Title: Quantum Fields and Strings 2021/2022 -Lecture 1

Speakers: Dan Wohns

Collection: Quantum Fields and Strings 2021/2022

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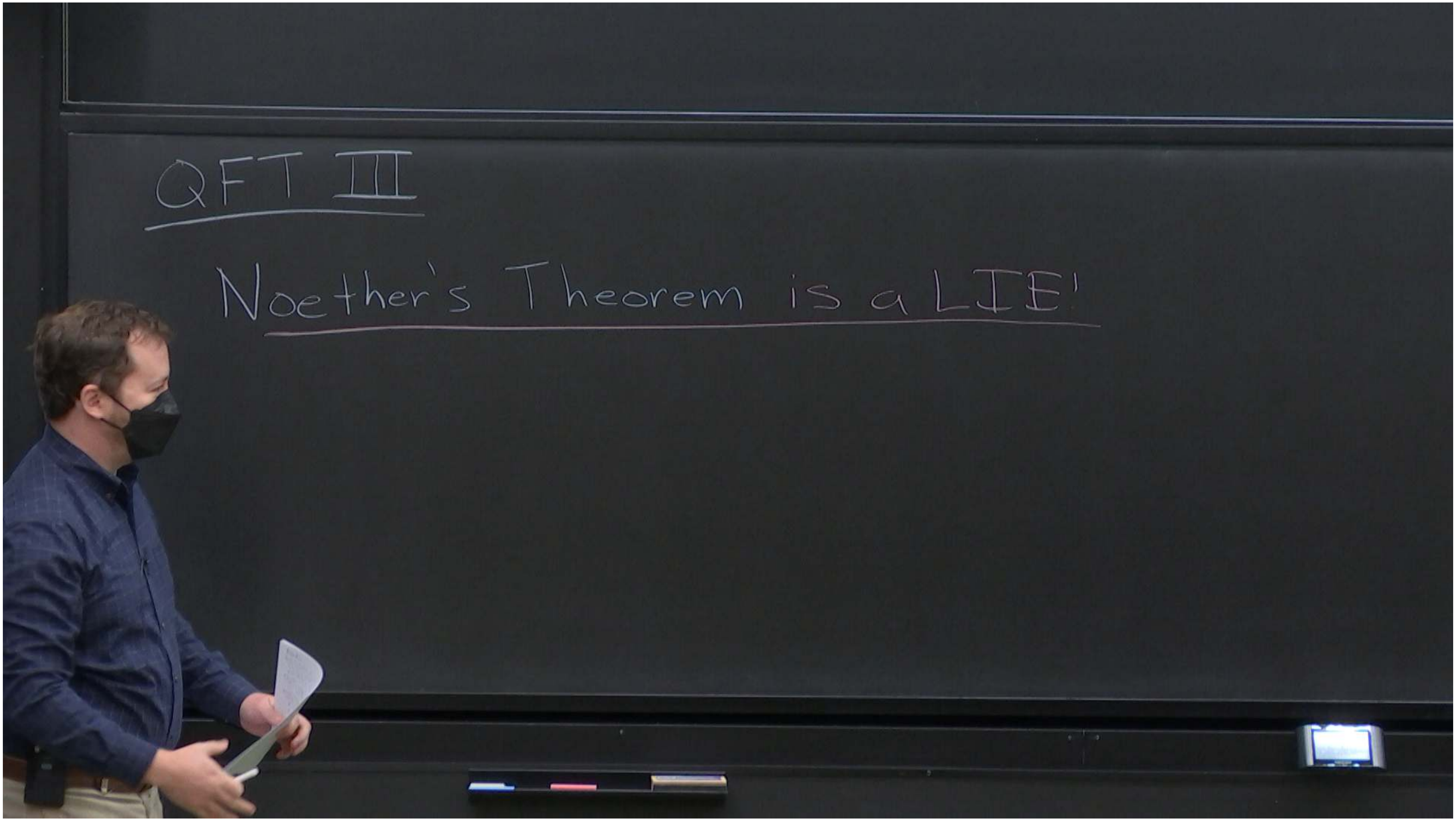
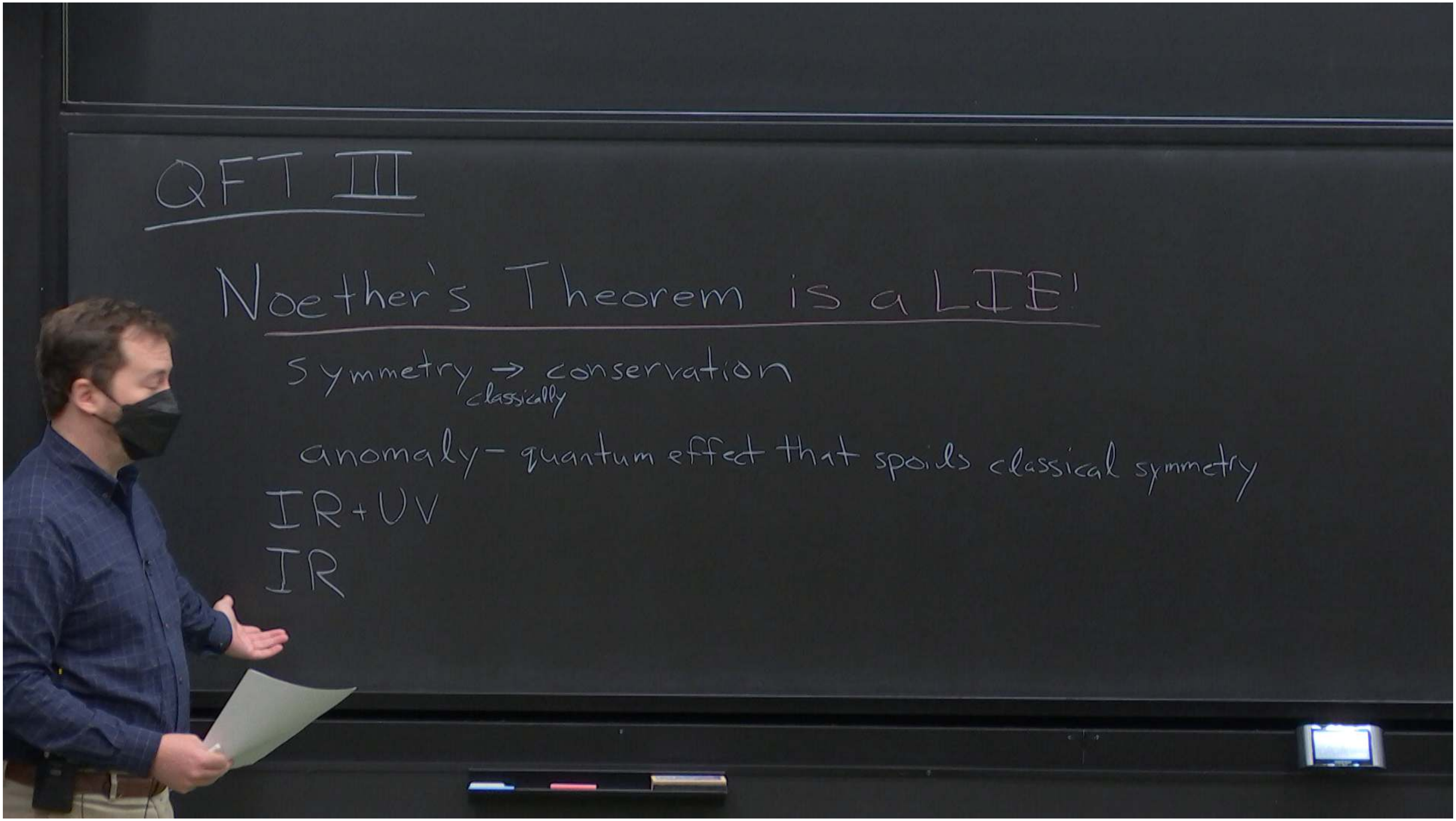
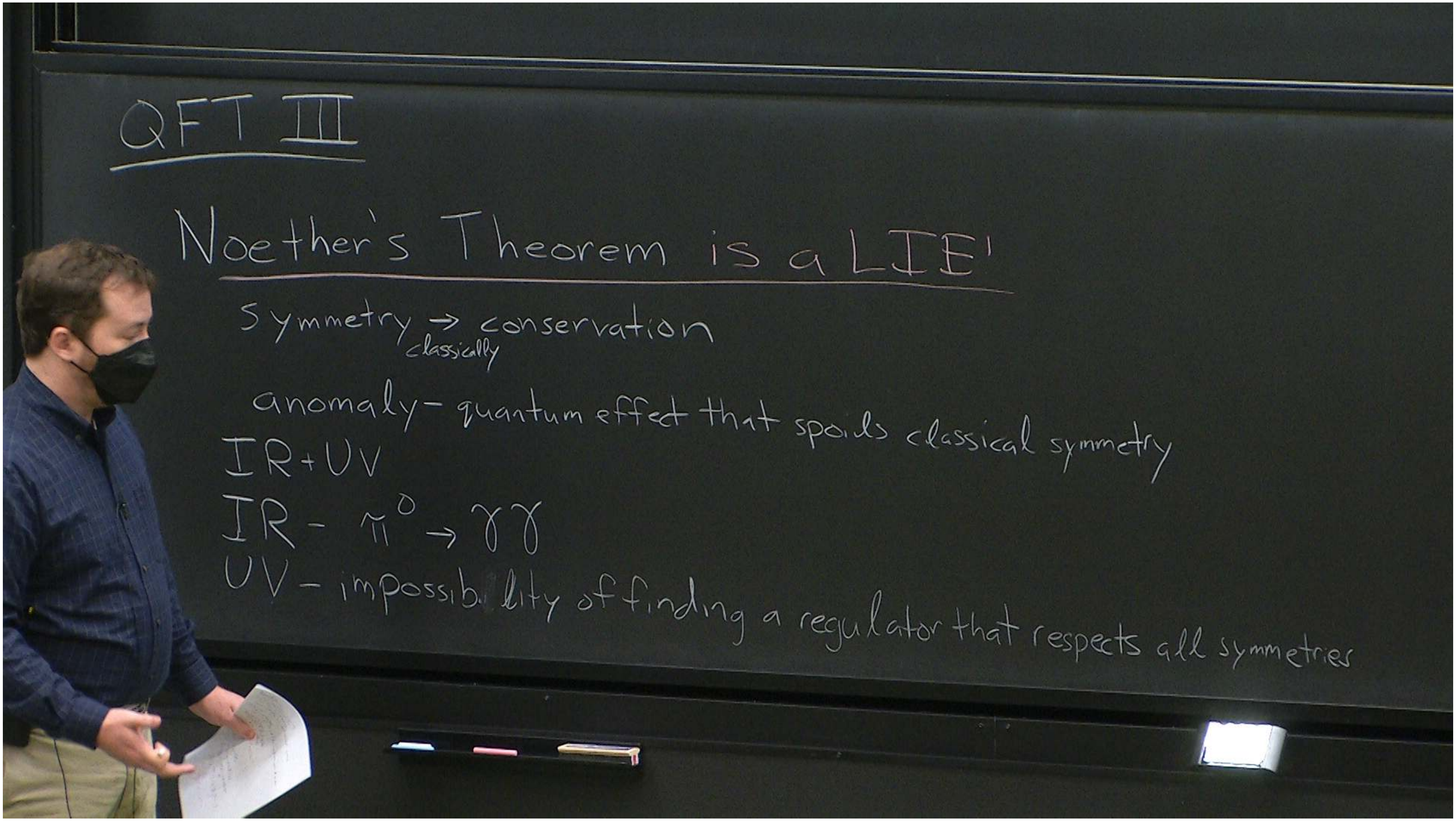
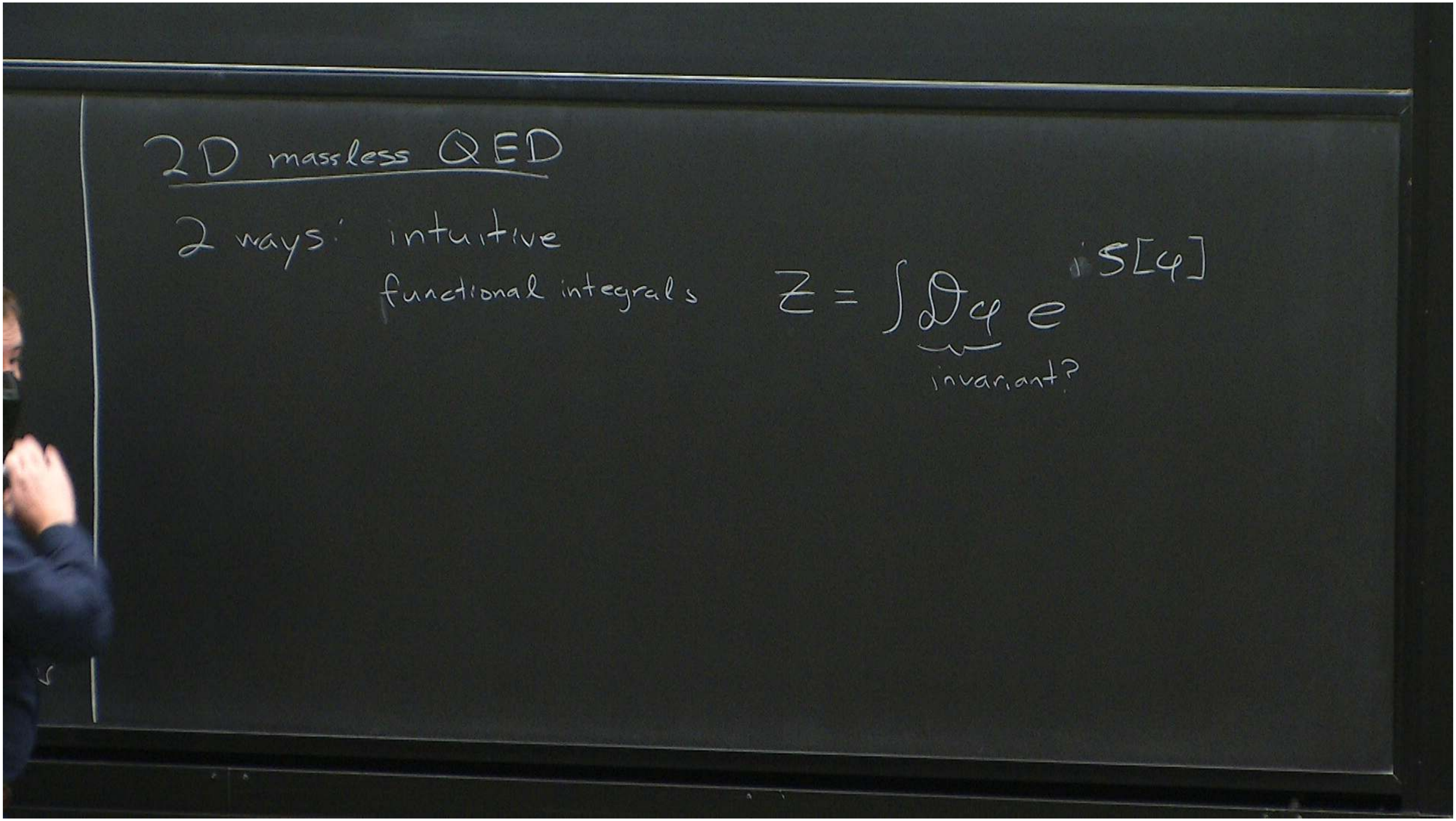
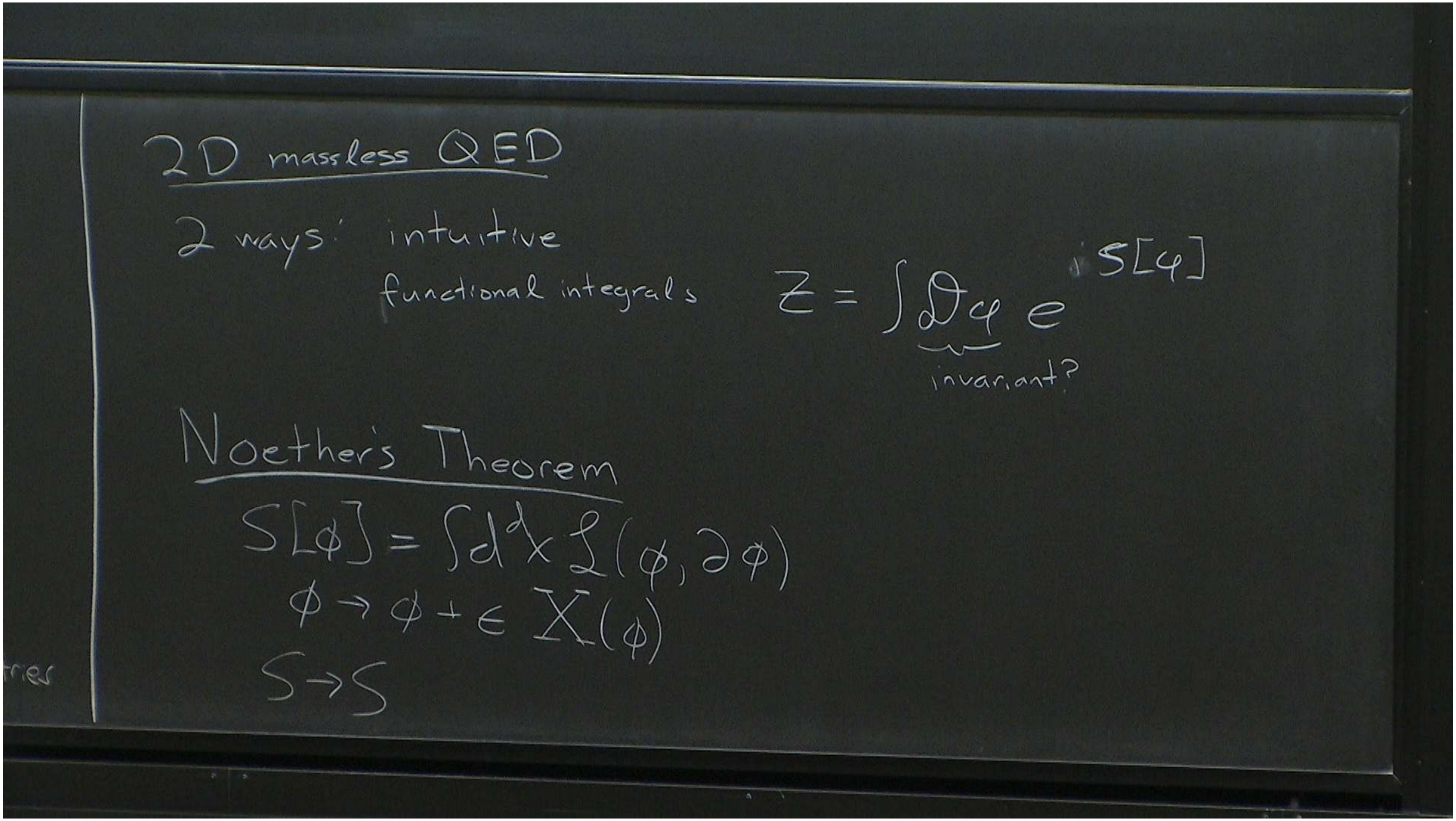

QFT III
Noether's Theorem is a LIE!


QFT III
Noether's Theorem is a LIE!
symmetry → conservation
classically
anomaly - quantum effect that spoils classical symmetry
IR+UV
IR


QFT III
Noether's Theorem is a LIE!
symmetry → conservation
classically
anomaly - quantum effect that spoils classical symmetry
IR + UV
IR - π⁰ → γγ
UV - impossibility of finding a regulator that respects all symmetries


2D massless QED
2 ways: intuitive
functional integrals
$Z = \int \mathcal{D}\varphi \, e^{iS[\varphi]}$
invariant?


2D massless QED
2 ways: intuitive
functional integrals
$Z = \int \mathcal{D}\varphi \, e^{iS[\varphi]}$
invariant?
Noether's Theorem
$S[\phi] = \int d^d x \, \mathcal{L}(\phi, \partial\phi)$
$\phi \to \phi + \epsilon X(\phi)$
$S \to S$

2D massless QED

2 ways: intuitive

functional integrals $\quad Z = \int \underbrace{\mathcal{D}\varphi}_{\text{invariant?}} \, e^{iS[\varphi]}$

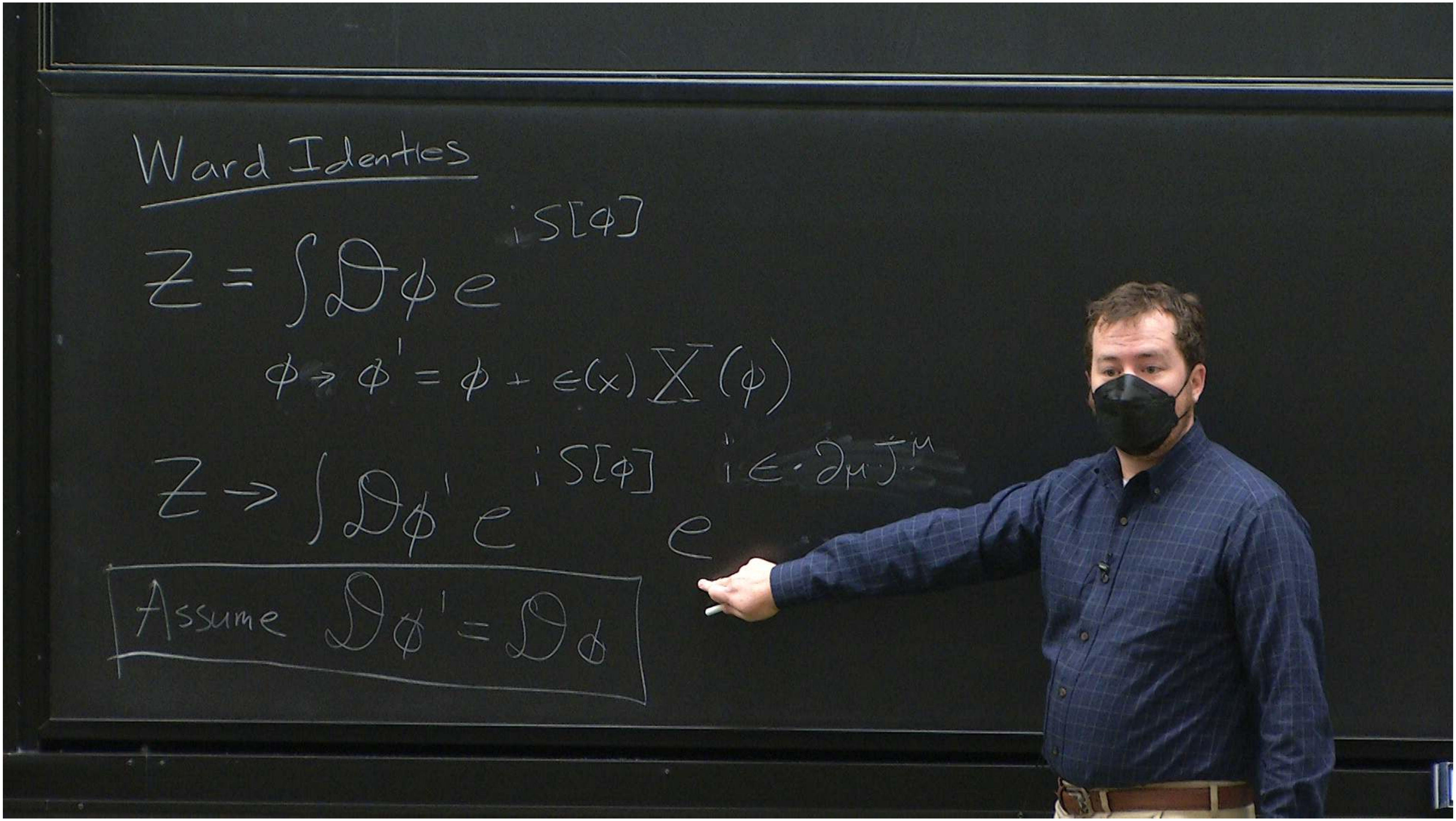
Noether's Theorem

$$S[\phi] = \int d^d x \, \mathcal{L}(\phi, \partial\phi)$$

$$\phi \to \phi + \epsilon(x) X(\phi)$$

$$S \to S + \int d^d x \, \epsilon(x) \, \partial_\mu J^\mu$$

tries


Ward Identies
$Z = \int \mathcal{D}\phi \, e^{iS[\phi]}$
$\phi \to \phi' = \phi + \epsilon(x) X(\phi)$
$Z \to \int \mathcal{D}\phi' \, e^{iS[\phi]} \, e^{i\epsilon \cdot \partial_\mu J^\mu}$
Assume $\mathcal{D}\phi' = \mathcal{D}\phi$

Ward Identities

$$Z = \int \mathcal{D}\phi \, e^{iS[\phi]}$$

$$\phi \to \phi' = \phi + \epsilon(x) \, \mathbb{X}(\phi)$$

$$Z \to \int \mathcal{D}\phi' \, e^{iS[\phi]} \, e^{i\epsilon \cdot \partial_\mu J^\mu} \approx \int \mathcal{D}\phi \, e^{iS[\phi]} \left(1 + i\epsilon \cdot \partial_\mu J^\mu + \dots \right)$$

Assume $\mathcal{D}\phi' = \mathcal{D}\phi$

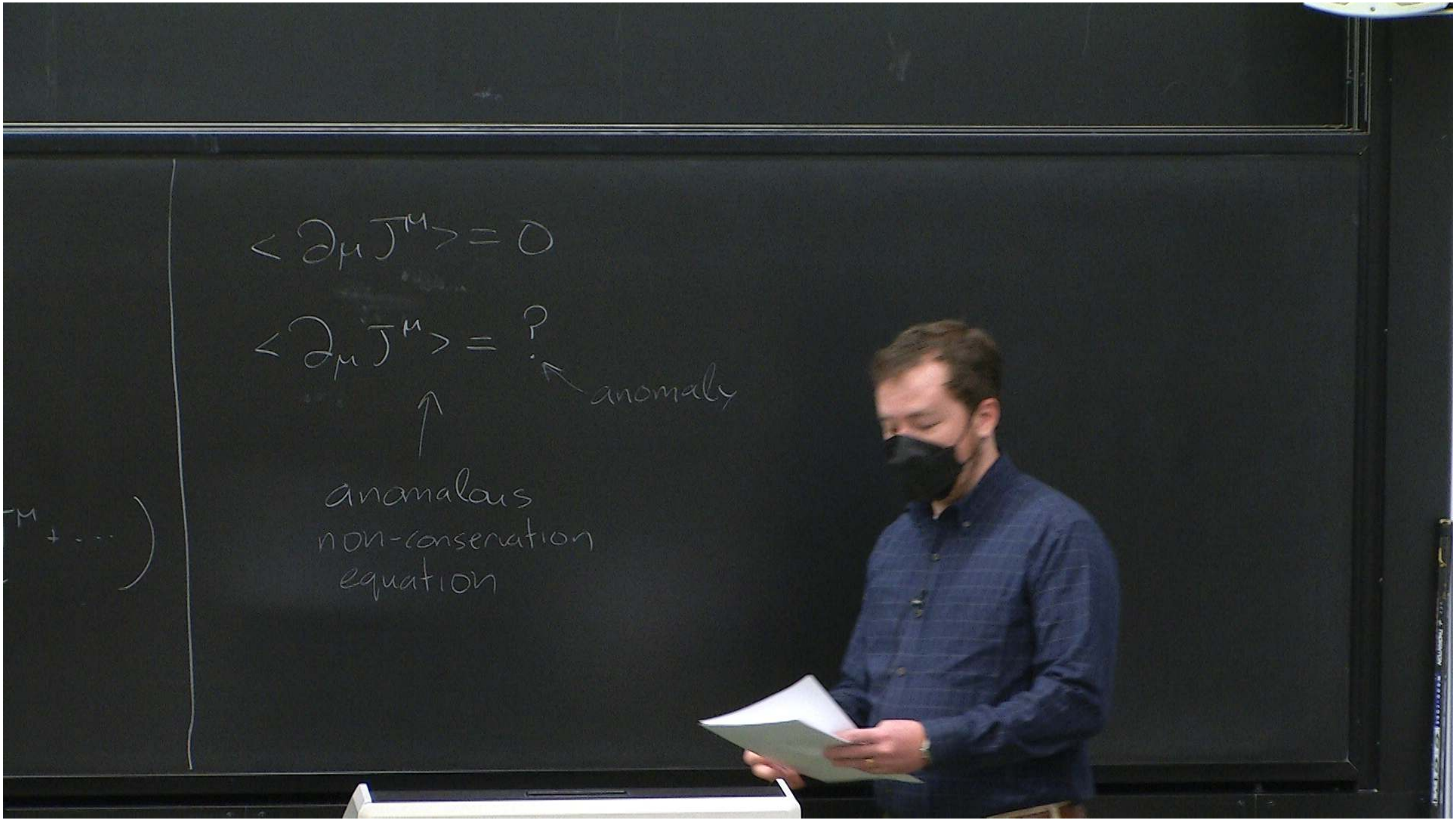
Ward Identities

$$Z = \int \mathcal{D}\phi \, e^{iS[\phi]}$$

$$\phi \to \phi' = \phi + \epsilon(x) \, X(\phi)$$

$$Z \to \int \mathcal{D}\phi' \, e^{iS[\phi]} \, e^{i\epsilon \cdot \partial_\mu J^\mu} \simeq \int \mathcal{D}\phi \, e^{iS[\phi]} \Big(1 + \underbrace{i\epsilon \cdot \partial_\mu J^\mu}_{=0} + \cdots \Big)$$

$$\boxed{\text{Assume } \mathcal{D}\phi' = \mathcal{D}\phi}$$


< ∂μ J^μ > = 0
< ∂μ J^μ > = ?
anomaly
anomalous
non-conservation
equation

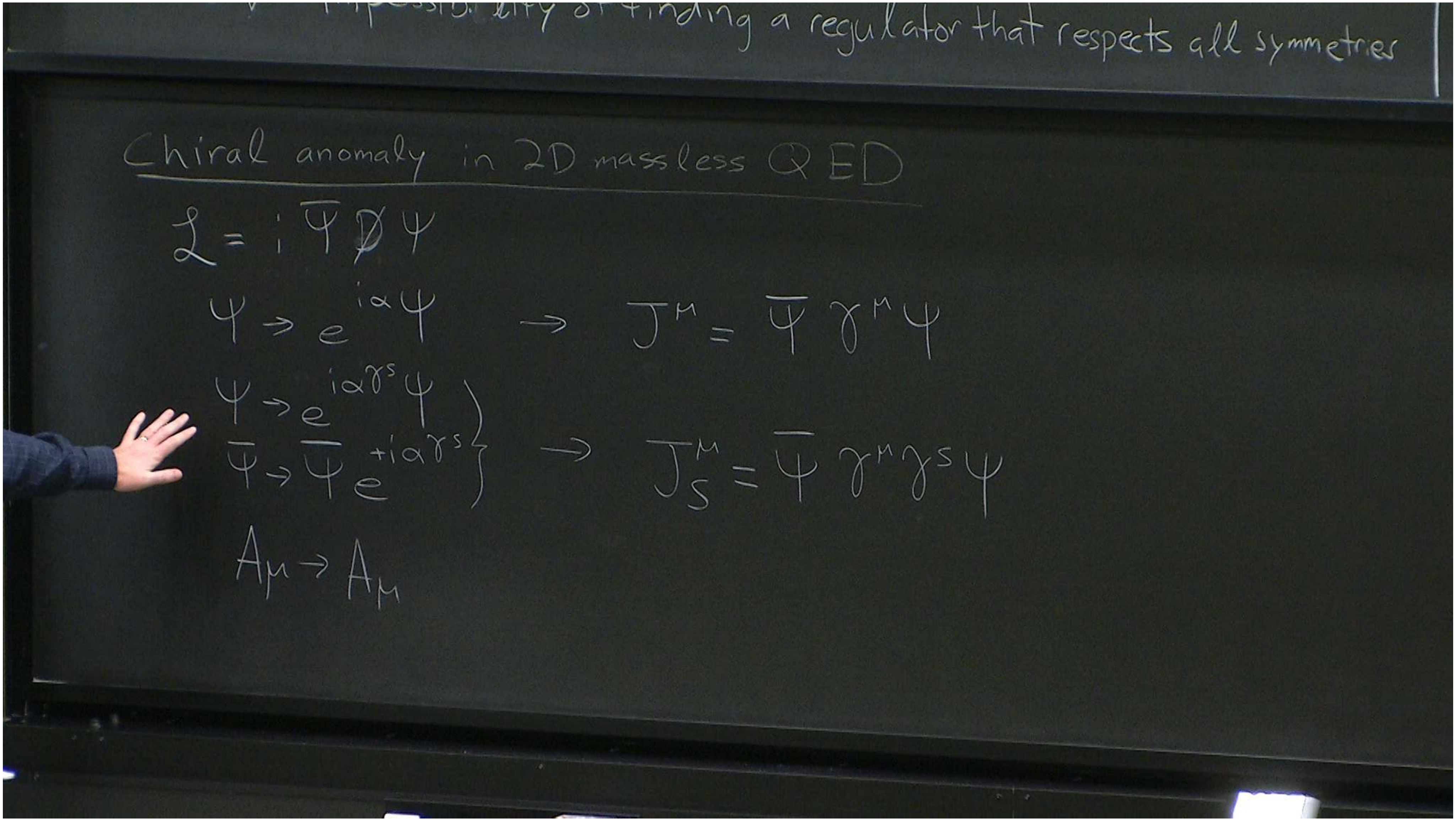
... impossibility of finding a regulator that respects all symmetries

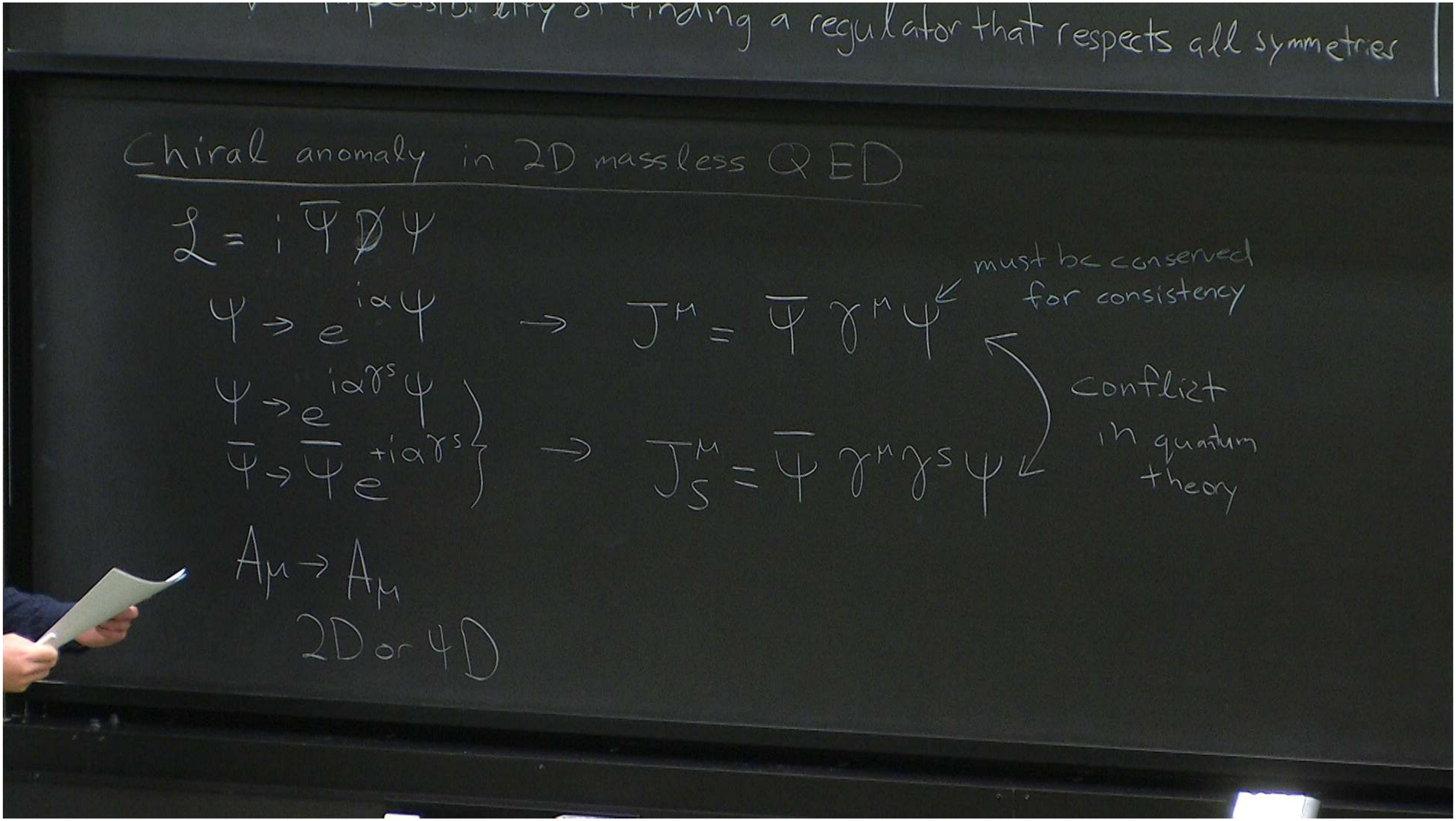
Chiral anomaly in 2D massless QED

$$\mathcal{L} = i\bar{\Psi}\not{\partial}\Psi$$

$$\Psi \to e^{i\alpha}\Psi \quad \longrightarrow \quad J^\mu = \bar{\Psi}\gamma^\mu\Psi$$

$$\left.\begin{aligned} \Psi &\to e^{i\alpha\gamma^5}\Psi \\ \bar{\Psi} &\to \bar{\Psi}e^{+i\alpha\gamma^5} \end{aligned}\right\} \quad \longrightarrow \quad J_5^\mu = \bar{\Psi}\gamma^\mu\gamma^5\Psi$$


finding a regulator that respects all symmetries
Chiral anomaly in 2D massless QED
$\mathcal{L} = i \bar{\Psi} \not{D} \Psi$
$\Psi \to e^{i\alpha} \Psi \quad \to \quad J^\mu = \bar{\Psi} \gamma^\mu \Psi$
$\Psi \to e^{i\alpha\gamma^5} \Psi$
$\bar{\Psi} \to \bar{\Psi} e^{+i\alpha\gamma^5} \quad \to \quad J_5^\mu = \bar{\Psi} \gamma^\mu \gamma^5 \Psi$
$A_\mu \to A_\mu$


finding a regulator that respects all symmetries
Chiral anomaly in 2D massless QED
$\mathcal{L} = i \bar{\Psi} \not{D} \Psi$
$\Psi \to e^{i\alpha} \Psi \to J^\mu = \bar{\Psi} \gamma^\mu \Psi$
must be conserved for consistency
$\Psi \to e^{i\alpha\gamma^5} \Psi$
$\bar{\Psi} \to \bar{\Psi} e^{+i\alpha\gamma^5} \to J_5^\mu = \bar{\Psi} \gamma^\mu \gamma^5 \Psi$
conflict in quantum theory
$A_\mu \to A_\mu$
2D or 4D

symmetries $S \to S + \int d^d x\, \epsilon(x)\, \partial_\mu J^\mu$.

Quantize fermion + treat A_μ classically

EOM $i(\partial_0 \pm \partial_1)\Psi = 0$ if $A_\mu = 0$

plane waves moving to left $+$, right $-$ with $v=1$

ll symmetries

$$S \to S + \int d^d x\, \epsilon(x)\, \partial_\mu J^\mu$$

Quantize fermion + treat A_μ classically

EOM $i(\partial_0 \pm \partial_1)\Psi = 0$ if $A_\mu = 0$

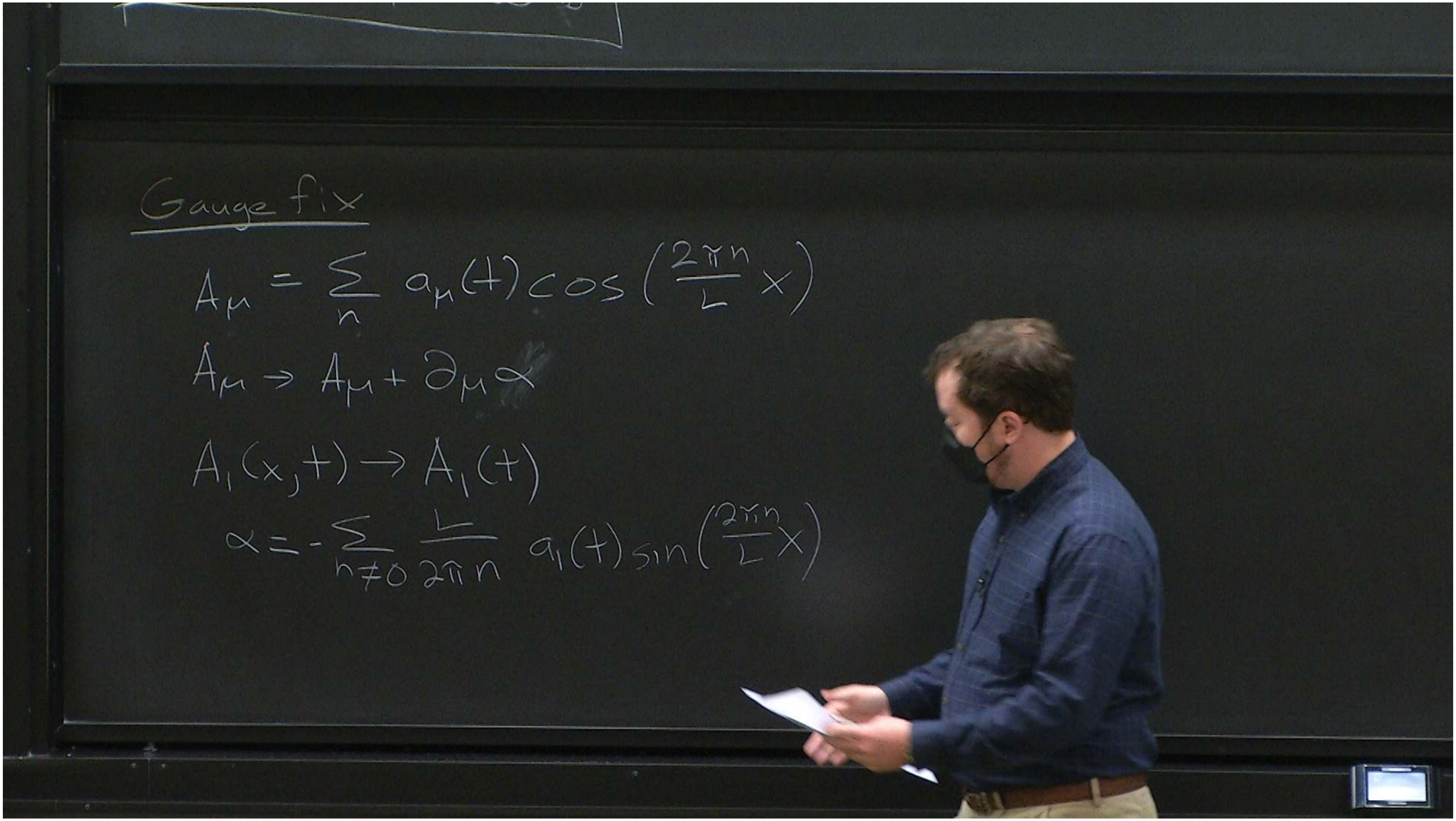
plane waves moving to left $+$ / right $-$ with $v=1$

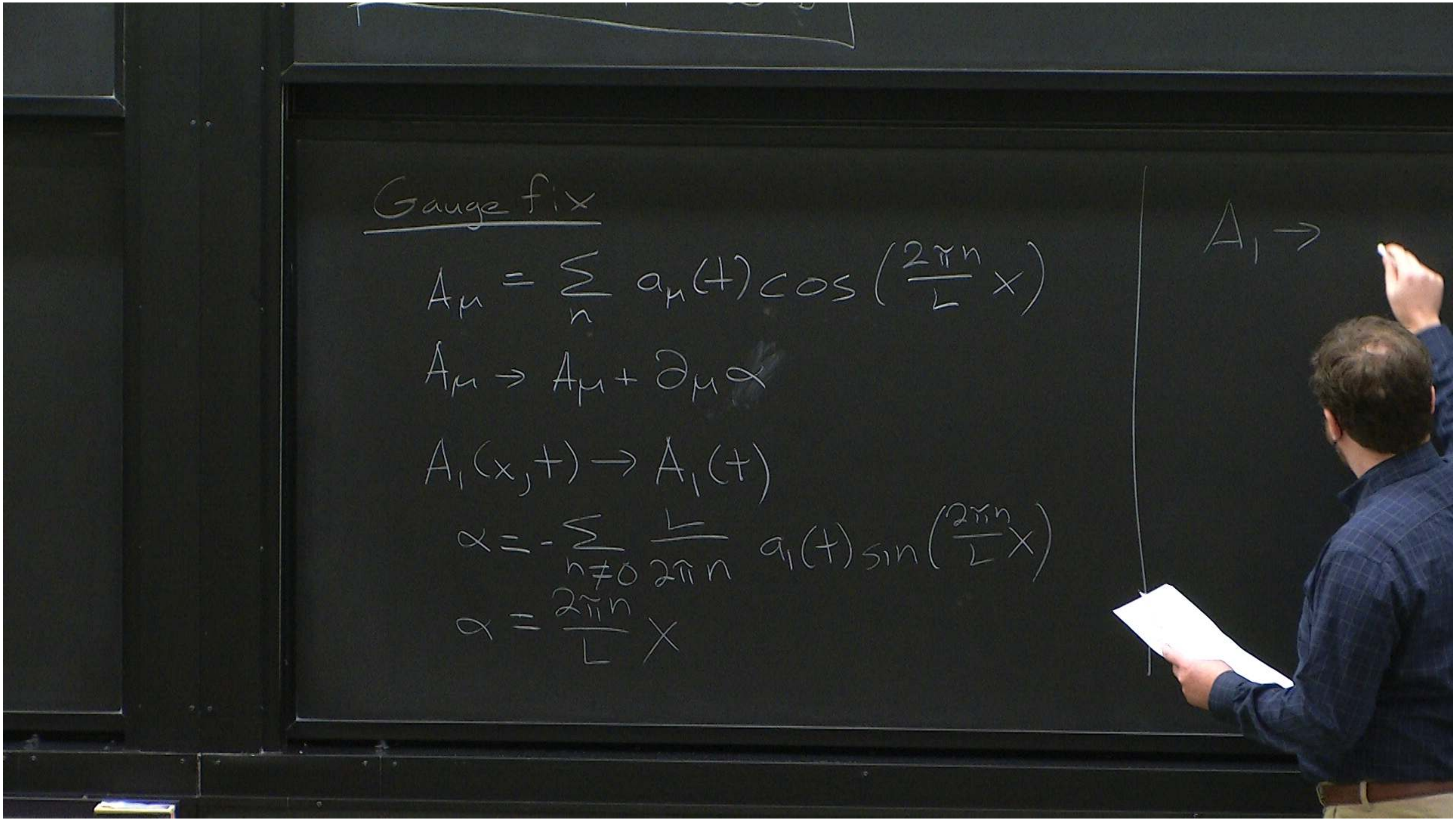
$E =$ constant $\to V_{coulomb} \propto r \to$ strong coupling

Fix compactifying with periodicity is L

$e_0 L \ll 1$

ved
ency
t
antum
ory


Gauge fix
$A_\mu = \sum_n a_\mu(t) \cos\left(\frac{2\pi n}{L} x\right)$
$A_\mu \to A_\mu + \partial_\mu \alpha$
$A_1(x,t) \to A_1(t)$
$\alpha = -\sum_{n \neq 0} \frac{L}{2\pi n} a_1(t) \sin\left(\frac{2\pi n}{L} x\right)$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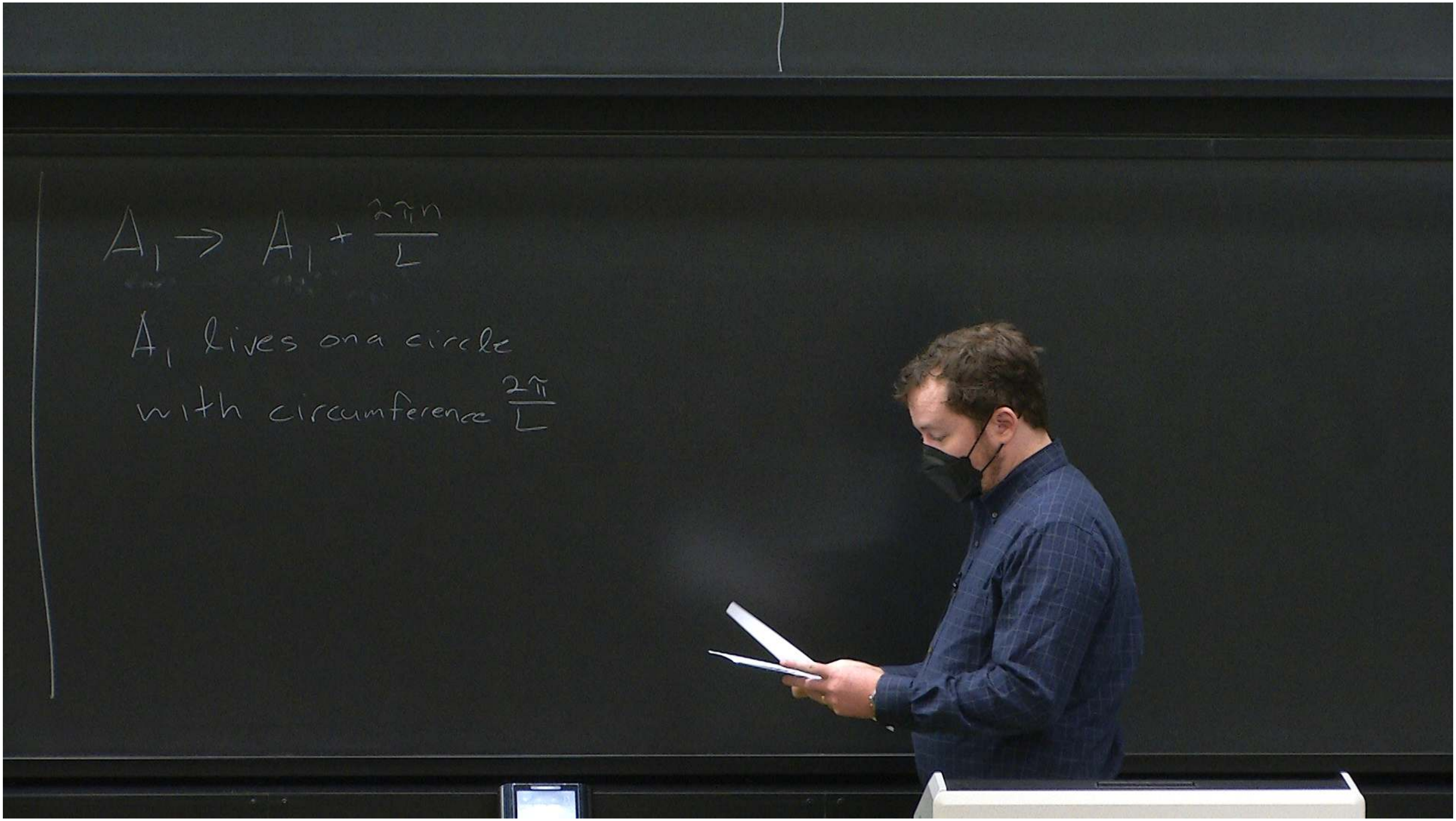
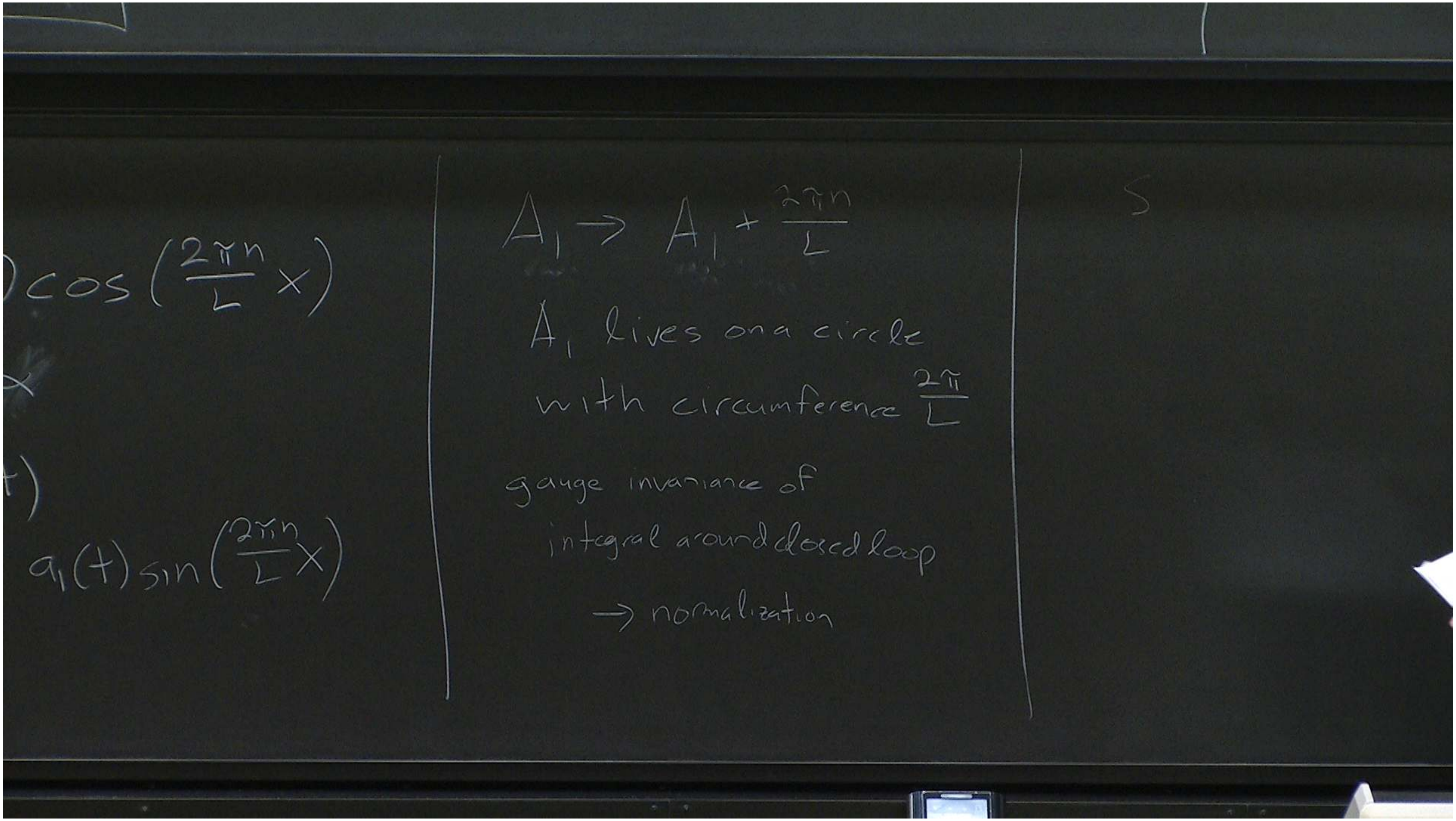
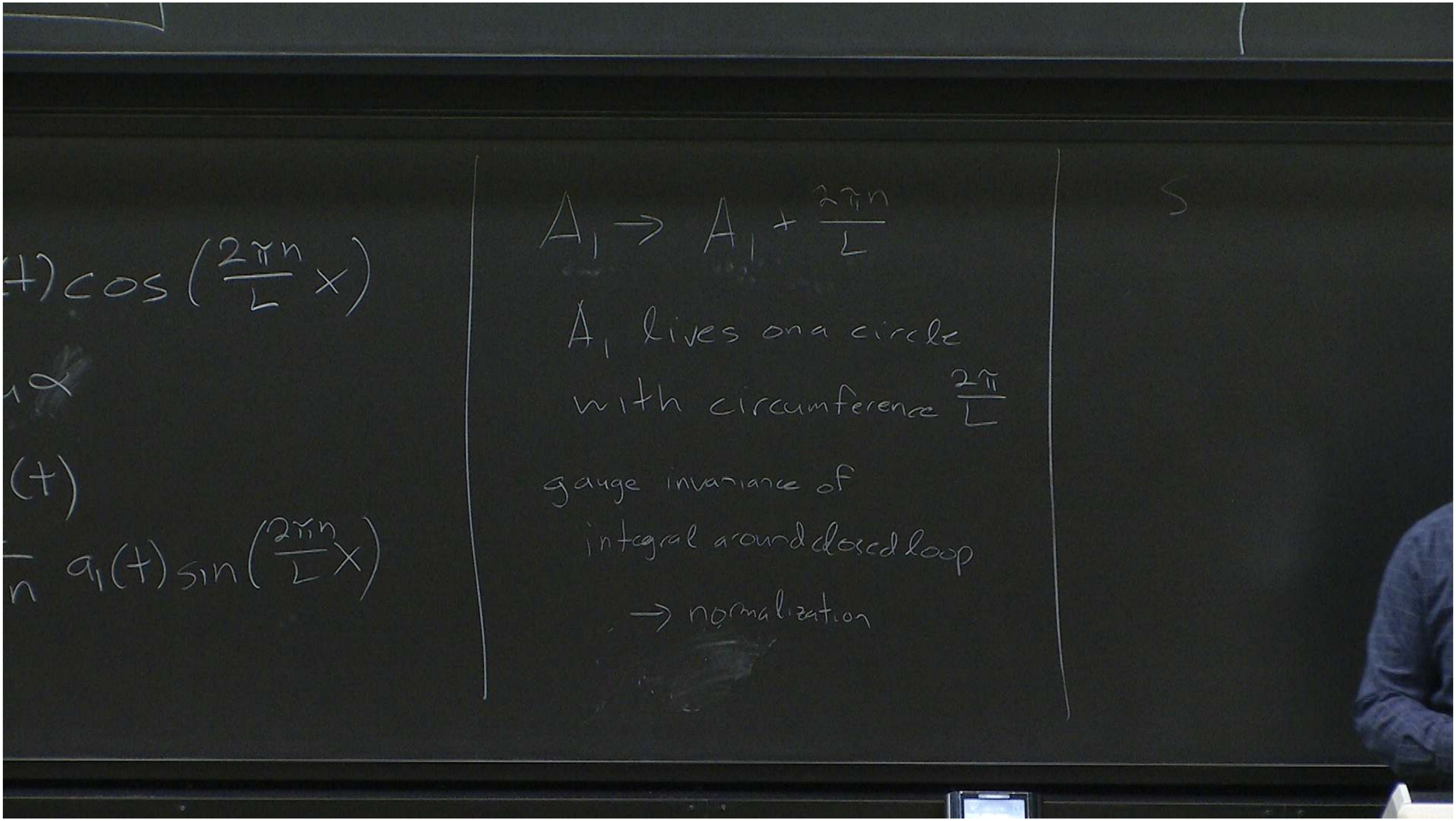
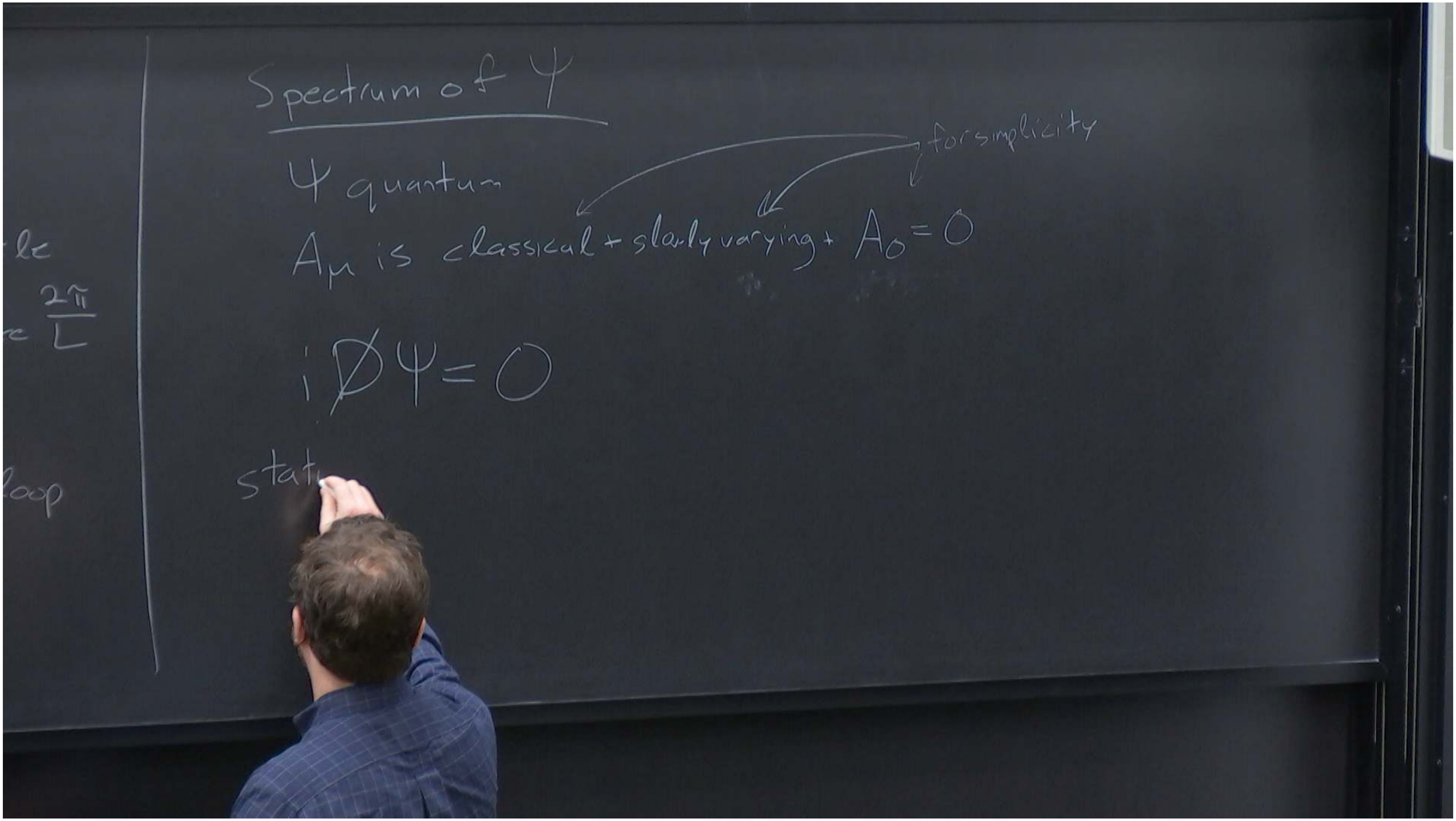
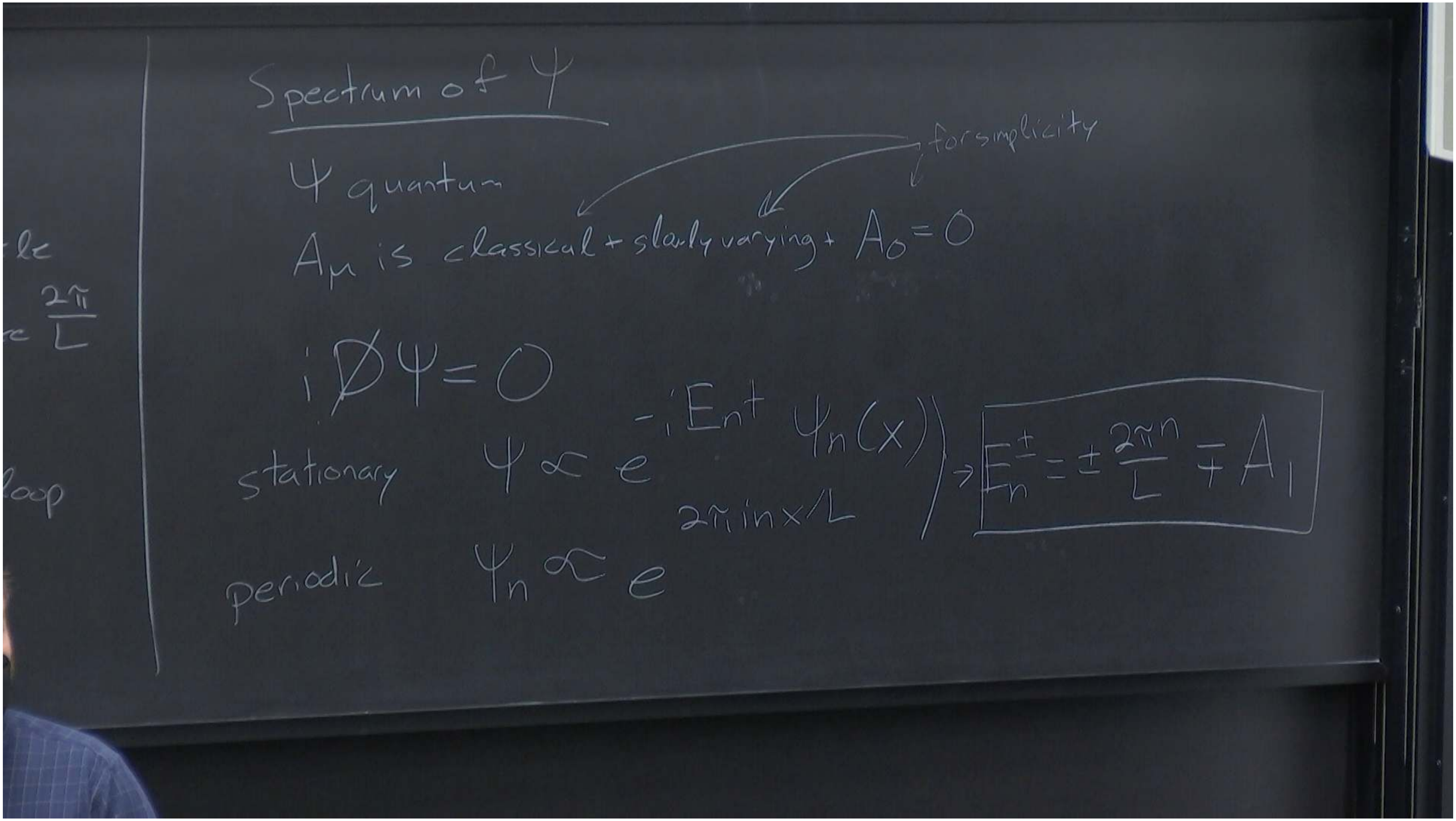
$\alpha = \frac{2\pi n}{L} x$
$A_1 \to$

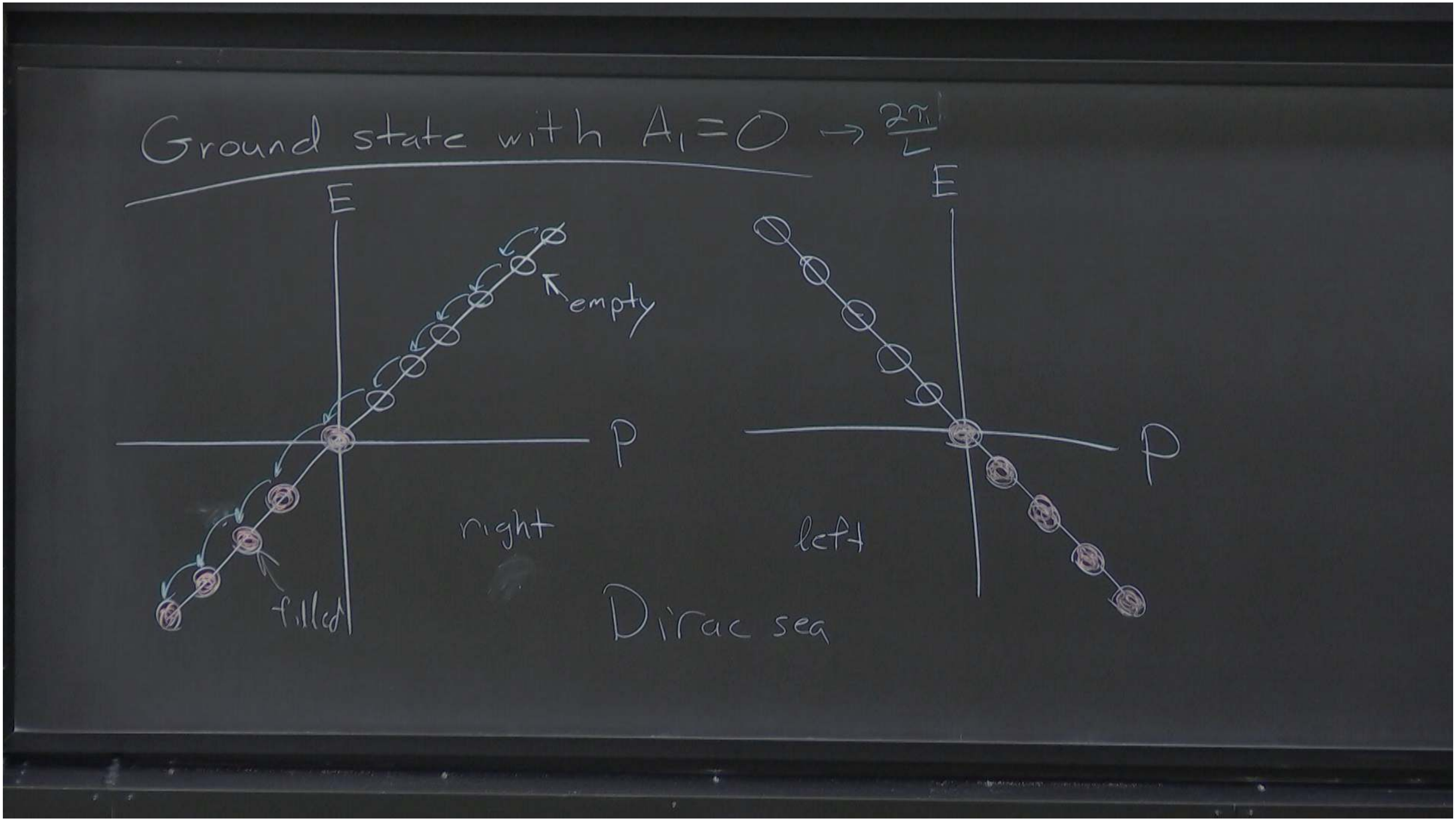

$A_1 \to A_1 + \frac{2\pi n}{L}$
A_1 lives on a circle
with circumference $\frac{2\pi}{L}$

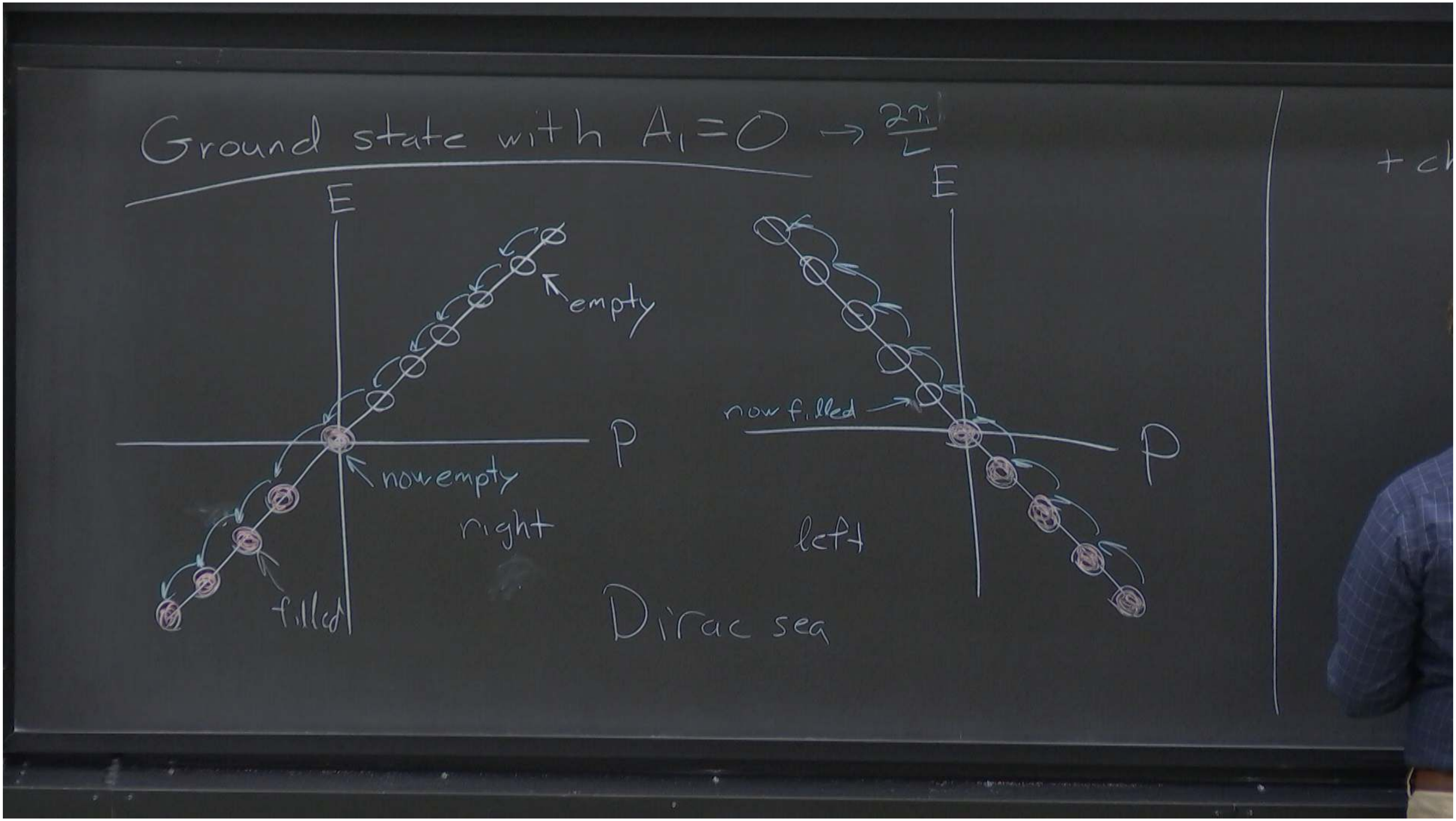

$A_1 \to A_1 + \frac{2\pi n}{L}$
A_1 lives on a circle
with circumference $\frac{2\pi}{L}$
gauge invariance of
integral around closed loop
→ normalization
$\cos(\frac{2\pi n}{L} x)$
$a_1(t) \sin(\frac{2\pi n}{L} x)$

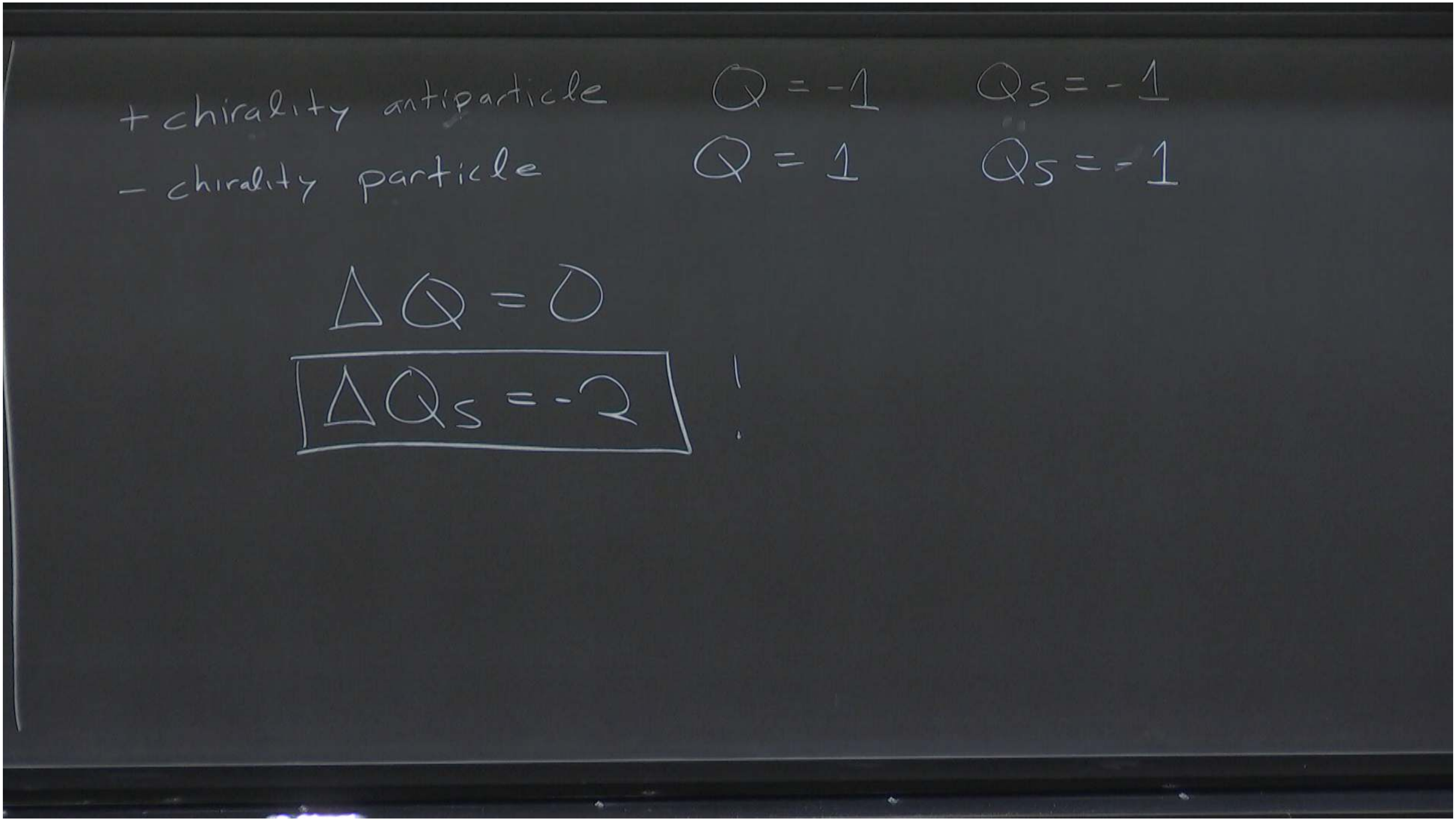

$A_1 \to A_1 + \frac{2\pi n}{L}$
A_1 lives on a circle
with circumference $\frac{2\pi}{L}$
gauge invariance of
integral around closed loop
→ normalization
$(t) \cos(\frac{2\pi n}{L} x)$
(t)
$a_1(t) \sin(\frac{2\pi n}{L} x)$


Spectrum of Ψ
Ψ quantum
for simplicity
Aμ is classical + slowly varying + A0 = 0
iD̸Ψ = 0
stat


Spectrum of ψ
ψ quantum
for simplicity
A_μ is classical + slowly varying + $A_0 = 0$
$i \not{D} \psi = 0$
stationary $\psi \propto e^{-iE_n t} \psi_n(x)$
periodic $\psi_n \propto e^{2\pi i n x/L}$
$E_n^\pm = \pm \frac{2\pi n}{L} \mp A_1$


Ground state with A1=0 → 2π/L
E
empty
P
right
filled
E
P
left
Dirac sea


Ground state with $A_1 = 0 \to \frac{2\pi}{L}$
E
P
empty
now empty
filled
right
E
P
now filled
left
Dirac sea


+ chirality antiparticle $Q = -1$ $Q_5 = -1$
− chirality particle $Q = 1$ $Q_5 = -1$
$\Delta Q = 0$
$\Delta Q_5 = -2$!