizon and associated causal structure

Plan

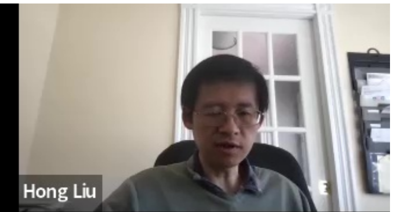
1. Outline the main results

2. Entanglement structure in relativistic quantum field theory: type III_1 von Neumann algebra

   Half-sided modular inclusions/translations

3. Constructions of Kruskal-like time evolutions from boundary

4. Discussions



A description of infalling observers

Will show that there exist "evolution operators" on the boundary

$$U(s) = e^{-iGs}, \; s \in \mathbb{R}, \quad G \text{ hermitian}$$

Properties:

1. G involves both R and L degrees of freedom
2. $$G \geq 0$$
3. $\Phi(X;s) \equiv U(-s)\phi(X)U(s),$ $X \in R$ region

takes it inside the horizon for sufficiently large s.

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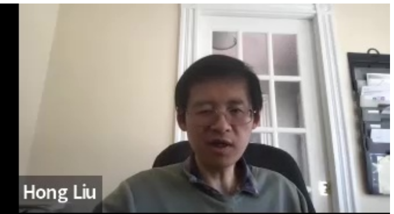
Sharp signatures of horizon, "generate" F and P regions from R and L.

An example for BTZ



For BTZ, such a U(s) can be worked out explicitly.

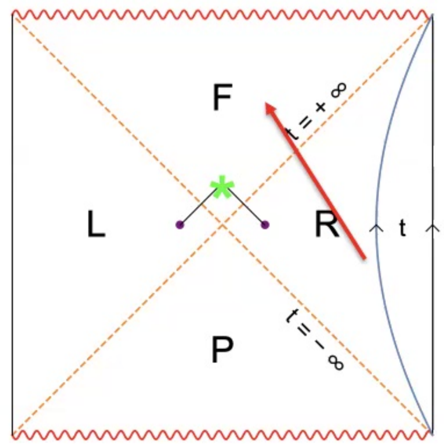
$$\Phi(X;s) \equiv U(-s)\phi(X)U(s), \quad X \in R$$

There exists an $s_0 > 0$

$s < s_0, \quad \Phi(X;s) \in \mathrm{CFT}_R$

$s > s_0, \quad \Phi(X;s) \in \mathrm{CFT}_R \otimes \mathrm{CFT}_L$

signature of a sharp horizon.



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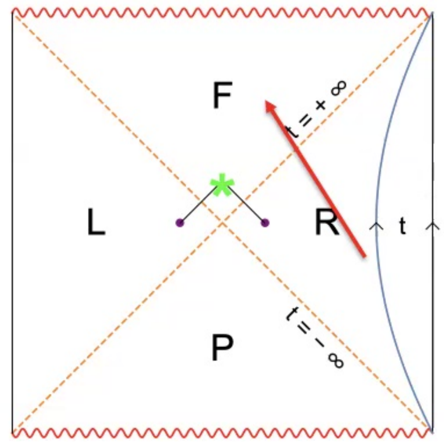
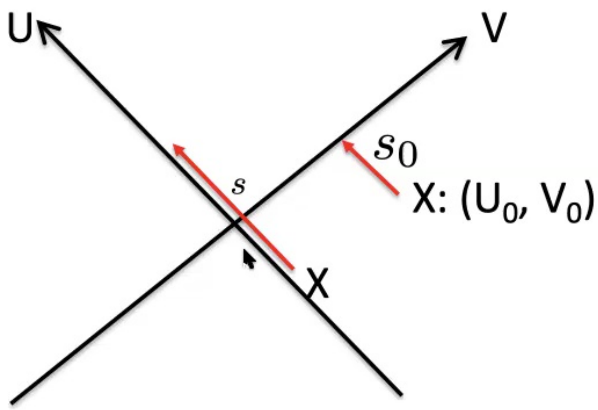


For any two quantum systems, if such U(s) and s_0 exist, we say they are causally connectable.

$$\Phi(X;s) \equiv U(-s)\phi(X)U(s), \quad X \in R$$



U, V: Kruskal null coordinates

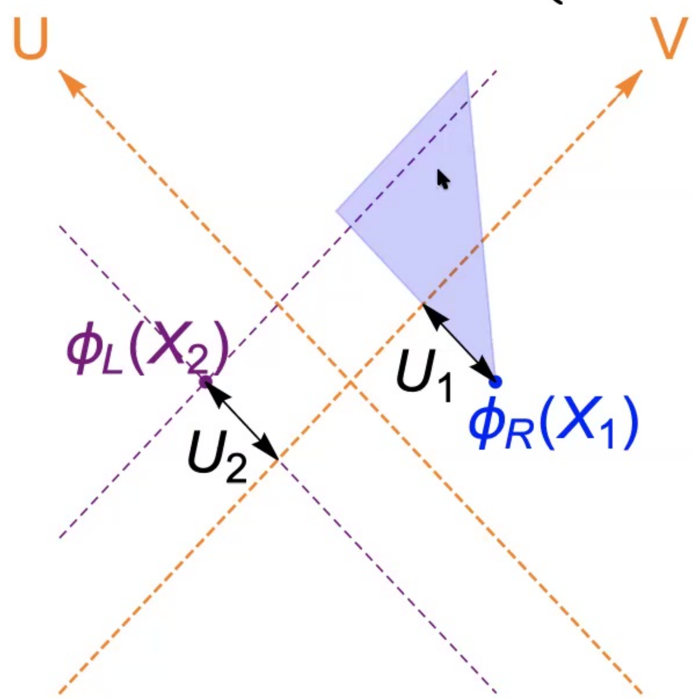
$$s_0 = -U_0$$

X near the horizon, local transformation: Kruskal null translation

General X: transformation is nonlocal, but respects the casual structure

Causal structure

$$[U^\dagger(s)\phi_R(X_1)U(s), \phi_L(X_2)] = \begin{cases} 0 & s < |U_1| + U_2 \\ \neq 0 & s > |U_1| + U_2 \end{cases}$$

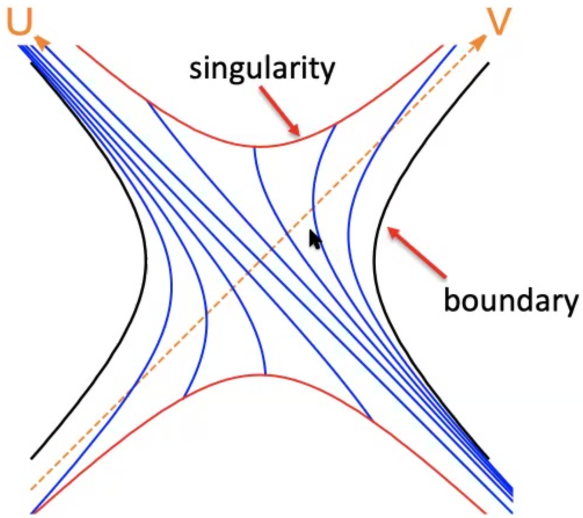


Flow pattern in the large mass limit

$$\Phi(X;s) \equiv U(-s)\phi(X)U(s) \propto \phi(X_s)$$

$$U_s = U_0 + s, \quad V_s = \frac{V_0}{1 - sV_0}$$

average over boundary spatial directions



Family of trajectories

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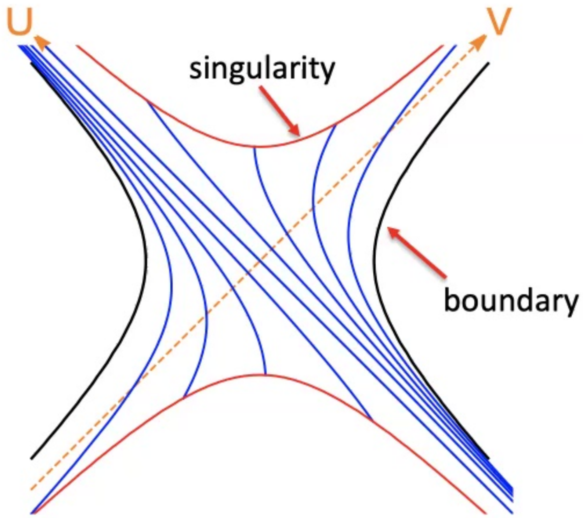

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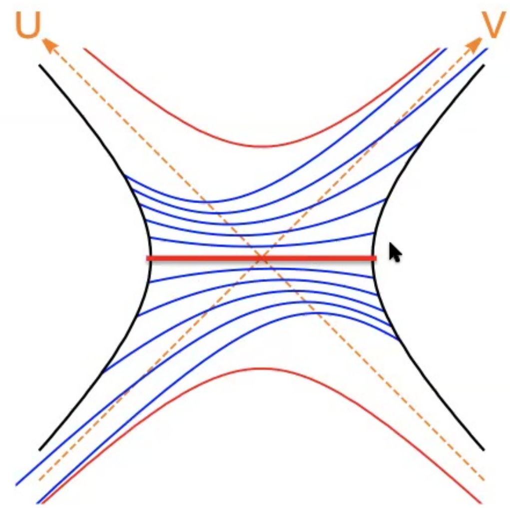
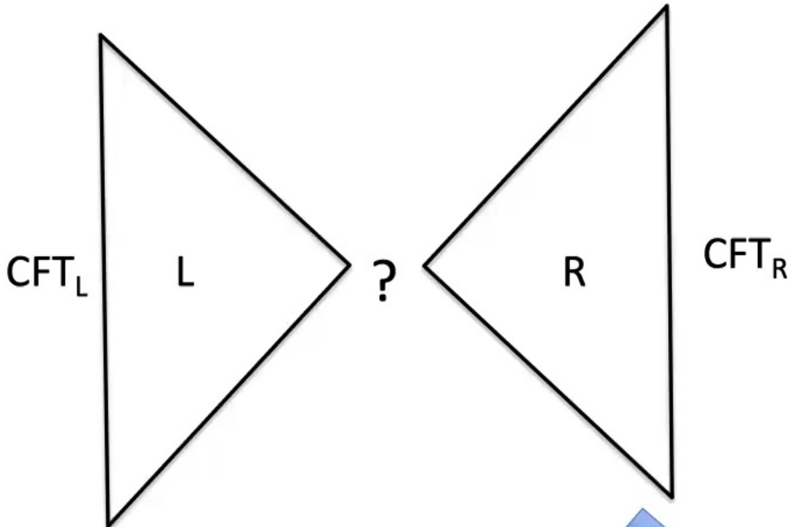
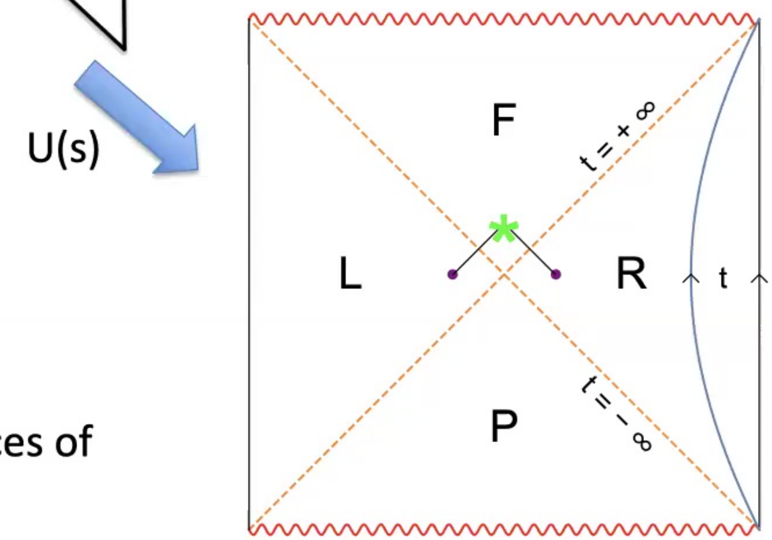


Constant-s slices

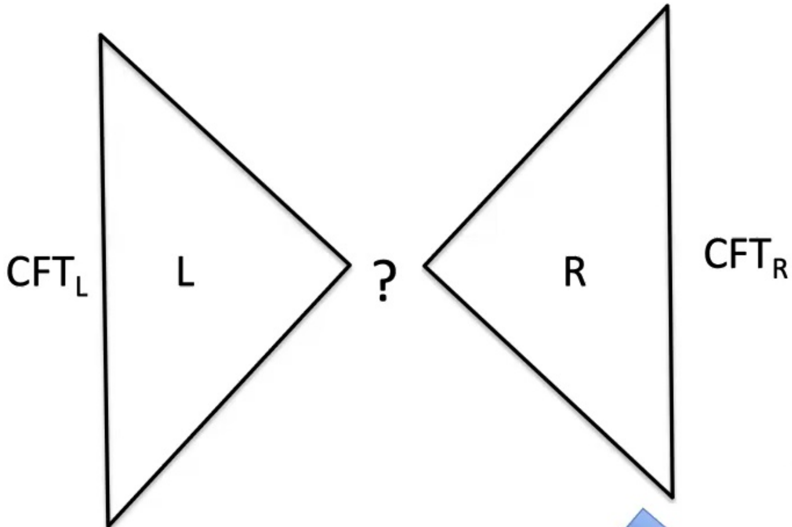


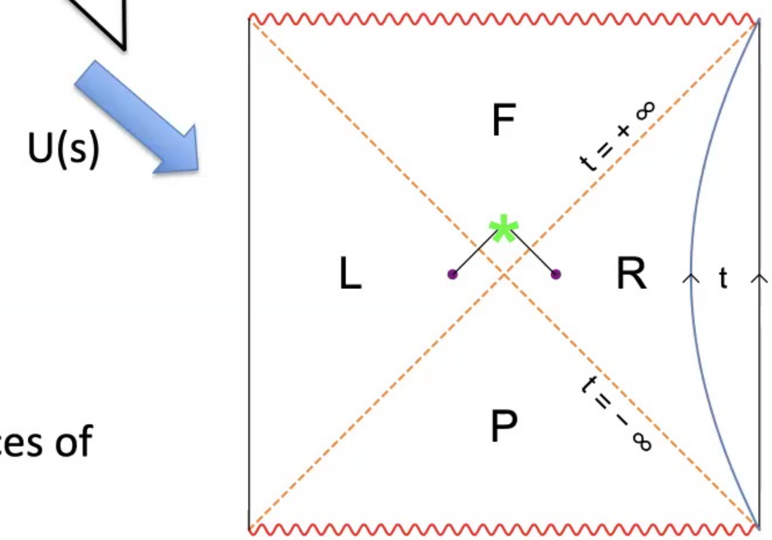


U(s) leads to a new
an emergent
infalling ”time”

Infinite number of choices of
such infalling times

Entanglement in relativistic QFT: III_1 von Neumann algebra





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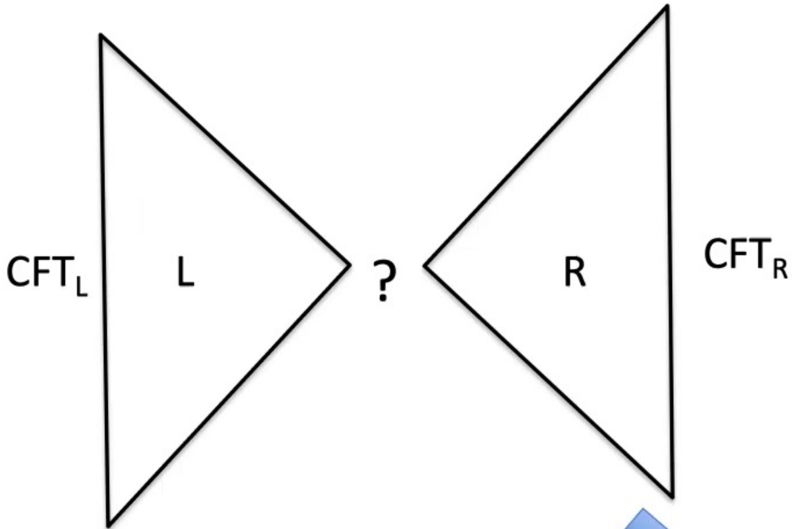

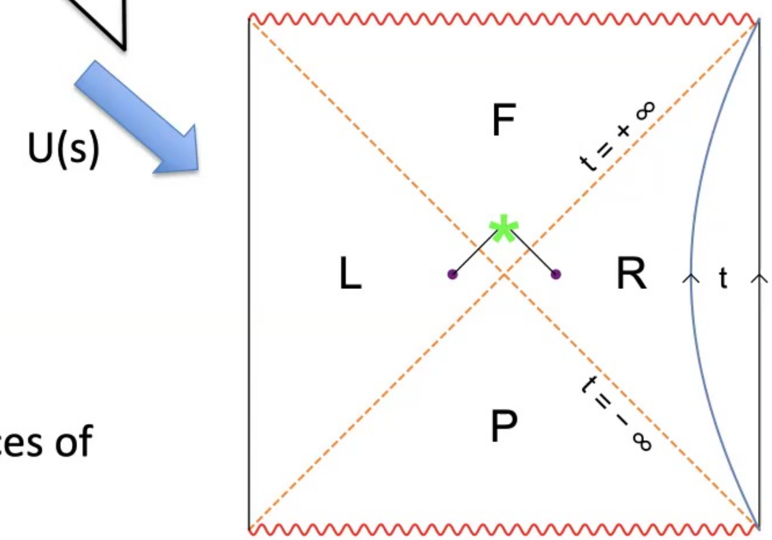

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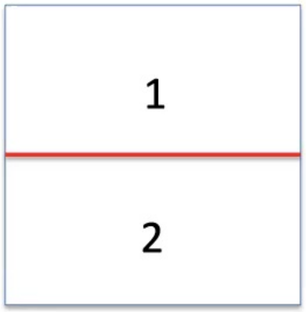
Entanglement of a quantum system



$$\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$$

$$\rho_1 = \mathrm{Tr}_2 |\Psi\rangle\langle\Psi|$$

$$S_1 = -\mathrm{Tr}_1 \rho_1 \log \rho_1$$

When ρ_1, ρ_2 are both full rank

Modular operator: $\Delta = \rho_2 \rho_1^{-1}$

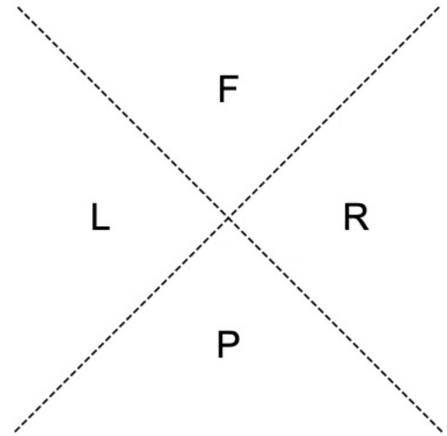
$$\Delta^{it} B(\mathcal{H}_1) \Delta^{-it} \in B(\mathcal{H}_1), \quad \Delta^{it} B(\mathcal{H}_2) \Delta^{-it} \in B(\mathcal{H}_2),$$

modular flow → highly entangled

Entanglement in QFT

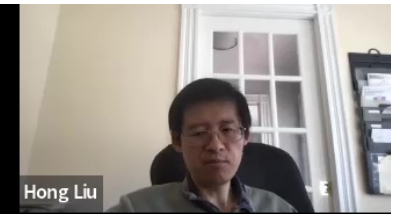
Consider a QFT in Minkowski spacetime.

It is often said the Minkowski vacuum state can be interpreted as a thermal field double state for the R and L Rindler patches.



Strictly speaking, the statement is only correct in the discretized theory.

They are some **fundamental differences** between the discrete and continuum cases.



Discrete	Continuum
Local Hilbert spaces for L and R $\mathcal{H} = \mathcal{H}_R \otimes \mathcal{H}_L$	no
Reduced density matrix	no
Finite entanglement entropy	Not defined (infinite)

modular operator and modular flows exist

Modular operator can be factorized	cannot
No sharp light cone	sharp light cone

Reasons: operator algebras in R region have different structures

Type I von Neumann algebra	Type III_1 von Neumann algebra

The story is general for relativistic QFTs:

For any local region, local operator algebra can be associated with a type III_1 vN algebra

Entanglement for any local region can be understood in terms of modular flows associated with such an algebraic structure.

Half-sided modular translation

Suppose $\mathcal{M}$ is a von Neumann algebra and the vector $|\Omega\rangle$ is cyclic and separating for $\mathcal{M}$

Suppose there exists a von Neumann subalgebra $\mathcal{N}$ of $\mathcal{M}$ with the properties:

$|\Omega\rangle$ is cyclic for $\mathcal{N}$

$$\Delta_{\mathcal{M}}^{-it} \mathcal{N} \Delta_{\mathcal{M}}^{it} \subset \mathcal{N}, \quad t \leq 0$$

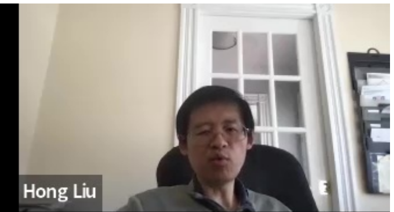
It can then be shown that for type III_1 there exists a unitary group U(s), with the following properties:

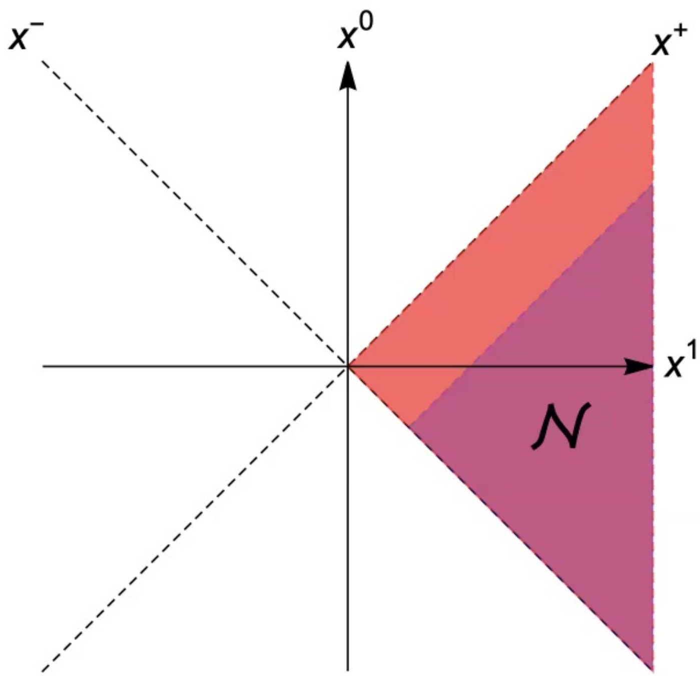
Borchers, Wiesbrock

$$U(s) = e^{-iGs}, \qquad G \geq 0$$

$$U(s)\Omega = \Omega, \quad \forall s \in \mathbb{R}$$

This can be used to generate "new" times!





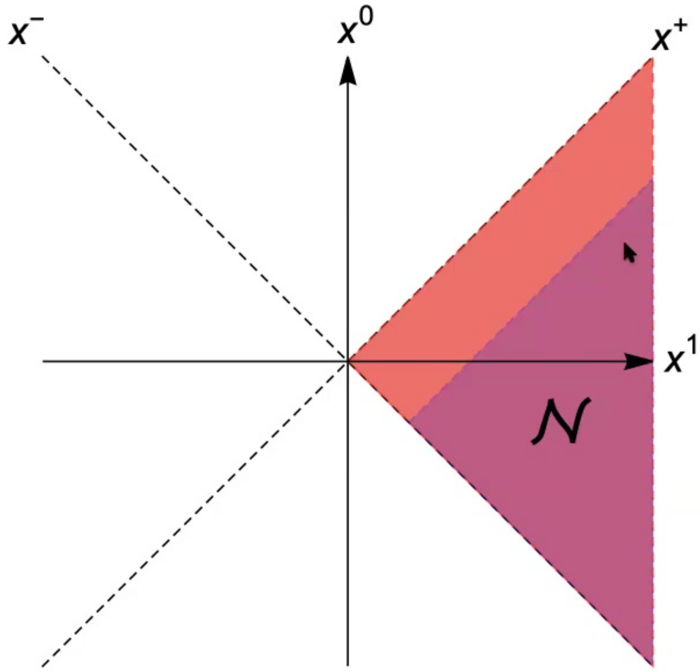
M: operator algebra in R region

$$U(s) = e^{-iGs}, \qquad G \geq 0$$

G generates translations along x^- direction

$$U^\dagger(s)\mathcal{M}U(s) \subseteq \mathcal{M}, \quad \forall s \leq 0\ . \quad \mathcal{N} = U^\dagger(-1)\mathcal{M}U(-1)$$

s> 0, “generate” F and P regions from the algebras of R and L regions



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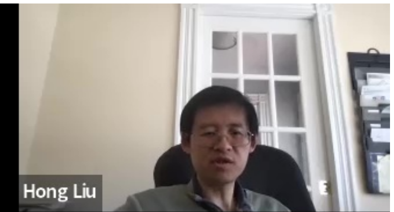
Starting with Rindler time in L and R, we obtain the Minkowski time!



Key: a type III_1 vN algebra and appropriately chosen subalgebras lead to new emergent times

Boundary constructions of emergent infalling times

Emergent type III_1 vN algebras

BH is described by CFT_R x CFT_L in the thermal field double state

At finite N, the (bounded) operator algebra of CFT_R or CFT_L is type I.

We argue there is an emergent type III_1 vN algebra in the large N limit

which leads to the emergence of a sharp horizon and the interior.

$\mathcal{A}_R$: algebra generated by single-trace operators of CFT_R

In the large N limit, there is another Hilbert space $\mathcal{H}_{\rm GNS}$: Hilbert space of small excitations around the thermal field double state.

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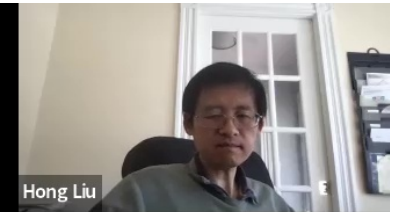
$\mathcal{M}_R$: action of $\mathcal{A}_R$ in $\mathcal{H}_{\text{GNS}}$

Conjecture: $\mathcal{M}_R$ and $\mathcal{M}_L$ are type III_1 vN algebras



Supports:
- Thermal spectral functions of single-trace operators
- Half-sided modular inclusion/translation structure
- Duality with bulk

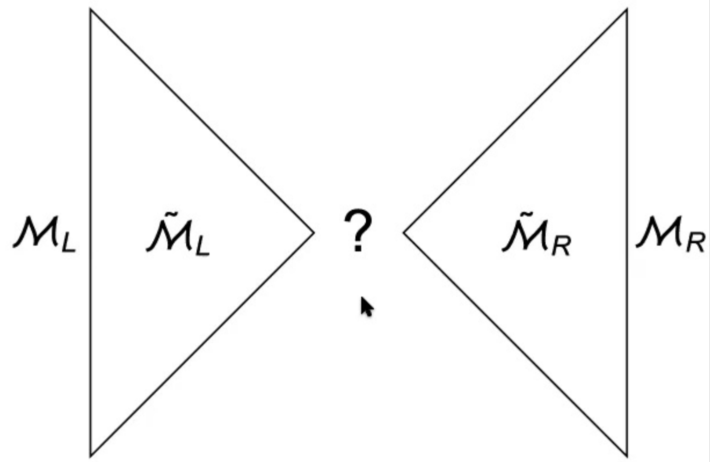
In the bulk: $\mathcal{H}_{\rm BH}^{(\rm Fock)}, \quad |HH\rangle, \quad \widetilde{\mathcal{M}}_R, \quad \widetilde{\mathcal{M}}_L$

Duality:

$$\mathcal{H}_{\rm GNS} = \mathcal{H}_{\rm BH}^{(\rm Fock)},$$

$$|\Omega\rangle = |HH\rangle,$$

$$\mathcal{M}_R = \widetilde{\mathcal{M}}_R, \quad \mathcal{M}_L = \widetilde{\mathcal{M}}_L$$



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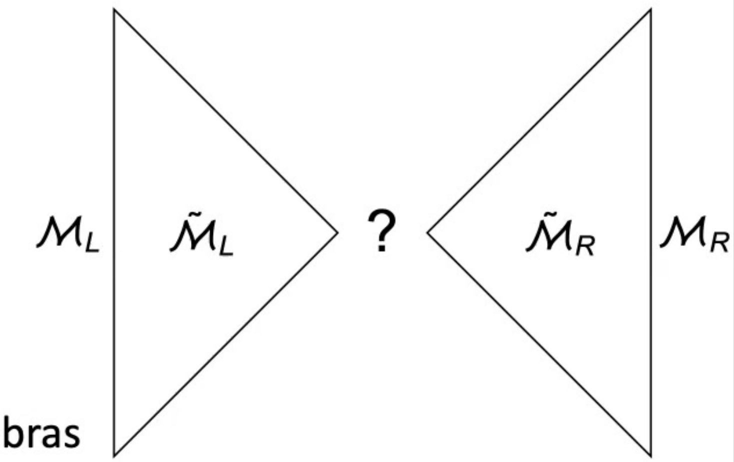
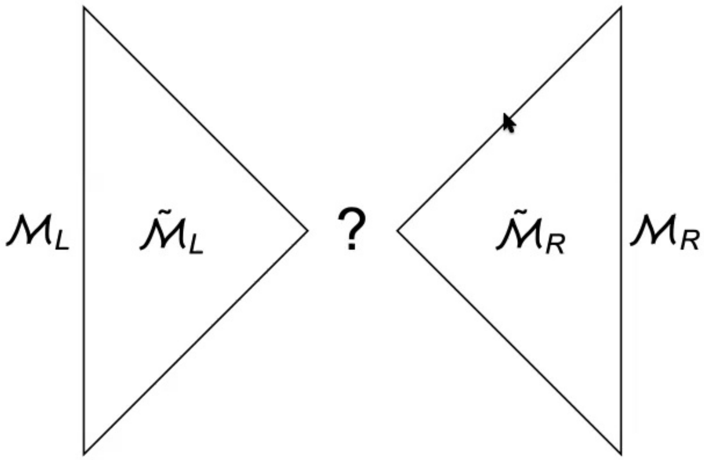
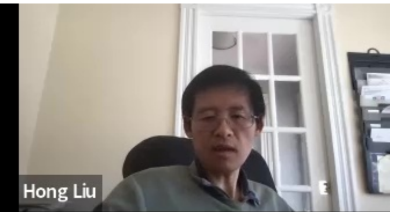


$\widetilde{\mathcal{M}}_R, \widetilde{\mathcal{M}}_L$ are type III_1 vN algebras



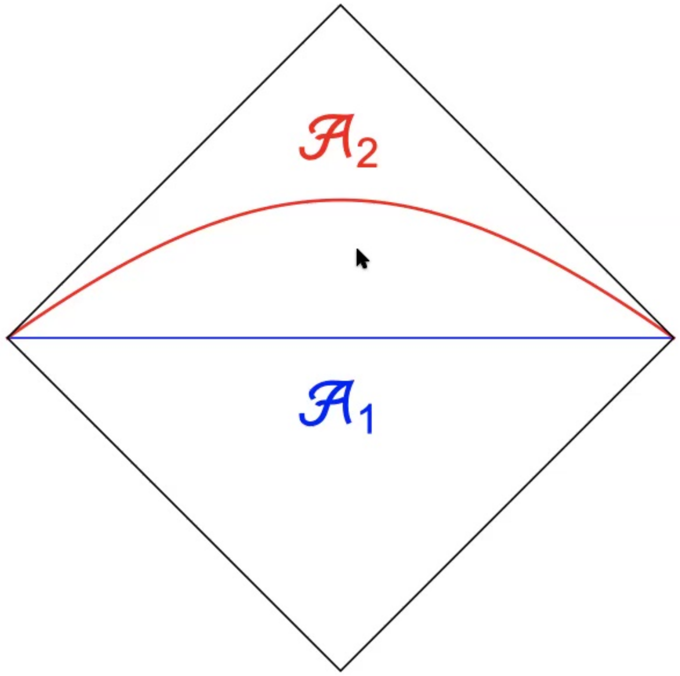


finding a U(s) boils down to finding an appropriate von Neumann subalgebra of $\mathcal{M}_R$

While theorems of half-sided modular translations ensure existence of a U(s), finding it explicitly in general is very difficult.

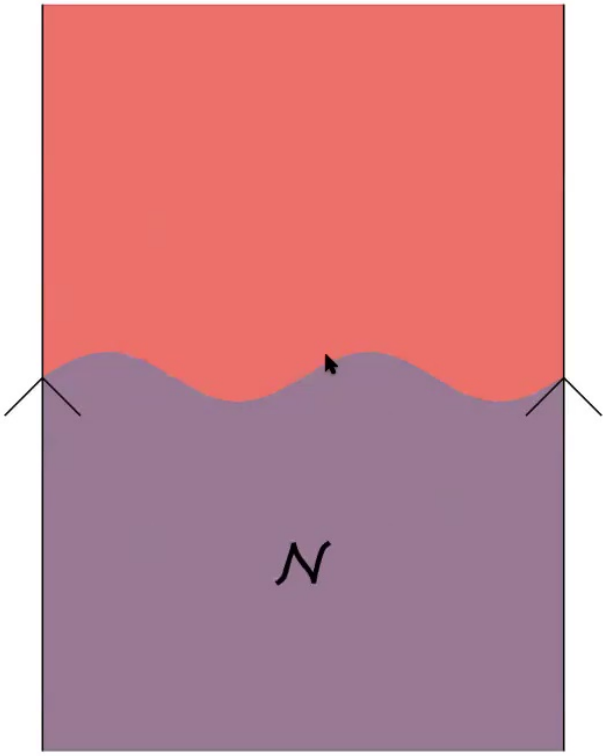
Here in the large N limit, algebra of single-trace operators in $\mathcal{H}_{\rm GNS}$ can be described by that of a generalized free theory.

In this case, the expression of U(s) has a universal form up to a phase factor which depends on specific algebras.

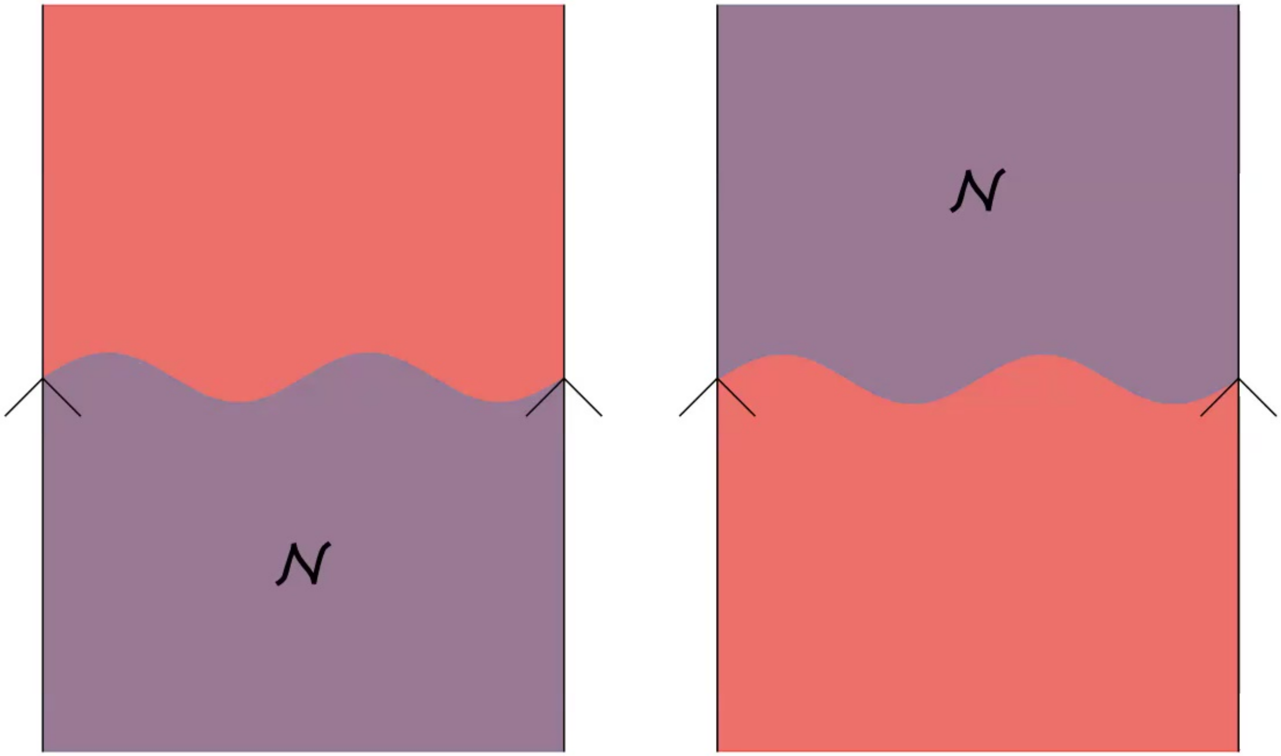


In a QFT, $\mathcal{A}_1 = \mathcal{A}_2$

But for algebras of single-trace operator, $\mathcal{A}_1 \neq \mathcal{A}_2$

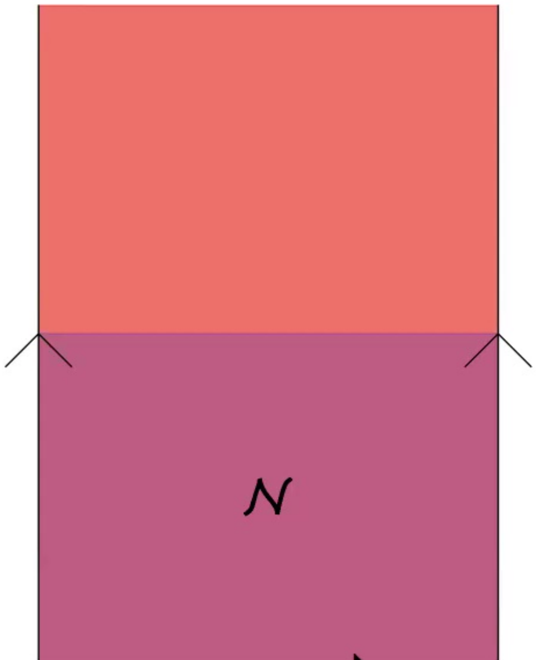


An infinite number of emergent times

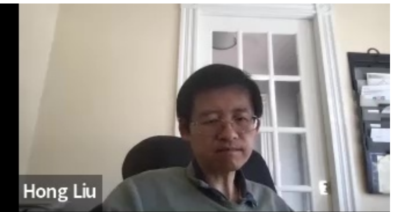


An infinite number of emergent times

Entanglement wedge of a subalgebra



Finding U(s) is now a strongly coupled problem



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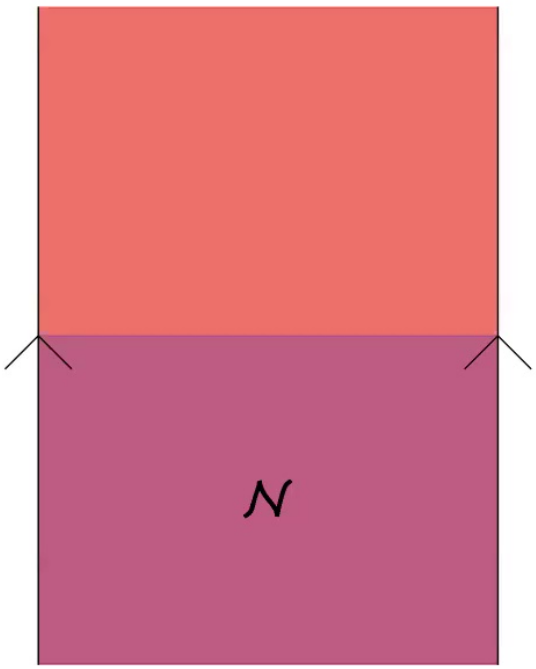
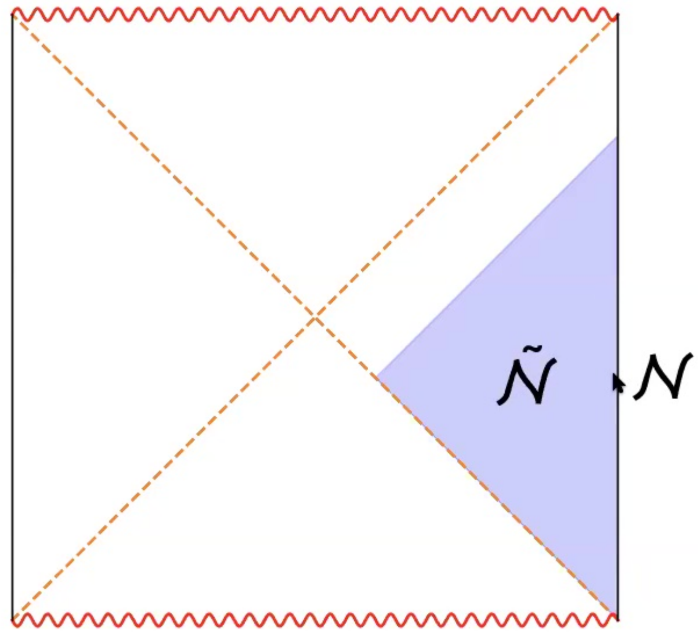
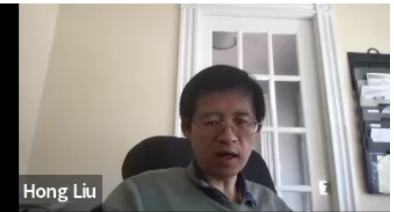
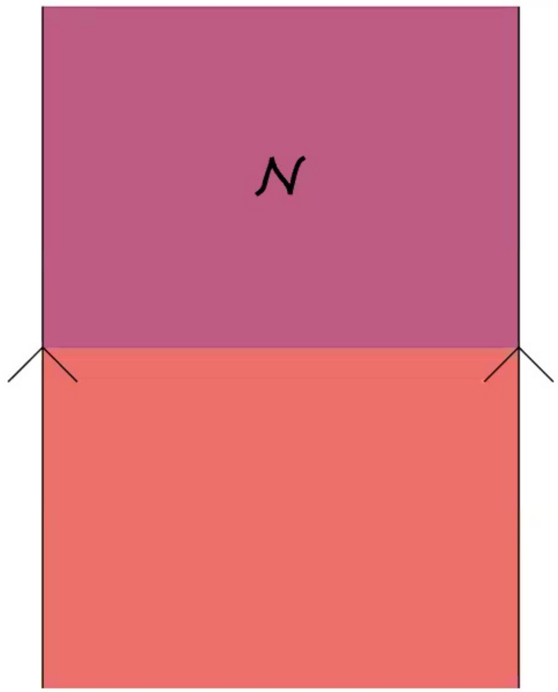


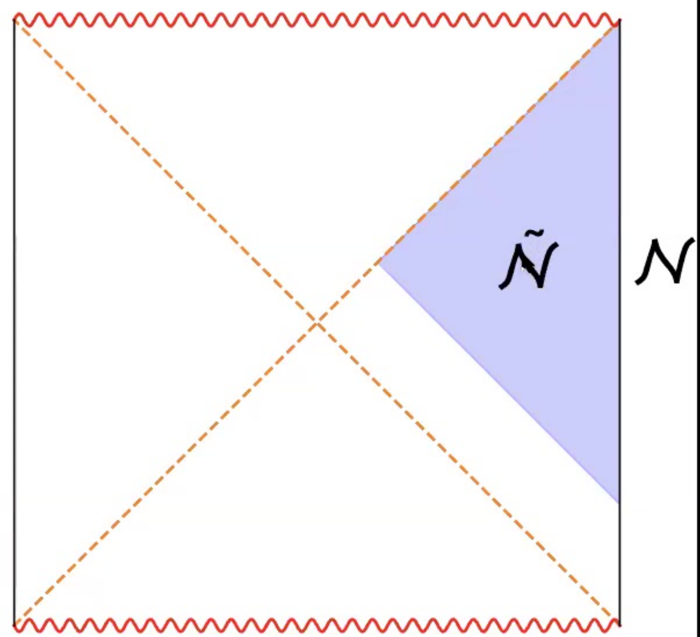


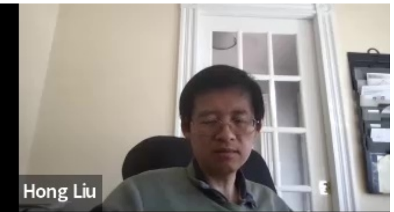


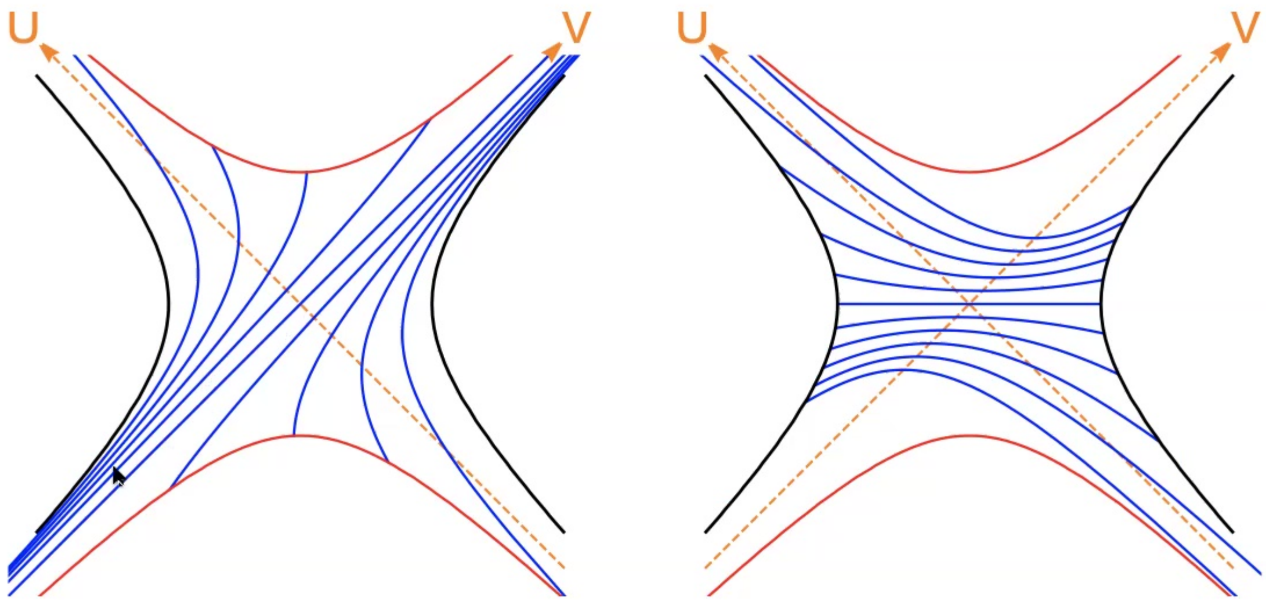
Finding U(s) is now a strongly coupled problem

Generation of Kruskal-like V-flow





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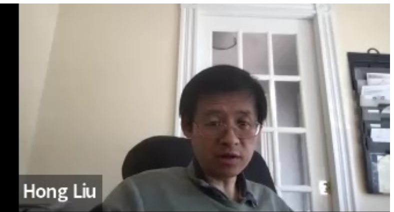
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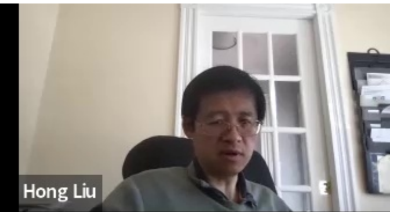

We can also consider compositions of such Kruskal-like U-type and V-type flows.

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