Title: Chaos in Celestial CFT

Speakers: Sabrina Pasterski

Series: Quantum Fields and Strings

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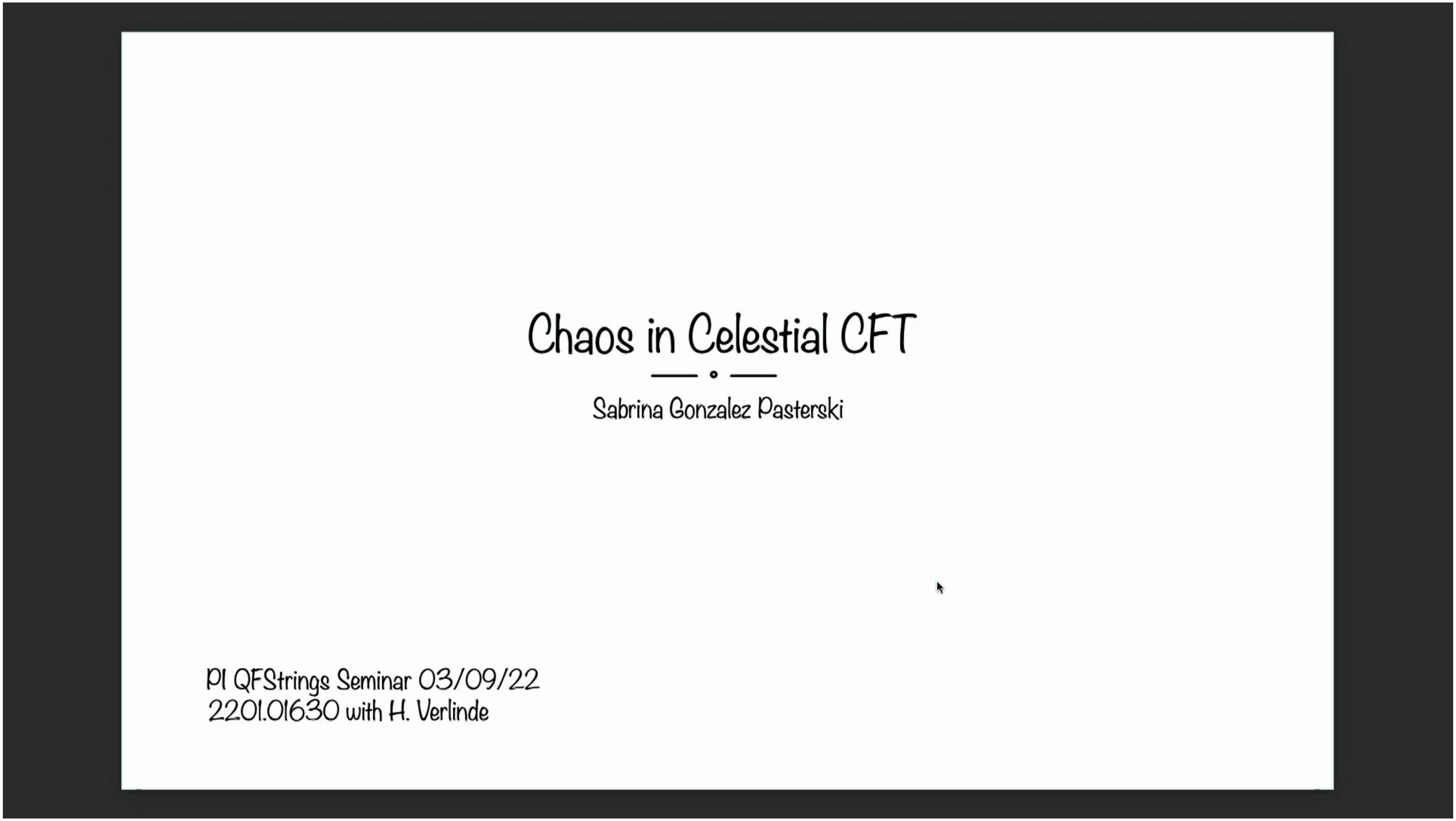
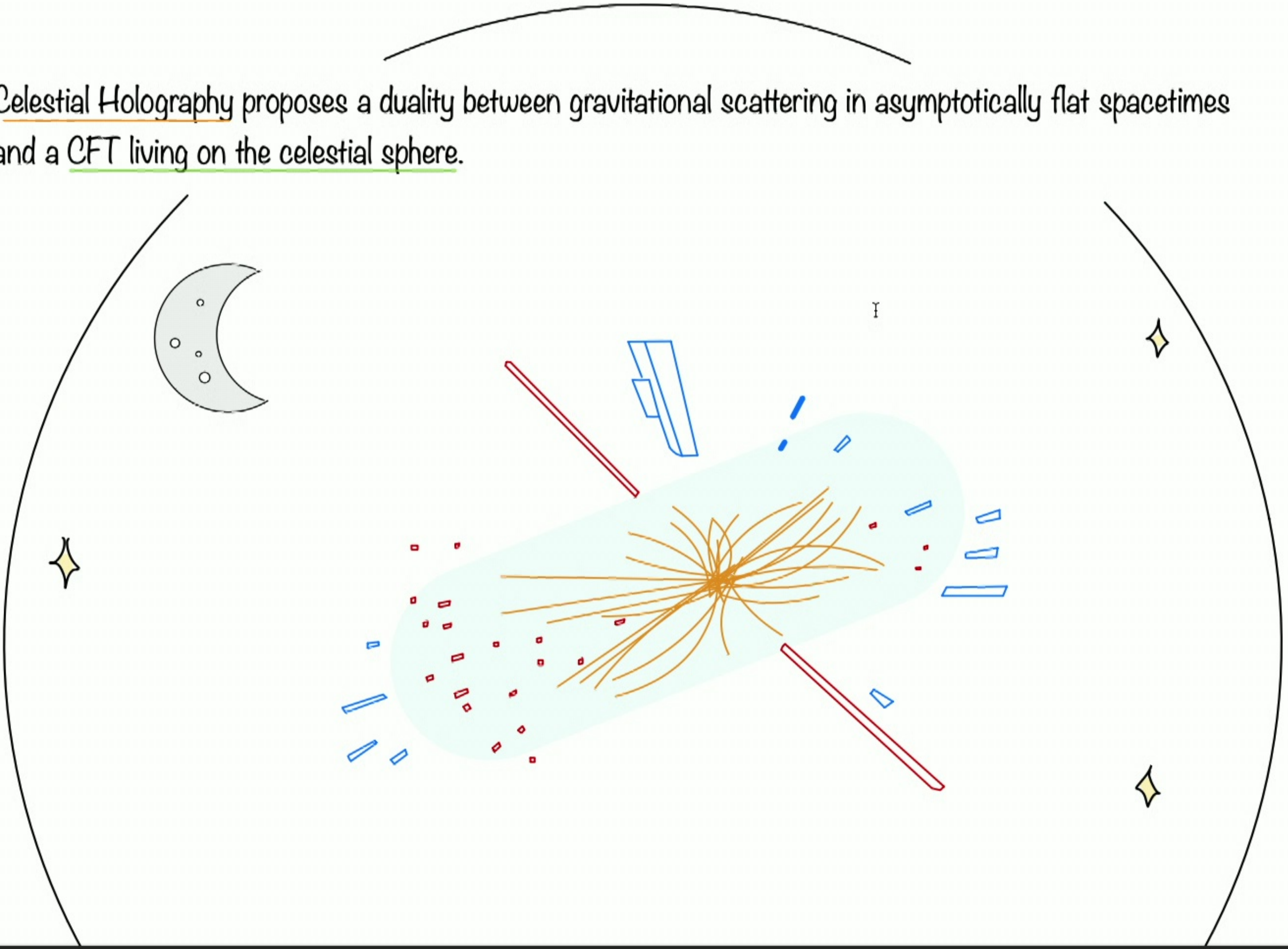


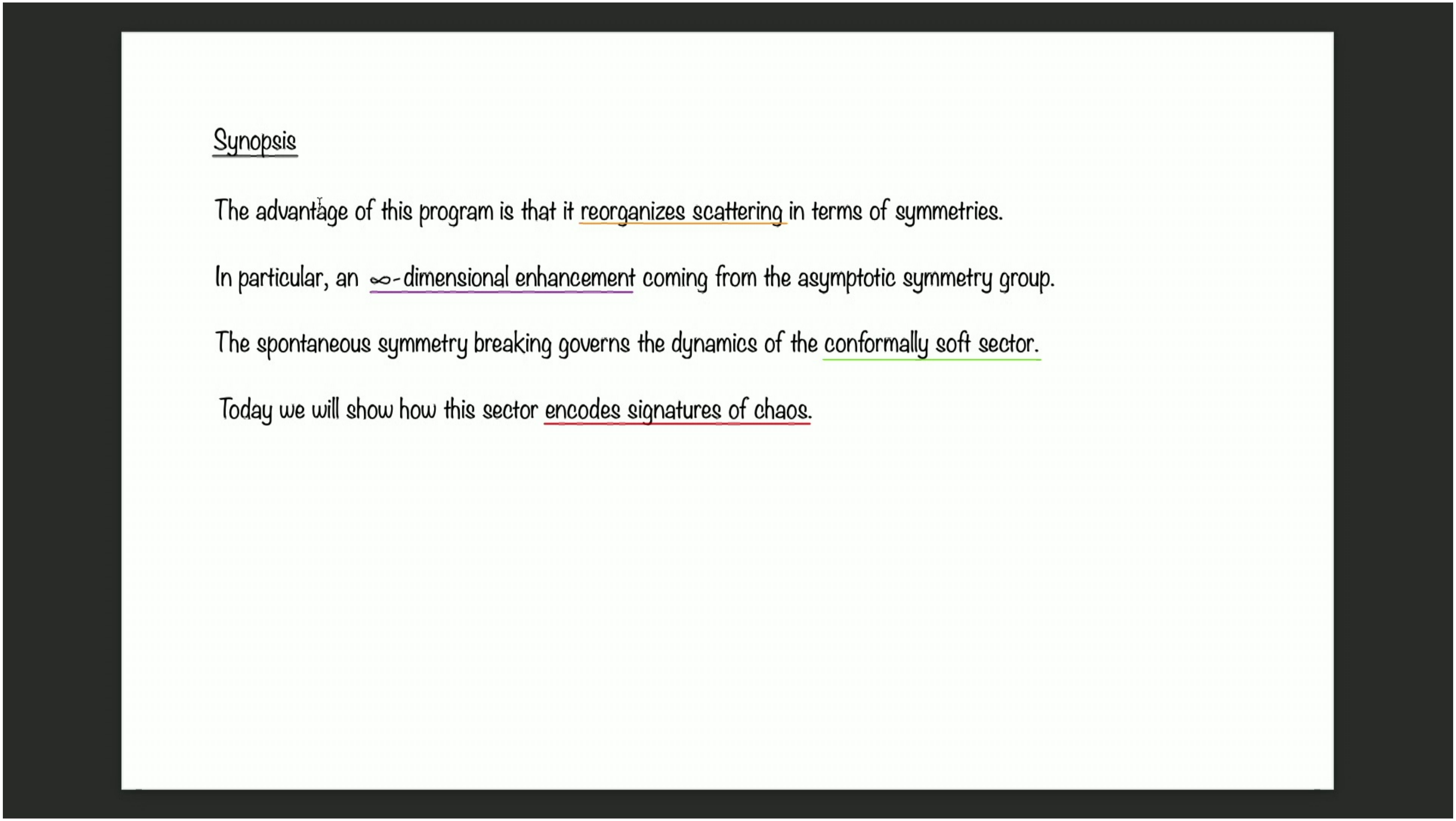
# Chaos in Celestial CFT

Sabrina Gonzalez Pasterski

PI QFStrings Seminar 03/09/22
2201.01630 with H. Verlinde

Celestial Holography proposes a duality between gravitational scattering in asymptotically flat spacetimes and a CFT living on the celestial sphere.





## Synopsis

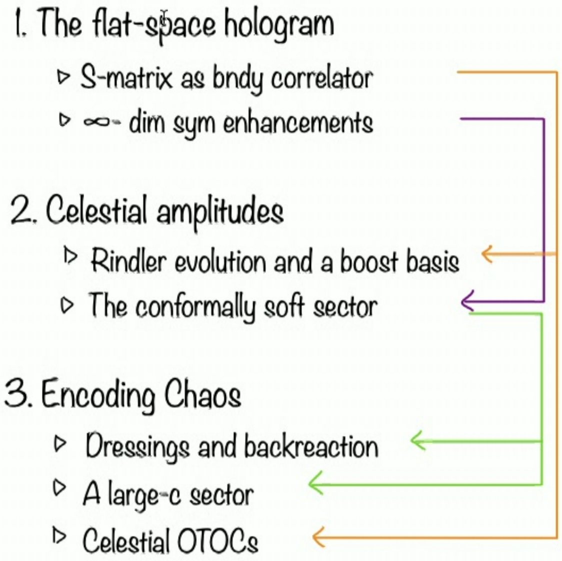
The advantage of this program is that it reorganizes scattering in terms of symmetries.

In particular, an $\infty$-dimensional enhancement coming from the asymptotic symmetry group.

The spontaneous symmetry breaking governs the dynamics of the conformally soft sector.

Today we will show how this sector encodes signatures of chaos.

## Outline



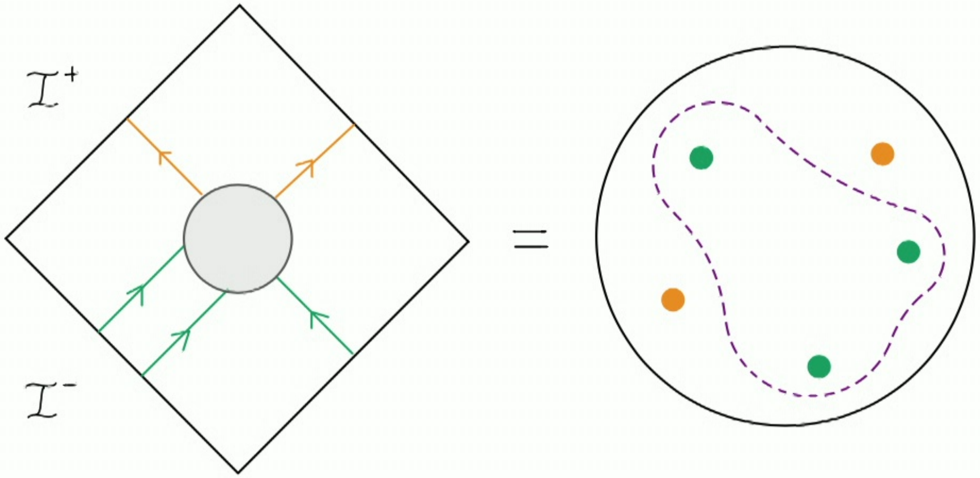
1. The flat-space hologram
   - ▷ S-matrix as bndy correlator
   - ▷ $\infty$- dim sym enhancements

2. Celestial amplitudes
   - ▷ Rindler evolution and a boost basis
   - ▷ The conformally soft sector

3. Encoding Chaos
   - ▷ Dressings and backreaction
   - ▷ A large-c sector
   - ▷ Celestial OTOCs

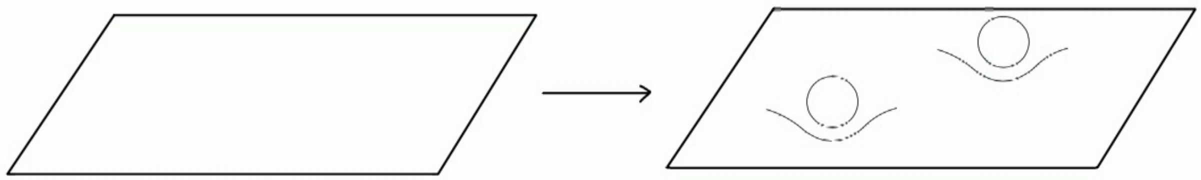
Goal: Apply the holographic principle to $\Lambda=0$ quantum gravity.

Plan: Celestial Holography proposes that the natural dual system lives on the celestial sphere.



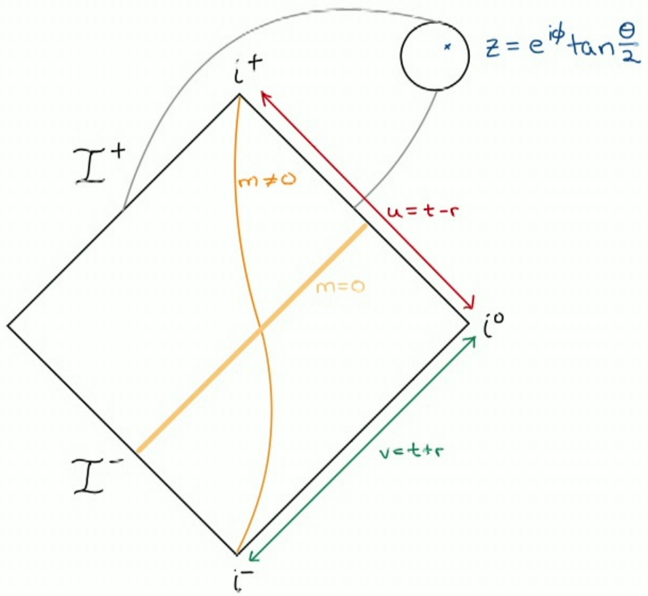
We are interested in scattering in asymptotically flat spacetimes.

$\Lambda = 0$ $T_{\mu\nu} \neq 0$



$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$

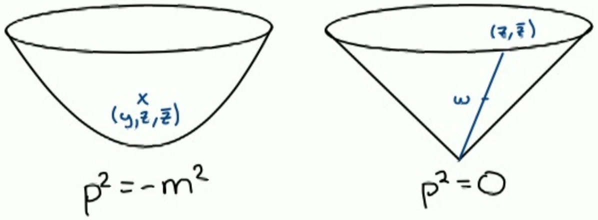
These spacetimes have the same asymptotic causal structure as Minkowski space.



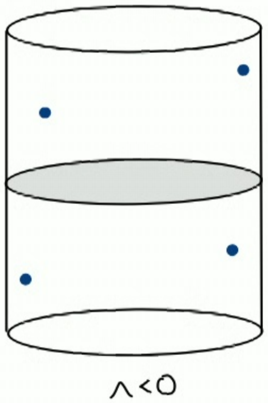
In particular massless excitations enter and exit along null hypersurfaces $\mathcal{I}^{\pm} \cong \mathbb{R} \times S^2$.
We will refer to this $S^2$ cross-section as the celestial sphere in what follows.

We can merge aspects of the two standard precedents for our hologram...

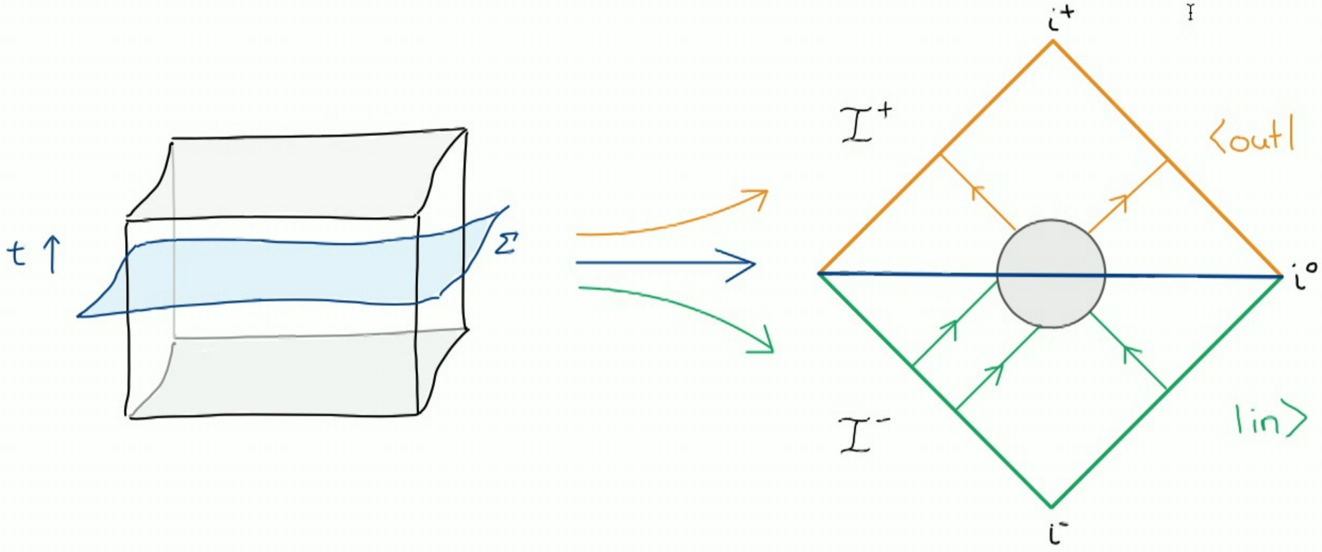
$\langle \text{out} | \mathcal{S} | \text{in} \rangle$



AdS/CFT



… by pushing our Cauchy slice to scri to prepare the in and out states with operators on the boundary.



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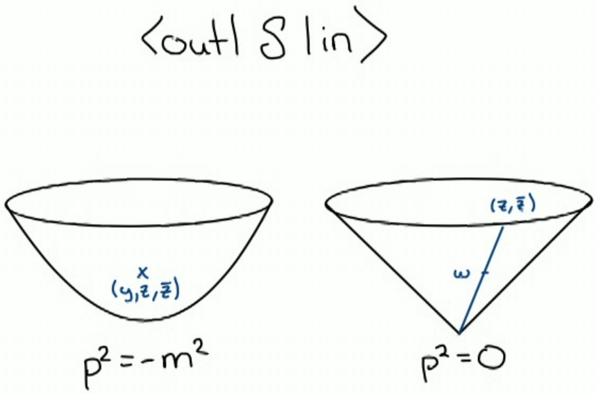

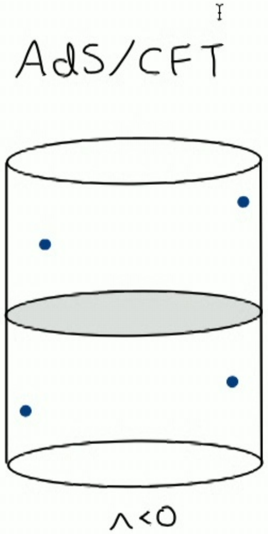

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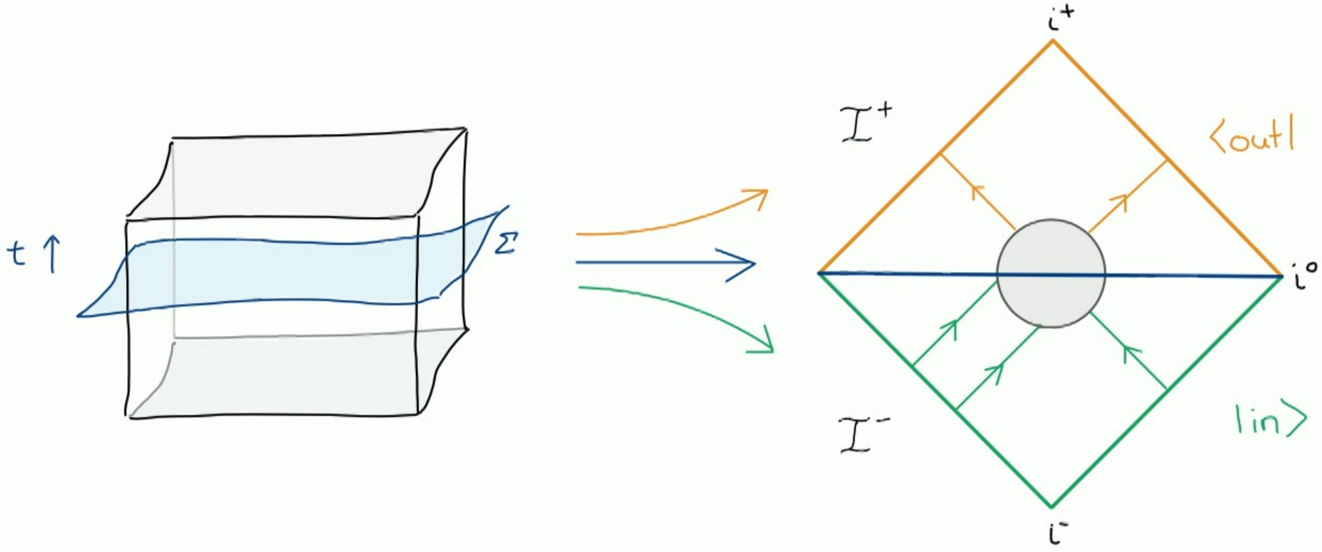
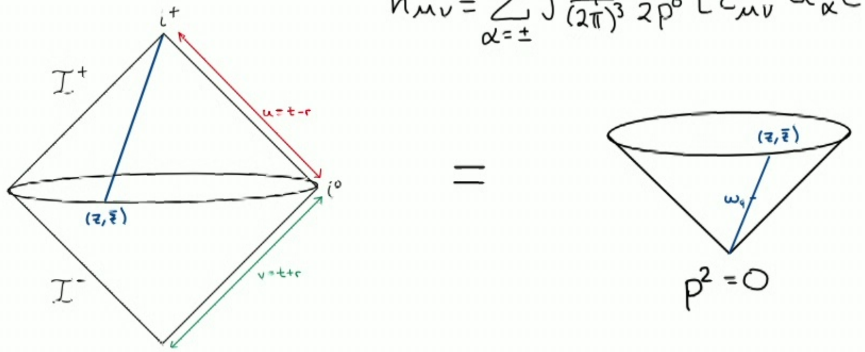


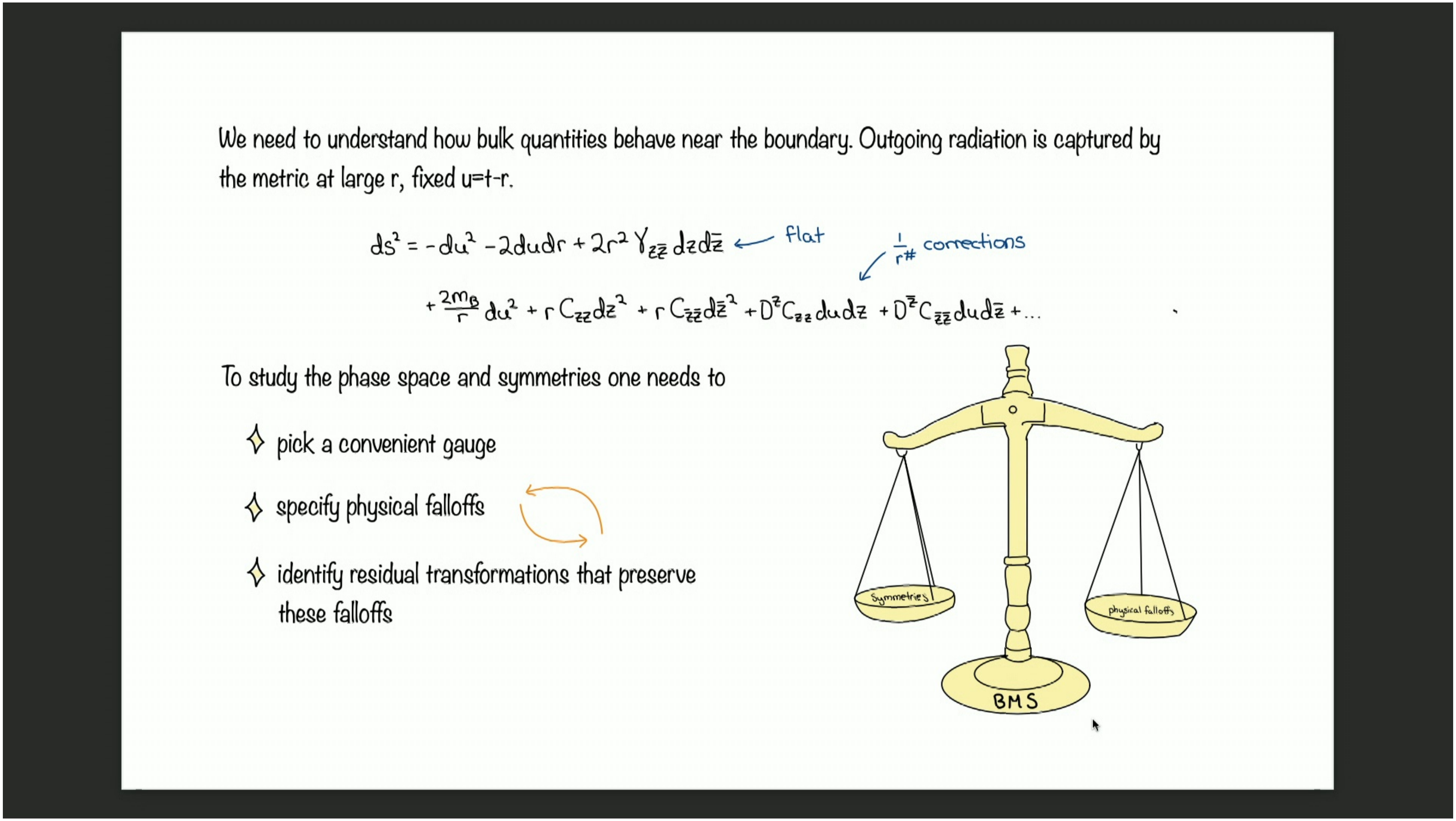
For example taking $r \to \infty$, u- fixed the plane wave localizes...

$$e^{ip\cdot X} = e^{-ip^0 u - ip^0 r(1-\cos\theta)} \longrightarrow e^{-ip^0 u} \times \frac{i}{p^0 r}\,\frac{\delta(\theta)}{\sin\theta}$$

... and we can prepare an m=0 momentum eigenstate with the boundary limit of our bulk operator smeared on a generator of $\mathcal{J}^+$.

$$h_{\mu\nu} = \sum_{\alpha=\pm}\int \frac{d^3p}{(2\pi)^3}\frac{1}{2p^0}\left[\varepsilon^{\alpha *}_{\mu\nu}\, a_\alpha e^{ip\cdot x} + \varepsilon^{\alpha}_{\mu\nu}\, a^\dagger_\alpha e^{-ip\cdot x}\right]$$





We need to understand how bulk quantities behave near the boundary. Outgoing radiation is captured by the metric at large r, fixed u=t-r.

$$ds^2 = -du^2 - 2dudr + 2r^2\gamma_{z\bar{z}}\,dz\,d\bar{z} \leftarrow \text{flat}$$

$$\swarrow \frac{1}{r^{\#}} \text{ corrections}$$

$$+\frac{2m_B}{r}du^2 + rC_{zz}dz^2 + rC_{\bar{z}\bar{z}}d\bar{z}^2 + D^zC_{zz}du\,dz + D^{\bar{z}}C_{\bar{z}\bar{z}}du\,d\bar{z} + \ldots$$

To study the phase space and symmetries one needs to

- pick a convenient gauge
- specify physical falloffs
- identify residual transformations that preserve these falloffs

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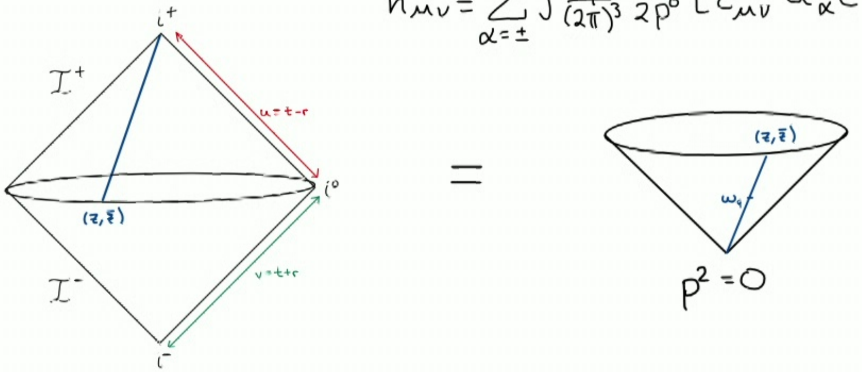

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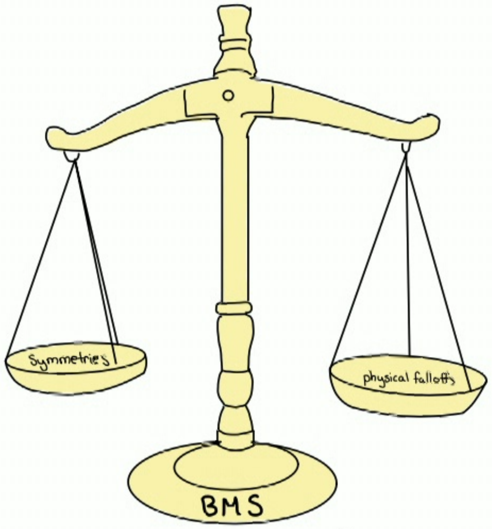


Residual diffeomorphisms that preserve the falloffs and act non-trivially on the asymptotic data are part of the Asymptotic Symmetry Group.

$$ASG = \frac{\text{Allowed Symmetries}}{\text{Trivial Symmetries}}$$

The ASG will be much larger than the group of isometries of any given spacetime within this class.

$$\begin{array}{lccc} & \text{Poincaré} & \subset & \text{BMS} \\ \#\text{generators:} & 10 & & \infty \end{array}$$

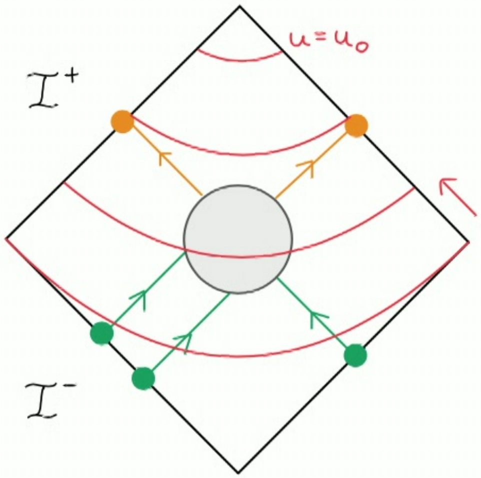
✧ Supertranslations induce angle-dependent shifts in the time coordinate

$$\xi|_{\mathcal{I}^+} = f(z,\bar{z})\,\partial_u$$

✧ Superrotations extend global conformal transformations to local CKVs.

$$\xi|_{\mathcal{I}^+} = Y^z(z)\,\partial_z + \frac{u}{2} D_z Y^z(z)\,\partial_u + c.c.$$

Now let's look at different ways of organizing our hologram. If we Fourier transform our in and out states we have correlators on a null 3-manifold.



Issues

- Connecting in and out
- Null time coordinate
- Charge non-conservation

Raju et. al & Carrollian perspectives

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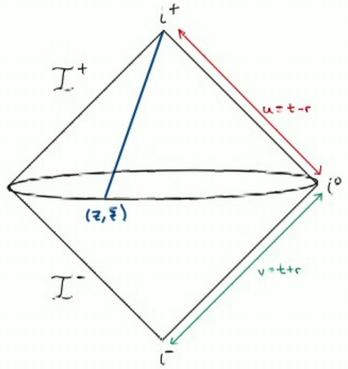

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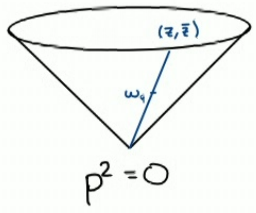

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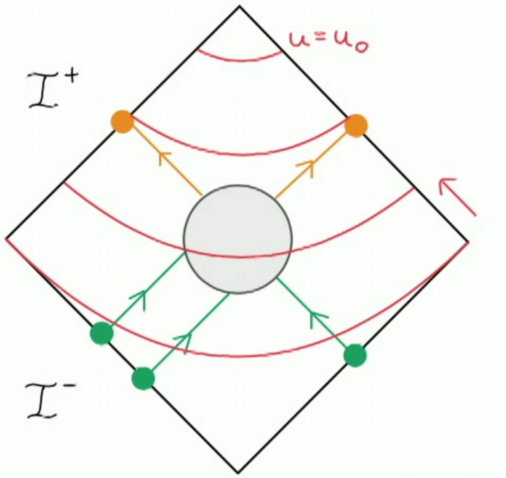

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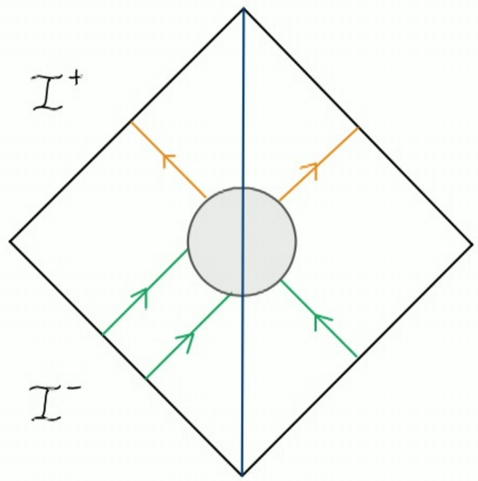
The Celestial program looks at Rindler evolution of the bulk and boundary. Can we see that the boost basis is a better way of presenting the flat space hologram?



Rindler = radial

Benefits

- ▷ Soft thms are canonical charges
- ▷ In and out come together antipodally
- ▷ Decoupling of primary descendants implies infinite tower of symmetries

vs checking universality of $\omega \to 0$ expansion
+ augmenting phase space w/ overleading

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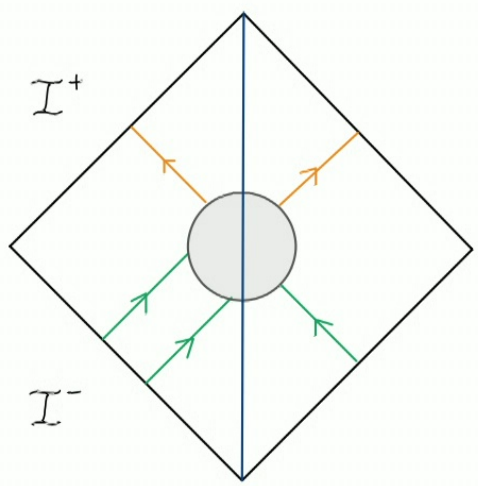

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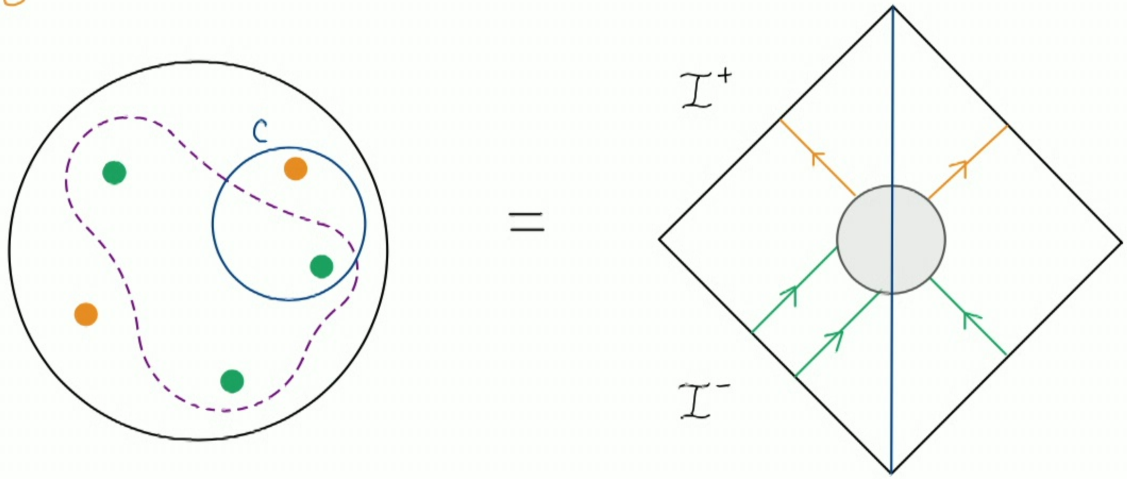
The charges in gauge theory are co-dimension 2. If I lift a path $C$ on the celestial sphere to a co-dimension 1 hypersurface $\Sigma_C$ of the bulk that runs along the generators of $\mathscr{I}$ I have

$$J_C(\lambda) = \int_{\partial\Sigma_C} *\kappa(\lambda)$$

charge generating transformation on Celestial State defined on contour $C$

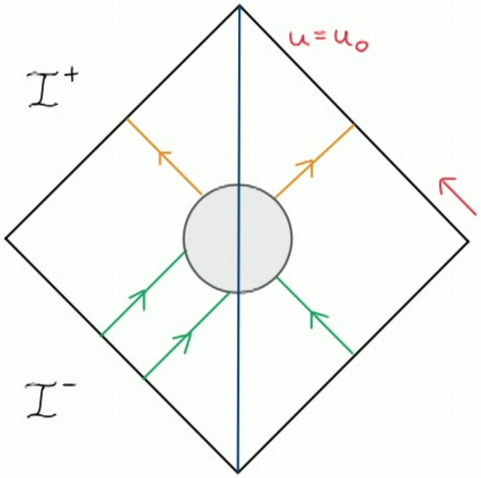
bulk 'charge' evaluated on hypersurface $\Sigma_C$

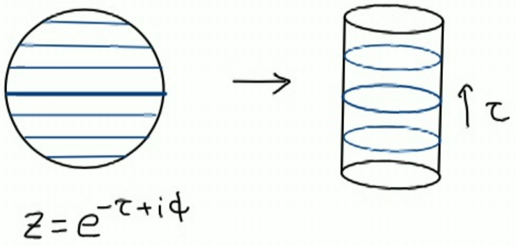


To be more concrete, let us consider the locus $X^3=0$. This splits the celestial sphere along its equator.

$$X^\mu = \left(u+r,\ r\,\frac{z+\bar{z}}{1+z\bar{z}},\ ir\,\frac{\bar{z}-z}{1+z\bar{z}},\ r\,\frac{1-z\bar{z}}{1+z\bar{z}}\right) \longrightarrow |z|^2=1$$

The boost image of this slice sweeps out a foliation of the bulk and boundary that respects our ability to quotient by the generators of $\mathcal{J}^\pm$.





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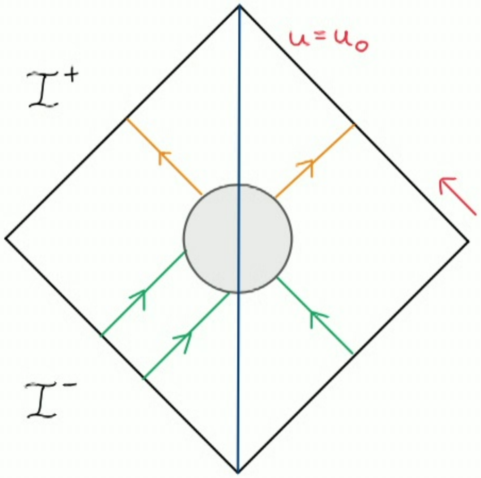
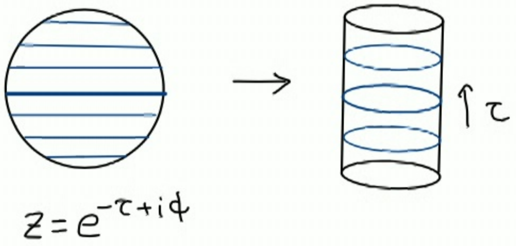




The energy conjugate to the Rindler time is diagonalized in the celestial basis. For massless states

$$\tilde{\mathcal{A}}(\Delta_i, z_i, \bar{z}_i) = \prod_{i=1}^{n}\left(\int_0^\infty d\omega\, \omega^{\Delta_i-1}\right) A(\omega_i, z_i, \bar{z}_i)$$

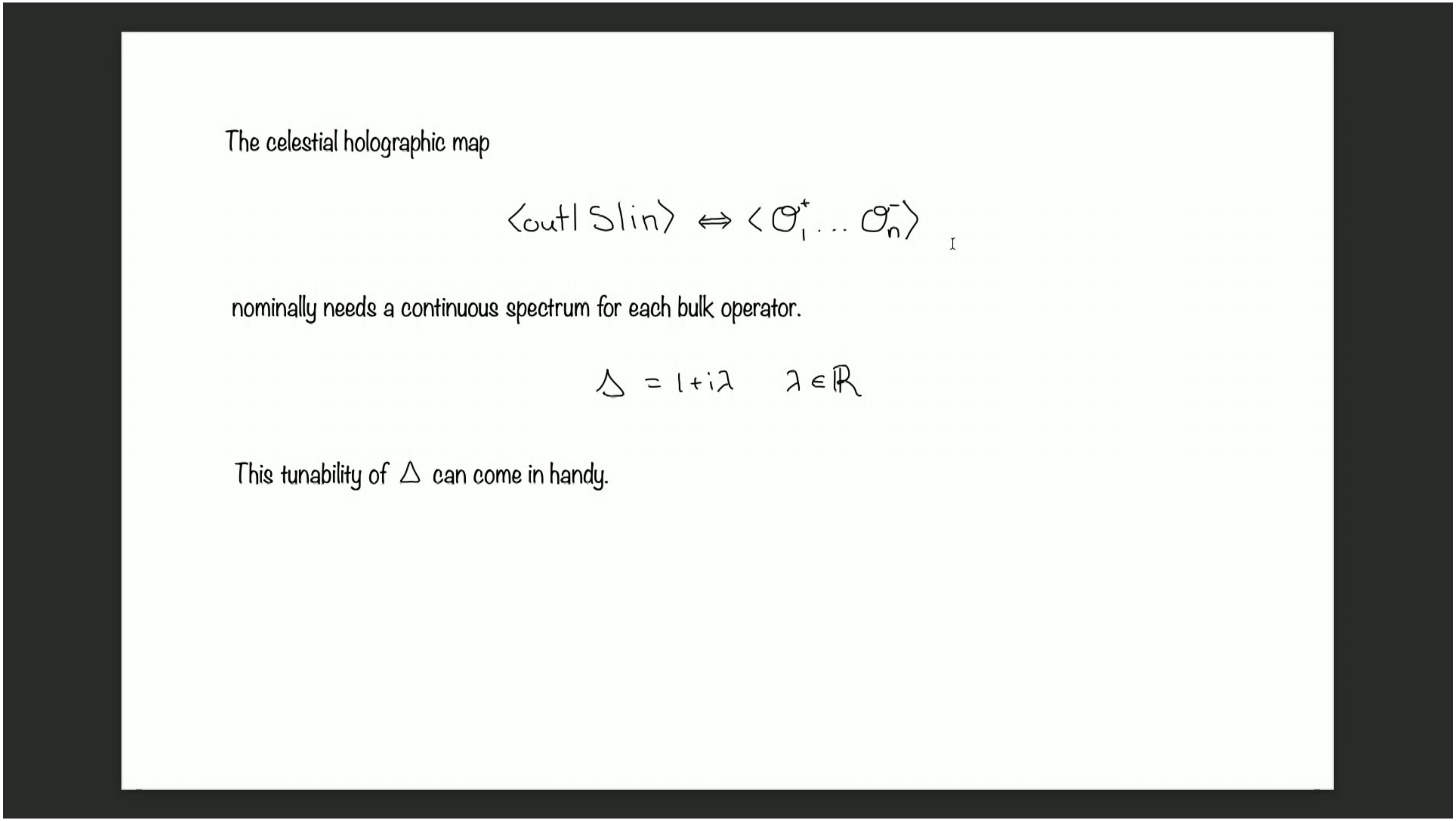
manifestly transforms like a correlator of quasi-primary under the Lorentz group.

$J_i$ = ith helicity

$$\tilde{\mathcal{A}}\left(\Delta_i, \frac{az_i+b}{cz_i+d}, \frac{\bar{a}\bar{z}_i+\bar{b}}{\bar{c}\bar{z}_i+\bar{d}}\right) = \prod_{i=1}^{n}\left[(cz_i+d)^{\Delta_i+J_i}(\bar{c}\bar{z}_i+\bar{d})^{\Delta_i-J_i}\right]\tilde{\mathcal{A}}(\Delta_i, z_i, \bar{z}_i)$$

These get promoted to Virasoro primaries when we couple to gravity.

What is the central charge?



## The celestial holographic map

$$\langle \text{out} | S | \text{in} \rangle \Leftrightarrow \langle \mathcal{O}_1^{+} \dots \mathcal{O}_n^{-} \rangle$$

nominally needs a continuous spectrum for each bulk operator.

$$\Delta = 1 + i\lambda \qquad \lambda \in \mathbb{R}$$

This tunability of $\Delta$ can come in handy.

Consider the $SL(2,\mathbb{C})$ multiplet structure. A primary state:

$$L_1|h,\bar{h}\rangle = \bar{L}_1|h,\bar{h}\rangle = 0 \qquad h=\tfrac{1}{2}(\Delta+J) \quad \bar{h}=\tfrac{1}{2}(\Delta-J)$$

will have a primary descendant at level-k when:

$$L_1(L_{-1})^k|h,\bar{h}\rangle = -k\overset{\swarrow 0}{(2h+k-1)}(L_{-1})^{k-1}|h,\bar{h}\rangle = 0$$

similarly for $\bar{L}_{-1}$. When both conditions are met we get nested primaries:

$$\begin{tikzcd}
& \left|\frac{1-k}{2},\frac{1-\bar{k}}{2}\right\rangle \arrow[dl, "(\bar{L}_{-1})^{\bar{k}}"'] \arrow[ddr, "(L_{-1})^k"] & \\
\left|\frac{1-k}{2},\frac{1+\bar{k}}{2}\right\rangle \arrow[ddr] & & \\
& & \left|\frac{1+k}{2},\frac{1-\bar{k}}{2}\right\rangle \arrow[dl] \\
& \left|\frac{1+k}{2},\frac{1+\bar{k}}{2}\right\rangle &
\end{tikzcd}$$

$k,\bar{k}\in\mathbb{Z}_>$

When we analytically continue to such $\Delta$ we find that the primary descendants decouple, as one would expect in a radially quantized 2D CFT.

$$H^{-2p+4}(z,\bar{z}) = \lim_{\varepsilon\to 0} \varepsilon\, \mathcal{O}_{-2p+4+\varepsilon,\,2}(z,\bar{z}) = \sum_{n=1-p}^{p-1} \frac{\bar{z}^{p-n-1}}{\Gamma(p-n)\Gamma(p+n)} W^{p}_{n}(z)$$

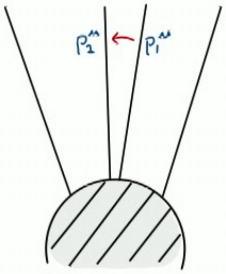
We call this the conformally soft sector. It contains an infinite tower of symmetry generators.

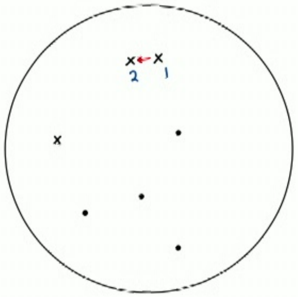
$$\hat{w}^{p}_{n} = \oint \frac{dz}{2\pi i} W^{p}_{n}(z) \qquad [\hat{w}^{p}_{m}, \hat{w}^{q}_{n}] = [m(q-1) - n(p-1)]\,\hat{w}^{p+q-2}_{m+n}$$

Indeed this radial evolution picture is further borne out by our ability to extract consistent symmetry algebras from the OPEs.

$$\mathcal{O}^{+}_{\Delta_1+2}(z_1,\bar{z}_1)\,\mathcal{O}^{+}_{\Delta_2+2}(z_2,\bar{z}_2) \sim -\frac{\kappa}{2}\frac{\bar{z}_{12}}{z_{12}}B(\Delta_1-1,\Delta_2-1)\;\mathcal{O}^{+}_{\Delta_1+\Delta_2+2}\;(z_2,\bar{z}_2)$$

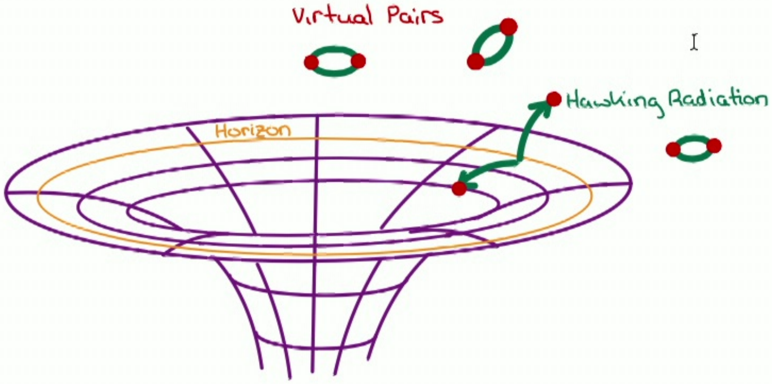
$$[A,B](z) = \oint_z \frac{dx}{2\pi i}\,A(x)B(z)$$





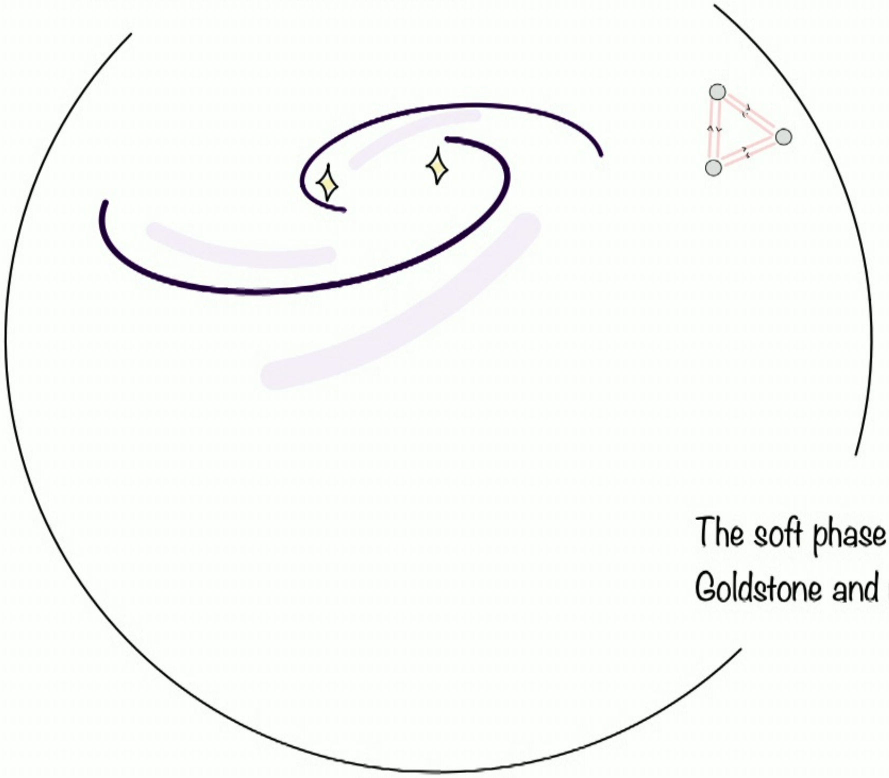
How should we analytically continue $z, \bar{z}$?

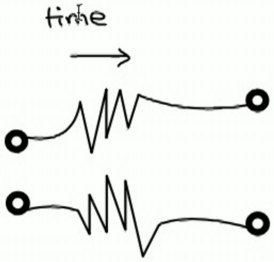
The CCFT currents correspond to symmetry enhancements in the bulk. These infinitely many symmetries should also constrain black hole evaporation.



In 2102.03850 we showed that super translation dressing modes capture non-trivial backreaction effects and associated Lyupanov behavior. Can we carry over what we've learned to CCFT?

Scattering processes are accompanied by radiation which induces dynamical vacuum transitions.

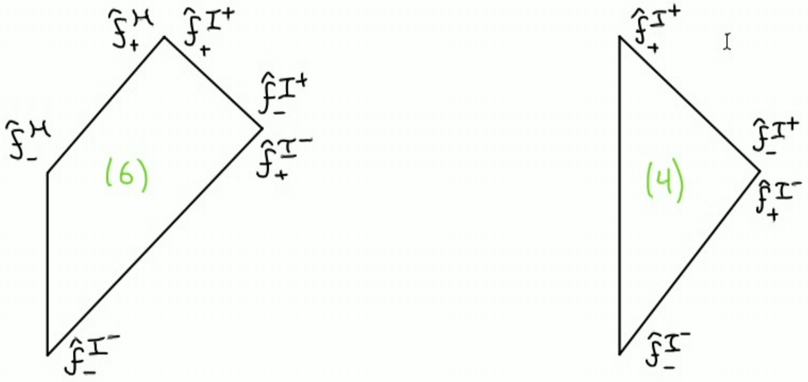




$$\Delta h_{zz} = 2r D_z^2(\hat{f}_+ - \hat{f}_-)$$

The soft phase space includes paired Goldstone and memory modes.

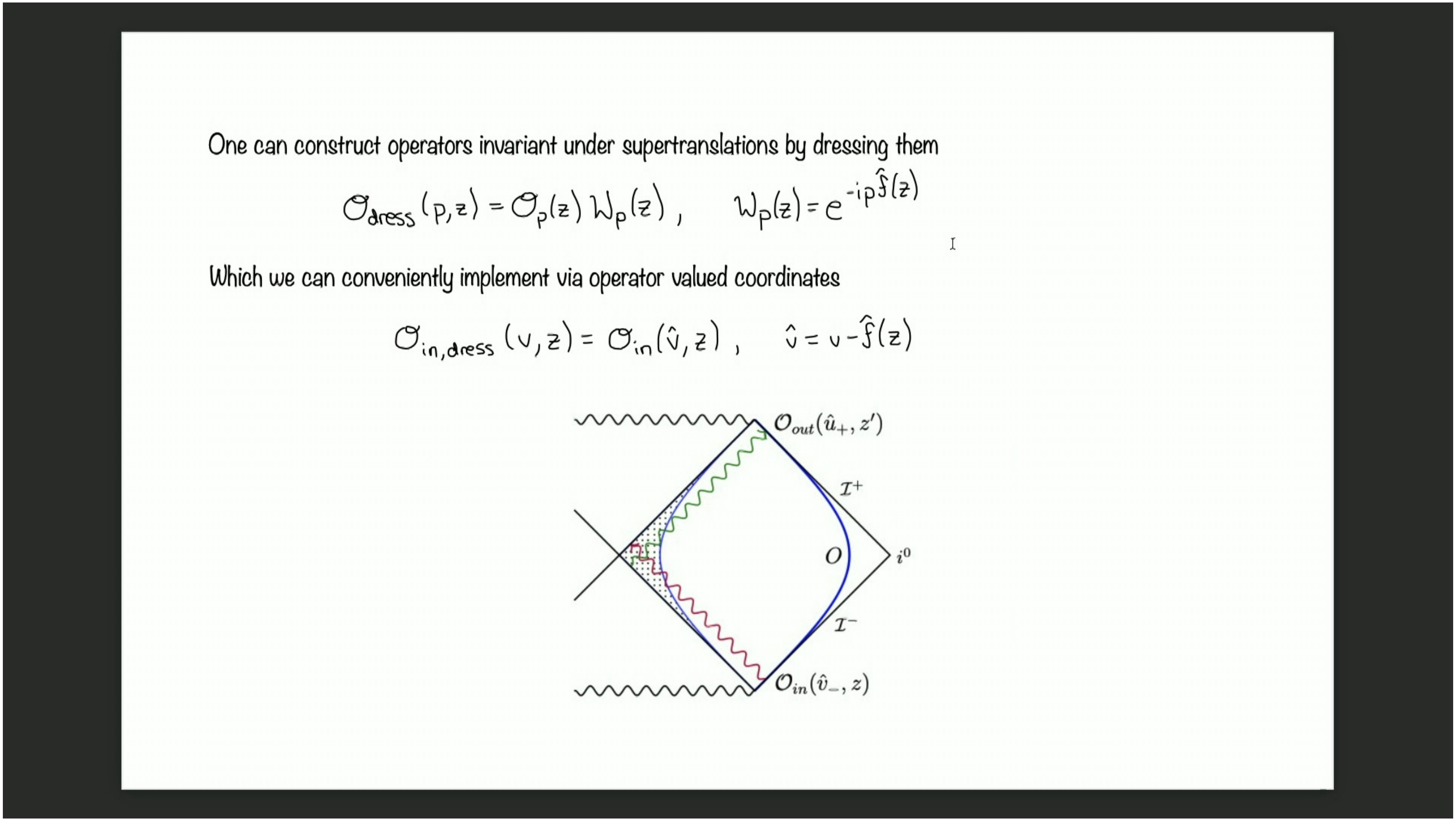
## Introducing a horizon enlarges the soft sector of phase space.



(-2) BMS invariance: $\hat{f}_\pm^{\mathcal{I}^-} \mapsto \hat{f}_\pm^{\mathcal{I}^-} + f$, $\hat{f}_\pm^{\mathcal{I}^+} \mapsto \hat{f}_\pm^{\mathcal{I}^+} + f$, $\hat{f}_\pm^{\mathcal{H}} \mapsto \hat{f}_\pm^{\mathcal{H}} + f$

(-2) CPT invariance: $\hat{f}_-^{\mathcal{I}^+} = \hat{f}_+^{\mathcal{I}^-}$

HPS: Black holes have soft hair!

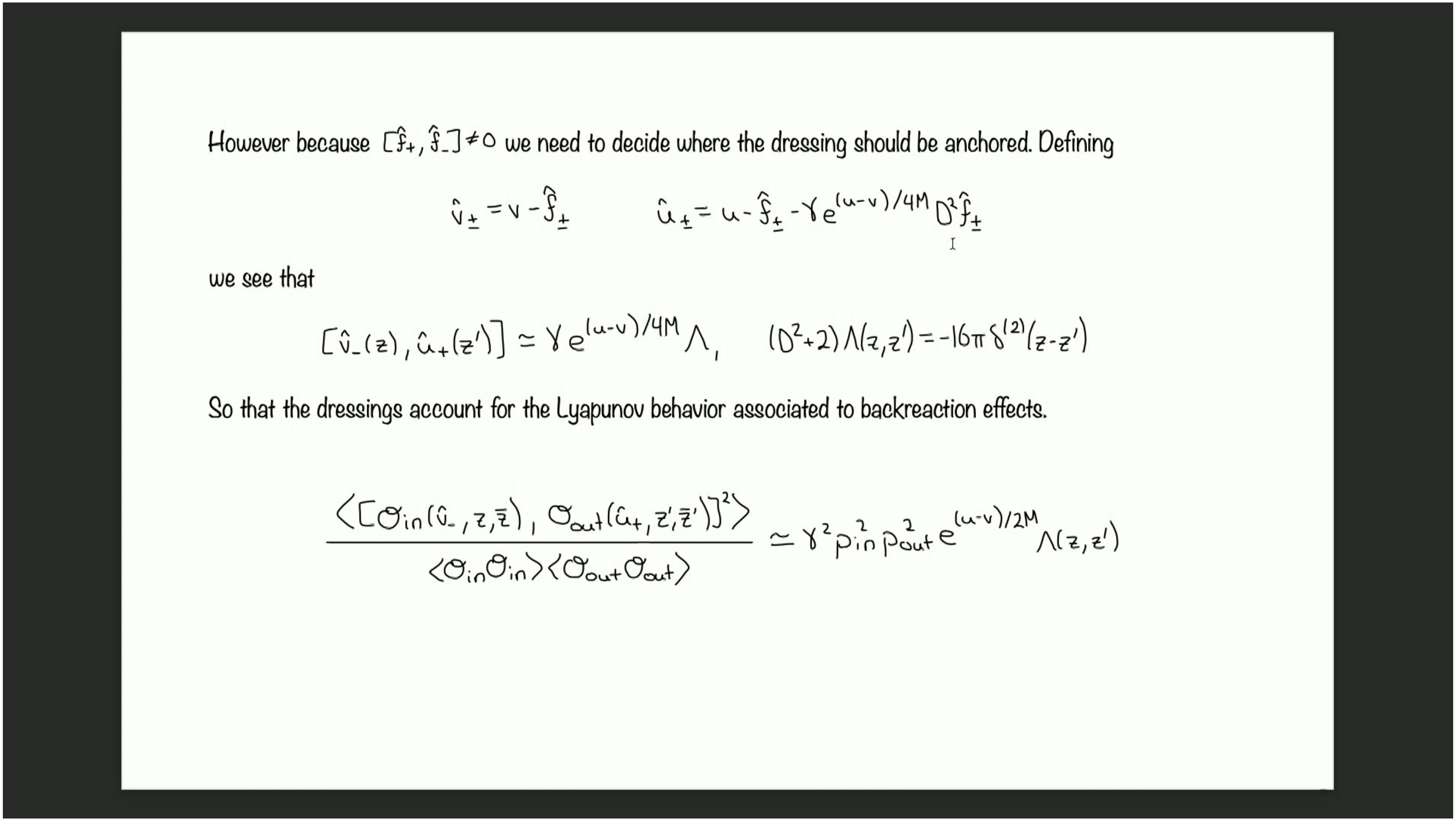


One can construct operators invariant under supertranslations by dressing them

$$\mathcal{O}_{dress}(p,z) = \mathcal{O}_p(z)\, W_p(z)\,, \qquad W_p(z) = e^{-ip\hat{f}(z)}$$

Which we can conveniently implement via operator valued coordinates

$$\mathcal{O}_{in,dress}(v,z) = \mathcal{O}_{in}(\hat{v},z)\,, \qquad \hat{v} = v - \hat{f}(z)$$



However because $[\hat{f}_+, \hat{f}_-] \neq 0$ we need to decide where the dressing should be anchored. Defining

$$\hat{v}_\pm = v - \hat{f}_\pm \qquad \hat{u}_\pm = u - \hat{f}_\pm - \gamma e^{(u-v)/4M} D^2 \hat{f}_\pm$$

we see that

$$[\hat{v}_-(z), \hat{u}_+(z')] \simeq \gamma e^{(u-v)/4M} \Lambda, \qquad (D^2+2)\Lambda(z,z') = -16\pi \delta^{(2)}(z-z')$$

So that the dressings account for the Lyapunov behavior associated to backreaction effects.

$$\frac{\langle [\mathcal{O}_{in}(\hat{v}_-, z, \bar{z}), \mathcal{O}_{out}(\hat{u}_+, z', \bar{z}')]^2 \rangle}{\langle \mathcal{O}_{in}\mathcal{O}_{in}\rangle \langle \mathcal{O}_{out}\mathcal{O}_{out}\rangle} \simeq \gamma^2 \rho_{in}^2 \rho_{out}^2 e^{(u-v)/2M} \Lambda(z,z')$$

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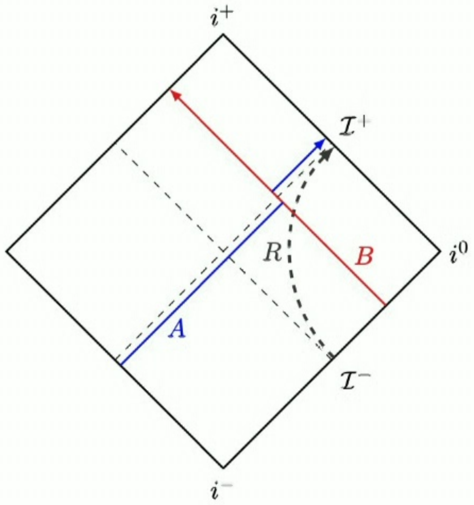
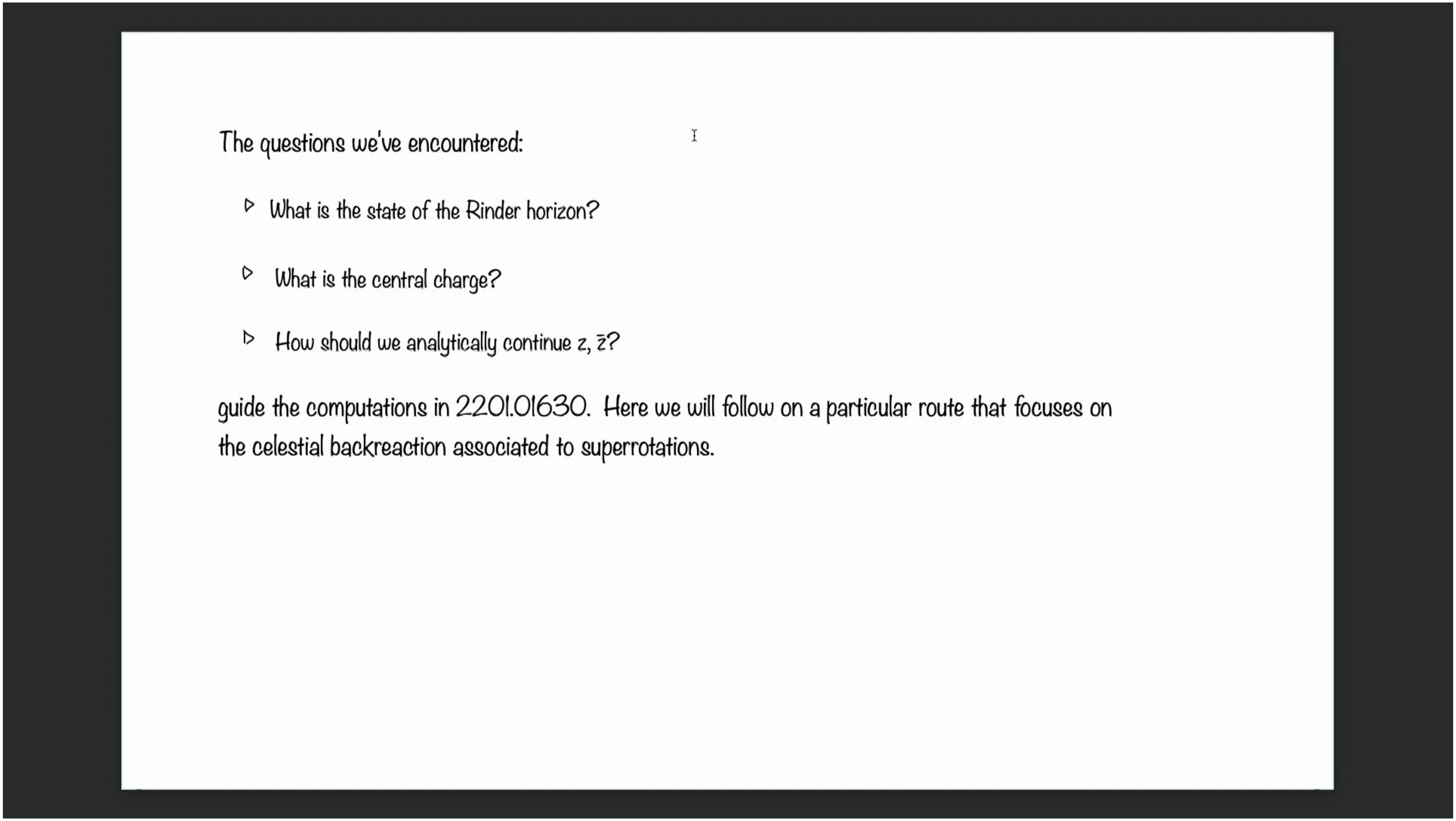
Can we identify similar behavior in CCFT? The fact that radial evolution in the CCFT maps to time evolution for a Rinder observer in the bulk hints at a way out.



Shockwave shifts can have drastic effects from the perspective of a fiducial Rinder observer.

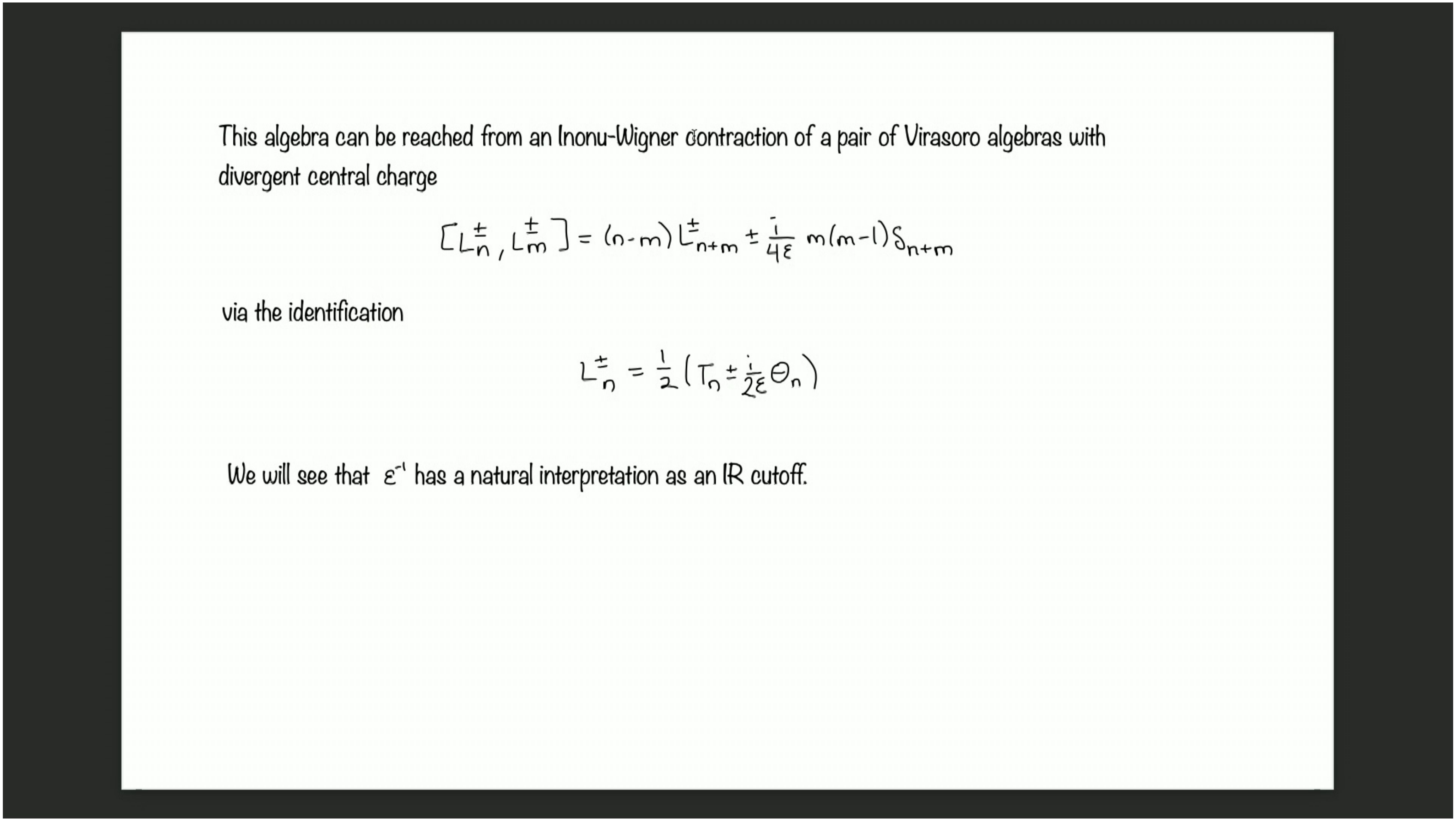
We can look for signatures of chaos without a horizon!



The questions we've encountered:

- What is the state of the Rinder horizon?
- What is the central charge?
- How should we analytically continue $z, \bar{z}$?

guide the computations in 2201.01630. Here we will follow on a particular route that focuses on the celestial backreaction associated to superrotations.



This algebra can be reached from an Inonu-Wigner contraction of a pair of Virasoro algebras with divergent central charge

$$[L^{\pm}_{n}, L^{\pm}_{m}] = (n-m)L^{\pm}_{n+m} \pm \frac{i}{4\varepsilon} m(m-1)\delta_{n+m}$$

via the identification

$$L^{\pm}_{n} = \frac{1}{2}\left(T_n \pm \frac{i}{2\varepsilon}\Theta_n\right)$$

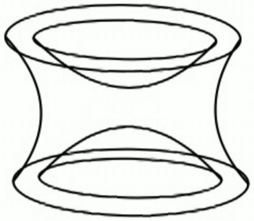
We will see that $\varepsilon^{-1}$ has a natural interpretation as an IR cutoff.

Recall super rotations preserve the leaves of a hyperbolic foliation of $\mathbb{R}^{1,3}$. Finitely superrotated vacua look like Bañados geometries on these leaves.

$$ds^2 = -d\eta^2 + \eta^2(d\rho^2 + (\sinh^2\rho + e^{-2\rho}V)2\gamma_{z\bar{z}}dz\,d\bar{z} + (1-e^{-2\rho})(\Theta^+_{zz}dz^2 + \Theta^+_{\bar{z}\bar{z}}d\bar{z}^2) \qquad i^+$$

$$ds^2 = d\tilde{\rho}^2 + \tilde{\rho}^2(-d\tilde{\eta}^2 + (\cosh^2\tilde{\eta} + e^{-2\tilde{\eta}}V)2\gamma_{z\bar{z}}dz\,d\bar{z} - (1+e^{-2\tilde{\eta}})(\Theta^-_{zz}dz^2 + \Theta^-_{\bar{z}\bar{z}}d\bar{z}^2) \qquad i^0$$

If we introduced an IR cutoff by setting the (A)dS radius to $R = \frac{2}{\varepsilon}$ we would have two stress tensors.



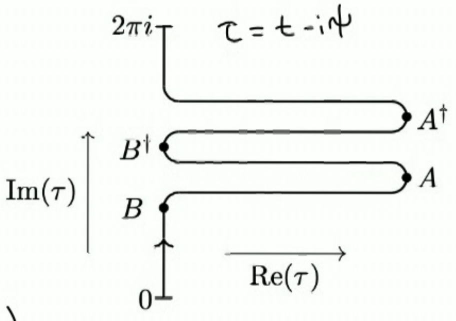
$$[L^\pm_n, L^\pm_m] = (n-m)L^\pm_{n+m} \pm \frac{i}{4\varepsilon}m(m-1)\delta_{n+m}$$

Restricting to a sector of operators that commute with the $L_{\bar{n}}$ gives a system with large c. This sector effectively includes a backreaction such that near a local operator the Goldstone mode Z(z) behaves as

$$-\frac{i}{2\varepsilon}\{Z,z\} = \frac{h}{z^2} \quad \Longleftrightarrow \quad Z(z) = z^{1-2i\varepsilon h}$$

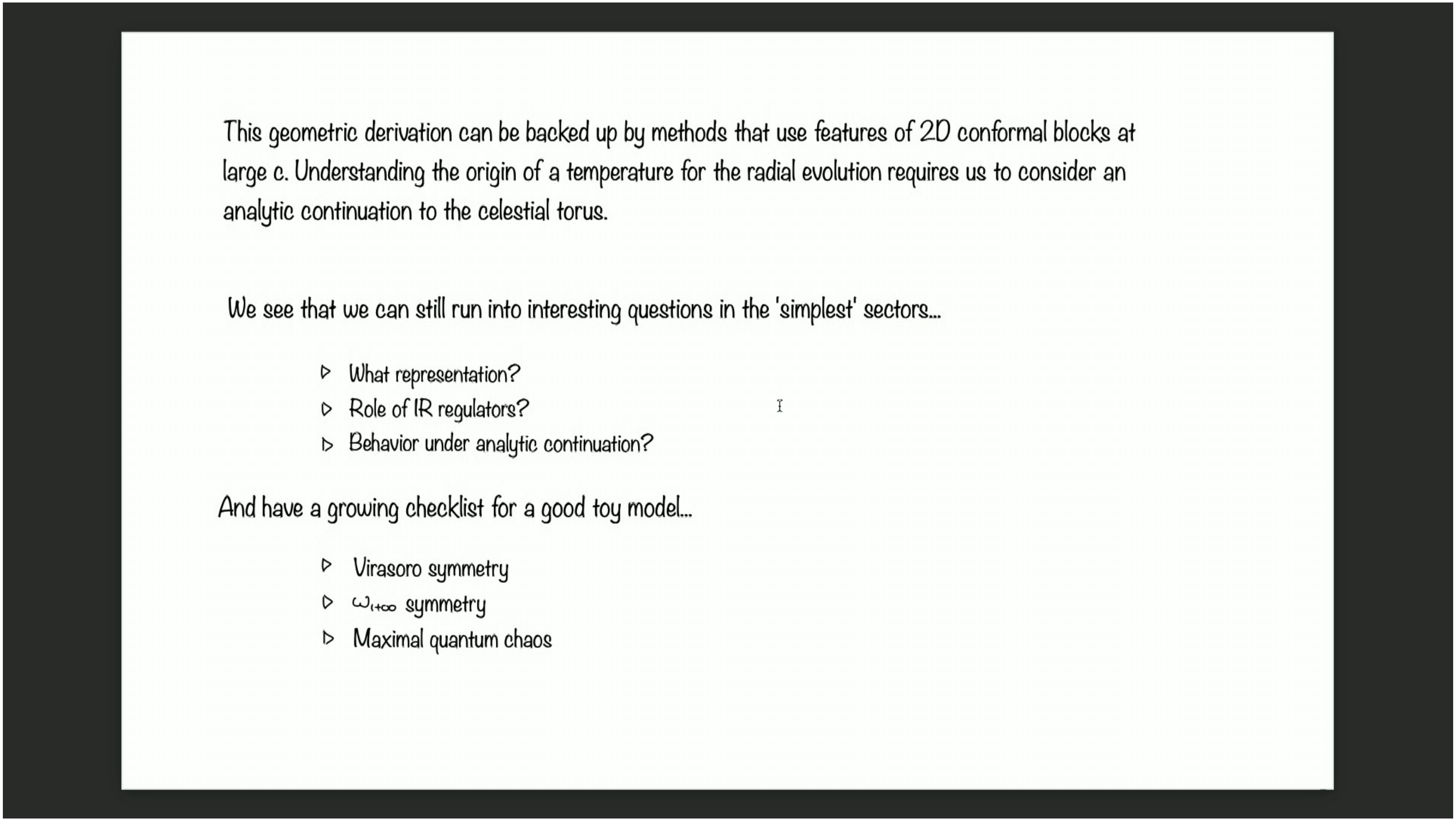
giving a defect on the celestial sphere. We can use this to evaluate monodromies when we move operators around one another for complexified

$$(z,\bar{z}) = (e^{-\tau+i\phi}, e^{-\tau-i\phi})$$



Which we interpret as Lyapunov behavior of Rinder time OTOCs.

$$[A(t_2), B(t_1)] \simeq 2\pi i\varepsilon\, e^{t_2-t_1}\partial_{t_1}B(t_1)\,\partial_{t_2}A(t_2)$$



This geometric derivation can be backed up by methods that use features of 2D conformal blocks at large c. Understanding the origin of a temperature for the radial evolution requires us to consider an analytic continuation to the celestial torus.

We see that we can still run into interesting questions in the 'simplest' sectors...

- What representation?
- Role of IR regulators?
- Behavior under analytic continuation?

And have a growing checklist for a good toy model...

- Virasoro symmetry
- $w_{1+\infty}$ symmetry
- Maximal quantum chaos

Thank You!