Title: Measurement-induced criticality and charge-sharpening transitions

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Series: Quantum Matter

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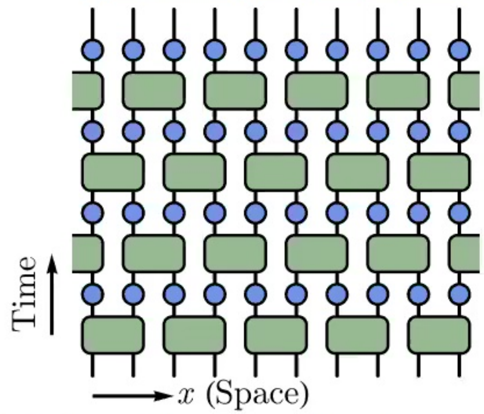
Abstract: Monitored quantum circuits (MRCs) exhibit a measurement-induced phase transition between area-law and volume-law entanglement scaling. In this talk, I will argue that MRCs with a conserved charge additionally exhibit two distinct volume-law entangled phases that cannot be characterized by equilibrium notions of symmetry-breaking or topological order, but rather by the non-equilibrium dynamics and steady-state distribution of charge fluctuations. These include a charge-fuzzy phase in which charge information is rapidly scrambled leading to slowly decaying spatial fluctuations of charge in the steady state, and a charge-sharp phase in which measurements collapse quantum fluctuations of charge without destroying the volume-law entanglement of neutral degrees of freedom. I will present some statistical mechanics and effective field theory approaches to such charge-sharpening transitions.

Zoom Link: https://pitp.zoom.us/meeting/register/tJcqc-ihqzMvHdW-YBm7mYd_XP9Amhypv5vO

Measurement-induced criticality & charge-sharpening transitions

Romain Vasseur
(UMass Amherst)

Quantum Matter Frontier Seminars



Agrawal, Zabalo, Chen, Wilson, Potter, Pixley, Gopalakrishnan, RV, 2107.10279
Barratt, Agrawal, Gopalakrishnan, Huse, RV, Potter, 2111.09336

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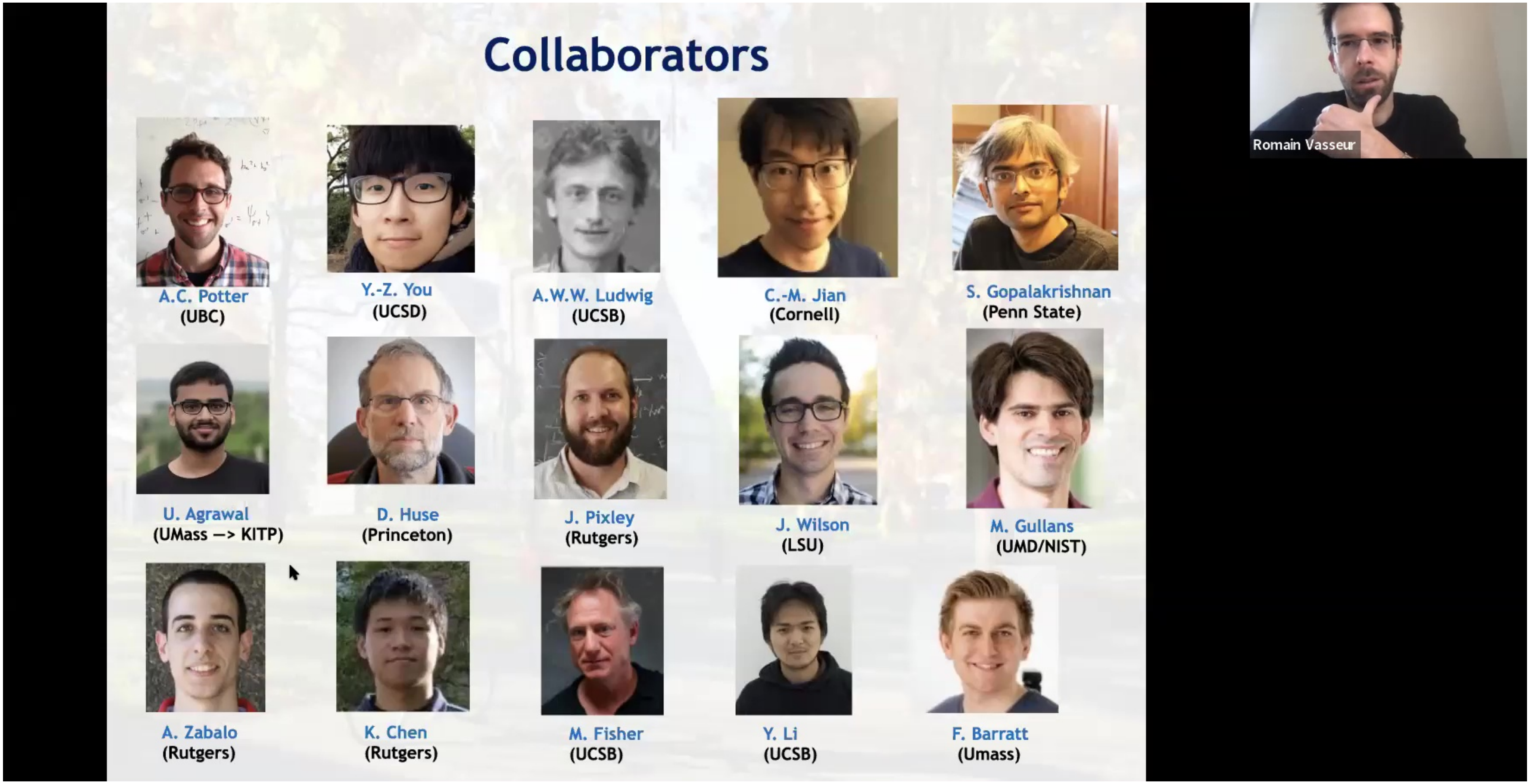
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M. Fisher
(UCSB)
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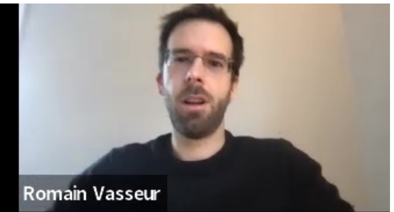
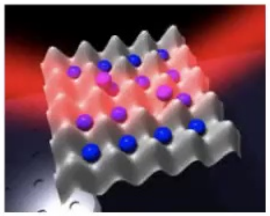
F. Barratt
(Umass)
Romain Vasseur

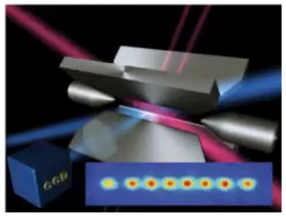
Non-equilibrium quantum systems



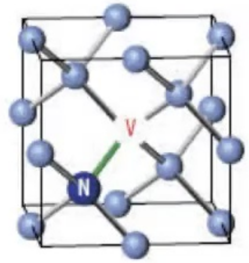
Ultracold atoms



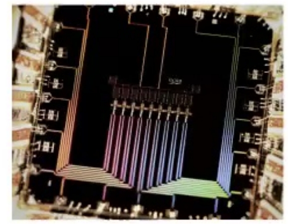
Trapped Ions



NV Centers



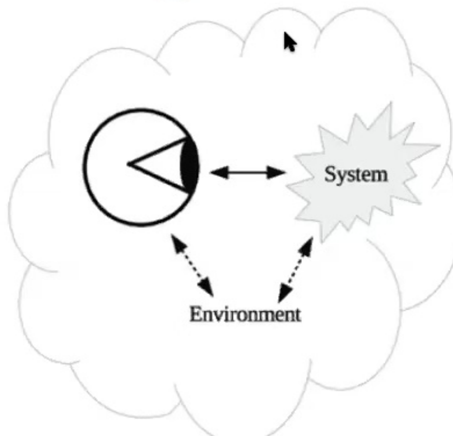
Superconducting Qubits



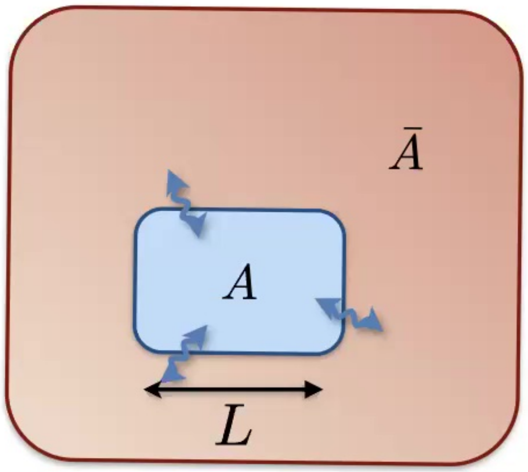
Many fundamental questions:

Thermalization, non-equilibrium steady states, emergent universal phenomena

Role of the environment?



Entanglement Dynamics and Thermalization



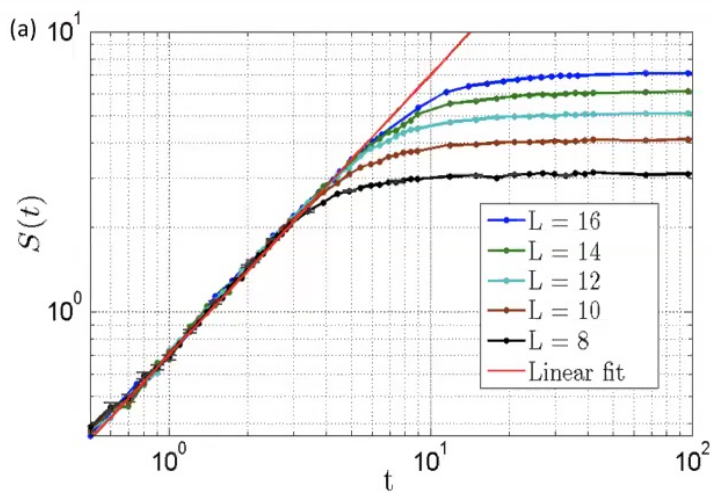
$$\rho_A(t \to \infty) \to \mathrm{e}^{-H/T_{\text{eff}}}$$

Highly (volume law) entangled at long times.

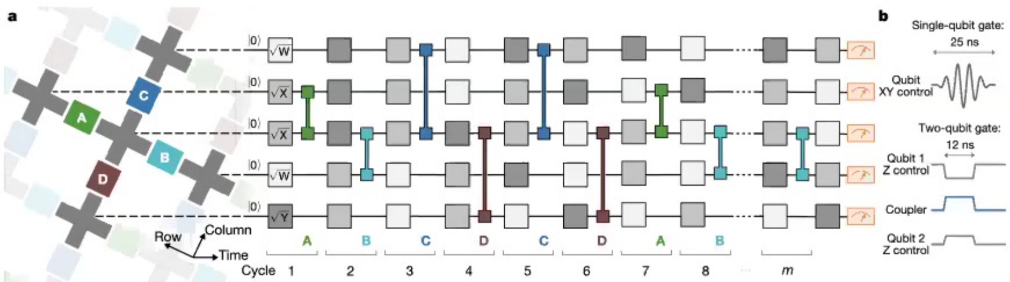
Linear growth vs time

$$S_A(t) \sim t$$

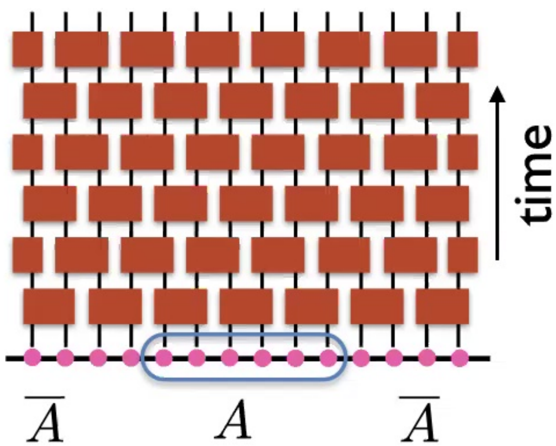
Cardy & Calabrese '05
Kim & Huse '13



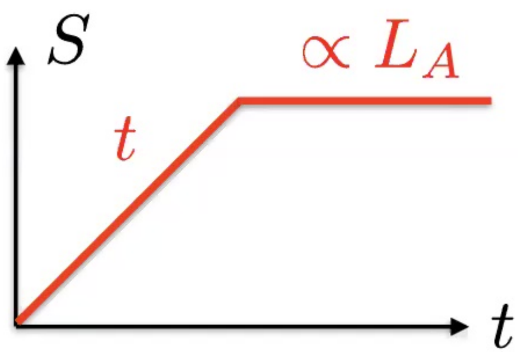
Random circuits, and entanglement growth



Google quantum advantage expt



$$\rho_A = \mathrm{tr}_{\overline{A}}\, |\psi(t)\rangle\langle\psi(t)| \rightarrow \mathbb{I}$$

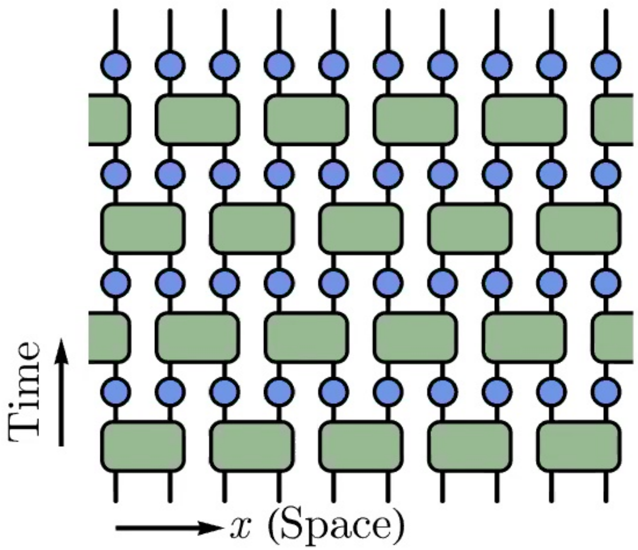


Nahum, Vijay & Haah PRX '17, '18

Measurement-induced transition

Related to quantum error correction problem:

Gullans and Huse, Bao, Choi, Altman, …

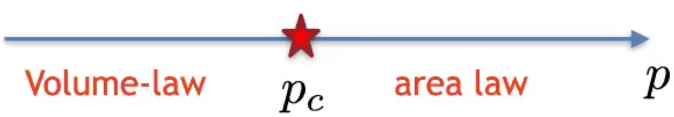


- Chaotic dynamics: Random unitary circuits (Haar or Clifford)

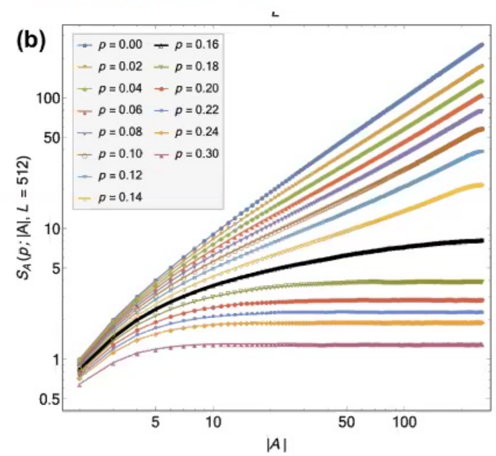
  Nahum, Vijay & Haah '17, …

- Local projective measurements with probability "p"

Competition between scrambling/chaotic dynamics and disentangling measurements

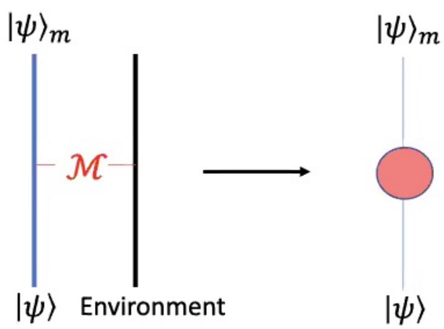


Skinner, Ruhman & Nahum '19
Li, Chen, Fisher '19



Open quantum systems: observer vs decoherence

Quantum Trajectories

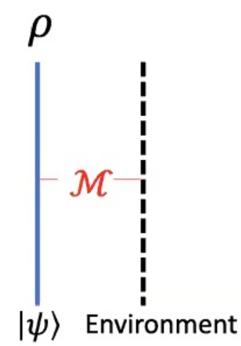


The wavefunction is **projected** to one of the eigen basis state $|m\rangle$

$$|\psi\rangle \to |\psi\rangle_m = \Pi_m|\psi\rangle$$

$p_m = \langle\psi|\Pi_m^2|\psi\rangle = \langle\psi_m|\psi_m\rangle$ is the Born probability.

Quantum Channel



Tracing out the environment gives a mixed density matrix for the system

$$\rho = \sum_{\{m\}} |\psi_m\rangle\langle\psi_m|$$

Quantum trajectories
vs
Quantum channels

- LINEAR QUANTITIES

$$f = \langle O \rangle$$

The trajectory average is given by:

$$[f] = \sum_m p_m \frac{\langle \psi_m | O | \psi_m \rangle}{\langle \psi_m | \psi_m \rangle}$$
$$= \sum_m \langle \psi_m | O | \psi_m \rangle$$
$$= \mathrm{Tr}\, \rho O = [f]_{channel}$$

- NON-LINEAR QUANTITIES

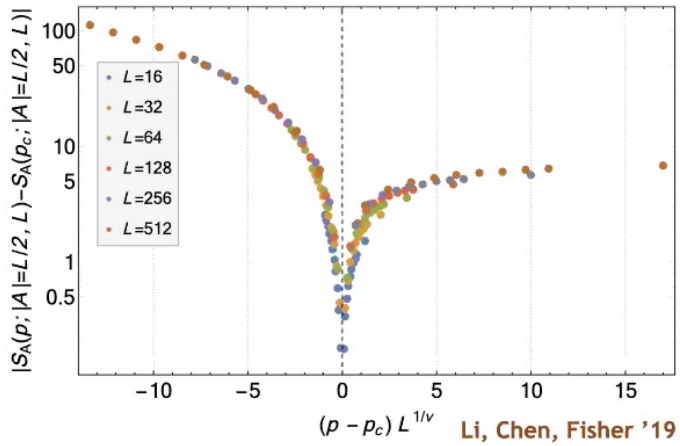
$$f = \langle O \rangle^2$$

The trajectory average is given by:

$$[f] = \sum_m p_m \frac{\langle \psi_m | O | \psi_m \rangle^2}{\langle \psi_m | \psi_m \rangle^2}$$
$$= \sum_m \frac{\langle \psi_m | O | \psi_m \rangle^2}{p_m}$$

Does not have an equivalent expression in terms of density matrix in quantum channel.

Measurements: numerical results



Li, Chen, Fisher '19

- Second order transition in all Renyi entropies

$$S_n = \frac{1}{1-n}\log \mathrm{tr}\rho_A^n$$

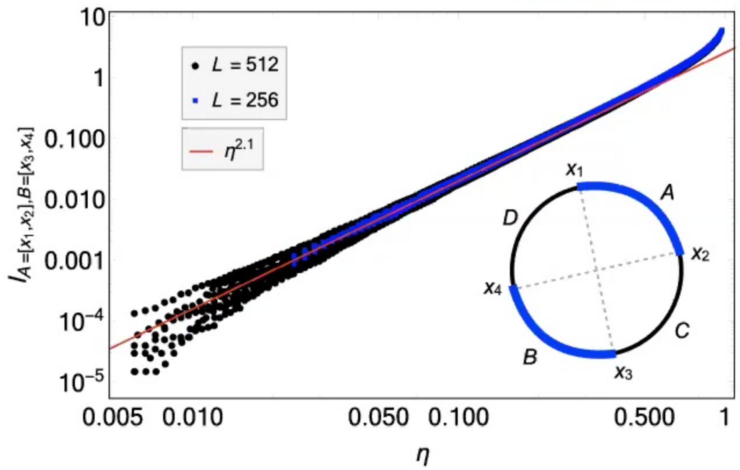
- Critical exponents (universality!)

$$\nu \simeq 1.3 \quad z = 1$$

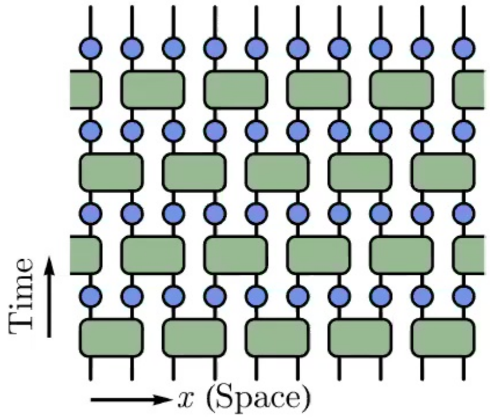
- conformal invariance (!)

Li, Chen, Ludwig, Fisher '19

$$I = S_A + S_B - S_{AB}$$
$$= f(\eta)$$



Replica trick and stat mech model



Replica trick:

$$\log \mathrm{tr}\rho_A^n = \lim_{k\to 0}\frac{\partial}{\partial k}(\mathrm{tr}\rho_A^n)^k$$

For n and k integers, average of $(\mathrm{tr}\rho_A^n)^k$

can be mapped onto the partition function of a 2D classical stat mech model defined on the circuit!

$$Z_0 = \left\langle \sum_m (\mathrm{tr}\rho)^Q \right\rangle_{\text{circuits}} = \sum_{\{g_i \in S_Q\}} \mathrm{e}^{-\mathcal{H}}$$

spins =

$$g_i \in S_{Q=nk+1}$$

Schur-Weyl duality: "spins" belong to commutant

Generalization to Clifford group: Li, RV, Fisher & Ludwig '21

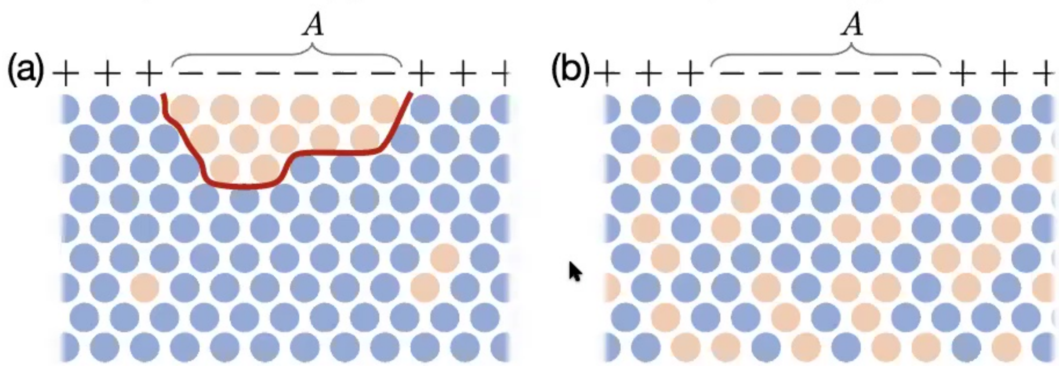
Classical domain wall picture

RV, A.C. Potter, Y-Z. You and A.W.W. Ludwig '19
C-M. Jiang, RV, Y-Z. You and A.W.W. Ludwig '20
Bao, Choi & Altman '20

Figure from Bao, Choi and Altman

Ferromagnetic (volume-law) phase

Paramagnetic (area-law) phase



"Minimal cut", RT formula

p

$$F_A - F_0 \sim L_A \qquad\qquad F_A - F_0 \sim \text{cst}$$

$$S_n = \frac{1}{n-1} \lim_{k \to 0} \frac{\partial}{\partial k} (F_A - F_0)$$

Entanglement ~ Free energy cost of domain wall

U(1) Symmetric circuits

- Symmetries will put constraints on the scrambling dynamics. E.g hydrodynamic modes will scramble differently to generic degree of freedoms.
  Khemani, Vishwanath, Huse, Phys. Rev. X 8, 031057 (2018)
  Rakovszky, Pollmann, and von Keyserlingk, Phys. Rev. X 8, 031058 (2018)

- E.g., Renyi entropies $S_{n>1}$ are known to grow sub ballistically in systems with U(1) symmetry. (without measurements)
  Rakovszky, Pollmann, and Von Keyserlingk, PRL 122, 250602 (2019)
  Yichen Huang, arXiv:1902.00977
  Zhou, Ludwig, PRR, 2, 033020 (2020)

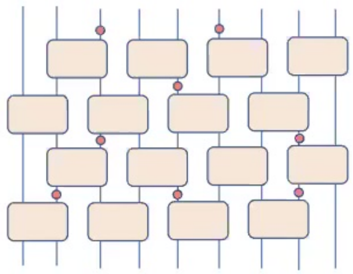
- I will focus on U(1) symmetries.

(techniques could be generalized to any abelian symmetry)

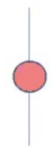
Previous works on $\mathbb{Z}_2$ circuits revealed different types of area law phases

Ippoliti et al PRX ’21, Lavasani, Alavirad & Barkeshli Nat Phys ’21, Sang & Hsieh Phys. Research ‘21

U(1) Symmetric circuits



Measurement of local charge S_z and qudit.



- The value of local charge is $0,1$.

$$U = \begin{pmatrix} U_{d^2\times d^2} & & \\ & U_{2d^2\times 2d^2} & \\ & & U_{d^2\times d^2} \end{pmatrix}$$

Local Hilbert space is:

$$\mathcal{H}_i = \mathcal{H}_d \otimes \mathcal{H}_2$$

Qudit d.o.f with no symmetry

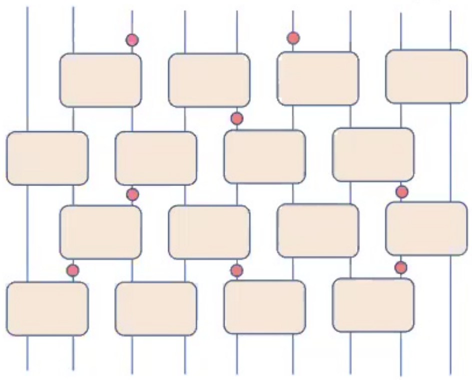
Qubit d.o.f with U(1) symmetry

- The large d limit has a mapping to a classical stat mech model; allows for efficient simulation (TEBD). The classical model also allows for an effective field theory description of the circuit.
- We find a charge sharpening transition ($p_{\#} \approx 0.2$) in addition to the entanglement transition ($p_c = 0.5$).

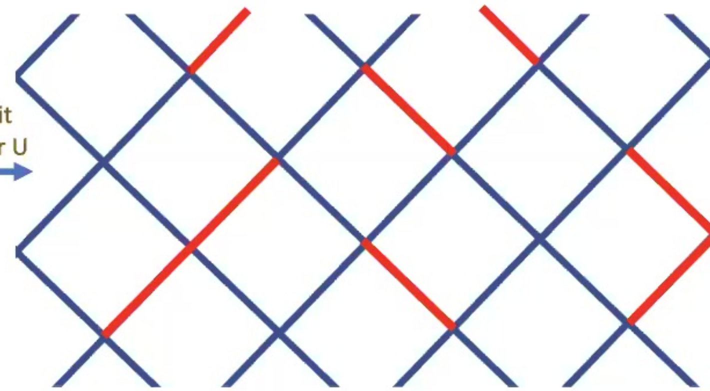
Mapping onto classical stochastic model

In $d \to \infty$ limit, the U(1) circuit can be mapped exactly to a classical stochastic model using replica trick.

$$\delta Q_{\{m\}}^2 \equiv \langle \psi_{\{m\}} | \hat{Q}^2 | \psi_{\{m\}} \rangle - \langle \psi_{\{m\}} | \hat{Q} | \psi_{\{m\}} \rangle^2$$

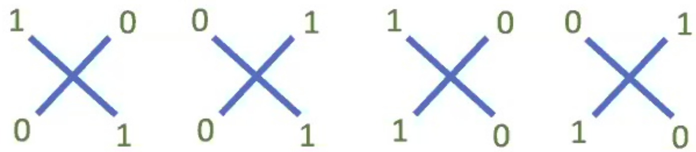


$d \to \infty$
Replica limit
Average over U

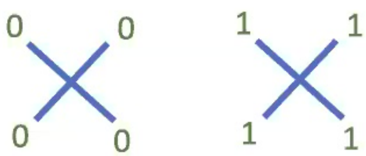


- Classical charge 0,1 lives on all links
- Measured links are fixed to measurement outcomes.

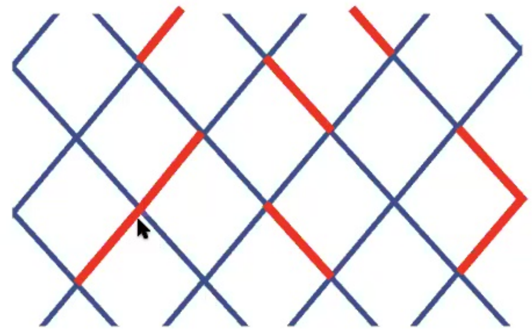
Mapping onto classical stochastic model



Weight = $\frac{1}{2}$



Weight = 1

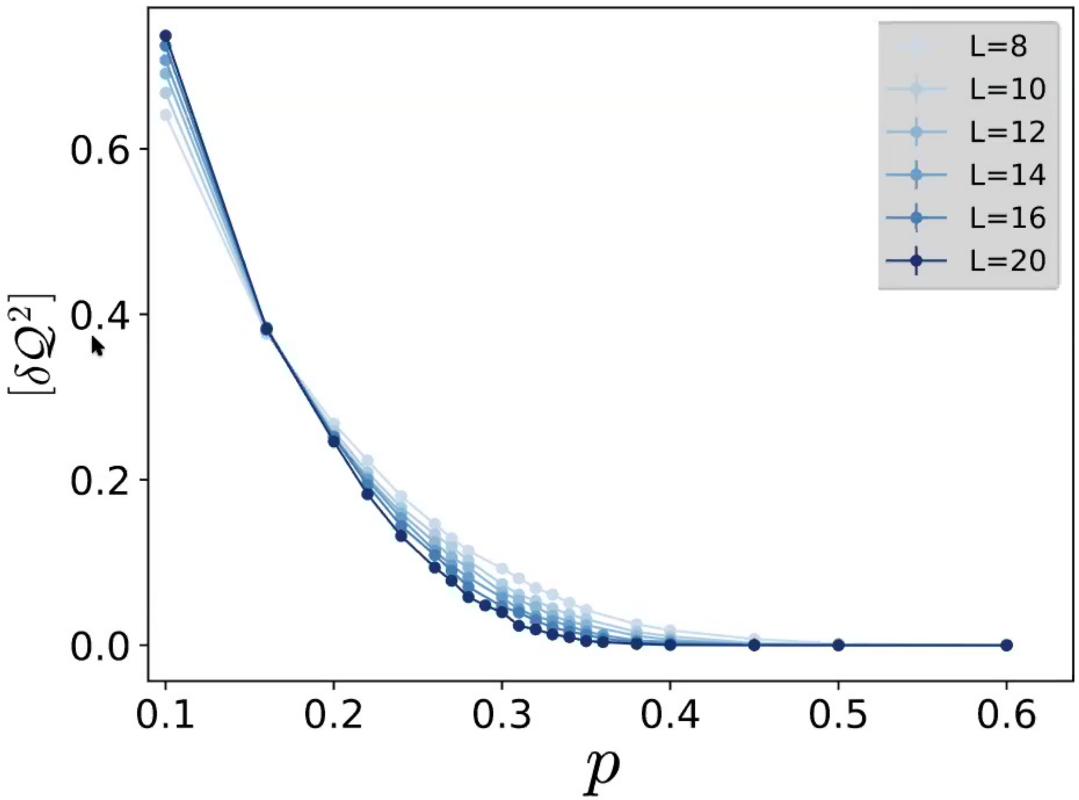


The dynamics of charge is described by 6-vertex model, also known as symmetric exclusion process.

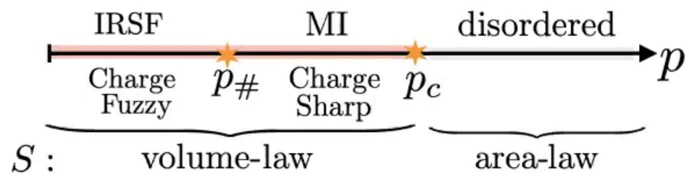
The charge on measured links are fixed; they are constrained to take the value given by outcome of the measurement

Charge sharpening: numerics

Transition inside the volume law (entangling) phase!



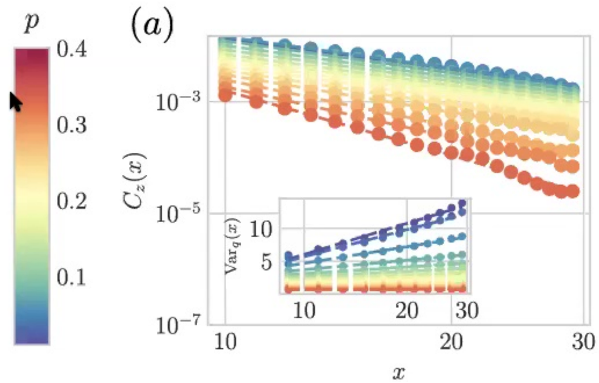
Dynamical ZZ correlations & charge variance



$$\boxed{C_z(x) \sim \begin{cases} \rho_s (a/x)^{-2} & p \le p_\# \\ e^{-x/\xi} & p > p_\# \end{cases} + \dots}$$

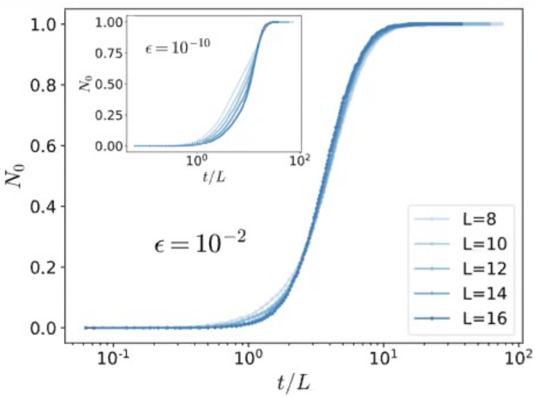
$$C_z(x) = \sum_m p_m \left[\langle \sigma_0^z \sigma_r^z \rangle_m - \langle \sigma_0^z \rangle_m \langle \sigma_r^z \rangle_m \right]$$

In fuzzy phase: $\delta Q^2 \sim \log L$



Central limit theorem estimates

- Let's evolve the system, but we don't know the global charge of the system.
- We perform measurements (with small enough p) and get a data list for outcomes, $\{m\}$.



$$Q_{estimate} = \frac{\sum_i m_i}{N} L.$$

To distinguish between states with close global charge, we need the error in $Q_{estimate}$ to be order 1,

$$\frac{L}{\sqrt{N}} < 1, or\ N > L^2.$$

Since $N \sim pLt$, we have a good estimate of the global charge for $t \gtrsim \frac{L}{p}$.

For small p, the charge fluctuations remain significant for until $t \sim \frac{L}{p}$

Effective field theory

- The stat mech model can be described by a disordered transfer matrix. Via replica trick we can write it as a transfer matrix over Q number of replicas, $\mathbb{T}^{(Q)}$.

Just as in quantum-classical correspondence, we can think of $\mathbb{T}^{(Q)}$ to be generated by a quantum Hamiltonian H.

- $H = H_0 + H_M$

$$H_0 = -J \sum_{a} \sum_{<ij>} S_{i,a} . S_{j,a}$$

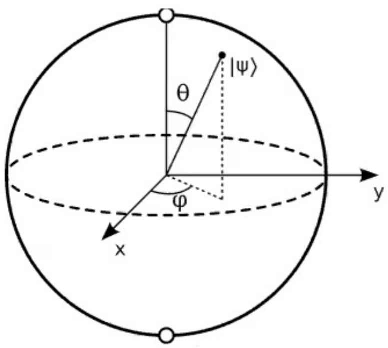
$$H_M = \gamma \sum_{a,b} \sum_{i} S^z_{i,a} \Pi_{ab} S^z_{i,b}$$

$\Pi_{ab} = \delta_{ab} - \frac{1}{Q}$ is projection to inter replica fluctuations

- Gapless with $z = 2 \Leftrightarrow$ Circuit Thouless time $t_{th} \sim L^2$.
- The replica symmetric mode is not touched by measurements $\Leftrightarrow$ Linear quantities are trivial in monitored circuits!

Effective field theory

$$\theta = \frac{\pi}{2} + \delta\theta,$$
$$\delta\theta = \delta\bar{\theta} + \Pi\delta\theta,$$
$$\phi = \bar{\phi} + \Pi\phi$$



$$H_0 = -J \sum_a \sum_{<ij>} S_{i,a}.S_{j,a}$$

$$H_M = \gamma \sum_{a,b} \sum_i S^z_{i,a} \Pi_{ab} S^z_{i,b}$$

$$\mathcal{L} = \overbrace{iS\delta\bar{\theta}\partial_\tau\bar{\phi} + \frac{S^2 J}{2}[(\nabla\delta\bar{\theta})^2 + (\nabla\bar{\phi})^2]}^{\text{Heisenberg Ferromagnet. Gapless and } z=2.}$$

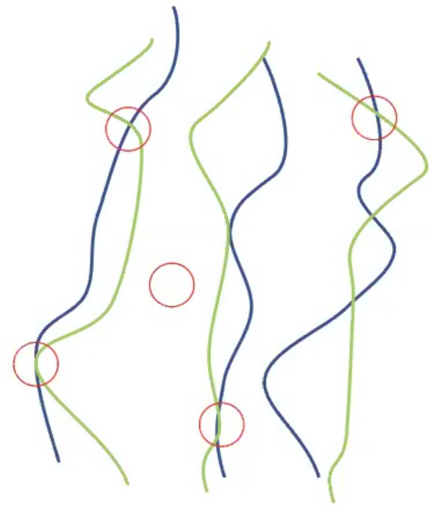
$$\underbrace{+\frac{S^2}{2}[\gamma^{-1}(\partial_\tau\Pi\phi)^2 + J(\nabla\Pi\phi)^2]}_{(Q-1)\text{ XY models! Undergoes KT transition with respect to } J/\gamma.}$$

Inter-replica superfluid

$$C_z(x) \sim \begin{cases} \rho_s (a/x)^{-2} & p \le p_\# \\ e^{-x/\xi} & p > p_\# \end{cases} + \dots$$

The effective H can thought of as hard-core bosons with replicas being decoupled from each other.

The replicas are constrained to agree on sites of measurements.



Inter replica superfluid (fuzzy phase):
When measurements are sparse or weak, the inter replica fluctuations are strong. These modes are in a ''superfluid'' phase.

Inter replica insulator (sharp phase):
Frequent or strong measurements restrict the dynamics of inter replica modes. This leads to an "insulating" phase.

$$\mathrm{Var}_q(x) = \sum_{0<i,j<x} \mathbb{E}\left[\langle \sigma_i^z \sigma_j^z \rangle_c\right] \approx \left\langle \left(\int_0^x \frac{d\vartheta}{\pi} \right)^2 \right\rangle = \frac{8\rho_s}{\pi} \log |x|/a$$

Conclusion

- New class of "entanglement transitions"
- Transition in ability of measurements to reveal a conserved charge inside the scrambled, entangling phase!
- Stat mech mappings allow for efficient TEBD + Effective Field theory approaches
- Higher d? More "practical" ways to observe the sharpening transition without postselection? Experiments? Relation to decoding problem/QEC?

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