Title: Harnessing S-duality in N=4 SYM and supergravity as SL(2,Z)-averaged strings

Speakers: Scott Collier

Series: Quantum Fields and Strings

Date: March 15, 2022 - 2:00 PM

URL: https://pirsa.org/22030031

Abstract: I will describe an approach to extracting the physical consequences of S-duality of four-dimensional N = 4 super Yang-Mills (SYM) and its string theory dual based on SL(2,Z) spectral theory. I will show that processing S-duality in this way leads to strong consequences for the CFT data, both perturbatively and non-perturbatively in all parameters. In large-N limits, I will argue for the existence and scaling of non-perturbative effects, both at large N and at strong 't Hooft coupling. An elegant benchmark for these techniques is a certain integrated stress-tensor multiplet four-point function, whose form I will elucidate. I will explain how the ensemble average of CFT observables over the N = 4 supersymmetric conformal manifold with respect to the Zamolodchikov measure is cleanly isolated by the spectral decomposition, and will show that the large-N limit of the ensemble average is equal to the strong-coupling limit of the observable in the planar theory, which is its value in type IIB supergravity on AdS_5 x S^5. This embeds an emergent averaged holographic duality within the conventional holographic paradigm.

Zoom link: https://pitp.zoom.us/j/95197874062?pwd=QU4vbXNNeFVmS0hNbTVLL24wdDBndz09

Harnessing S-duality in $\mathcal{N} = 4$ SYM
&
supergravity as $SL(2,\mathbb{Z})$-averaged strings

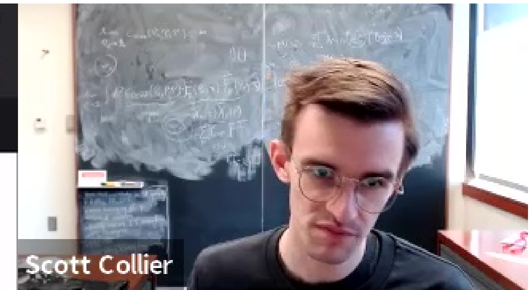
Scott Collier
Princeton PCTS

Perimeter Institute
2022/03/15

PRINCETON CENTER FOR THEORETICAL SCIENCE

based on [2201.05093] with Eric Perlmutter (CEA Saclay)

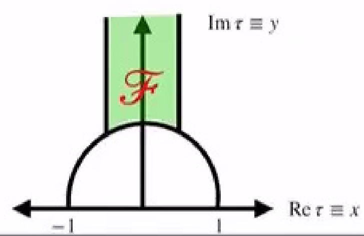
14:03 Tue Mar 15



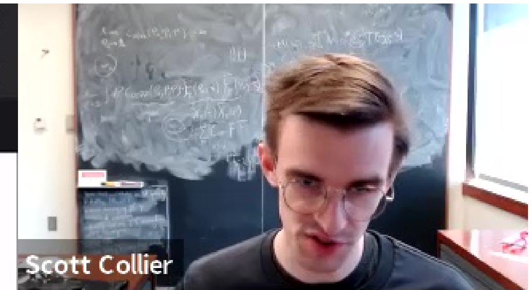
Motivating remarks

- $\mathcal{N} = 4$ SYM is a beautiful theory from a multitude of perspectives
  - Maximal SUSY $\Rightarrow$ rigid structure of CFT data
  - Approaches: perturbation theory, bootstrap, localization, integrability
  - Via AdS/CFT, it furnishes our most robust and explicit definition of non-perturbative quantum gravity
- It enjoys **S-duality** [Montenen, Olive; Witten, Olive; Osborn; Sen]:
  - Local CFT observables $\mathcal{O}(\tau)$ are invariant under $SL(2,\mathbb{Z})$ transformations: $\mathcal{O}(\tau) = \mathcal{O}(\gamma\tau)$
  - An **inherently non-perturbative duality**; obscured in the 't Hooft limit
  - A symmetry that has not been fully used!

$$\tau = \frac{\theta}{2\pi} + \frac{4\pi i}{g_{\text{YM}}^2} \mapsto \gamma\tau = \frac{a\tau + b}{c\tau + d}, \quad \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,\mathbb{Z})$$



2 / 40



Motivating remarks

- In practice, very little concrete information about the **modular structure** of CFT observables $\mathcal{O}(\tau)$ is known to complement perturbation theory

> **Idea**: Bake in modular invariance at the outset.
> Reduce CFT observables to their dynamical content and systematically explore the consequences of S-duality.

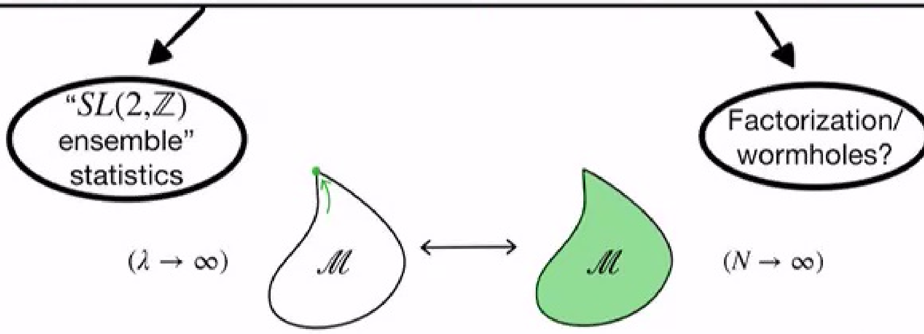
- Facilitated by a robust $SL(2,\mathbb{Z})$ spectral theory
  - **Simple calculations** yield a wealth of insights into the structure of perturbation theory and of the instanton expansion

- Structure is especially rigid at large N in the 't Hooft limit with $\lambda \equiv N g_{\mathrm{YM}}^2$ held fixed
  - $SL(2,\mathbb{Z})$ invariance **implies the existence of non-perturbative effects** at large N and at strong coupling $(\lambda \gg 1)$

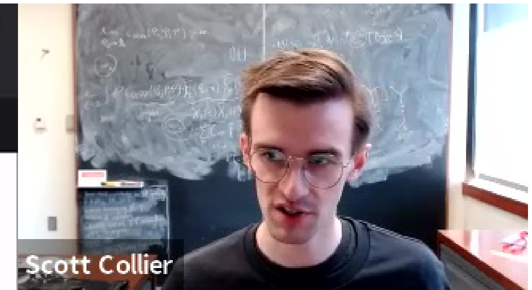
Motivating remarks

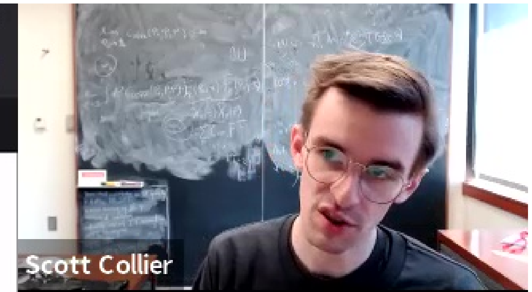
- At strong coupling $(\lambda \gg 1)$ $\mathcal{N}=4$ SYM is famously dual to type IIB supergravity on AdS$_5 \times S^5$, with a prescribed set of stringy α' corrections
- Remarkably, S-duality has something to say about this:

$$\boxed{\text{The large-}N\text{ limit of ensemble averaged } \mathcal{N}=4 \text{ SYM is the strong-coupling limit of the planar theory, i.e. AdS}_5\times S^5 \text{ supergravity}}$$



- An **emergent averaged holographic duality** within string theory



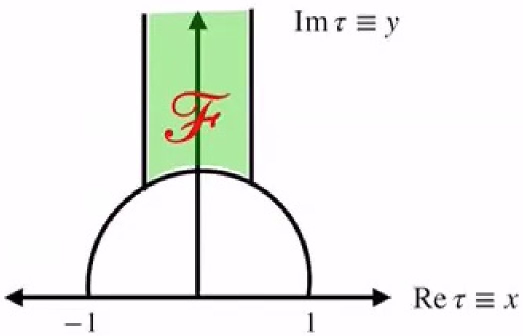


Outline

1. $SL(2,\mathbb{Z})$ spectral theory for $\mathcal{N} = 4$ SYM observables
2. Example: integrated stress-tensor multiplet four-point function
3. Perturbation theory and the analytic structure of spectral overlaps
4. Large-N and the 't Hooft limit
5. Supergravity as an emergent ensemble average
6. Statistics of the "$SL(2,\mathbb{Z})$ ensemble"
7. Remarks on AdS/CFT and wormholes

1. $SL(2,\mathbb{Z})$ spectral theory

The fundamental domain $\mathscr{F}$ of $SL(2,\mathbb{Z})$:



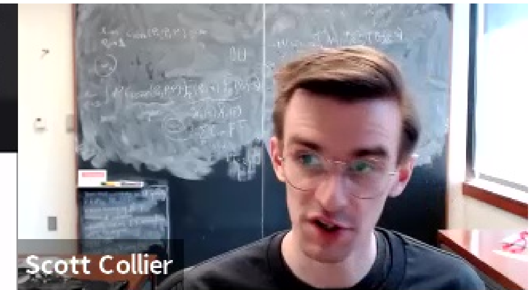
$$\mathscr{F} = \left\{ \tau = x + iy \in \mathbb{H} \,\middle|\, -\frac{1}{2} \leq x \leq \frac{1}{2}, |\tau| \geq 1 \right\}$$

$$x = \frac{\theta}{2\pi}$$

$$y = \frac{4\pi}{g_{\mathrm{YM}}^2}$$

- Natural metric on moduli space:
$$ds^2 = \frac{dx^2 + dy^2}{y^2}$$

- Laplacian: $\Delta_\tau = -y^2(\partial_x^2 + \partial_y^2)$



14:10 Tue Mar 15

84%

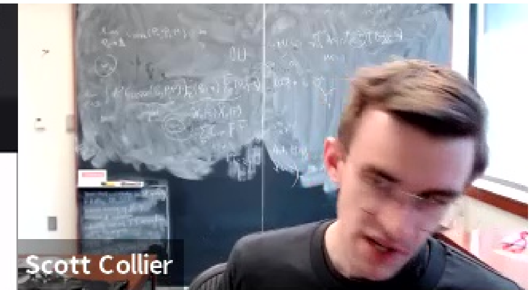
$SL(2,\mathbb{Z})$ spectral theory

- There is a natural **inner product** on this space:

$$(f,g) = \int_{\mathcal{F}} \frac{dxdy}{y^2}\, f(\tau)\overline{g(\tau)}$$

- Basic idea: decompose $\mathcal{O}(\tau)$ into eigenfunctions of Δ_τ using this inner product
  - This is possible when $||\mathcal{O}||^2 = (\mathcal{O},\mathcal{O}) < \infty$:
  - This is generic for $\mathcal{N} = 4$ SYM observables $\mathcal{O}(\tau)$ (cusp $\tau \to i\infty$: free limit)

7 / 40



14:11 Tue Mar 15

Spectral resolution of Δ_τ

$$\boxed{\mathcal{O}(\tau) = \overline{\mathcal{O}} + \int_{\mathrm{Re}\, s=\frac{1}{2}} \frac{ds}{4\pi i} (\mathcal{O}, E_s)\, E_s(\tau) + \sum_{n=1}^{\infty} (\mathcal{O}, \phi_n)\, \phi_n(\tau)}$$

The complete basis of eigenfunctions of Δ_τ splits into three branches:

(1) The constant function:

$$\Delta_\tau 1 = 0$$

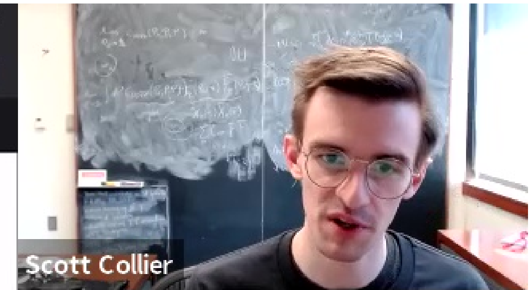
(2) Continuous branch: **real analytic Eisenstein series**

$$\Delta_\tau E_s(\tau) = s(1-s)E_s(\tau), \quad s \in \frac{1}{2} + i\mathbb{R}$$

(3) (Infinite) discrete branch: **Maass cusp forms**

$$\Delta_\tau \phi_n(\tau) = \left(\frac{1}{4} + t_n^2\right) \phi_n(\tau), \quad 0 < t_1 < t_2 < \dots$$

8 / 40



14:12 Tue Mar 15 83%

Real analytic Eisenstein series

- Defined via a Poincaré series

$$E_s(\tau) = \sum_{\gamma \in PSL(2,\mathbb{Z})/\mathbb{Z}} \mathrm{Im}(\gamma\tau)^s, \quad \mathrm{Re}(s) > 1$$

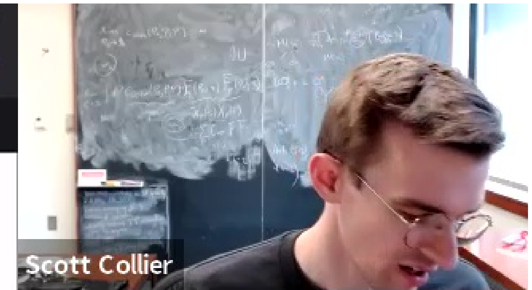
- Satisfies the functional equation

$$E_s^*(\tau) \equiv \Lambda(s) E_s(\tau) = E_{1-s}^*(\tau) \qquad \big(\Lambda(s) \equiv \pi^{-s}\Gamma(s)\zeta(2s) = \Lambda(1/2 - s)\big)$$

- Admits a **meromorphic continuation** to the entire complex s plane

$$E_s^*(\tau) = \underbrace{\Lambda(s) y^s + \Lambda(1-s) y^{1-s}}_{E_{s,0}^*(y)} + \sum_{k=1}^{\infty} 2\cos(2\pi k x) \underbrace{\frac{2\sigma_{2s-1}(k)}{k^{s-\frac{1}{2}}}\sqrt{y} K_{s-\frac{1}{2}}(2\pi k y)}_{E_{s,k}^*(y) \sim e^{-2\pi k y}}$$

9 / 40



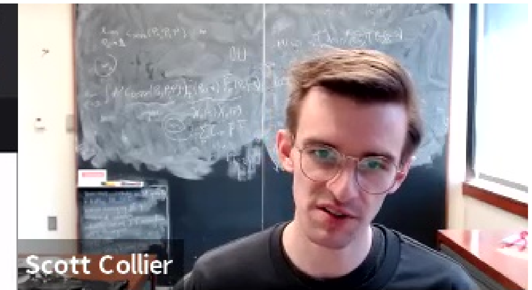
Maass cusp forms

- The discrete branch is much more **wild & mysterious**
- Related to the energy eigenstates of the quantum mechanics of a particle propagating on $\mathscr{F}$, a **classically chaotic system**
- Functional form is similar to the continuous branch

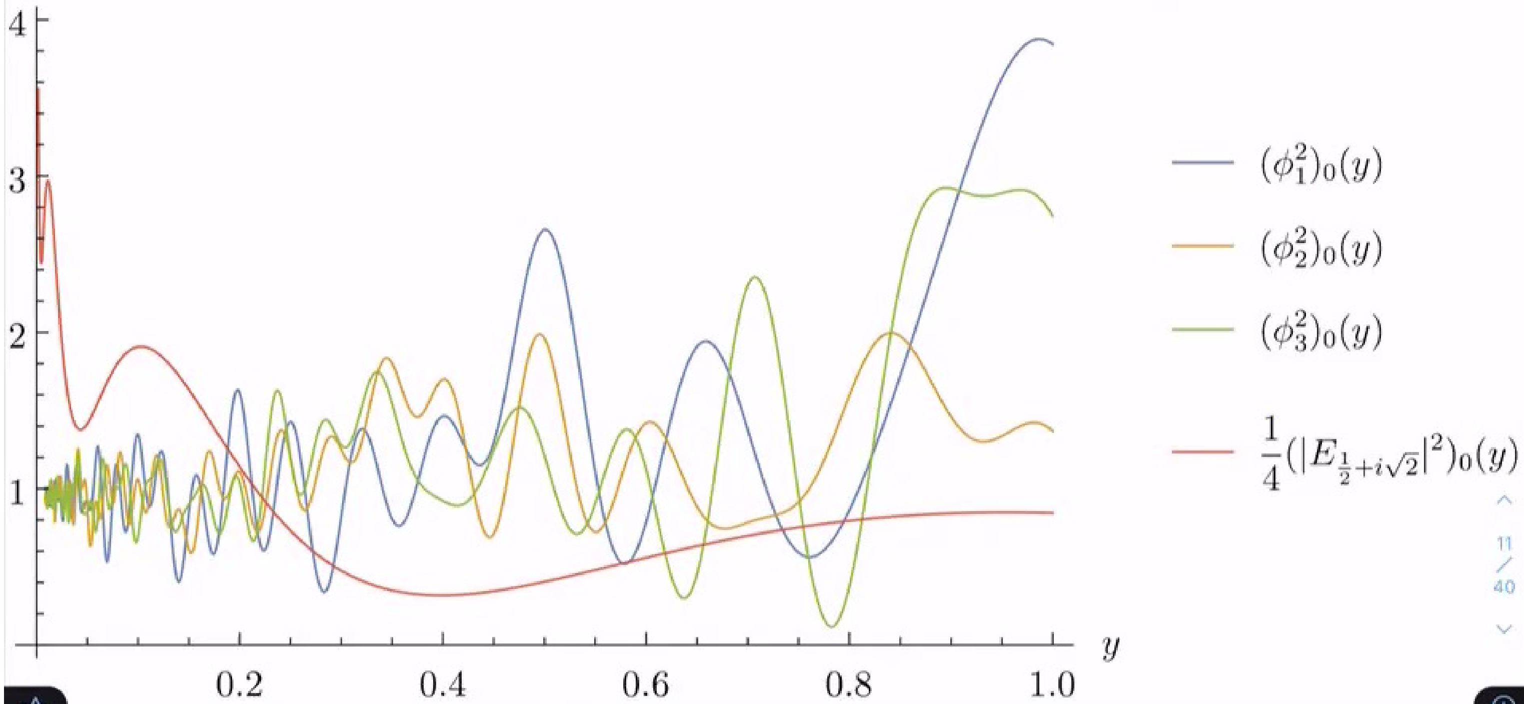
$$\phi_n(\tau) = \sum_{k=1}^{\infty} a_k^{(n)} \cos(2\pi k x)\sqrt{y}K_{it_n}(2\pi k y)$$

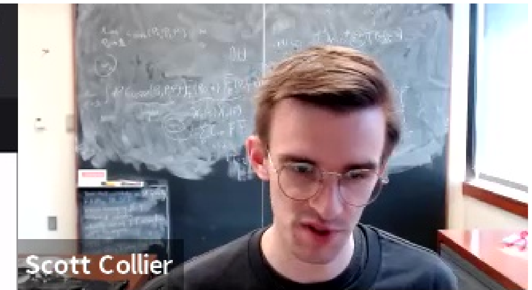
- The eigenvalues $\{t_n\}$ and Fourier coefficients $\{a_k^{(n)}\}$ are sporadic real numbers with very interesting statistics ("**arithmetic quantum chaos**" [Sarnak])

10 / 40



Eisenstein series vs. Maass cusp forms





Spectral decomposition of CFT observables

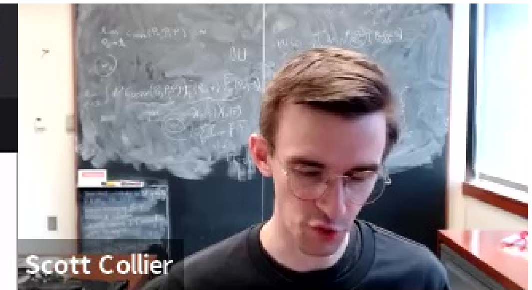
CFT observables admit a **unique decomposition** in this basis:

$$\boxed{\mathcal{O}(\tau) = \overline{\mathcal{O}} + \int_{\mathrm{Re}\, s=\frac{1}{2}} \frac{ds}{4\pi i} (\mathcal{O}, E_s)\, E_s(\tau) + \sum_{n=1}^{\infty} (\mathcal{O}, \phi_n)\, \phi_n(\tau)}$$

where:

$\overline{\mathcal{O}} = \mathrm{vol}(\mathcal{F})^{-1} \int_{\mathcal{F}} \frac{dxdy}{y^2} \mathcal{O}(\tau) = \mathrm{Res}_{s=1}(\mathcal{O}, E_{\bar{s}})$ is the **modular average**

$(\mathcal{O}, E_s) = \int_{\mathcal{F}} \frac{dxdy}{y^2} \mathcal{O}(\tau)\overline{E_s(\tau)} = \int_0^\infty \frac{dy}{y^2} y^{\bar{s}} \mathcal{O}_0(y)$ is a **Mellin transform of the zeromode**



14:18 Tue Mar 15 82%

Spectral decomposition of CFT observables

Consequences:

(1) The overlap $(\mathcal{O}, E_s)$ satisfies a **functional equation**

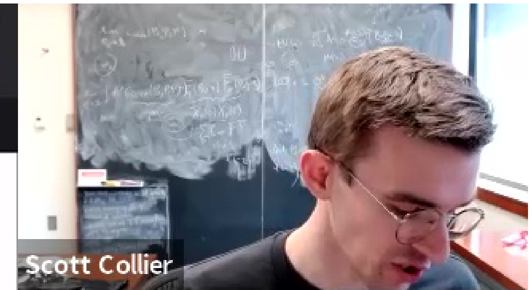
$$\{\mathcal{O}, E_s\} \equiv \frac{(\mathcal{O}, E_s)}{\Lambda(s)} = \{\mathcal{O}, E_{1-s}\}$$

(2) The constant term $\overline{\mathcal{O}}$ is the **ensemble average** over the conformal manifold (wrt the Zamolodchikov measure)

$$\langle \mathcal{O} \rangle \equiv \mathrm{vol}(\mathcal{M})^{-1} \int_{\mathcal{M}} \underbrace{d\mu_{\mathcal{M}}}_{\frac{dxdy}{y^2}} \mathcal{O}(\tau) = \overline{\mathcal{O}} \quad \text{by virtue of maximal SUSY}$$

(3) "**Instantons are redundant**:" $\mathcal{O}(\tau) = \mathcal{O}_0(y) + \sum_{k=1}^{\infty} 2\cos(2\pi k x)\mathcal{O}_k(y)$

13 / 40



Instantons are redundant

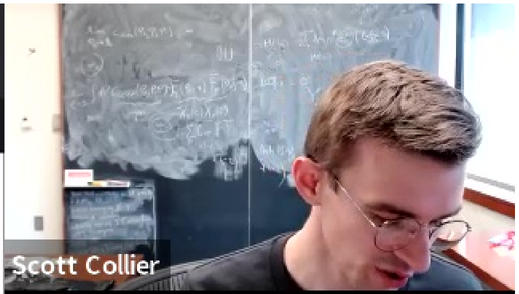
$$\mathcal{O}_0(y)$$

$$\downarrow$$

$$\{\mathcal{O}, E_s\}$$

$$\downarrow$$

$$\mathcal{O}_k(y) = \int_{\mathrm{Re}\, s=\frac{1}{2}} \frac{ds}{4\pi i} \{\mathcal{O}, E_s\} E^*_{s,k}(y)$$



14:20 Tue Mar 15

2. Integrated correlator

- "CFT observables:" **non-perturbatively well-defined**

  e.g.: correlators of protected operators, spectrum of the dilatation operator, structure constants
  not, e.g.: "anomalous dimension of the Konishi"

- A rare and beautiful example: integrated $\mathcal{O}_{20'}$ four-point function of [Binder Chester Pufu Wang 2019]

SUSY-preserving measure

$$\mathcal{G}_N(\tau) = \int du dv\, \rho(u,v) T_N(u,v)$$

(Dynamical part of) 20' 4-pt function

$$= -\frac{1}{4}\Delta_\tau\, \partial_m^2 \log Z_N^{S^4}(m,\tau)\Big|_{m=0}$$

Mass-deformed sphere free energy (localization)

14:23 Tue Mar 15

Integrated correlator

- Remarkably, $\mathcal{G}_N(\tau)$ is conjecturally known for all N, τ (!) [Dorigoni Green Wen 2021]

$$\boxed{\mathcal{G}_N(\tau) = \frac{1}{2} \sum_{(m,n)\in\mathbb{Z}^2} \int_0^\infty d\xi\, B_N(\xi)\, \exp\left(-\frac{\pi\xi}{y} |m\tau + n|^2\right)}$$

- The kernel $B_N(\xi)$ is determined by recursion from $N = 2$ via a Laplace difference equation satisfied by $\mathcal{G}_N(\tau)$

- Another useful representation

$$\mathcal{G}_N(\tau) = \frac{N(N-1)}{8} - \frac{1}{2}\sum_{s=2}^{\infty} (-)^s c_s^{(N)} E_s^*(\tau)$$

e.g. for $SU(2)$, $c_s^{(2)} = s(1-s)(2s-1)^2$

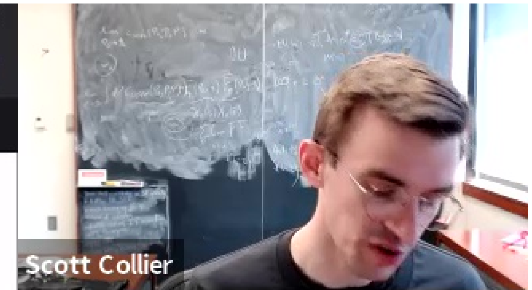
16 / 40

Integrated correlator

- We claim that this integrated correlator is given **exactly** by the following spectral decomposition

$$\boxed{\mathcal{G}_N(\tau) = \underbrace{\frac{N(N-1)}{4}}_{\langle \mathcal{G}_N \rangle} + \int_{\mathrm{Re}\, s=\frac{1}{2}} \underbrace{\frac{ds}{4\pi i}\frac{\pi}{\sin \pi s} c_s^{(N)}}_{\{\mathcal{G}_N, E_s\}} E_s^*(\tau)}$$

- In particular: $(\mathcal{G}_N, \phi_n) = 0$



14:25 Tue Mar 15 80%

3. Perturbation theory and the analytic structure of spectral overlaps

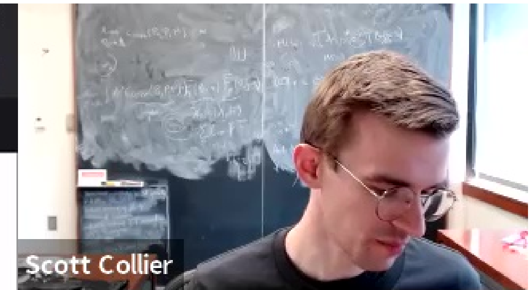
- Consistency of the weak-coupling $(y \gg 1)$ expansion constrains $(\mathcal{O}, E_s)$ (recall: $y = 4\pi/g_{\mathrm{YM}}^2$)

$$\mathcal{O}_0(y) = \langle \mathcal{O} \rangle + \int_{\mathrm{Re}\, s=\frac{1}{2}} \frac{ds}{4\pi i} \{\mathcal{O}, E_s\}(\Lambda(s)y^s + \Lambda(1-s)y^{1-s})$$

- Perturbation theory: upon contour deformation, **only non-negative integer powers of y, no logarithms**

- Large-$|s|$ asymptotics of $\{\mathcal{O}, E_s\}$ encodes asymptotics of perturbation theory

18 / 40



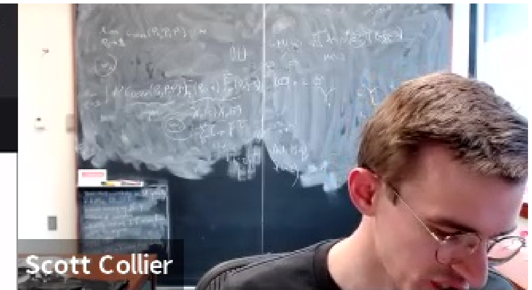
Perturbation theory and the analytic structure of spectral overlaps

- Claim: the most general $\{\mathcal{O}, E_s\}$ consistent with perturbation theory takes the form

$$\boxed{\{\mathcal{O}, E_s\} = \underbrace{\frac{\pi}{\sin \pi s} s(1-s) f_{\mathrm{p}}(s)}_{\text{perturbative} \sim y^{-n}} + \underbrace{f_{\mathrm{np}}(s)}_{\text{non-perturbative} \sim (q\bar{q})^n}}$$

where both $f_{\mathrm{p}}(s), f_{\mathrm{np}}(s)$ are:

- symmetric under reflection $s \to 1-s$
- regular for $s \in \mathbb{C}$ away from $s = 0,1$
- such that $\{\mathcal{O}, E_s\} = 0$ for $s = 1/2$



The optimal simplicity of $\mathscr{G}_2(\tau)$

Recall the $SU(2)$ integrated correlator $\mathscr{G}_2(\tau)$:

$$\boxed{\begin{aligned}(\mathscr{G}_2, \phi_n) &= 0 \\ f_{\mathrm{np}}(s) &= 0 \\ f_{\mathrm{p}}(s) &= (2s-1)^2\end{aligned}}$$

An **optimally simple observable** consistent with $SL(2,\mathbb{Z})$-invariance and perturbation theory!

14:30 Tue Mar 15 78%

The $SL(2,\mathbb{Z})$ Borel transform

- It is often the case the $\mathcal{N}=4$ SYM observables have **Borel-summable perturbative series**:

$$\mathcal{O}_0(y) \approx \sum_{n=1}^{\infty} (-)^n c_n y^{-n}, \qquad (c_n = -n(n+1)\Lambda(n+1/2) f_{\mathrm{p}}(n+1))$$

e.g. suppose $c_n \sim (\pi R)^{-n} n!$

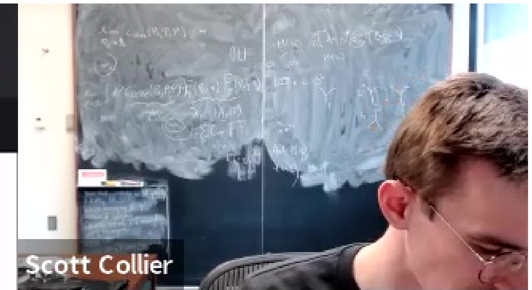
- Convenient to define the "$SL(2,\mathbb{Z})$ **Borel transform**":

$$\boxed{B[\xi] = \sum_{n=0}^{\infty} \frac{(-)^n c_n}{\Lambda(n+\frac{1}{2})} \xi^{n+1}} \text{ (radius of convergence } R\text{)}$$

- The **resummation** of the perturbative series is neatly obtained by inverting this transform:

$$\mathcal{O}_0(y) = y^{1/2} \int_0^{\infty} \frac{d\xi}{\xi^{3/2}} \left(\frac{\theta_3(y\xi) - 1}{2} \right) B[\xi]$$

21 / 40

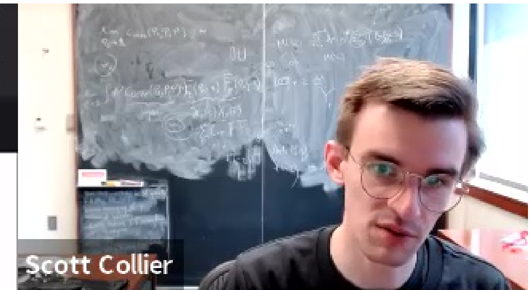


The $SL(2,\mathbb{Z})$ Borel transform

- When $(\mathcal{O}, \phi_n) = 0$, can reconstruct $\mathcal{O}(\tau)$ from $\mathcal{O}_0(y)$ in an elegant way
- Sum the resummation of the perturbative series over $SL(2,\mathbb{Z})$ images:

$$\boxed{\begin{aligned}\mathcal{O}(\tau) &= \frac{1}{2}\sum_{\gamma\in PSL(2,\mathbb{Z})/\mathbb{Z}} \mathcal{O}_0(\mathrm{Im}(\gamma\tau)) \\ &= \frac{1}{4}\sum_{(m,n)\in\mathbb{Z}^2}\int_0^\infty \frac{d\xi}{\xi} B[\xi]\, \exp\left(-\frac{\pi\xi}{y}|m\tau+n|^2\right)\end{aligned}}$$

- Thus there is a **lattice integral representation** for rather general* CFT observables
- The kernel is the $SL(2,\mathbb{Z})$ **Borel transform of the zeromode**!



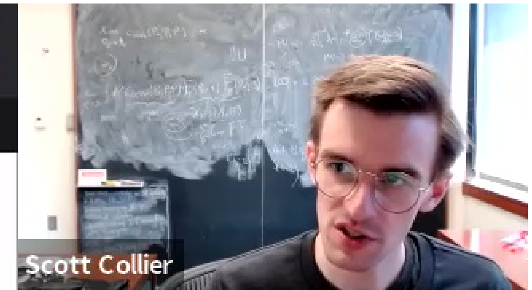
The redundancy of instantons

- If the Borel transform of the perturbative series of $\mathcal{O}_0(y)$ has **radius of convergence** R, then that of the $k > 0$ instanton sectors is also Borel summable with radius of convergence:

$$\boxed{R_k = R\left(1 + \frac{k}{R}\right)^2}$$

- $SL(2,\mathbb{Z})$ **relates all instanton sectors** in a simple way

- Violation of this can be taken as a sharp signal that $(\mathcal{O}, \phi_n) \neq 0$



4. Large-N and the 't Hooft limit

- We will mostly be concerned with the **'t Hooft limit**

$$N \to \infty, \text{ with } \lambda = N g_{\mathrm{YM}}^2 = \frac{4\pi N}{y} \text{ held fixed}$$

- The perturbative $1/N$ expansion organizes into a genus expansion

$$\mathcal{O}_0(y) = \sum_{g=0}^{\infty} N^{2-2g} \mathcal{O}_0^{(g)}(\lambda) \qquad (k\text{-instantons are non-perturbatively suppressed } \sim e^{-8\pi^2 kN/\lambda})$$

- We then consider the $1/N$ expansion of the spectral overlaps (set $f_{\mathrm{np}}(s) = 0$ for now):

$$\mathcal{O}_0(y) = \langle \mathcal{O} \rangle + \sum_{g=0}^{\infty} N^{2-2g} \int \frac{ds}{2\pi i} \frac{\pi}{\sin \pi s} s(1-s)\left(\Lambda(s)\lambda^{-s} + \Lambda(1-s) N^{1-2s} \lambda^{s-1}\right) f_{\mathrm{p}}^{(g)}(1-s)$$

24 / 40

14:38 Tue Mar 15 76%

Large-N and the 't Hooft limit

- At the end of the day, one finds the following for the general form of the **weak-coupling** ($\lambda \ll 1$) expansion:

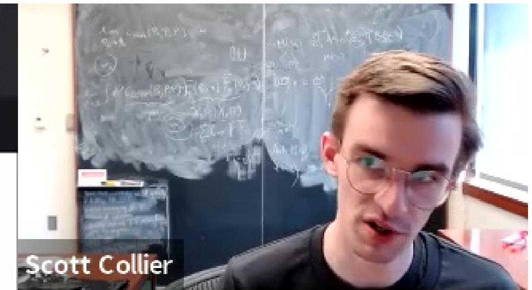
$$\boxed{\mathcal{O}_0(y) \approx \sum_{g=0}^{\infty} N^{2-2g} \sum_{m=1}^{\infty} \mathsf{R}^{(g)}_{-m-\frac{1}{2}} \lambda^m}, \text{ where } \mathsf{R}^{(g)}_m = \mathrm{Res}_{s=\frac{1}{2}+m}\left(\frac{\pi}{\sin \pi s} s(1-s)\Lambda(s) f_{\mathrm{p}}^{(g)}(1-s)\right)$$

- While at **strong coupling** ($\lambda \gg 1$) we have:

$$\boxed{\mathcal{O}_0(y) \approx \mathsf{C}(N) - \frac{1}{2}\sum_{g=0}^{\infty} N^{2-2g} \sum_{m=1}^{\infty} \mathsf{R}^{(g)}_m \left(\lambda^{-m-\frac{1}{2}} + \underbrace{\frac{\Lambda(\frac{1}{2}-m)}{\Lambda(\frac{1}{2}+m)} N^{-2m} \lambda^{m-\frac{1}{2}}}_{\substack{\text{Needed for renormalization of } 1/N \text{ expansion at strong-coupling} \\ \text{(stringy regularization of loop divergences in sugra)}}}\right)}$$

where $\mathsf{C}(N) = \langle \mathcal{O} \rangle - \frac{1}{2}\sum_{g=0}^{\infty} N^{1-2g} f_{\mathrm{p}}^{(g)}(0)$

25 / 40



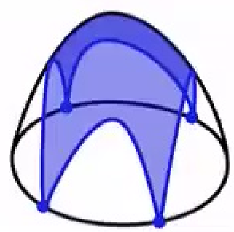
Non-perturbative effects implied by $SL(2,\mathbb{Z})$

- A surprising consequence of modular invariance:
  - Convergence of the weak-coupling expansion of $\mathcal{O}(\tau)$ **implies non-perturbative effects** both at $N \gg 1$ and at strong coupling **(at fixed orders in the genus expansion)**
  - The **non-perturbative scale** is set by the radius of convergence $|\lambda| < \lambda_*$ of the weak-coupling expansion

- In $\mathcal{N} = 4$ SYM, $\lambda_* = \pi^2$ is generic.
  This implies the non-perturbative scales Λ_λ^2 and $\Lambda_{\lambda_S}^2$ (for $\lambda \gg 1$ and $\lambda_S \gg 1$ respectively):

$$\begin{cases} \Lambda_\lambda & = e^{-\sqrt{\lambda}} = e^{-2\pi T_{\mathrm{F1}}} \\ \Lambda_{\lambda_S} & = e^{-\sqrt{\lambda_S}} = e^{-2\pi T_{\mathrm{D1}}} \end{cases}, \text{ where } \lambda_S = \frac{(4\pi N)^2}{\lambda} \text{ is an } \textbf{S-dual 't Hooft coupling}$$

- **Worldsheet instanton effects**



[Bargheer Coronado Vieira 2019]

14:42 Tue Mar 15

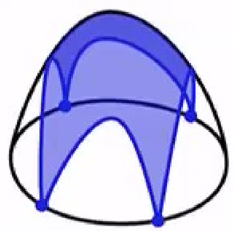
Non-perturbative effects implied by $SL(2,\mathbb{Z})$

- A surprising consequence of modular invariance:
  - Convergence of the weak-coupling expansion of $\mathcal{O}(\tau)$ **implies non-perturbative effects** both at $N \gg 1$ and at strong coupling **(at fixed orders in the genus expansion)**
  - The **non-perturbative scale** is set by the radius of convergence $|\lambda| < \lambda_*$ of the weak-coupling expansion

- In $\mathcal{N}=4$ SYM, $\lambda_* = \pi^2$ is generic.
  This implies the non-perturbative scales Λ_λ^2 and $\Lambda_{\lambda_S}^2$ (for $\lambda \gg 1$ and $\lambda_S \gg 1$ respectively):

$$\begin{cases} \Lambda_\lambda & = e^{-\sqrt{\lambda}} = e^{-2\pi T_{\mathrm{F1}}} \\ \Lambda_{\lambda_S} & = e^{-\sqrt{\lambda_S}} = e^{-2\pi T_{\mathrm{D1}}} \end{cases}, \text{ where } \lambda_S = \frac{(4\pi N)^2}{\lambda} \text{ is an \textbf{S-dual 't Hooft coupling}}$$

- **Worldsheet instanton effects**



[Bargheer Coronado Vieira 2019]

26 / 40

14:45 Tue Mar 15 74%

Allowing for instanton-anti-instanton effects

- Essentially the exact same structure persists for $f_{\mathrm{np}}(s) \neq 0$.

  - one lesson: at strong-coupling, **integer powers of** $1/\lambda \Leftrightarrow f_{\mathrm{np}}(s) \neq 0$

- At the end of the day, the strong-coupling expansion is slightly modified

$$\boxed{\mathcal{O}_0(y) \approx \mathrm{C}(N) - \sum_{g=0}^{\infty} N^{2-2g} \sum_{m=0}^{\infty} \left(\mathrm{a}_m^{(g)} \lambda^{-\frac{m+3}{2}} + \mathrm{b}_{\alpha_m}^{(g)} N^{-2-2m} \lambda^{\alpha_m} \right)}$$

where $\mathrm{a}_m^{(g)}$ and $\mathrm{b}_m^{(g)}$ are computable residues.

27 / 40

14:47 Tue Mar 15

74%

5. Supergravity as an emergent ensemble average

- The general expression for the strong-coupling expansion has striking consequences

- Given the large-N expansion of the average (turning $f_{\text{np}}(s)$ off for clarity):

$$\langle \mathcal{O} \rangle = \frac{1}{2} \sum_{g=0}^{\infty} N^{2-2g} \Bigg(\underbrace{f_{\text{p}}^{(g)}(1)}_{\langle\langle \mathcal{O}^{(g)} \rangle\rangle \equiv \frac{1}{2} f_{\text{p}}^{(g)}(1)} + N^{-1} f_{\text{p}}^{(g)}(0) \Bigg)$$

- Then the large-N limit of the average is the only term that survives the strong-coupling limit:

$$\boxed{\mathcal{O}_0(y) \approx \sum_{g=0}^{\infty} N^{2-2g} \left[\langle\langle \mathcal{O}^{(g)} \rangle\rangle - \sum_{m=0}^{\infty} \left(\mathsf{a}_m^{(g)} \lambda^{-m-\frac{3}{2}} + \mathsf{b}_m^{(g)} N^{-2-2m} \lambda^{m+\frac{1}{2}} \right) \right]}$$

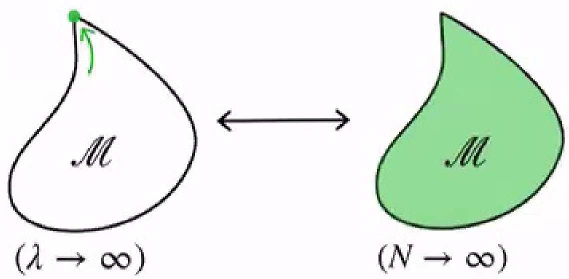
28 / 40

14:48 Tue Mar 15

Supergravity as an emergent ensemble average

- In other words, we have arrived at a large-N equivalence between **strong coupling** and **ensemble averaging** in $\mathcal{N} = 4$ SYM:

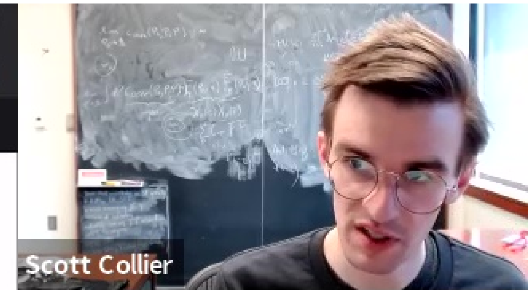
$$\boxed{\langle \mathcal{O} \rangle \overset{!}{=} \mathcal{O}(\lambda \to \infty) = \mathcal{O}_{\text{sugra}}}$$



- This extends to all loop orders:

$$\mathcal{O}_0^{(g)}(\lambda \to \infty) = \langle\langle \mathcal{O}^{(g)} \rangle\rangle$$

29 / 40



14:50 Tue Mar 15

Supergravity as an emergent ensemble average

- Obviously, the traditional holographic paradigm is left entirely intact

- Only the strongly-coupled theory at large N, dual to bulk supergravity, **emerges** as an ensemble average
  - As in low-dimension dualities, **averaging generates a simple (extremal) bulk dual**

- Perhaps there are scenarios where the average can be studied more easily than the strongly-coupled observable
  (cf. **averaged spectral gap** at large central charge in Narain ensemble average [Afkhami-Jeddi Cohn Hartman Tajdini 2020])

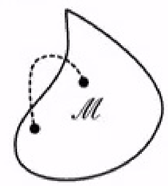
- Immediate corollary:
  - Any observable that diverges as $\lambda \to \infty$ cannot be modular invariant (e.g. Konishi anomalous dimension)

30 / 40

6. Statistics of the $SL(2,\mathbb{Z})$ ensemble

- We are thus motivated to study the **statistics** of CFT observables in the $SL(2,\mathbb{Z})$ ensemble
- We have seen that the spectral decomposition cleanly picks out the ensemble average

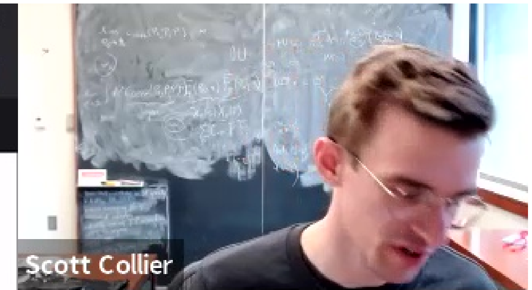
$$\mathcal{O}(\tau) = \langle \mathcal{O} \rangle + \underbrace{\mathcal{O}_{\text{spec}}(\tau)}_{\langle \mathcal{O}_{\text{spec}} \rangle = 0}$$



- e.g. the **variance** is straightforwardly computed in terms of the spectral overlaps:

$$\boxed{\begin{aligned} \mathscr{V}(\mathcal{O}) &= \langle \mathcal{O}^2 \rangle - \langle \mathcal{O} \rangle^2 \\ &= \langle \mathcal{O}_{\text{spec}}^2 \rangle \\ &= \text{vol}(\mathscr{F})^{-1} \left[\int_{\text{Re}\, s = \frac{1}{2}} \frac{ds}{4\pi i} \,|(\mathcal{O}, E_s)|^2 + \sum_{n=1}^{\infty} (\mathcal{O}, \phi_n)^2 \right] \end{aligned}}$$

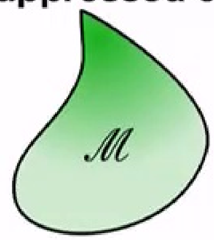
"Second moment in spectral space"



The variance at large N

- At large N, the variance is **parametrically suppressed** compared to the mean-squared (but still puzzlingly large):

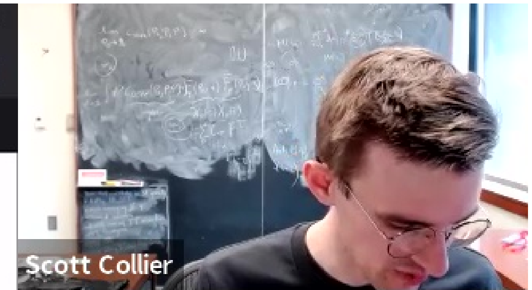
$$\frac{\mathscr{V}(\mathscr{O})}{\langle \mathscr{O} \rangle^2} \sim \frac{1}{N}$$



$$\left(N^2 \sim \frac{1}{G_{\mathrm{N}}} \right)$$

- Moreover, the $1/N$ expansion is **quadruple-factorially divergent**, leading to non-perturbative effects in powers of Λ_N^2, where

$$\Lambda_N \equiv e^{-4\sqrt{\pi N}}$$



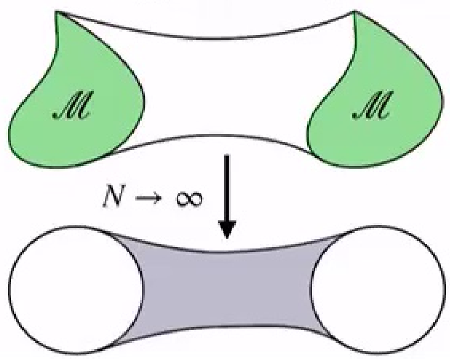
7. Wormholes in moduli space & emergent averaged holography

- Our results are especially interesting in light of recent developments in low-dimensional quantum gravity
  - 2d: JT gravity $\longleftrightarrow$ double-scaled random matrix integral [Saad Shenker Stanford 2019]
  - 3d: exotic "$U(1)$ gravity" $\longleftrightarrow$ averaged Narain lattice CFT [Maloney Witten; Afkhami-Jeddi Cohn Hartman Tajdini 2020]
- **Spacetime wormholes** play an important role in these dualities
- Leads to some natural questions in conventional holographic dualities:
  - What (other) bulk ingredients are needed for factorization of multi-boundary observables? [Saad Shenker Stanford Yao; Blommaert Kruthoff; Blommaert Iliesiu Kruthoff 2021]
  - In a product of observables in two decoupled CFTs, why are there contributions that appear to correlate them? (When and how do they appear?)
    Where are wormholes in the bootstrap?

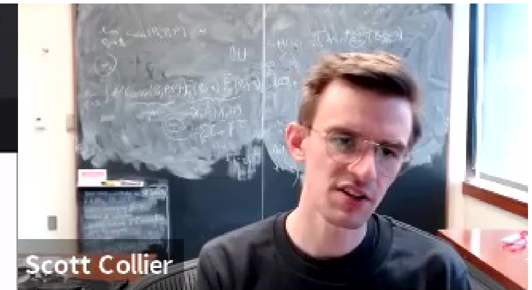
14:57 Tue Mar 15 71%

Wormholes in moduli space

- The large-N equivalence $\langle \mathcal{O} \rangle = \mathcal{O}_{\text{sugra}}$ suggests a **gravitational manifestation of** $SL(2,\mathbb{Z})$ **ensemble statistics**
  - How much does semiclassical $AdS_5 \times S^5$ string theory "know" about them?
- Note: we are not trying to argue that wormholes dominate multi-boundary observables in semiclassical $AdS_5 \times S^5$ (they don't, in general)
  - But rather, attempting to address whether and why wormholes may appear as part of the (unaveraged) semiclassical bulk theory (and how they do so consistently with UV completeness)



34 / 40



14:58 Tue Mar 15 71%

Wormholes in moduli space

- Our results resonate with recent work on factorization in low-dimensional models of quantum gravity [Saad Shenker Stanford Yao; Blommaert Kruthoff; Mukhametzhanov; Blommaert Iliesiu Kruthoff 2021]

$$\mathcal{O}(\tau) = \underbrace{\langle \mathcal{O} \rangle}_{\sim \mathcal{O}_{\text{sugra}}} + \underbrace{\int \frac{ds}{4\pi i}(\mathcal{O}, E_s)E_s(\tau) + \sum_{n=1}^{\infty}(\mathcal{O}, \phi_n)\phi_n(\tau)}_{\mathcal{O}_{\text{spec}}(\tau)\text{: "half-wormhole"}}$$

- To probe ensemble statistics, consider higher powers of $\mathcal{O}_{\text{spec}}(\tau)$:

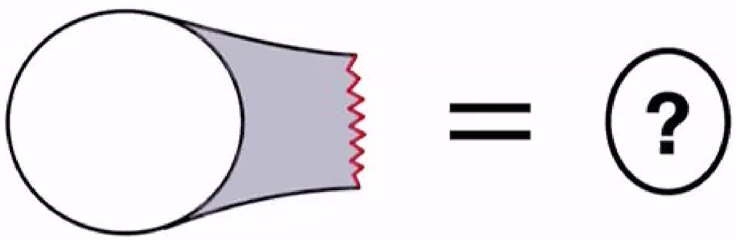
$$\boxed{\mathcal{O}_{\text{spec}}(\tau)^2 = \underbrace{\text{vol}(\mathcal{F})^{-1}\left[\int \frac{ds}{4\pi i}|(\mathcal{O}, E_s)|^2 + \sum_{n=1}^{\infty}(\mathcal{O}, \phi_n)^2\right]}_{\mathcal{V}(\mathcal{O})\text{: "wormhole"}} + \underbrace{\int \frac{ds}{4\pi i}(\mathcal{O}^2_{\text{spec}}, E_s)E_s(\tau) + \sum_{n=1}^{\infty}(\mathcal{O}^2_{\text{spec}}, \phi_n)\phi_n(\tau)}_{\text{coupling-dependent fluctuations around } \mathcal{V}(\mathcal{O})}}$$

35 / 40

14:59 Tue Mar 15

Wormholes in moduli space

- We have **not** identified the bulk configurations/stringy degrees of freedom that give the noisy contributions responsible for factorization for any particular observable



- Rather, we have shown concretely how ensemble statistics (& correspondingly connected bulk configurations) can play a role even in a fixed instance of $\mathcal{N} = 4$ SYM
  - Meaningful holographically because $\langle \mathcal{O} \rangle = \mathcal{O}_{\text{sugra}}$
  - Does the variance have a spacetime wormhole interpretation in semiclassical gravity? Does the answer depend on the type of observable? [Schlenker Witten 2022]

36 / 40

Future directions

- Obvious, urgent question: to what extent does this story generalize to other string/M-theory vacua?
  - Automorphic averages over U-duality groups? Ensemble averages over conformal manifolds? Over all directions in moduli space, or just the "gravity direction"?
- Other observables (and other S-duality groups):
  - Other integrated correlators, e.g. $\langle 22pp \rangle$
  - Ambitiously: unintegrated correlators
  - Observables particularly sensitive to statistics of high-energy states (thermal observables?)
  - Extremal correlators $\langle \mathcal{O}_p \overline{\mathcal{O}}_p \rangle$ in 4d $\mathcal{N} = 2$ SQCD
- Better understand worldsheet and spacetime perspectives on non-Borel summability of $\mathrm{AdS}_5 \times S^5$ string theory and the semiclassical bulk meaning of the variance
- "Bootstrapping in spectral space"
- Statistics of CFT data over conformal manifolds (cf. [Afkhami-Jeddi Ashmore Córdova 2021])
- Cusp forms and **arithmetic chaos** in $\mathcal{N} = 4$ SYM