Title: Goldilocks modes in celestial CFT

Speakers: Andrea Puhm

Series: Quantum Fields and Strings

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Abstract: In this talk I will consider massless scattering from the point of view of the position, momentum, and celestial bases with a view to advancing a holographic principle for asymptotically flat spacetimes. Within the soft sector, these different languages highlight distinct aspects of the 'infrared triangle': quantum field theory soft theorems arise in the limit of vanishing energy, memory effects are described via shifts of fields along retarded time, and celestial symmetry algebras are realized via currents that appear at special values of the conformal dimension. The latter are determined by the global conformal multiplets in celestial CFT referred to as 'celestial diamonds'. These diamonds degenerate beyond the leading universal soft modes and the standard interpretation of the infrared triangle breaks down: we have neither an obvious asymptotic symmetry nor a Goldstone mode but we do have a soft theorem, and hence a version of a memory effect. I will discuss various aspects of celestial CFT surrounding these Goldstone-like, or Goldilocks, modes and their canonically paired memory modes. They play an important role in constraining celestial OPEs and for understanding the interpretation and implications of the (semi-)infinite tower of tree-level symmetry currents which may pose powerful constraints on consistent low energy effective field theories.

Zoom Link: https://pitp.zoom.us/j/95785822777?pwd=cVJWYVJBS1E3aDJPT1ZSTmlZbzVQQT09

GOLDILOCKS MODES
AND THE THREE SCATTERING BASES

BASED ON WORK WITH
SABRINA PASTERSKI, LAURA DONNAY, EMILIO TREVISANI, YORGO PANO

ANDREA PUHM

SOLVAY WORKSHOP 'SELECTED TOPICS IN QUANTUM GRAVITY', 22 FEBRUARY 2022

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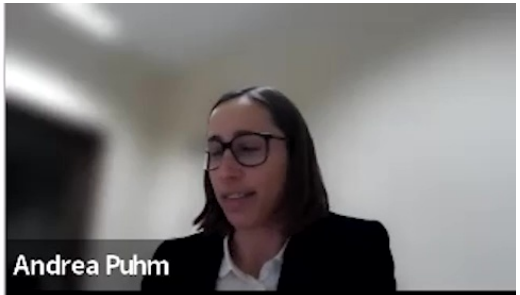
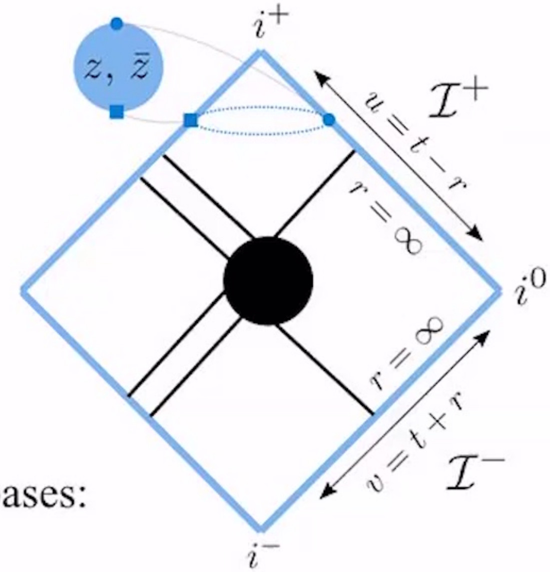


S-matrix

Gravitational S-matrix:
hologram for quantum gravity in asymptotically flat spacetimes



Initial and final states for massless particles prepared in terms of data at past and future null infinity $\mathscr{I}^-$ and $\mathscr{I}^+$.

Asymptotic data can be presented in different bases:

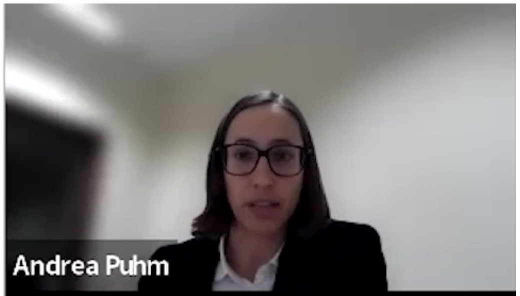
$$u \overset{\mathscr{F}}{\longleftrightarrow} \omega \overset{\mathscr{M}}{\longleftrightarrow} \Delta$$

Fourier transform

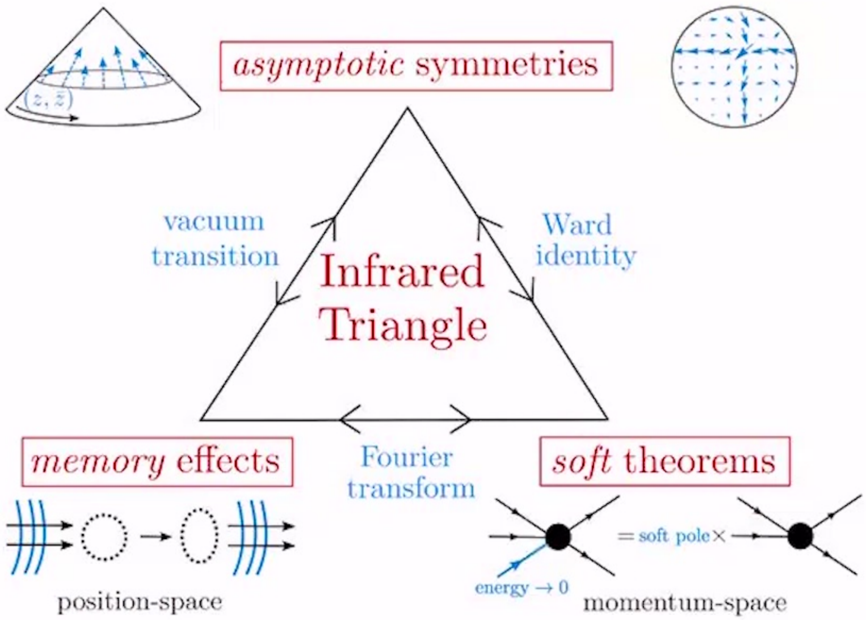
Mellin transform

$$\hat{f}(u) = \int_{-\infty}^{+\infty} d\omega e^{i\omega u} f(\omega) \qquad \phi(\Delta) = \int_{0}^{\infty} d\omega \omega^{\Delta-1} f(\omega)$$

Each basis highlights distinct aspects of the infrared sector.



Soft/memory/symmetry



asymptotic symmetries

vacuum transition

Ward identity

Infrared Triangle

memory effects

Fourier transform

soft theorems

position-space

momentum-space

Soft theorems and memory effects are related by a basis change:

$$u \xleftrightarrow{\mathscr{F}} \omega$$

⇒ uncover potentially observational signatures of soft physics

S-matrix as celestial amplitude

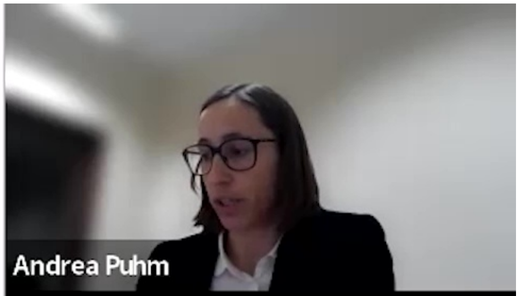
What are all the symmetries of nature?

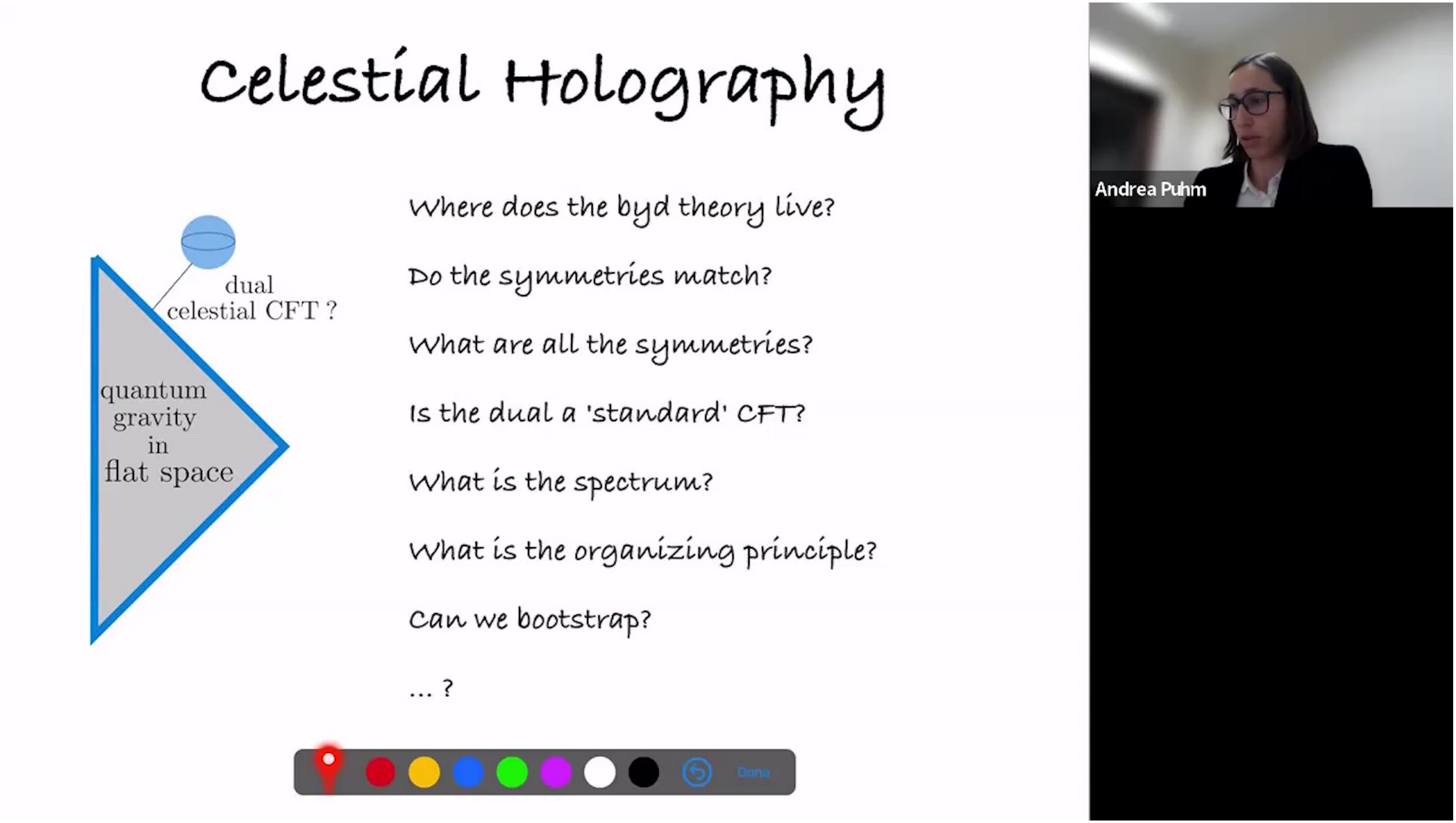
Aim of celestial holographer: make max # of symmetries manifest to determine constraints on consistent S-matrix ⇒ Δ basis!

$$\omega \xleftrightarrow{\mathscr{M}} \Delta$$

$$\langle p_1^{out} \ldots p_n^{out} \,|\, \mathcal{S} \,|\, p_1^{in} \ldots p_n^{in} \rangle \qquad \langle \mathcal{O}^{\epsilon_1}_{\Delta_1,J_1}(w_1, \bar{w}_1) \ldots \mathcal{O}^{\epsilon_n}_{\Delta_n,J_n}(w_n, \bar{w}_n) \rangle$$

⇒ uncover hidden structures / symmetries of amplitudes





Celestial Holography

Where does the byd theory live?

Do the symmetries match?

What are all the symmetries?

Is the dual a 'standard' CFT?

What is the spectrum?

What is the organizing principle?

Can we bootstrap?

... ?

Outline

1. Three bases for scattering
2. Soft limit of scattering
3. Celestial Triangle
4. Celestial symmetries & memories
5. Goldilocks modes
6. Outlook

S-matrix

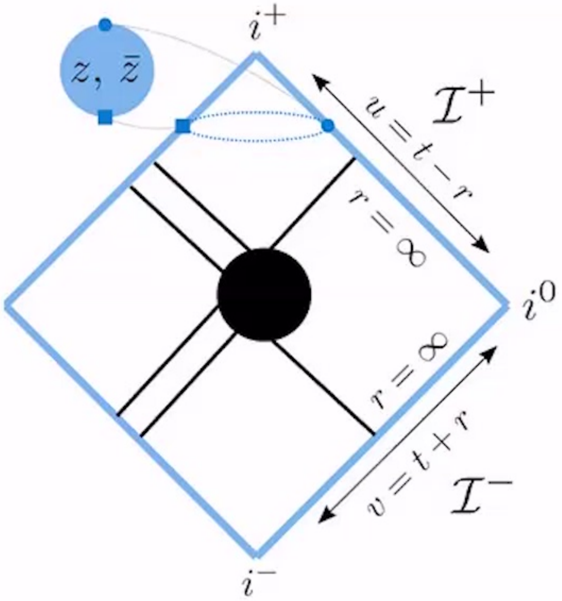
basic observable of quantum gravity in asymptotically flat spacetimes

Holographic flavor:

defined in terms of on-shell data

$$p_i^\mu = \epsilon_i \omega_i (1 + z_i \bar{z}_i, z_i + \bar{z}_i, i(\bar{z}_i - z_i), 1 - z_i \bar{z}_i)$$

energy ω_i, direction z_i
sign ϵ_i for in (-) and out (+)



single particle state:

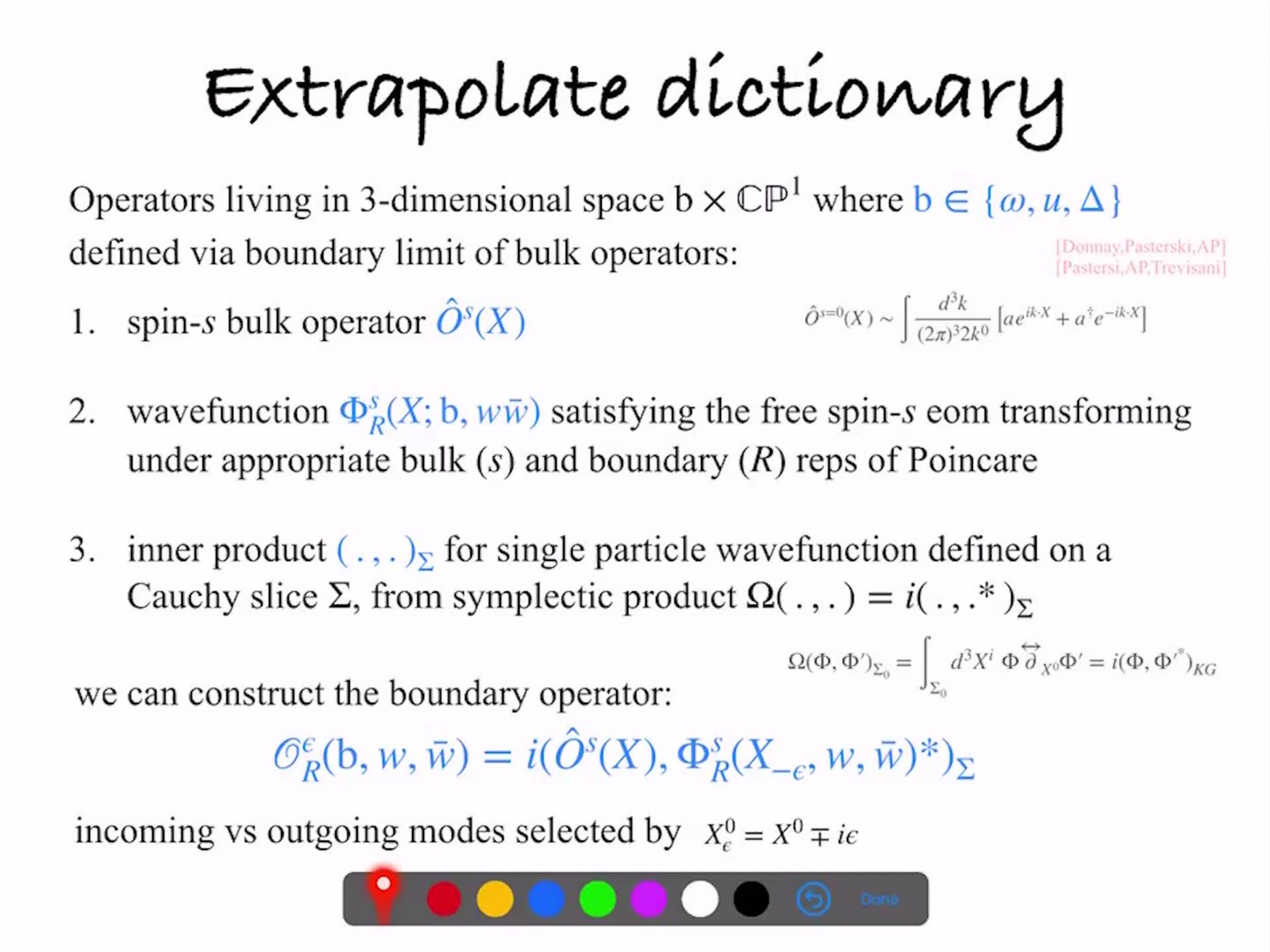
$$|\omega_i, z_i, \bar{z}_i\rangle = a^\dagger(\omega_i, z_i, \bar{z}_i)|0\rangle$$

Scattering amplitude:

$$\mathscr{A}(\omega_i, z_i, \bar{z}_i) = A(\omega_i, z_i, \bar{z}_i)\delta^{(4)}(\sum_i p_i^\mu)$$

$$\omega_i \nearrow u_i \text{ ...null time}$$
$$\omega_i \searrow \Delta_i \text{ ...boost weight}$$

r Rindler energy



Extrapolate dictionary

Operators living in 3-dimensional space $\mathrm{b} \times \mathbb{CP}^1$ where $\mathrm{b} \in \{\omega, u, \Delta\}$ defined via boundary limit of bulk operators:

[Donnay,Pasterski,AP]
[Pastersi,AP,Trevisani]

1. spin-s bulk operator $\hat{O}^s(X)$

$$\hat{O}^{s=0}(X) \sim \int \frac{d^3k}{(2\pi)^3 2k^0}\left[a e^{ik\cdot X} + a^\dagger e^{-ik\cdot X}\right]$$

2. wavefunction $\Phi^s_R(X; \mathrm{b}, w\bar{w})$ satisfying the free spin-s eom transforming under appropriate bulk (s) and boundary (R) reps of Poincare

3. inner product $(\,.\,,.\,)_\Sigma$ for single particle wavefunction defined on a Cauchy slice Σ, from symplectic product $\Omega(\,.\,,.\,) = i(\,.\,,.^*\,)_\Sigma$

$$\Omega(\Phi, \Phi')_{\Sigma_0} = \int_{\Sigma_0} d^3X^i\ \Phi \overleftrightarrow{\partial}_{X^0} \Phi' = i(\Phi, \Phi'^*)_{KG}$$

we can construct the boundary operator:

$$\mathcal{O}^\epsilon_R(\mathrm{b}, w, \bar{w}) = i(\hat{O}^s(X), \Phi^s_R(X_{-\epsilon}, w, \bar{w})^*)_\Sigma$$

incoming vs outgoing modes selected by $X^0_\epsilon = X^0 \mp i\epsilon$

Three bases for scattering

position momentum boost

$$A(u) \overset{\mathscr{F}}{\longleftrightarrow} A(\omega) \overset{\mathscr{M}}{\longleftrightarrow} A(\Delta)$$

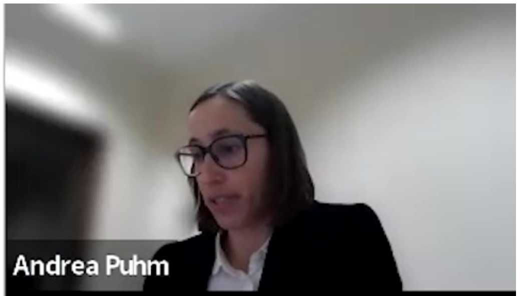
memory effects soft theorems symmetries

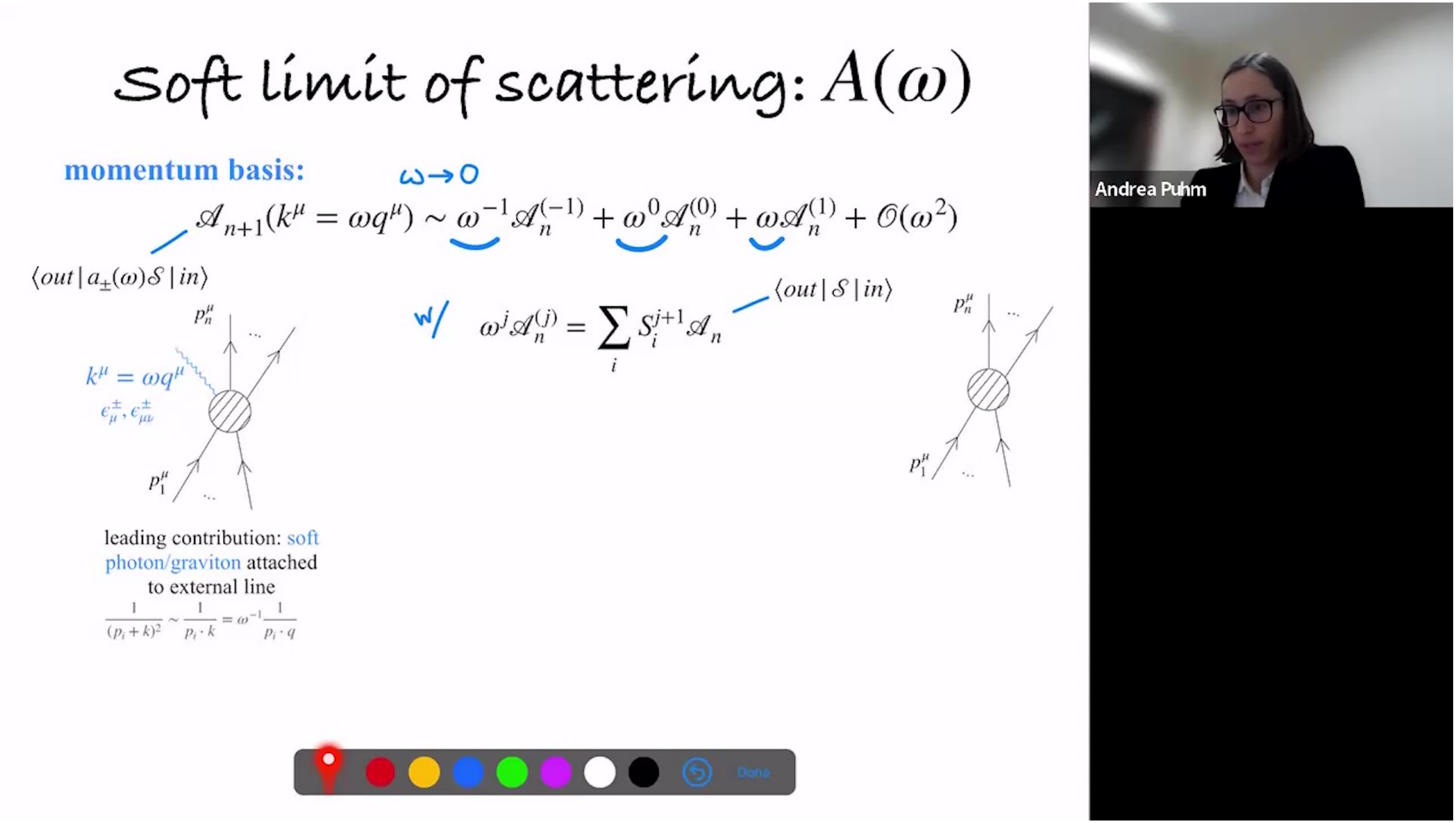
$$\int_{-\infty}^{+\infty} du\, n^\mu \mathscr{J}_\mu = 0$$

$$\langle out \,|\, a_\pm(\omega)\mathcal{S} \,|\, in\rangle = \sum_i \sum_{k\in\{in,out\}} S_k^{(i)\pm}\langle out\,|\,\mathcal{S}\,|\,in\rangle$$

$$\lim_{\Delta_i\to -n}(\Delta_i+n)\langle \mathcal{O}^{\epsilon_1}_{\Delta_1,J_1}\cdots\mathcal{O}^{\pm}_{\Delta_i,J_i}\cdots\mathcal{O}^{\epsilon_n}_{\Delta_n,J_n}\rangle = \sum_{k\in\{in,out\}}\hat{S}_k^{(i)\pm}\langle \mathcal{O}^{\epsilon_1}_{\Delta_1,J_1}\cdots\mathcal{O}^{\epsilon_n}_{\Delta_n,J_n}\rangle$$

Each highlight distinct aspects of infrared physics.





Soft limit of scattering: $A(\omega)$

momentum basis:

$\omega \to 0$

$$\mathscr{A}_{n+1}(k^\mu = \omega q^\mu) \sim \underbrace{\omega^{-1}}_{}\mathscr{A}_n^{(-1)} + \underbrace{\omega^0}_{}\mathscr{A}_n^{(0)} + \underbrace{\omega}_{}\mathscr{A}_n^{(1)} + \mathcal{O}(\omega^2)$$

$\langle out \,|\, a_\pm(\omega)\mathcal{S} \,|\, in\rangle$

w/ $$\omega^j \mathscr{A}_n^{(j)} = \sum_i S_i^{j+1} \mathscr{A}_n$$

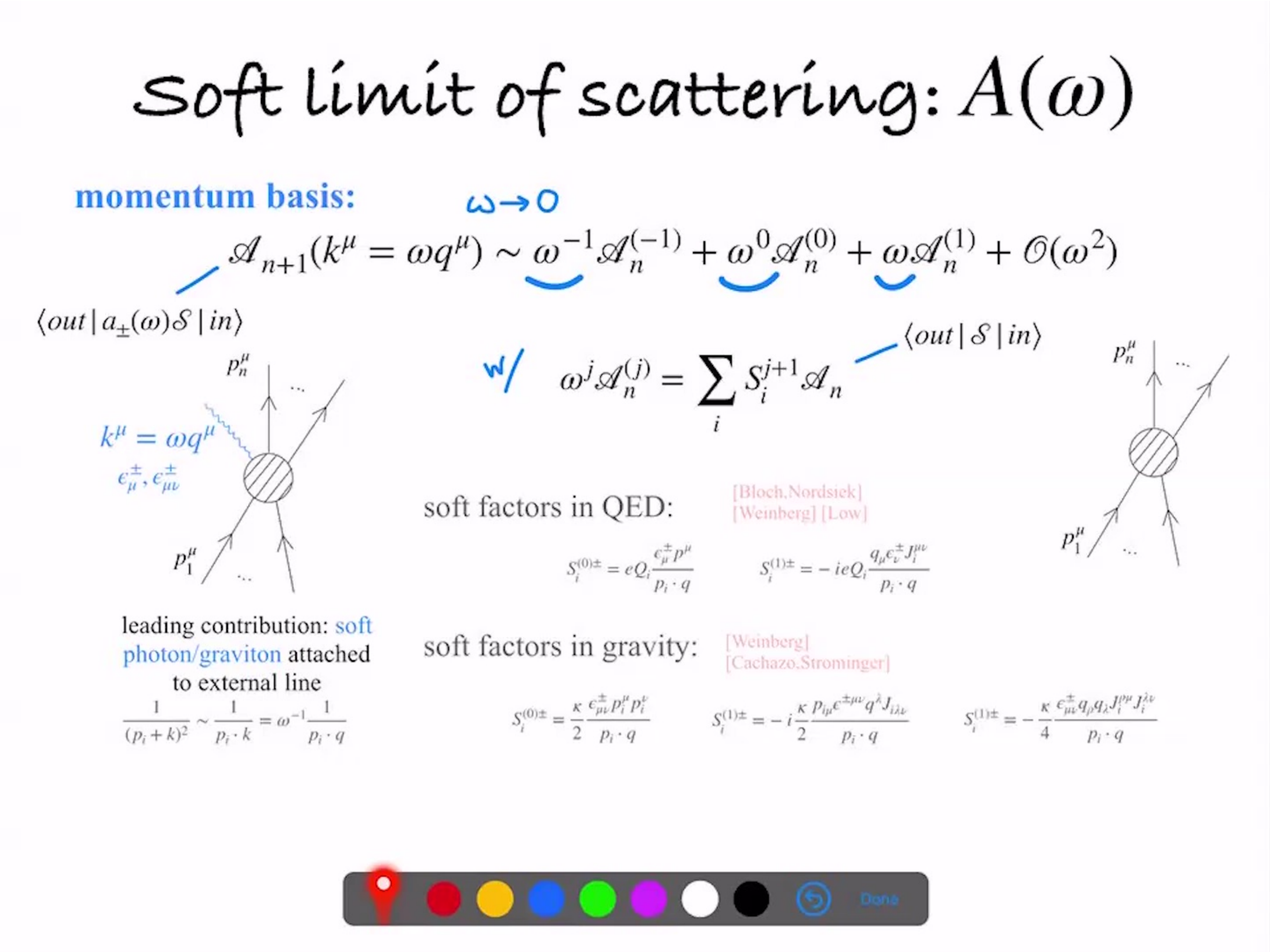
$\langle out \,|\, \mathcal{S} \,|\, in\rangle$

$k^\mu = \omega q^\mu$

$\epsilon_\mu^\pm, \epsilon_{\mu\nu}^\pm$

leading contribution: soft photon/graviton attached to external line

$$\frac{1}{(p_i+k)^2} \sim \frac{1}{p_i \cdot k} = \omega^{-1}\frac{1}{p_i \cdot q}$$



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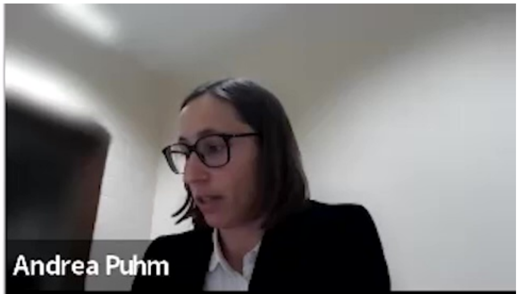
soft factors in QED: [Bloch,Nordsiek] [Weinberg] [Low]

$$S_i^{(0)\pm} = eQ_i\frac{\epsilon_\mu^\pm p^\mu}{p_i \cdot q} \qquad S_i^{(1)\pm} = -ieQ_i\frac{q_\mu \epsilon_\nu^\pm J_i^{\mu\nu}}{p_i \cdot q}$$

soft factors in gravity: [Weinberg] [Cachazo,Strominger]

$$S_i^{(0)\pm} = \frac{\kappa}{2}\frac{\epsilon_{\mu\nu}^\pm p_i^\mu p_i^\nu}{p_i \cdot q} \qquad S_i^{(1)\pm} = -i\frac{\kappa}{2}\frac{p_{i\mu}\epsilon^{\pm\mu\nu}q^\lambda J_{i\lambda\nu}}{p_i \cdot q} \qquad S_i^{(1)\pm} = -\frac{\kappa}{4}\frac{\epsilon_{\mu\nu}^\pm q_\rho q_\lambda J_i^{\rho\mu}J_i^{\lambda\nu}}{p_i \cdot q}$$



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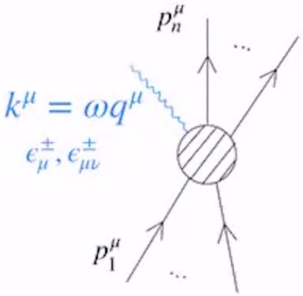
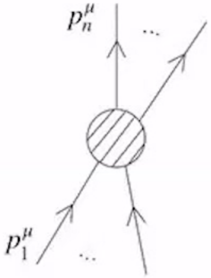
$\langle out \,|\, a_\pm(\omega)\mathscr{S} \,|\, in\rangle$



w/ $$\omega^j \mathscr{A}_n^{(j)} = \sum_i S_i^{j+1}\mathscr{A}_n$$

$\langle out \,|\, \mathscr{S} \,|\, in\rangle$



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1-loop corrections: [Laddha,Sen] [Sahoo,Sen] [Ghosh,Sahoo]

$$\mathscr{A}_{n+1} \sim \omega^{-1}\mathscr{A}_n^{(-1)} + \underline{\log\omega}\,\overline{\mathscr{A}}_n^{(0)} + \omega^0\mathscr{A}_n^{(0)} + \underline{\omega\log\omega}\,\overline{\mathscr{A}}_n + \omega\mathscr{A}_n^{(1)} + \mathscr{O}(\omega^2\log\omega)$$

Soft limit of scattering: $A(\Delta)$

boost basis: Integration over all energies: soft factorization?
Yes, but organized in boost weight poles!

$\nearrow$ $w_{1+\infty}$ symmetry (more later)

cutoff separating IR and UV behavior

$m = -1, 0, 1, \ldots$

tree-level soft theorems encoded in *simple* poles

$$\int_0^{\omega_*} d\omega\,\omega^{\Delta-1+m} = \frac{\omega_*^{\Delta+m}}{\Delta+m}$$

[Fan,Fotopoulos,Taylor]
[Nandan,Schreiber,Volovich,Zlotnikov]
[Pate,Strominger,Raclariu]
[Adamo,Mason,Sharma] [AP] [Guevara]

[Donnay,Pasterski,AP]

1-loop logs encoded in *double* poles

$$\int_0^{\omega_*} d\omega\,\omega^{\Delta-1+m}\log\omega = \frac{\partial}{\partial\Delta}\int_0^{\omega_*} d\omega\,\omega^{\Delta-1+m} = -\frac{\omega_*^{\Delta+m}}{(\Delta+m)^2} + \mathcal{O}\left(\frac{1}{\Delta+m}\right)$$

Soft limit of scattering: $A(u)$

position basis: For every soft theorem a memory effect corresponding to expectation value of operator appearing in soft charge.

Semi-classical interpretation of soft physics and memory most apparent:

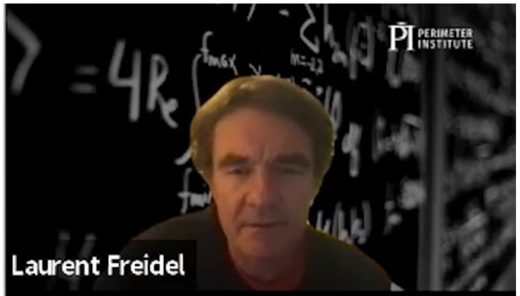
[Laddha,Sen] [Sahoo,Sen]
[Ghosh,Sahoo]

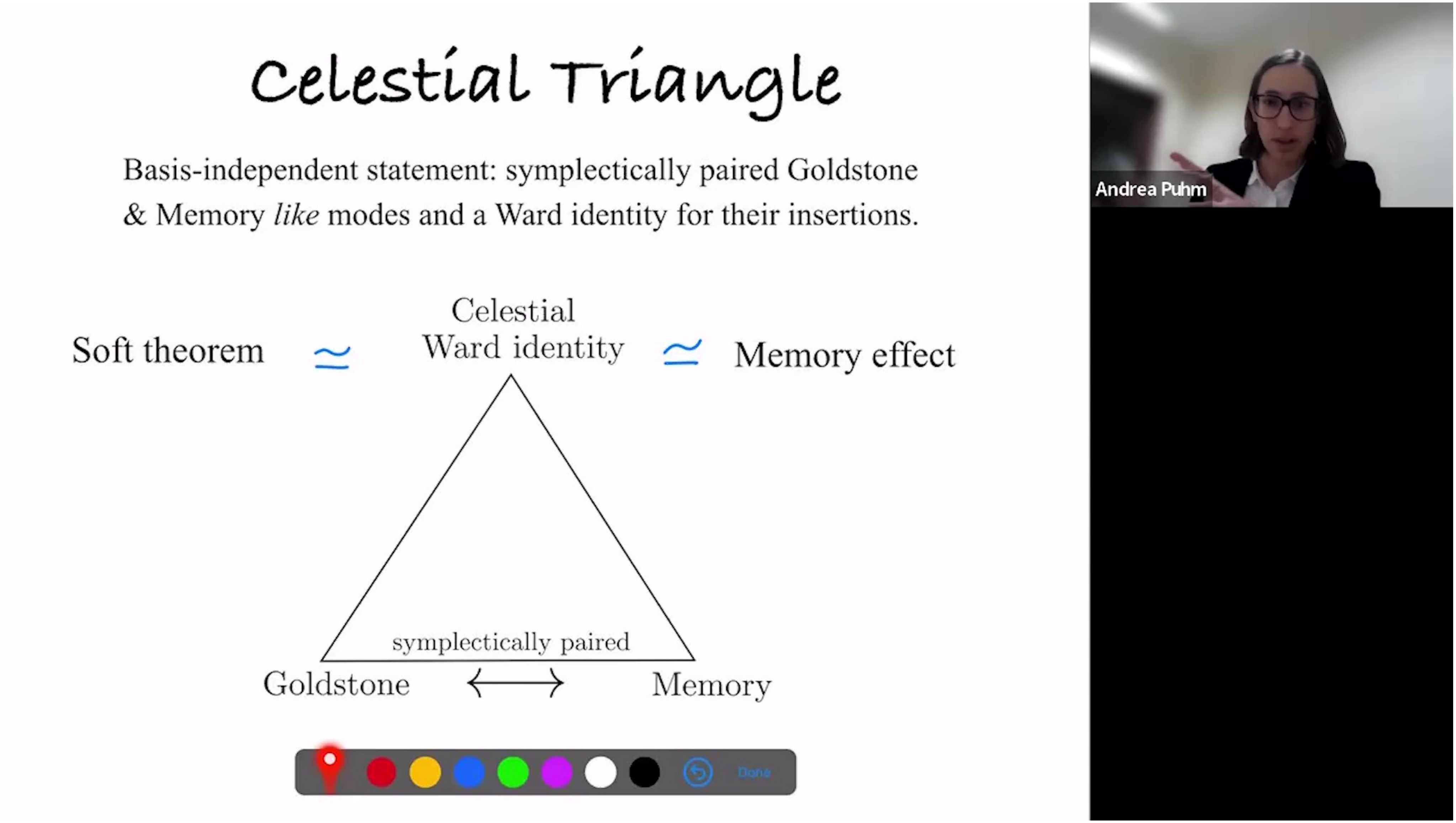
$$\omega^{n-1} \overset{\mathscr{F}}{\longleftrightarrow} \partial_u^n \theta(u)$$

shift in metric or gauge field suppressed by r^{-n} compared to radiative order

Even logarithmic corrections expected when taking into account deflection of external scattering states due to long-range interactions:

$$\log|\omega| \overset{\mathscr{F}}{\longleftrightarrow} -\frac{1}{2|u|} - \gamma_E \delta(u)$$





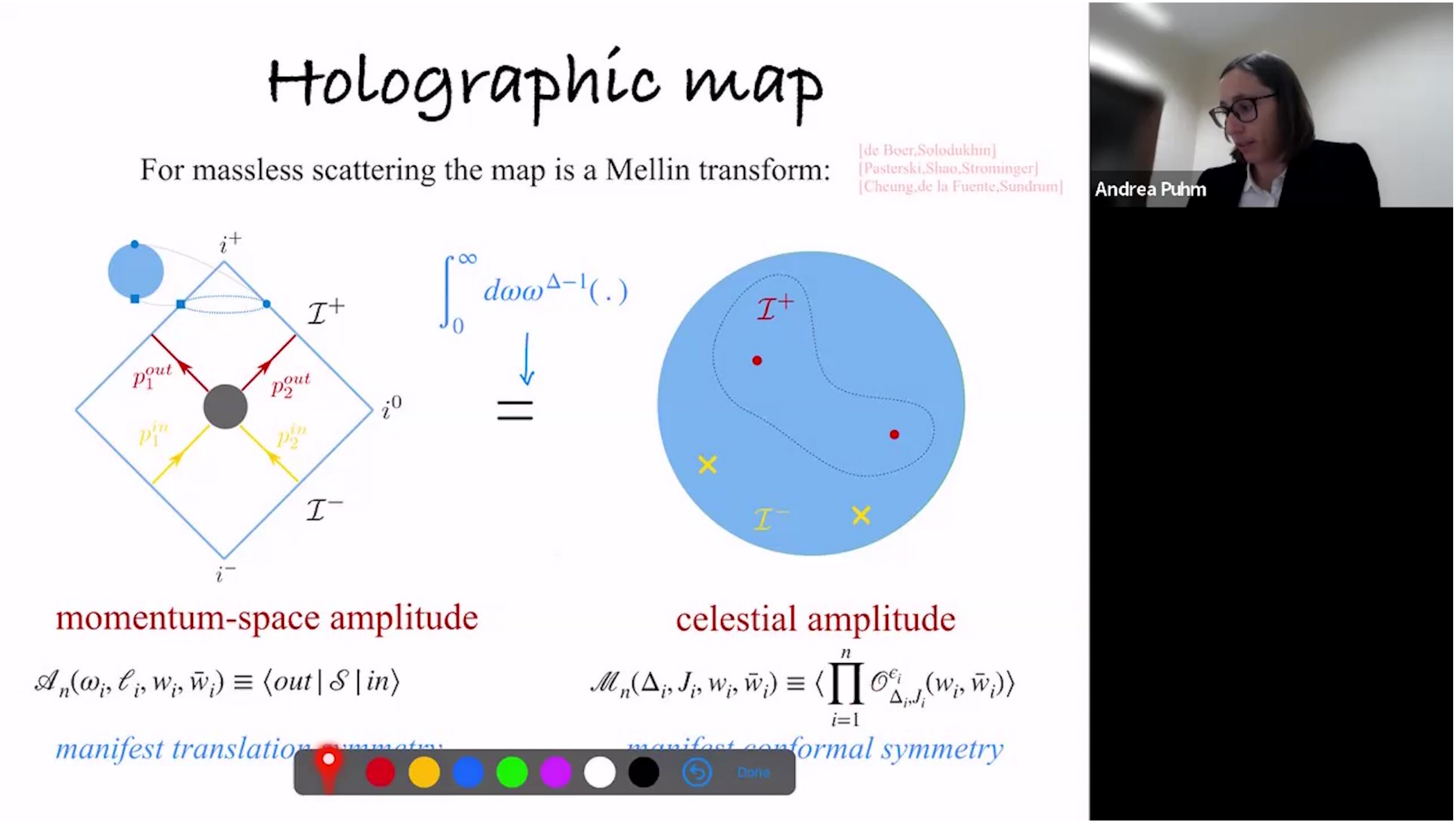
Celestial Triangle

Basis-independent statement: symplectically paired Goldstone & Memory *like* modes and a Ward identity for their insertions.

Soft theorem $\simeq$ Celestial Ward identity $\simeq$ Memory effect

symplectically paired

Goldstone $\longleftrightarrow$ Memory



Holographic map

For massless scattering the map is a Mellin transform:

[de Boer,Solodukhin]
[Pasterski,Shao,Strominger]
[Cheung,de la Fuente,Sundrum]

$$\int_0^\infty d\omega\,\omega^{\Delta-1}(\,.\,)$$

$$\downarrow$$

$$=$$

momentum-space amplitude

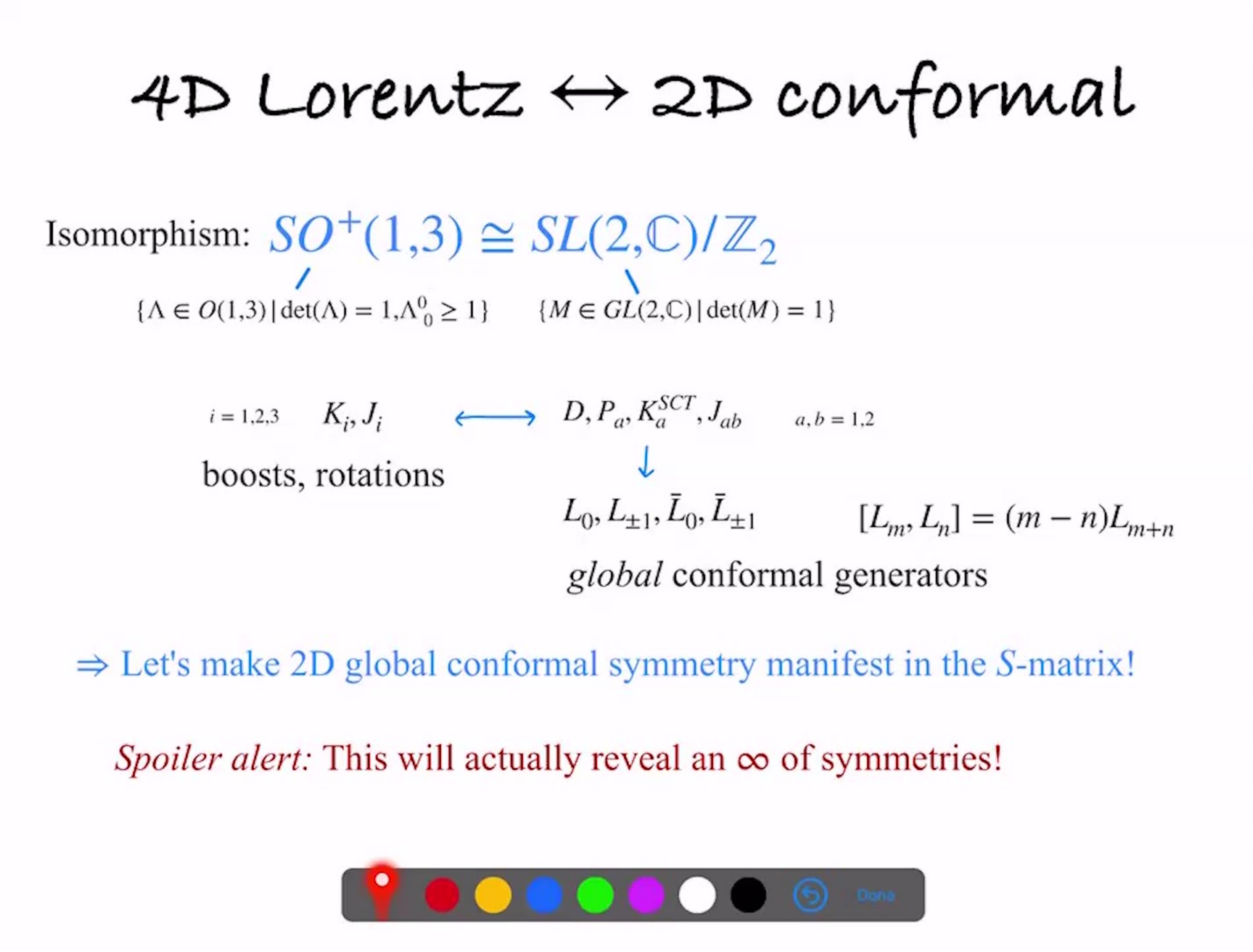
$$\mathcal{A}_n(\omega_i, \ell_i, w_i, \bar{w}_i) \equiv \langle out \,|\, \mathcal{S} \,|\, in \rangle$$

manifest translation symmetry

celestial amplitude

$$\mathcal{M}_n(\Delta_i, J_i, w_i, \bar{w}_i) \equiv \langle \prod_{i=1}^{n} \mathcal{O}^{\epsilon_i}_{\Delta_i, J_i}(w_i, \bar{w}_i) \rangle$$

manifest conformal symmetry



4D Lorentz ↔ 2D conformal

Isomorphism: $SO^+(1,3) \cong SL(2,\mathbb{C})/\mathbb{Z}_2$

$\{\Lambda \in O(1,3)\,|\,\det(\Lambda) = 1, \Lambda^0_{\ 0} \geq 1\}$ $\{M \in GL(2,\mathbb{C})\,|\,\det(M) = 1\}$

$$\underset{i=1,2,3}{} \quad K_i, J_i \quad \longleftrightarrow \quad D, P_a, K_a^{SCT}, J_{ab} \quad a,b = 1,2$$

boosts, rotations

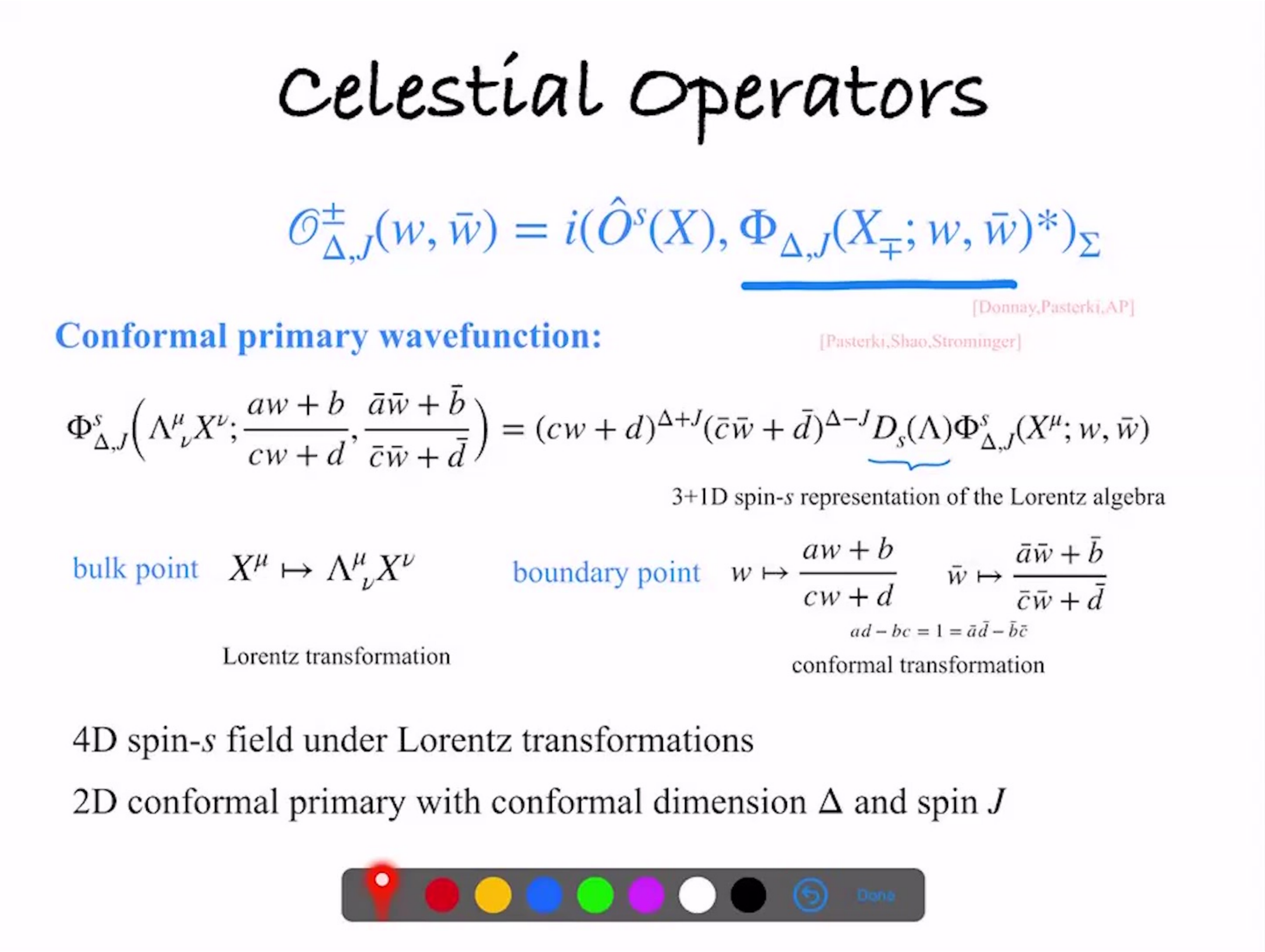
$$\downarrow$$

$$L_0, L_{\pm 1}, \bar{L}_0, \bar{L}_{\pm 1} \qquad [L_m, L_n] = (m-n)L_{m+n}$$

global conformal generators

⇒ Let's make 2D global conformal symmetry manifest in the S-matrix!

Spoiler alert: This will actually reveal an ∞ of symmetries!



Celestial Operators

$$\mathcal{O}^{\pm}_{\Delta,J}(w,\bar{w}) = i(\hat{O}^s(X), \underline{\Phi_{\Delta,J}(X_{\mp};w,\bar{w})^*})_{\Sigma}$$

[Donnay,Pasterki,AP]

[Pasterki,Shao,Strominger]

Conformal primary wavefunction:

$$\Phi^s_{\Delta,J}\left(\Lambda^\mu{}_\nu X^\nu; \frac{aw+b}{cw+d}, \frac{\bar{a}\bar{w}+\bar{b}}{\bar{c}\bar{w}+\bar{d}}\right) = (cw+d)^{\Delta+J}(\bar{c}\bar{w}+\bar{d})^{\Delta-J}\underbrace{D_s(\Lambda)}_{\text{3+1D spin-}s\text{ representation of the Lorentz algebra}}\Phi^s_{\Delta,J}(X^\mu;w,\bar{w})$$

bulk point $X^\mu \mapsto \Lambda^\mu{}_\nu X^\nu$

Lorentz transformation

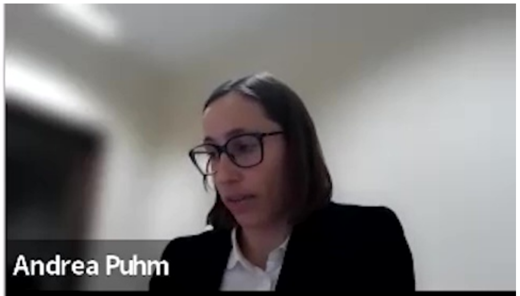
boundary point $w \mapsto \dfrac{aw+b}{cw+d}$ $\bar{w} \mapsto \dfrac{\bar{a}\bar{w}+\bar{b}}{\bar{c}\bar{w}+\bar{d}}$

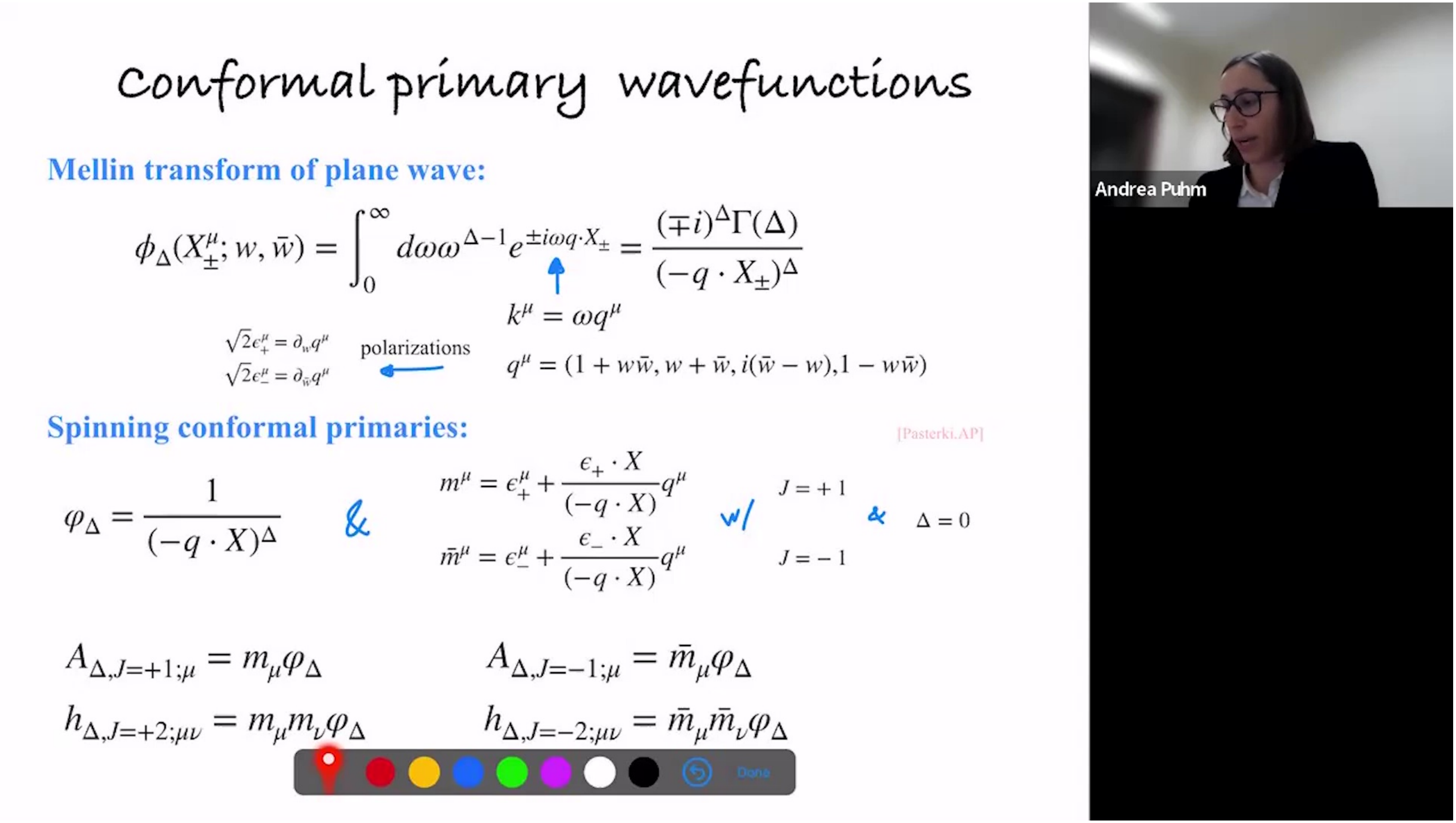
$ad-bc=1=\bar{a}\bar{d}-\bar{b}\bar{c}$

conformal transformation

4D spin-s field under Lorentz transformations

2D conformal primary with conformal dimension Δ and spin J





Conformal primary wavefunctions

Mellin transform of plane wave:

$$\phi_\Delta(X^\mu_\pm; w, \bar{w}) = \int_0^\infty d\omega\, \omega^{\Delta-1} e^{\pm i\omega q\cdot X_\pm} = \frac{(\mp i)^\Delta \Gamma(\Delta)}{(-q\cdot X_\pm)^\Delta}$$

$$\uparrow$$

$$k^\mu = \omega q^\mu$$

$\sqrt{2}\epsilon^\mu_+ = \partial_w q^\mu$

$\sqrt{2}\epsilon^\mu_- = \partial_{\bar{w}} q^\mu$

$\leftarrow$ polarizations

$$q^\mu = (1 + w\bar{w}, w + \bar{w}, i(\bar{w} - w), 1 - w\bar{w})$$

Spinning conformal primaries:

[Pasterki.AP]

$$\varphi_\Delta = \frac{1}{(-q\cdot X)^\Delta} \quad \& \quad \begin{aligned} m^\mu &= \epsilon^\mu_+ + \frac{\epsilon_+\cdot X}{(-q\cdot X)} q^\mu \\ \bar{m}^\mu &= \epsilon^\mu_- + \frac{\epsilon_-\cdot X}{(-q\cdot X)} q^\mu \end{aligned} \quad \text{w/} \quad \begin{aligned} J &= +1 \\ J &= -1 \end{aligned} \quad \& \quad \Delta = 0$$

$$A_{\Delta, J=+1;\mu} = m_\mu \varphi_\Delta \qquad A_{\Delta, J=-1;\mu} = \bar{m}_\mu \varphi_\Delta$$

$$h_{\Delta, J=+2;\mu\nu} = m_\mu m_\nu \varphi_\Delta \qquad h_{\Delta, J=-2;\mu\nu} = \bar{m}_\mu \bar{m}_\nu \varphi_\Delta$$

Spectrum

$\int \frac{d\omega}{\omega}\omega^{\Delta}$ *what is* Δ? $\Delta \in \mathbb{R}, \mathbb{Z}, \mathbb{C},$ *free parameter?*

Conformal primary wavefunctions $\Phi_{\Delta,J}(X; w, \bar{w})$ [Pasterki,Shao]

form a δ-fct normalizable basis when the conformal dimension is

$\Delta \in 1 + i\mathbb{R}$ ... principal continuous series of the $SL(2,\mathbb{C})$ Lorentz group.

$$\widetilde{\mathcal{O}}_{2-\Delta,-J}(w,\bar{w}) = \frac{k_{\Delta,J}}{2\pi}\int d^2w' \frac{\mathcal{O}_{\Delta,J}(w',\bar{w}')}{(w-w')^{2-\Delta-J}(\bar{w}-\bar{w}')^{2-\Delta+J}}$$

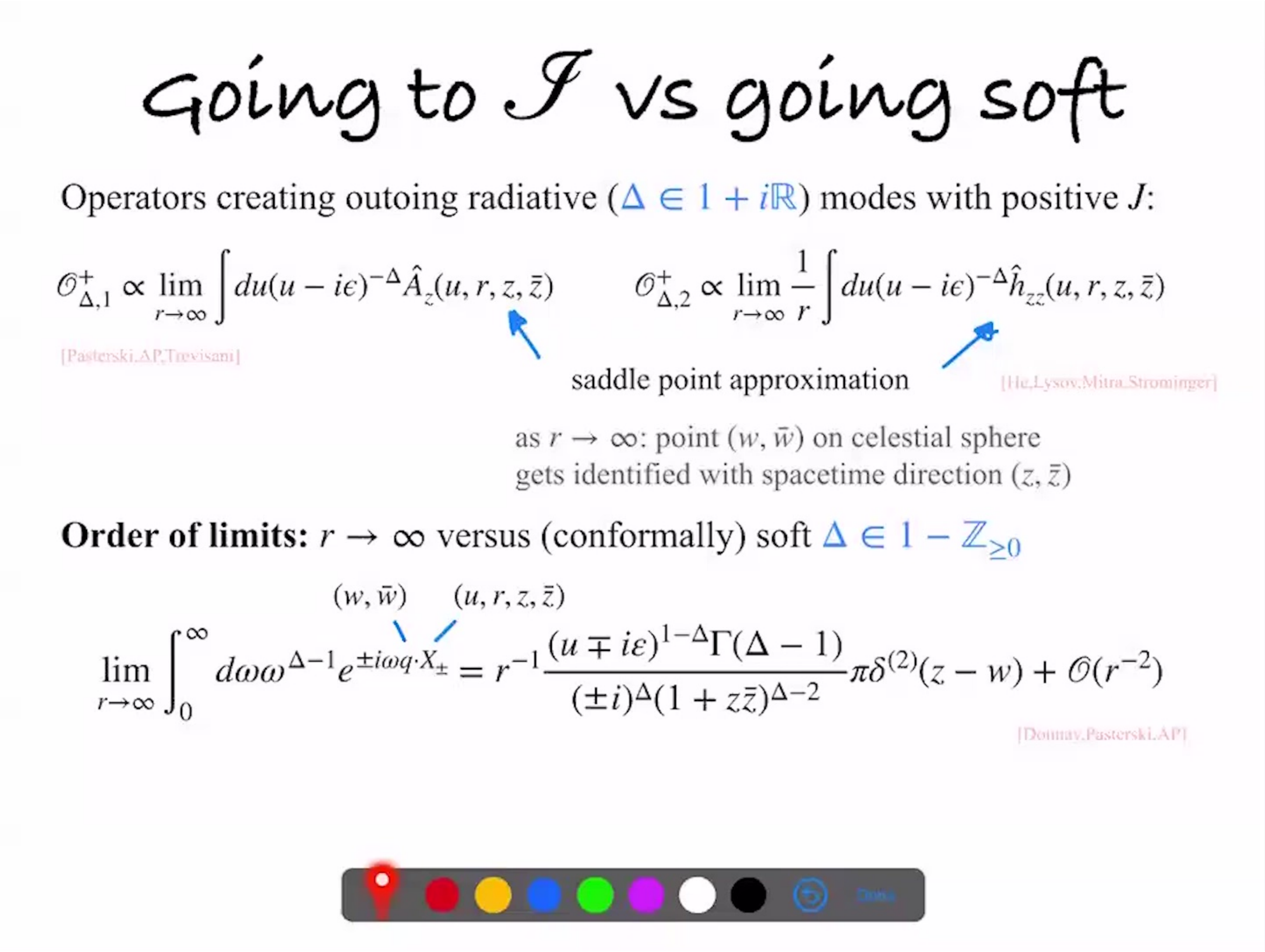
Conformal primary *shadow* wavefunctions $\widetilde{\Phi}_{2-\Delta,-J}(X; w, \bar{w})$

form equally good δ-fct normalizable basis when $\Delta \in 1 + i\mathbb{R}$.

Asymptotic symmetries and *conformally* soft factorization of celestial amplitudes for $\Delta \in 1 - \mathbb{Z}_{\geq 0}$ $(s \in \mathbb{Z})$!

Obtained by analytic continuation off the principal series.

[Donnay,Pasterski,AP]



Going to $\mathscr{I}$ vs going soft

Operators creating outoing radiative ($\Delta \in 1 + i\mathbb{R}$) modes with positive J:

$$\mathcal{O}^{+}_{\Delta,1} \propto \lim_{r\to\infty} \int du (u - i\epsilon)^{-\Delta} \hat{A}_z(u, r, z, \bar{z}) \qquad \mathcal{O}^{+}_{\Delta,2} \propto \lim_{r\to\infty} \frac{1}{r} \int du (u - i\epsilon)^{-\Delta} \hat{h}_{zz}(u, r, z, \bar{z})$$

[Pasterski,AP,Trevisani]

saddle point approximation

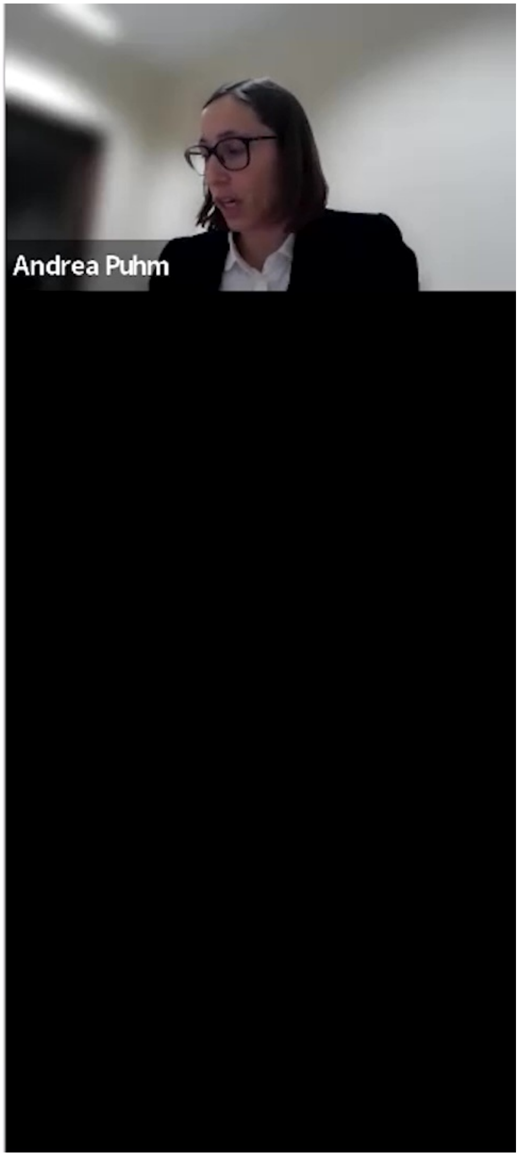
[He,Lysov,Mitra,Strominger]

as $r \to \infty$: point $(w, \bar{w})$ on celestial sphere gets identified with spacetime direction $(z, \bar{z})$

Order of limits: $r \to \infty$ versus (conformally) soft $\Delta \in 1 - \mathbb{Z}_{\geq 0}$

$$\lim_{r\to\infty} \int_0^\infty d\omega\, \omega^{\Delta-1} e^{\pm i\omega \overset{(w,\bar{w})}{q} \cdot \overset{(u,r,z,\bar{z})}{X_\pm}} = r^{-1} \frac{(u \mp i\varepsilon)^{1-\Delta}\Gamma(\Delta - 1)}{(\pm i)^{\Delta}(1 + z\bar{z})^{\Delta-2}} \pi \delta^{(2)}(z - w) + \mathcal{O}(r^{-2})$$

[Donnay,Pasterski,AP]



Going to $\mathscr{I}$ vs going soft

Operators creating outoing radiative ($\Delta \in 1 + i\mathbb{R}$) modes with positive J:

$$\mathcal{O}^{+}_{\Delta,1} \propto \lim_{r\to\infty} \int du (u - i\epsilon)^{-\Delta} \hat{A}_z(u, r, z, \bar{z}) \qquad \mathcal{O}^{+}_{\Delta,2} \propto \lim_{r\to\infty} \frac{1}{r} \int du (u - i\epsilon)^{-\Delta} \hat{h}_{zz}(u, r, z, \bar{z})$$

[Pasterski,AP,Trevisani]

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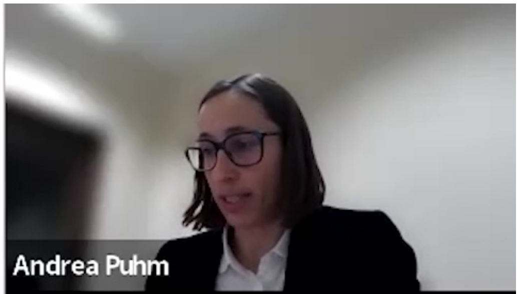
$(w, \bar{w})$ $\quad$ $(u, r, z, \bar{z})$

$$\lim_{r\to\infty} \int_0^\infty d\omega \omega^{\Delta-1} e^{\pm i\omega q \cdot X_\pm} = r^{-1} \frac{(u \mp i\varepsilon)^{1-\Delta}\Gamma(\Delta - 1)}{(\pm i)^\Delta (1 + z\bar{z})^{\Delta-2}} \pi \delta^{(2)}(z - w) + \mathcal{O}(r^{-2})$$

[Donnay,Pasterski,AP]

Note: The saddle point approximation (first large r, then soft) is already implicit in 'soft theorem = Ward identity' relation.

(We can nevertheless keep track of first conformally soft, then large r terms.)



Conformal Goldstones

$$\mathcal{O}_{\Delta,J} = \Omega(\hat{O}, \Phi_{\Delta,J})_{\Sigma \to \mathcal{I}}$$

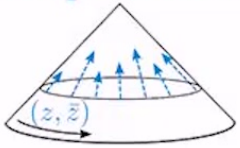
canonical charge for asymptotic symmetry
for values of Δ for which $\Phi_{\Delta,J}$ is pure gauge

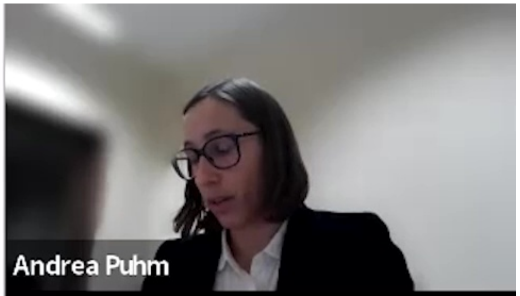
[Donnay, AP, Strominger]
[Donnay,Pasterski,AP]

Pure gauge wavefunctions:

$$A^{\mathrm{G}}_{\Delta=1,J;\mu} = \nabla_\mu \Lambda_{gauge} \quad \rightarrow \text{ large gauge transformation}$$

$$h^{\mathrm{G}}_{\Delta=1,J;\mu\nu} = \nabla_{(\mu}\xi^1_{\nu)} = \nabla_\mu \nabla_\nu \Lambda_{gravity} \quad \rightarrow \text{ BMS supertranslation}$$





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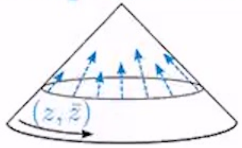


$h^{\mathrm{G}}_{\Delta=0,J;\mu\nu} = \nabla_{(\mu}\xi^0_{\nu)}$

$\widetilde{h}^{\mathrm{G}}_{\Delta=2,J;\mu\nu} = \nabla_{(\mu}\xi^2_{\nu)}$

→ [Campiglia,Laddha] Diff(S^2) superrotation (related by shadow transform) [Donnay,Pasterski,AP]

Virasoro [Barnich,Troessaert]

Asymptotic symmetries

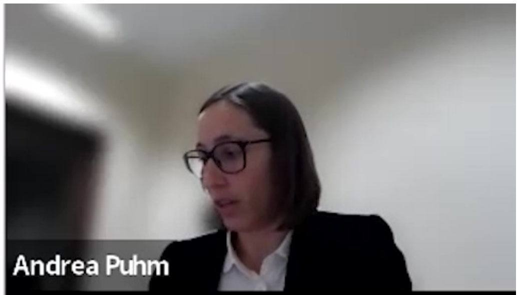
4D (asymptotic) symmetries generated by 2D celestial currents!

$$\mathcal{O}_{\Delta,J} = \Omega(\hat{O}, \Phi_{\Delta,J})_{\Sigma\to\mathscr{I}}$$

asymptotic symmetry generator   pure gauge $\Delta \in \frac{1}{2}\mathbb{Z}$

$s = \lvert J \rvert$	Δ	$\tilde{\Delta} = 2 - \Delta$	energetically soft pole	celestial current	asymptotic symmetry
1	1	1	ω^{-1}	$\mathscr{J}$	large U(1)
$\frac{3}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	$\omega^{-\frac{1}{2}}$	$\mathscr{S}$ & $\tilde{\mathscr{S}}$	large SUSY
2	1	1	ω^{-1}	$\mathscr{P}$	supertranslation
2	0	2	ω^{0}	$\mathscr{T}$ & $\tilde{\mathscr{T}}$	superrotation

2D stress tensor! [Kapec,Mitra,Raclariu,Strominger]



Conformally soft photon

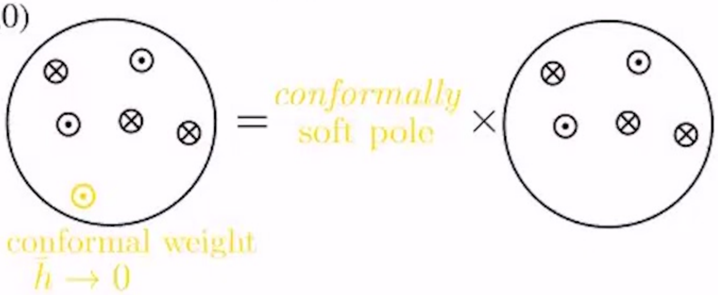
- Leading soft photon:

$$\lim_{\Delta \to 1} (\Delta - 1)\mathscr{M}_n(\Delta, J = +1, w, \bar{w}; \Delta_i, w_i, \bar{w}_i) = \sum_{k=1}^{n} \frac{Q_k}{w - w_k} \mathscr{M}_n(\Delta_i, w_i, \bar{w}_i)$$

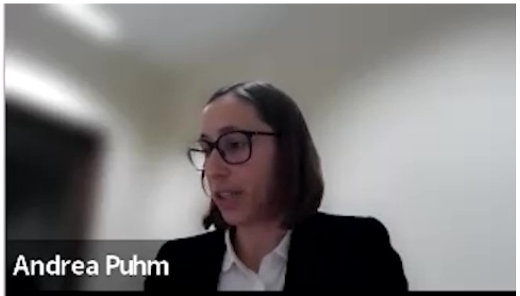
$$\langle J_w \mathcal{O}_{\Delta_1, J_1} \cdots \mathcal{O}_{\Delta_n, J_n} \rangle = \sum_{k=1}^{n} \frac{Q_k}{w - w_k} \langle \mathcal{O}_{\Delta_1, J_1} \cdots \mathcal{O}_{\Delta_n, J_n} \rangle$$

$(h, \bar{h}) = (1,0)$

[Fan,Fotopoulos,Taylor]
[Nandan,Schreiber,Volovich,Zlotnikov]
[Pate,Strominger,Raclariu]
[Adamo,Mason,Sharma]



Celestial analogue of 4D soft theorems ⇒ 2D Ward identities



Conformally soft graviton

- Leading soft graviton:

[Strominger] [He,Lysov,Mitra,Strominger]
[Adamo,Mason,Scharma] [AP] [Guevara]

$$\lim_{\Delta\to 1}(\Delta-1)\mathscr{M}_n(\Delta, J=+2, w, \bar{w}; \Delta_i, w_i, \bar{w}_i) = -\frac{\kappa}{2}\sum_{k=1}^{n}\frac{\bar{w}-\bar{w}_k}{w-w_k}\mathscr{M}_n(\Delta_k+1, w_k, \bar{w}_k)$$

$$\xrightarrow{\partial_{\bar{w}}} \langle P_w \mathcal{O}^{\epsilon_1}_{\Delta_1,J_1}\cdots\mathcal{O}^{\epsilon_n}_{\Delta_n,J_n}\rangle = -\frac{\kappa}{2}\sum_{k=1}^{n}\frac{1}{w-w_k}\langle \mathcal{O}^{\epsilon_1}_{\Delta_1,J_1}\cdots\mathcal{O}^{+}_{\Delta_k+1,J_k}\cdots\mathcal{O}^{\epsilon_n}_{\Delta_n,J_n}\rangle$$

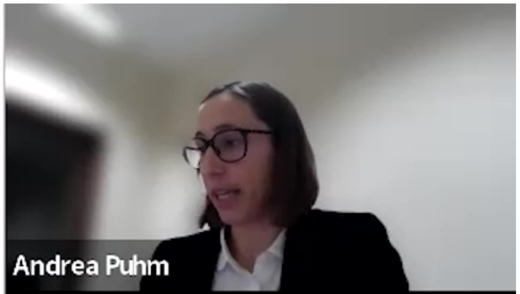
- Sub-leading soft graviton:

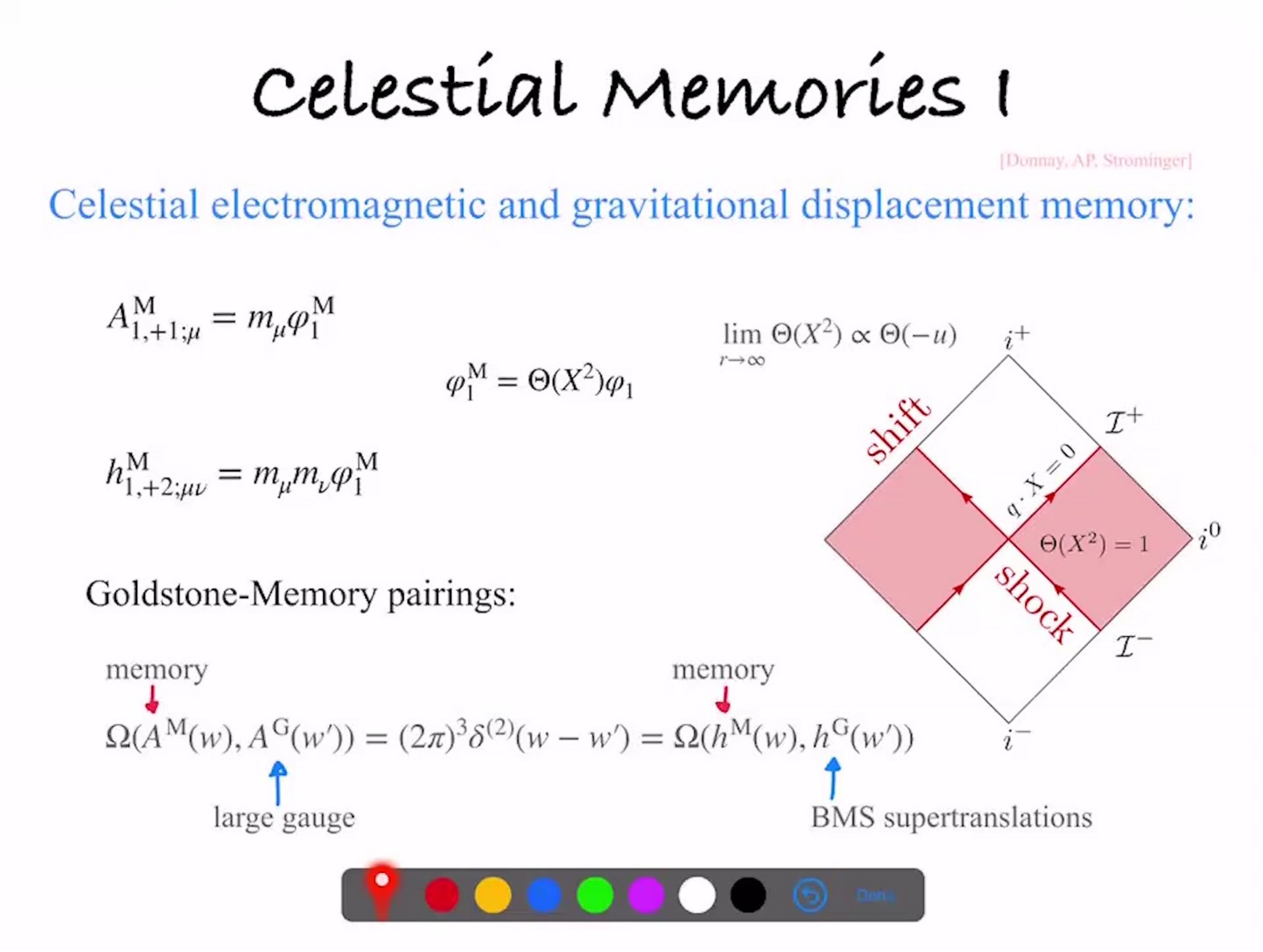
[Kapec,Mitra,Raclariu,Strominger]
[Adamo,Mason,Scharma] [Guevara]

shadow of $\Delta = 0$:

$$\langle \mathcal{T}_{ww}\mathcal{O}^{\epsilon_1}_{\Delta_1,J_1}\cdots\mathcal{O}^{\epsilon_n}_{\Delta_n,J_n}\rangle = \sum_{k=1}^{n}\left[\frac{h_j}{(w-w_j)^2}+\frac{\partial_{w_j}}{w-w_j}\right]\langle \mathcal{O}^{\epsilon_1}_{\Delta_1,J_1}\cdots\mathcal{O}^{\epsilon_n}_{\Delta_n,J_n}\rangle$$

Celestial analogue of 4D soft theorems ⇒ 2D Ward identities





Celestial Memories I

[Donnay, AP, Strominger]

Celestial electromagnetic and gravitational displacement memory:

$$A^{\mathrm{M}}_{1,+1;\mu} = m_\mu \varphi^{\mathrm{M}}_1$$

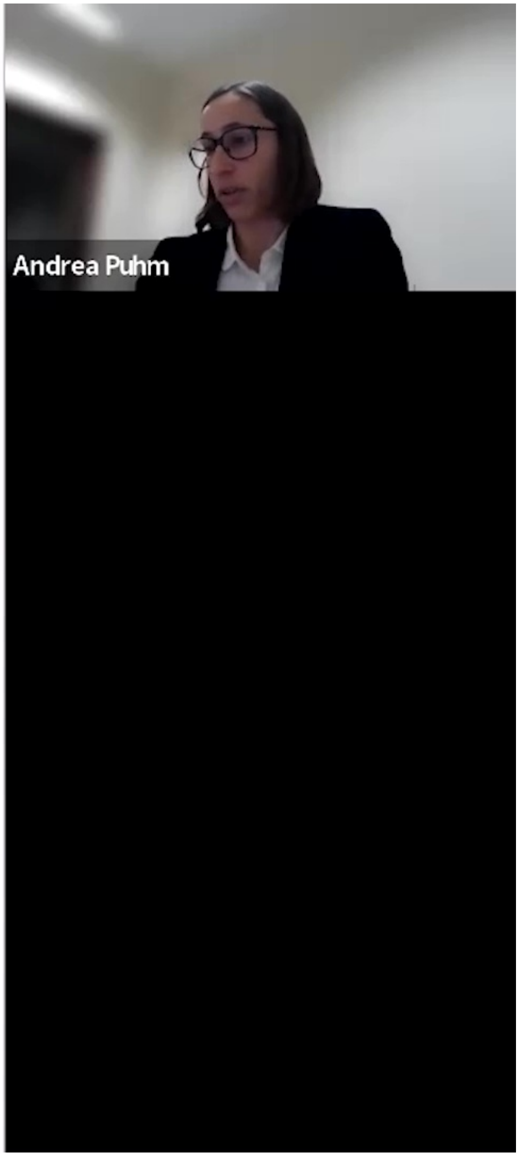
$$\varphi^{\mathrm{M}}_1 = \Theta(X^2)\varphi_1$$

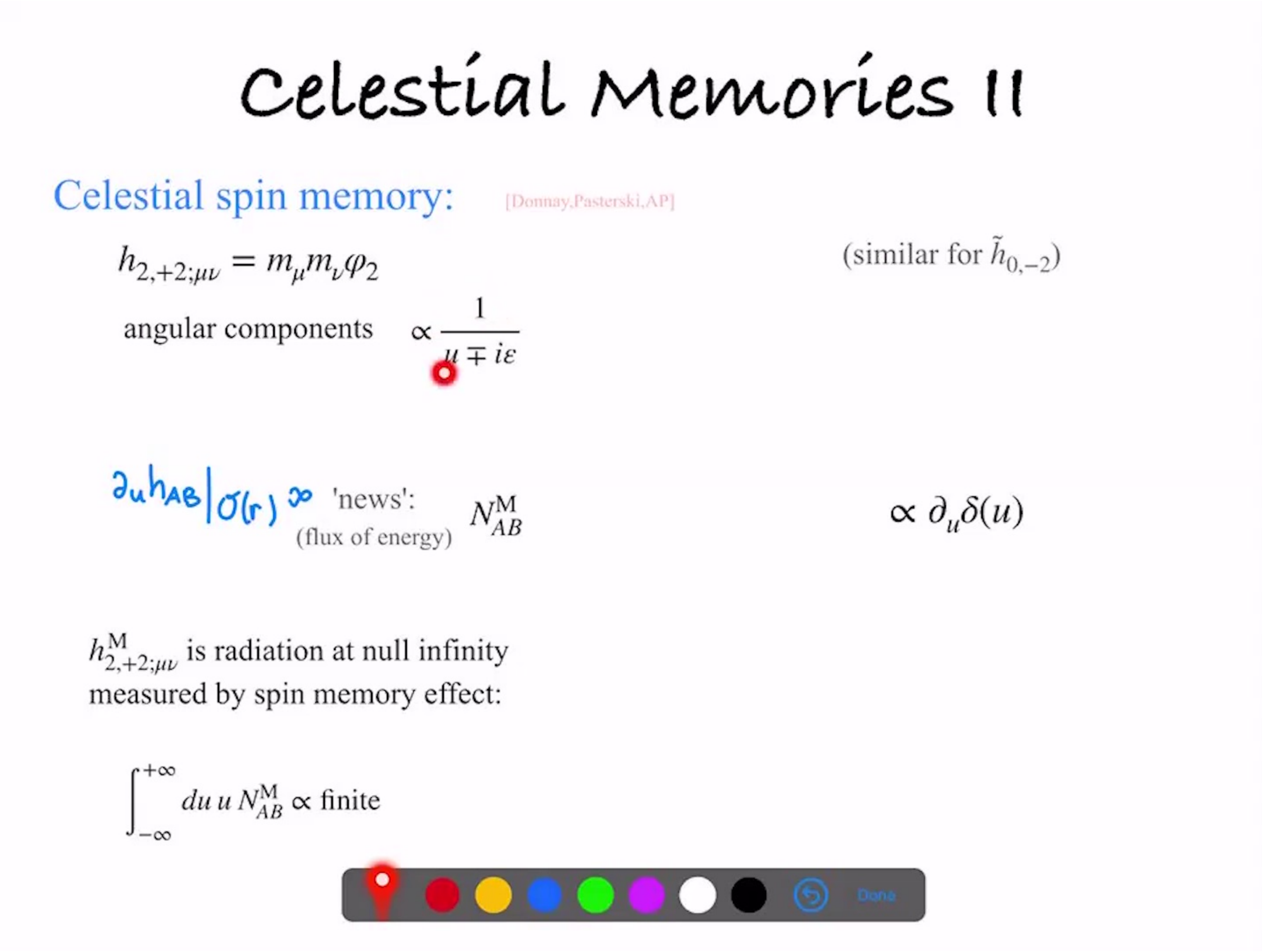
$$\lim_{r\to\infty}\Theta(X^2) \propto \Theta(-u)$$

$$h^{\mathrm{M}}_{1,+2;\mu\nu} = m_\mu m_\nu \varphi^{\mathrm{M}}_1$$

Goldstone-Memory pairings:

$$\Omega(\overset{\text{memory}}{\overset{\downarrow}{A^{\mathrm{M}}}}(w), \underset{\text{large gauge}}{\underset{\uparrow}{A^{\mathrm{G}}}}(w')) = (2\pi)^3\delta^{(2)}(w-w') = \Omega(\overset{\text{memory}}{\overset{\downarrow}{h^{\mathrm{M}}}}(w), \underset{\text{BMS supertranslations}}{\underset{\uparrow}{h^{\mathrm{G}}}}(w'))$$





Celestial Memories II

Celestial spin memory: [Donnay,Pasterski,AP]

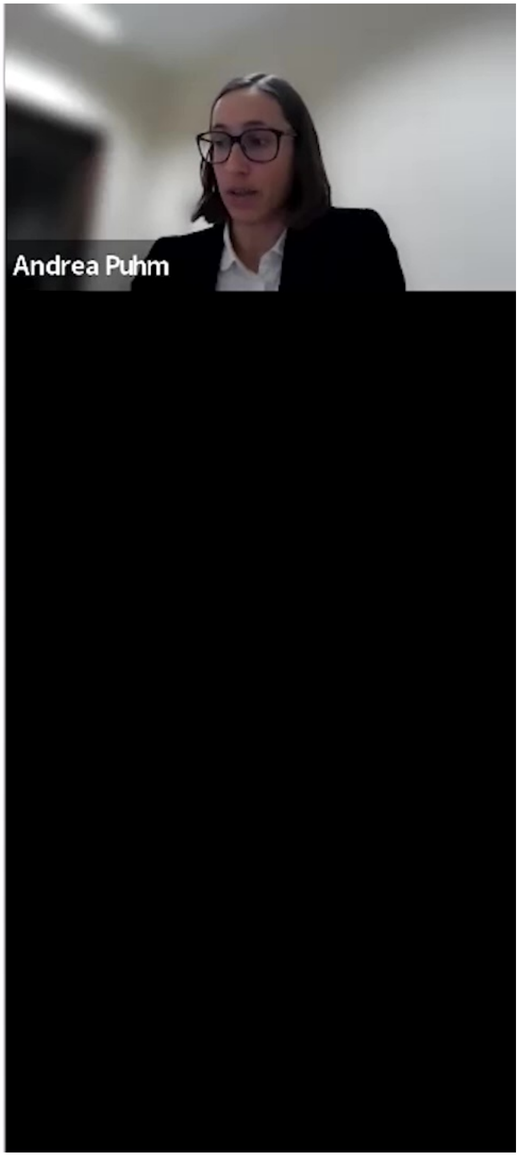
$h_{2,+2;\mu\nu} = m_\mu m_\nu \varphi_2$ (similar for $\tilde{h}_{0,-2}$)

angular components $\propto \frac{1}{u \mp i\varepsilon}$

$\partial_u h_{AB}|_{\sigma(r)} \infty$ 'news': (flux of energy) N^{M}_{AB} $\propto \partial_u \delta(u)$

$h^{\mathrm{M}}_{2,+2;\mu\nu}$ is radiation at null infinity measured by spin memory effect:

$$\int_{-\infty}^{+\infty} du\, u\, N^{\mathrm{M}}_{AB} \propto \text{finite}$$



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(similar for $\tilde{h}_{0,-2}$)

angular components $\propto \dfrac{1}{u \mp i\varepsilon}$

Admixture of incoming (-) and outgoing (+) modes:

$\partial_u h_{AB}\big|_{\mathcal{O}(r)} \propto$ 'news': (flux of energy)

$$N^{\mathrm{M}}_{AB} = \lim_{\varepsilon\to 0} \frac{1}{2\pi i}(N^{+}_{AB} - N^{-}_{AB}) \quad \propto \partial_u \delta(u)$$

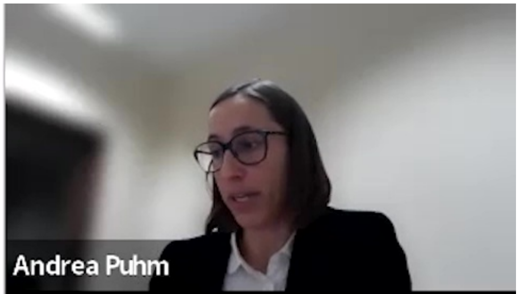
$h^{\mathrm{M}}_{2,+2;\mu\nu}$ is radiation at null infinity measured by spin memory effect:

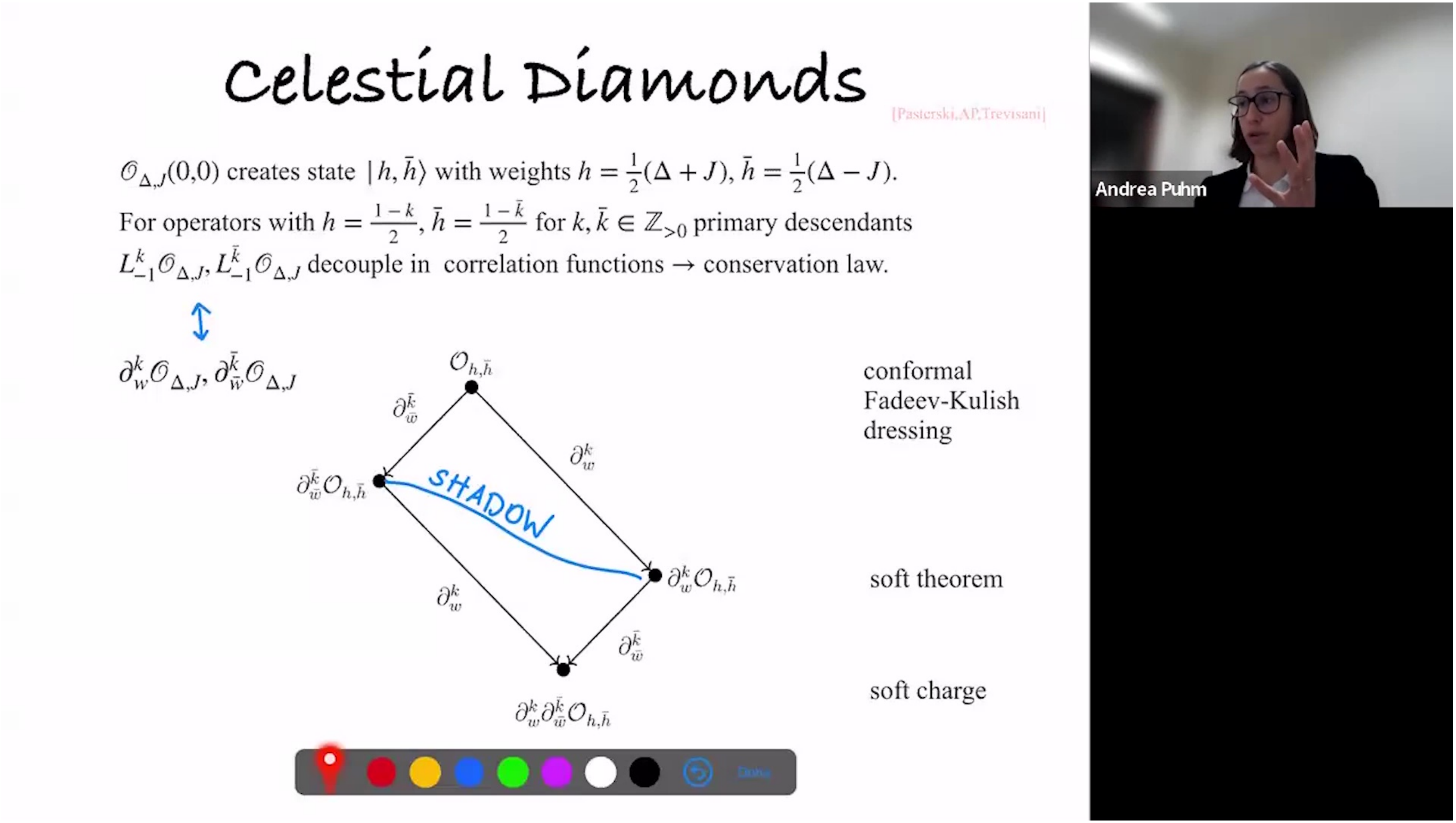
$$\int_{-\infty}^{+\infty} du\, u\, N^{\mathrm{M}}_{\bar{z}\bar{z}} \propto \frac{\delta^{(2)}(z-w)}{(1+z\bar{z})^2}$$

$$\Omega(\mathrm{h}^{\mathrm{M}}_{2,+2}(\mathrm{w}), \mathrm{h}^{\mathrm{G}}_{0,-2}(\mathrm{w}')) = \frac{4\pi^2}{3}\delta^{(2)}(\mathrm{w}-\mathrm{w}')$$

$$\Omega(h^{\mathrm{M}}_{2,+2}(w), \tilde{h}^{\mathrm{G}}_{2,+2}(w')) = \frac{2\pi}{3}\frac{1}{(w-w')^4}$$

\`dual stress tensor'





Celestial Diamonds

[Pasterski,AP,Trevisani]

$\mathcal{O}_{\Delta,J}(0,0)$ creates state $|h,\bar{h}\rangle$ with weights $h=\frac{1}{2}(\Delta+J)$, $\bar{h}=\frac{1}{2}(\Delta-J)$.

For operators with $h=\frac{1-k}{2}$, $\bar{h}=\frac{1-\bar{k}}{2}$ for $k,\bar{k}\in\mathbb{Z}_{>0}$ primary descendants $L_{-1}^{k}\mathcal{O}_{\Delta,J}, L_{-1}^{\bar{k}}\mathcal{O}_{\Delta,J}$ decouple in correlation functions → conservation law.

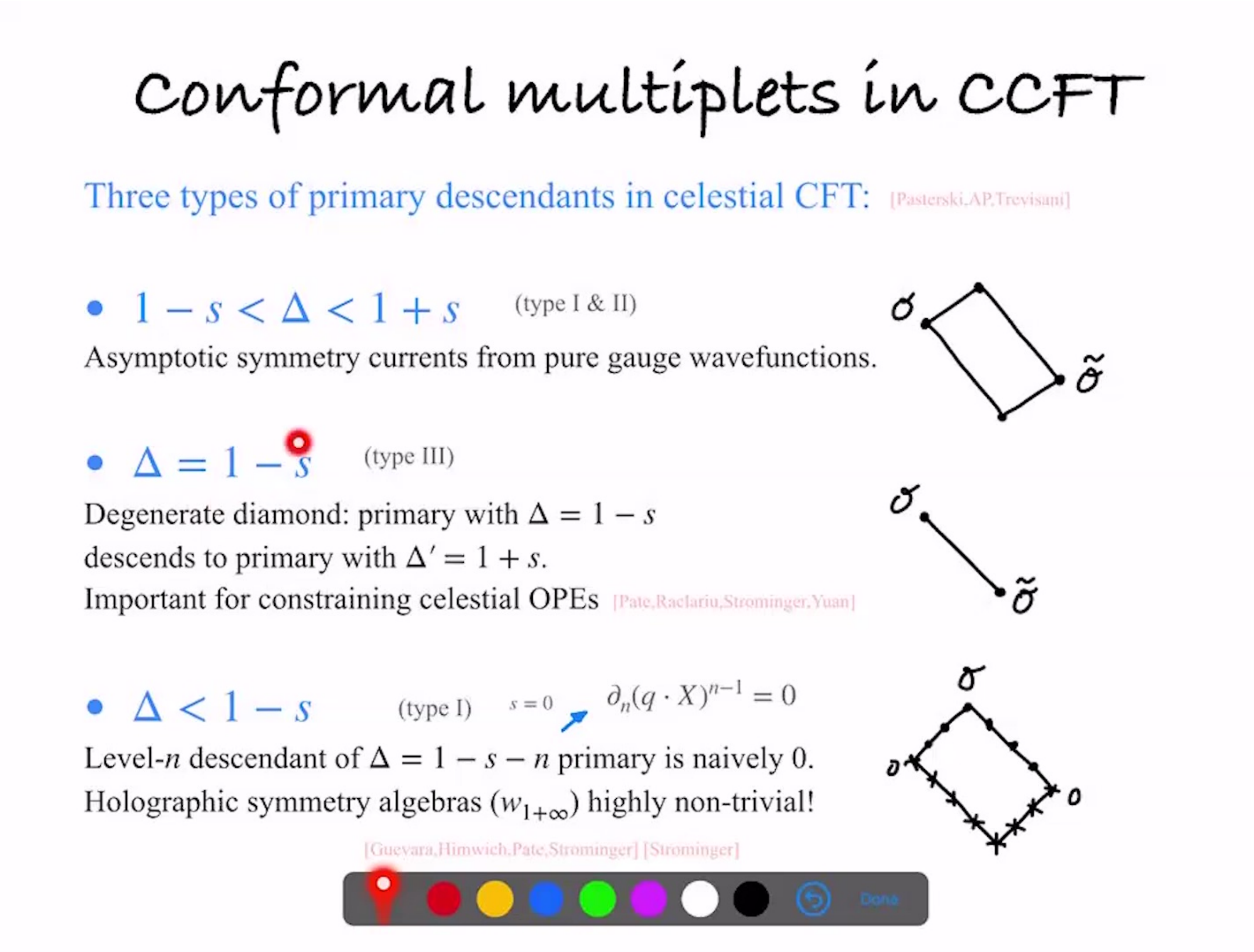
↕

$\partial_w^k\mathcal{O}_{\Delta,J}, \partial_{\bar{w}}^{\bar{k}}\mathcal{O}_{\Delta,J}$

conformal
Fadeev-Kulish
dressing

soft theorem

soft charge



Conformal multiplets in CCFT

Three types of primary descendants in celestial CFT: [Pasterski,AP,Trevisani]

- $1 - s < \Delta < 1 + s$ (type I & II)

Asymptotic symmetry currents from pure gauge wavefunctions.

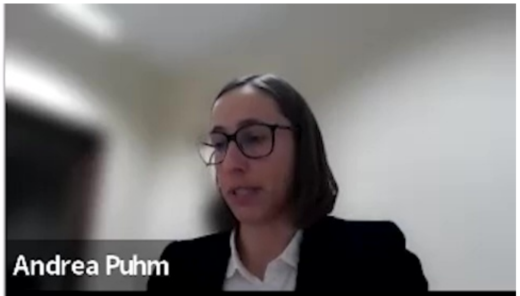
- $\Delta = 1 - s$ (type III)

Degenerate diamond: primary with $\Delta = 1 - s$
descends to primary with $\Delta' = 1 + s$.
Important for constraining celestial OPEs [Pate,Raclariu,Strominger,Yuan]

- $\Delta < 1 - s$ (type I) $s = 0$ $\partial_n (q \cdot X)^{n-1} = 0$

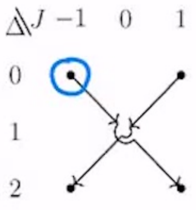
Level-n descendant of $\Delta = 1 - s - n$ primary is naively 0.
Holographic symmetry algebras ($w_{1+\infty}$) highly non-trivial!

[Guevara,Himwich,Pate,Strominger] [Strominger]



Goldilocks Modes

Sub-leading photon:



$A_{0,-1;\mu} = \bar{m}_\mu$

[Donnay,Pasterski,AP]

$$A^{G}_{0,-1;z} = -\lim_{\Delta\to 0}\frac{u^{1-\Delta}}{\Delta}\overset{\propto\,\delta^{(2)}(z-w)}{\boxed{D_z^2\bar{y}^z}} + u\bar{W}_z + \mathcal{O}(r^{-1})$$

$$A^{G}_{0,-1;\bar{z}} = \underline{r\bar{y}_{\bar{z}}} + u\bar{W}_{\bar{z}} + \mathcal{O}(r^{-1})$$

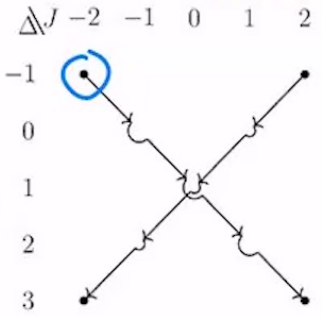
Precisely kernel of soft charge identified by [Campiglia,Laddha] in their overleading gauge symmetry proposal. See also [Compere].

[Pasterski,AP,Trevisani]

$$\partial_w^{2s}\mathcal{O}_{\Delta=1-s,J=+s} \propto \tilde{\mathcal{O}}_{2-\Delta,-s}$$

primary descendant = shadow primary

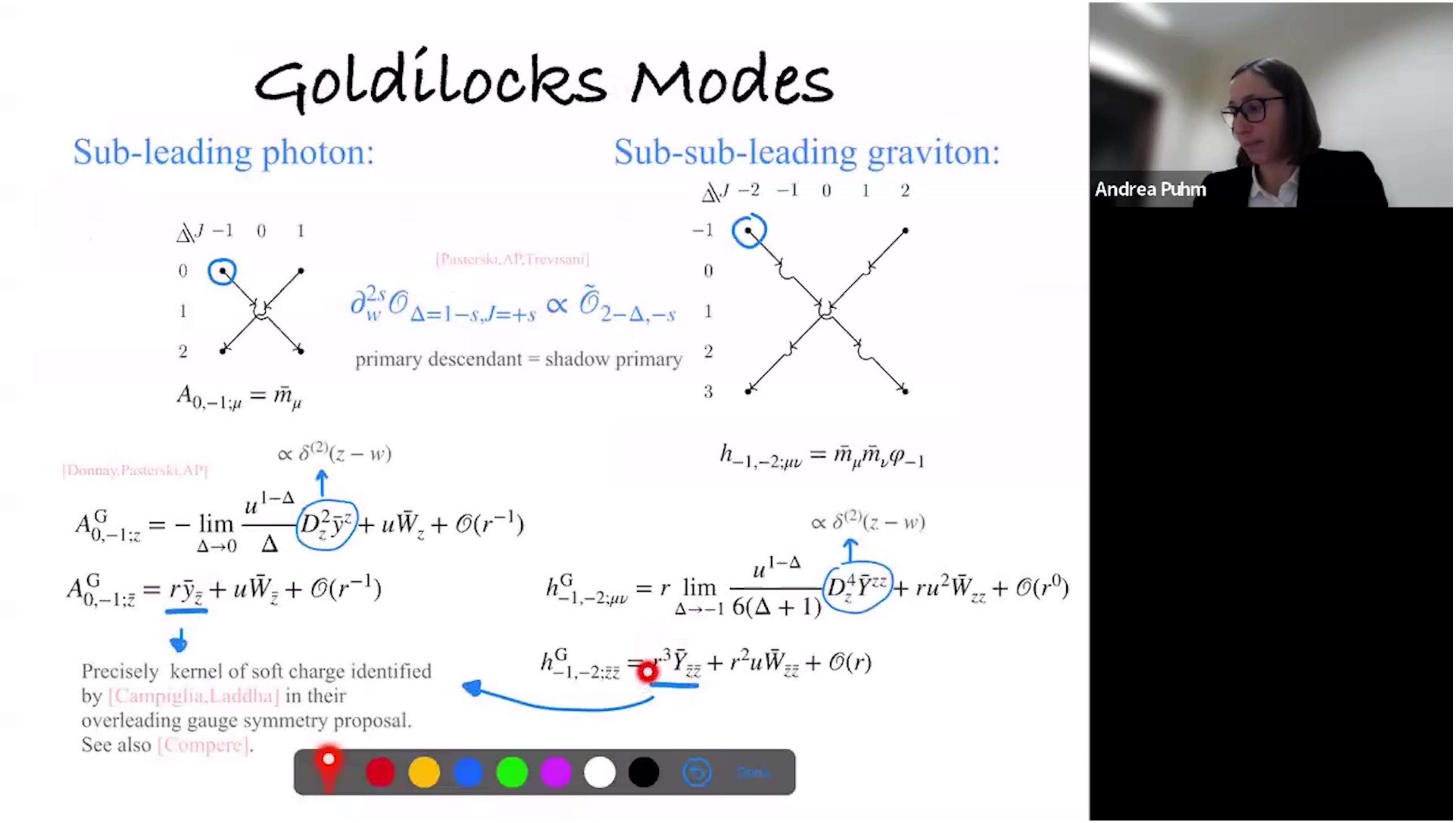
Sub-sub-leading graviton:



$h_{-1,-2;\mu\nu} = \bar{m}_\mu\bar{m}_\nu\varphi_{-1}$

$$h^{G}_{-1,-2;\mu\nu} = r\lim_{\Delta\to -1}\frac{u^{1-\Delta}}{6(\Delta+1)}\overset{\propto\,\delta^{(2)}(z-w)}{\boxed{D_z^4\bar{Y}^{zz}}} + ru^2\bar{W}_{zz} + \mathcal{O}(r^0)$$

$$h^{G}_{-1,-2;\bar{z}\bar{z}} = \underline{r^3\bar{Y}_{\bar{z}\bar{z}}} + r^2u\bar{W}_{\bar{z}\bar{z}} + \mathcal{O}(r)$$



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Precisely kernel of soft charge identified by [Campiglia,Laddha] in their overleading gauge symmetry proposal. See also [Compere].

Sub-leading Soft Charge

[Donnay,Pasterski,AP]

- First large $r \to \infty$, then conformally soft $\Delta \to 0$:

  $\Rightarrow$ contact terms

Mellin transformed creation and annihilation operators $a_{\Delta,\pm s}, a^{\dagger}_{\Delta,\pm s}$

First $\int_{-\infty}^{+\infty} du$, then $\Delta \to 0$:

$$\mathcal{O}_{\Delta\to 0,-1} \ni \lim_{\Delta\to 0}\frac{1}{\Delta}\int du d^2z\, u^{1-\Delta}\boxed{D_z^2\bar{y}^z}\overleftrightarrow{\partial}_u \hat{A}_{\bar{z}}^{(0)} \longrightarrow \mathcal{O}_{0,-1} \ni ie[a_{0,-1} + a^{\dagger}_{0,-1}]$$

Inserted in celestial amplitudes: subleading soft photon theorem.

- First conformally soft $\Delta \to 0$, then large $r \to \infty$:

  $\Rightarrow$ non-contact terms

naively divergent soft charge $\quad \propto \lim\limits_{r\to\infty} r \int du \partial_u \hat{A}_z^{(0)}$

r × soft charge for leading soft theorem!

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r × soft charge for leading soft theorem!

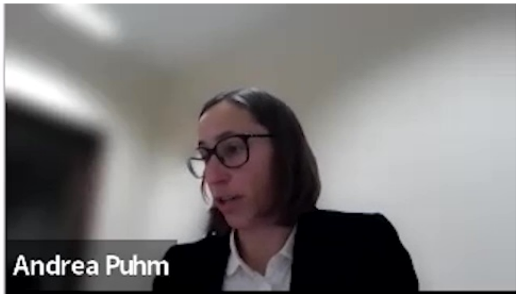
renormalization of symplectic product:
(ambiguity in covariant phase space formalism)

[Compere,Fiorucci,Ruzziconi]

$$Q^{ren}_{0,-1} = -2\int d^2zdu\bar{W}_z u\partial_u\hat{A}^{(0)}_{\bar{z}}$$

$\Rightarrow$ analogous story for (super) gravity

[Pano,Pasterski,AP]



Memories for Goldilocks

[Donnay,Pasterski,AP]

Sub-leading photon memory mode:

$$A_{2,+1;\mu} = m_\mu \varphi_2$$

$$A^{\mathrm{M}}_{2,+1;z} = \mathcal{O}(r^{-1}) \qquad A^{\mathrm{M}}_{2,+1;\bar{z}} = -\,\underline{\delta(u)}(1+z\bar{z})^{-1}\pi\delta^{(2)}(z-w) + \mathcal{O}(r^{-1})$$

Goldilocks-Memory pairings: $\Omega(A^{\mathrm{M}}_{2,+1}(w), A^{\mathrm{G}}_{0,-1}(w')|_{\text{saddle pt}} \propto \lim_{\Delta\to 0}\frac{1}{\Delta}\delta^{(2)}(w-w')$

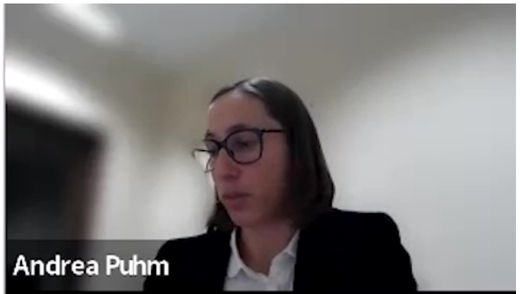
Sub-sub-leading graviton memory mode:

$$h_{3,+2;\mu\nu} = m_\mu m_\nu \varphi_3$$

$$h^{\mathrm{M}}_{3,+2;zz} = \mathcal{O}(r^{-2}) \qquad h^{\mathrm{M}}_{3,+2;\bar{z}\bar{z}} = \frac{r}{8}\underline{\partial_u\delta(u)}(1+z\bar{z})\pi\delta^{(2)}(z-w) + \mathcal{O}(r^{0})$$

Goldilocks-Memory pairings: $\Omega(h^{\mathrm{M}}$[illegible]$_{2}(w), h^{\mathrm{G}}_{-1,+2}(w')|_{\text{saddle pt}} \propto \lim_{\Delta\to -1}\frac{1}{\Delta+1}\delta^{(2)}(w-w')$

Done



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Sub-leading photon memory mode:

$$A_{2,+1;\mu} = m_\mu \varphi_2$$

$$A^{\mathrm{M}}_{2,+1;z} = \mathcal{O}(r^{-1}) \qquad A^{\mathrm{M}}_{2,+1;\bar{z}} = -\underline{\delta(u)}(1+z\bar{z})^{-1}\pi\delta^{(2)}(z-w) + \mathcal{O}(r^{-1})$$

Goldilocks-Memory pairings:

$$\Omega(A^{\mathrm{M}}_{2,+1}(w), A^{\mathrm{G}}_{0,-1}(w')\,|_{\text{saddle pt}} \propto \lim_{\Delta\to 0}\frac{1}{\Delta}\delta^{(2)}(w-w')$$

$$\Omega(A^{\mathrm{M}}_{2,+1}(w), \tilde{A}^{\mathrm{G}}_{2,+1}(w'))\,|_{\text{sh}\,\circ\,\text{saddle pt}} \propto \frac{1}{(w-w')^3(w-w')}$$

off-diagonal level for sub-leading soft photon current

Sub-sub-leading graviton memory mode:

$$h_{3,+2;\mu\nu} = m_\mu m_\nu \varphi_3$$

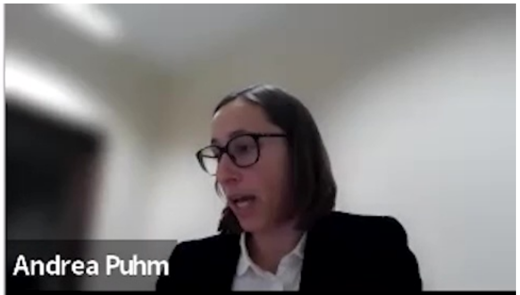
$$h^{\mathrm{M}}_{3,+2;zz} = \mathcal{O}(r^{-2}) \qquad h^{\mathrm{M}}_{3,+2;\bar{z}\bar{z}} = \frac{r}{8}\underline{\partial_u\delta(u)}(1+z\bar{z})\pi\delta^{(2)}(z-w) + \mathcal{O}(r^0)$$

Goldilocks-Memory pairings:

$$\Omega(h^{\mathrm{M}}_{3,+2}(w), h^{\mathrm{G}}_{-1,+2}(w')\,|_{\text{saddle pt}} \propto \lim_{\Delta\to -1}\frac{1}{\Delta+1}\delta^{(2)}(w-w')$$

$$\Omega(h^{\mathrm{M}}_{3,+2}(w), \tilde{h}^{\mathrm{G}}_{3,+2}(w')\,|_{\text{sh}\,\circ\,\text{saddle pt}} \propto \frac{1}{(w-w')^5(\bar{w}-\bar{w}')}$$

off-diagonal level for sub-sub-leading soft graviton current



Over-leading symmetries?

[Donnay,Pasterski,AP]

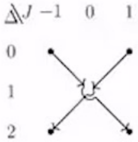
While the $1-s<\Delta<1+s$ Goldstones wavefunctions are pure gauge the $\Delta=1-s$ Goldilocks are not but nevertheless ∃ soft theorems.

Can we recast the overleading (compared to radiative) large r behavior

$A^{\mathrm{G}}_{0,-1;\bar{z}} \propto r$ & $h^{\mathrm{G}}_{-1,-2;\bar{z}\bar{z}} \propto r^2$ *as a gauge symmetry?*

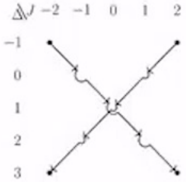
Combinations of $J=+1$ and $J=-1$ sub-leading soft photons descendants: $\propto \nabla_\mu \Lambda'_{gauge} + \ldots$

$\mathcal{O}(r)$



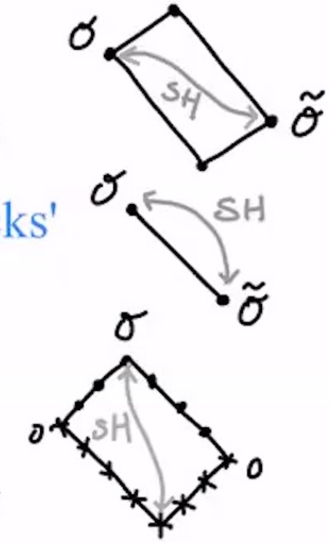
Combinations of $J=+2$ and $J=-2$ sub-sub-leading soft gravitons descendants: $\propto \nabla_\mu \nabla_\nu \Lambda'_{gravity} + \ldots$

$\mathcal{O}(r^3)$



Consistent with over-leading gauge symmetries of [Campiglia,Laddha]

Celestial symmetries



- $1-s<\Delta<1+s$: Goldstones

$$\left(\widetilde{\partial_w^k \mathcal{O}_{1-\frac{k+\bar{k}}{2}}}\right) \propto \partial_{\bar{w}}^{\bar{k}} \mathcal{O}_{1-\frac{k+\bar{k}}{2}}$$

- $\Delta=1-s$: Goldstone-like 'Goldilocks'

$$\partial_w^{2s} \mathcal{O}_{\Delta=1-s,J=+s} \propto \tilde{\mathcal{O}}_{2-\Delta,-s}$$

- $\Delta<1-s$: Goldstone-like tower

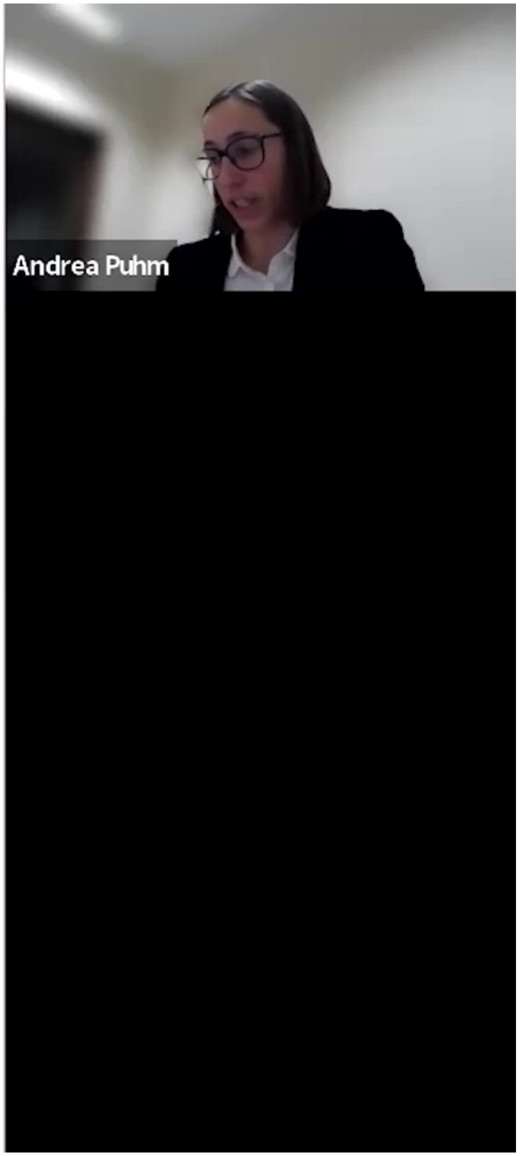
$$\partial_w^k \partial_{\bar{w}}^{\bar{k}} \mathcal{O}_{\Delta+\epsilon,J} \propto \epsilon \tilde{\mathcal{O}}_{2-\Delta,-J}$$

The tower of celestial symmetry generators in gauge theory and gravity

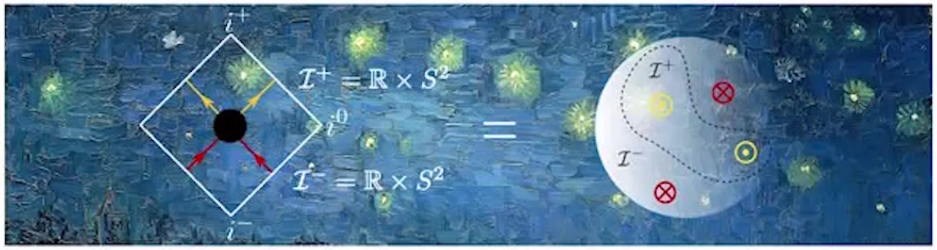
$$R^\Delta = \lim_{\epsilon\to 0} \epsilon \mathcal{O}_{\Delta+\epsilon,\pm 1} \qquad H^\Delta = \lim_{\epsilon\to 0} \epsilon \mathcal{O}_{\Delta+\epsilon,\pm 2}$$

$\Delta = 1,0,-1,\ldots$ $\qquad$ $\Delta = 2,1,0,-1,\ldots$

form closed algebra! Strominger] [Strominger]



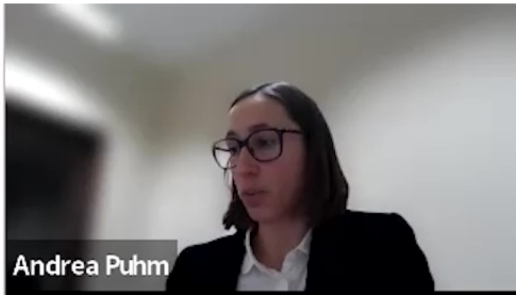
Summary

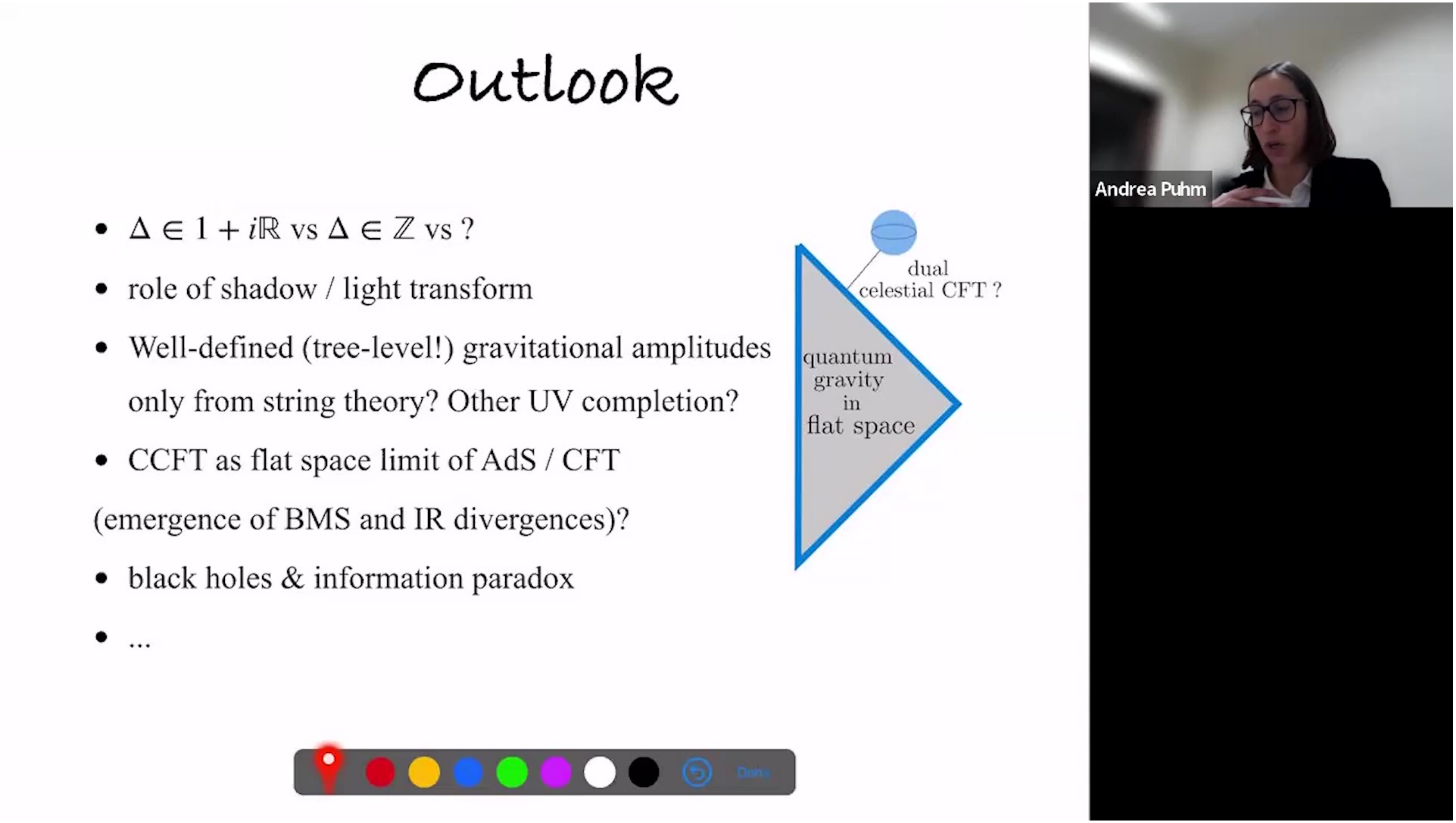


Celestial amplitudes organize scattering in novel way:

(by virtue of going to a boost weight/Rindler energy Δ basis)

- power of symmetry to organize (conformally) soft sector
- 4D asymptotic symmetries from 2D celestial currents
- soft expansion → conformally soft poles
- manifests ∞ many (soft) symmetries
- soft towers form holographic symmetry algebra
- anti-Wilsonian paradigm (probing all energy scales)
- ...





Outlook

- $\Delta \in 1 + i\mathbb{R}$ vs $\Delta \in \mathbb{Z}$ vs ?
- role of shadow / light transform
- Well-defined (tree-level!) gravitational amplitudes only from string theory? Other UV completion?
- CCFT as flat space limit of AdS / CFT

(emergence of BMS and IR divergences)?

- black holes & information paradox
- ...