

Title: Cooling quantum systems with quantum information processing

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Abstract: The field of quantum information provides fundamental insight into central open questions in quantum thermodynamics and quantum many-body physics, such as the characterization of the influence of quantum effects on the flow of energy and information. These insights have inspired new methods for cooling physical systems at the quantum scale using tools from quantum information processing. These protocols not only provide an essentially different way to cool, but also go beyond conventional cooling techniques, bringing important applications for quantum technologies. In this talk, I will first review the basic ideas of algorithmic cooling and give analytical results for the achievable cooling limits for the conventional heat-bath version. Then, I will show how the limits can be circumvented by using quantum correlations. In one algorithm I take advantage of correlations that can be created during the rethermalization step with the heat-bath and in another I use correlations present in the initial state induced by the internal interactions of the system. Finally, I will present a recently fully characterized quantum property of quantum many-body systems, in which entanglement in low-energy eigenstates can obstruct local outgoing energy flows.



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Cooling quantum systems with quantum information processing

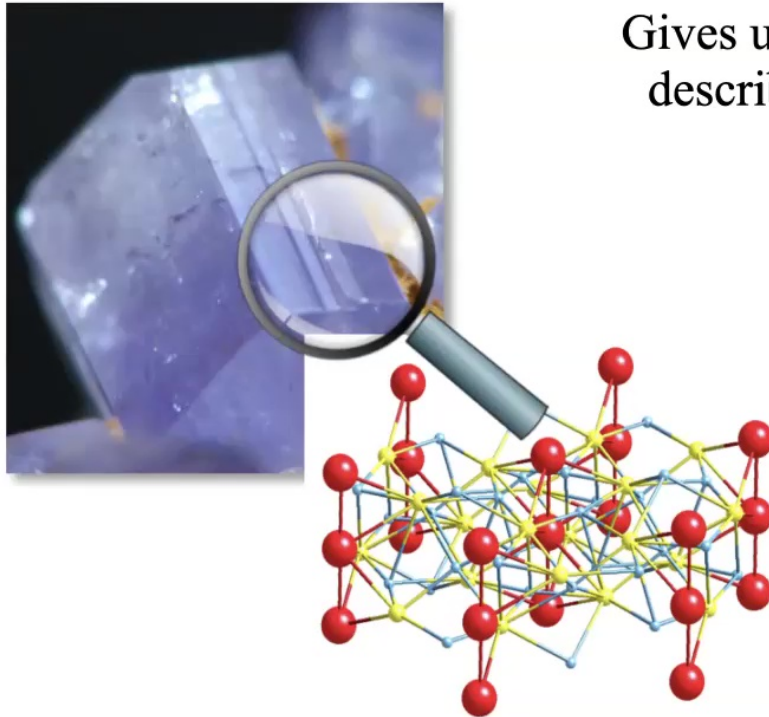
Nayeli A. Rodríguez-Briones

Feb 14, 2022

Introduction

Quantum Information Processing

Gives us an efficient mathematical language to describe and understand quantum physics in terms of information evolution.



Fundamental quantum physics



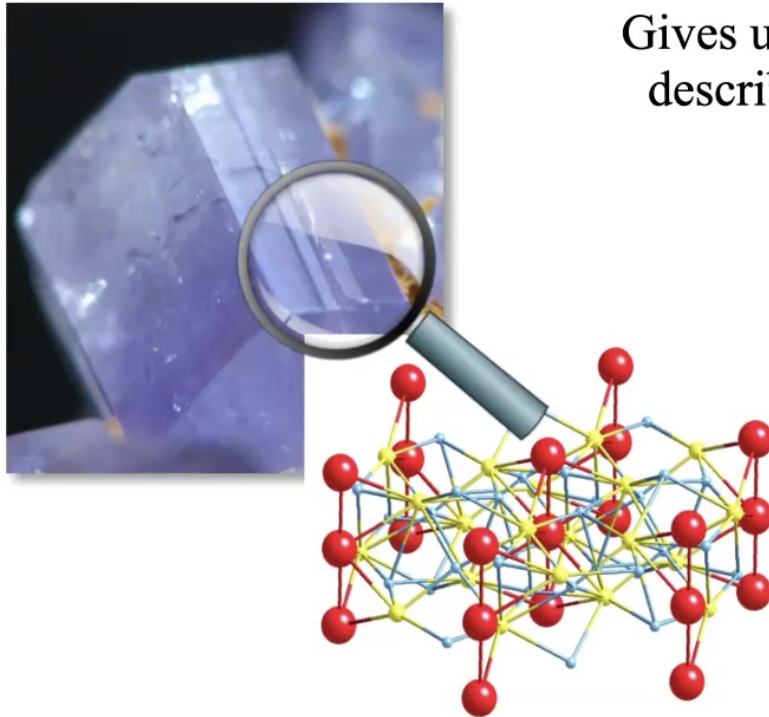
Controllable quantum simulation tools

- Quantum Gates
- Simulation of interactions
- Quantum measurements
- State preparation

Introduction

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Fundamental quantum physics



Controllable quantum simulation tools

Quantum
Thermodynamics

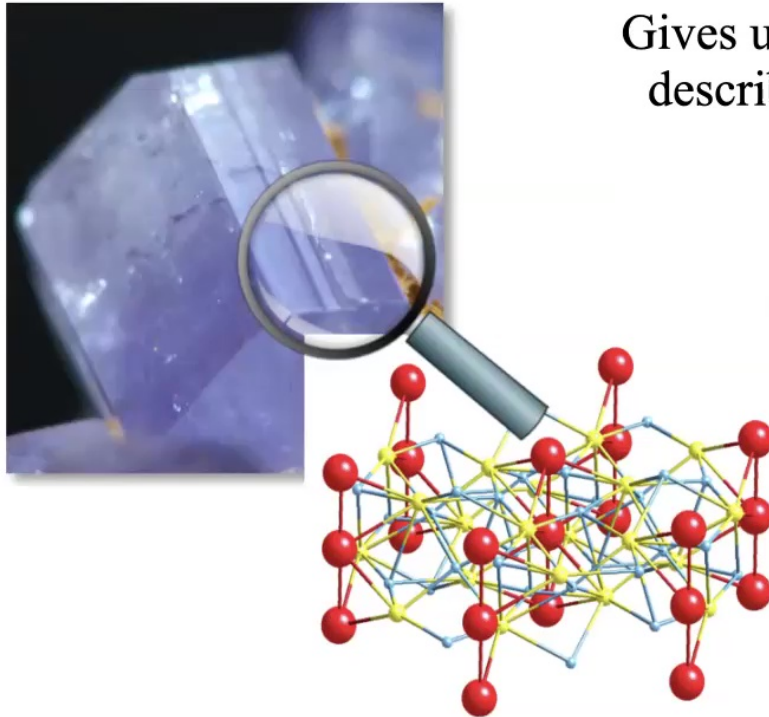
Generalize classical thermodynamics to capture quantum phenomena

- Quantum effects on energy and information flows
- Emergent behavior coming from entanglement
- Failure of global thermalization (many-body localization)

Introduction

Quantum Information Processing

Gives us an efficient mathematical language to describe and understand quantum physics in terms of information evolution.



Fundamental quantum physics



Controllable quantum
simulation tools

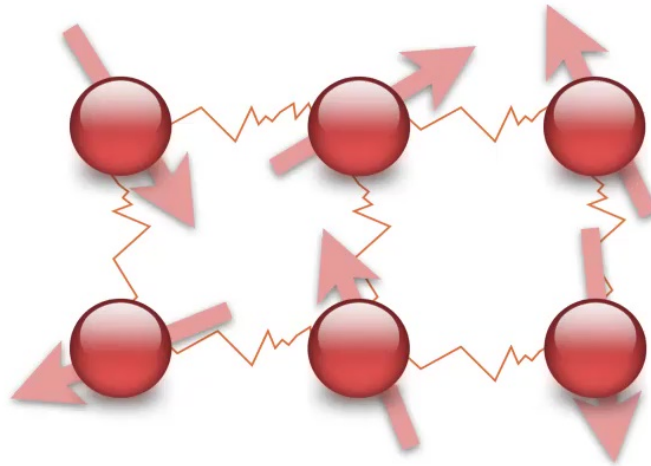
Quantum
Thermodynamics

Quantum Gravity

Quantum Matter

(...)

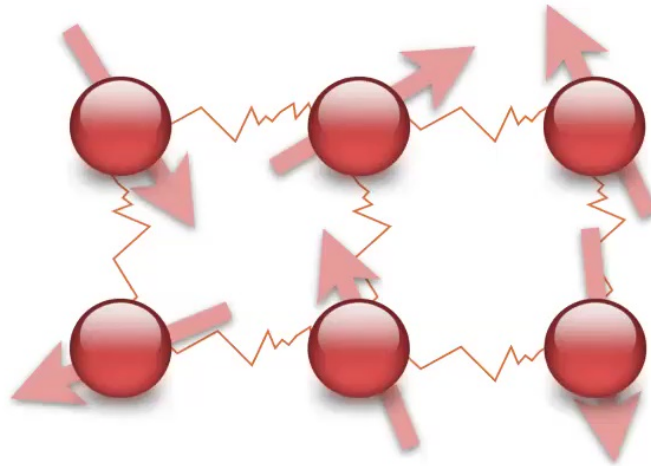
Why should we care about low energy quantum many-body systems?



They provide a unique setting for exploration:

- Emergence of genuine quantum behavior
- New insights for open questions about properties coming from entanglement between particles
- Phase transitions
- Critical behavior
- Applications for quantum technologies

Understanding the full scope of the quantum phenomena remains a big challenge due to its complexity.



Theoretically: Problems of general many-body Hamiltonian systems at low temperature are computationally hard

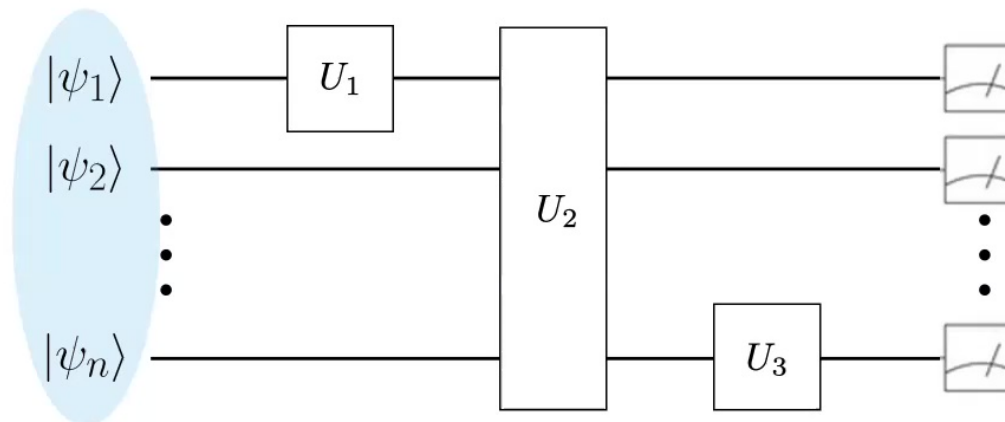
Experimentally: It is crucial and difficult to find new cooling techniques that go beyond current cooling limits

Applications in quantum science technologies

Preparation of highly pure quantum states

Quantum computers need a constant supply of highly pure states.

- As initial states for most quantum algorithms
- For quantum error correction: to prepare auxiliary qubits that satisfy the fault tolerance threshold for quantum computing

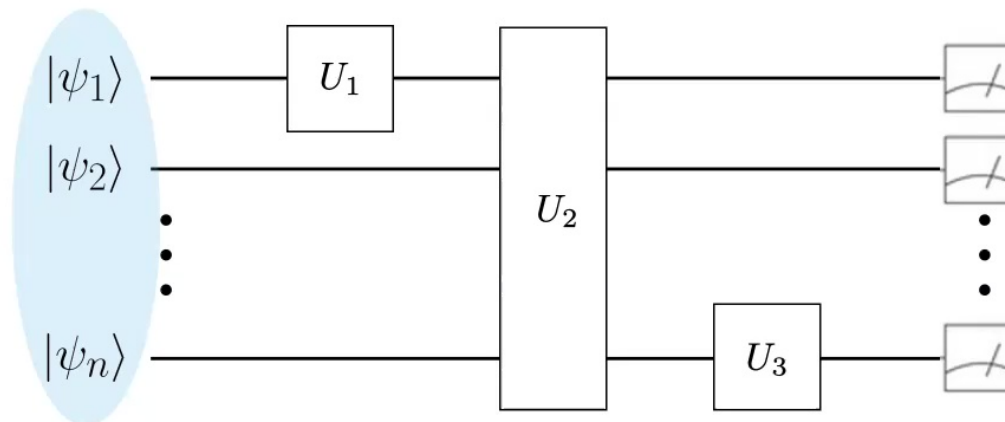


Applications in quantum science technologies

Preparation of highly pure quantum states

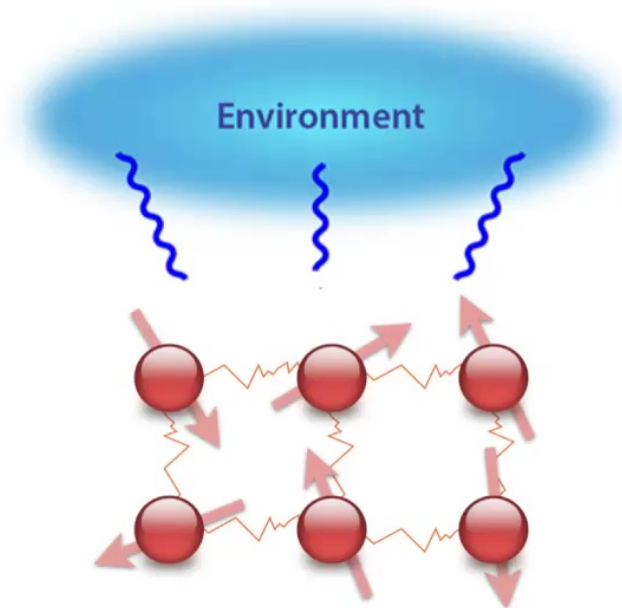
Quantum computers need a constant supply of highly pure states.

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Cooling techniques help to achieve the required purity to satisfy the fault tolerance threshold

What is the conventional cooling?



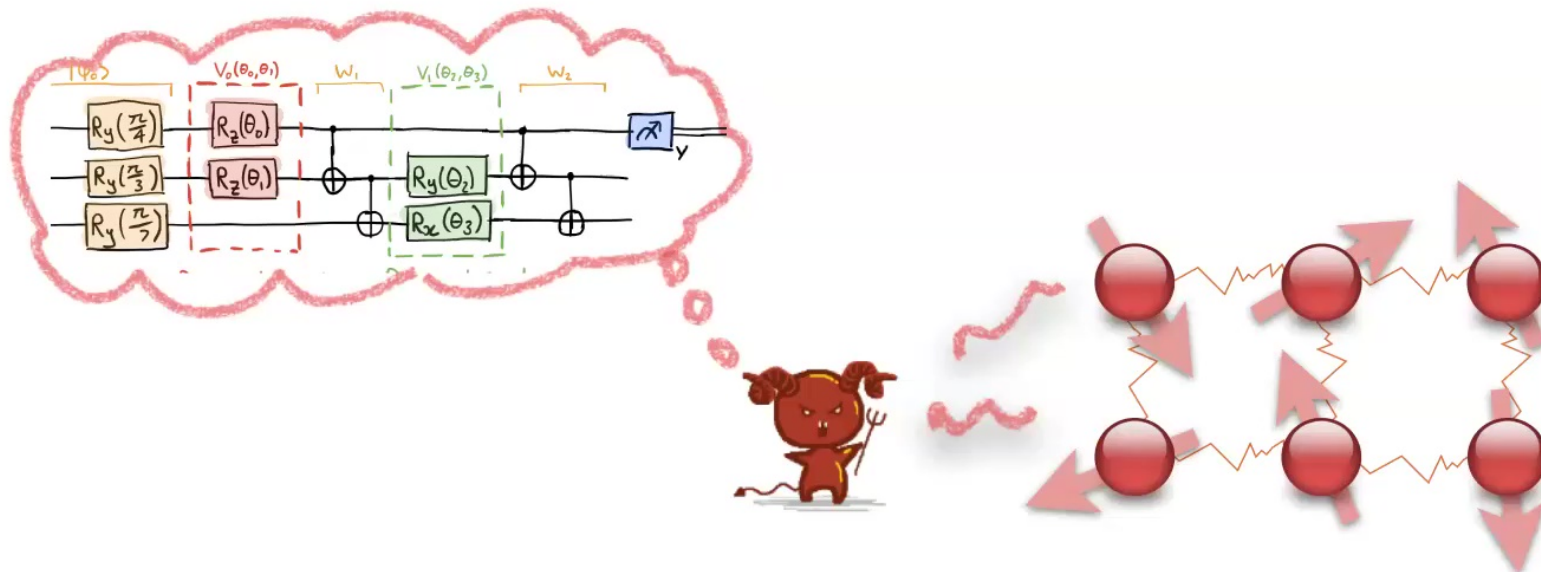
Conventional cooling: dissipative process

Couple the system to a heat reservoir and allow system to thermalize under dissipative dynamics

Device dependent process

How can we use the tools from QIP?

Information processing \longrightarrow Entropy manipulation



Quantum information theory can provide new fundamentally different ways to **decrease the temperature of quantum** systems in an **algorithmic way** by manipulating entropy.

Outline



Intro Algorithmic Cooling Methods

Contributions

Role of correlations
in cooling mechanisms

Improving cooling limits?

- Correlated relaxation processes between qubit-environment
- Quantum correlations within the system

New properties of quantum many-body systems

- Entanglement can prevent outgoing energy flows

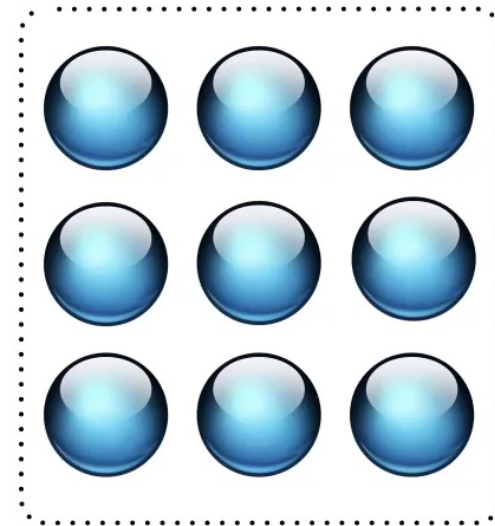
Algorithmic Cooling

How can we cool a set of qubits using **unitary** operations and contact with a heat-reservoir?

Target qubit
↓



Qubit system



Heat-bath

(and projective measurements not allowed. *)

For the target qubit:



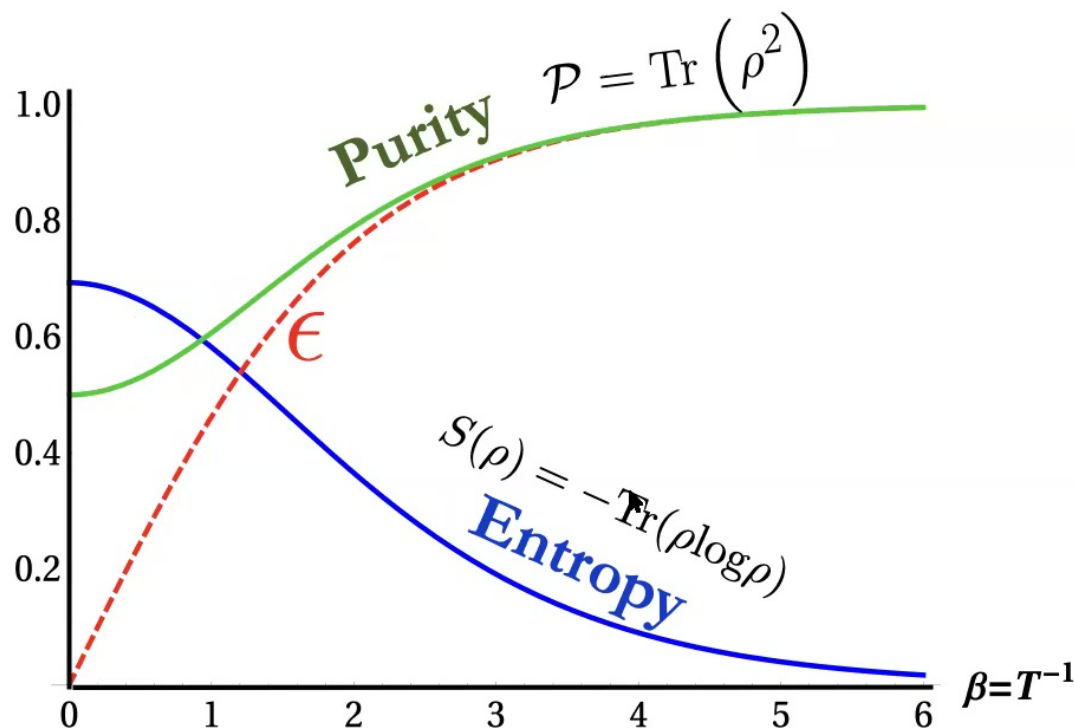
$$\rho = \begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{pmatrix}$$

Local thermal state

$$\rho = \frac{e^{-\beta H}}{Z}$$

$$H = -\frac{1}{2}\omega\sigma_z$$

$$\rho = \frac{1}{2} \begin{pmatrix} 1 + \epsilon & 0 \\ 0 & 1 - \epsilon \end{pmatrix}$$

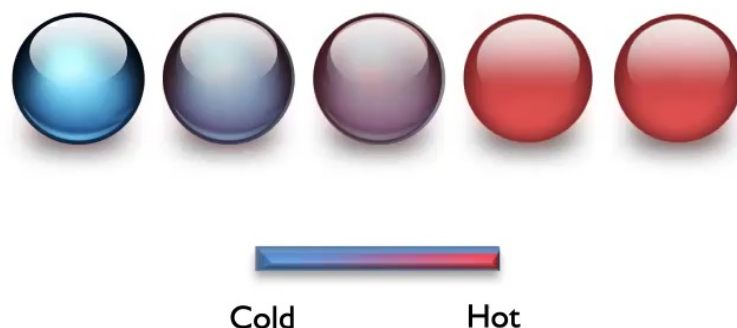


Algorithmic Cooling (AC)



“A quantum mechanical heat engine”
Schulman and Vazirani, 1990

Algorithmic Cooling (AC)

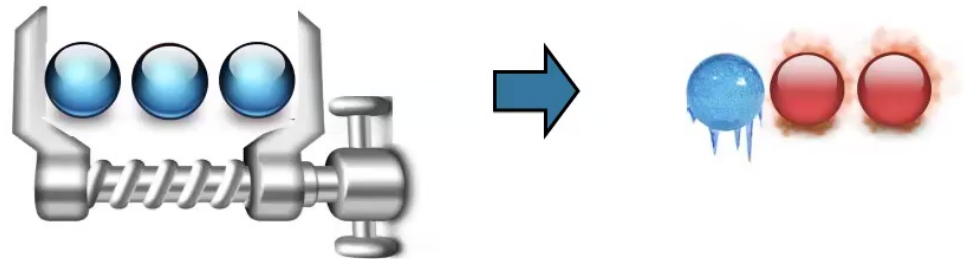


The idea is to re-distribute the entropy within the group of qubits using a unitary operation U .

$$\rho_{1\text{st}} = \text{Tr}_{\overline{1\text{st}}} (U \rho_{\text{total}} U^\dagger) \quad S(\rho_{1\text{ST}}) \rightarrow 0$$

Entropy compression

Simplest example:



State of each qubit

$$\rho_{\epsilon_0} = \frac{1}{2} \begin{pmatrix} 1 + \epsilon_0 & 0 \\ 0 & 1 - \epsilon_0 \end{pmatrix}$$

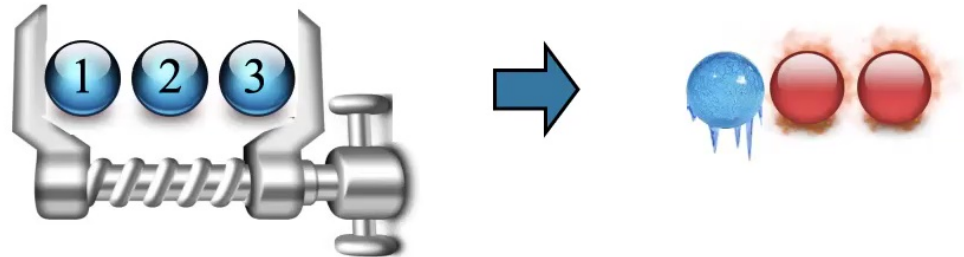
$$0 \leq \epsilon_0 \leq 1$$

Diagonal of the full state:

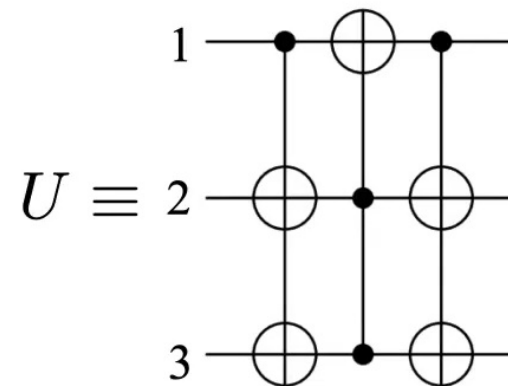
$$\text{diag}(\rho_{\epsilon}^{\otimes 3}) = \frac{1}{2^3}$$

$(1 + \epsilon_0)^3$	000
$(1 + \epsilon_0)^2 (1 - \epsilon_0)$	001
$(1 + \epsilon_0)^2 (1 - \epsilon_0)$	010
$(1 + \epsilon_0) (1 - \epsilon_0)^2$	011
$(1 + \epsilon_0)^2 (1 - \epsilon_0)$	100
$(1 + \epsilon_0) (1 - \epsilon_0)^2$	101
$(1 + \epsilon_0) (1 - \epsilon_0)^2$	110
$(1 - \epsilon_0)^3$	111

Simplest example



$$|011\rangle \longleftrightarrow |100\rangle$$



Example, when $\beta = \frac{1}{k_B T_0} \ll 1$:

Target qubit
effective temperature

$$T_0 \rightarrow \frac{2}{3}T_0$$

Second and third qubit effective
temperatures

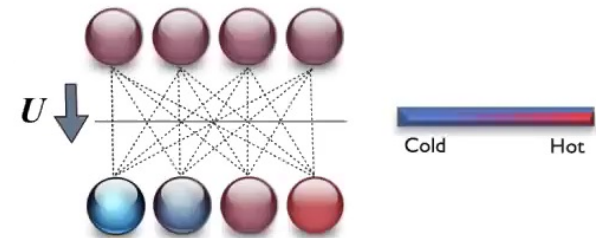
$$T_0 \rightarrow 2T_0$$

The Partner Pairing Algorithm (PPA-HBAC)

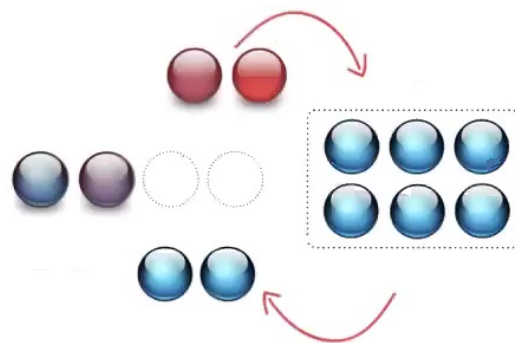
PPA-HBAC is the iteration of the two steps:

① **Entropy
Compression Step.**

$$\rho \rightarrow \rho' = U \rho U^\dagger$$



② **Refresh
Step.**



$$\rho' \rightarrow \rho'' = \text{Tr}_m(\rho) \otimes \rho_{\epsilon_b}^{\otimes m}$$

L. J. Schulman, T. Mor, and Y. Weinstein, Phys. Rev. Lett. 94, 120501 (2005).

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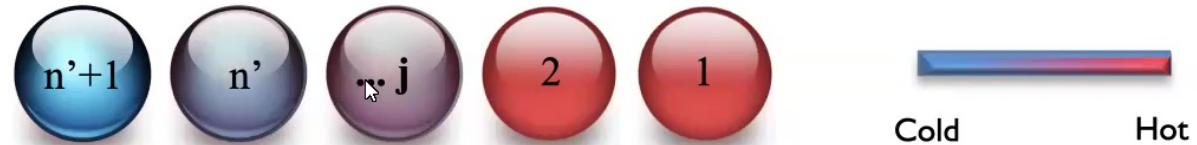
Limits of entropy compression using unitary dynamics for a close system

- Shannon Entropy bound (Limited by unitary dynamics)
- Conservation of eigenvalues of the total system
- If initial state is highly mixed, it requires a lot of resources

Ex. $\epsilon_b \sim 10^{-5} \longrightarrow 10^{12}$ qubits

Cooling Limit of the PPA-HBAC

Steady State

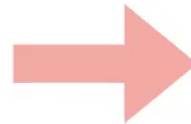


$$\rho_T^\infty = \bigotimes_{i=1}^{n'+1} \rho_{j^{th}}^\infty = \bigotimes_{i=1}^{n'+1} \frac{1}{2} \begin{pmatrix} 1 + \epsilon_j^\infty & 0 \\ 0 & 1 - \epsilon_j^\infty \end{pmatrix}$$

$$\epsilon_j^\infty = \frac{(1 + \epsilon_b)^{m2^{j-1}} - (1 - \epsilon_b)^{m2^{j-1}}}{(1 + \epsilon_b)^{m2^{j-1}} + (1 - \epsilon_b)^{m2^{j-1}}}$$

Temperature of the target qubit

$$T_\infty = \frac{T_b}{2^{n-2}}$$



For $n = 2$
there is no improvement
beyond the bath temperature
 $T_\infty = T_b$

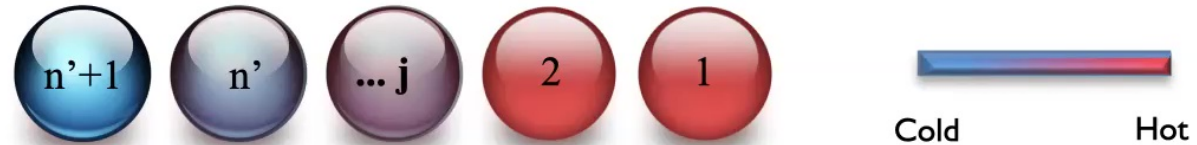
What is the theoretical **cooling limit** of these algorithmic cooling methods?

This question had been a longstanding open problem for more than a decade.



Cooling Limit of the PPA-HBAC

Steady State

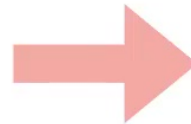


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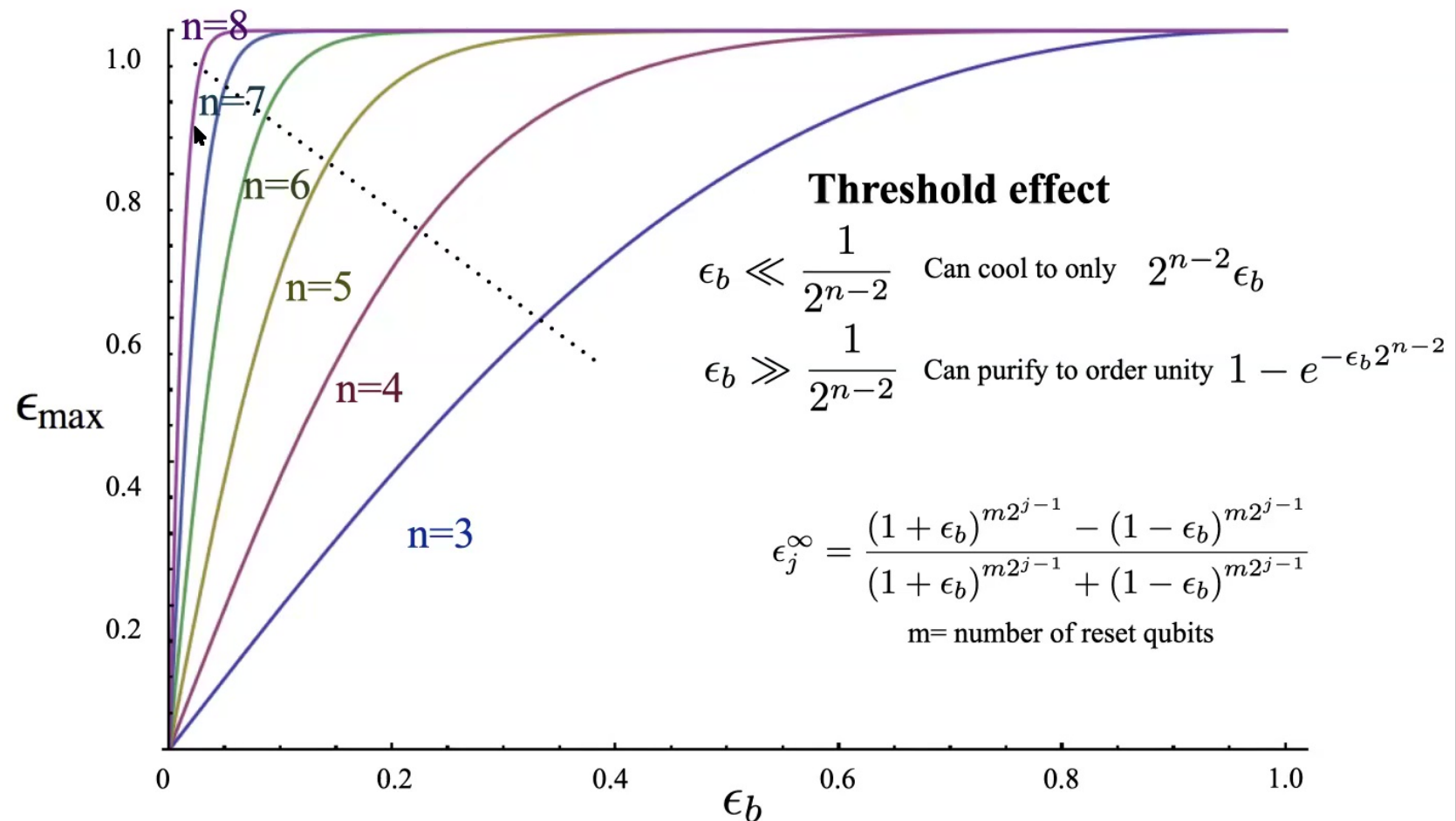
Temperature of the target qubit

$$T_\infty = \frac{T_b}{2^{n-2}}$$



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 $T_\infty = T_b$

Cooling limit of PPA-HBAC



N Rodríguez-Briones and R Laflamme. Phys. Rev. Lett. 116,170501 (2016)

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Is this the fundamental cooling limit of these techniques?

It was widely believed that PPA-HBAC gave the optimal HBAC method...

L. Schulman, T. Mor, and Y. Weinstein, Phys. Rev. Lett. 94, 120501 (2005)

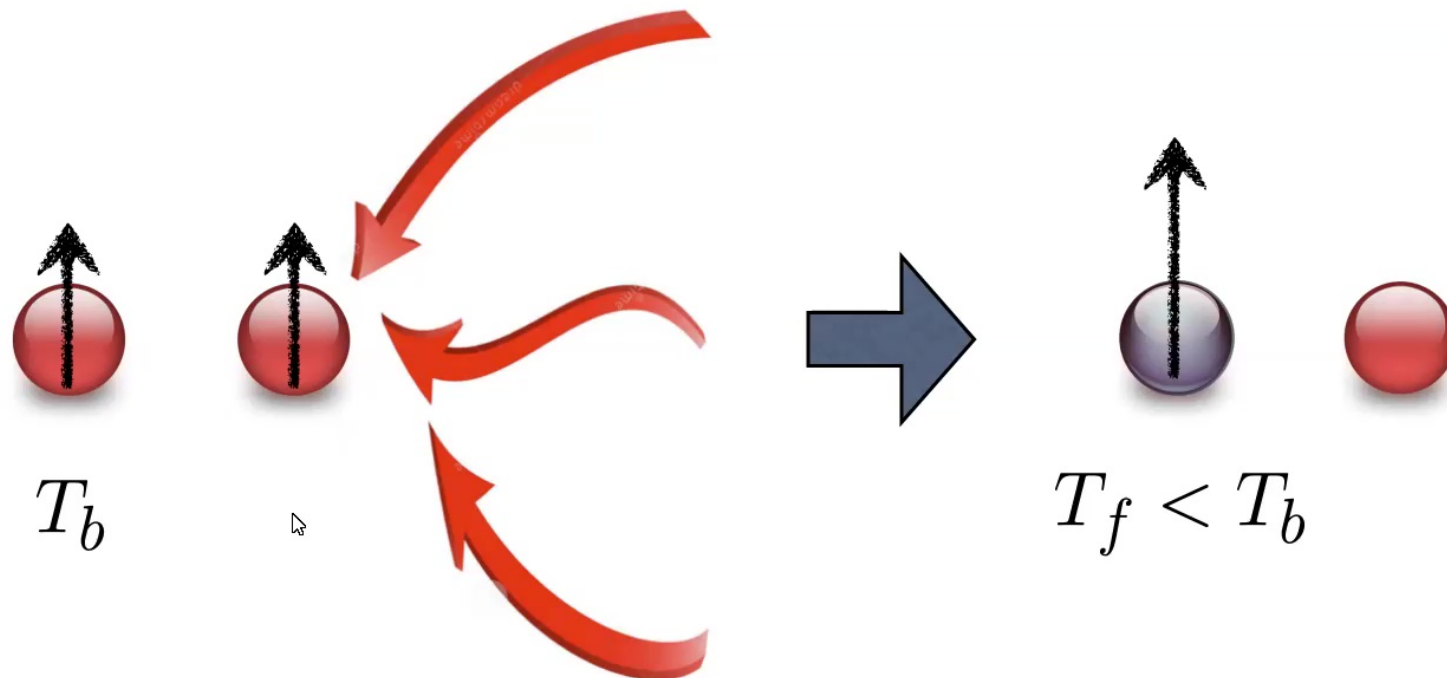
S. Raesi and M. Mosca, Phys. Rev. Lett. 114, 100404 (2016)

However, we found a way to
circumvent this cooling limit by
taking advantage of correlations

N. Rodríguez-Briones, E. Martín Martínez, A. Kempf, R. Laflamme Phys. Rev. Lett. 119 (5), 050502, 2017

N. Rodríguez-Briones, J Li, X Peng, T Mor, Y Weinstein, R Laflamme, NJP 19 (11), 113047, 2018

Nuclear Overhauser Effect (NOE)



How to reconcile this with algorithmic cooling?

A. Overhauser, Phys. Rev. 89, 689 (1953).

Dipolar relaxation process

Solomon equations

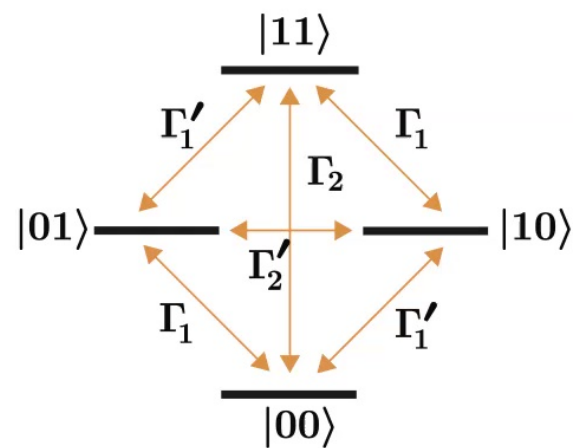
$$\frac{d\langle Z^1 \rangle}{dt} = -R_1(\langle Z^1 \rangle - \langle Z^1 \rangle_0) - R_{12}(\langle Z^2 \rangle - \langle Z^2 \rangle_0)$$

Individual-relaxation

$$R_1 = \Gamma'_2 + 2\Gamma_1 + \Gamma_2$$

Cross-relaxation

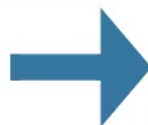
$$R_{12} = \Gamma_2 - \Gamma'_2.$$



NOE effect

For $\frac{d\langle Z^1 \rangle}{dt} = 0$ with $\langle Z^2 \rangle \approx 0$

$$\langle Z^1 \rangle \rightarrow \langle Z^1 \rangle_0 + \frac{R_{12}}{R_1} \langle Z^2 \rangle_0$$



$$\epsilon_{target} = \left(1 + \frac{R_{12}}{R_1}\right) \epsilon_b,$$

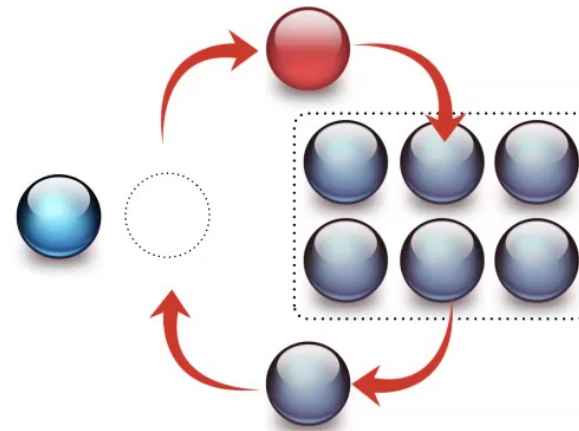
$$\Gamma_2 \ll \Gamma_1, \Gamma'_1, \Gamma'_2$$

Why can't PPA achieve this cooling?

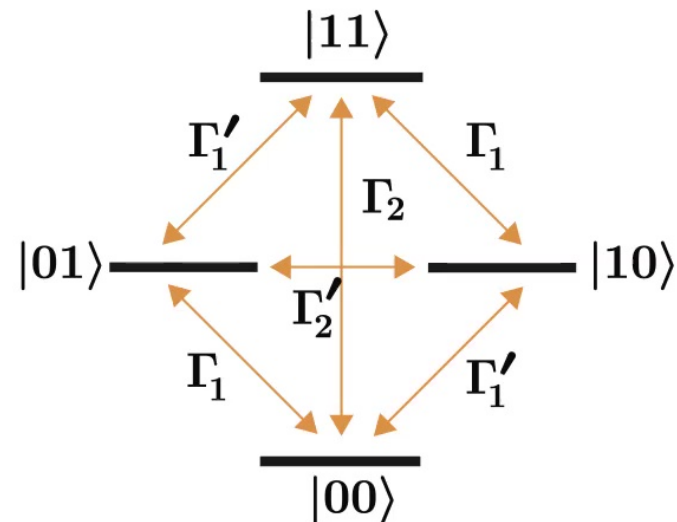
PPA-HBAC

Qubit reset step

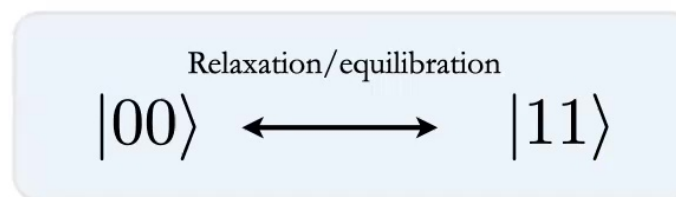
$$\rho_{12} \rightarrow \text{Tr}_2 (\rho_{12}) \otimes \rho_{\epsilon_b}$$



Relaxation diagram for a two-qubit system



Reset of states Γ_2



Kraus Operators

$$A_1^{(n=2)} = \sqrt{p_2} |00\rangle \langle 00|$$

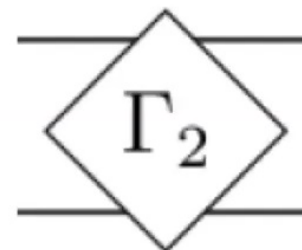
$$A_2^{(n=2)} = \sqrt{p_2} |00\rangle \langle 11|$$

$$A_3^{(n=2)} = \sqrt{1 - p_2} |11\rangle \langle 11|$$

$$A_4^{(n=2)} = \sqrt{1 - p_2} |11\rangle \langle 00|$$

$$A_5^{(n=2)} = |01\rangle \langle 01|$$

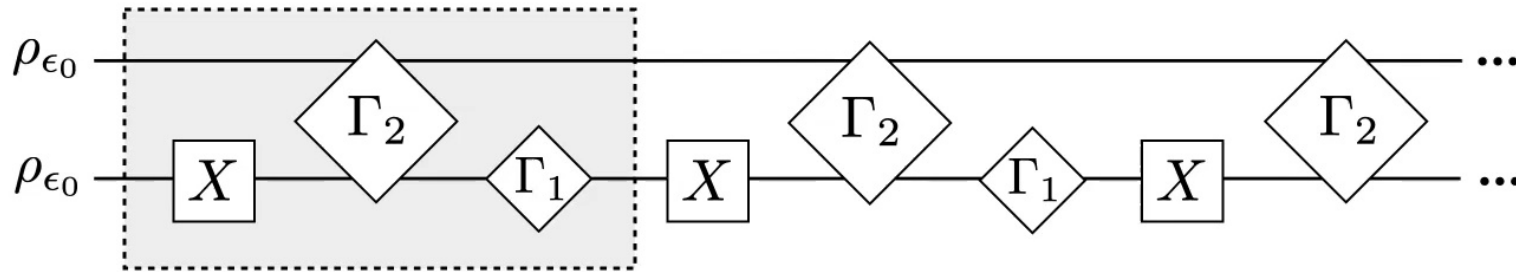
$$A_6^{(n=2)} = |10\rangle \langle 10|$$



$$P_2 = \frac{e^{2\xi}}{2 \cosh(2\xi)}$$

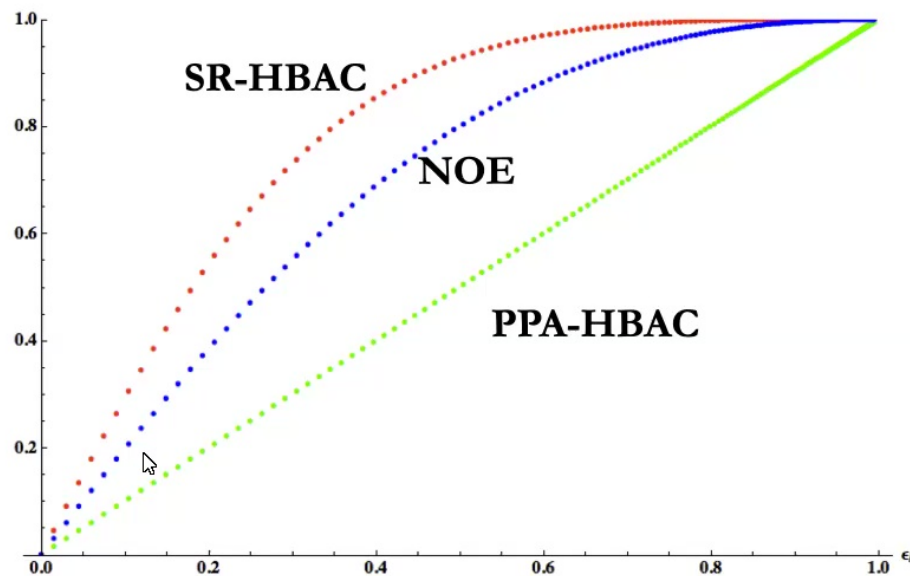
$$\xi = \tanh^{-1}(\epsilon_b)$$

SR-HBAC algorithm, for n=2



Round_(n=2)

$\epsilon_{\max}^{(n=2)}$



$$\epsilon_{\max}^{(n=2)} = \tanh(3\xi)$$

$$\xi = \tanh^{-1}(\epsilon_b)$$

$$\epsilon_k^{(n=2)} = \left(3 - \frac{1}{2^{k-1}}\right) \epsilon_b$$

SR-HBAC algorithm for n qubits

Final temperatures

SR-HBAC

$$T_{\infty} = \frac{T_b}{2^n - 1}$$

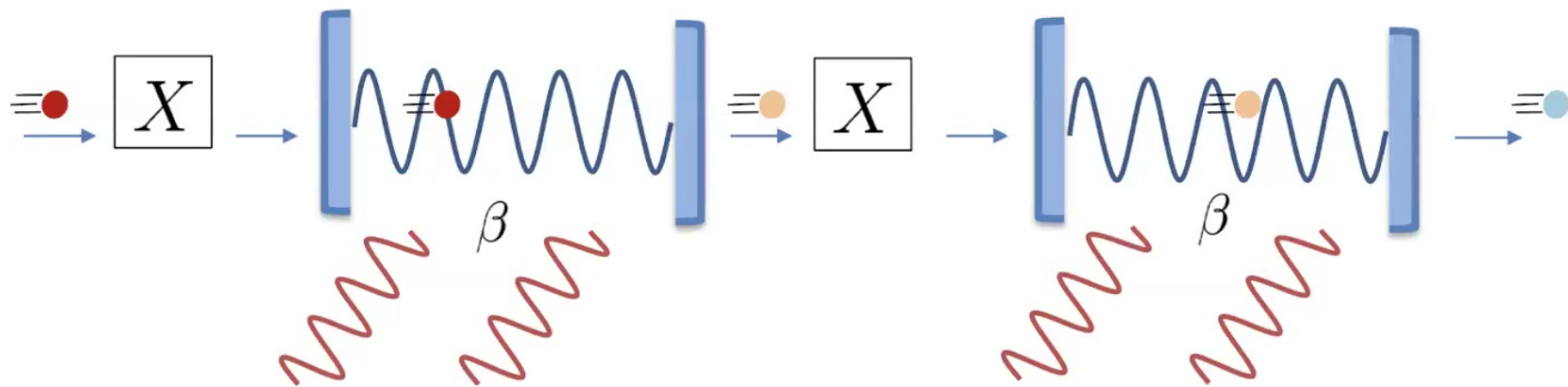
PPA-HBAC

$$T_{\infty} = \frac{T_b}{2^{n-2}}$$

- Circumventing previous cooling limit by taking advantage of crossed-relaxation processes
- Providing new tools for algorithm cooling protocols
- Inspiring other research groups to design new cooling protocols

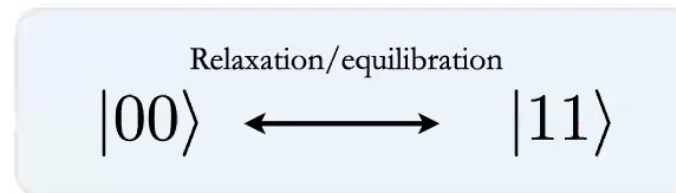
Alvaro M. Alhambra et al. pushes this new tool to its limits by finding optimal strategies under general engineered thermalization processes. This optimization achieves polarization of order 1 exponentially fast with the number of rounds.

They show that such enhancements can only be seen when the thermalization dynamics are non-Markovian.



Alhambra, Á. M., Lostaglio, M., & Perry, C. (2019) Quantum, 3, 188.

Reset of states Γ_2



Kraus Operators

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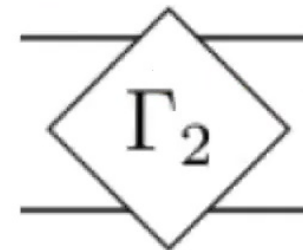
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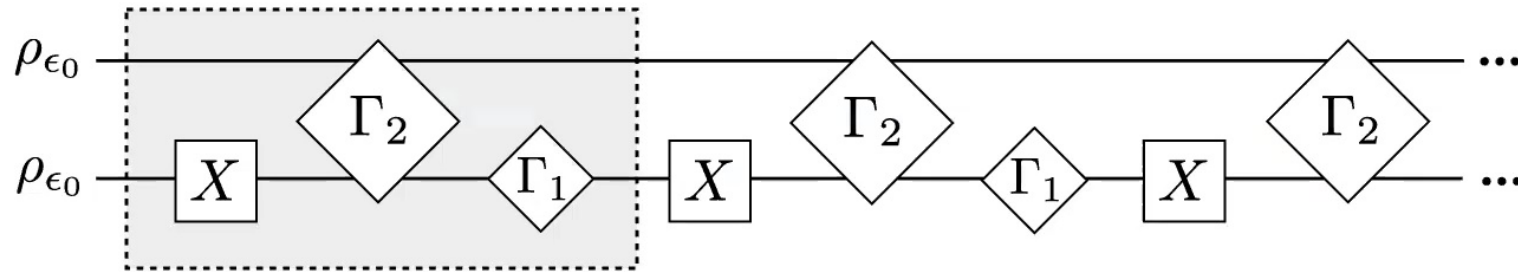
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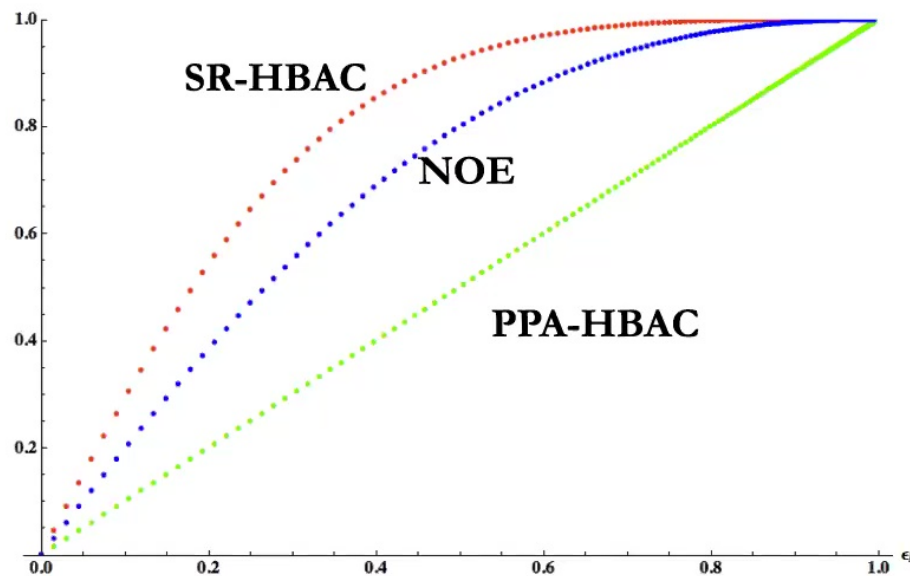
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$$\xi = \tanh^{-1}(\epsilon_b)$$

SR-HBAC algorithm, for n=2



Round_(n=2)
 $\epsilon_{\max}^{(n=2)}$



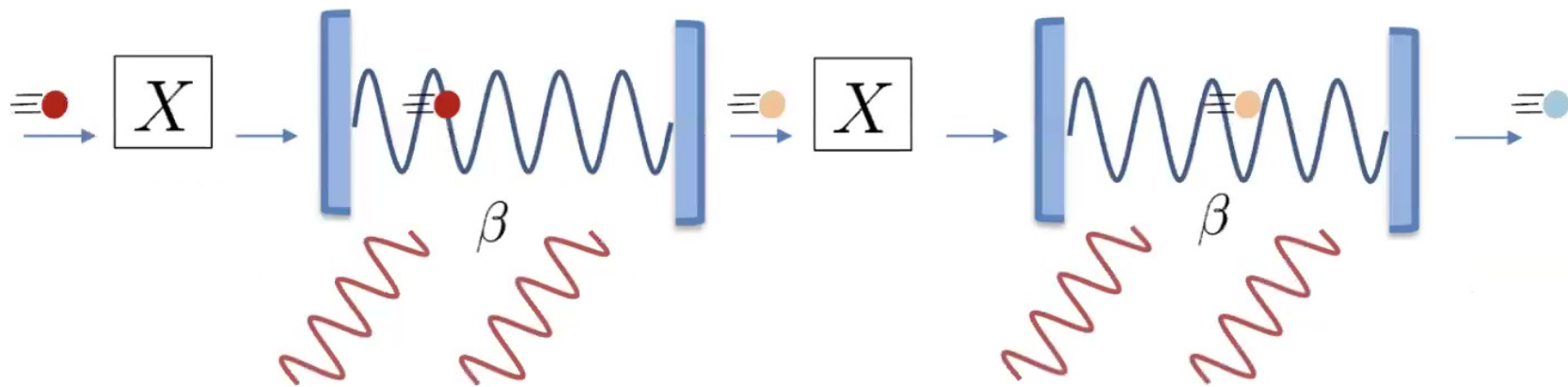
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SR-HBAC algorithm for n qubits

Final temperatures

SR-HBAC

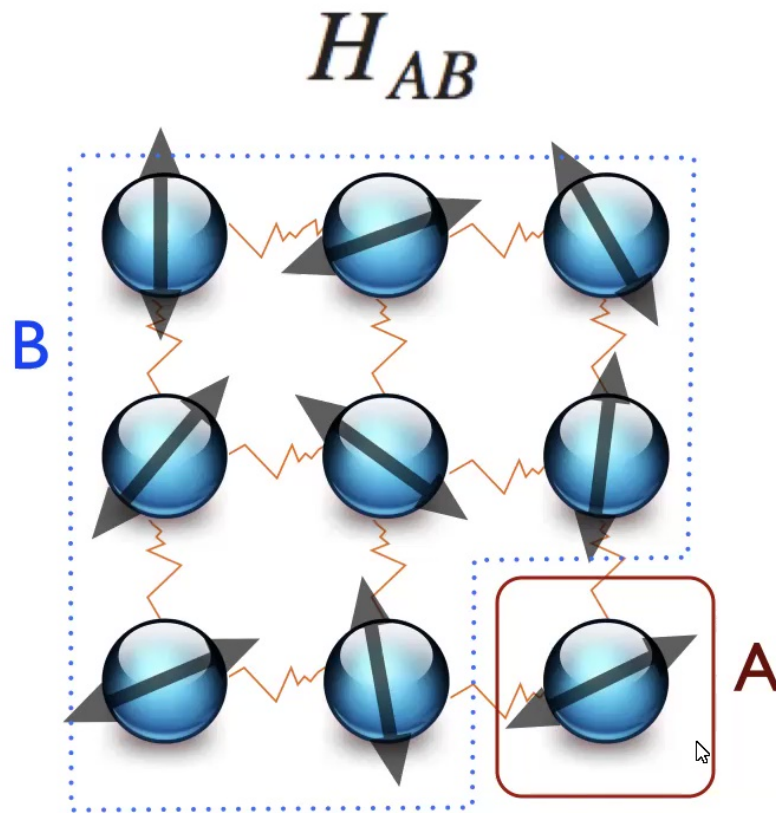
$$T_{\infty} = \frac{T_b}{2^n - 1}$$

PPA-HBAC

$$T_{\infty} = \frac{T_b}{2^{n-2}}$$

- Circumventing previous cooling limit by taking advantage of crossed-relaxation processes
- Providing new tools for algorithm cooling protocols
- Inspiring other research groups to design new cooling protocols

Can we use pre-existing
correlations within the system to
improve cooling?

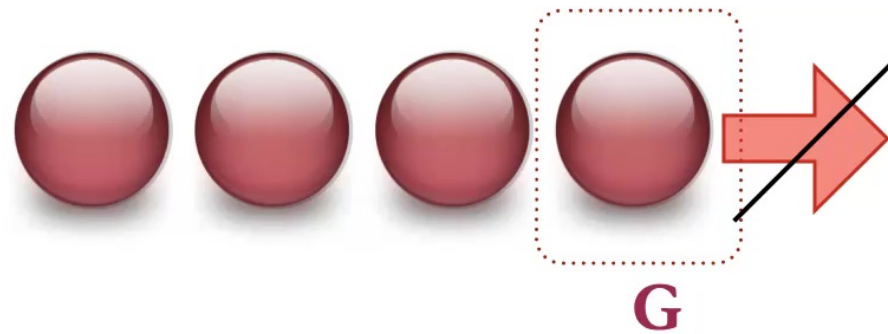


How could a Hamiltonian with an entangled ground state affect the extraction of energy?



Strong local passive states

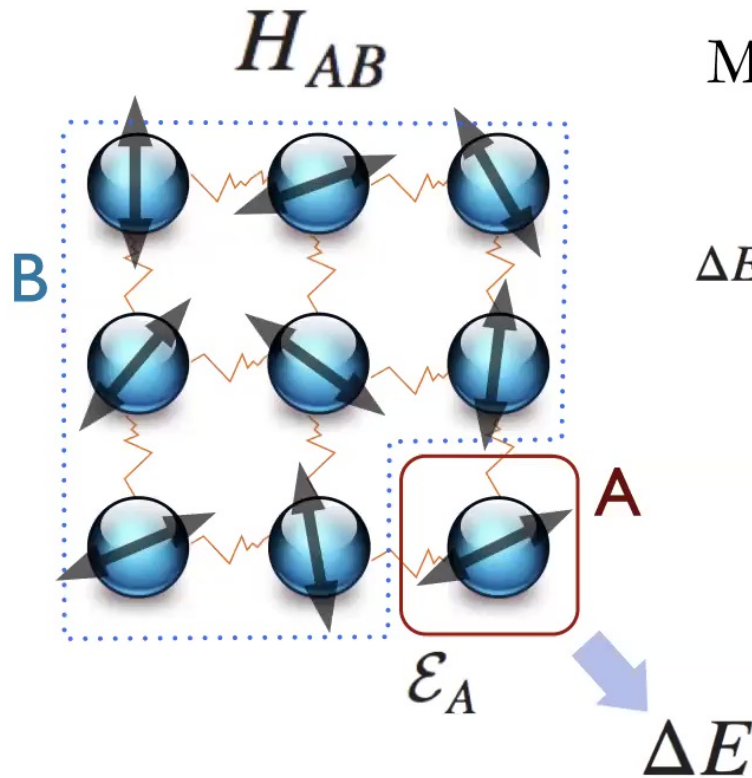
A system state is defined to be strong-local passive with respect to a subsystem, if no **general** quantum operation **G** applied **locally** on the subsystem can extract positive energy.



$$\Delta E(\rho) = \text{Tr} [H (I \otimes G) \rho] - \text{Tr} [H \rho]$$

$$\forall G, \Delta E \geq 0$$

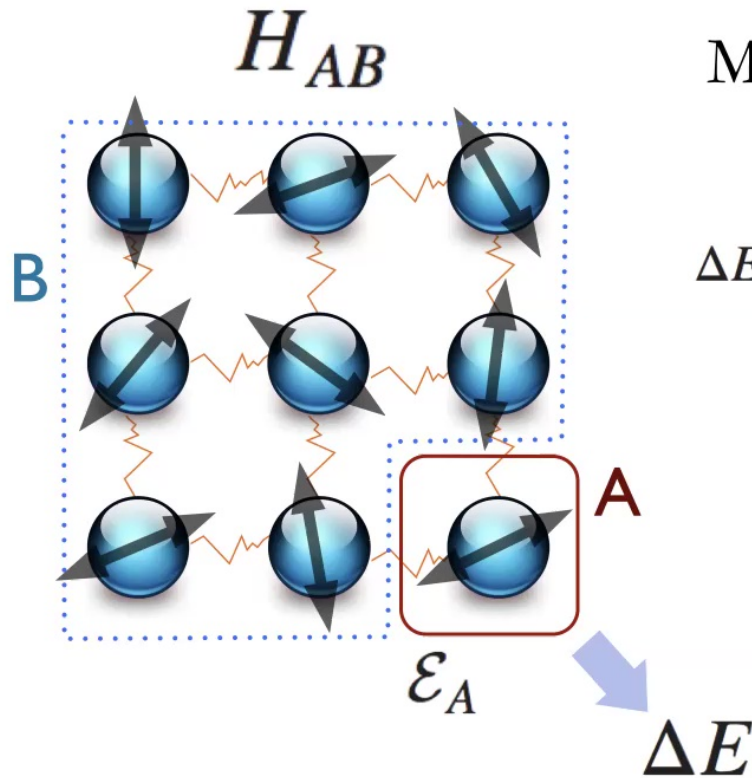
How to fully characterize this strong-local passivity?



Maximum extractable energy under general local operations on A:

$$\Delta E_{(A)B} := \min_{\mathcal{E}_A} \text{Tr}[H_{AB}(\mathcal{E}_A \otimes \mathcal{I}_B)\rho_{AB}] - \text{Tr}[H_{AB}\rho_{AB}]$$

How to fully characterize this strong-local passivity?



Maximum extractable energy under general local operations on A:

$$\Delta E_{(A)B} := \min_{\mathcal{E}_A} \text{Tr}[H_{AB}(\mathcal{E}_A \otimes \mathcal{I}_B)\rho_{AB}] - \text{Tr}[H_{AB}\rho_{AB}]$$

When is it a passive state?

$$\Delta E_{(A)B} = \Delta E_{(A)B}^{\mathcal{I}_A} = 0$$

The problem of finding the conditions for the CP-local passivity becomes an optimization problem over all \mathcal{E}_A , such that for the pair $\{\rho_{AB}, H_{AB}\}$ satisfies

$$\Delta E_{(A)B} = \Delta E_{(A)B}^{\mathcal{I}_A} = 0$$

Semi-definite programming

$$\alpha = \inf \{ \text{Tr}(CX) : \Phi(X) = B, X \geq 0 \}$$

↳ Allowing us to obtain a theorem with the necessary and sufficient conditions

Corollary: For thermal states

If the ground state is entangled max-rank with respect to A and B and not degenerate.

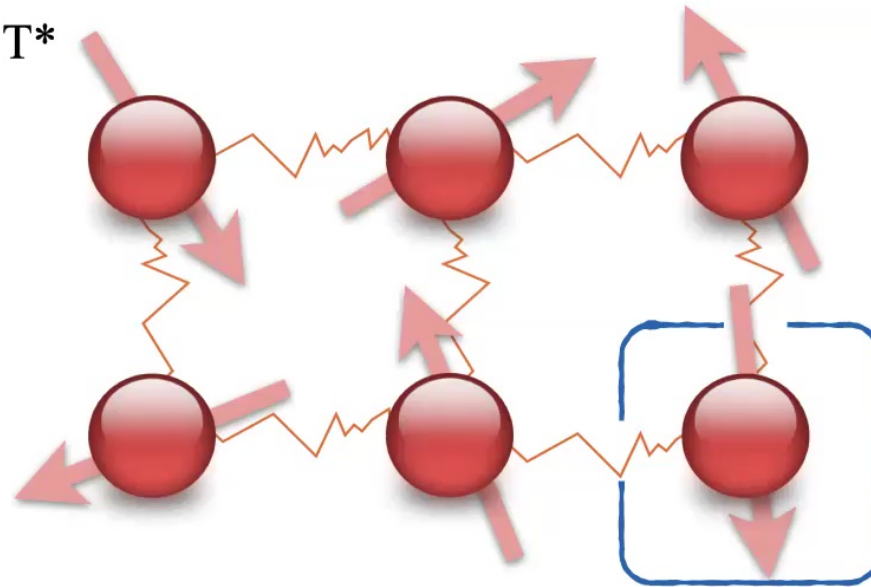


$\exists T_* > 0$, such that if $T \leq T_*$,

the system state is strong-local passive

Corollary: For thermal states

Hot system at
temperature $T < T^*$

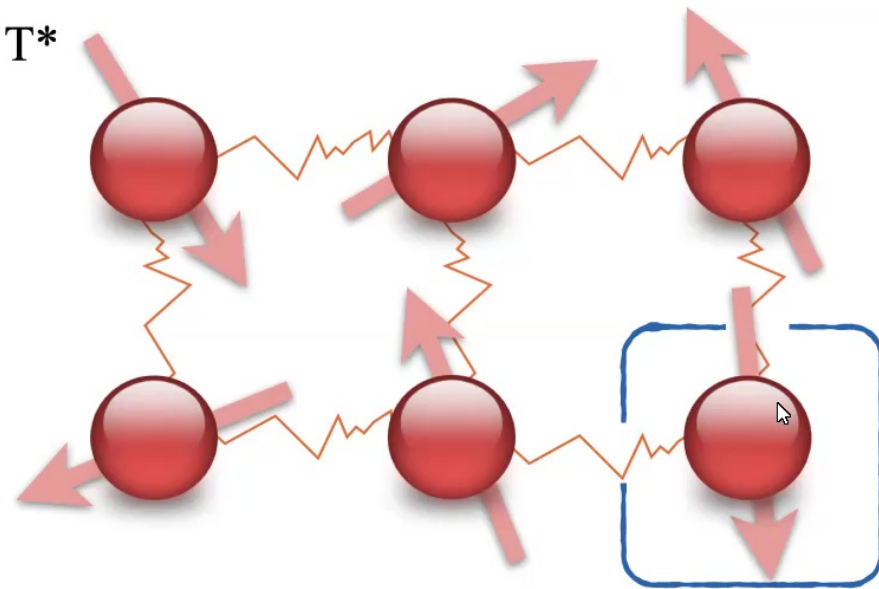


A Alhambra, G Styliaris, [NA Rodriguez Briones](#), J Sikora, Martín-Martínez. **Phys Rev Lett.** 123 (19), 190601 (2019)

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Corollary: For thermal states

Hot system at
temperature $T < T^*$



Strong point contact
in a short duration



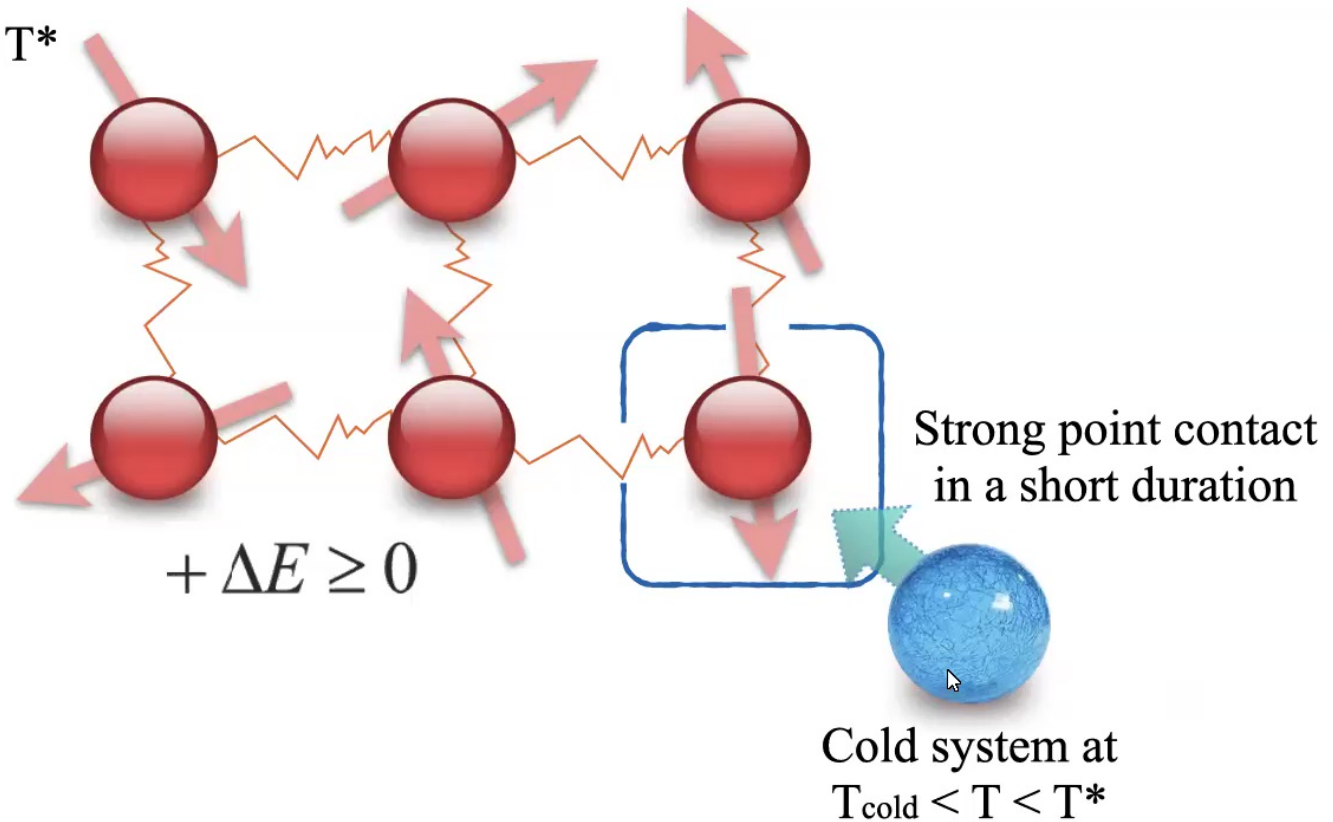
Cold system at
 $T_{\text{cold}} < T < T^*$

A Alhambra, G Styliaris, [NA Rodriguez Briones](#), J Sikora, Martín-Martínez. **Phys Rev Lett.** 123 (19), 190601 (2019)

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Corollary: For thermal states

Hot system at
temperature $T < T^*$



$$+ \Delta E \geq 0$$

$$- \Delta E \leq 0$$

A Alhambra, G Styliaris, [NA Rodriguez Briones](#), J Sikora, Martín-Martínez. **Phys Rev Lett.** 123 (19), 190601 (2019)

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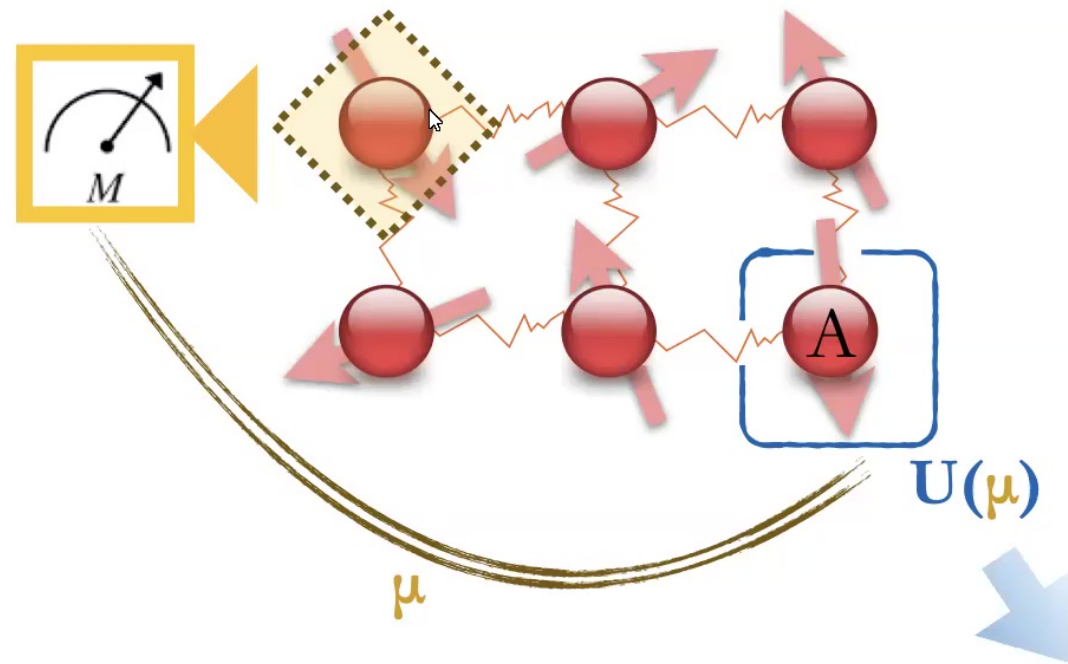
Bad news for local cooling of
interacting systems?

How can we break strong local passivity? and taking advantage of pre-existing quantum correlations

Using an idea from quantum field theory, a method called **quantum energy teleportation (QET)** to consume correlations to activate energy extraction locally by means of local operations and classical communication (LOCC).



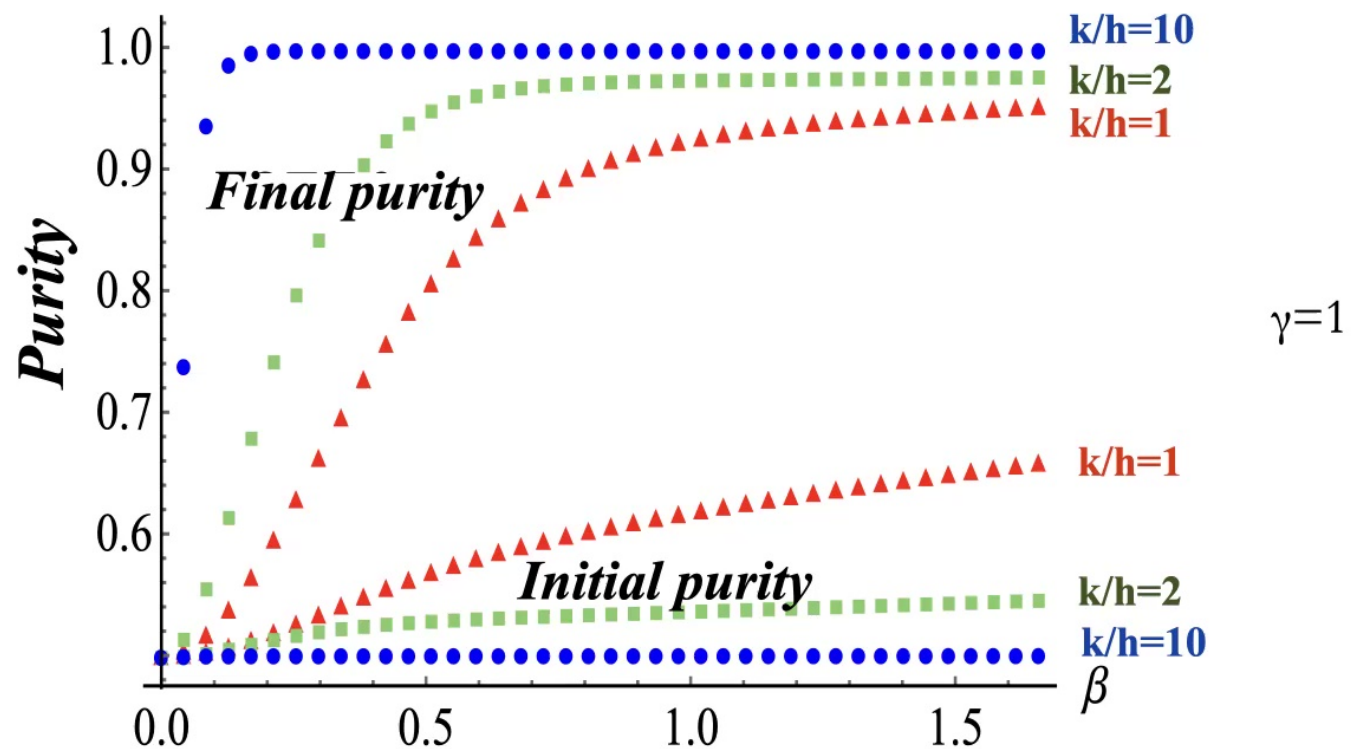
Breaking Strong-Local Passivity with QET



- **Step I.** Local measurement of a distant subsystem, with operators that commute with the interaction Hamiltonians
- **Step II.** Communicate the outcome to subsystem A.
- **Step III.** Based on the outcome, choose a unitary operation.

Results for Minimal QET

$$\mathbf{H} = \kappa \left(\frac{1+\gamma}{2} \sigma_1^x \sigma_2^x + \frac{1-\gamma}{2} \sigma_1^y \sigma_2^y \right) + \sigma_1^z + \sigma_2^z$$



N. Rodríguez-Briones, E. Martín-Martínez, A. Kempf, R. Laflamme Phys. Rev. Lett. 119 (5), 050502

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Conclusions

Correlations play an important role in improving algorithmic cooling methods

We circumvent previous limits by using correlations:

- We take advantage of correlations that can be created during the re-thermalization step with the heat-bath.
- We use correlations present in the initial state induced by the internal interactions of the system.

Furthermore, we have fully characterized the strong-local passivity (necessary and sufficient conditions)

[1] [NA Rodríguez-Briones](#), R Laflamme. **PRL** 116 (17), 170501, (2016)

[4] [NA Rodríguez-Briones](#), E Martín-Martínez, A Kempf, R Laflamme. **PRL** 119 (5), 050502

[5] A Alhambra, G Styliaris, [NA Rodríguez Briones](#), J Sikora, E. Martín Martínez, **PRL** 123 (19), 190601 (2019)

[3] [NA Rodríguez-Briones](#), J Li, X Peng, T Mor, Y Weinstein, R Laflamme. **NJP** 19 (11), 113047

[2] DK Park, [NA Rodríguez-Briones](#), G Feng, R Rahimi, J Baugh, R Laflamme. Book chapter at Electron Spin Resonance (ESR) Based Quantum Computing, 227-255

Future Research Program

Goal: To use quantum information theory to elucidate central open questions about quantum thermalization phenomena and information dynamics.

Specifically:

- To derive new tools by combining resource theories and the tools from heat-bath algorithmic cooling
 - Explore the influence of quantum effects on the flow of energy and information
 - Gain insights into the nature of thermalization phenomena vs many-body localization and their phase transitions
 - Strong local passivity life times and LOCC to study the energy flows in highly correlated systems
- Apply the new results to create a new class of state preparation protocols
- Propose and explore new perspectives to tackle open questions in other fields of physics related to the transfer of information/energy in correlated systems. Examples include the study of black-holes, quantum matter problems, among others.