Title: AdS/CFT

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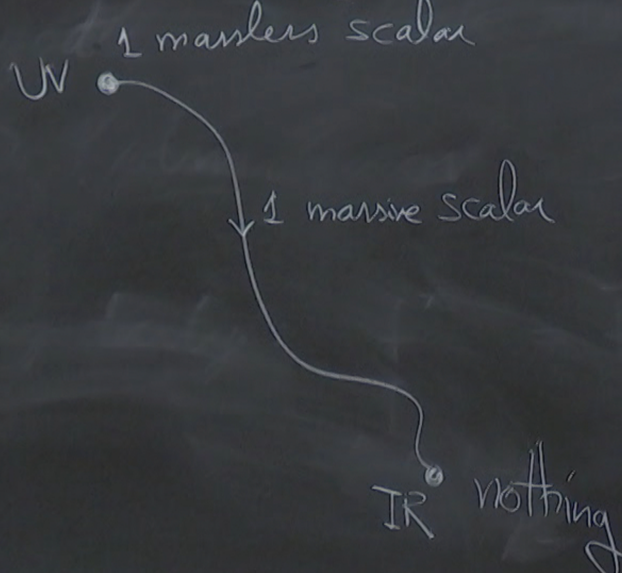
$$G(r) = \int \frac{d^d p}{(2\pi)^d} \frac{1}{p^2+m^2} e^{ip\cdot x} \simeq \begin{cases} e^{-mr} & , \; mr > \\ \dfrac{1}{r^{d-2}} & , \; mr < \end{cases}$$



$\underbrace{\dfrac{1}{r^{d-2}}}$ 2pt function of a scalar operator in a CFT with $\Delta = \dfrac{d-2}{2}$

$\gg 1$

< 1

$$\langle O_1(x_1)\cdots\rangle_{\Omega^2 g} = \Omega^{-\Delta_1}(x_1)\cdots\langle O_1(x_1)\cdots\rangle_g$$

CFT def.

$$\Downarrow$$

$$\langle O_1(\tilde{x}_1)\cdots\rangle_{\mathbb{R}^d} = \Omega^{-\Delta_1}(x_1)\cdots\langle O_1(x_1)\cdots\rangle_{\mathbb{R}^d}$$

$\tilde{x}^\mu$ a conf. transf s.t. $d\tilde{x}^\mu d\tilde{x}_\mu = \Omega^2 dx_\mu dx^\mu$

$$\langle \mathcal{O}_{\Delta_1}(\tilde{x}_1)\, \mathcal{O}_{\Delta_2}(\tilde{x}_2) \rangle_{\mathbb{R}^d} = \Omega^{-\Delta_1}(x_1)\, \Omega^{-\Delta_2}(x_2)$$

$$= 0 \text{ unless } \Delta_1 = \Delta_2$$

if $\Delta_1 = \Delta_2$, $\langle \mathcal{O}_1(\tilde{x}_1) \ldots \rangle = [\Omega(x_1)\Omega(x_2)]^{-\Delta} \langle$

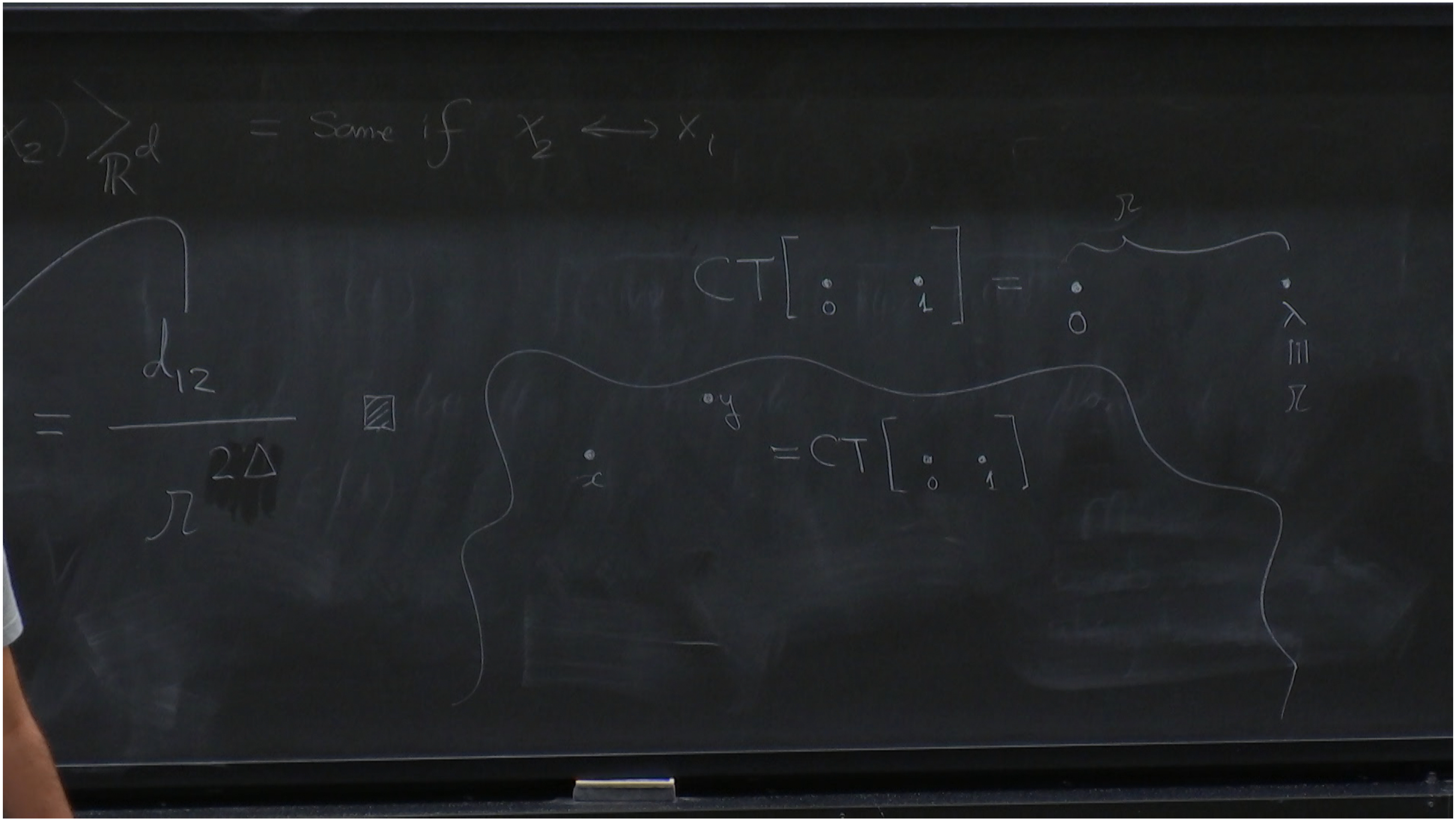
$$\tilde{x} = \lambda x \quad \text{with } \lambda = \Omega$$

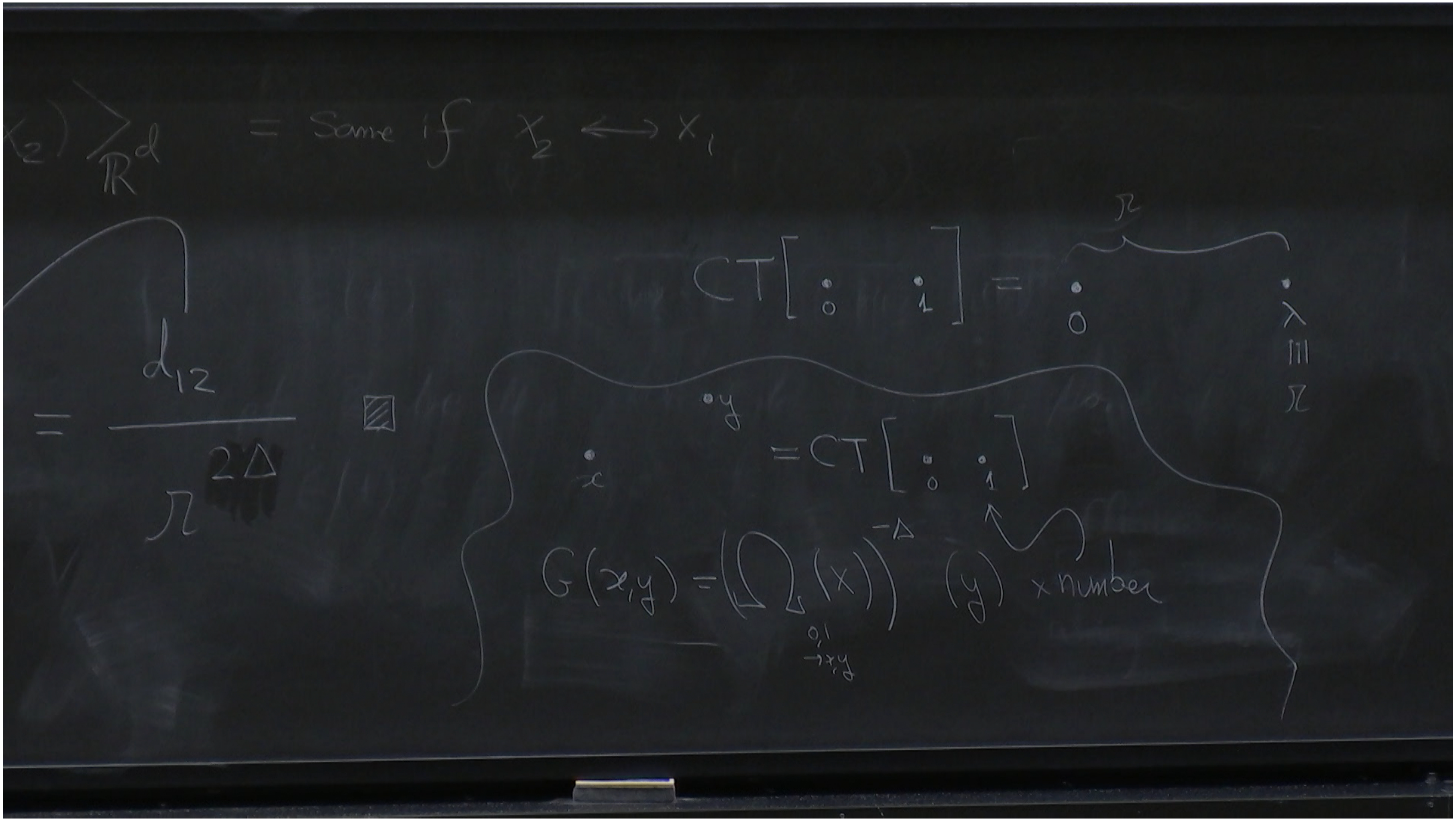
$$\Omega^{-\Delta_1}(x_1)\,\Omega^{-\Delta_2}(x_2)\,\langle O_{\Delta_1}(x_1)\,O_{\Delta_2}(x_2)\rangle_{\mathbb{R}^d} \quad = \text{Same if}$$

0 unless $\Delta_1 = \Delta_2$

$$= \left[\Omega(x_1)\,\Omega(x_2)\right]^{-\Delta} \langle \overbrace{O_1(x_1)\dots}^{x_1=0,\ x_2=1} \rangle = \quad d_{12}$$

with $\lambda = r \longrightarrow \Omega = \lambda = r$

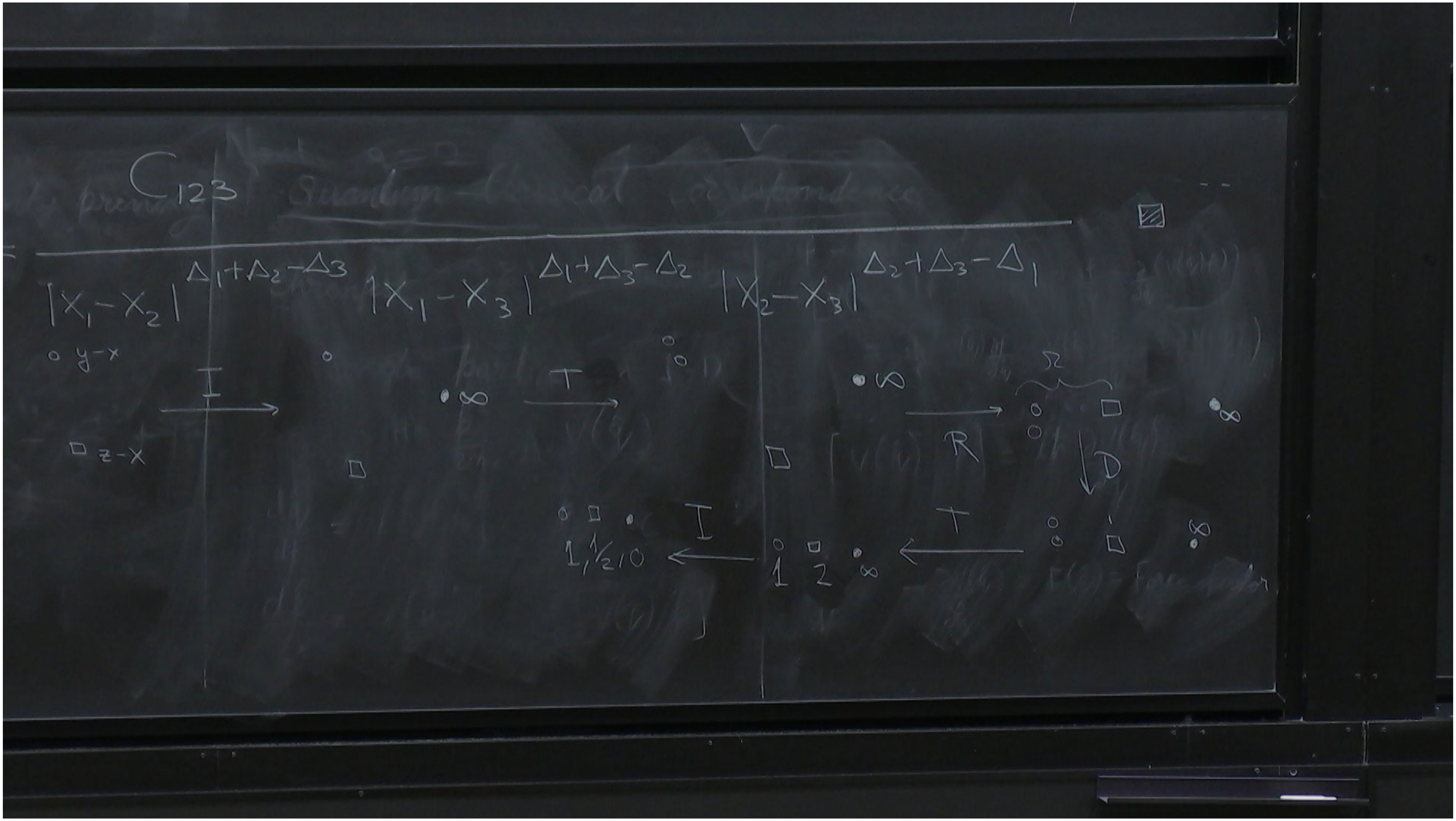

= Same if $x_2 \longleftrightarrow x_1$
$\mathbb{R}^d$
$= \dfrac{d_{12}}{\Omega^{2\Delta}}$
CT[0 1] = 0 λ ≡ Ω
Ω
y
x
= CT[0 1]


= Same if x2 ⟷ x1
R^d
d12
Ω 2Δ
CT[0 1] =
Ω
0 λ
Ω
y
x
= CT[0 1]
G(x,y) = (Ω(x))^-Δ (y) × number
0,1
→ x,y

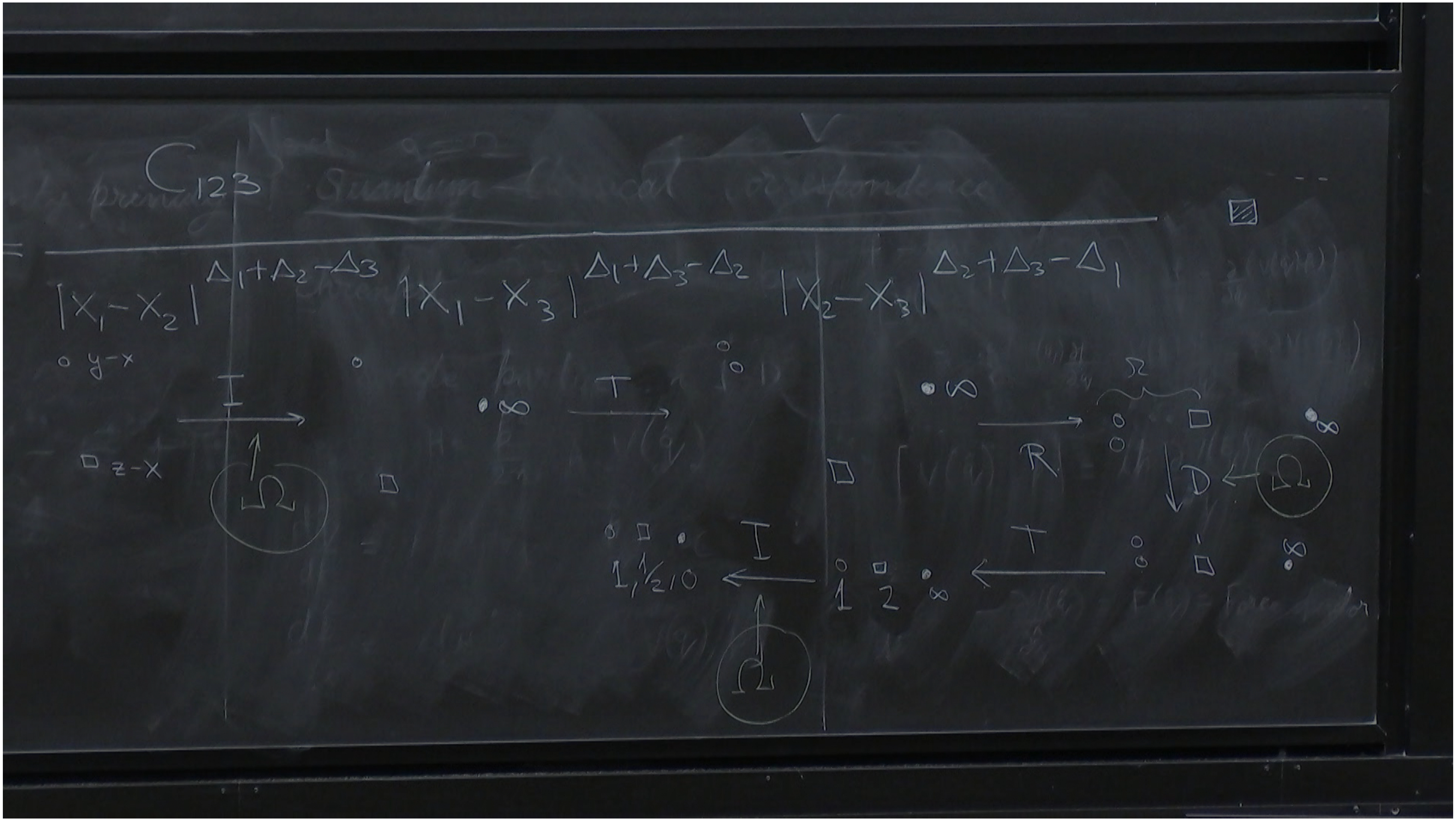
For 3pt, similarly,

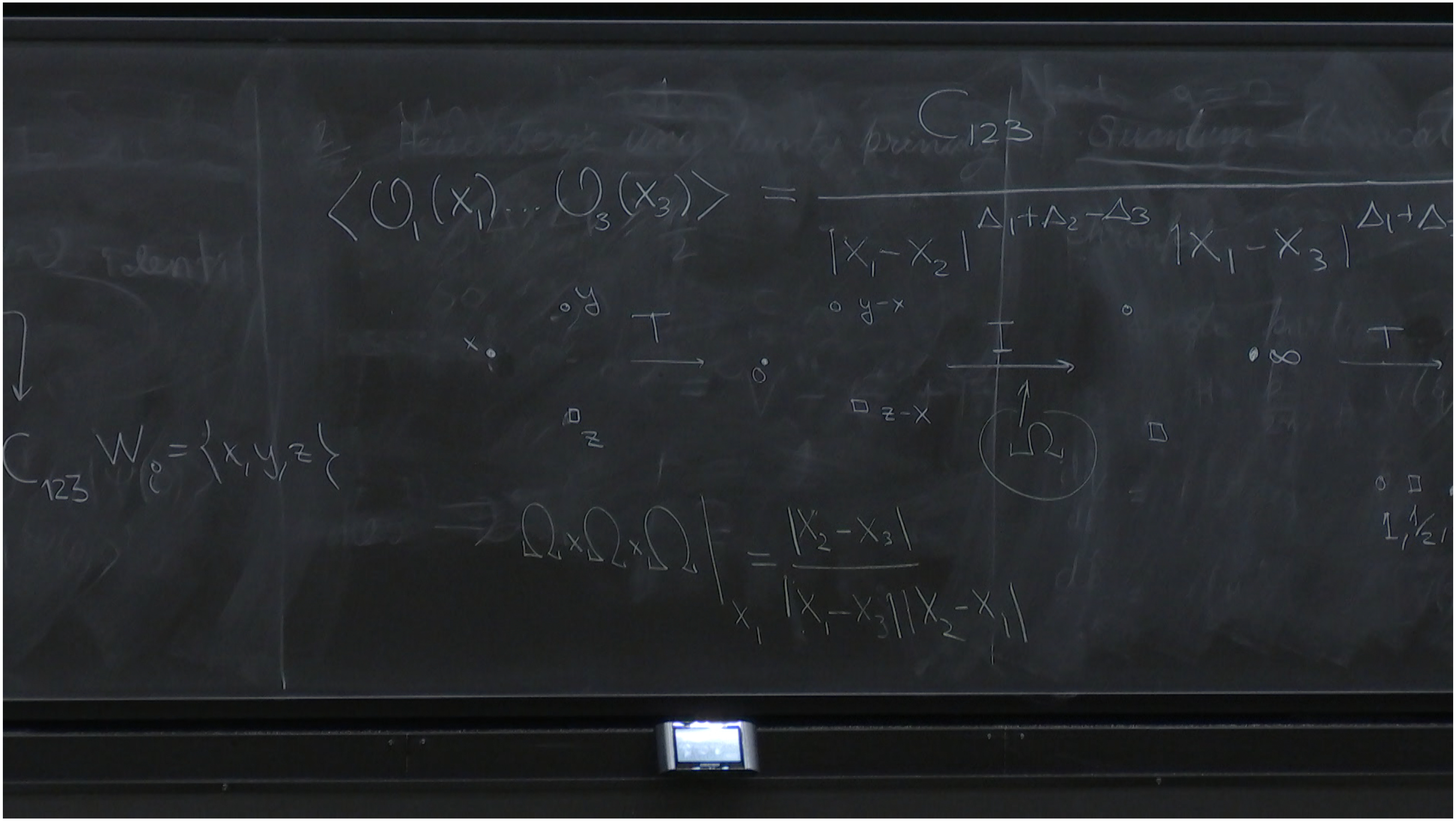
$$x \bullet \quad \bullet\, y \quad \bullet\, z \;=\; CT\left[\ \underset{0}{\bullet}\quad \underset{1/2}{\bullet}\quad \underset{1}{\bullet}\ \right]$$

$$G(x,y,z) = \prod_{i=1}^{3} \Omega_{\substack{0,\,1/2,\,1 \\ \downarrow \\ x,\,y,\,z}} \left(W_i\right)^{-\Delta_i} \times C_{123} \qquad W_i = \{x, y, z\}$$



C123
Quantum Classical Correspondence
|X1 − X2|^(Δ1+Δ2−Δ3) |X1 − X3|^(Δ1+Δ3−Δ2) |X2 − X3|^(Δ2+Δ3−Δ1)
y − x
z − x
I
T
R
D
1, 1/2, 0
1 2 ∞


C123
|X1 - X2|^(Δ1+Δ2-Δ3) |X1 - X3|^(Δ1+Δ3-Δ2) |X2 - X3|^(Δ2+Δ3-Δ1)
y-x
z-x
I
T
R
D
Ω
1, 1/2, 0
1 2 ∞



C123
⟨O1(x1)...O3(x3)⟩ = C123 / |x1−x2|^(Δ1+Δ2−Δ3) |x1−x3|^(Δ1+Δ
C123 Wi = {x, y, z}
Ω1×Ω2×Ω3 |x1 = |x2−x3| / |x1−x3||x2−x1|

$$\langle \mathcal{O}_1(x_1)\cdots\mathcal{O}_3(x_3)\rangle = \frac{C_{123}}{|x_1-x_2|^{\Delta_1+\Delta_2-\Delta_3}\,|x_1-x_3|^{\Delta_1+\Delta_3-\cdots}}$$

C_{123} $W_i = \{x, y, z\}$

$x\circ$, $\circ\, y$, $\square\, z$ $\xrightarrow{\ T\ }$ $0\circ$, $\circ\, y-x$, $\square\, z-x$ $\xrightarrow{\ I\ }$ $\circ$, $\bullet\,\infty$, $\square$ $\xrightarrow{\ T\ }$ $\circ\ \square$ $1, 1/2, \ldots$

Ω

$$\Rightarrow \Omega_1 \times \Omega_2 \times \Omega_3 \Big|_{x_1} = \frac{|x_2-x_3|}{|x_1-x_3|\,|x_2-x_1|}$$

etc

For 3pt, similarly,

y

$x \quad = CT\left[\begin{array}{ccc} \cdot & \cdot & \cdot \\ 0 & 1/2 & 1 \end{array}\right]$

z

$$G(x,y,z) = \boxed{\prod_{i=1}^{3} \Omega_{\underset{\underset{x,y,z}{\downarrow}}{0,1/2,1}} (W_i)^{-\Delta_i} \times C_{123}} \qquad W_i = \{x,y,z\}$$

$\langle \mathcal{O}_1(x_1) \ldots \mathcal{O}_3(x$

y

x

z

$\Rightarrow \Omega \times \Omega \times$

So far CFT data $\equiv \{\Delta_i, C_{ijk}\}$ $\rightsquigarrow$ All

CLAIM : actually this gives us all n-pt fn.

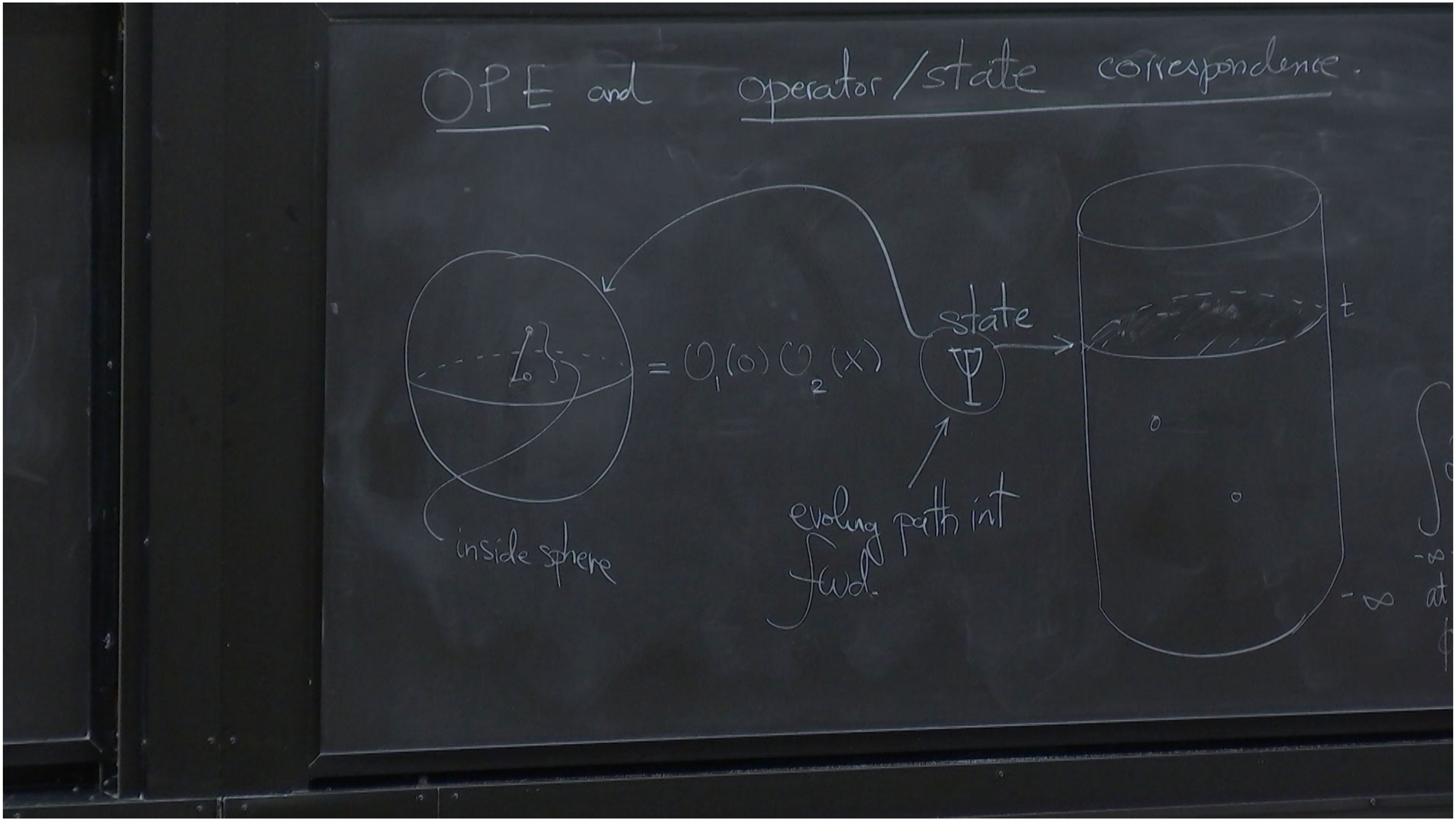
$$\langle \mathcal{O}_1(x_1) \ldots \mathcal{O}_4(x_4) \rangle_{\substack{\text{all} \\ \Delta_i = \Delta}} = \frac{1}{(x_1 - x_2)^{2\Delta} (x_3 - x_4)^{2\Delta}}$$

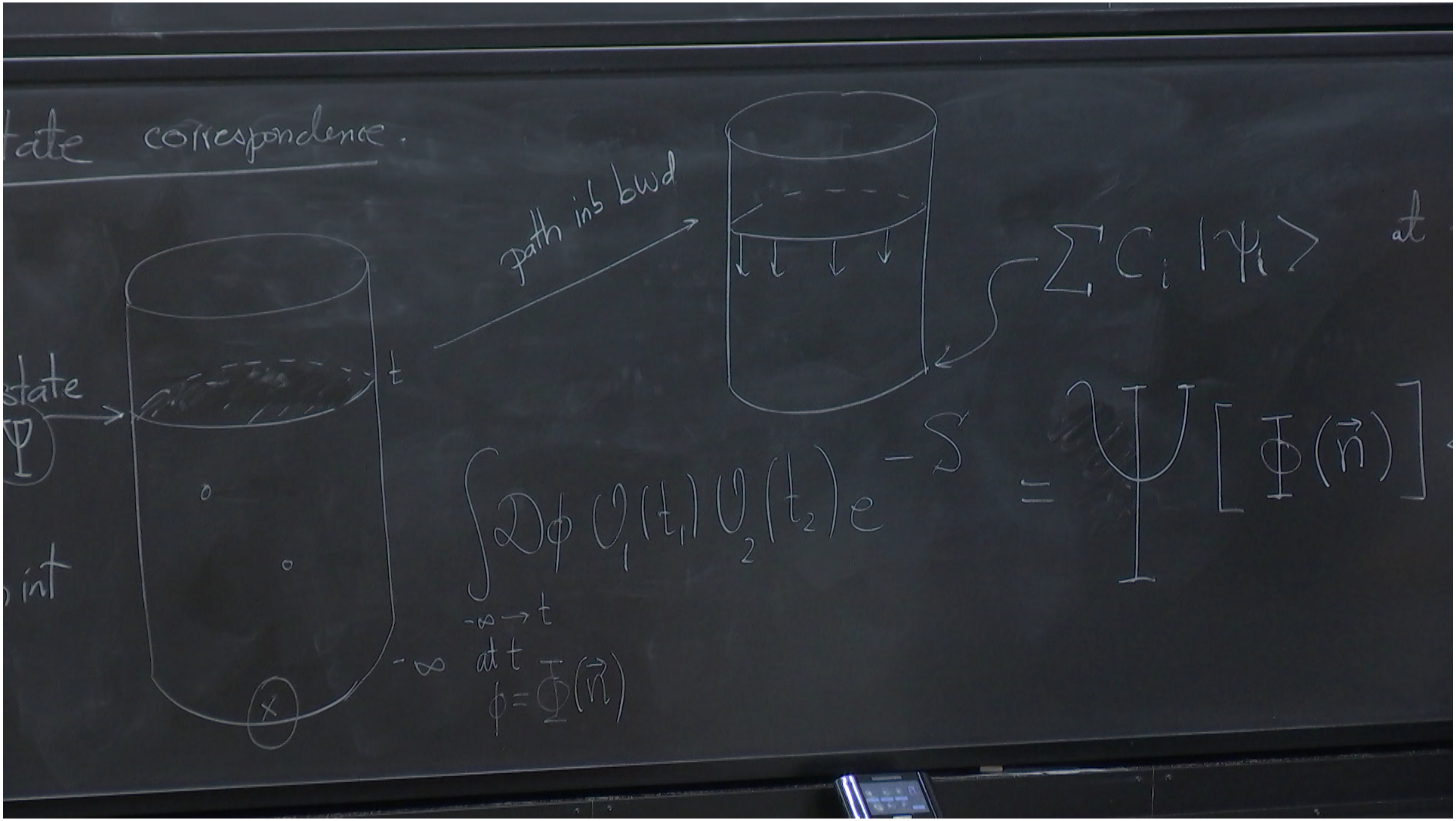
& 3pt ✓

cross-ratios, inv. under CT.

interesting dynamics!

$$\frac{(x_3-x_4)^2}{\ldots)^2(x_2-x_4)^2}, \quad \infty$$


OPE and operator/state correspondence.
= O₁(0) O₂(x)
state
Ψ
t
inside sphere
evolving path int fwd.
-∞



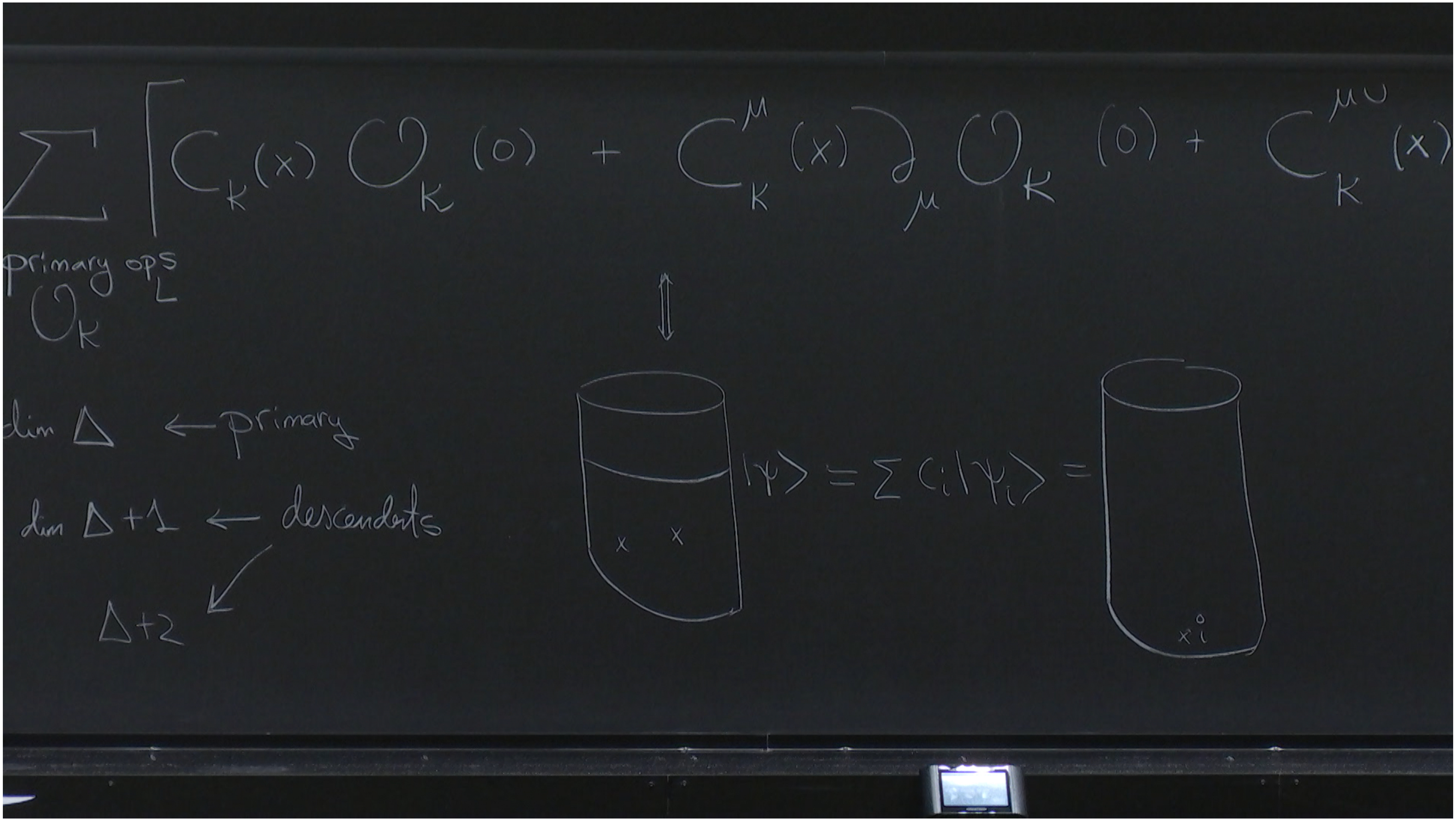
tate correspondence.
path int bwd
state
Ψ
t
int
-∞
Σ c_i |ψ_i> at
∫Dφ O_1(t_1) O_2(t_2) e^{-S} = Ψ[Φ(n⃗)]
-∞ → t
at t
φ = Φ(n⃗)

$$\mathcal{O}_1(x)\,\mathcal{O}_2(0) = \sum_{\substack{\text{primary ops} \\ \mathcal{O}_k}} C_k(x)\,\mathcal{O}_k(0) + C^{\mu}_k$$

$\partial\tilde{\mathcal{O}} \neq \mathcal{O}$ with dim Δ ← primary

$\partial_\mu \mathcal{O}$ has dim $\Delta + 1$ ← descendents

$\partial_{\mu_1}\partial_{\mu_2}\mathcal{O}_2$ $\quad \Delta + 2$

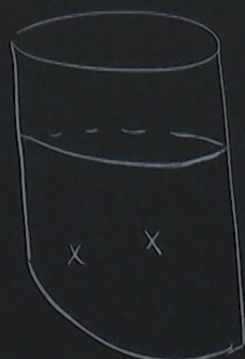
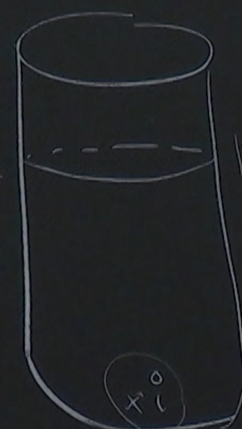

$\sum \left[C_k(x)\, \mathcal{O}_k(0) + C_k^{\mu}(x)\, \partial_\mu \mathcal{O}_k(0) + C_k^{\mu\nu}(x) \right.$
primary ops
$\mathcal{O}_k$
dim Δ ← primary
dim $\Delta + 1$ ← descendants
$\Delta + 2$
$|\psi\rangle = \sum c_i |\psi_i\rangle =$
x_i

$$\ldots_K(x)\, \mathcal{O}_K(0) + C^{\mu}_K(x)\, \partial_\mu \mathcal{O}_K(0) + C^{\mu\nu}_K(x)\, \partial_\mu \partial_\nu (\ldots$$

$\Updownarrow$

primary

descendants

 $|\psi\rangle = \sum c_i |\psi_i\rangle =$  $|\psi\rangle$

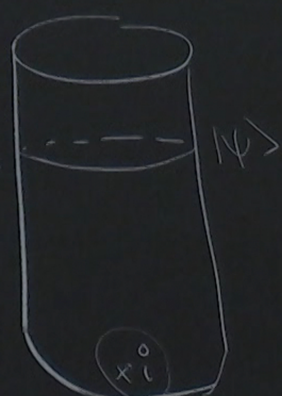
$$\ldots_\mu \mathcal{O}_k(0) + C_k^{\mu\nu}(x)\,\partial_\mu \partial_\nu \mathcal{O}_k(0) + \cdots \Big]$$

$$\Downarrow$$

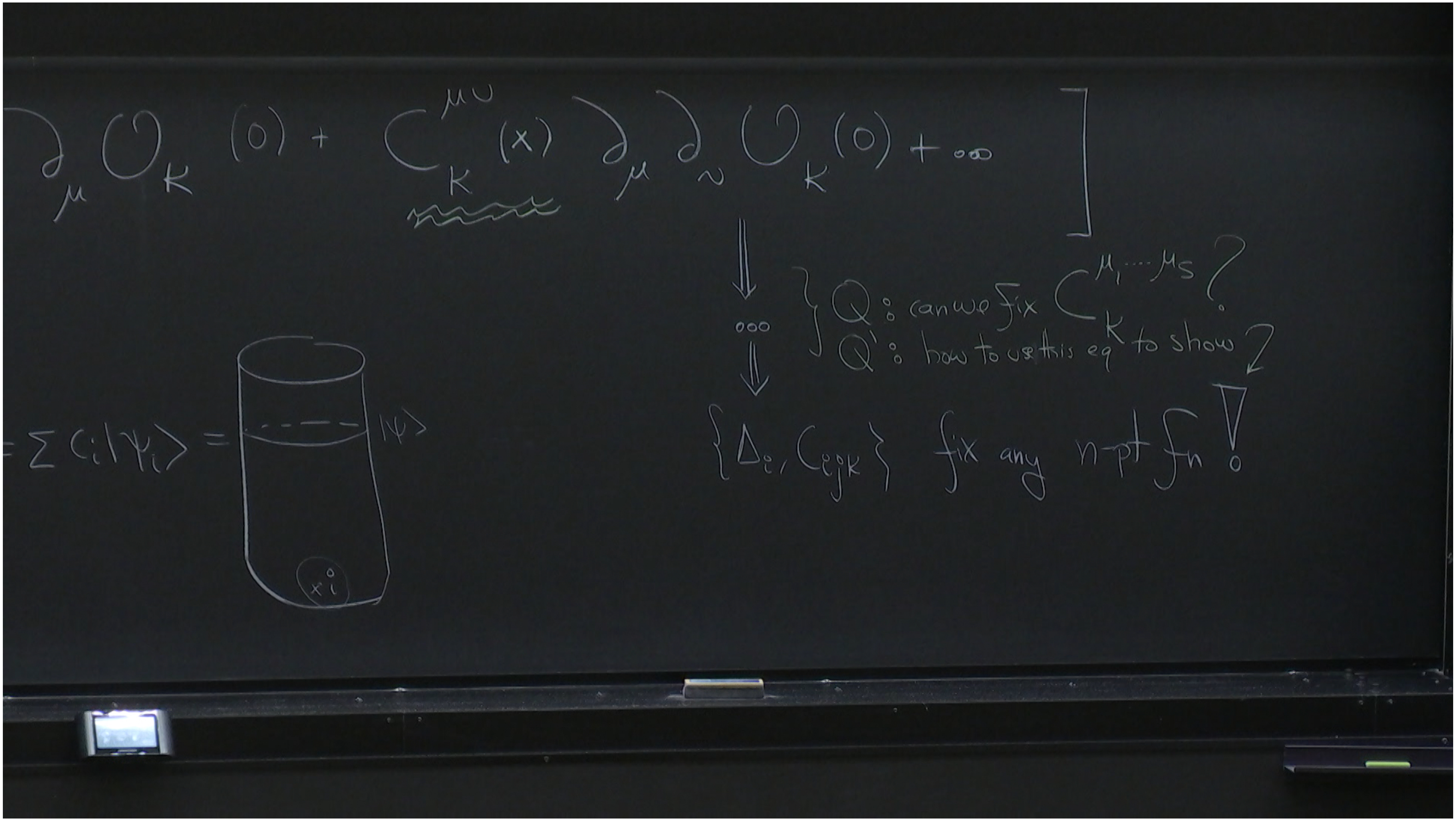
$$\cdots$$

$$\Downarrow$$

$$\ldots_i|\psi_i\rangle =$$



$\{\Delta_i, C_{ijk}\}$ fix any n-pt fn!


∂μ O_K (0) + C^{μν}_K (x) ∂μ ∂ν O_K (0) + ...
Q: can we fix C^{μ1....μs}_K ?
Q: how to use this eq to show ?
{Δi, Cijk} fix any n-pt fn !
= Σ ci |ψi⟩ =
|ψ⟩
xi