Title: Probing Black Holes with Gravitational Waves and Shadows

Speakers: Sebastian Volkel

Series: Strong Gravity

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Abstract: In this talk I will outline recent attempts to probe black holes in the strong gravity regime. The access to gravitational wave emission from binary black hole mergers and images of supermassive black holes allow for new tests of general relativity. After reviewing recent activities, I will outline how quasi-normal modes and shadow images can be used to study possible deviations from general relativity. Finally, I will discuss open problems that need to be addressed in the future.

PROBING BLACK HOLES WITH GRAVITATIONAL WAVES AND SHADOWS

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Strong Gravity Seminar
Perimeter Institute for Theoretical Physics
24 February 2022

ERC-2018-COG GRAMS 815673

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Motivation for Gravity beyond Einstein

General relativity (GR) passed all tests with *flying colors*:

- many “**classical tests**” for more than 100 years
- binary pulsar systems studied for half a century (e.g. Hulse-Taylor system)
- “stellar” mass compact objects with **gravitational waves** by LIGO/Virgo
- supermassive **black hole shadow** with the Event Horizon Telescope

Open problems may challenge GR:

- origin of **dark energy** and dark matter
- the **Hubble tension** (early – late time cosmology)
- **quantum gravity** in general (information loss problem, firewalls)

Exploring Gravity with Black Holes

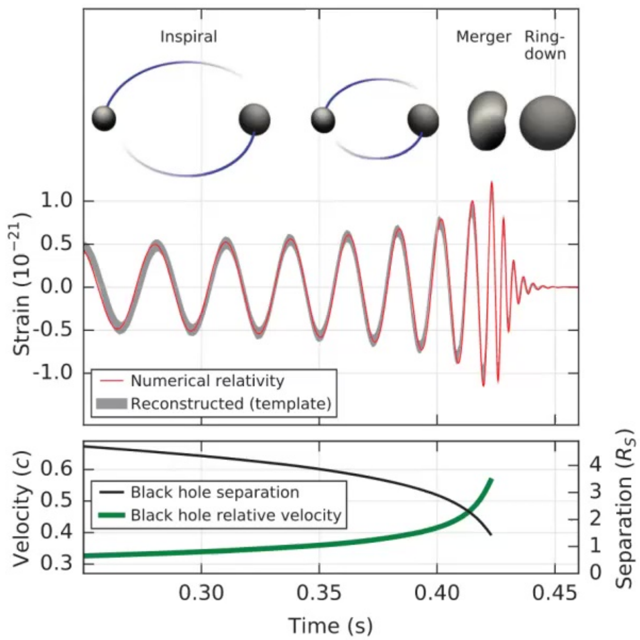
Black holes (BHs):

- most extreme gravitational objects one expects in current universe
- **strong field dynamics** are one current frontier
- binary black hole mergers **increasingly accessible** (~ 100 mergers LIGO/Virgo)

Clear Expectations from GR:

- no-hair/uniqueness theorems predict **Kerr(-Newman)** BHs
- perturbed BHs ring with characteristic **quasi-normal modes** (QNMs)
- binary merger ringdown regime allows for **BH spectroscopy**

Birth of Gravitational Wave Astronomy



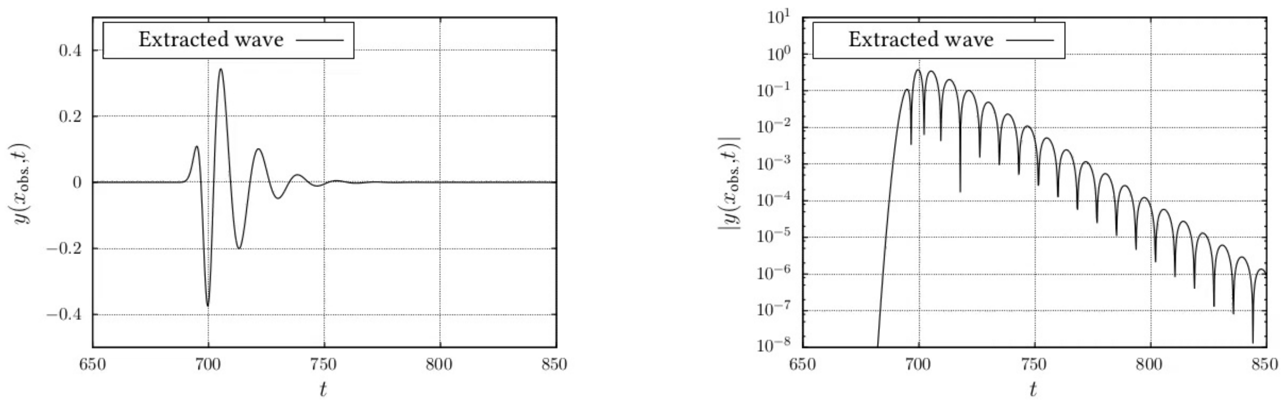
Stages of a binary black hole merger for GW150914, taken from Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration) Phys. Rev. Lett. 116, 061102, https://doi.org/10.1103/PhysRevLett.116.061102

WHAT ARE (QUASI-)NORMAL MODES?

Basically a sum of (damped) oscillations:

$$y(t) = \sum_{l,m,n} A_{l,m,n} \exp\left(i\omega_{l,m,n} t\right), \tag{1}$$

- excitation coefficients depend on initial data (how one perturbs the system)
- quasi-normal modes (QNM) (frequencies) **only** depend on the unperturbed system (inherent property)



Time evolution of perturbations that scattered with a black hole as observed far away.

The Simplest Case

Qualitatively, most gravitational features can already be found from:

$$\Box_{g_{\mathrm{BH}}} \varphi = (-g)^{-1/2} \partial_\mu \left[(-g)^{1/2} g^{\mu\nu} \partial_\nu \right] \varphi = 0 \tag{2}$$

After several steps one finds
(Schwarzschild, spherical harmonics, Fourier transform, tortoise coordinate r^*)

$$\frac{\mathrm{d}^2}{\mathrm{d}r^{*2}} \psi + \left[\omega^2 - V_l(r) \right] \psi = 0, \tag{3}$$

with

$$V_l(r) = \left(1 - \frac{2M}{r}\right) \left(\frac{l(l+1)}{r^2} + \frac{2M}{r^3} \right), \qquad \partial_r = \left(-\frac{g_{rr}}{g_{tt}} \right)^{1/2} \partial_{r^*}. \tag{4}$$

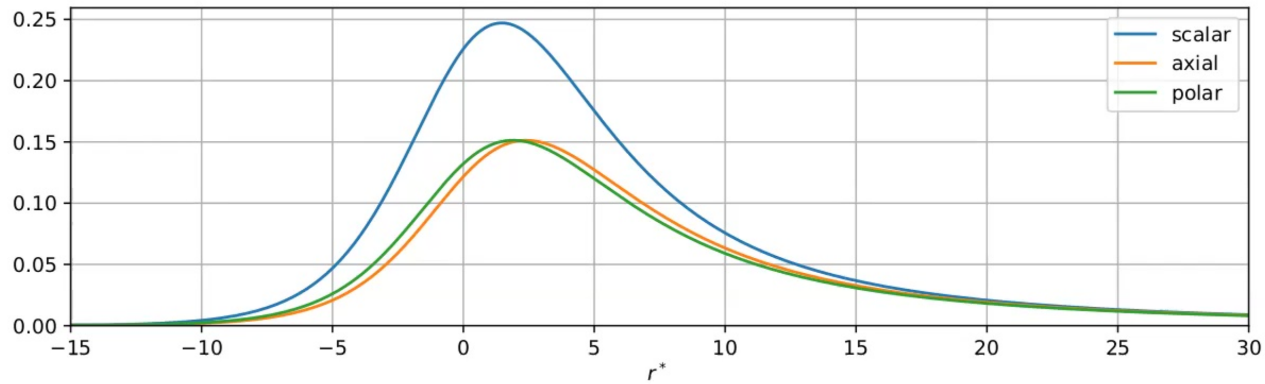
Assuming purely outgoing/ingoing boundary conditions defines **QNM spectrum** ω.

PERTURBATION POTENTIALS

$$V_l^{\text{scalar}}(r) = \left(1 - \frac{2M}{r}\right)\left(\frac{l(l+1)}{r^2} + \frac{2M}{r^3}\right),$$

Regge-Wheeler: $$V_l^{\text{axial}}(r) = \left(1 - \frac{2M}{r}\right)\left(\frac{l(l+1)}{r^2} - \frac{6M}{r^3}\right),$$

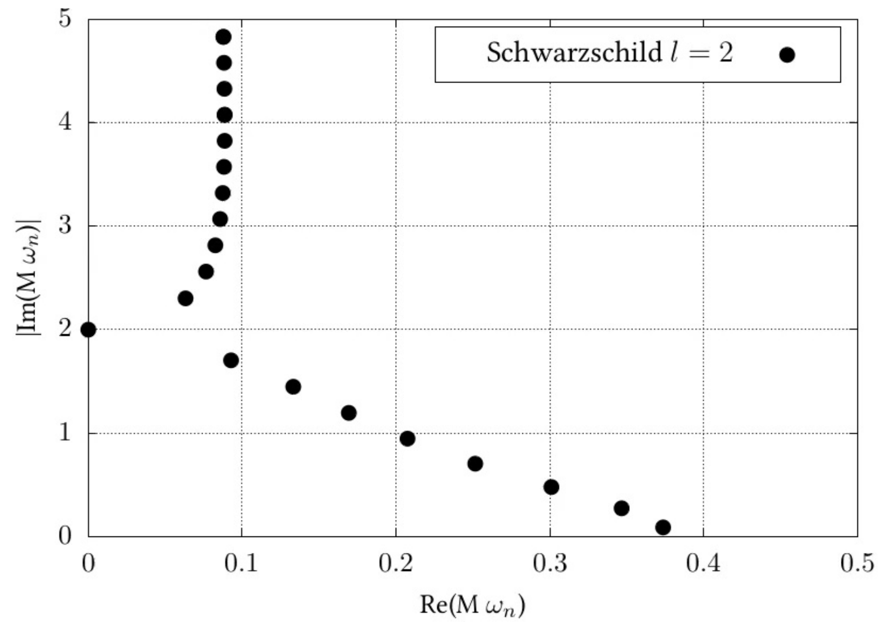
Zerilli: $$V_l^{\text{polar}}(r) = \left(1 - \frac{2M}{r}\right)\frac{2\lambda^2(\lambda+1)r^3 + 6\lambda^2 M r^2 + 18\lambda M^2 r + 18M^3}{r^3(\lambda r + 3M)^2}.$$



Perturbation potentials for $l = 2$ as function of the tortoise coordinate r^*.

SCHWARZSCHILD SPECTRUM

Axial and polar potentials are isospectral!



Axial/polar quasi-normal modes on the complex plane for $l = 2$. Asymptotic branch continues parallel to y-axis. Shown spectrum based on Leaver's method.

Gravitational Perturbations

Theory dependent: gravitational theory needs to be specified

In GR:

- Schwarzschild: Regge-Wheeler (1957), Zerilli (1970)
- Kerr: Teukolsky (1972)

Theories beyond GR can in principle:

- include Schwarzschild BHs and have **same** perturbations
- include Schwarzschild BHs and have **different** perturbations
- have non-Schwarzschild BHs/perturbations
- not much known for rotating BH perturbations
- or do whatever . . .

Some Comments

Qualitative features of GR:

- perturbations are separable
- no additional fields or couplings
- axial and polar perturbations are isospectral

Phenomenological modifications:

- potential terms are being modified
- couplings between known and additional fields are introduced

Theory dependent modifications:

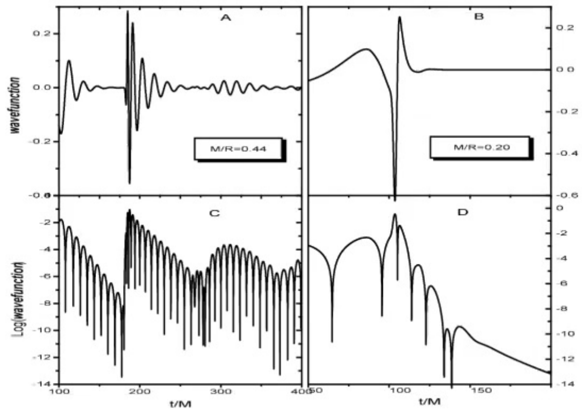
- "just" start from a theory, compute background and perturbations
- clear connections between observations, metric and theory

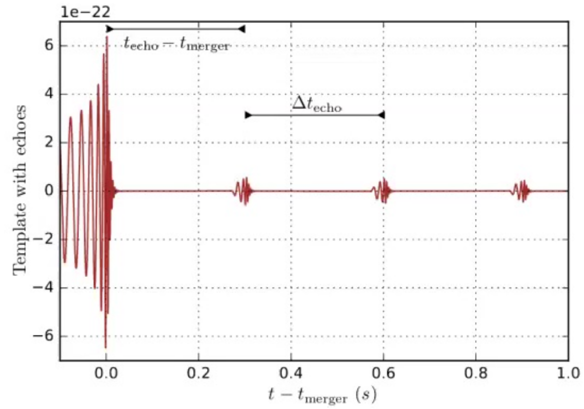
EXOTIC COMPACT OBJECTS

Popular and disputed hot topic: "echoes"

- modified horizon conditions or ultra compact objects reflect GWs
- repeated reflections would occur generically, smoking gun effect

Evolution **black hole** like at **early times**, but **different at late times**:





Left: Constant density stars, Kokkotas, Les Houches School of Physics, (1995).
Right: Horizon scale reflection, Abedi, Dykaar and Afshordi, Phys. Rev. D 96, 082004 (2017).

Works on the Inverse QNM Problem

Under what assumptions can one recover properties of the metric?

Can one recover the potential or couplings to additional fields?

PROJECT I

Basic overview of SV and Barausse, PRD 102 084025 (2020)

Axial like perturbations with Rezzolla-Zhidenko metric [1] background

$$\frac{\mathrm{d}^2}{\mathrm{d}r^{*2}} Z + \left[\omega^2 - V_l(r)\right] Z = 0 \,, \tag{5}$$

also include parametrized modification of the potential (K)

$$V_l(r) = \frac{l(l+1)}{r^2} N^2(r) - \frac{K}{r} \frac{\mathrm{d}}{\mathrm{d}r^*} \frac{N^2(r)}{B(r)} \,, \tag{6}$$

compute QNMs with higher order WKB method[2].

[1] Rezzolla and Zhidenko PRD 90 084009 (2014)
[2] Konoplya, PRD 68 024018, (2003)

BACKGROUND METRIC

We use the Rezzolla-Zhidenko (RZ) metric[3]

- parametrization for spherically symmetric and static black holes
- continued fraction expansion for $\tilde{A}(x)$ and $\tilde{B}(x)$
- relation to PPN parameters β and γ possible

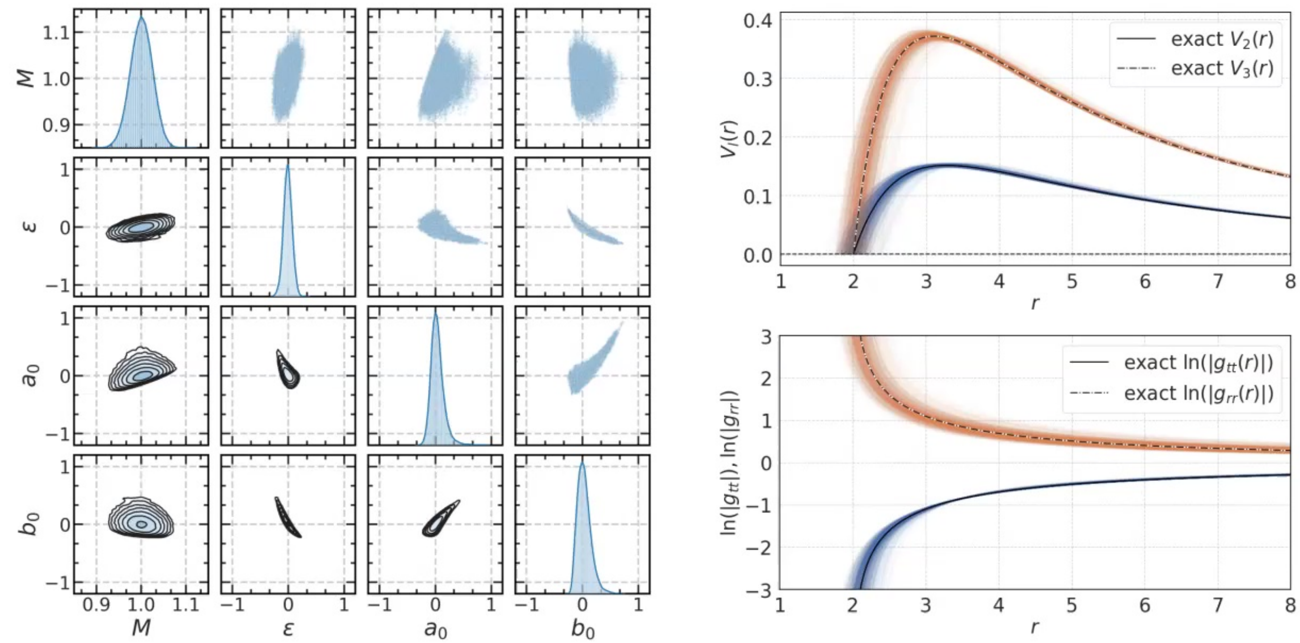
$$\mathrm{d}s^2 = -N^2(r)\mathrm{d}t^2 + \frac{B^2(r)}{N^2(r)}\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2, \qquad x \equiv 1 - \frac{r_0}{r}, \qquad N^2 = xA(x), \tag{7}$$

$$A(x) = 1 - \varepsilon(1-x) + (a_0 - \varepsilon)(1-x)^2 + \tilde{A}(x)(1-x)^3, \tag{8}$$

$$B(x) = 1 + b_0(1-x) + \tilde{B}(x)(1-x)^2. \tag{9}$$

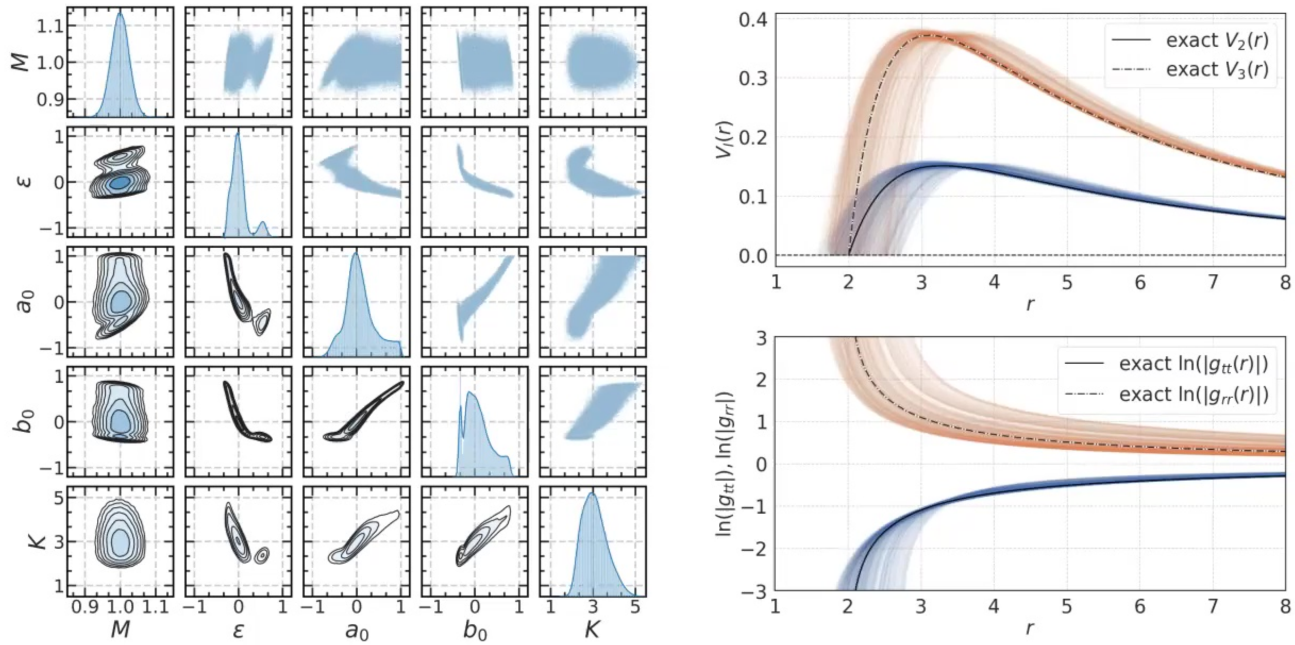
[3] Rezzolla, Zhidenko, Phys. Rev. D 90, 084009, (2014)

Project I



Results for using $l = 2,3$ and $n = 0,1$ QNM spectrum with $\pm 1\,\%$ relative error. Left: MCMC parameter estimation. Right top: Exact (black lines) and reconstructed (color lines) potentials $V_2(r)$ and $V_3(r)$. Right bottom: Exact (black lines) and reconstructed (color lines) metric functions $g_{tt}(r)$ and $g_{rr}(r)$.
SV and E. Barausse, PRD 102 084025 (2020).

PROJECT I



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METHODS OVERVIEW

Basic overview of SV, Franchini and Barausse, `arXiv:2202.08655`

Coupled equations are prototype for rotation and alternative theories

$$f\frac{\mathrm{d}}{\mathrm{d}r}\left(f\frac{\mathrm{d}\mathbf{\Phi}}{\mathrm{d}r}\right)+\left[\omega^2-f\mathbf{V}\right]\mathbf{\Phi}=0, \tag{11}$$

with $f(r)=1-r_0/r$, with r_0 being the location of the event horizon and

$$\mathbf{\Phi}=\left[\Phi^{\text{scalar}},\Phi^{\text{polar}},\Phi^{\text{axial}},\dots\right]. \tag{12}$$

Methods Overview

Parametrization proposed in Refs. [4, 5]

$$V_{ij} = V_{ij}^{\rm GR} + \delta V_{ij}, \qquad \delta V_{ij} = \frac{1}{r_H^2} \sum_{k=0}^{\infty} \alpha_{ij}^{(k)} \left(\frac{r_H}{r}\right)^k . \tag{13}$$

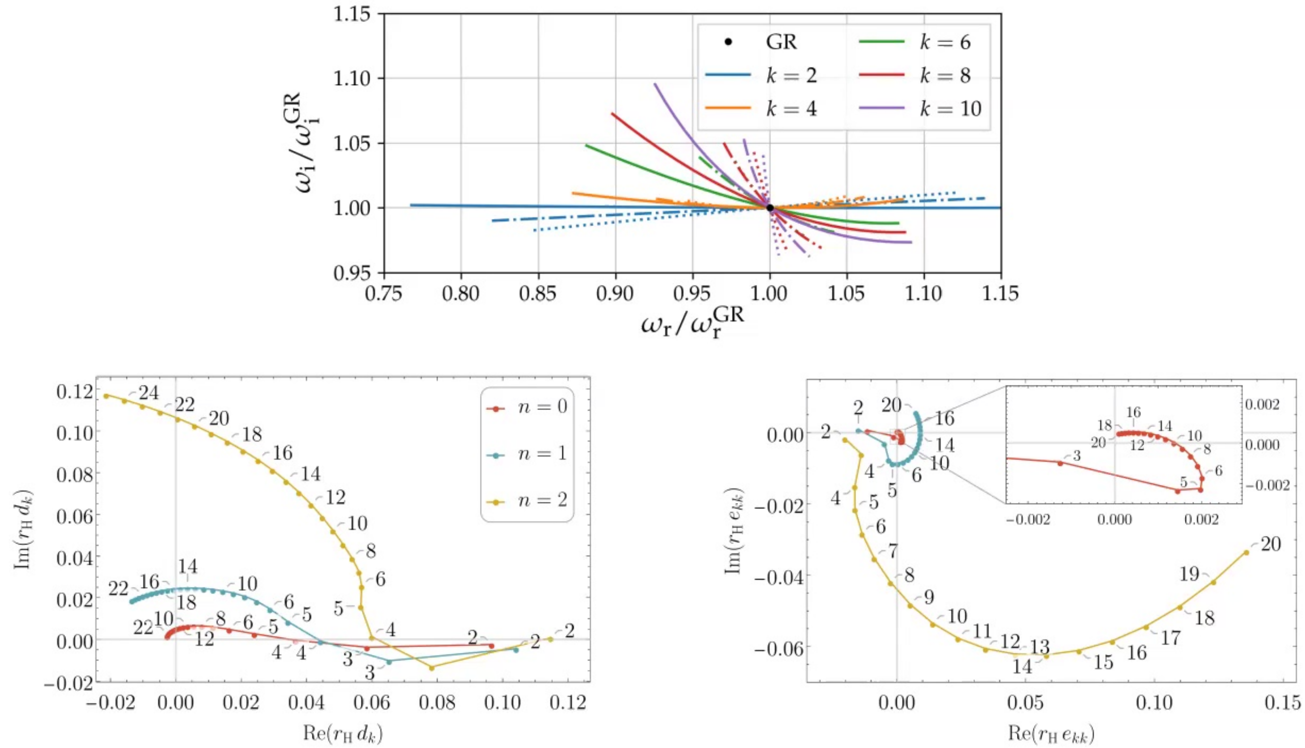
Approximate expression for QNMs given by

$$\omega = \omega^0 + \alpha_{ij}^{(k)} d_{(k)}^{ij} + \alpha_{ij}^{(k)} \partial_\omega \alpha_{pq}^{(s)} d_{(k)}^{ij} d_{(s)}^{pq} + \frac{1}{2} \alpha_{ij}^{(k)} \alpha_{pq}^{(s)} e_{(ks)}^{ijpq} . \tag{14}$$

We extended the coefficients to include the $n = 1, 2$ overtones.

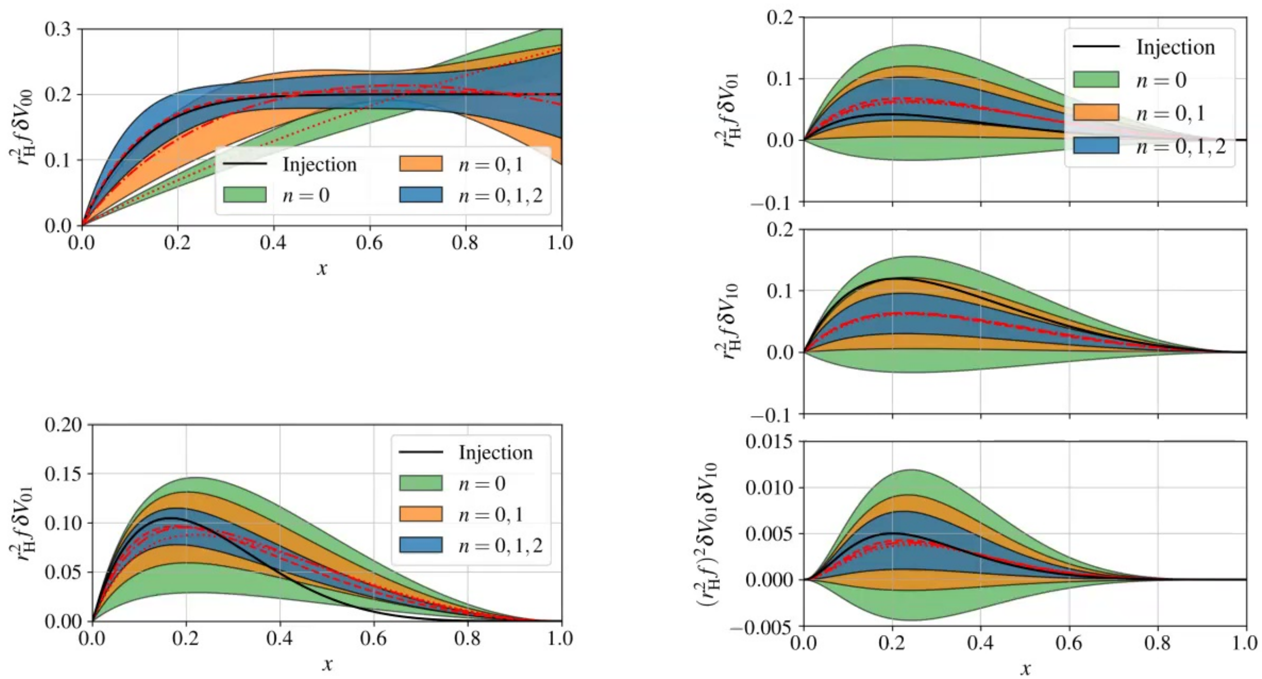
[4] Cardoso, Kimura, Maselli, et al., PRD 99 104077 (2019)
[5] McManus, Berti, Macedo, et al., PRD 100 044061 (2019)

COEFFICIENTS



Modifications of the axial QNM spectrum with respect to GR as function of individual basis functions for the potential only (top). Linear (bottom left) and diagonal quadratic (bottom right) coefficients.

Inverse Problem

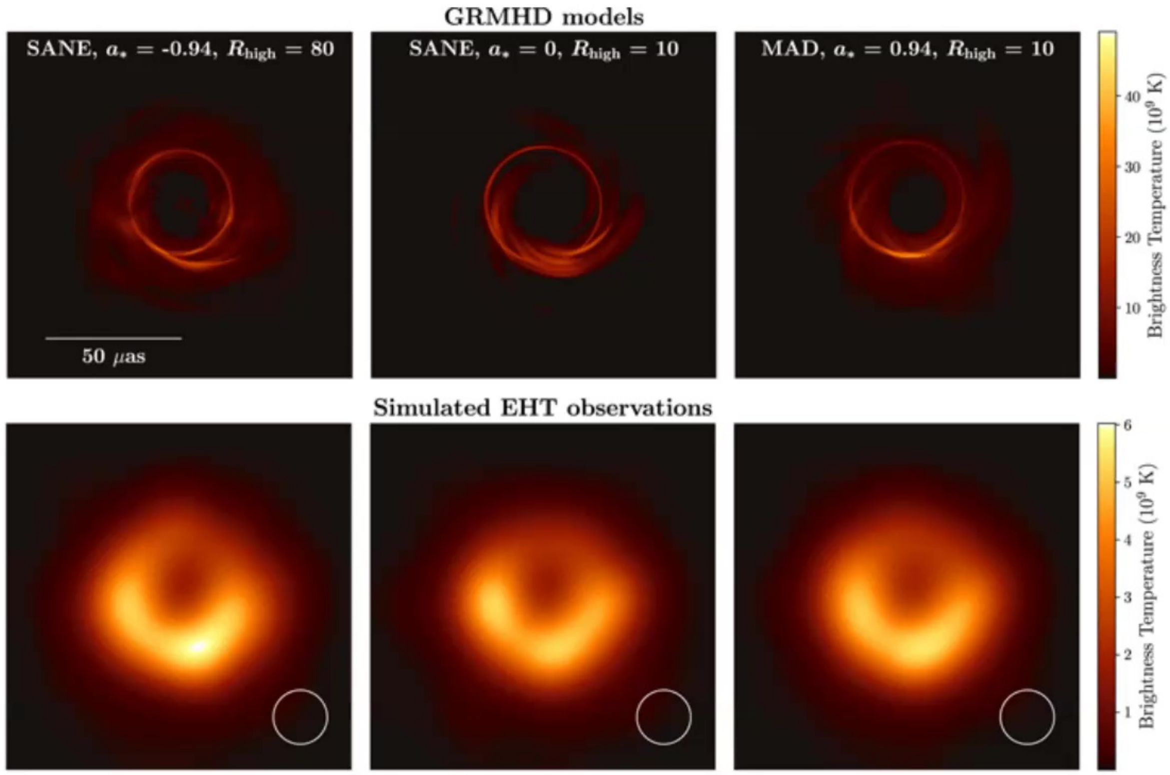


Injection and PCA reconstruction of parametrized deviations of the axial potential and possible presence of coupling functions to a scalar field. **Top left**: only axial potential. **Bottom left**: symmetric coupling functions injecting dCS. **Right:** Two independent coupling functions and their product.

Imaging Black Holes

Black hole from the movie *Interstellar*.

EHT RESULTS



Best fitting reconstructions for the M87* observations. Taken from *The Event Horizon Telescope Collaboration et al* 2019 ApJL 875 L1

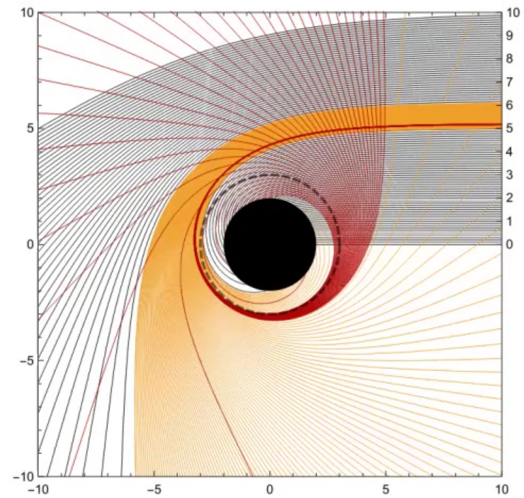
How to Compute an Image?

- solve geodesic equation

$$\frac{\mathrm{d}^2 x^\mu}{\mathrm{d}s^2} + \Gamma^\mu_{\alpha\beta} \frac{\mathrm{d}x^\alpha}{\mathrm{d}s} \frac{\mathrm{d}x^\beta}{\mathrm{d}s} = 0$$

- choose matter distribution (accretion disc) and appropriate radiative transfer equations

- state-of-the-art simulations: general relativistic magneto hydrodynamics (GRMHD).



Geodesics around Schwarzschild black holes. Taken from *Gralla et al.*, Phys. Rev. D 100, 024018 (2019)

EHT tests of the strong-field regime of General Relativity

Using the BH image to test GR?[6]

- involved data analysis, GRMHD simulations, feature extraction,...
- EHT: shadow size robust and identified in image as predicted ($\sim$ 17%)[7]
- Recently: claiming gravitational tests beyond first PN order [8]

How robust are shadow-size measurements to test GR?[9]

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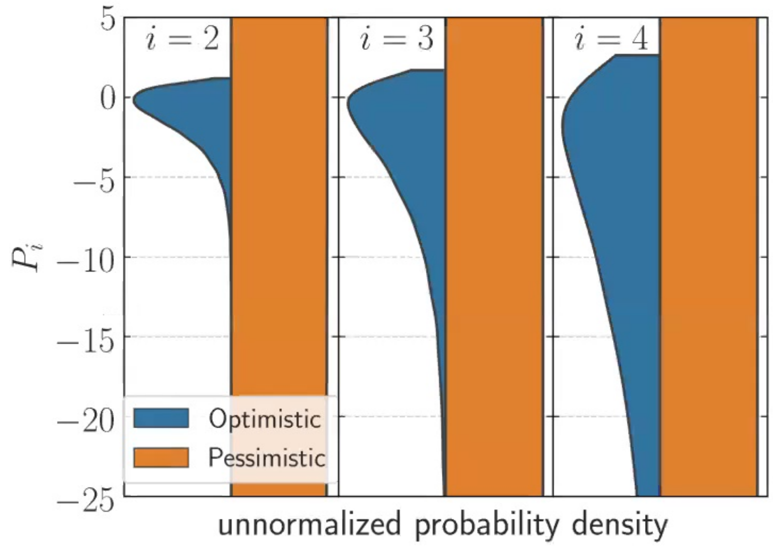
[6] E.g., Cunha, Herdeiro, & Radu, PRL 123, 011101, (more Refs. in paper!)
[7] The Event Horizon Telescope Collaboration et al 2019 ApJL 875 L1
[8] Psaltis et al. (EHT Collaboration), PRL 125, 141104
[9] SV, Barausse, Franchini, and Broderick, CQG 38 (2021) 21, 21LT01, arXiv:2011.06812

Constraints on PN parameters

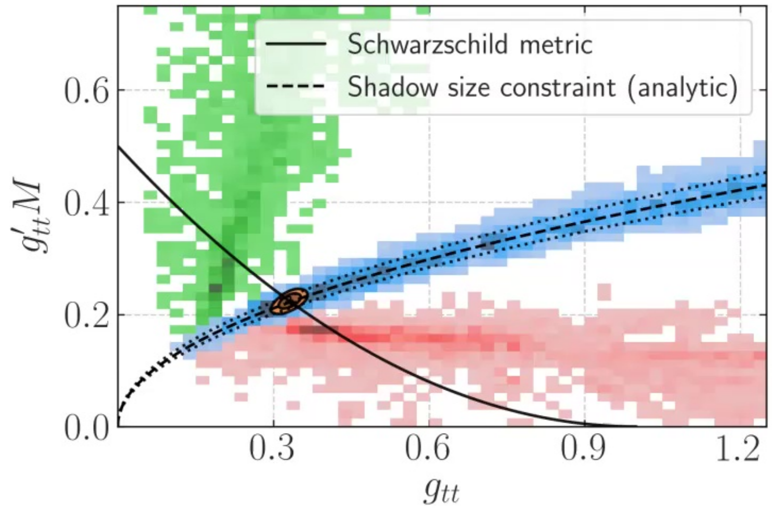
PN metric: $-g_{tt} = 1 - \frac{2M}{r} + \sum_{i=1}^{\infty} P_i \left(\frac{M}{r}\right)^{i+1}$,

"shadow-size": $b_{\text{ph}} = \frac{r_{\text{ph}}}{\sqrt{-g_{tt}(r_{\text{ph}})}}$



Optimistic (blue) and conservative (orange) posterior distributions for the PN coefficients P_i using the 17% relative error margin for the observed shadow-size of M87* as observed by the EHT collaboration with the expected shadow-size predicted by the independent measurement of the BH mass coming from stellar dynamics.

Constraints on the Metric



Posteriors for g_{tt} and g'_{tt}, produced by our MCMC sampler in the case of shadow-size measurement with 5% error, and five free parameters $\varepsilon, a_0, a_1, a_2, a_3$. The posteriors are evaluated at $r = r_{\rm ph}$ (blue), $r = 0.85\, r_{\rm ph}$ (green) and $r = 1.15\, r_{\rm ph}$ (red). Analytic predictions for g_{tt} and g'_{tt} at $r_{\rm ph}$ are provided by the black dashed line and by the black dotted ones (1σ error bars). For comparison, the Schwarzschild metric is shown for $r \in [2M, \infty]$ (black solid). To demonstrate the close relation between QNM and shadow-size measurements, we also show the MCMC results assuming a $l = 2, n = 0$ QNM measured with 5% accuracy (orange).

Can one use the entire image?

Many studies (on shadow size) “throw away” information:

- image is a combination of geometry and “astrophysics”
- realistic inverse problem extremely difficult

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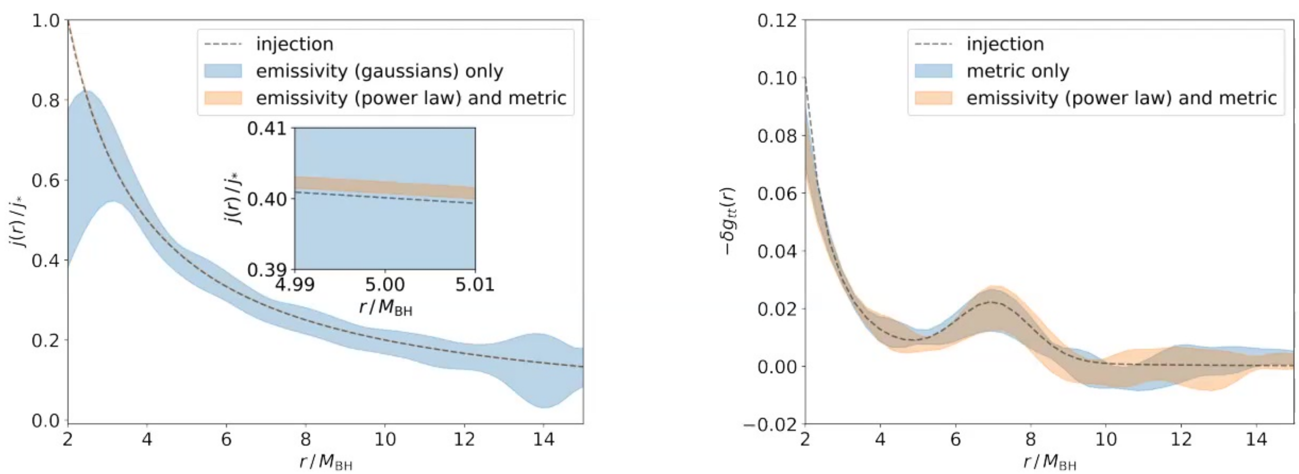
Proof of principle approach in Lara, SV, Barausse, PRD, 104 12, 124041, (2021)

- spherical symmetry, simple accretion model
- expand metric and astrophysics in a general basis
- work out a linear problem to apply principal component analysis (PCA)
- **vary metric and astrophysics at the same time**

RESULTS



Left: Reconstruction of emissivity $j(r)$ for spherical accretion. **Right:** Reconstruction of metric GR deviation $\delta g_{tt}(r)$.

Quasi-Normal Modes:

- in principle clean test, no uncertain astrophysics
- increasing, precise data becomes available (quest of overtones)
- theory specific tests for rotating black holes very hard
- parametrized background metrics exist, but “dynamics” adhoc

Shadow Images:

- can be rather theory agnostic (assuming geodesics)
- in principle astrophysics and geometry can be distinguished
- realistic GRMHD accretion physics even in GR very involved
- not many observed images available, also in foreseeable future limited