

Title: Asymptotically safe gravity and beyond

Speakers: Masatoshi Yamada

Series: Quantum Gravity

Date: March 31, 2022 - 2:30 PM

URL: <https://pirsa.org/22030030>

Abstract: In this talk, we first introduce the basic idea of asymptotic safety and discuss its application to quantum gravity. We compute non-perturbative flow equations for the couplings of quantum gravity in fourth order of a derivative expansion. It is shown that the beta functions admit two possible fixed points: One is the asymptotically safe fixed point, and the other is the asymptotically free one. The corresponding critical exponents to these fixed points are evaluated.

Next, we argue that asymptotically safe gravity could be an effective theory emerging from the spontaneous symmetry breaking in an ultraviolet (UV) theory. We consider a UV theory invariant under SO(4) local Lorentz symmetry and diffeomorphism. In particular, we impose the degenerate limit (zero eigenvalues of vierbein) on the action and then show that only spinor fields can be dynamical. We will discuss prospects whether or not this UV theory could be a fundamental theory underlying asymptotically safe gravity.

Zoom Link: <https://pitp.zoom.us/j/94422371986?pwd=a25pS3cyUmJkNWJHL3hic1NhNlozQT09>



Masatoshi Yamada

Asymptotically safe gravity and beyond

Masatoshi Yamada

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Based on

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JHEP 03 (2022) 130

Perimeter Institute@Zoom

Quantum gravity



Masatoshi Yamada

- One says
 - Quantum gravity based on the Einstein-Hilbert action is not renormalizable.
 - Because the Newton constant is mass-dimension -2.

Quantum gravity



- One says
 - Quantum gravity based on the Einstein-Hilbert action is not renormalizable.
 - perturbatively*
 - Because the Newton constant is mass-dimension -2.

Contents

- Asymptotically safe gravity:
 - non-perturbatively renormalizable quantum gravity
 - Universality class at fixed point
 - Duality between asymptotically safe and free theories
 - Fermion-induced spacetime

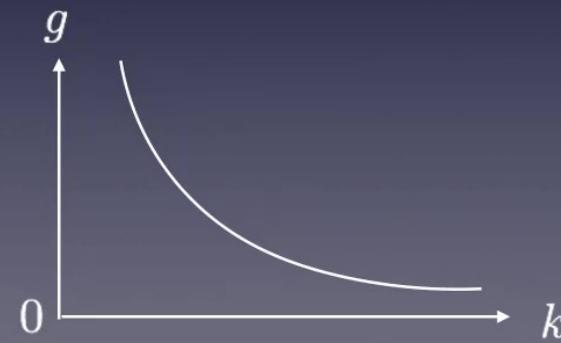
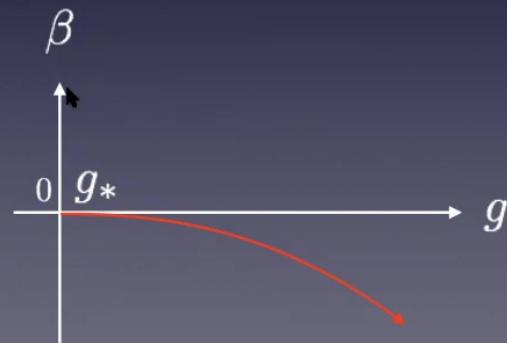




Asymptotic freedom

- Asymptotic freedom

$$\partial_t g = -\beta_0 g^3, \quad \beta_0 > 0$$

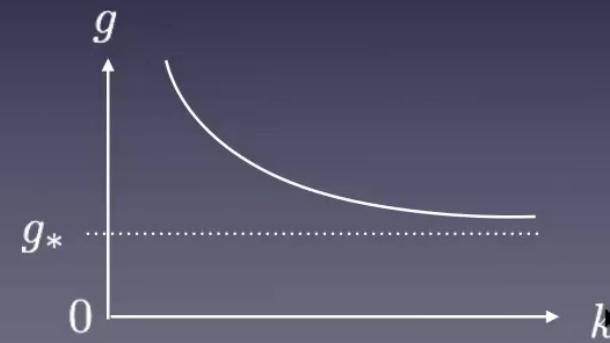
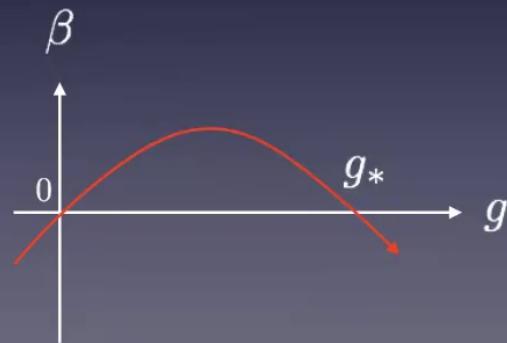


Asymptotic safety

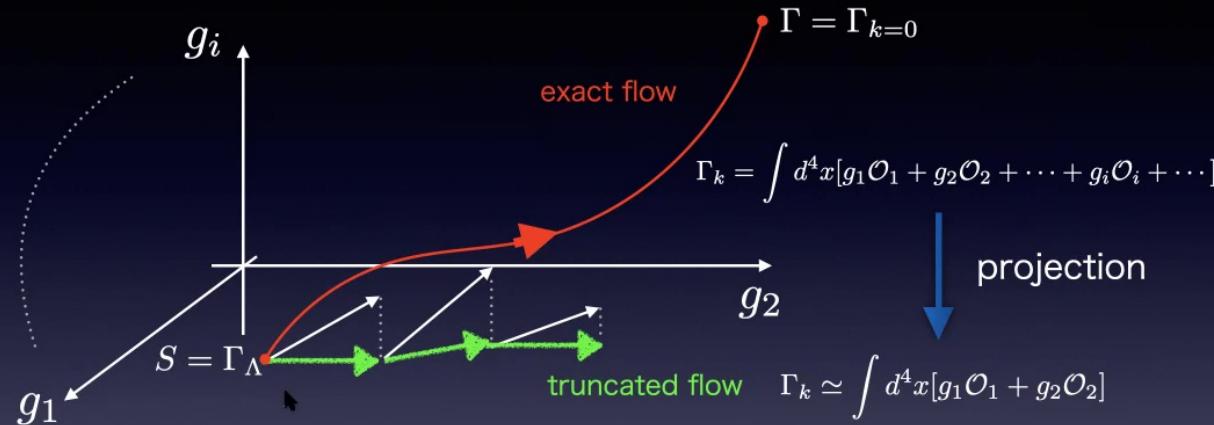


- Asymptotic **safety** $g = G_N k^2$

$$\partial_t g = 2g - \beta_0 g^2, \quad \beta_0 > 0$$

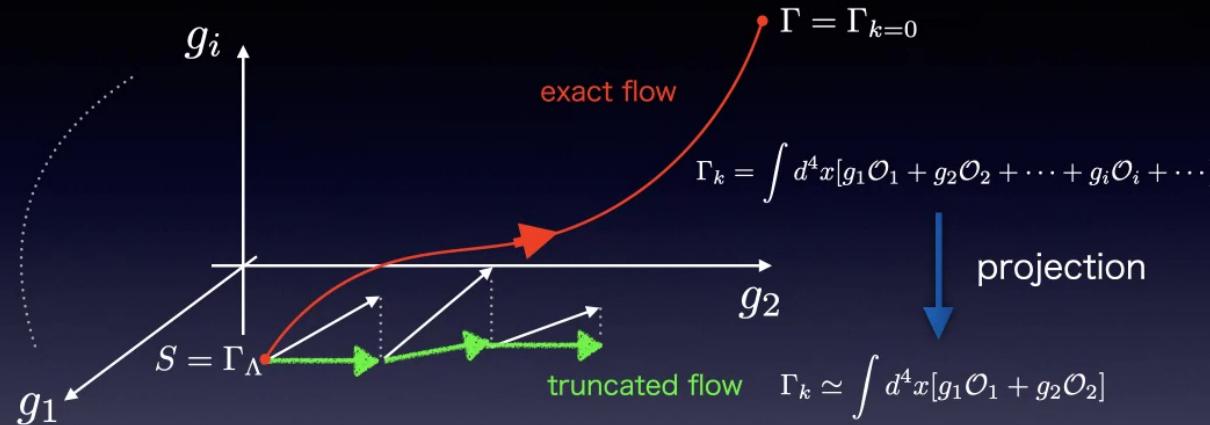


Functional renormalization group



Wetterich equation $k\partial_k \Gamma_k = \frac{1}{2} \text{Str}[(\Gamma_k^{(2)} + R_k)^{-1} k\partial_k R_k]$

Functional renormalization group



Wetterich equation $k\partial_k \Gamma_k = \frac{1}{2} \text{Str}[(\Gamma_k^{(2)} + R_k)^{-1} k\partial_k R_k]$

$$k\partial_k \Gamma_k = \int d^4x \left[\frac{(k\partial_k g_1) \mathcal{O}_1}{\beta_1(g)} + \frac{(k\partial_k g_2) \mathcal{O}_2}{\beta_2(g)} + \dots + \frac{(k\partial_k g_i) \mathcal{O}_i}{\beta_i(g)} + \dots \right]$$

Fixed point $k\partial_k \Gamma_k^* = 0 \longrightarrow \beta_i(g^*) = 0$



Critical exponent

$$k \frac{dg_i}{dk} = \beta_i(g)$$

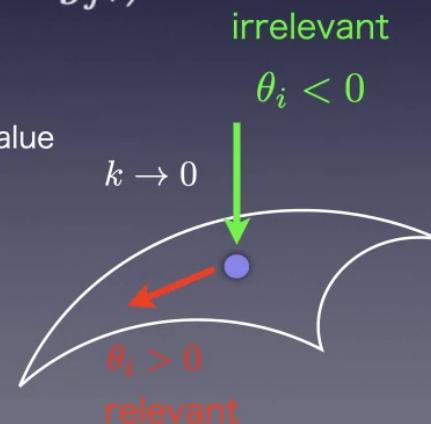
- RG eq. around FP g^*

$$k \frac{dg_i}{dk} \simeq \beta_i(g_*) + \left. \frac{\partial \beta_i}{\partial g_j} \right|_{g=g_*} (g_j - g_{j*})$$

- eigenvalue

- Solution of RG eq.

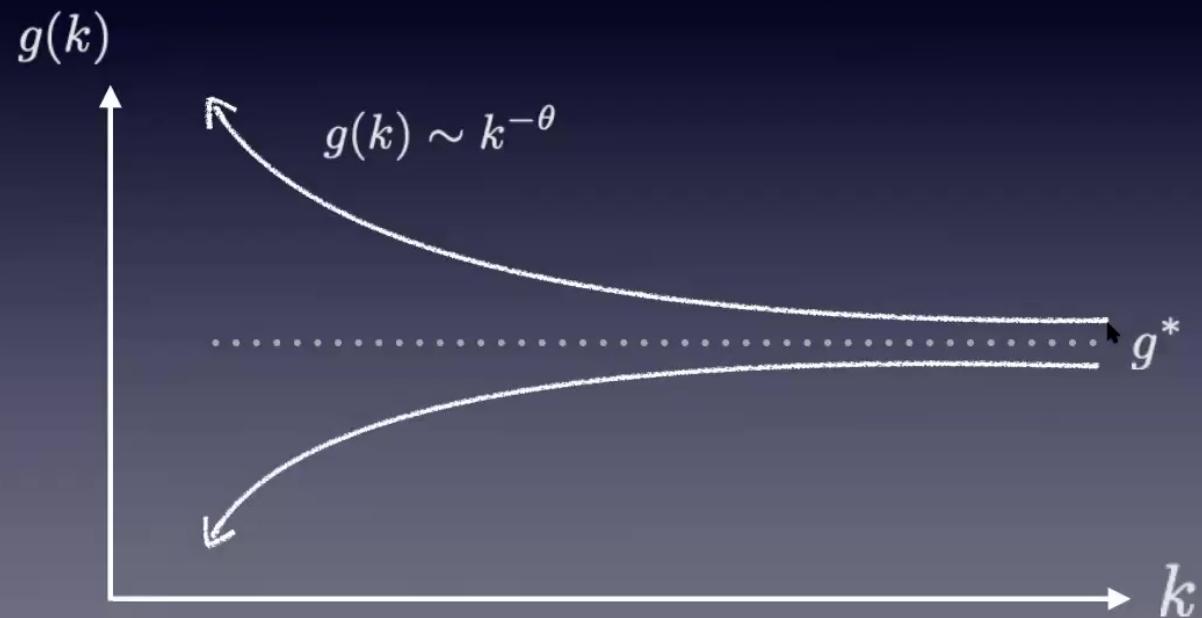
$$g_i(k) = g_i^* + \sum_j^N \zeta_j^i \left(\frac{k}{\Lambda} \right)^{-\theta_j}$$



Relevant: $\theta > 0$



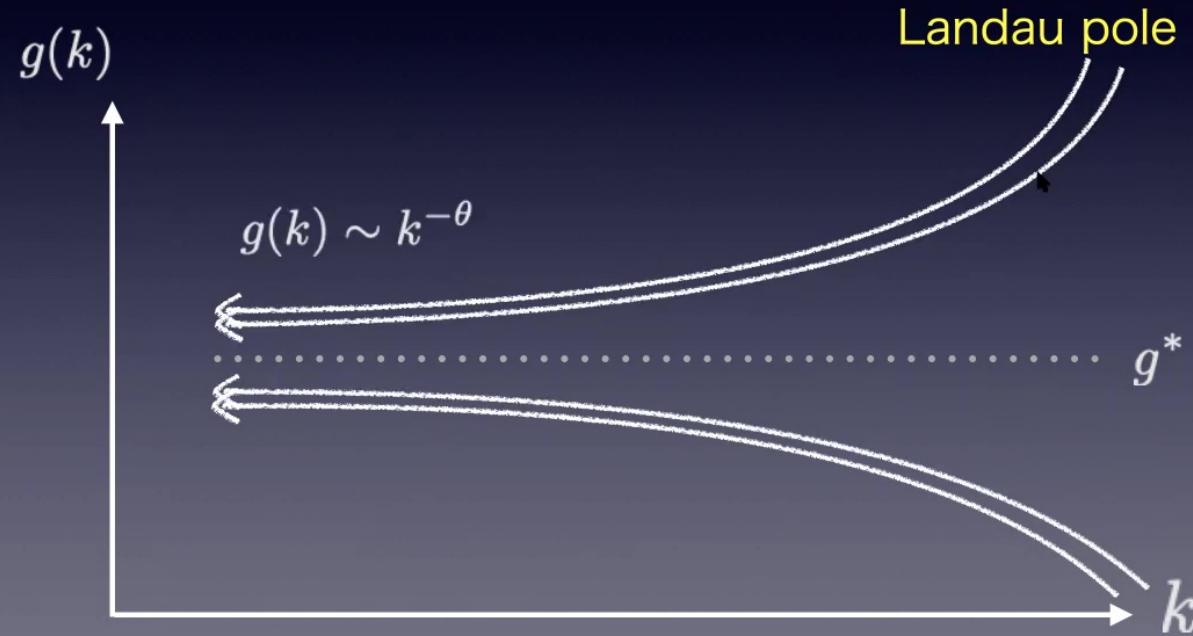
- Free parameter



Irrelevant $\theta < 0$



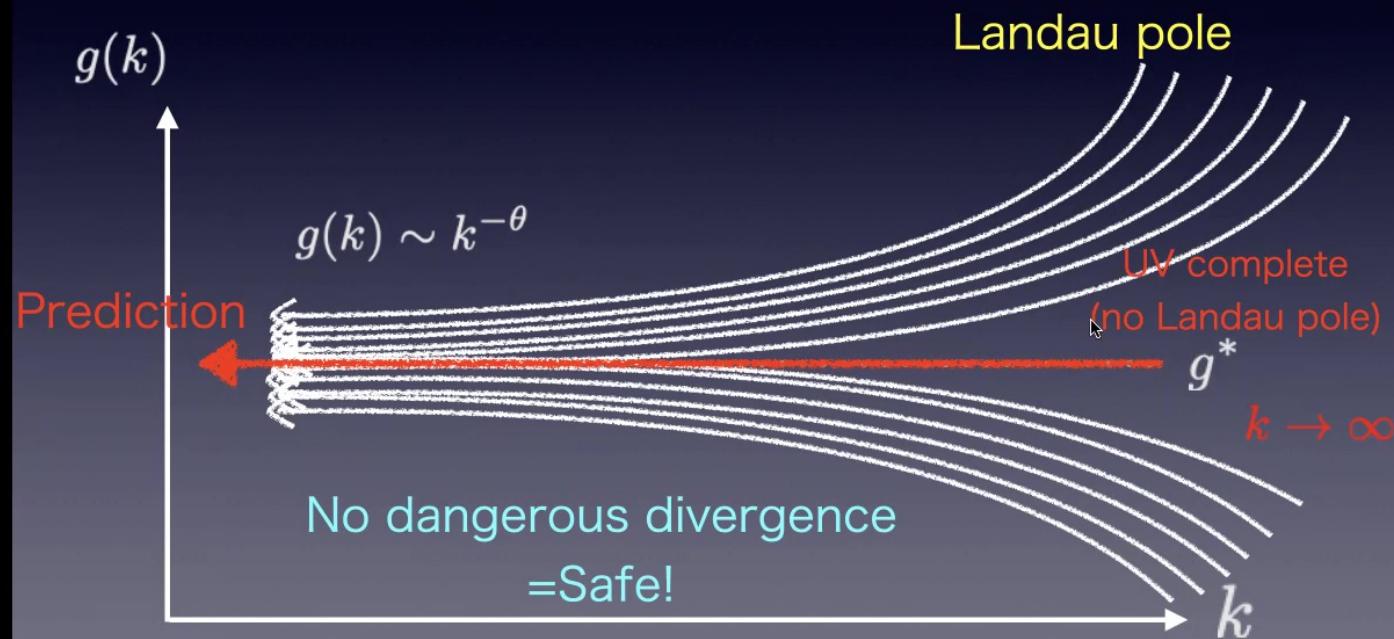
- Predictable parameter



Irrelevant $\theta < 0$



- Predictable parameter



Gravitational system



- Effective action for pure gravity

$$\Gamma_k = \int d^4x \sqrt{g} \left[V - \frac{M_p^2}{2} R + CR^2 - DC_{\mu\nu\rho\sigma}^2 + \dots \right]$$

Einstein-Hilbert truncation e.g. scholarpedia

$$\beta_g(g, \lambda) = (2 + \eta_N)g,$$

$$\beta_\lambda(g, \lambda) = -(2 - \eta_N)\lambda - \frac{g}{\pi} \left[5 \ln(1 - 2\lambda) - 2\zeta(3) + \frac{5}{4} \eta_N \right],$$

with **anomalous dimension** induced by quantum gravity

$$\eta_N(g, \lambda) = -\frac{2g}{6\pi + 5g} \left[\frac{18}{1 - 2\lambda} + 5 \ln(1 - 2\lambda) - \zeta(2) + 6 \right].$$

$$g_N = \frac{k^2}{8\pi M_p^2} \\ = k^2 G_N$$

$$\lambda = \frac{V}{8\pi k^2 M_p^2}$$

Gravitational system



- Fixed point

$$\beta_g = \beta_\lambda = 0 \rightarrow (g_*, \lambda_*) = (0, 0), (0.378, 0.340)$$

- Critical exponents

$$\theta_i = -\text{eig} \begin{pmatrix} \frac{\partial \beta_g}{\partial g} & \frac{\partial \beta_g}{\partial \lambda} \\ \frac{\partial \beta_\lambda}{\partial g} & \frac{\partial \beta_\lambda}{\partial \lambda} \end{pmatrix} \Big|_{g=g_*, \lambda=\lambda_*}$$

$$(\theta_g, \theta_\lambda) = (-2, 2), (2.141 + 3.438i, 2.141 - 3.438i)$$

Higher derivative truncation



- Inclusion of higher dimensional operators

S.Saswato, C. Wetterich, **MY**, JHEP 03 (2022) 130

$$\Gamma_k = \int d^4x \sqrt{g} \left[V - \frac{M_p^2}{2} R + C R^2 - D C_{\mu\nu\rho\sigma}^2 \right]$$

- Asymptotically safe fixed point

$$v_* = (V/k^4)_* \neq 0 \quad w_* = (M_p^2/(2k^2))_* \neq 0 \quad C_* \neq 0 \quad D_* \neq 0$$

Graviton propagator: $G_h \sim \frac{1}{-v_* + w_* p^2 + D_* p^4}$

- Asymptotically free fixed point

$$v_* = (V/k^4)_* \neq 0 \quad w_* = (M_p^2/(2k^2))_* \neq 0 \quad C_*^{-1} = D_*^{-1} = 0$$

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coupling $\sim k^{-\theta}$

- Asymptotically safe fixed point

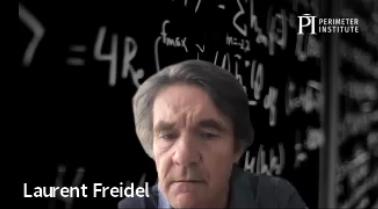
$$v_* = (V/k^4)_* \neq 0 \quad w_* = (M_p^2/(2k^2))_* \neq 0 \quad C_* \neq 0 \quad D_* \neq 0$$

$$\theta_1 = 3.1 \quad \theta_2 = 2.4 \quad \theta_3 = 10.9 \quad \theta_4 = -88.1$$

- Asymptotically free fixed point

$$v_* = (V/k^4)_* \neq 0 \quad w_* = (M_p^2/(2k^2))_* \neq 0 \quad C_*^{-1} = D_*^{-1} = 0$$

$$\theta_1 = 4 \quad \theta_2 = 2 \quad \theta_3 = \theta_4 = 0$$



Higher derivative truncation

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Gravitational system



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- Fixed point

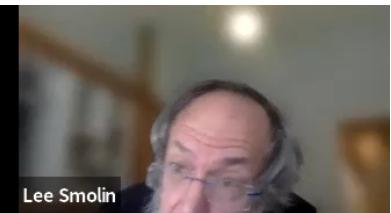
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Higher derivative truncation



Lee Smolin

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Is asymptotically safe theory ultimate?



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Asymptotically safe theories



- D=3 non-linear σ model
- D=3 Gross-Neveu model
- D=5 Yang-Mills theory???
- D=4 gravity???

These theories are perturbatively NON-renormalizable
but (could be) NON-perturbatively renormalizable.

Unitarity??

Asymptotically safe theories



- D=3 non-linear σ model \Leftrightarrow linear σ model
- D=3 Gross-Neveu model \Leftrightarrow Higgs-Yukawa model
- D=5 Yang-Mills theory???
- D=4 gravity???

These theories are perturbatively NON-renormalizable
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Unitarity??

An Example of asymptotically safe theories



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- Non-linear σ model in 3 dim.
 - Scalar theory with a field constraint $\langle \phi^i \phi^j \rangle = f_\pi^2 \delta^{ij}$
 - Symmetry breaking $O(N) \rightarrow O(N-1)$ in the linear σ model.
- $$\phi^i = (\sigma, \pi^1, \dots, \pi^{N-1}) \quad \langle \sigma \rangle = f_\pi$$
- Describes dynamics of massless NG bosons (pions). $S[\pi^i]$
- Perturbatively **non**-renormalizable

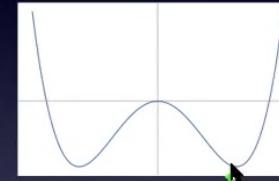
An Example of asymptotically safe theories

- Linear σ model

$$S = \int d^3x \left[\frac{1}{2}(\partial_\mu \phi^i)^2 - \frac{m^2}{2}(\phi^i)^2 - \frac{\lambda}{4}(\phi^i)^4 \right]$$

$$\phi^i = (\sigma, \pi^1, \dots, \pi^{N-1})$$

$$\langle \phi^i \phi^j \rangle = f_\pi^2 \delta^{ij}$$



$$f_\pi = \langle \phi \rangle = \sqrt{-2m^2/\lambda}$$

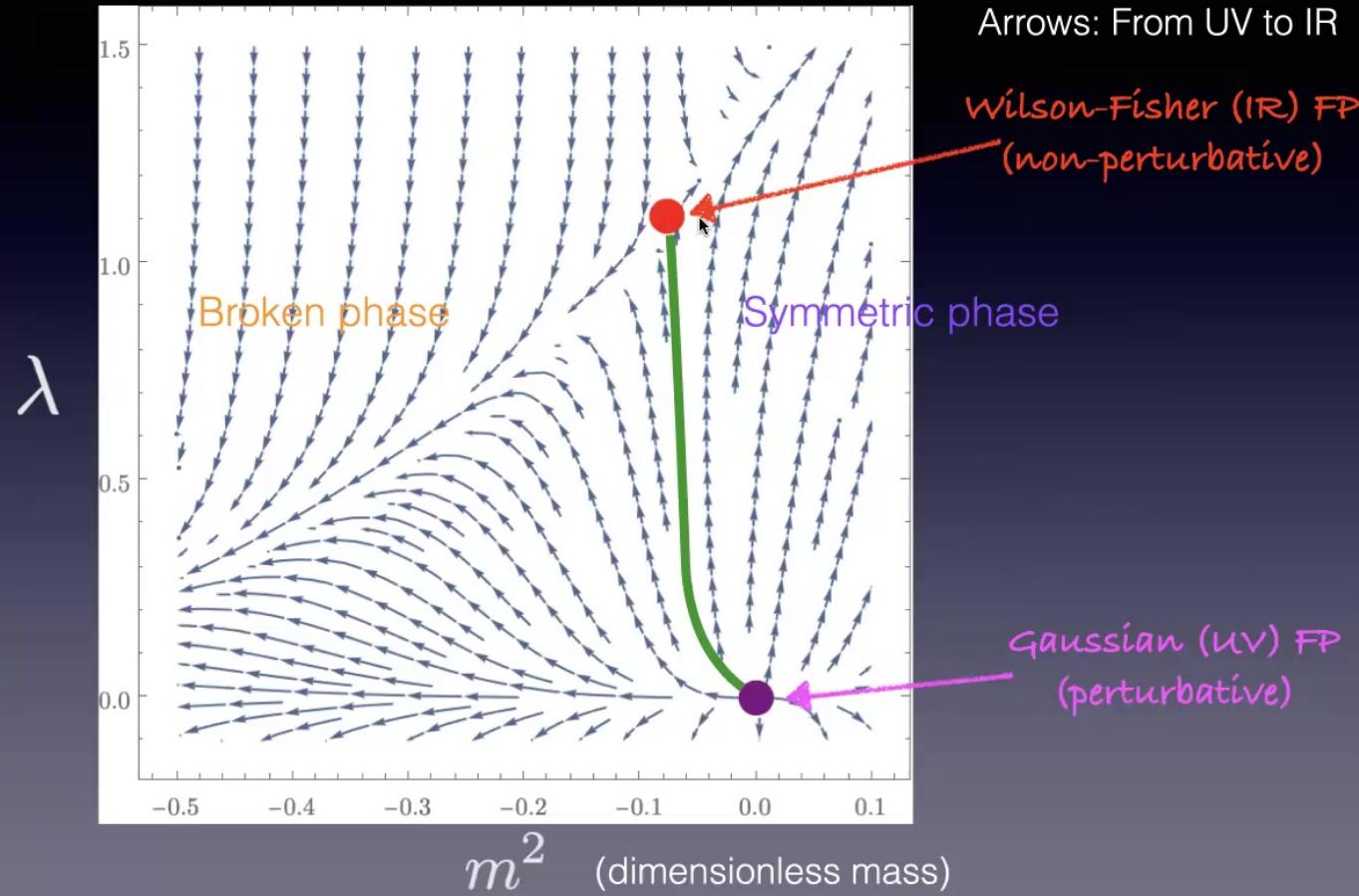
- Non-linear σ model

$$S = \int d^3x \left[\frac{f_\pi^2}{2}(\partial_\mu \pi^i)^2 + \frac{a}{2}(\pi^i \partial_\mu \pi^i)^2 + \dots \right]$$

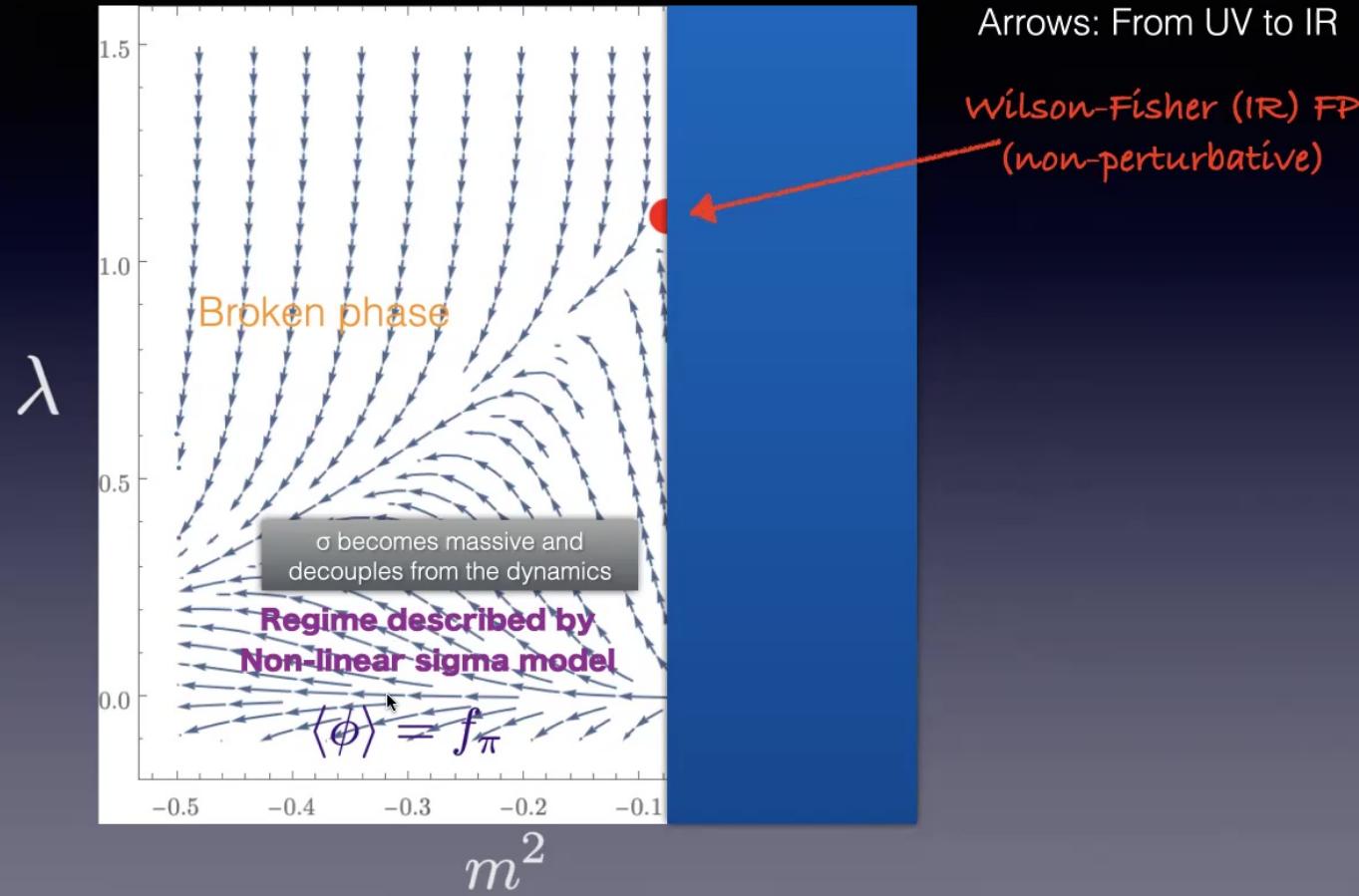


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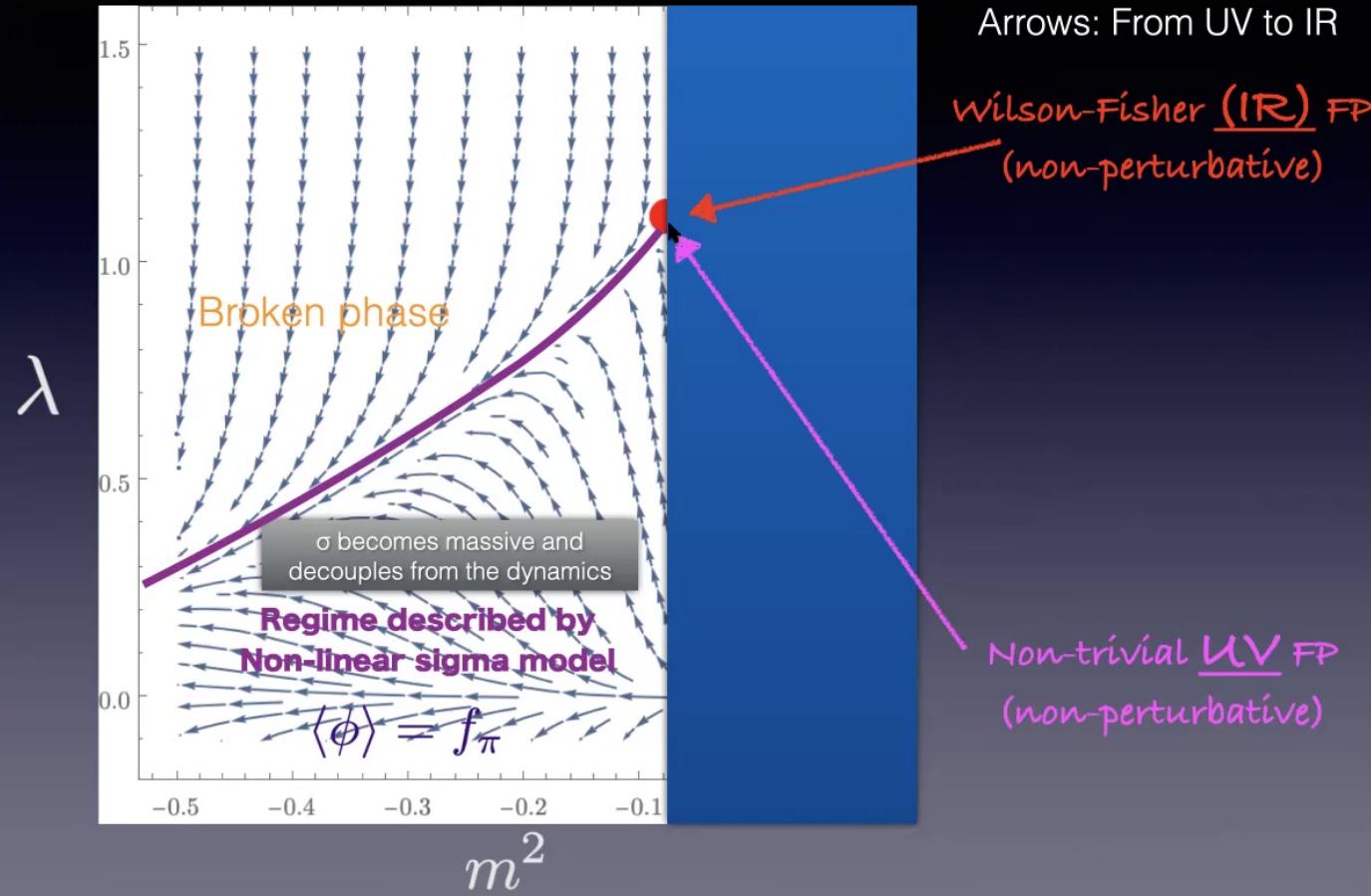
Phase diagram of 3d linear σ model



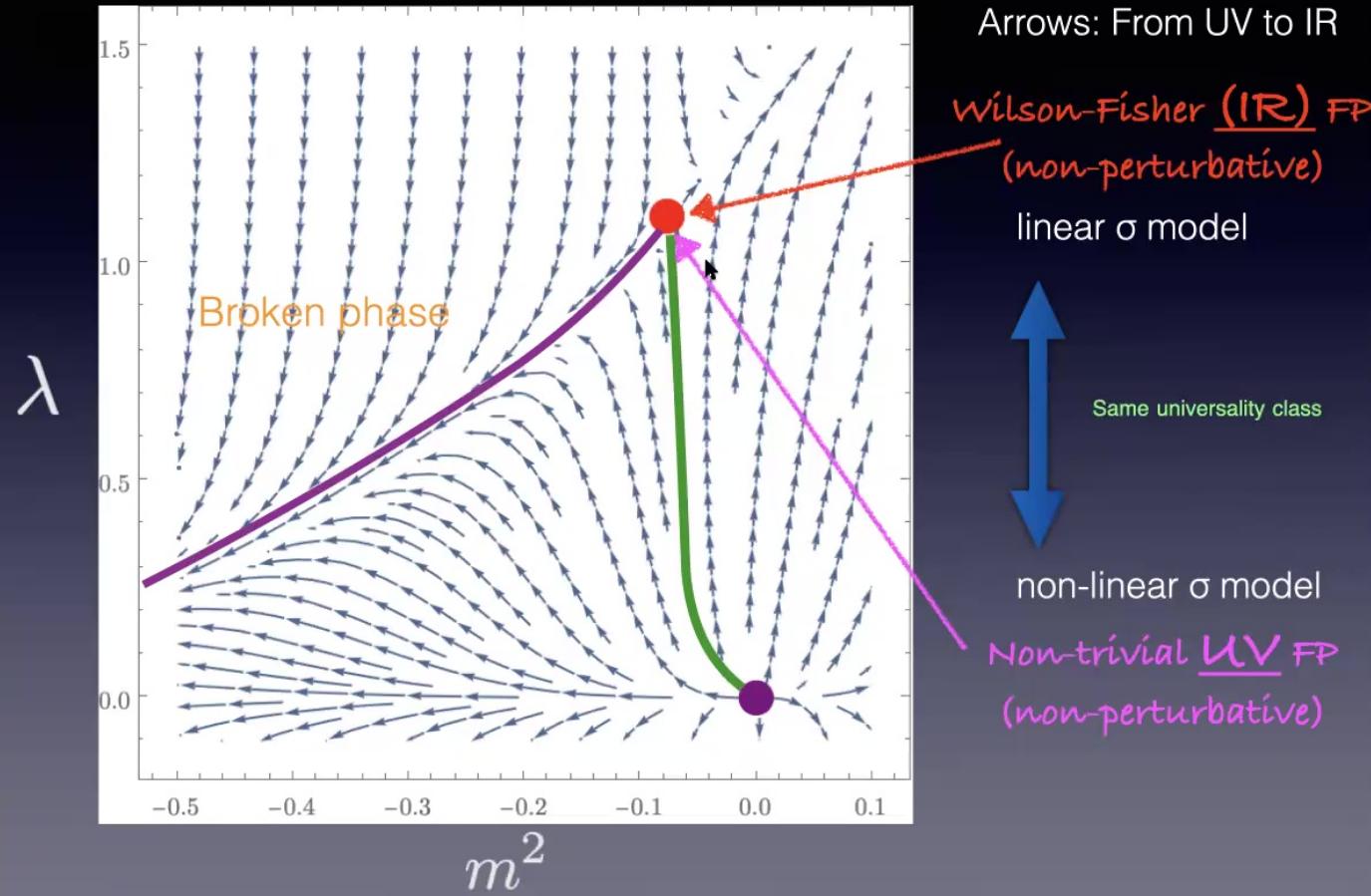
Phase diagram of 3d linear σ model



Phase diagram of 3d linear σ model



Phase diagram of 3d linear σ model



To summarize



Non-linear σ model in 3 dim
 $O(N-1)$

- Perturbatively non-renormalizable
- Asymptotically safe (UV FP)
- Constraint on fields

$$\langle \phi^i \phi^j \rangle = f_\pi^2 \delta^{ij}$$

Same universality class

$O(N)$ linear σ model in 3 dim

- Perturbatively renormalizable
- Unitary (at Gaussian FP)
- Asymptotically free (Gaussian FP)
- IR fixed point (Wilson-Fisher FP)



Asymptotically safe gravity

- Perturbatively non-renormalizable
- Asymptotically safe (UV FP)
- Constraint on fields

$$g_{\mu\alpha} g^{\alpha\nu} = \delta_\mu^\nu$$

?

How to formulate?



- Metric theories are diffeomorphism invariant.



- In this work, we consider local Lorentz $SO(1,3)$:



First-order formalism



- Based on $SO(1,3)$ local Lorentz symmetry (and diff.)
 - Vierbein $e_\mu{}^a$
 - Local-Lorentz (LL) gauge field $(A_\mu)^a{}_b$
 - Minimal action (Einstein-Hilbert)

$$S = \int d^4x e \left[-\Lambda + \frac{M^2}{2} e_a{}^\mu e_b{}^\nu F^{ab}{}_{\mu\nu} \right]$$

$$F^a{}_{b\mu\nu} = (\partial A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu])^a{}_b$$

Skippable



First-order formalism

$$S = \int d^4x e \left[-\Lambda + \frac{M^2}{2} e_a{}^\mu e_b{}^\nu F^{ab}{}_{\mu\nu} \right]$$

- Equation of motion $\overset{\nwarrow}{(A_\mu)^a}_b = e_\nu{}^a D_\mu \overset{\textcolor{red}{\nwarrow}}{e^\nu}_b$
 - Obtain the EH action in the vierbein formalism
 - Introducing inverse vierbein breaks $SO(1,3)_{\text{local}}$ symmetry.
- Kinetic term of LL gauge field

$$\frac{1}{4} F^{ab}{}_{\mu\nu} F_{ab}{}^{\mu\nu} + \dots \rightarrow R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + \dots$$

Degenerate limit



- Non-linear σ model: $O(N-1)$ invariant
 - Constraint on fields $\langle \phi^i \phi^j \rangle = f_\pi^2 \delta^{ij}$
 - $f_\pi^2 \rightarrow 0$: symmetric phase ($O(N)$ invariant)
- Gravity in first-order formalism
 - Constrain on metric $g_{\mu\alpha} g^{\alpha\nu} = \delta_\mu^\nu$
 - $C \rightarrow 0$: symmetric phase ($SO(1,3)$ invariant).
 - More precisely, $\det(e^a{}_\mu) = 0$

Model with degenerate limit



- Including matters, at a certain scale,

$$S = \int d^4x e \left[-V + \frac{M^2}{2} e_a{}^\mu e_b{}^\nu F^{ab}{}_{\mu\nu} - \frac{Z_\psi}{2} (\bar{\psi} e_a{}^\mu \gamma^a D_\mu \psi + \text{h.c.}) \right]$$
$$D_\mu = \partial_\mu - ig_L (A_\mu)^{ab} \Sigma_{ab} + \dots$$

- Invariant under $\text{SO}(1,3)_{\text{local}} \times \text{diff.}$
- Only fermions are dynamical!
- No kinetic terms of vierbein, gauge fields, scalar fields.
- These fields would be dynamical via fermion quantum corrections.

Inverse metric



- Metric $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$
 - Canonical normalization $g_{\mu\nu} = \bar{g}_{\mu\nu} + \frac{1}{M_P} h_{\mu\nu}$
 - Inverse metric $g_{\mu\alpha} g^{\alpha\nu} = \delta_\mu^\nu$

$$g^{\mu\nu} = \bar{g}^{\mu\nu} - M_P h^{\mu\nu} + M_P^2 h^\mu{}_\alpha h^{\alpha\nu} + \dots$$

Degenerate limit



- Non-linear σ model: $O(N-1)$ invariant

- Constraint on fields $\langle \phi^i \phi^j \rangle = f_\pi^2 \delta^{ij}$
 - $f_\pi^2 \rightarrow 0$: symmetric phase ($O(N)$ invariant)

- Gravity in first-order formalism

$$\langle e^a{}_\mu \rangle = C \delta_\mu^a$$

$$\bar{g}_{\mu\nu} \propto C^2$$

- Constrain on metric $g_{\mu\alpha} g^{\alpha\nu} = \delta_\mu^\nu$
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 - More precisely, $\det(e^a{}_\mu) = 0$

Model with degenerate limit



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Spontaneous local Lorentz symmetry breaking



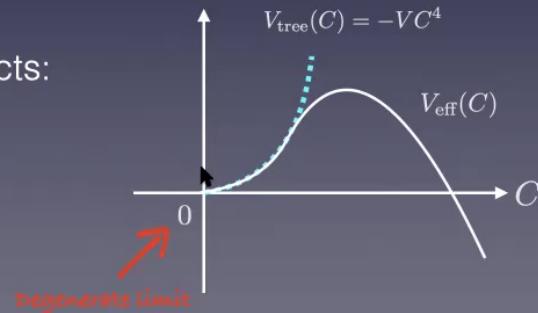
- $\text{SO}(1,3)_{\text{local}} \times \text{diff.}$
- Generation of expectation value of vierbein
- A possible solution would be a flat spacetime.

$$\langle e^a{}_\mu \rangle = C \delta_\mu^a$$

- Effective potential from spinor loop effects:

$$V_{\text{eff}}(C) = -VC^4 - \frac{(CM)^4}{2(4\pi)^2} \log(C^2 M^2 / \mu^2)$$

- Precise analysis is in progress.





Lee Smolin

Model with degenerate limit

- Including matters, at a certain scale,

$$S = \int d^4x e \left[-V + \frac{M^2}{2} e_a{}^\mu e_b{}^\nu F^{ab}{}_{\mu\nu} - \frac{Z_\psi}{2} (\bar{\psi} e_a{}^\mu \gamma^a D_\mu \psi + \text{h.c.}) \right]$$
$$D_\mu = \partial_\mu - ig_L (A_\mu)^{ab} \Sigma_{ab} + \dots$$

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- Only fermions are dynamical!
- No kinetic terms of vierbein, gauge fields, scalar fields.
- These fields would be dynamical via fermion quantum corrections.

Model with null limit



- Including matters, at a certain scale,

$$S = \int d^4x e \left[-V + \frac{M^2}{2} e_a{}^\mu e_b{}^\nu F^{ab}{}_{\mu\nu} - \frac{Z_\psi}{2} (\bar{\psi} e_a{}^\mu \gamma^a D_\mu \psi + \text{h.c.}) \right]$$

$$D_\mu = \partial_\mu - ig_L (A_\mu)^{ab} \Sigma_{ab} + \dots$$

- $e = \frac{1}{4!} \epsilon_{abcd} \epsilon^{\mu\nu\rho\sigma} e^a{}_\mu e^b{}_\nu e^c{}_\rho e^d{}_\sigma \sim C^4 \rightarrow 0$
- $ee_a{}^\mu = \frac{1}{3!} \epsilon_{abcd} \epsilon^{\mu\nu\rho\sigma} e^b{}_\nu e^c{}_\rho e^d{}_\sigma \sim C^3 \rightarrow 0$
- $ee_{[a}{}^\mu e_{b]}{}^\nu = \frac{1}{2!2!} \epsilon_{abcd} \epsilon^{\mu\nu\rho\sigma} e^c{}_\rho e^d{}_\sigma \sim C^2 \rightarrow 0$
- AntiSym $[ee_a{}^\mu e_b{}^\nu e_c{}^\rho] = \epsilon_{abcd} \epsilon^{\mu\nu\rho\sigma} e^d{}_\sigma \propto C \rightarrow 0$ No invariant term
- AntiSym $[ee_a{}^\mu e_b{}^\nu e_c{}^\rho e_d{}^\sigma] = \epsilon_{abcd} \epsilon^{\mu\nu\rho\sigma}$ Topological $\epsilon_{abcd} \epsilon^{\mu\nu\rho\sigma} F^{ab}{}_{\mu\nu} F^{cd}{}_{\rho\sigma}$

Spontaneous local Lorentz symmetry breaking

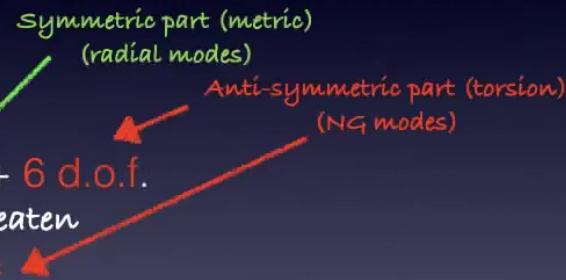


- Local Lorentz gauge symmetry is broken.

- Degrees of freedom (d.o.f.):

- Vierbein e_μ^a : 16 d.o.f. = 10 + 6 d.o.f.

- LL gauge field $(A_\mu)^a_b$: 6 d.o.f.

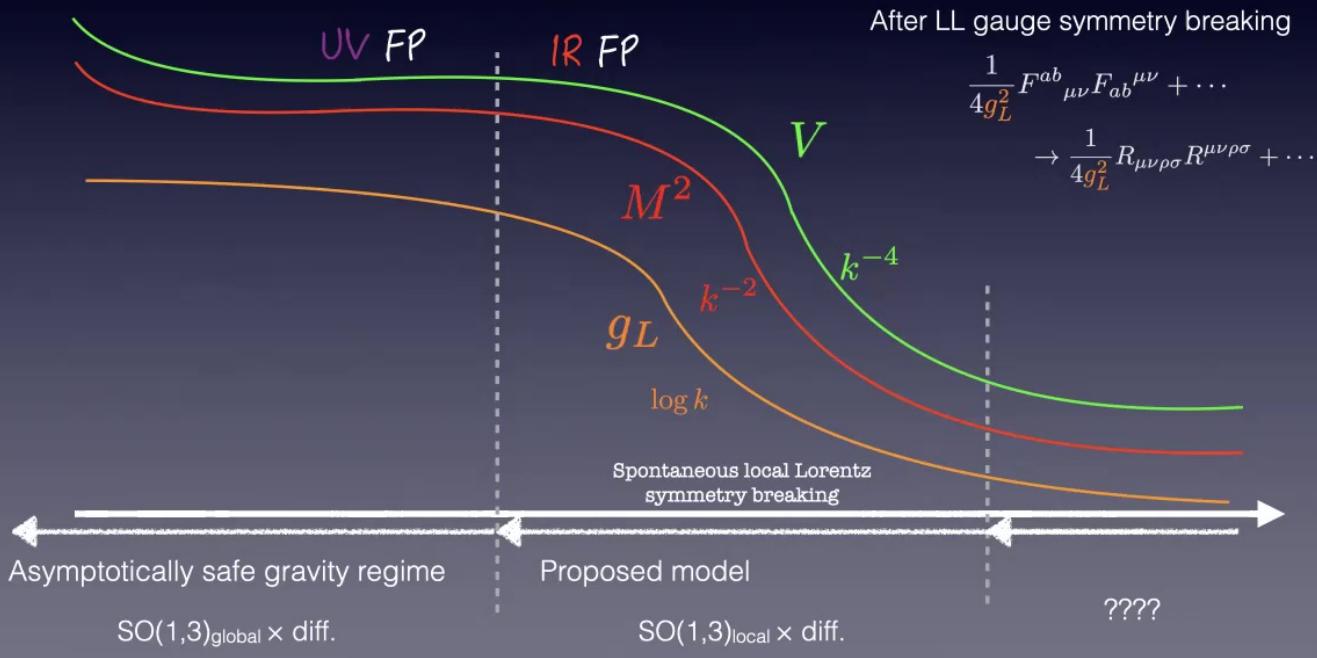


- LL gauge bosons become massive and decouple.
- The symmetry parts (radial modes) are still massless thanks to diif..

RG flow (ideal)



$$S = \int d^4x e \left[-V + \frac{M^2}{2} e_a^\mu e_b^\nu F^{ab}_{\mu\nu} - \frac{Z_\psi}{2} (\bar{\psi} e_a^\mu \gamma^a (\partial_\mu - i g_L A_\mu) \psi + h.c.) \right]$$



Summary

- Asymptotically safe gravity:
 - Non-perturbatively renormalizable
 - Unitarity??
 - Indicate **the existence of new degrees of freedom.**
- Einstein-Cartan theory with degenerate limit
- Connection to low energy quantum gravity

