Title: From UV to IR and back

Speakers: Claudia de Rham

Series: Quantum Gravity

Date: March 24, 2022 - 10:30 AM

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Abstract: By considering the photon-graviton scattering amplitude at low-energy including loops from the Standard Model, I will argue how we can infer constraints on the Regge behaviour of the gravitational scattering amplitude. I will also comment on implications to other gauge fields in four and higher dimensions and application to the Weak Gravity Conjecture and connections with causality considerations.

Zoom Link: https://pitp.zoom.us/j/91414100213?pwd=Z3g5SmRPb2xjUXEweGtjUC83WCtKdz09


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THE ROYAL SOCIETY
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Perimeter Institute
24th March 2022
PERIMETER INSTITUTE
FOR THEORETICAL PHYSICS
From UV to IR and back
Imperial College
London
Claudia de Rham

Special thanks to amazing collaborators
Lasma Alberte (@ Imperial)
Mariana Carrillo Gonzalez (@ Imperial)
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Scott Melville (@ Cambridge)
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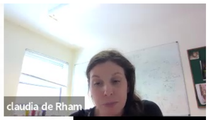
Johannes Noller (@ Portsmouth)
Andrew Tolley (@ Imperial)
Jun Zhang (@ ICTP Asia-Pacific)
Shuang-Yong Zhou (@ USTC)
Speed of GWs & Causality: 1806.09417, 1909.00881, 2007.01847, 2112.05031, 2112.05054
Gravitational EFTS: 2005.13923, 2107.11384,
Positivity Bounds: 2007.12667, 2012.05798, 2103.06855, 2111.09226
Models of Dark Energy: 2108.12892, 2110.14327, …
claudia de Rham



Effective Field Theory

Energy

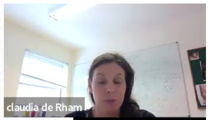
UV completion of Quantum Gravity

???

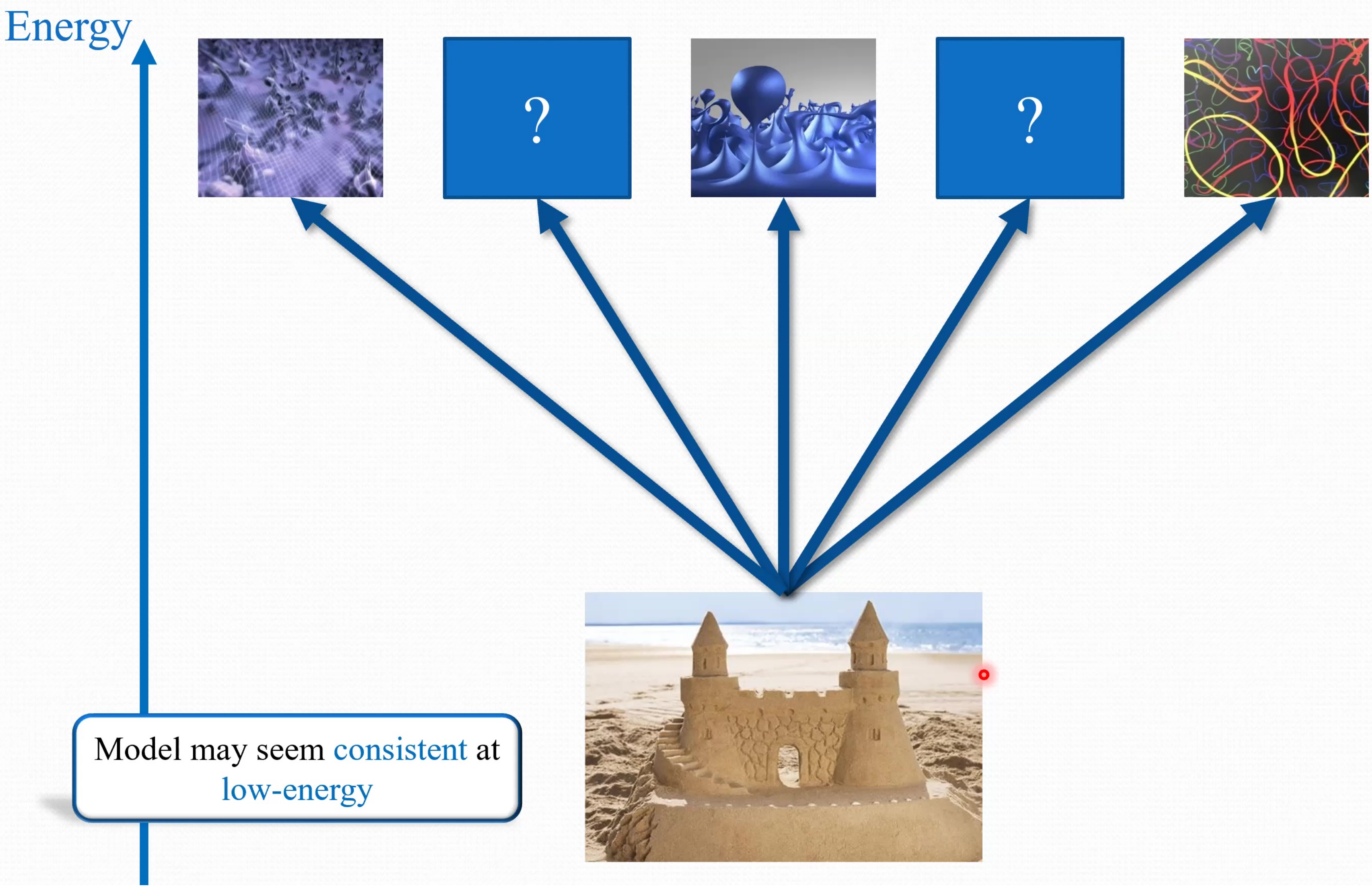
Low-energy EFT

E.g.: GR + irrelevant operators
+ (B)SM fields
+ light degrees of freedom

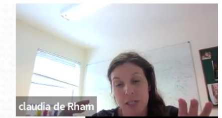
???



High-energy completion



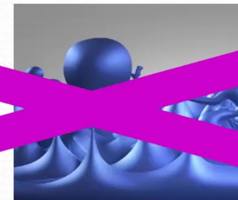
Model may seem consistent at low-energy



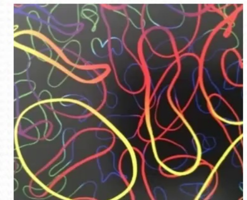
High-energy completion

Energy

?



?

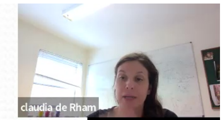


Yet we know that some models would never EVER be able to enjoy a standard UV completion

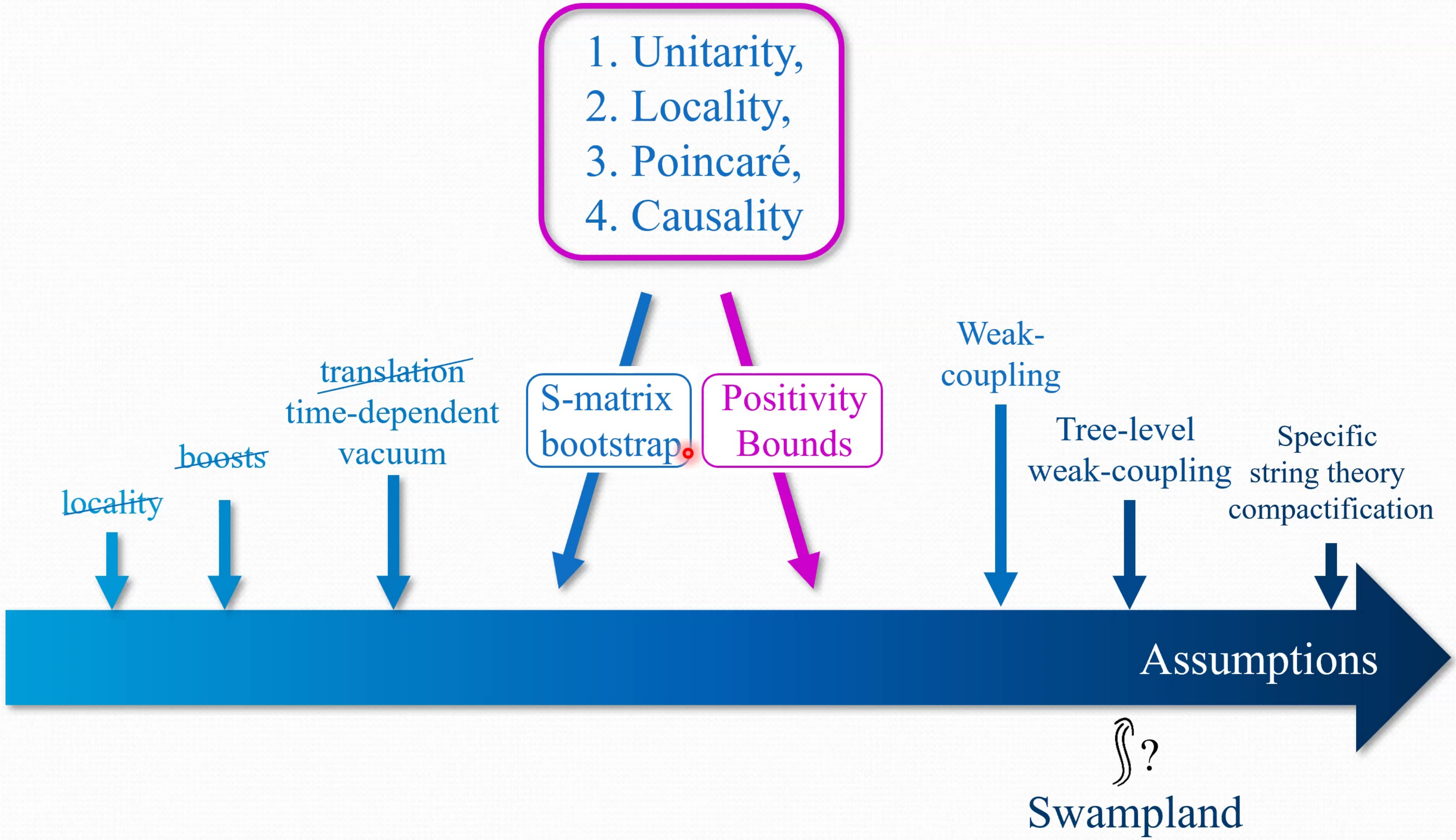
60 orders of magnitude

Model may seem consistent at low-energy

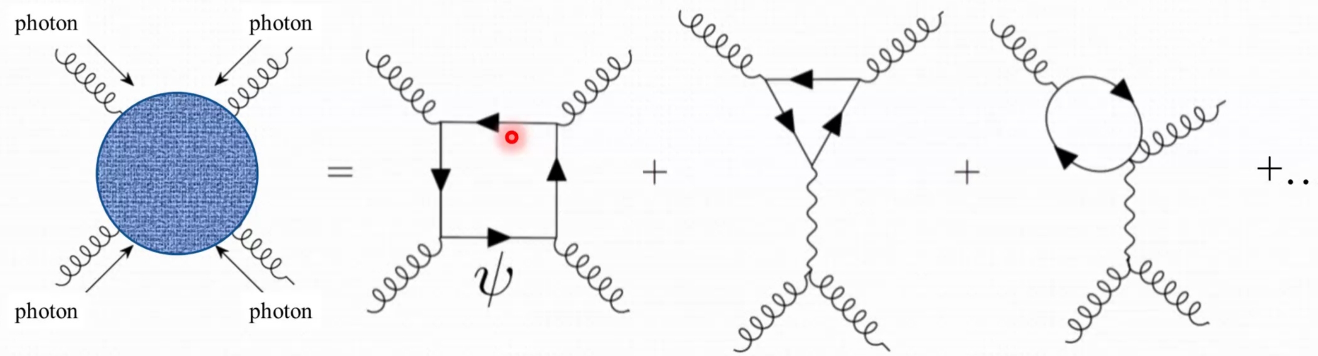
Its **not** that a theory with parameter $\alpha = 0$ is ruled out but $|\alpha| > 10^{-60}$ is allowed

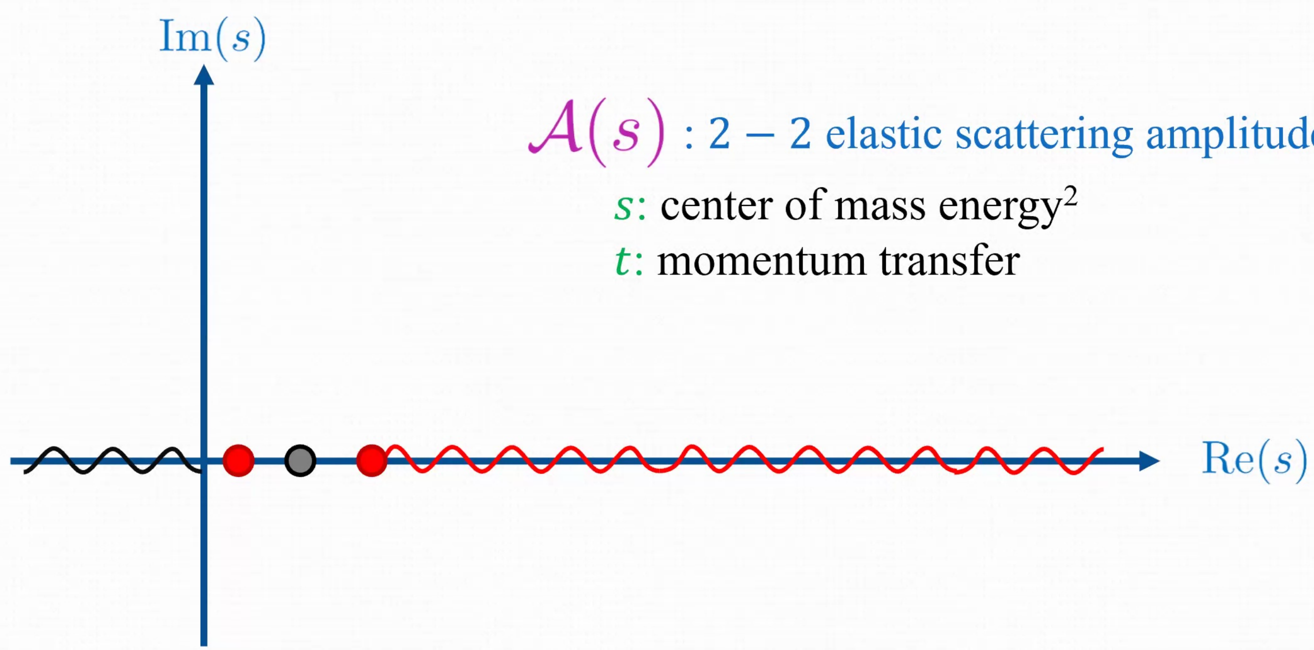


Assumptions on the UV Completion



Low-energy Scattering

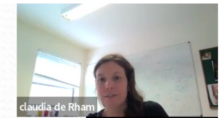




$\mathcal{A}(s)$: 2 – 2 elastic scattering amplitude

s: center of mass energy^2

t: momentum transfer



Low-energy Scattering

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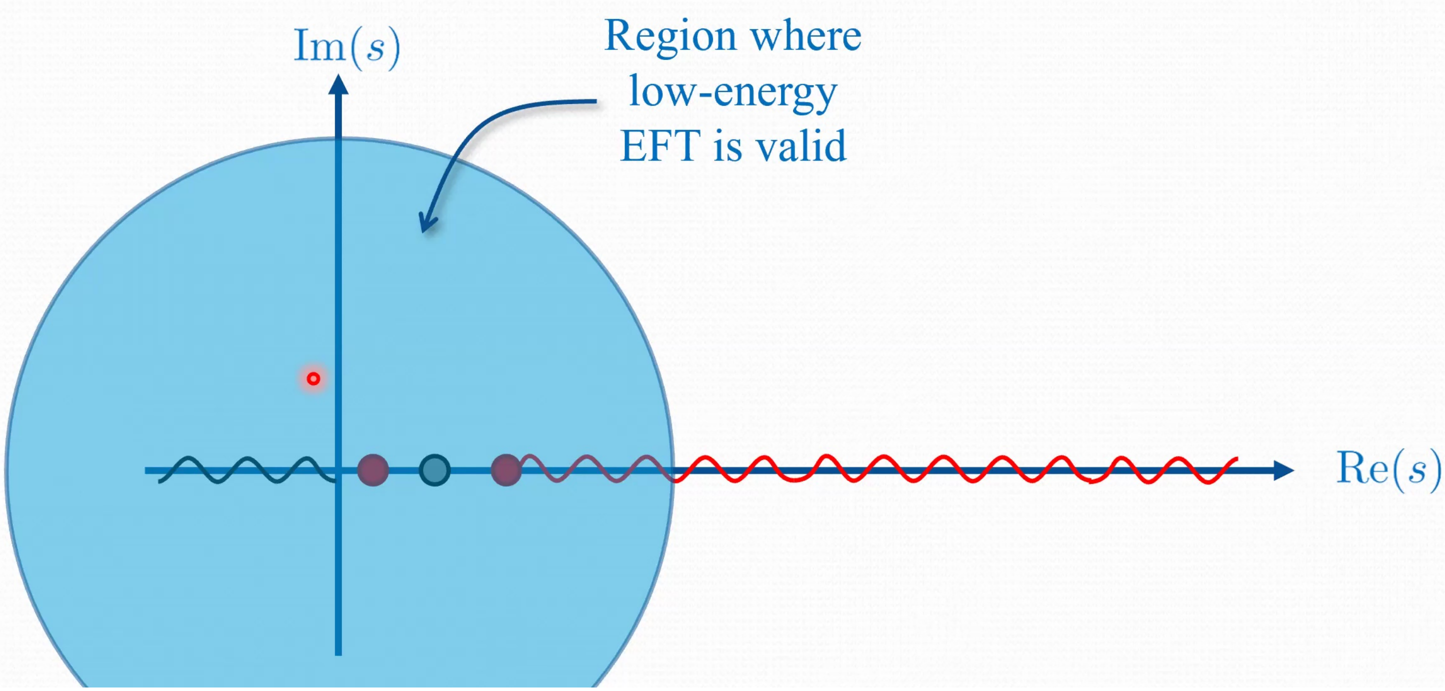

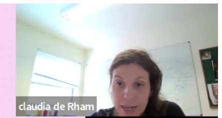

Low-energy Scattering

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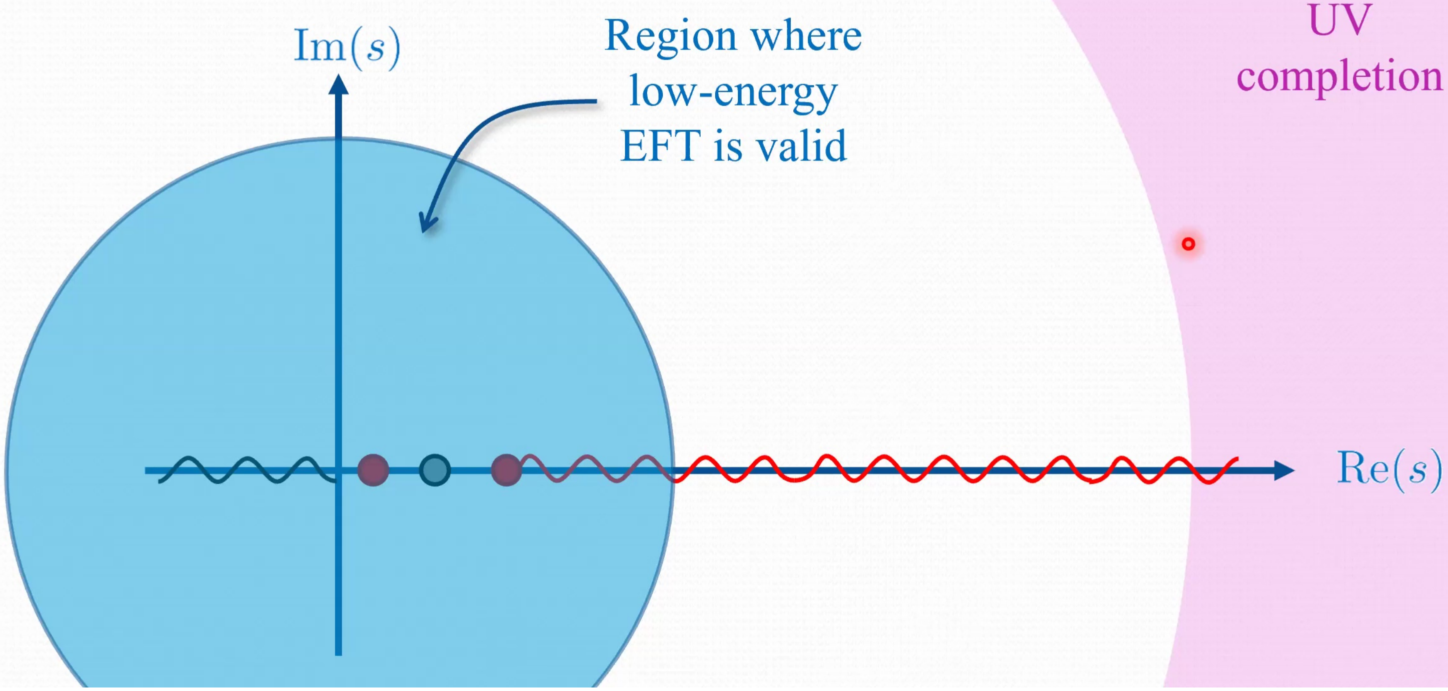
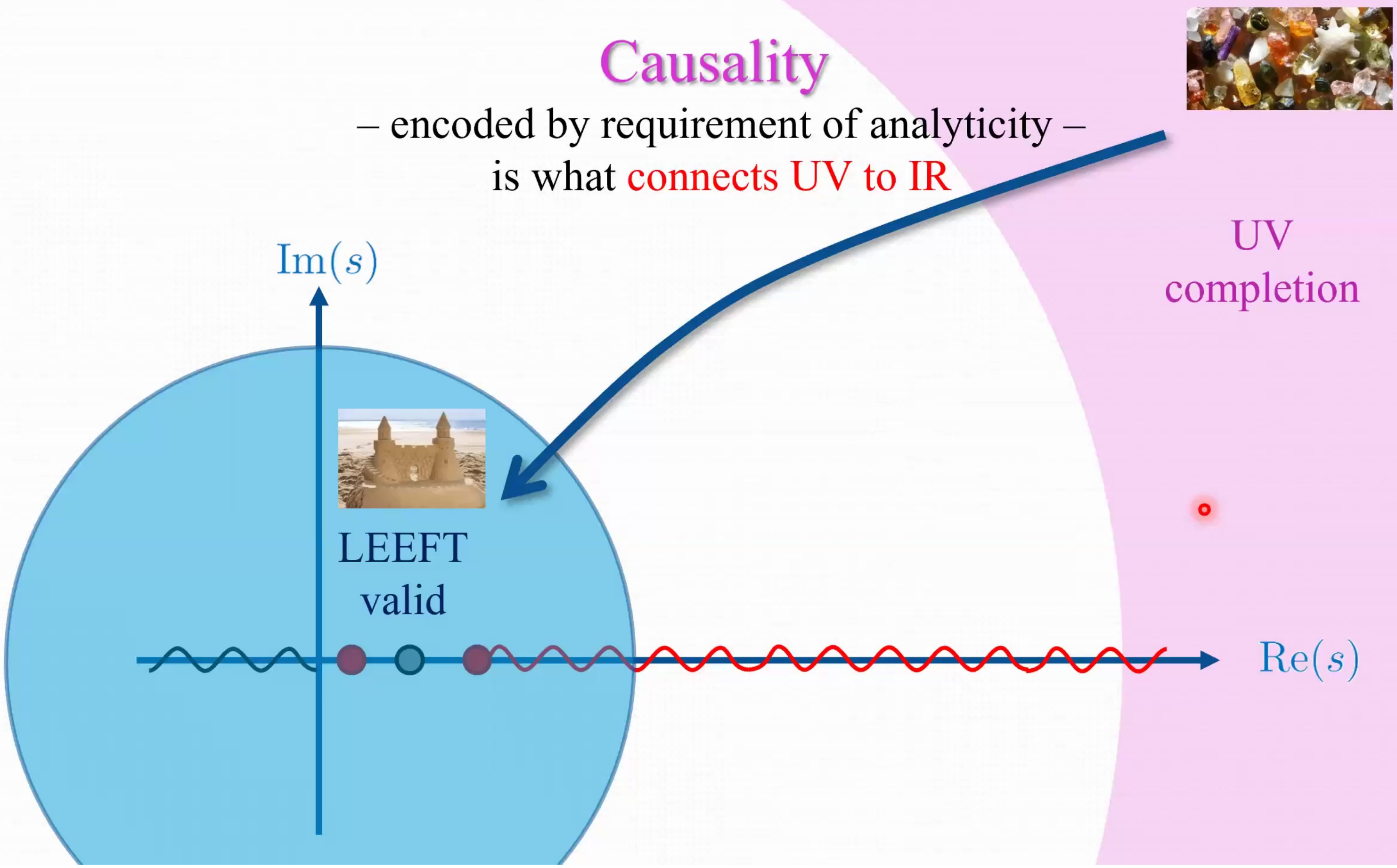
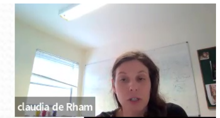


Analyticity - Causality

Causality

– encoded by requirement of analyticity –
is what connects UV to IR

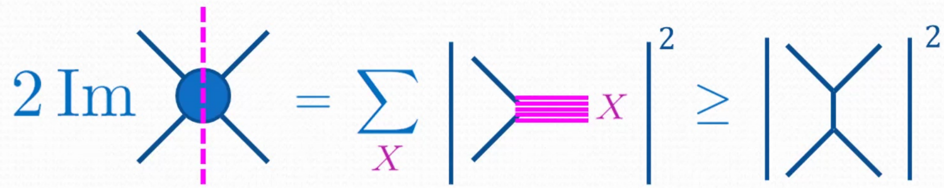


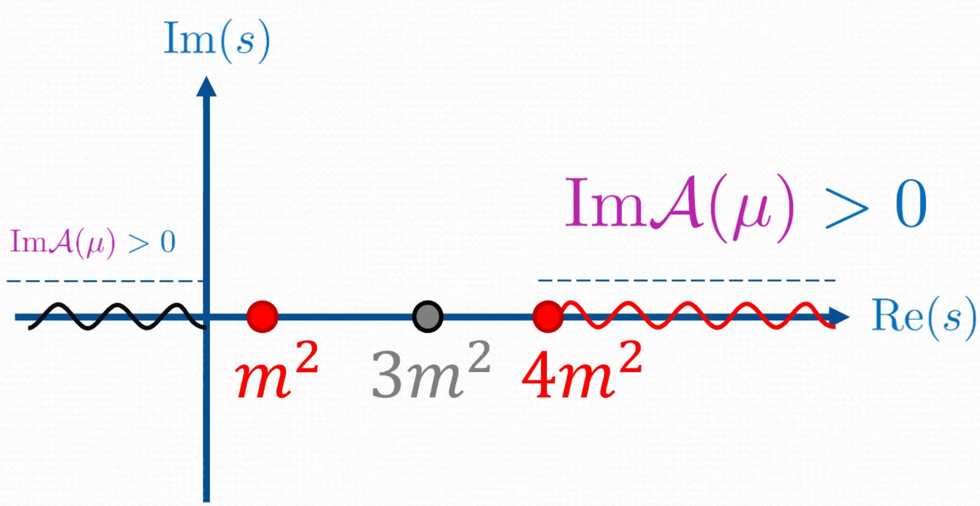


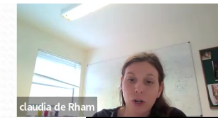
Non-Gravitational EFT

UV completion
- ✓ Unitary (optical theorem)
- ✓ Lorentz invariant (crossing symmetry)

$\mathcal{A}$: 2 – 2 elastic scattering amplitude





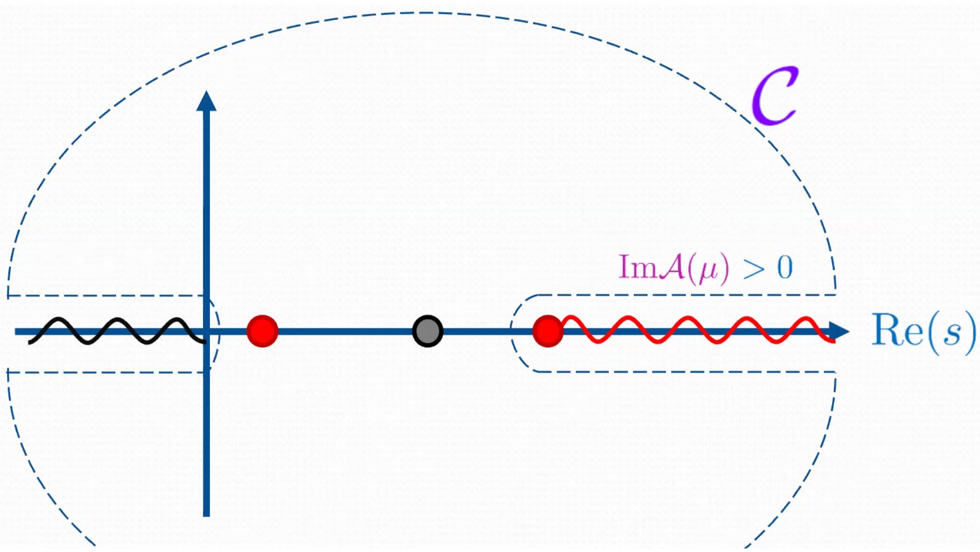


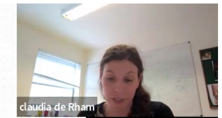
Non-Gravitational EFT

UV completion

- ✓ Unitary (optical theorem)
- ✓ Lorentz invariant (crossing symmetry)
- ✓ CAUSAL (analyticity)

$$\mathcal{A}''(s)\big|_{t=0} = \oint_{\mathcal{C}} \frac{\mathcal{A}(\mu)}{(\mu - s)^3} + \text{poles} + \text{cross-symmetric}$$





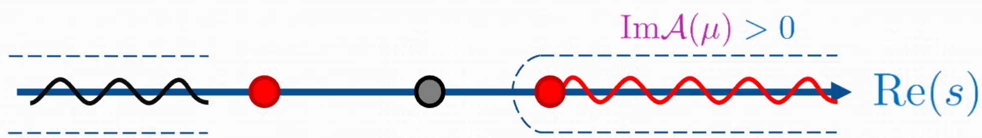
Non-Gravitational EFT

UV completion

- ✓ Unitary (optical theorem)
- ✓ Lorentz invariant (crossing symmetry)
- ✓ CAUSAL (analyticity)
- ✓ Local (Froissart Bound) $\lim_{|s|\to\infty} s^{-2}\mathcal{A}(s) = 0$

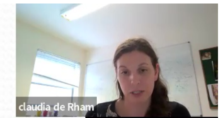
$$\mathcal{A}''(s)\Big|_{t=0} = \int_{4m^2}^{\infty} \frac{\mathrm{Im}\mathcal{A}(\mu)}{(\mu - s)^3} + \text{cross-symmetric}$$

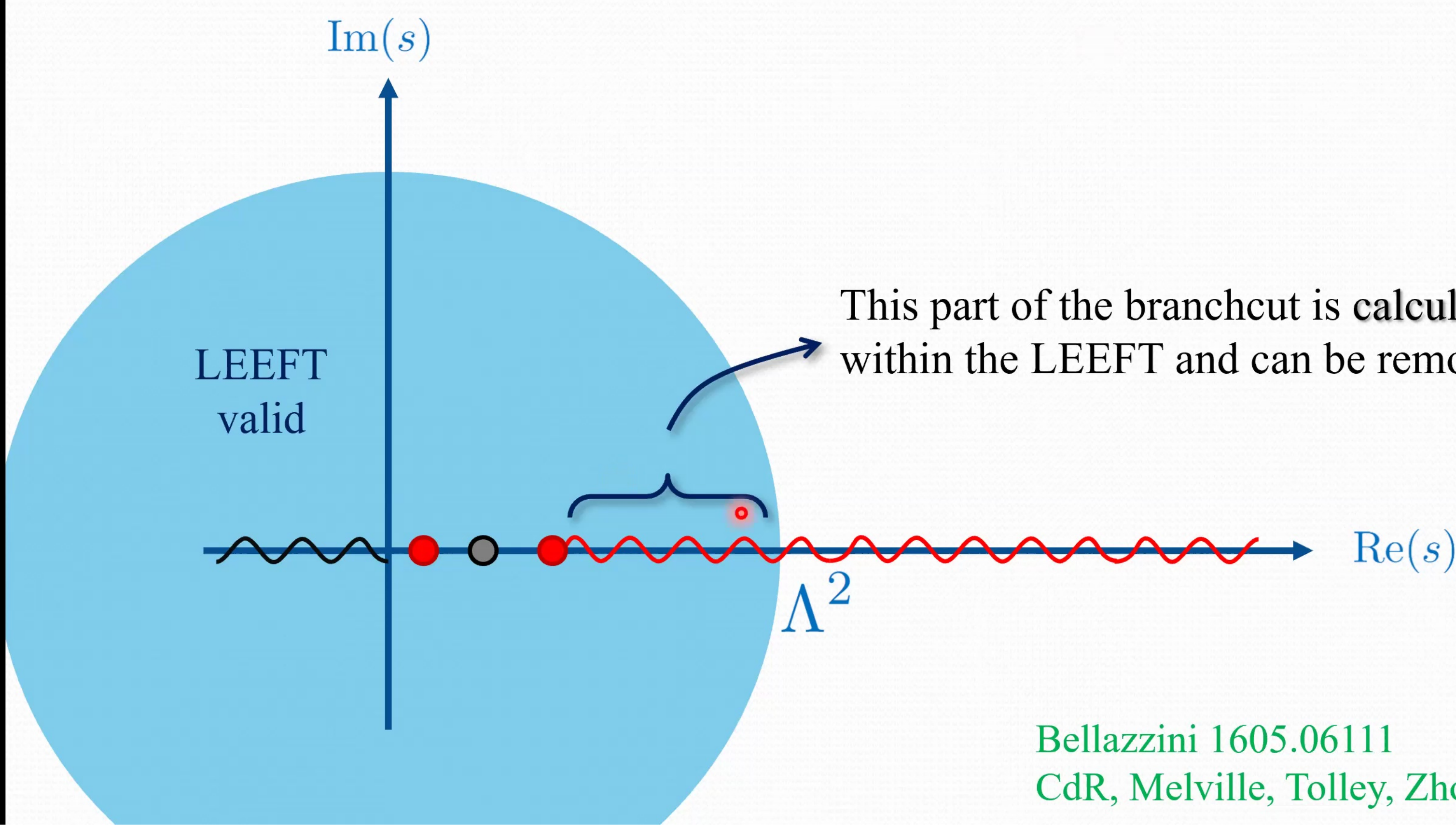
$$\left.\frac{\mathrm{d}^2\mathcal{A}(s,t)}{\mathrm{d}s^2}\right|_{t=0} > 0$$



Pham and Truong 1985
Ananthanarayan, Toublan and Wanders, 1994
Adams et. al. 2006

Improved Positivity Bounds



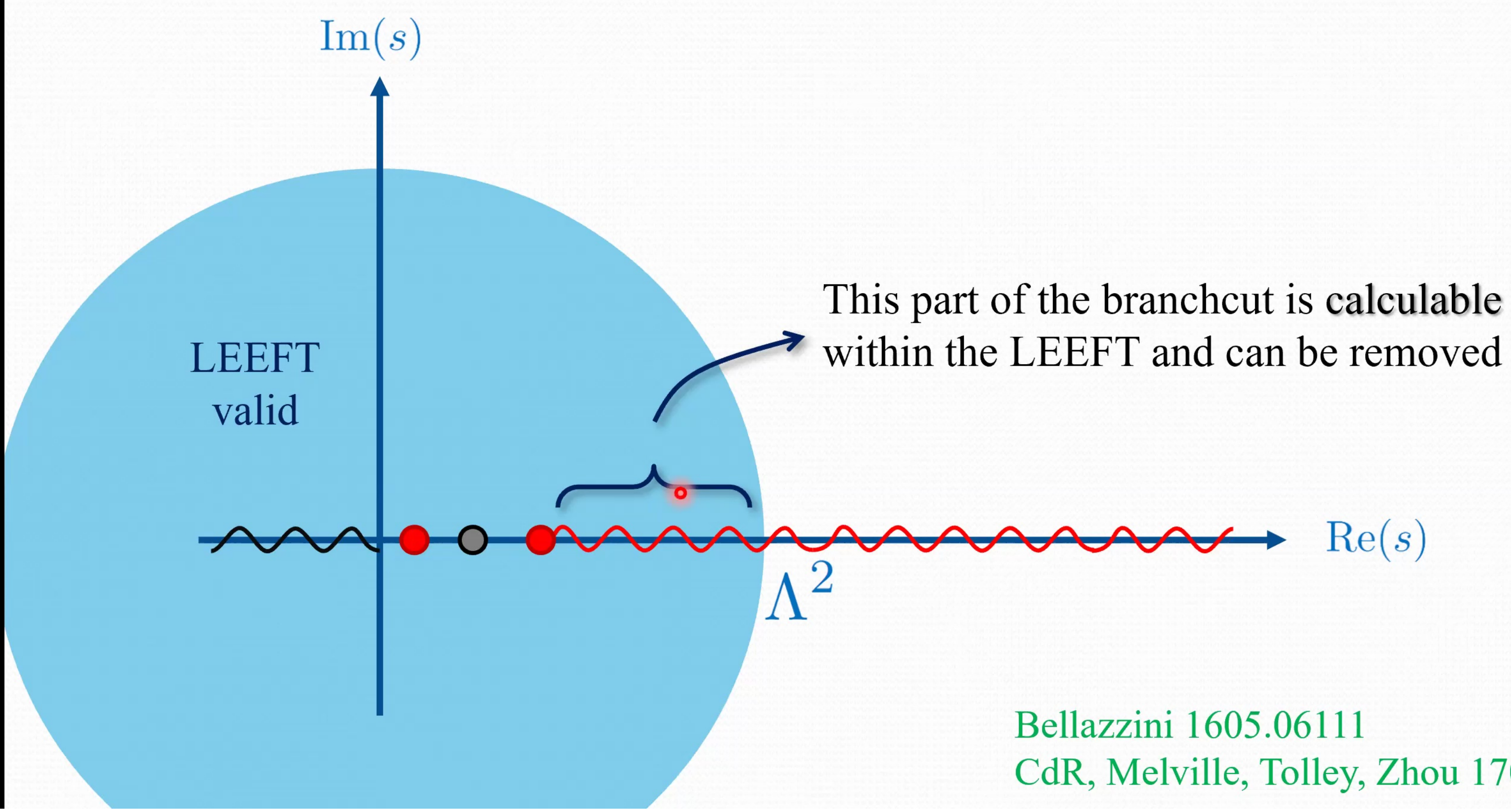


This part of the branchcut is **calculable** within the LEEFT and can be removed

Bellazzini 1605.06111
CdR, Melville, Tolley, Zhou 1702.08577

Improved Positivity Bounds

$$\mathcal{A}''(s) > \int_{4m^2}^{\Lambda^2} \mathrm{d}\mu \frac{\mathrm{Im}\mathcal{A}(\mu)}{(\mu - s)^3} + \text{cross-symmetric} > 0$$



Bellazzini 1605.06111
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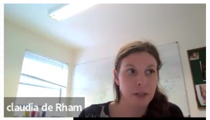
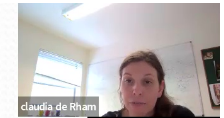
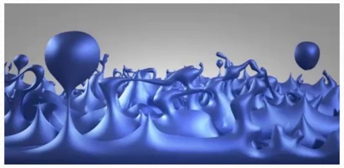




CoM Energy

Non-Gravitational EFT



UV completion

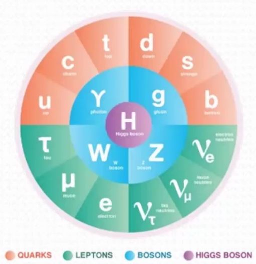
- ✓ Unitary (optical theorem)
- ✓ Lorentz invariant (crossing symmetry)
- ✓ CAUSAL (analyticity)
- ✓ Local (Froissart Bound) $\lim_{|s|\to\infty} s^{-2}\mathcal{A}(s)=0$

$$\frac{\partial^n}{\partial t^n}\mathrm{Im}\mathcal{A}(s,t)>0$$

positivity bounds

(applied to low-energy scattering amplitude)

Low-energy EFT



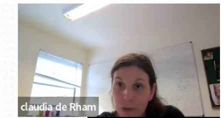
Pham and Truong 1985
Ananthanarayan, Toublan and Wanders, 1994
Adams et. al. 2006

$$\left.\frac{\mathrm{d}^2\mathcal{A}(s,t)}{\mathrm{d}s^2}\right|_{t=0}>0$$

+ infinite number of other bounds

CdR, Melville, Tolley & Zhou, 1702.06134 + …

+ new fully crossing ones from Arkani-Hamed et.al., Bellazzini et.al. & Tolley et.al.



3. Dim-8 operators in VBS

The SMEFT Lagrangian is given by

$$\mathscr{L}_{\text{SMEFT}} = \mathscr{L}_{\text{SM}} + \sum_i \frac{c_i \mathscr{O}_i^{(6)}}{\Lambda^2} + \sum_i \frac{f_i \mathscr{O}_i^{(8)}}{\Lambda^4} + \cdots, \tag{3.1}$$

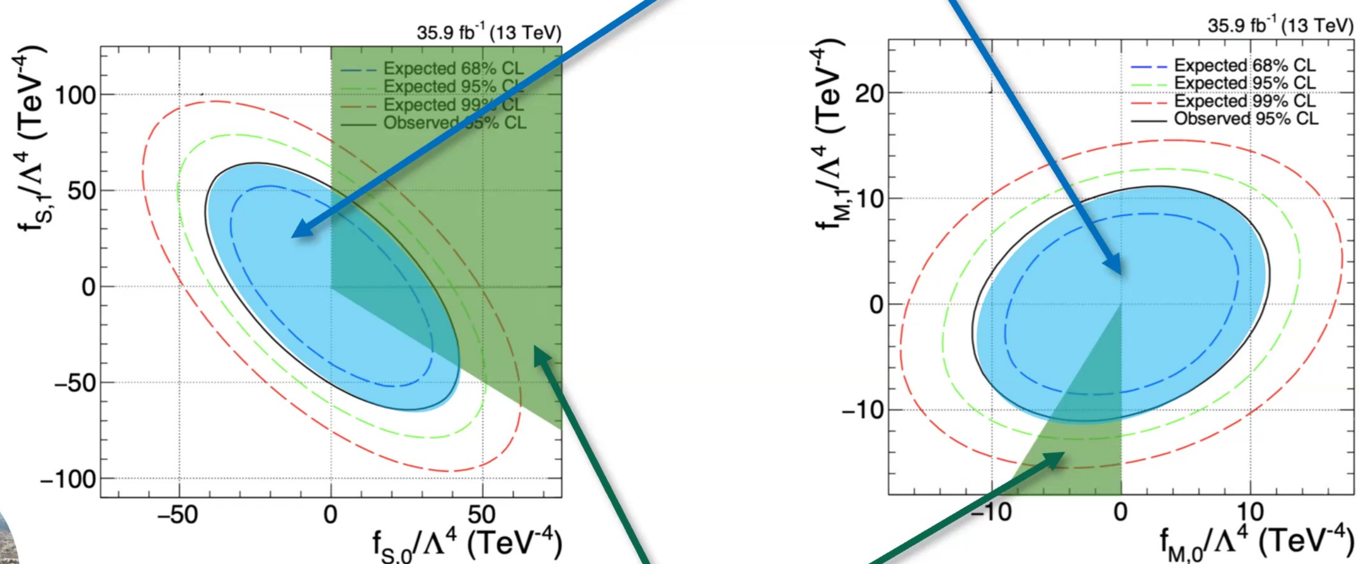
where Λ is the cutoff and $\mathscr{L}_{\text{SM}}$ is the Standard Model Lagrangian. SMEFT shares the same particle content, gauge group structure and global symmetries as the Standard Model. We focus on vector boson scattering, and the relevant dim-8 operators are [6]

$$\begin{aligned}
&\mathscr{O}_{S,0} = [(D_\mu\Phi)^\dagger D_\nu\Phi][(D^\mu\Phi)^\dagger D^\nu\Phi] && \\
&\mathscr{O}_{S,1} = [(D_\mu\Phi)^\dagger D^\mu\Phi][(D_\nu\Phi)^\dagger D^\nu\Phi] && \mathscr{O}_{T,0} = \mathrm{Tr}\left[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\right]\mathrm{Tr}\left[\hat{W}_{\alpha\beta}\hat{W}^{\alpha\beta}\right] \\
&\mathscr{O}_{S,2} = [(D_\mu\Phi)^\dagger D_\nu\Phi][(D^\nu\Phi)^\dagger D^\mu\Phi] && \mathscr{O}_{T,1} = \mathrm{Tr}\left[\hat{W}_{\alpha\nu}\hat{W}^{\mu\beta}\right]\mathrm{Tr}\left[\hat{W}_{\mu\beta}\hat{W}^{\alpha\nu}\right] \\
&\mathscr{O}_{M,0} = \mathrm{Tr}\left[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\right]\left[(D_\beta\Phi)^\dagger D^\beta\Phi\right] && \mathscr{O}_{T,2} = \mathrm{Tr}\left[\hat{W}_{\alpha\mu}\hat{W}^{\mu\beta}\right]\mathrm{Tr}\left[\hat{W}_{\beta\nu}\hat{W}^{\nu\alpha}\right] \\
&\mathscr{O}_{M,1} = \mathrm{Tr}\left[\hat{W}_{\mu\nu}\hat{W}^{\nu\beta}\right]\left[(D_\beta\Phi)^\dagger D^\mu\Phi\right] && \mathscr{O}_{T,5} = \mathrm{Tr}\left[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\right]\hat{B}_{\alpha\beta}\hat{B}^{\alpha\beta} \\
&\mathscr{O}_{M,2} = \left[\hat{B}_{\mu\nu}\hat{B}^{\mu\nu}\right]\left[(D_\beta\Phi)^\dagger D^\beta\Phi\right] && \mathscr{O}_{T,6} = \mathrm{Tr}\left[\hat{W}_{\alpha\nu}\hat{W}^{\mu\beta}\right]\hat{B}_{\mu\beta}\hat{B}^{\alpha\nu} \\
&\mathscr{O}_{M,3} = \left[\hat{B}_{\mu\nu}\hat{B}^{\nu\beta}\right]\left[(D_\beta\Phi)^\dagger D^\mu\Phi\right] && \mathscr{O}_{T,7} = \mathrm{Tr}\left[\hat{W}_{\alpha\mu}\hat{W}^{\mu\beta}\right]\hat{B}_{\beta\nu}\hat{B}^{\nu\alpha} \\
&\mathscr{O}_{M,4} = \left[(D_\mu\Phi)^\dagger\hat{W}_{\beta\nu}D^\mu\Phi\right]\hat{B}^{\beta\nu} && \mathscr{O}_{T,8} = \hat{B}_{\mu\nu}\hat{B}^{\mu\nu}\hat{B}_{\alpha\beta}\hat{B}^{\alpha\beta} \\
&\mathscr{O}_{M,5} = \tfrac{1}{2}\left[(D_\mu\Phi)^\dagger\hat{W}_{\beta\nu}D^\nu\Phi\right]\hat{B}^{\beta\mu} + hc && \mathscr{O}_{T,9} = \hat{B}_{\alpha\mu}\hat{B}^{\mu\beta}\hat{B}_{\beta\nu}\hat{B}^{\nu\alpha}, \\
&\mathscr{O}_{M,7} = \left[(D_\mu\Phi)^\dagger\hat{W}_{\beta\nu}\hat{W}^{\beta\mu}D^\nu\Phi\right] &&
\end{aligned} \tag{3.2}$$

where $\hat{W}^{\mu\nu} \equiv ig\frac{\sigma^I}{2}W^{I,\mu\nu}$, $\hat{B}^{\mu\nu} \equiv ig'\frac{1}{2}B^{\mu\nu}$ and the Higgs field should be expanded around the broken vacuum $\Phi = \left(0, \frac{v+h}{\sqrt{2}}\right)^T$.

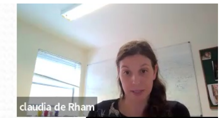
e.g. Positivity bounds on dim-8 SMEFT operators

Regions compatible with data



Regions that can enjoy a standard high energy completion

Zhang&Zhou, 2018, Zhou 2019,2020



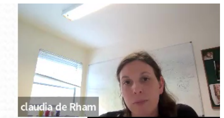
Gravitational EFTs

- ✓ Unitary (optical theorem)
- ✓ Lorentz invariant (crossing symmetry)
- ✓ CAUSAL (analyticity)
- ✓ Local (Froissart Bound)

positivity bounds

(sub)luminal
sound speed

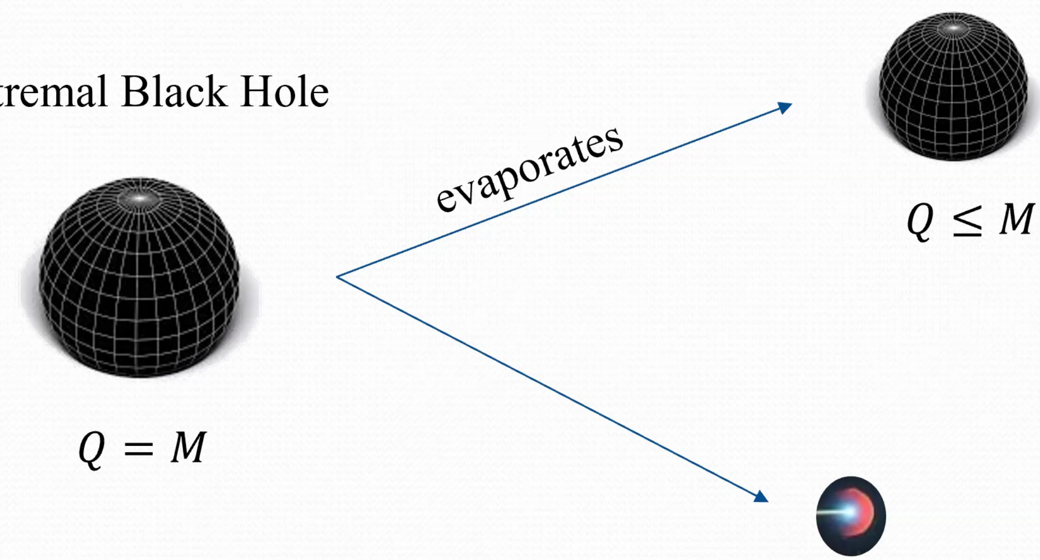
Surely all these subtleties are
suppressed…
Why should I care?



Motivation 1: Weak Gravity Conjecture

Consider a new BSM U(1) $\mathcal{L} = \frac{M_{\rm Pl}^2}{2} R - \frac{1}{4} F_{\mu\nu}^2$

Extremal Black Hole



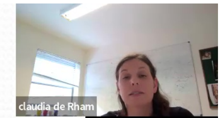
$Q = M$

$Q \leq M$ (to avoid naked singularity)

(in units of $\sqrt{2}$ Mpl)

For every $U(1)$, there must be a massive charged particle with $Q \geq M$

Arkani-Hamed, Motl, Nicolis, Cumrun, 2007



U(1) EFT

$$\mathcal{L} = \frac{M_{\rm Pl}^2}{2} R - \frac{1}{4} F_{\mu\nu}^2 + a \frac{q^4}{m^4} \left(F_{\mu\nu}^2\right)^2 + b \frac{2q^2}{m^2} F_{\mu\nu} F_{\alpha\beta} W^{\mu\alpha\nu\beta} + \cdots$$

Redresses BH mass and charge

New BH extremality condition is: $\left(\frac{\sqrt{2}|Q|}{M/M_{\rm Pl}}\right)_{\rm extr} = 1 + \frac{|\#|}{M^2/M_{\rm Pl}^2} \mathcal{C}(a,b) + \cdots$

If $\mathcal{C}(a,b) > 0$ ➡ Extremal BHs can evaporate *without* leading to naked singularity and *without* postulating a massive charged particle with $Q \geq M$ (other than BHs themselves).

Do positivity bounds impose $C > 0$?

Kats, Motl & M. Padi, hep-th/0606100
Cheung & Remmen, 1407.7865
Hamada, Noumi & Shiu, 1810.03637

Motivation 2: EFT of Gravity

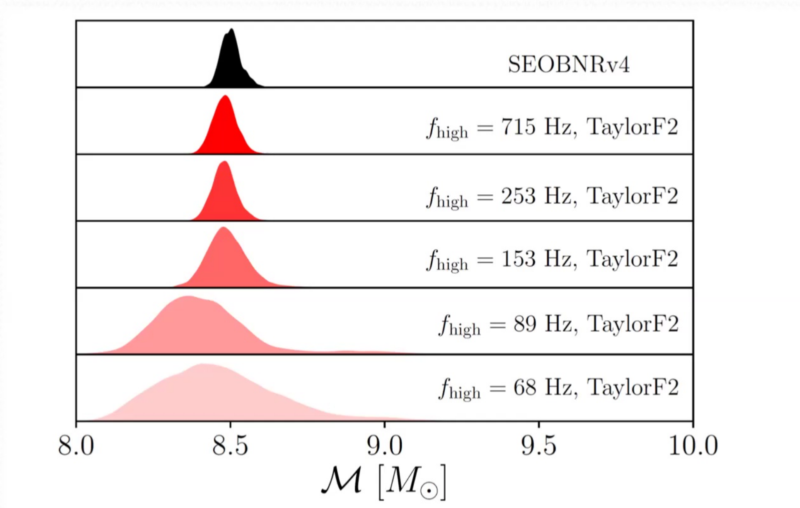
Interesting GWs constraints in dim-8 EFT

$$S_{\text{eff}} = 2M_{\text{pl}}^2 \int d^4x \sqrt{-g}\left(-R + \frac{C^2}{\Lambda^6} + \frac{\tilde{C}^2}{\tilde{\Lambda}^6} + \frac{C\tilde{C}}{\Lambda_-^6}\right) + \dots ,$$

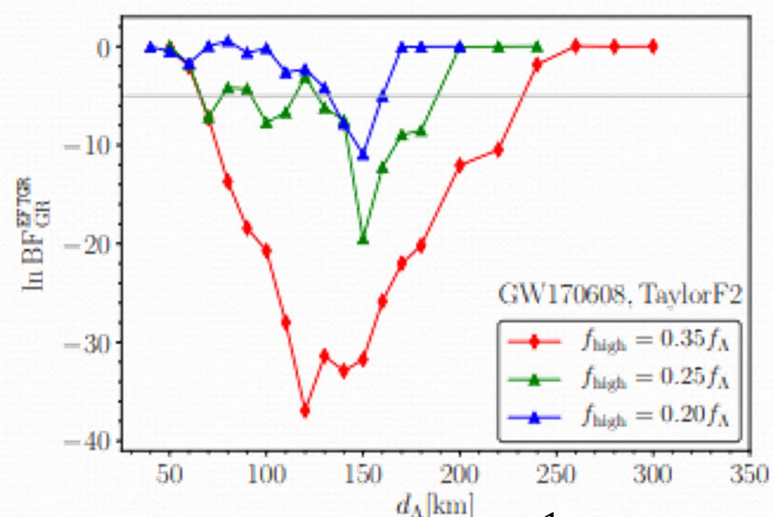
$$C \equiv R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta} ,$$

$$\tilde{C} \equiv R_{\alpha\beta\gamma\delta}\, \epsilon^{\alpha\beta}{}_{\mu\nu}\, R^{\mu\nu\gamma\delta} ,$$

For GW170608, with corresponding cutoff of $\Lambda \sim 10^{-13}$eV



chirp mass posterior density distribution



Λ^{-1}

Sennett, Brito, Buonanno, Gorbenko, Senatore, 1912.09917

Motivation 3: Constraining Models of DE

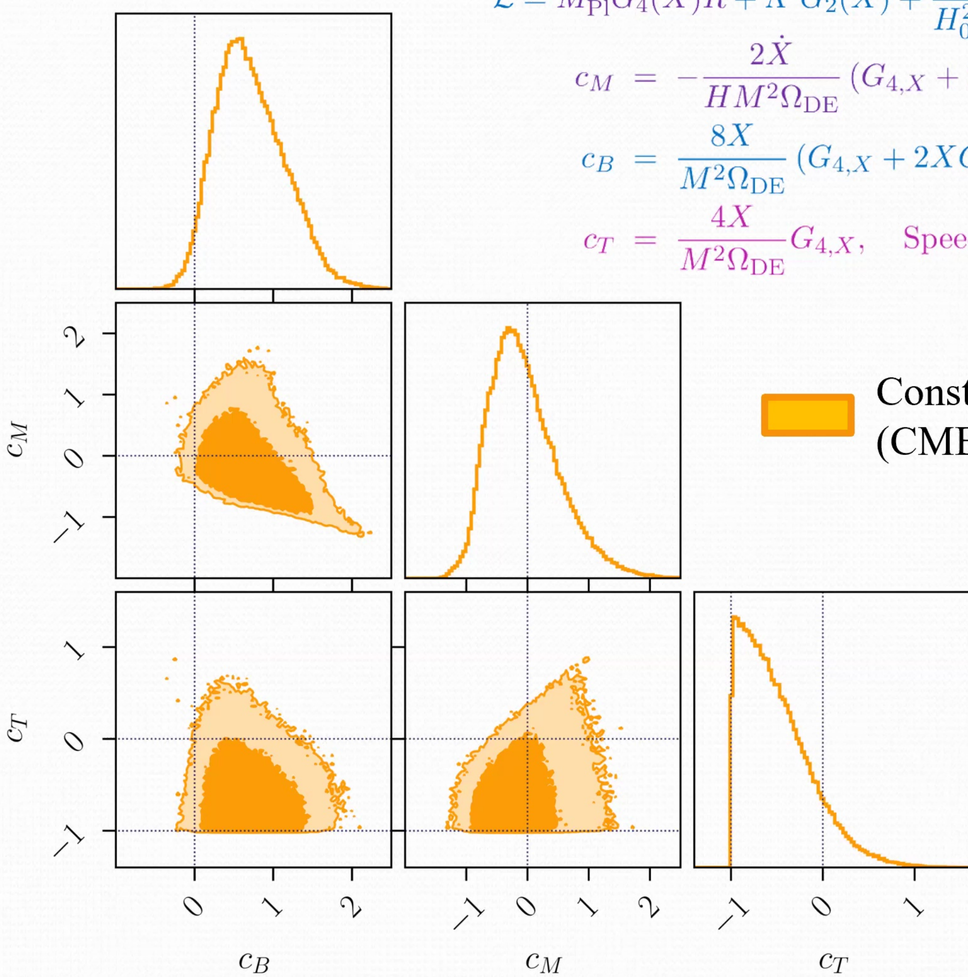
Example of DE/MG: quartic Horndeski with parameters $c_{B,M,T}$

$$\mathcal{L} = M_{\rm Pl}^2 G_4(X) R + \Lambda^4 G_2(X) + \frac{1}{H_0^2} G_{4,X}(X)\left((\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right) + \mathcal{L}_{\rm matter}(\psi, g)$$

$$c_M = -\frac{2\dot{X}}{HM^2\Omega_{\rm DE}}\left(G_{4,X} + 2XG_{4,XX}\right), \quad \text{Effective Planck Mass}$$

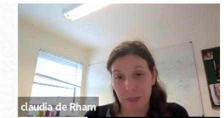
$$c_B = \frac{8X}{M^2\Omega_{\rm DE}}\left(G_{4,X} + 2XG_{4,XX}\right), \quad \text{"Braiding" (scalar/tensor mixing)}$$

$$c_T = \frac{4X}{M^2\Omega_{\rm DE}} G_{4,X}, \quad \text{Speed of GWs} - 1 \qquad M^2 = 2\left(G_4 - 2XG_{4,X}\right)$$



Constraints from observations (CMB, BAO, redshift space distortion,…)

CdR, Melville and Noller 2103.06855



Motivation 3: Constraining Models of DE

Example of DE/MG:
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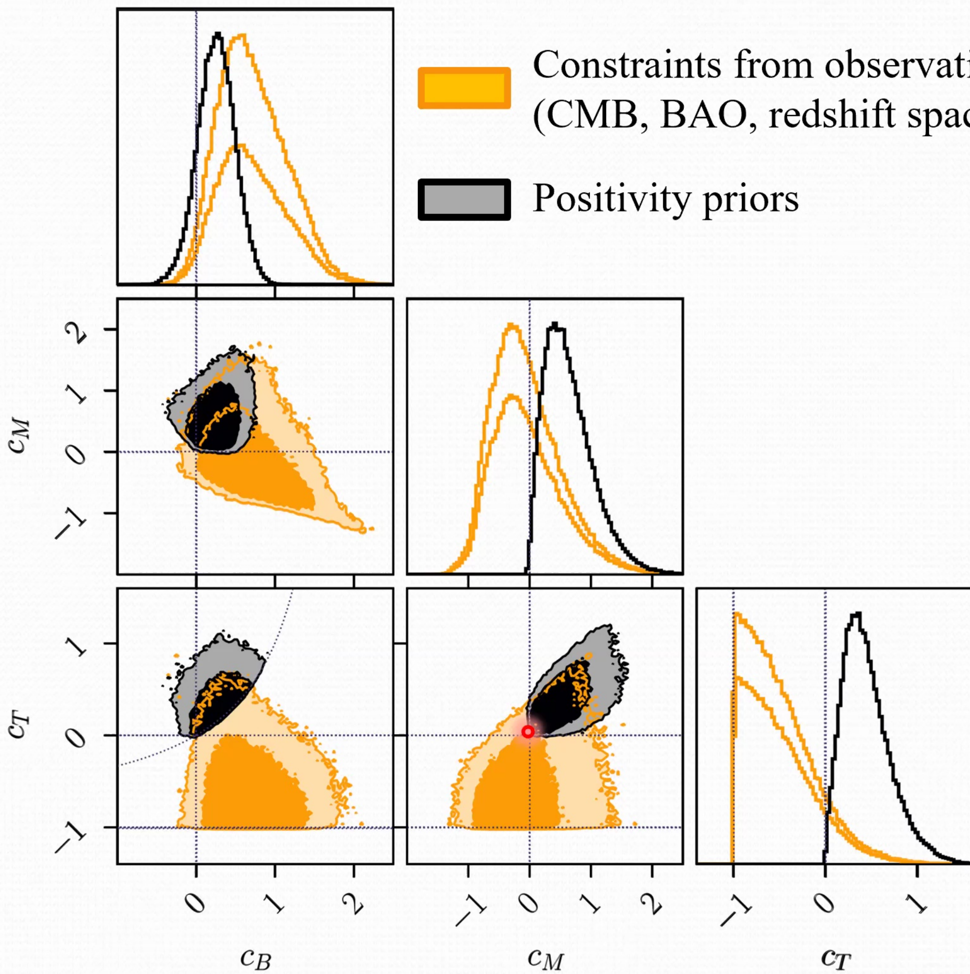


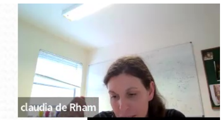
Constraints from observations (CMB, BAO, redshift space distortion,…)

Positivity priors

Note: To be taken with a pinch of salt

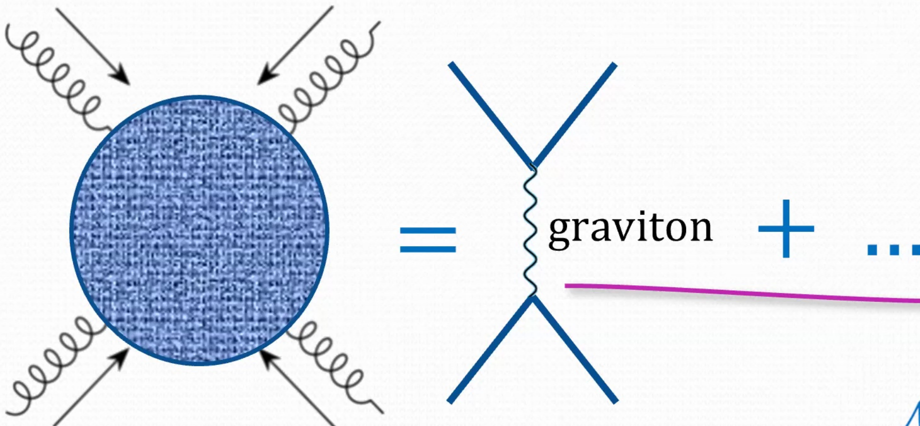
still a lot assumptions been made…

CdR, Melville and Noller 2103.06855



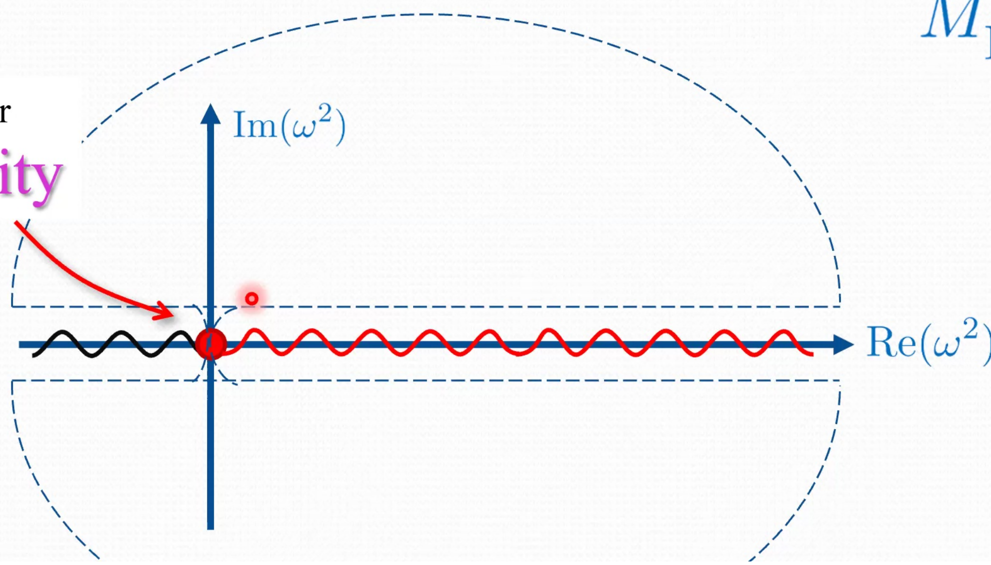
Unitarity + Causality with Gravity

Graviton exchange spoils analyticity



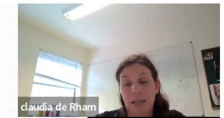
$$\mathcal{A} \sim -\frac{s^2}{M_{\rm Pl}^2 t} + \frac{\mathcal{C}\, s^2}{m^4}$$

no room for **analyticity**



$\mathcal{C} > 0$???

Unitarity + Causality with Gravity



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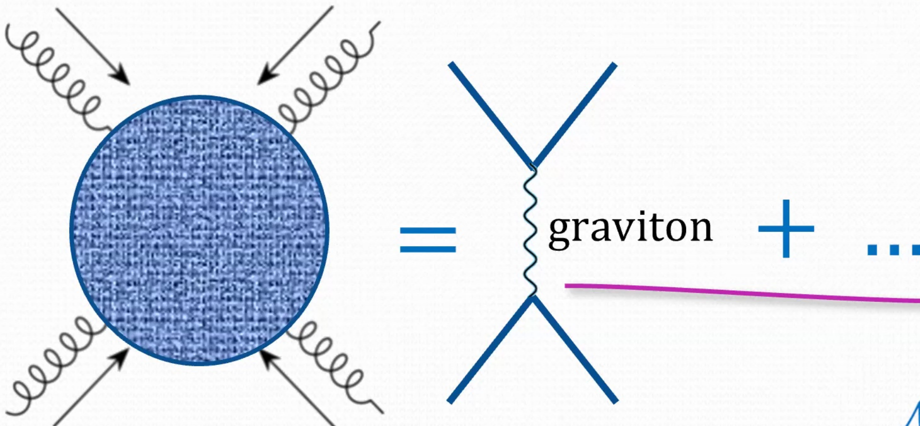

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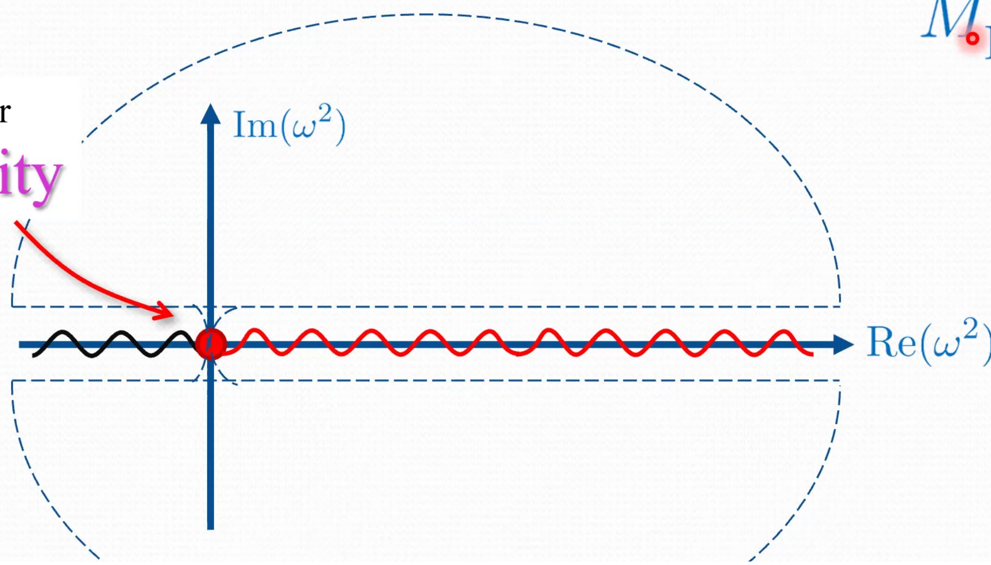


$\mathcal{C} > 0$???

Gravitational bounds

$$\mathcal{A} \sim -\frac{s^2}{M_{\rm Pl}^{D-2}\, t} + \mathcal{C}\frac{s^2}{m^D} + \cdots \quad \Longrightarrow \quad \cancel{\mathcal{C} > 0}$$

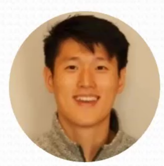
Conjectured (based on QED examples and causality arguments) that gravity generically allows for some mild negativity

$$\mathcal{C} \gtrsim -\left(\frac{m}{M_{\rm Pl}}\right)^{D-2}$$

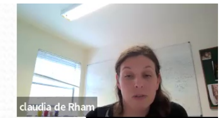
with Alberte, Jaitly and Tolley
2007.12667, 2012.05798

Consistent with results from "Sharp Boundaries for the Swampland"
Caron-Huot, Mazac, Rastelli & Simmons-Duffin, 2102.08951

Can be derived using "Infrared Causality",

Aoibheann Margalit  Calvin Chen  Andrew Tolley  2112.05031

arguments based on "Asymptotic Causality" would fail
see also See also CdR, Tolley & Zhang 2112.05054



Switch Gravity On

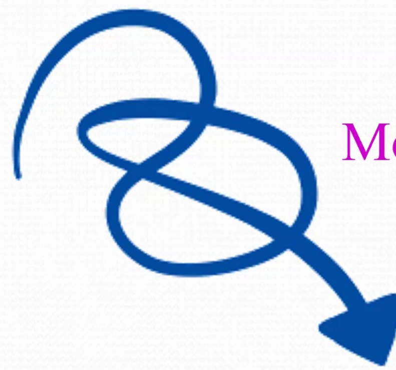
- ✓ Unitary
- ? ✓ Lorentz invariant (crossing symmetry)
- ? CAUSAL (analyticity)
- ? Local (Froissart-like Bound) $\lim_{|s|\to\infty} s^{-2}\mathcal{A}(s) = 0$

Causality ⟷ Analyticity ?

Massless loops ⟷ Gaplessness !

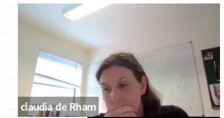
Massless spin-2 ⟷ Froissart-like bound ?

Massless spin-2 ⟷ t-channel pole !



More assumptions

Positivity-like bounds



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- Graviton loops are Mpl suppressed
- Spin<2 massless loops do not contribute the s^2 unless non-local IR-UV mixing

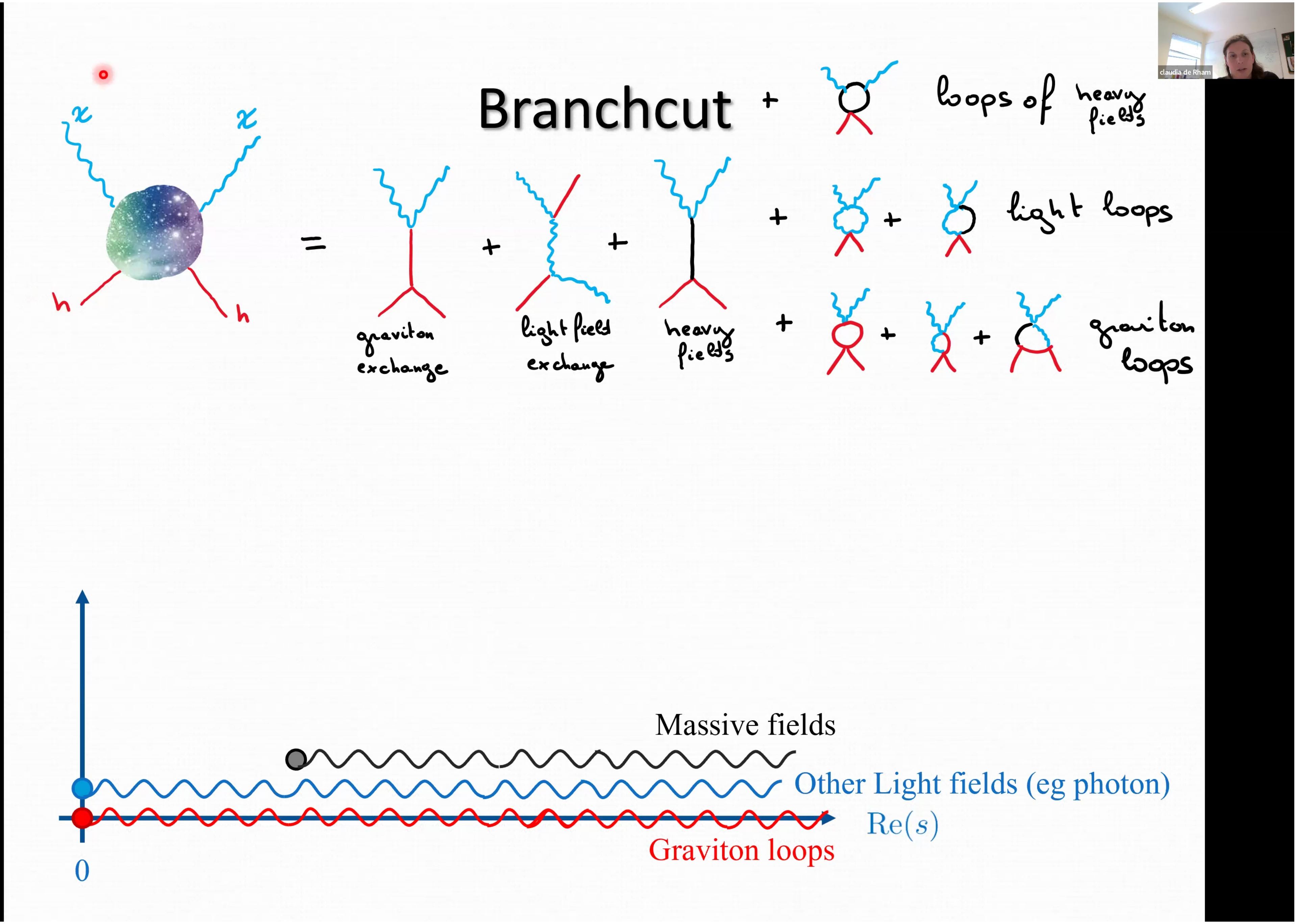
2. Amplitude respects Froissart-like Bound $\lim_{|s|\to\infty} s^{-2}\mathcal{A}(s) = 0$

- Expected to be valid for instance in string theory in $D \geq 5$
- For $D = 4$, IR divergences makes it more subtle

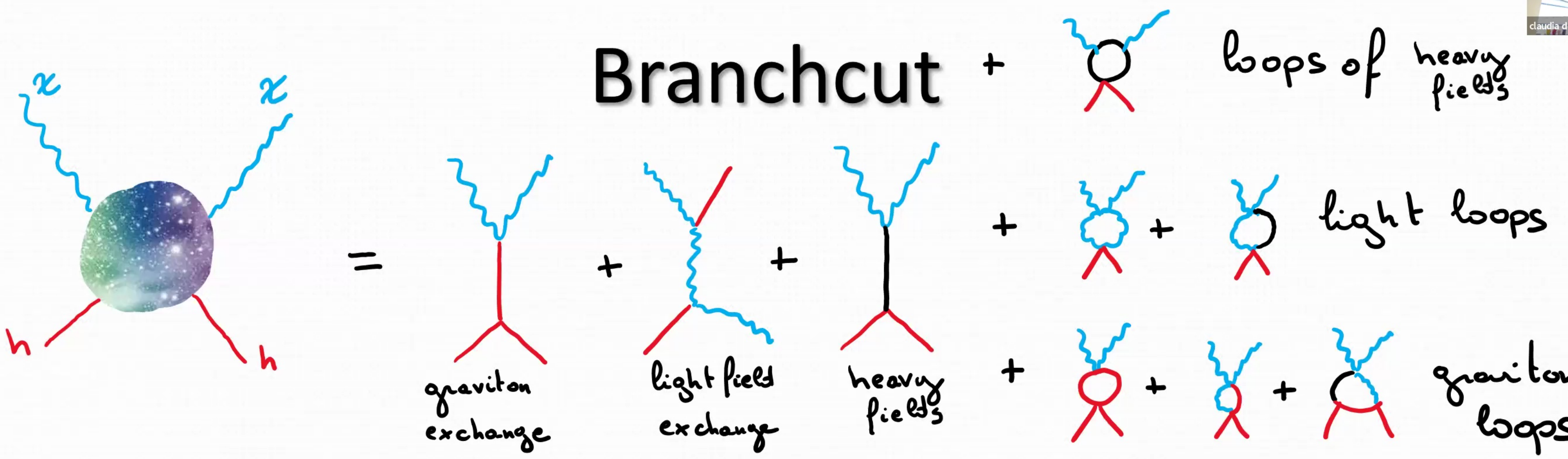
→ Regge behaviour $\lim_{s\to\infty} \mathrm{Disc}\,\mathcal{A}(s,t) = r(t)\Lambda_r^4 \left(\frac{s}{\Lambda_r^2}\right)^{\alpha(t)}$

- Arises in weakly coupled string theory
- Can be argued for more universally if amplitude satisfies a dispersion relation in physical region with 2 subtractions

3. Analyticity away from real s-axis, 4. Unitarity, 5. Crossing symmetry, …
in 1[st] Riemann sheet

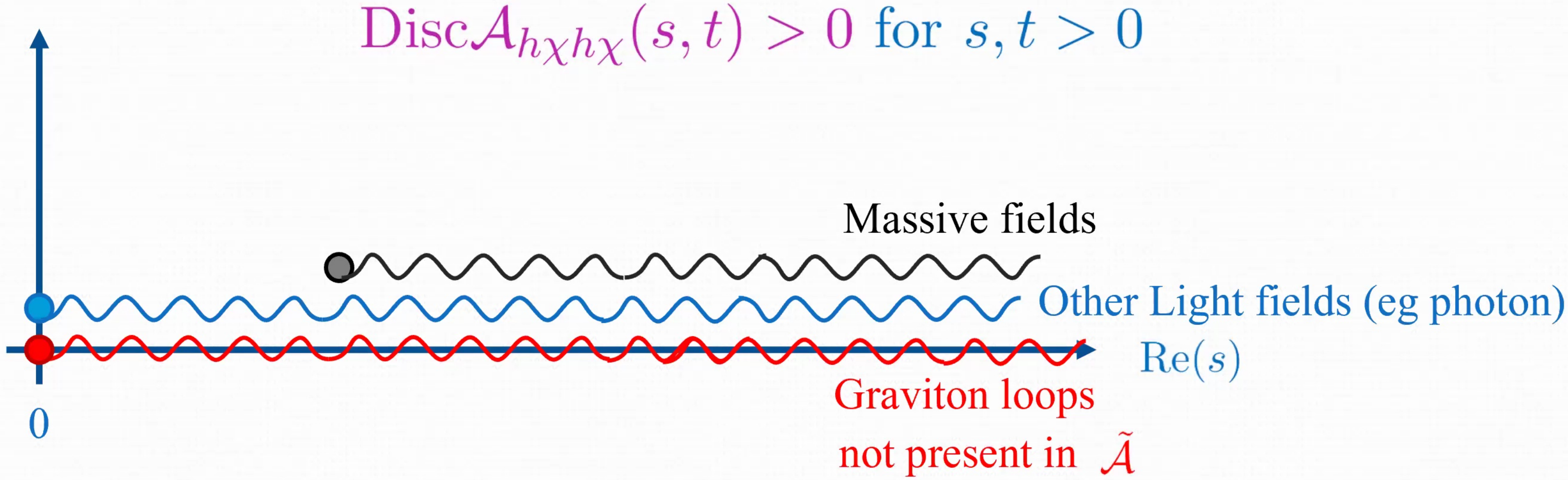

Branchcut
loops of heavy fields
light loops
graviton loops
graviton exchange
light field exchange
heavy fields
Massive fields
Other Light fields (eg photon)
Re(s)
Graviton loops
0
claudia de Rham

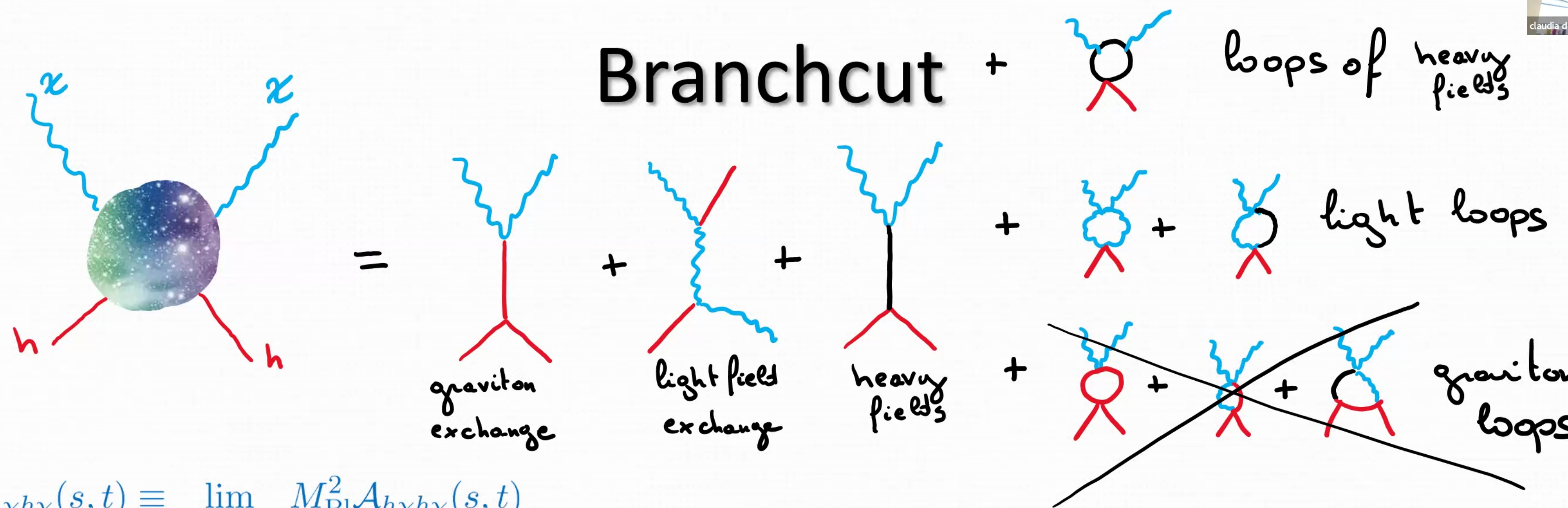
Branchcut



$$\tilde{\mathcal{A}}_{h\chi h\chi}(s,t) \equiv \lim_{M_{\rm Pl}\to\infty} M_{\rm Pl}^2 \mathcal{A}_{h\chi h\chi}(s,t)$$

$$\text{Disc}\tilde{\mathcal{A}}_{h\chi h\chi}(s,t) > 0 \text{ for } s,t>0$$





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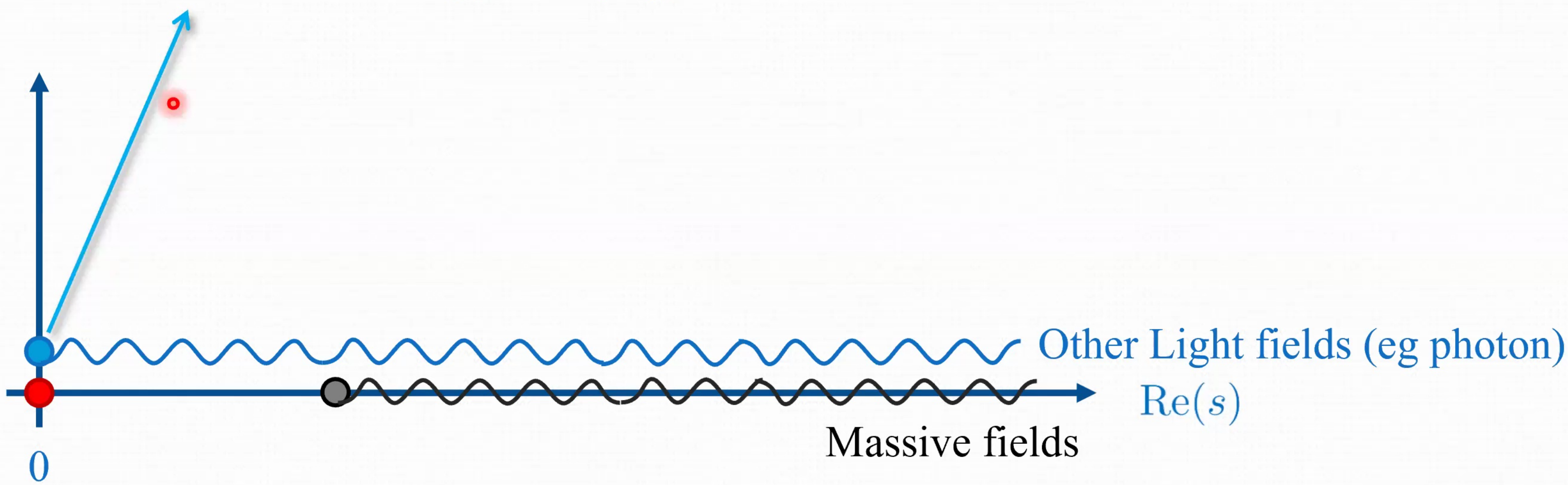
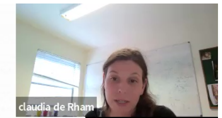
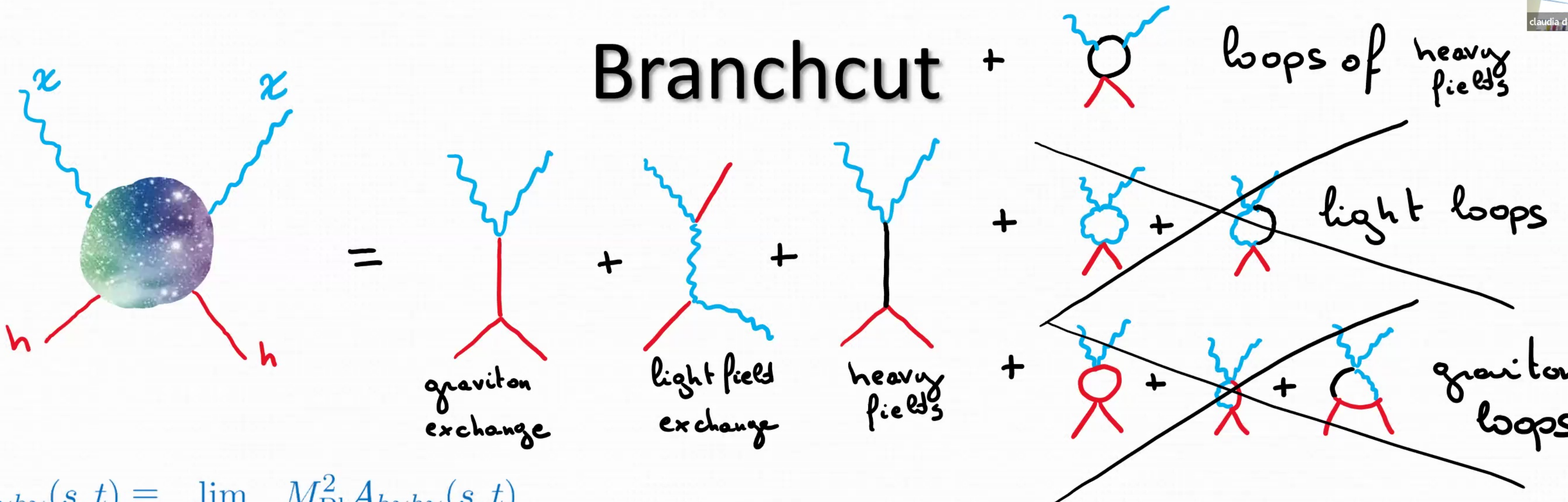
Poles from other (spin< 2) light fields are harmless





Branchcut



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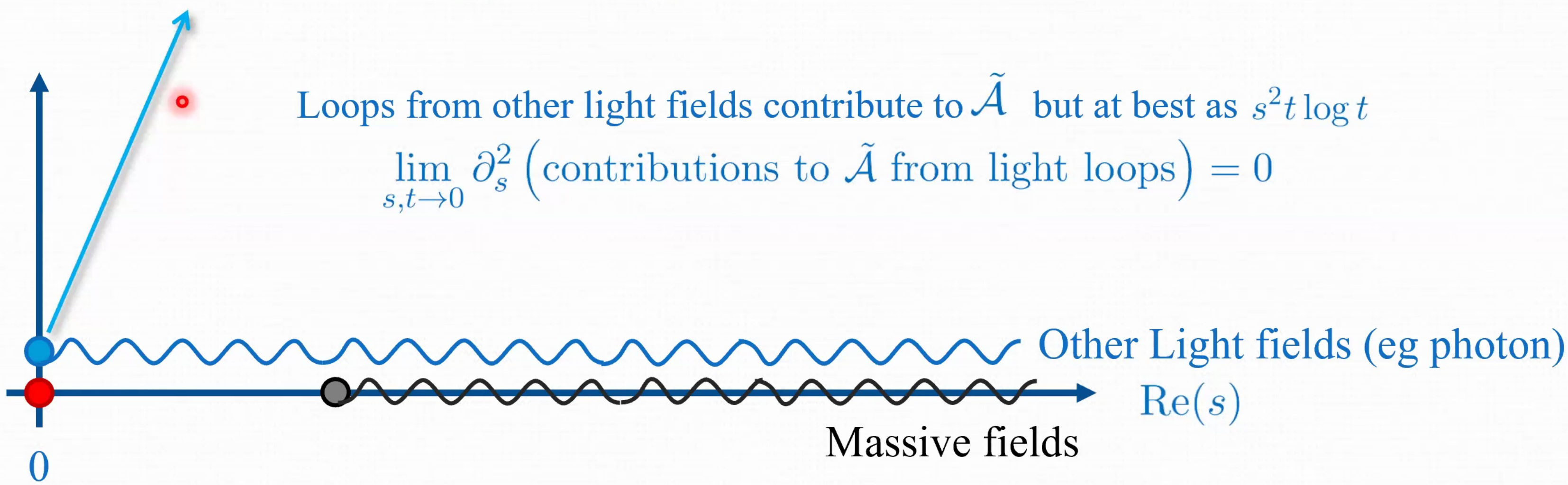
Loops from other light fields contribute to $\tilde{\mathcal{A}}$ but at best as $s^2 t \log t$

$$\lim_{s,t\to 0} \partial_s^2 \left(\text{contributions to } \tilde{\mathcal{A}} \text{ from light loops}\right) = 0$$



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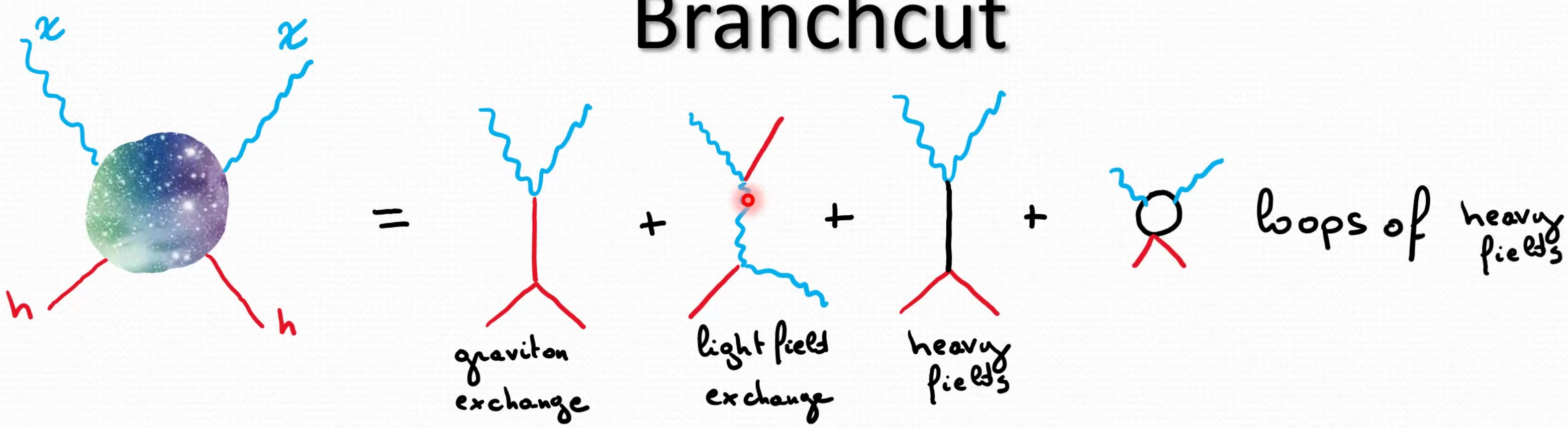

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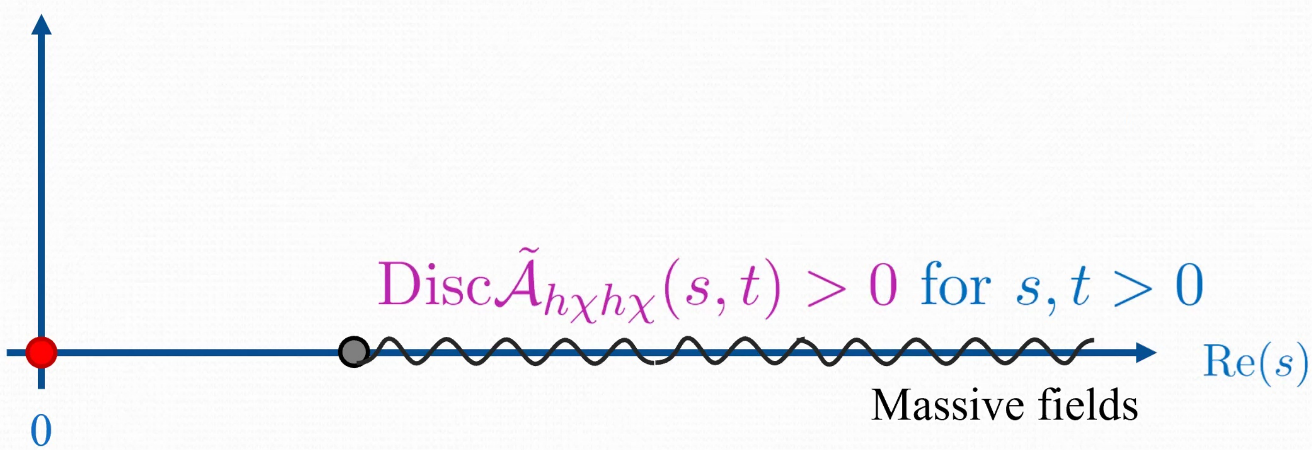

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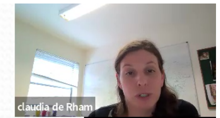


Two Subtractions

Amplitude should admit a dispersion relation with 3 subtractions

$$\lim_{|s|\to\infty} s^{-3}\tilde{\mathcal{A}} = 0 \text{ for } t<0 \text{ and } t>0$$

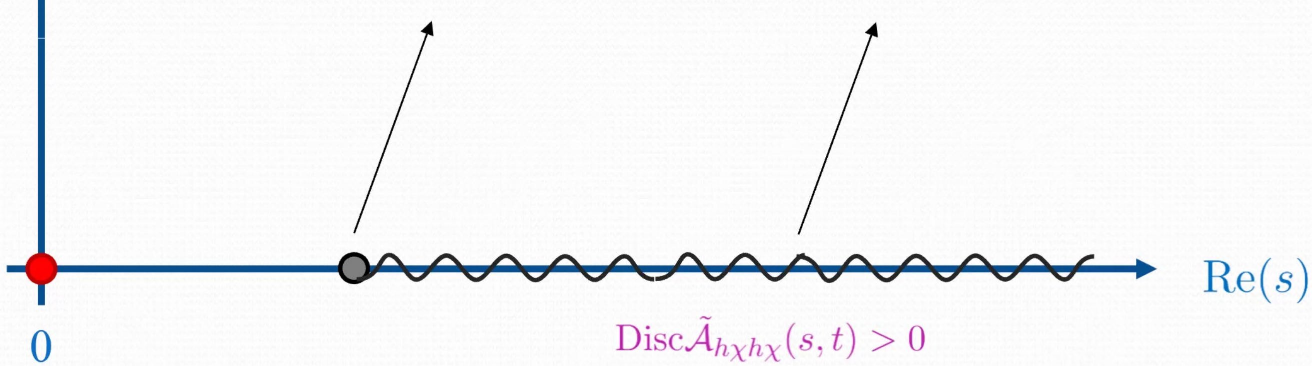
(assuming no massless loops)

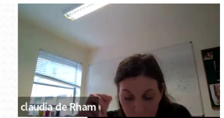
In $D \geq 5$, we would expect from perturbative string theory

$$\lim_{|s|\to\infty} s^{-2}\tilde{\mathcal{A}} = 0 \text{ for } t<0$$

take it as an assumption in $D = 4$

$$\tilde{\mathcal{A}}_s(s,t) = a_s(t) + b_s(t)s + \frac{s^2}{\pi}\int_0^\infty \mathrm{d}\mu \frac{\mathrm{Disc}\tilde{\mathcal{A}}_s(\mu,t)}{\mu^2(\mu-s)} + (s \leftrightarrow u) + (s,u \text{ poles})$$





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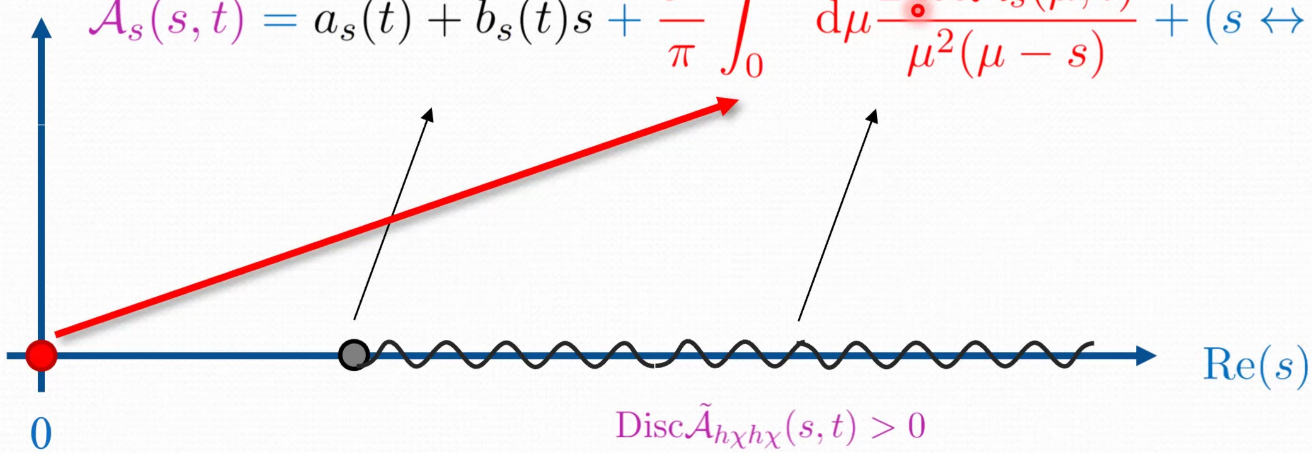
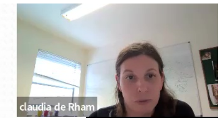




Regge behavior

$$\mathcal{A} = \frac{\lambda(t)}{m^2 - s} + s^2 \int_{4m^2}^{\infty} d\mu \, \frac{\mathrm{Im}\, A(\mu, t)}{\mu^2(\mu - s)} + s \to u$$

$\lambda(t) \sim a + bt + ct^2$

$$\lim_{t \to m^2} \sim s^2 \int_{4m^2}^{\infty} d\mu \, \frac{\mathrm{Im}\,(A(\mu, m^2))}{\mu^2(\mu - s)} \sim \frac{\lambda(s)}{m^2 - t}$$

$\lambda(s) \to a + bs + cs^2$

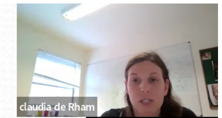
pole must come from $\int^{\infty} d\mu$

* $\underline{\text{for } t < m^2}$ $\quad \int^{\infty} d\mu$ finite $\Rightarrow \lim_{|\mu| \to \infty} \mathrm{Im}\, A < \mu^{②}$
* $\underline{\text{for } t = m^2}$ $\quad \int^{\infty} d\mu$ diverge $\quad \lim_{|\mu| \to \infty} \mathrm{Im}\, A \sim \mu^{②}$
* $\underline{\text{for } t > m^2}$ $\quad$ diverge but would converge with one extra subtraction $\quad \lim_{|\mu| \to \infty} \mathrm{Im}\, A < \mu^{③}$

Switch of power ≡ Regge behavior

Implicit as soon as assume Froissart bound.

about $t \sim m^2$, $\lim_{|\mu| \to \infty} \mathrm{Im}\, A(\mu, t) \sim \mu^{\alpha(t)}$, $\quad \alpha(t) \simeq \underbrace{\alpha(m^2)}_{=2} + (t - m^2)\,\alpha'(m^2)$



Regge Assumption for massless state

near $t = 0$

$$\lim_{\mu \to +\infty} \text{Disc}\, \tilde{A}_{s,u}(\mu, t) = r_{s,u}(t) \Lambda_r^4 \left(\frac{\mu}{\Lambda_r^2} \right)^{\alpha_{s,u}(t)}$$

$\Lambda_r \gg$ other scales

$r_{s,u}(t)$ residue of Regge pole

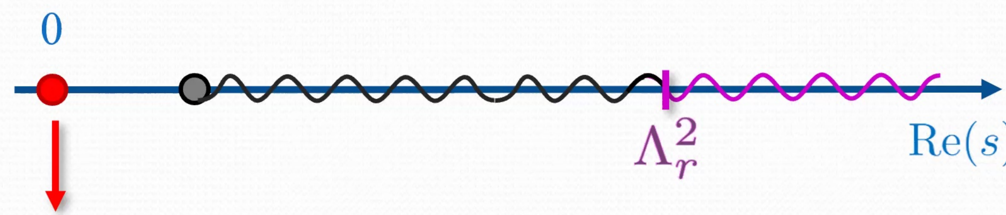
$\alpha_{s,u}(t) = 2 + \alpha'_{s,u} t + \cdots$

Regge slope

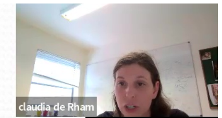
t −dependence of Regge residue and slope typically expected to enter at string scale M_* or scale $\gg$ SM physics

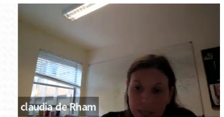
Subtracting Regge

$\alpha_{s,u}(t) = 2 + \alpha'_{s,u} t + \cdots$



$$\tilde{\mathcal{A}}_s(s,t) = a_s(t) + b_s(t)s + \frac{s^2}{\pi}\int_0^\infty \mathrm{d}\mu \frac{\mathrm{Disc}\tilde{\mathcal{A}}_s(\mu,t)}{\mu^2(\mu-s)} + (s \leftrightarrow u) + (s,u \text{ poles})$$





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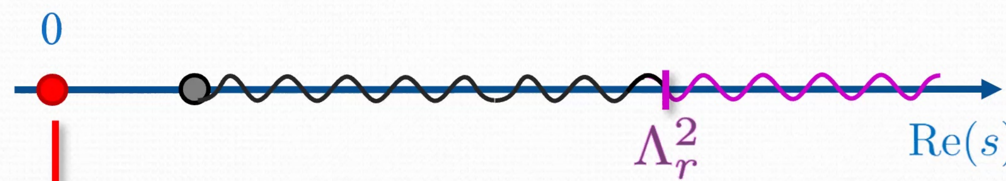

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3 subtractions

$$= \frac{s^3}{\pi}\int_0^\infty \mathrm{d}\mu \frac{\mathrm{Disc}\tilde{\mathcal{A}}_s}{\mu^3(\mu - s)} + \frac{s^2}{\pi}\int_0^\infty \mathrm{d}\mu \frac{\mathrm{Disc}\tilde{\mathcal{A}}_s}{\mu^3}$$

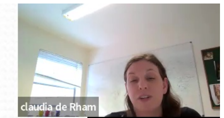
Finite & positive

Regge behaviour

$$\frac{s^2}{\pi}\int_0^{\Lambda_r^2} \mathrm{d}\mu \frac{\mathrm{Disc}\tilde{\mathcal{A}}_s}{\mu^3} + \frac{s^2}{\pi}\int_{\Lambda_r^2}^\infty \mathrm{d}\mu \frac{\mu^{\alpha_s(t)-3}}{\Lambda_r^{2\alpha_s(t)-4}} r_s(t)$$

$$= \frac{s^2}{\pi}\frac{r_s(t)}{2-\alpha_s(t)}$$

$$= -\frac{r_s(0)}{\pi\alpha'_s(0)} s^2 \left(\frac{1}{t} + -2(\ln r_s)' + (\ln \alpha'_s)' + \mathcal{O}(t)\right)$$



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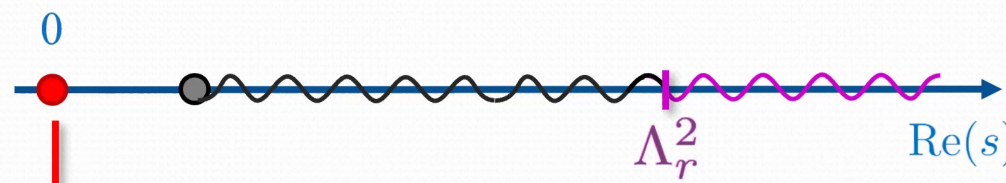

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$$\partial_s^2 \hat{\mathcal{A}}\Big|_{t=0} > -2(\ln r)' + (\ln \alpha')'$$

typically assumed

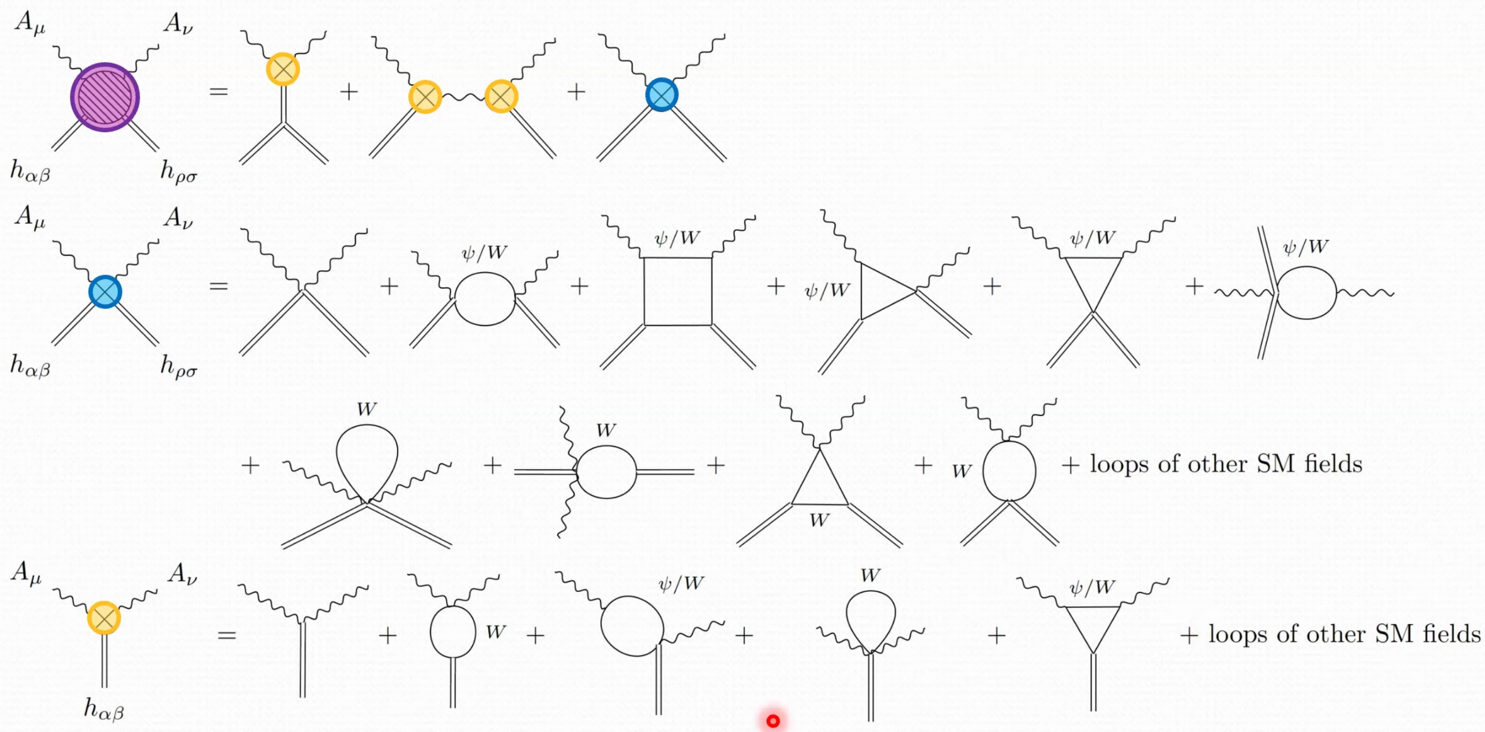
$\sim M_*^{-4} \ll$ low-energy physics

pole subtracted

Related to details of Regge behaviour

hAhA

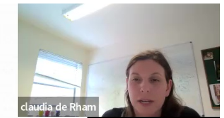
(graviton-photon scattering in SM)



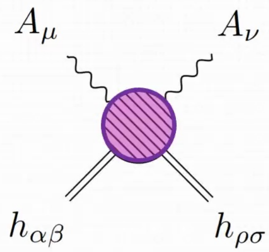
$$\mathcal{L} = -\frac{M_{\rm Pl}^2}{2} R - \frac{1}{4} F_{\mu\nu}^2 + b F_{\mu\nu} F_{\alpha\beta} R^{\mu\nu\alpha\beta} + \mathcal{O}_{\rm dim \geq 8}$$

Contribution to s^2 dominated by lightest charged particle

$$b \sim -\frac{q_e^2}{1440 \pi^2 m_e^2} \left(1 + \mathcal{O}\left(\frac{m_e^2}{m_{\rm Heavier}^2} \right) \right)$$



hAhA: graviton-photon scattering



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$$b \sim \frac{q_e^2}{1440\pi^2 m_e^2}$$

$$|i\rangle = |f\rangle = \frac{1}{\sqrt{2}}\left(|{+1}\rangle_\gamma + \mathrm{Sign}(b)\,|{-1}\rangle_\gamma\right) \otimes |{\pm 2}\rangle_{\rm graviton}$$

$$\partial_s^2 \hat{\mathcal{A}}\Big|_{t=0} = -4|b| > -2(\ln r)' + (\ln \alpha')'$$

From assumption of Regge behaviour

Lasma Alberte  Sumer Jaitly  Andrew Tolley 

2111.09226

Reverse Engineering

$$\partial_s^2 \hat{\mathcal{A}}\Big|_{t=0} = -\frac{q_e^2}{360\pi^2 m_e^2} > -2(\ln r)' + (\ln \alpha')'$$

1. Causality/Locality

2. Light Loops

3. New Physics?

4. Regge Slope

$$(\ln \alpha')' \sim -\frac{q_e^2}{m_e^2} \sim -\frac{10^{38}}{M_{\rm Pl}^2}$$

5. Regge Residue

$$\lim_{s\to\infty} \text{Disc}\,\mathcal{A}(s,t) = r(t)\Lambda_r^4 \left(\frac{s}{\Lambda_r^2}\right)^{\alpha(t)}$$

$$(\ln r)' \sim \frac{q_e^2}{m_e^2} \sim \frac{10^{38}}{M_{\rm Pl}^2}$$

Summary: IR lessons to UV physics

- Accounting for SM loops in photon-graviton amplitude leads to negative amplitude

$$\partial_s^2 \hat{\mathcal{A}}\Big|_{t=0} = -\frac{q_e^2}{360\pi^2 m_e^2}$$

- Amount of negativity, is precisely relevant to WGC positivity proofs

- Could indicate a violation of Froissart bound or non-local IR/UV mixing. “Dramatic” outcome in higher-dim.

- Assuming Froissart-like bound, UV Regge slope or residue controlled by scale of IR loops
(higher order corrections cannot be ignored)

- Amount of negativity tightly linked with infrared causality