Title: Quantum States of Matter with Fractal Symmetries: Theory and realization with Rydberg atoms

Speakers: Cenke Xu

Series: Quantum Matter

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Abstract: In recent years, generalizations of the notion of symmetry have significantly broadened our view on states of matter. We will discuss some recent progress of understanding and realizing the "fractal symmetry", where the symmetric charge i.e. the generator of the symmetry is defined on a fractal subset of the system with a noninteger or more generally irrational Hausdorff dimension. We will introduce a series of models with exotic fractal symmetries, which can in general be deduced from a "Pascal Triangle" (also called Yang Hui Triangle in ancient China) symmetry. We will discuss their various features including quantum phase transitions. We will also discuss the potential realization of these phases and phase transitions in experimental systems, such as the highly tunable platform of Rydberg atoms.

Zoom Link: https://pitp.zoom.us/meeting/register/tJcqc-ihqzMvHdW-YBm7mYd_XP9Amhypv5vO


# *Quantum States of Matter with Fractal Symmetries: Theory and Experimental Platform*


***Cenke Xu***

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*Quantum States of Matter with Fractal Symmetries: Theory and Experimental Platform*


**Content:**

1, fractal and subsystem symmetry, and SSB of fractal symmetry (defined as "fractal order");

2, "Pascal triangle (Yang Hui triangle)" fractal symmetries and realizations, which unifies a series of fractal symmetries;

3, quick review of Rydberg atoms;

4, realizing fractal symmetry and fractal order with Rydberg atoms.

Reference:
arXiv:2108.07765, arXiv:2110.02237

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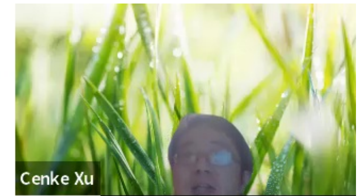
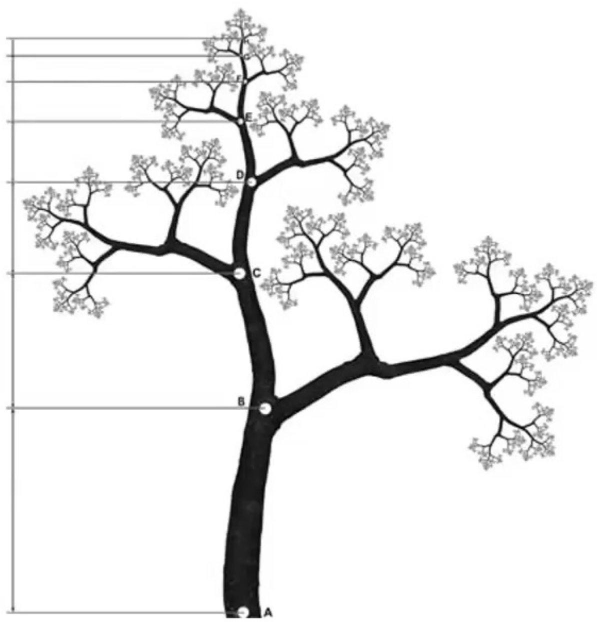
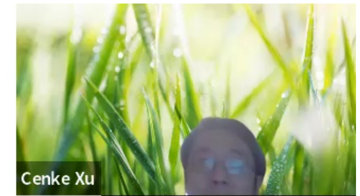



### Fractals in nature:

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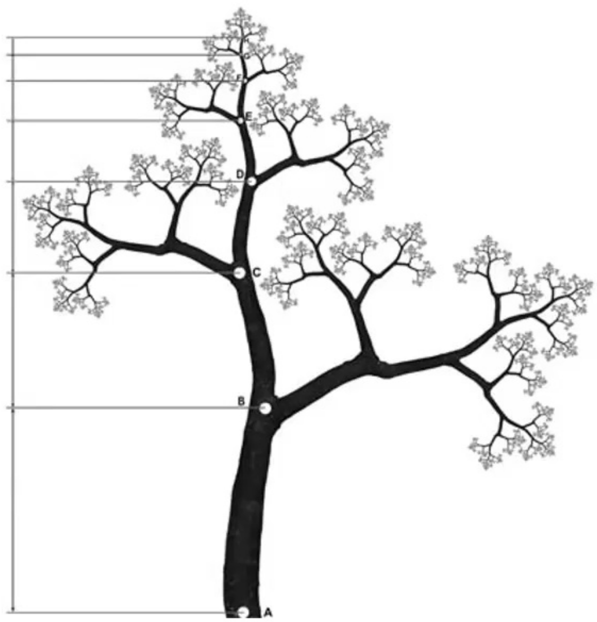


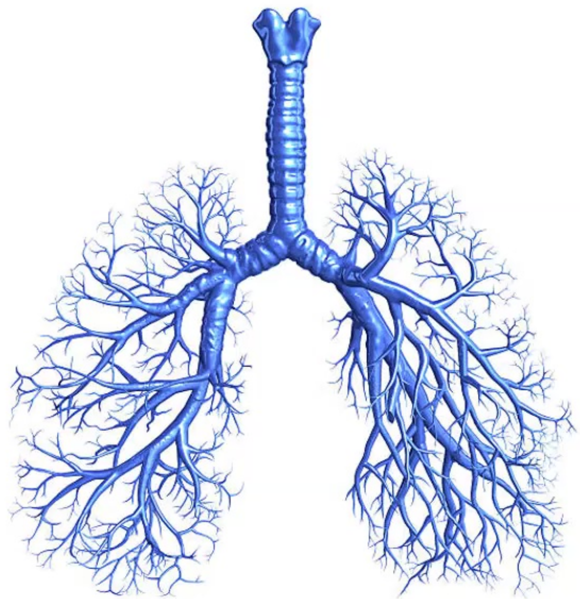

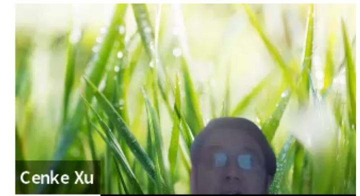


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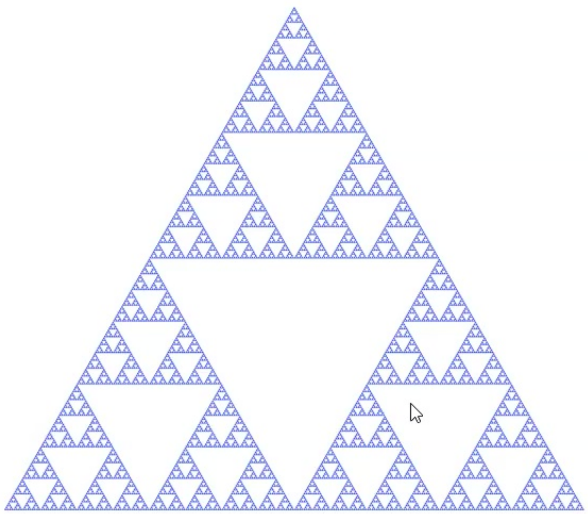

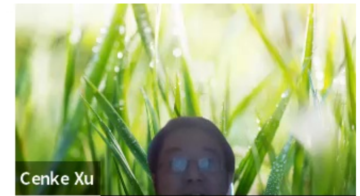


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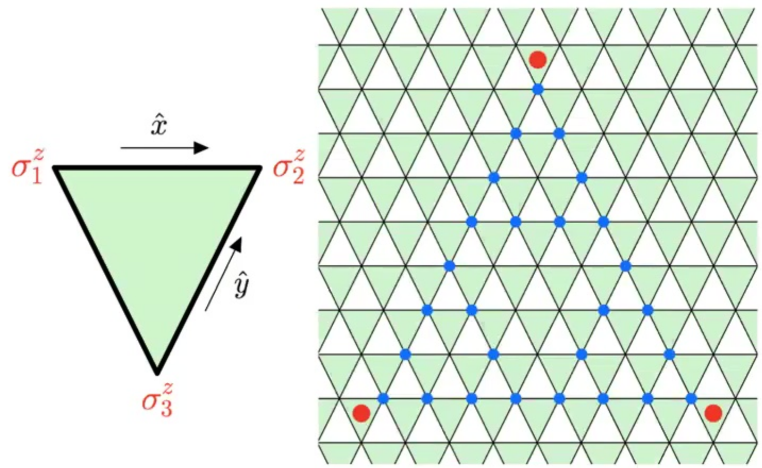
$$H_{\mathrm{ST}} = \sum_{\bigtriangledown} -K\sigma_1^z\sigma_2^z\sigma_3^z,$$




Newman-Moore (1999)-Yoshida (2013)

1. All down-facing triangles favor to have either “3-up” spins, or “2-down, 1-up” spins. Large number of excited states.

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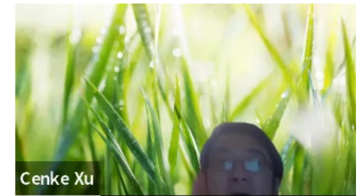


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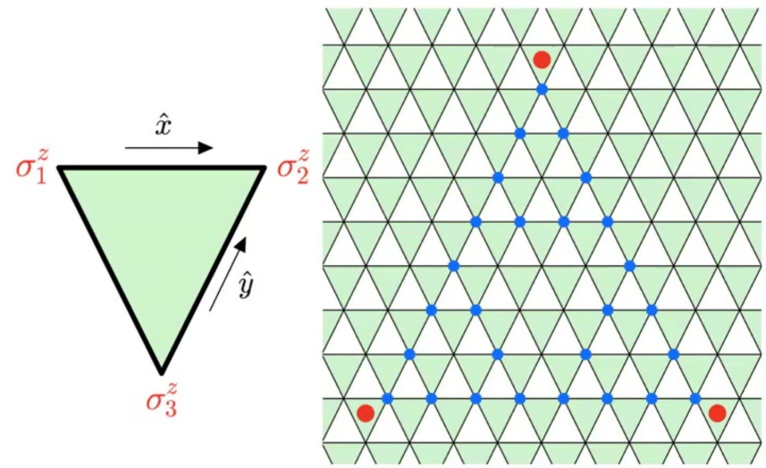


2, no phase transition at finite temperature; the three-spin correlation function is short ranged by still have fractal structure.

$$C(r) = (1-2p)^{r^{\log 3/\log 2}}$$

$$r = 2^m$$

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Cenke Xu

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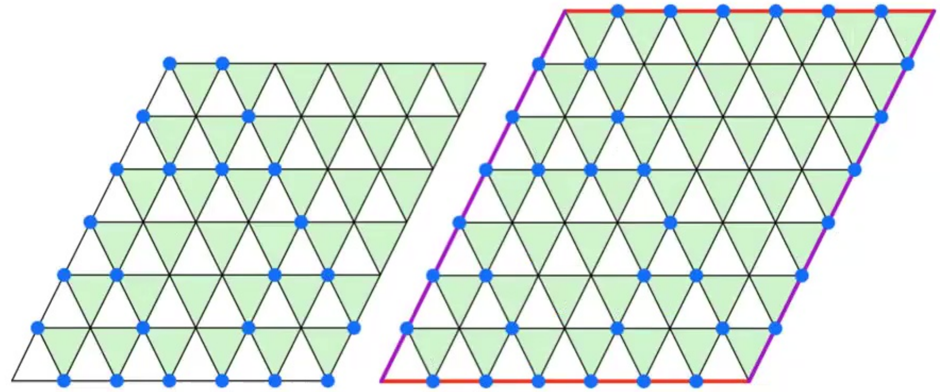

3, fractal symmetry: for a lattice (periodic bc) with size $L^2$, with $L = 2^k - 1$, the model has an explicit symmetry: the Hamiltonian is invariant under flipping spins on a Sierpinski triangle.

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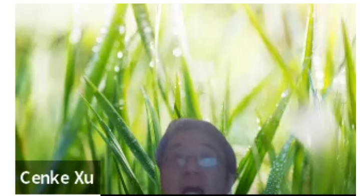
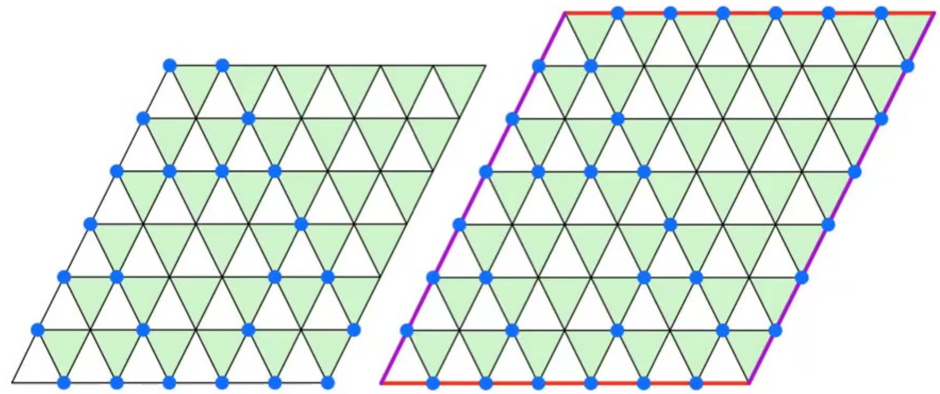
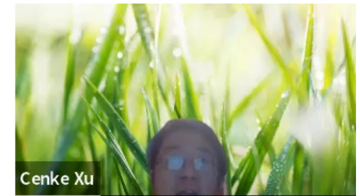



$$H_{q\mathrm{ST}} = \sum_{\bigtriangledown} -K\sigma_1^z\sigma_2^z\sigma_3^z - \sum_j h\sigma_j^x.$$



4, quantum version of fractal symmetry: for a lattice (periodic bc) with size $L^2$, with $L = 2^k - 1$, Prod $\sigma^x$ on the Sierpinski triangle is a conserved quantity.

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## Subsystem symmetry:

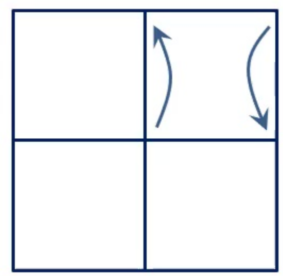
Early examples: 1, 2+1d Bose metal, Paramekanti, Balents, Fisher, 2002 (PBF)

$$\mathcal{H}_{\square} = \frac{U}{2}\sum_{\mathbf{r}}(n_{\mathbf{r}} - \bar{n})^2 - K\sum_{\mathbf{r}}\cos(\Delta_{xy}\phi_{\mathbf{r}}),$$

$$\Delta_{xy}\phi_{\mathbf{r}} \equiv \phi_{\mathbf{r}} - \phi_{\mathbf{r}+\hat{\mathbf{x}}} - \phi_{\mathbf{r}+\hat{\mathbf{y}}} + \phi_{\mathbf{r}+\hat{\mathbf{x}}+\hat{\mathbf{y}}}.$$

$$\phi_{\mathbf{r}} \to \phi_{\mathbf{r}} + f(x) + g(y)$$

1d symmetry, boson number $n$ along each line (both directions) are conserved, as $e^{i\phi}$ is the creation operator of a boson. The system behaves a lot like 1d boson, i.e. no superfluid phase, but a power-law algebraic phase of dipoles.



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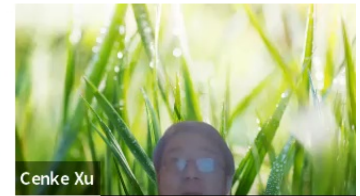



## Subsystem symmetry:

Early examples: 2, 2+1d Ising plaquette model (Xu, Moore, 2003),

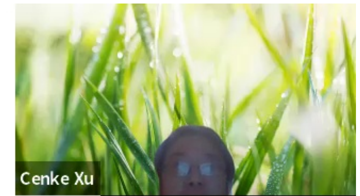
$$H = -K \sum_{\square} \sigma_1^z \sigma_2^z \sigma_3^z \sigma_4^z - h \sum_i \sigma_i^x.$$

Also has 1d subsystem symmetry which flips the spins on each line, along both directions. Prod $\sigma^x$ along each line is a conserved quantity. Also behaves like 1d quantum Ising model:

1, self-duality at zero temperature;

2, quantum phase transition at $h = K$ (between an ordered phase at $K > h$ with spontaneous subsystem symmetry breaking, and a disordered phase at $h > K$);

3, no classical phase transition at finite temperature.

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Subsystem symmetry + “Topological” order: Fracton topological order. Type-I: conserved charges on an ordinary subset of the lattice such as a line, or plane (Chamon’s model, X-cube model, etc.); type-II, conserved charges on a fractal subset of the lattice (Haah’s code) Review of fracton: arXiv:1803.11196, Nandkishore, Hermele; arXiv:2001.01722, Pretko, You, Chen

This talk will focus on type-II subsystem symmetries. The simple model with the type-II subsystem symmetry is the one we discussed: Newman-Moore (1999)-Yoshida (2013) model, or the Sierspinki-triangle model.

$$H_{\mathrm{ST}} = \sum_{\bigtriangledown} -K\sigma_1^z\sigma_2^z\sigma_3^z,$$

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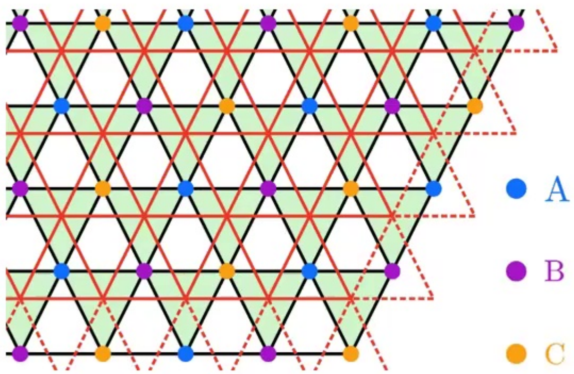
$$H_{q\mathrm{ST}} = \sum_{\bigtriangledown} -K\sigma_1^z\sigma_2^z\sigma_3^z - \sum_j h\sigma_j^x .$$

5, self-duality (arXiv:2105.05851, Zhou, Zhang, Pollmann, You); like the 1+1d quantum Ising model, and the 2+1d quantum plaquette model with subsystem symmetry: there is (likely) a quantum phase transition at $h = K$.



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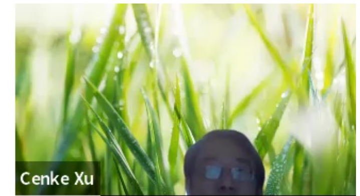


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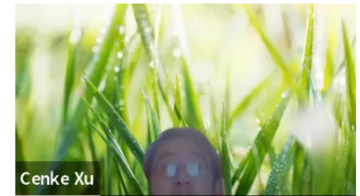
6, the quantum phase transition at $h=K$ separates two phases:
$K > h$ is a spontaneous symmetry breaking phase of the fractal symmetry, or "fractal ordered" phase, with large ground state degeneracy with system size $L = 2^k - 1$, and nonzero $\langle \sigma^z \rangle$ in the ground state;
$K < h$ is a disordered phase with a nondegenerate ground state and zero $\langle \sigma^z \rangle$ .

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$$H_{q\mathrm{ST}} = \sum_{\triangledown} -K\sigma_1^z\sigma_2^z\sigma_3^z - \sum_j h\sigma_j^x.$$

7, nature of the quantum phase transition at $h = K$ ?

Theoretically, no well established field theory for spontaneous breaking of fractal symmetry yet, no RG analysis…

Numerically, earlier result suggests a first order transition (Vasiloiu, et.al. arXiv:1911.11739); more recent work suggests a continuous phase transition (Zhou, et.al. arXiv:2105.05851).

$$G_x(r) = \langle \sigma^x(0)\sigma^x(r)\rangle \sim \frac{1}{r^{2\Delta}}, \quad \Delta = \ln 3/\ln 2$$

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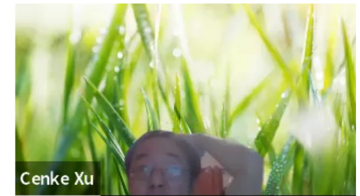
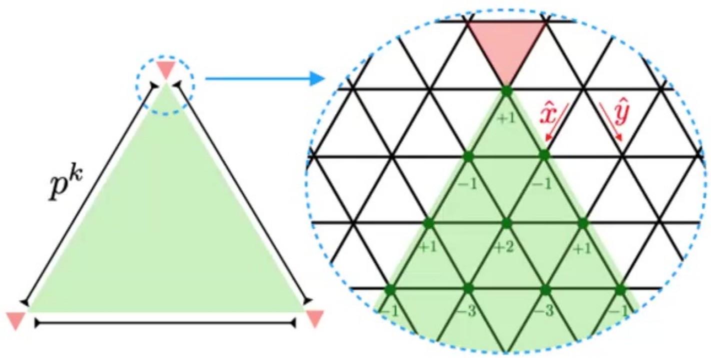


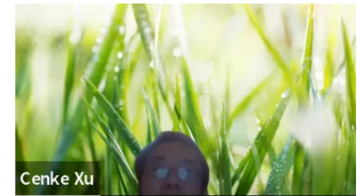

A U(1) generalization of the Sierpinski triangle model has a "Pascal triangle" "symmetry" (Myerson-Jain, et.al. arXiv:2110.02237)

$$H_p = \sum_{\bigtriangledown} -t\cos(\theta_1 + \theta_2 + \theta_3)$$

"Pascal triangle" (Yang Hui triangle) transformation:




$$\theta_i \longrightarrow \theta_i + \frac{2\pi}{p} m_i$$

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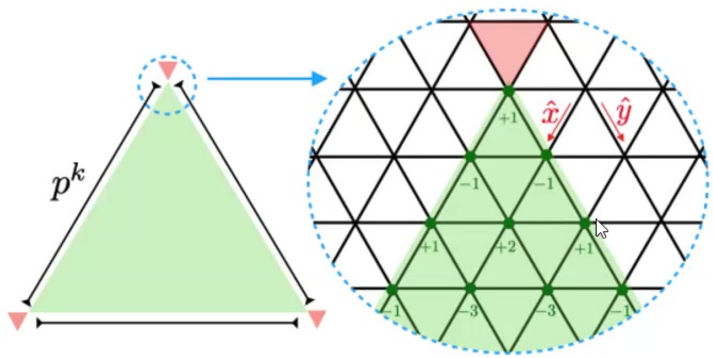


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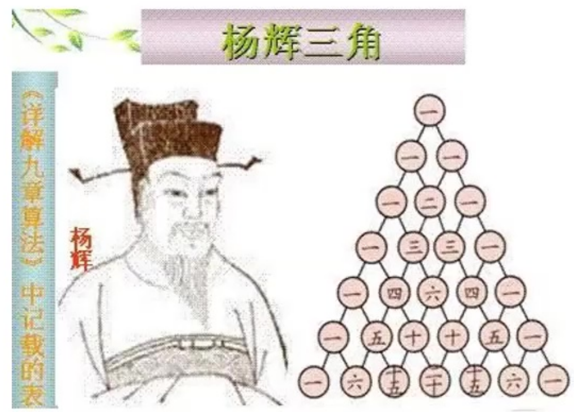

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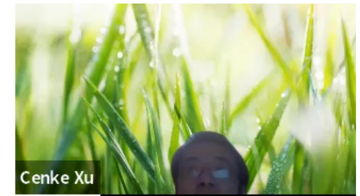
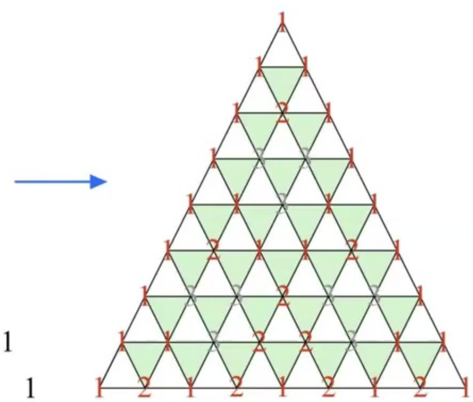



A $Z_p$ generalization of the Sierpinski triangle model each has a different fractal symmetry (Myerson-Jain, et.al. arXiv:2110.02237), with fractal dimension

$$\mathcal{C}_3(r = p^k) \sim e^{-\alpha r^{d_H}} \qquad d_H = \frac{\ln(p(p+1)/2)}{\ln p}$$

All these fractal symmetries can be deduced from the Pascal triangle

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1



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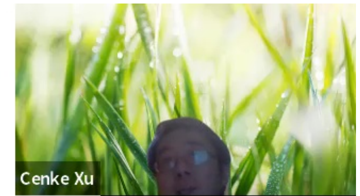


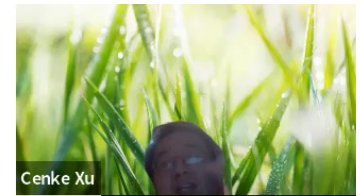

Closer look at the U(1) quantum Pascal triangle model:

$$H_{Q-\mathrm{U}(1)} = \sum_{\bigtriangledown} -t\cos\left(\theta_1 + \theta_2 + \theta_3\right) + \sum_j \frac{U}{2} n_j^2,$$

The full symmetries: a global U(1) x U(1) symmetry besides the fractal symmetry. The classical ground states in the limit $U$=0 spontaneously breaks all symmetries.

Is the SSB (order) stable against quantum fluctuation at finite $U$ ? is there an "order-disorder" quantum phase transition at finite $U$ ?

"Standard" procedure: assuming the semiclassical order, go to the dual picture, analyze the stability of the semiclassical order in the dual theory. Example: the SSB of the 1-form symmetry of U(1) gauge field is instable against confinement, this can be shown in the dual theory.

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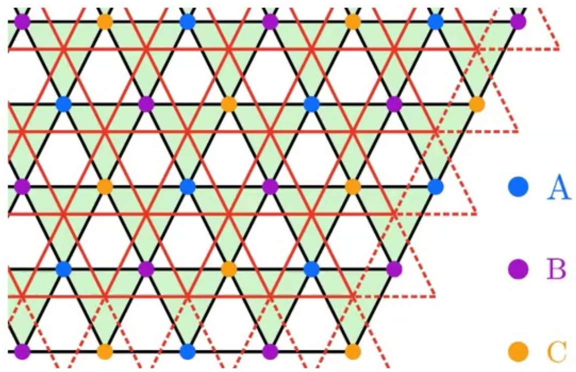


The Gaussian theory of the semiclassical order is "self-dual" (like the Sierspinki triangle model); the semiclassical order is destroyed by weak quantum fluctuation $U$.

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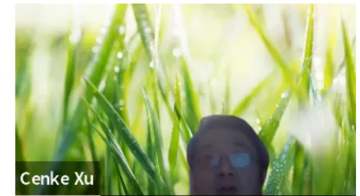


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1, a standard way of analyzing the $Z_N$ clock model, is to start with the U(1) model and turn on $Z_N$ anisotropy; the algebraic phase of the U(1) model reduces to the $Z_N$ ordered phase. But in the current case, this approach may be complicated by extra fractal symmetries.

2, though the generic U(1) model has no semiclassical order (or quasi-long range order), there is a fine-tuned multi-critical point, like the "Rokhsar-Kivelson point" of the quantum dimer model. (Myerson-Jain etl.al. in progress)

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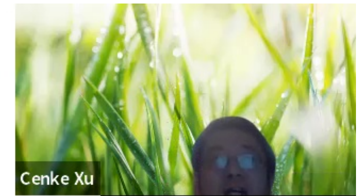


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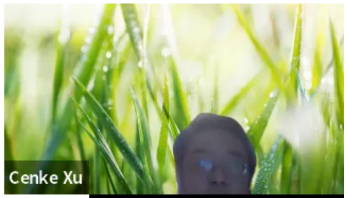
3, Open question: a "field theory" (low energy effective theory) with the fractal symmetry ?

For type-I models, one can usually start the discussion with a "field theory", which captures a lot of key physics. For example, the 2+1d Bose metal, 2002, PBF

$$\mathcal{H}_{\Box} = \frac{U}{2}\sum_{\mathbf{r}}(n_{\mathbf{r}}-\bar{n})^2 - K\sum_{\mathbf{r}}\cos(\Delta_{xy}\phi_{\mathbf{r}}),$$

$$\mathcal{L} = \frac{1}{2K}\left((\partial_\tau\phi)^2 + (\partial_x\partial_y\phi)^2\right)$$

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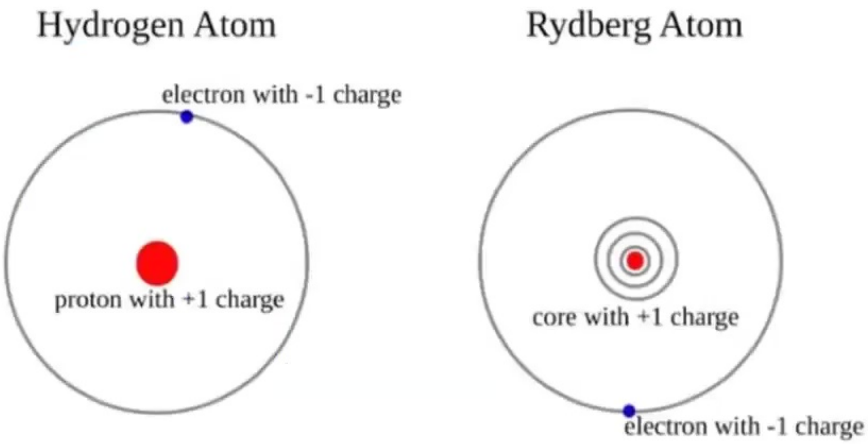
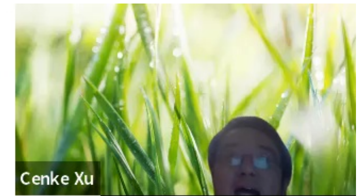
## Rydberg atoms:

A highly excited state of an atom, one electron is excited to a state with large principal quantum number.




$$E_n = -\frac{E_0}{n^2} \qquad E_n = -\frac{E_0}{(n-\delta_{n,l})^2}$$






## Rydberg atoms:

A highly excited state of an atom, one electron is excited to a state with large principal quantum number.

Two Rydberg atoms interact strongly through dipole moment fluctuation, the Van der Waals potential, which scales very strongly with the principal quantum number:

$$V(r) \sim \frac{C}{r^6}, \quad C \sim n^{11}$$

Dipole-dipole interaction decays as $1/r^3$. The Rydberg state itself may not have dipole moment; the $1/r^6$ interaction is a second order perturbation.

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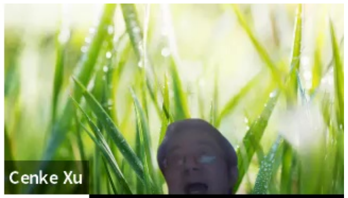



### Rydberg atoms:

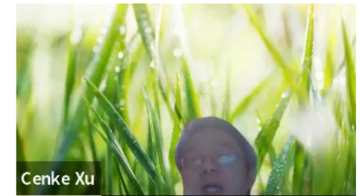
Consider an atom-photon coupled system (cavity QED), and atom is coupled with photons whose energy is close to the resonant frequency between ground state and Rydberg state. Define a number operators:

$$|\hat{n}=0\rangle = |g, N_\gamma + 1\rangle, \quad |\hat{n}=1\rangle = |r, N_\gamma\rangle.$$

For a multi-atom system that is arranged in an array, or a lattice:

$$H = \sum_i \Omega_i \sigma_i^x + H_0, \quad H_0 = -\sum_i \delta_i \hat{n}_i + \sum_{ij} V_{ij} \hat{n}_i \hat{n}_j,$$

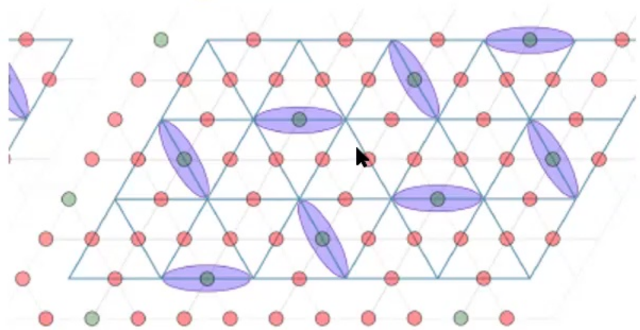
Review article on Rydberg atoms: Browaeys, Lahaye, Nature Physics, 16, 132–142 (2020)




## *Quantum States of Matter with Fractal Symmetries: Theory and Experimental Platform*

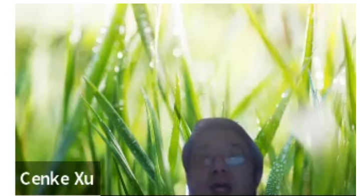
**Realizing strongly correlated phases with Rydberg atoms**

Example: $Z_2$ spin liquid/topological order, Samajdar, et.al. 2020, Verresen et.al. 2020, Semeghini, et.al. 2021



by tuning parameters in the Hamiltonian, neighboring Rydberg atoms are strongly suppressed; the allowed configurations of Rydberg atoms are equivalent to a quantum dimer model on a triangular lattice, which can host a $Z_2$ topological ordered state.

$$H_{\mathrm{eff}} = -\frac{3\Omega^6}{32\delta^5} \sum_{\hexagon} \left( |\hexagon\rangle\langle\hexagon| + h.c. \right)$$

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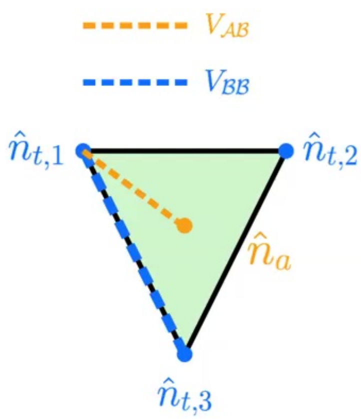
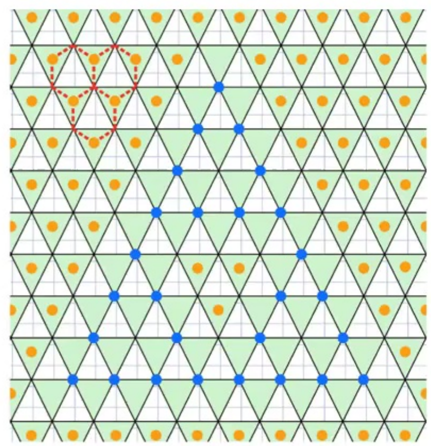
## Can we realize the quantum Sierpinski triangle model?

$$H_{q\mathrm{ST}} = \sum_{\triangledown} -K\sigma_1^z\sigma_2^z\sigma_3^z - \sum_j h\sigma_j^x.$$

The difficulty is the 3-body interaction; as condensed matter systems are generally dominated by 2-body interactions. Let's consider a honeycomb lattice, which is a decorated triangular lattice:

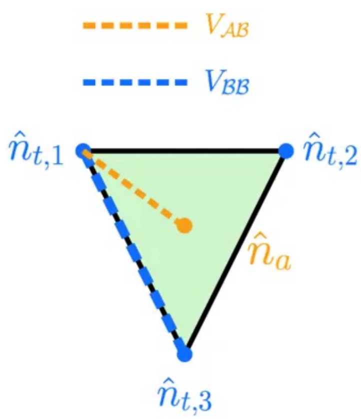


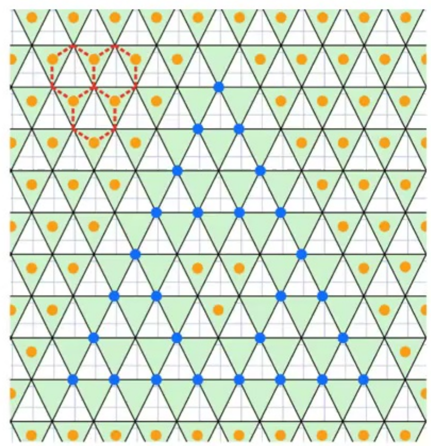



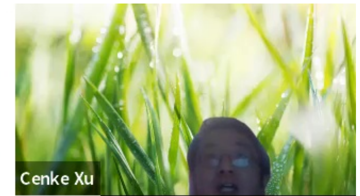
## *Quantum States of Matter with Fractal Symmetries: Theory and Experimental Platform*

$$H_{0,a} = V\left(2\hat{n}_a + \hat{n}_{t,1} + \hat{n}_{t,2} + \hat{n}_{t,3} - 2\right)^2 + \sum_{i=1}^{3} v\hat{n}_a\hat{n}_{t,i}$$

We trap "target" atoms on sublattice B of the honeycomb lattice (vertices of the triangular lattice); and "auxiliary" atoms on sublattice A (centers of down-triangles). Target atoms and auxiliary atoms have principal quantum numbers $n_B$ and $n_A$ respectively.



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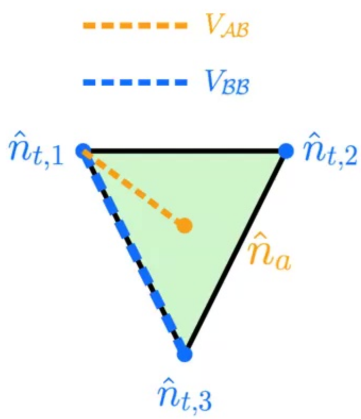
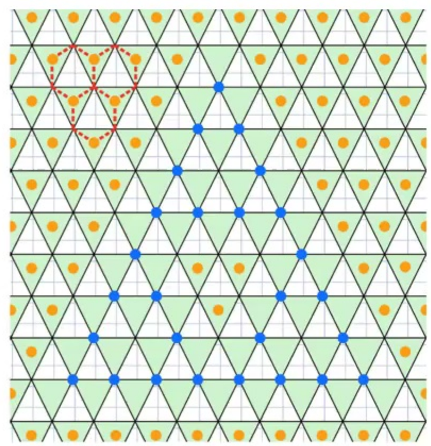
$$H_{0,a} = V\left(2\hat{n}_a + \hat{n}_{t,1} + \hat{n}_{t,2} + \hat{n}_{t,3} - 2\right)^2 + \sum_{i=1}^{3} v\hat{n}_a\hat{n}_{t,i}$$

Ground states of $H_0$: ⟶ $\sigma = 1 - 2\hat{n}_t$ ⟶ Ground states of $H_{\mathrm{ST}}$:

(1) $\hat{n}_a = 1,\ \hat{n}_{t,i} = 0;$ (1) $\sigma_1^z = \sigma_2^z = \sigma_3^z = +1;$

or (2) $\hat{n}_a = 0,$ two of $\hat{n}_{t,i} = 1.$ or (2) Two of $\sigma_i^z = -1.$



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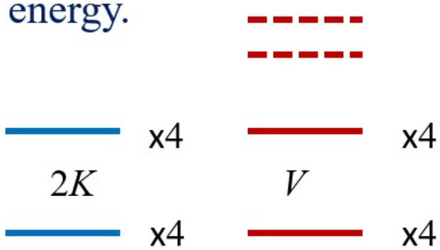
$$H_{0,a} = V\left(2\hat{n}_a + \hat{n}_{t,1} + \hat{n}_{t,2} + \hat{n}_{t,3} - 2\right)^2 + \sum_{i=1}^{3} v\hat{n}_a\hat{n}_{t,i}$$

All the states of the spin model is mapped to the low energy Hilbert space of the atomic system with the same degeneracy;
All the extra states of the atomic system have higher energy.




$$\sigma^z = (+1,+1,+1), \quad \text{Energy} = -K$$

$$\rightarrow \quad (\hat{n}_{t,i}; \hat{n}_a) = (0,0,0;1), \quad \text{Energy} = 0;$$

$$\sigma^z = (-1,-1,+1), \quad \text{Energy} = -K,$$

$$\rightarrow \quad (\hat{n}_{t,i}; \hat{n}_a) = (1,1,0;0), \quad \text{Energy} = 0;$$

$$\sigma^z = (-1,+1,+1), \quad \text{Energy} = +K,$$

$$\rightarrow \quad (\hat{n}_{t,i}; \hat{n}_a) = (1,0,0;0), \quad \text{Energy} = V;$$

$$\sigma^z = (-1,-1,-1), \quad \text{Energy} = +K$$

$$\rightarrow \quad (\hat{n}_{t,i}; \hat{n}_a) = (1,1,1;0), \quad \text{Energy} = V.$$

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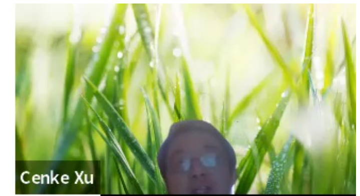
$$\begin{aligned} H_{0,a} &= V\left(2\hat{n}_a + \hat{n}_{t,1} + \hat{n}_{t,2} + \hat{n}_{t,3} - 2\right)^2 + \sum_{i=1}^{3} v\hat{n}_a\hat{n}_{t,i} \\ &\sim V\left(\sum_{i=1}^{3} 4\hat{n}_a\hat{n}_{t,i} + \sum_{i<j} 2\hat{n}_{t,i}\hat{n}_{t,j} - 4\hat{n}_a - \sum_{i=1}^{3} 3\hat{n}_{t,i}\right) \\ &+ \sum_{i=1}^{3} v\hat{n}_a\hat{n}_{t,i}\cdots . \end{aligned} \tag{5}$$

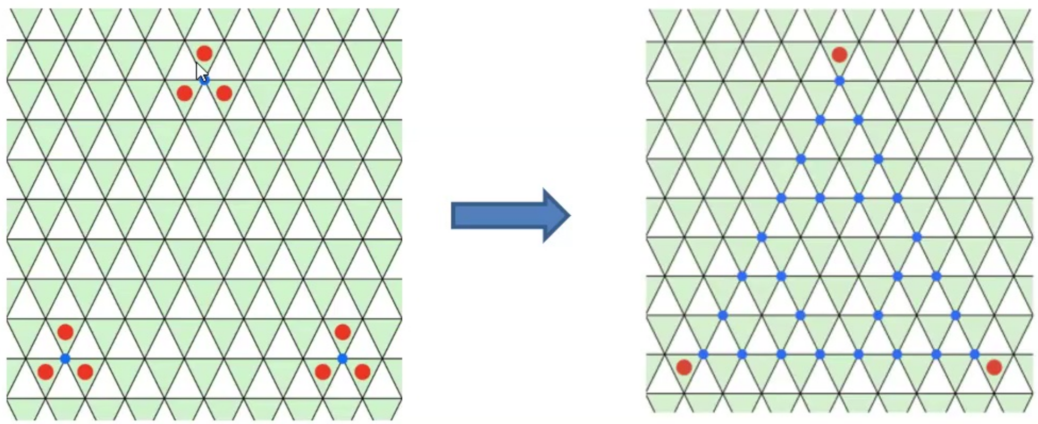
The physics described can be realized by choosing (for example) $n_B$ = 113 and $n_A$ = 76 for potassium atoms. Because the VdW interaction between Rydberg atoms decays fast with distance, perturbations (further neighbor interactions, etc.) are small. For example:

$$\frac{V'_{\mathcal{BB}}}{V_{\mathcal{BB}}} = \frac{1}{(\sqrt{3})^6} \sim 0.037, \quad \frac{V'_{\mathcal{AB}}}{V_{\mathcal{BB}}} = \frac{1}{2^6}\frac{V_{\mathcal{AB}}}{V_{\mathcal{BB}}} \sim 0.041.$$

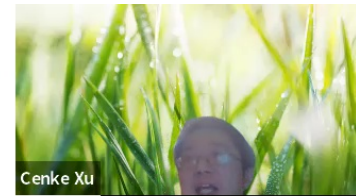



## *Quantum States of Matter with Fractal Symmetries: Theory and Experimental Platform*

Experiment visualization of fractal shape: slow manipulation on the corners of the Sierpinski triangle can spontaneously generate a Sierpinski-triangle shape of excitations.



Fast manipulation of parameters can probe quantum dynamics of the system. Fractal subsystem symmetry can lead to exotic dynamics.

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$$H = \sum_i \Omega_i \sigma_i^x + H_0, \quad H_0 = -\sum_i \delta_i \hat{n}_i + \sum_{ij} V_{ij} \hat{n}_i \hat{n}_j,$$

Turn on the Rabi oscillation term can potentially drive a quantum phase transition like the one in the quantum Sierpinski triangle model.

Experimentally, one can take "snapshots" of the configurations of the atoms; by averaging over many snapshots $< \sigma\, \sigma\, \sigma ... >$ we obtain the correlation function.

**Comment:**

1, no longer rely on perturbation to generate multi-spin interaction;

2, Perturbations (such as ordinary Ising interaction) may be relevant or irrelevant at the quantum phase transition. ($Z_N$ anisotropy irrelevant at U(1) WF fixed point for $N > 3$.)

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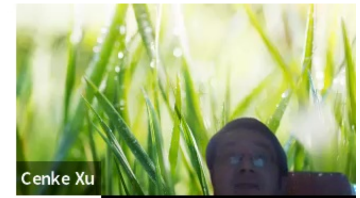
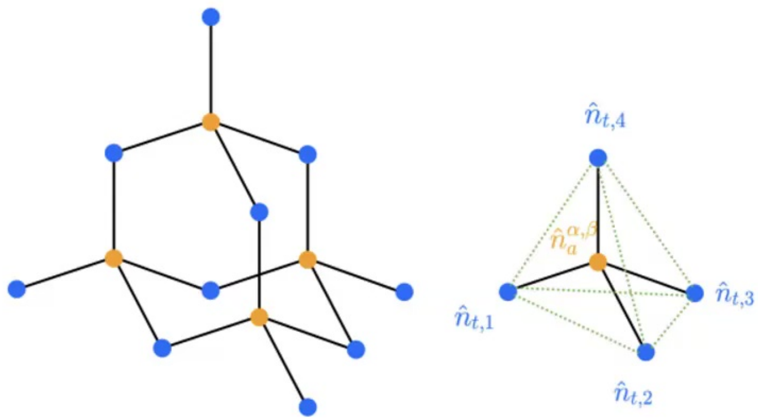



## Models with other fractal structure

$$H_{\text{tetra}} = \sum_{\text{tetra}} -K\sigma_1^z\sigma_2^z\sigma_3^z\sigma_4^z.$$




The tetrahedron model also has a fractal symmetry, and its excitations have a fractal geometry like the Sierpinski triangle model. The four body interaction can still be simulated through two body VdW interaction between Rydberg atoms after decorating the center of each tetrahedron with an auxiliary atom.

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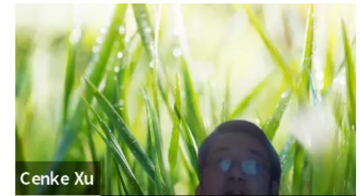
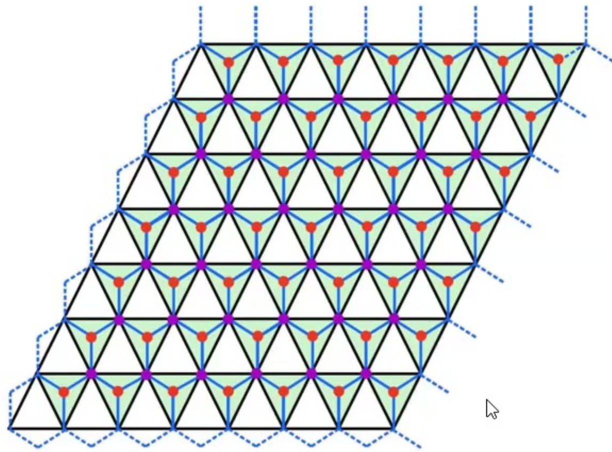



"Realization" of the U(1) quantum Pascal triangle model:




$$
\begin{aligned}
H_0 &= \sum_{R\in\mathcal{A}} \sum_{b=1}^{3} J\left(S_R^x s_{R,b}^x + S_R^y s_{R,b}^y\right) - D(S_R^z)^2 \\
&= \sum_{R\in\mathcal{A}} \sum_{b=1}^{3} \frac{J}{2}\left(S_R^+ s_{R,b}^- + S_R^- s_{R,b}^+\right) - D(S_R^z)^2 (23)
\end{aligned}
$$

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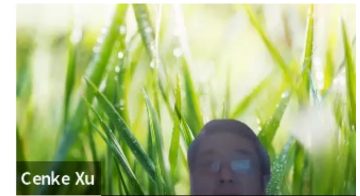
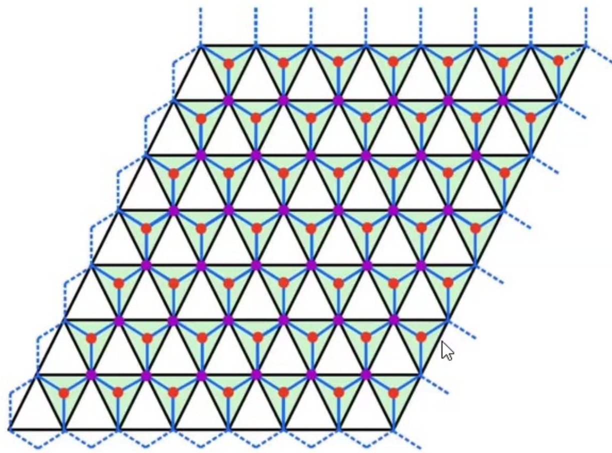



"Realization" of the U(1) quantum Pascal triangle model:



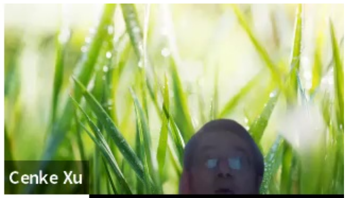
● $\mathcal{A}$ sublattice Spin-3/2

● $\mathcal{B}$ sublattice Spin-1/2

$$H_{\text{eff},0} = \sum_{R\in\mathcal{A}} K\left((S_R^+)^3 s_{R,1}^- s_{R,2}^- s_{R,3}^- + (S_R^-)^3 s_{R,1}^+ s_{R,2}^+ s_{R,3}^+\right),$$

$$K \sim \frac{J^3}{D^2}. \qquad (25)$$

$$H_{\text{eff}} = \sum_{R\in\mathcal{A}} -t\cos\left(3\theta_R - \theta_{R,1} - \theta_{R,2} - \theta_{R,3}\right)$$

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**Summary:**

1, quantum fractal models, especially their quantum phase transitions involving fractal subsystem symmetries and their spontaneous breaking, call for a new theoretical paradigm;

2, models with generalized "Pascal triangle" symmetries, and their quantum phase diagram;

3, realization of the quantum Sierpinski triangle model which involves 3-body interaction, through Rydberg atoms with only 2-body interaction

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