Title: Ultralocality and the robustness of slow contraction to cosmic initial conditions

Speakers: Anna Ijjas

Series: Strong Gravity

Date: February 17, 2022 - 1:00 PM

URL: https://pirsa.org/22020063

Abstract: I will discuss the detailed process by which slow contraction smooths and flattens the universe using an improved numerical relativity code that accepts initial conditions with non-perturbative deviations from homogeneity and isotropy along two independent spatial directions. Contrary to common descriptions of the early universe, I will show that the geometry first rapidly converges to an inhomogeneous, spatially-curved, and anisotropic ultralocal state in which all spatial gradient contributions to the equations of motion decrease as an exponential in time to negligible values. This is followed by a second stage in which the geometry converges to a homogeneous, spatially flat, and isotropic spacetime. In particular, the decay appears to follow the same history whether the entire spacetime or only parts of it are smoothed by the end of slow contraction.

Zoom Link: https://pitp.zoom.us/j/95441238892?pwd=TUh4Mjh1MHJ6TDNCL0V1NUk5WWFZQT09

Ultralocality and the robustness of slow contraction to cosmic initial conditions

based on

arXiv: 2201.03752, 2109.09768, 2104.12293; 2103.00584; 2006.04999; 2006.01172

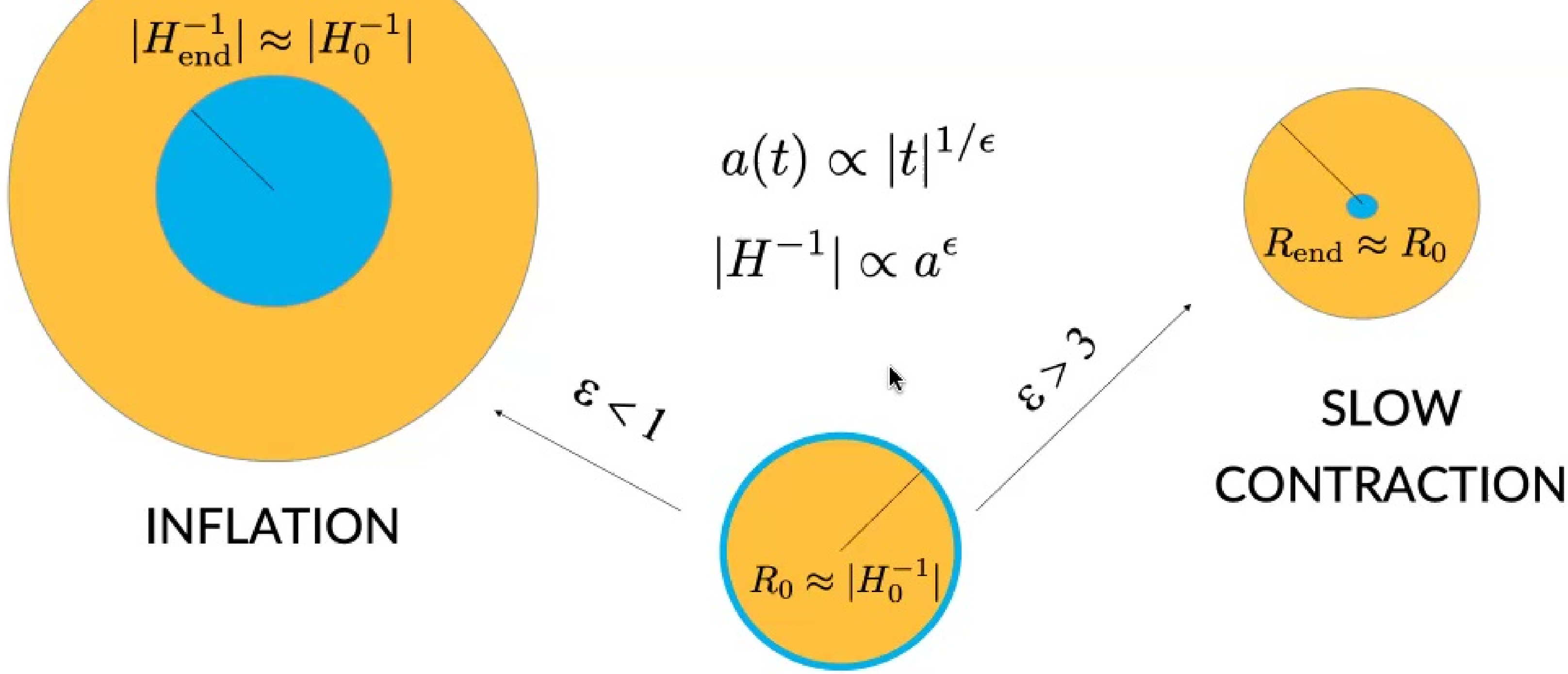
w/W.G. Cook, E. Y. Davies, I. Glushchenko, T. Kist, A.P. Sullivan

D. Garfinkle, F. Pretorius and P.J. Steinhardt

Anna Ijjas

NYU

Conventional Picture of Smoothing



$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3}\left(\frac{\rho_m^0}{a^3} + \frac{\rho_r^0}{a^4}\right) - \frac{k}{a^2} + \frac{\sigma^2}{a^6} + \frac{1}{3}\left(\frac{\rho_\phi^0}{a^{2\epsilon}}\right)$$

$$\epsilon = 3 \times \frac{\frac{1}{2}\dot{\phi}^2}{\frac{1}{2}\dot{\phi}^2 + V(\phi)}$$

$$R_{\alpha\beta} = T_{\alpha\beta} - \tfrac{1}{2} g_{\alpha\beta} T_{\lambda}{}^{\lambda}$$
$$\nabla^{\beta} T_{\alpha\beta} = 0$$
$$T_{\alpha\beta} \equiv \nabla_\alpha \phi \nabla_\beta \phi - \left(\tfrac{1}{2} \nabla_\lambda \phi \nabla^\lambda \phi + V_0 e^{-\phi/m} \right) g_{\alpha\beta}$$

$$\xrightarrow{\ ?\ }$$

$$3H^2 = \frac{1}{2}\dot{\phi}^2 + V_0 e^{-\phi/m}$$
$$-2\frac{\dot{H}}{H^2} = \frac{\dot{\phi}^2}{H^2} = \frac{1}{m^2}$$

\`hard' problem of cosmic initial conditions:

How \`big' is the basin of attraction of the Friedmann-scalar ODE fixed point?

mathematical challenge + computational challenge

Orthonormal tetrad formulation

$$\{x_0, x_1, x_2, x_3\} \rightarrow \{e_0, e_1, e_2, e_3\}$$

metric: $e_\alpha \cdot e_\beta = \eta_{\alpha\beta}$

variables: ${e_\alpha}^\mu$, $\gamma_{\alpha\beta\delta} \equiv e_\alpha \cdot \nabla_\delta e_\beta$

To evolve Einstein-scalar PDE system:
choose appropriate tetrad frame & coordinates

frame gauge fixing:

- Fermi propagated axes
- hypersurface orthogonal timelike congruence

coordinate gauge fixing:

- zero shift + constant mean curvature slicing
- time parametrization: $e^t = \Theta$

Evolution scheme

$$
\begin{aligned}
\partial_t \bar{E}_a{}^i &= -\Big(\mathcal{N}-1\Big)\bar{E}_a{}^i - \mathcal{N}\,\bar{\Sigma}_a{}^b\bar{E}_b{}^i,\\
\partial_t \bar{\Sigma}_{ab} &= -\Big(3\mathcal{N}-1\Big)\bar{\Sigma}_{ab} - \mathcal{N}\Big(2\bar{n}_{\langle a}{}^c\bar{n}_{b\rangle c} - \bar{n}^c{}_c\bar{n}_{\langle ab\rangle} - \bar{S}_{\langle a}\bar{S}_{b\rangle}\Big) + \bar{E}_{\langle a}{}^i\partial_i\Big(\bar{E}_{b\rangle}{}^i\partial_i\mathcal{N}\Big)\\
&\quad - \mathcal{N}\Big(\bar{E}_{\langle a}{}^i\partial_i\bar{A}_{b\rangle} - \epsilon^{cd}{}_{(a}\Big(\bar{E}_c{}^i\partial_i\bar{n}_{b)d} - 2\bar{A}_c\bar{n}_{b)d}\Big)\Big) + \epsilon^{cd}{}_{(a}\bar{n}_{b)d}\bar{E}_c{}^i\partial_i\mathcal{N} + \bar{A}_{\langle a}\bar{E}_{b\rangle}{}^i\partial_i\mathcal{N},\\
\partial_t \bar{n}_{ab} &= -\Big(\mathcal{N}-1\Big)\bar{n}_{ab} + \mathcal{N}\Big(2\bar{n}_{(a}{}^c\bar{\Sigma}_{b)c} - \epsilon^{cd}{}_{(a}\bar{E}_c{}^i\partial_i\bar{\Sigma}_{b)d}\Big) - \epsilon^{cd}{}_{(a}\bar{\Sigma}_{b)d}\bar{E}_c{}^i\partial_i\mathcal{N},\\
\partial_t \bar{A}_a &= -\Big(\mathcal{N}-1\Big)\bar{A}_a - \mathcal{N}\Big(\bar{\Sigma}_a{}^b\bar{A}_b - \tfrac{1}{2}\bar{E}_b{}^i\partial_i\bar{\Sigma}_a{}^b\Big) - \bar{E}_a{}^i\partial_i\mathcal{N} + \tfrac{1}{2}\bar{\Sigma}_a{}^b\bar{E}_b{}^i\partial_i\mathcal{N},\\
\partial_t \phi &= \mathcal{N}\,\bar{W},\\
\partial_t \bar{W} &= -\Big(3\mathcal{N}-1\Big)\bar{W} - \mathcal{N}\Big(\bar{V}_{,\phi} + 2\bar{A}^a\bar{S}_a - \bar{E}_a{}^i\partial_i\bar{S}^a\Big) + \bar{S}^a\bar{E}_a{}^i\partial_i\mathcal{N},\\
\partial_t \bar{S}_a &= -\Big(\mathcal{N}-1\Big)\bar{S}_a - \mathcal{N}\Big(\bar{\Sigma}_a{}^b\bar{S}_b - \bar{E}_a{}^i\partial_i\bar{W}\Big) + \bar{W}\bar{E}_a{}^i\partial_i\mathcal{N},
\end{aligned}
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Evolution scheme

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\end{aligned}
$$

$$
-\bar{E}^a{}_i\partial^i\left(\bar{E}_a{}^j\partial_j\mathcal{N}\right) + 2\bar{A}^a\bar{E}_a{}^i\partial_i\mathcal{N} + \mathcal{N}\left(3 + \bar{\Sigma}_{ab}\bar{\Sigma}^{ab} + \bar{W}^2 - \bar{V}\right) = 3
$$

Initial data

on t_0-hypersurface with CMC $1/\Theta_0$:

geometric variables

$$\{\bar{E}_a{}^i, \bar{n}_{ab}, \bar{A}_a, \bar{\Sigma}_{ab}\}$$

fixed by choosing

$$g_{ij} \equiv \psi^4(x,t_0)\delta_{ij}$$

fixed by Hamiltonian constraint

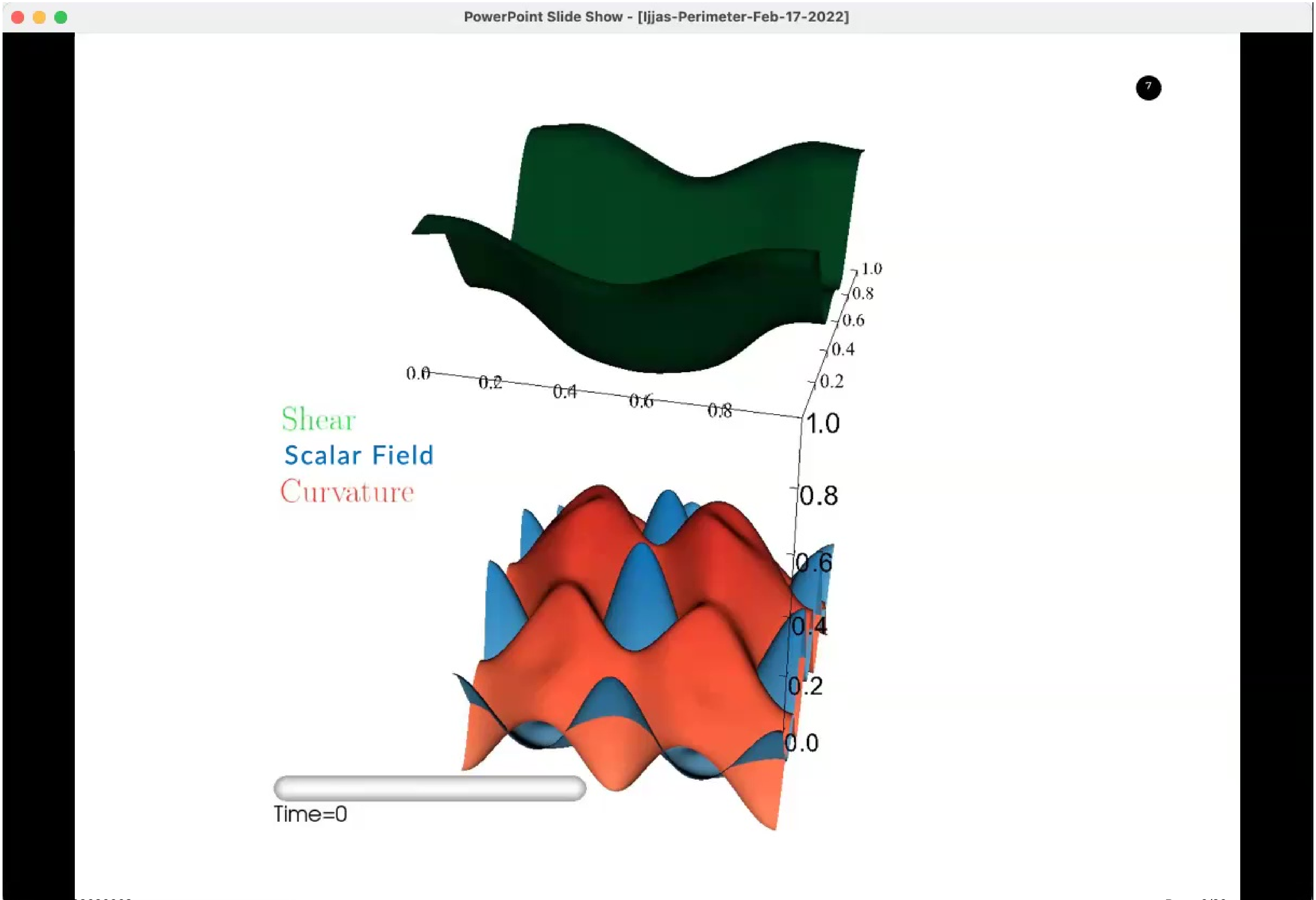
freely chosen vacuum contribution, `rest' fixed by momentum constraint

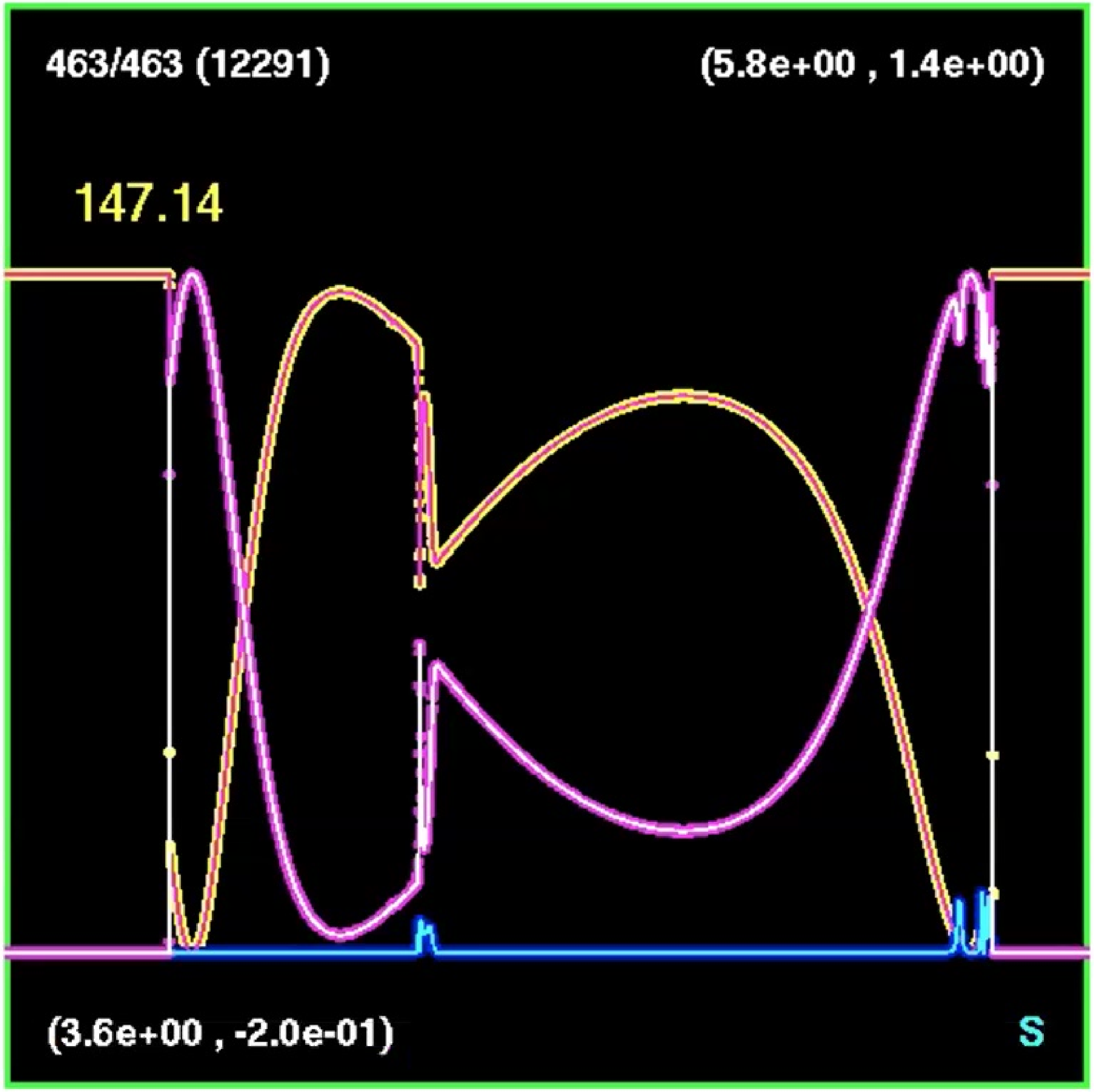
scalar-field variables

$$\{\phi(x,t_0), Q(x,t_0)\}$$

freely chosen

our numerical scheme allows for varying ALL freely specifiable variables


PowerPoint Slide Show - [Ijjas-Perimeter-Feb-17-2022]
7
Shear
Scalar Field
Curvature
0.0
0.2
0.4
0.6
0.8
1.0
Time=0


463/463 (12291)
(5.8e+00 , 1.4e+00)
147.14
(3.6e+00 , -2.0e-01)
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Ultralocality

arXiv: 2201.03752, 2103.00584

$$\partial_t \bar{n}_{ab} = -\Big(\mathcal{N}-1\Big)\bar{n}_{ab} + \mathcal{N}\Big(2\bar{n}_{(a}{}^{c}\bar{\Sigma}_{b)c} - \epsilon^{cd}{}_{(a}\bar{E}_{c}{}^{i}\partial_i\bar{\Sigma}_{b)d}\Big) - \epsilon^{cd}{}_{(a}\bar{\Sigma}_{b)d}\bar{E}_{c}{}^{i}\partial_i\mathcal{N}$$

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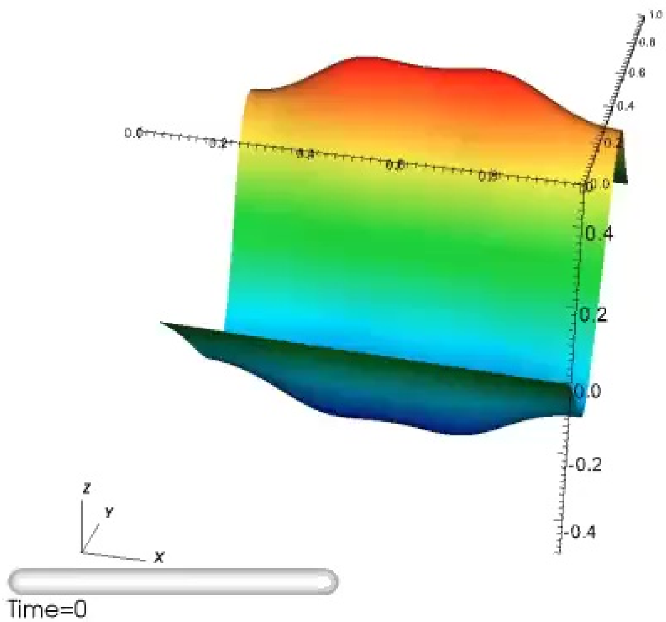

r.h.s. – all terms

Ultralocality

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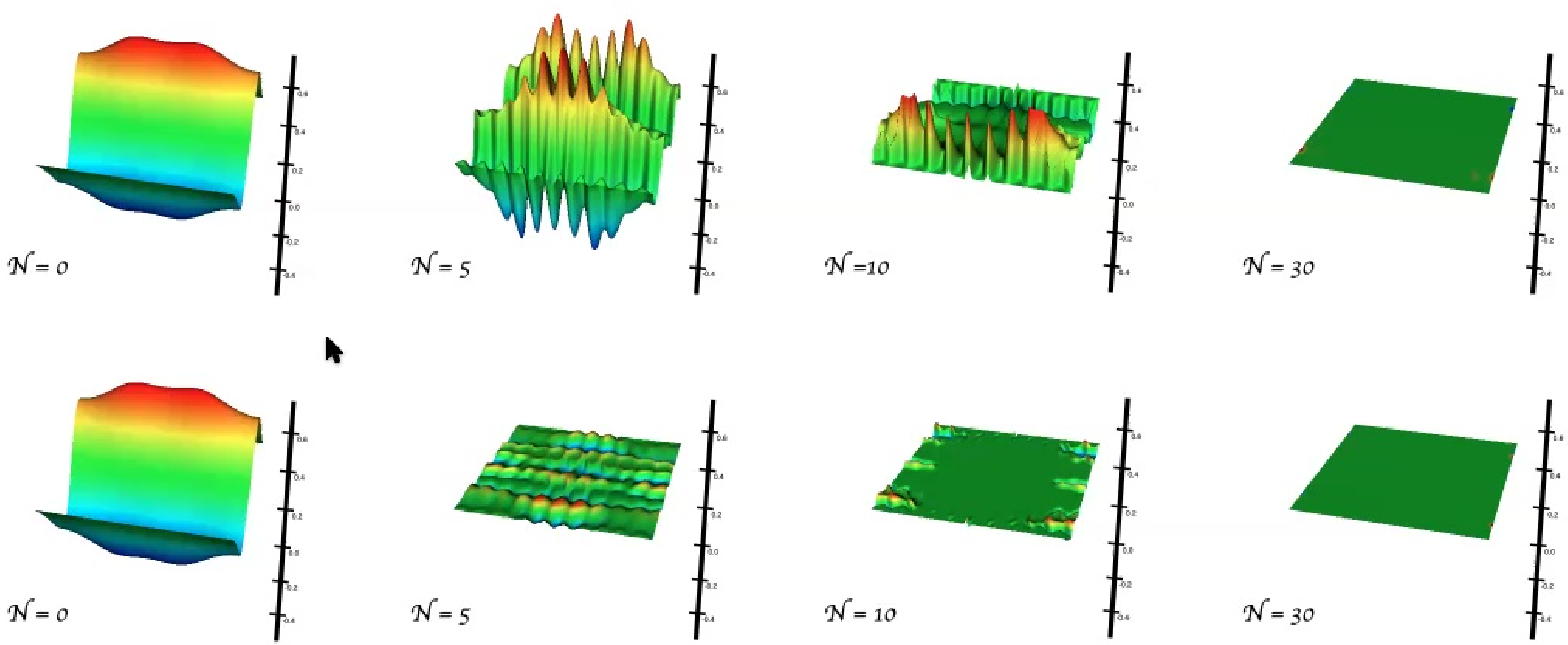

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Ultralocality

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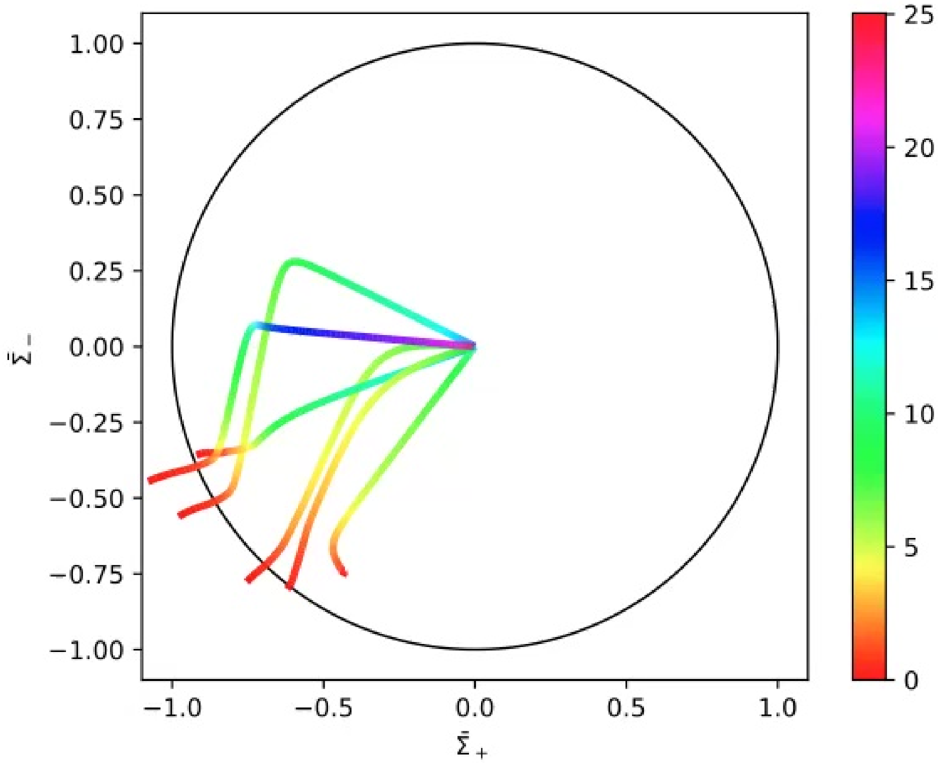

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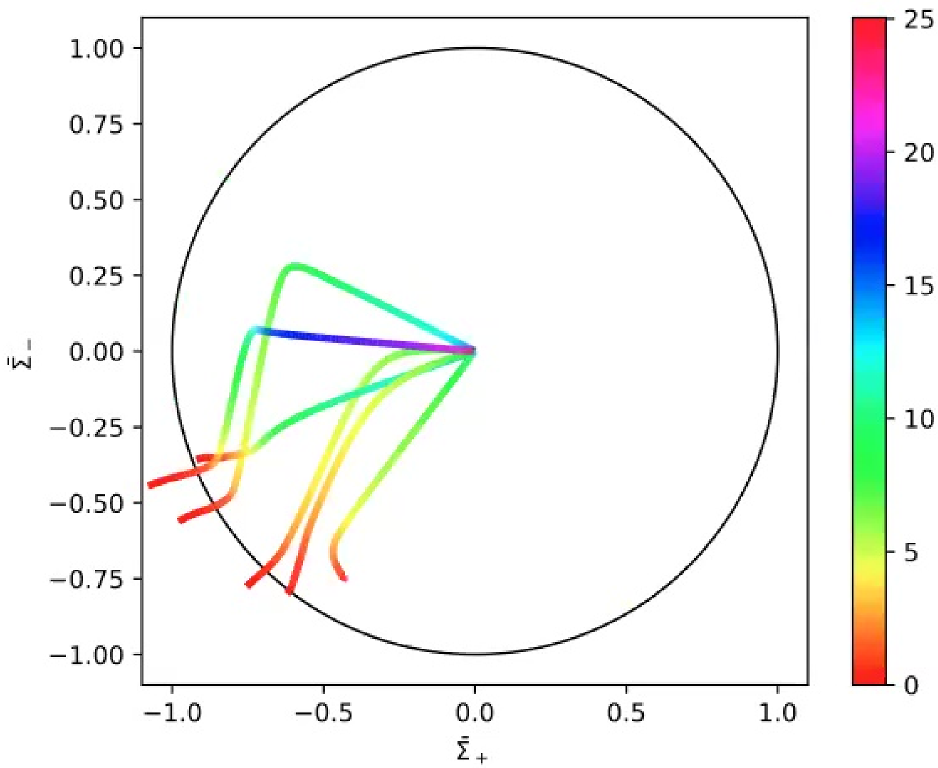

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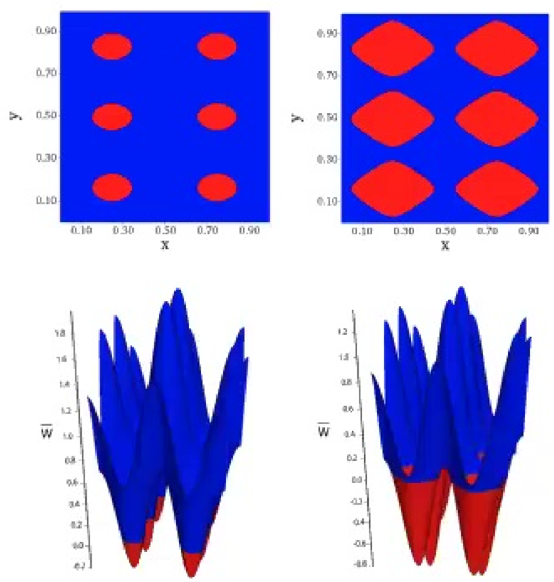

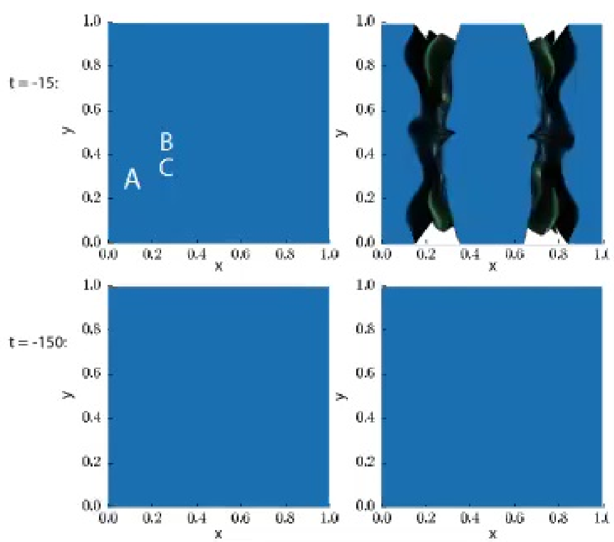

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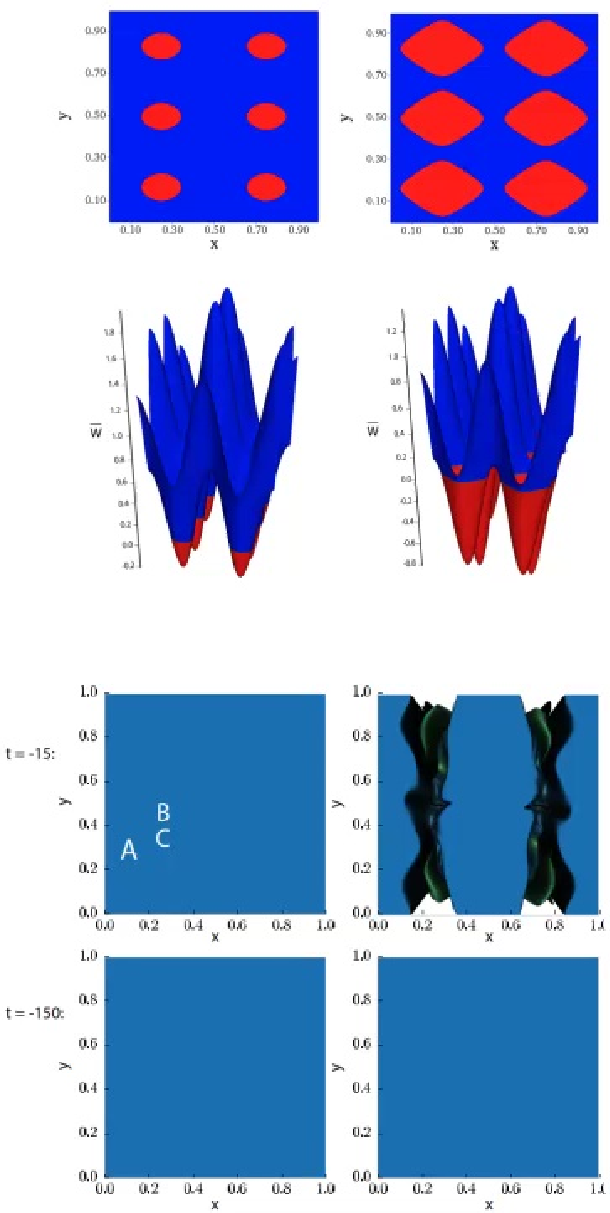

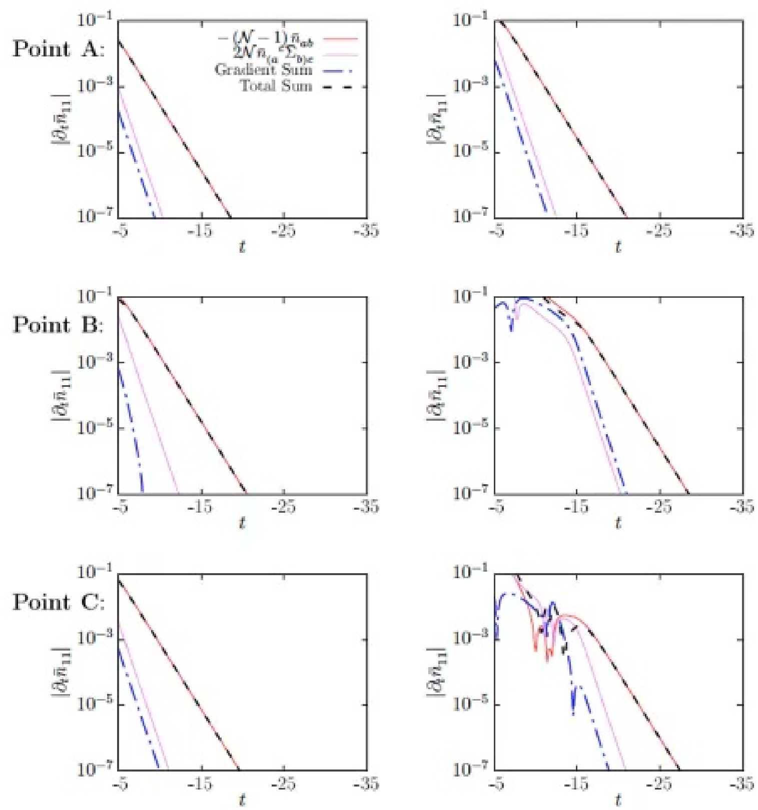

from analytics to computation & back

- numerical `experiments' yield evidence for ultralocality
- analytics: scaling FRW solution is `stable' attractor in the ultralocal limit and has a large basin of attraction

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Key ideas of the analytic argument:

- work in ultra-local limit as established by numerical analysis

13

Key ideas of the analytic argument:

- work in ultra-local limit as established by numerical analysis
- notice: $\epsilon_a{}^{bc}\bar{n}_b{}^d\bar{\Sigma}_{cd} = 0,$
  in particular, remaining geometric variables commute and share eigenvalues
- show: eigenvectors at t_0 remain eigenvectors at any later time
- show: only `stable' stationary points of the eigenvalue dynamics are FRW and Kasner-like

$\mathcal{N}$	σ_1	σ_2	ν_1	ν_2	ν_3	$\frac{1}{2}\bar{W}^2$	$\bar{V}$
$\frac{1}{\varepsilon}$	0	0	0	0	0	$\varepsilon > 0$	$3-\varepsilon$
$\frac{1}{3}$	$\neq 0$	$\neq 0$	0	0	0	$3-(\sigma_1^2+\sigma_1\sigma_2+\sigma_2^2)$	0
$\frac{1}{3}$	-1	-1	$\neq 0$	ν_1	0	0	0
1	0	0	$\pm 2\sqrt{\frac{1}{\varepsilon}-1}$	ν_1	ν_1	$\frac{1}{\varepsilon} > 1$	$\bar{W}^2$
1	0	0	$\neq 0$	0	ν_1	$\frac{1}{2}\varepsilon = \frac{1}{2}$	$\bar{W}^2$
$\frac{\varepsilon+8}{9\varepsilon}$	$\frac{4-4\varepsilon}{\varepsilon+8}$	$\frac{2\varepsilon-2}{\varepsilon+8}$	$\pm 6\frac{\sqrt{(1-\varepsilon)(\varepsilon-4)}}{\varepsilon+8}$	0	0	$1 < \frac{81\varepsilon}{(\varepsilon+8)^2} < 4$	$\frac{54\cdot(4-\varepsilon)}{(\varepsilon+8)^2}$
$\frac{\varepsilon+2}{3\varepsilon} < 1$	$\frac{1-\varepsilon}{2+\varepsilon}$	$\frac{2\varepsilon-2}{2+\varepsilon}$	$\pm 3\frac{\sqrt{\varepsilon-1}}{\varepsilon+2}$	0	$-\nu_1$	$\frac{9\varepsilon}{(2+\varepsilon)^2}$	$\frac{18\varepsilon}{(2+\varepsilon)^2}$

For the FRW fixed point, the basin of attraction is large.

arXiv: 2006.04999; 2006.01172

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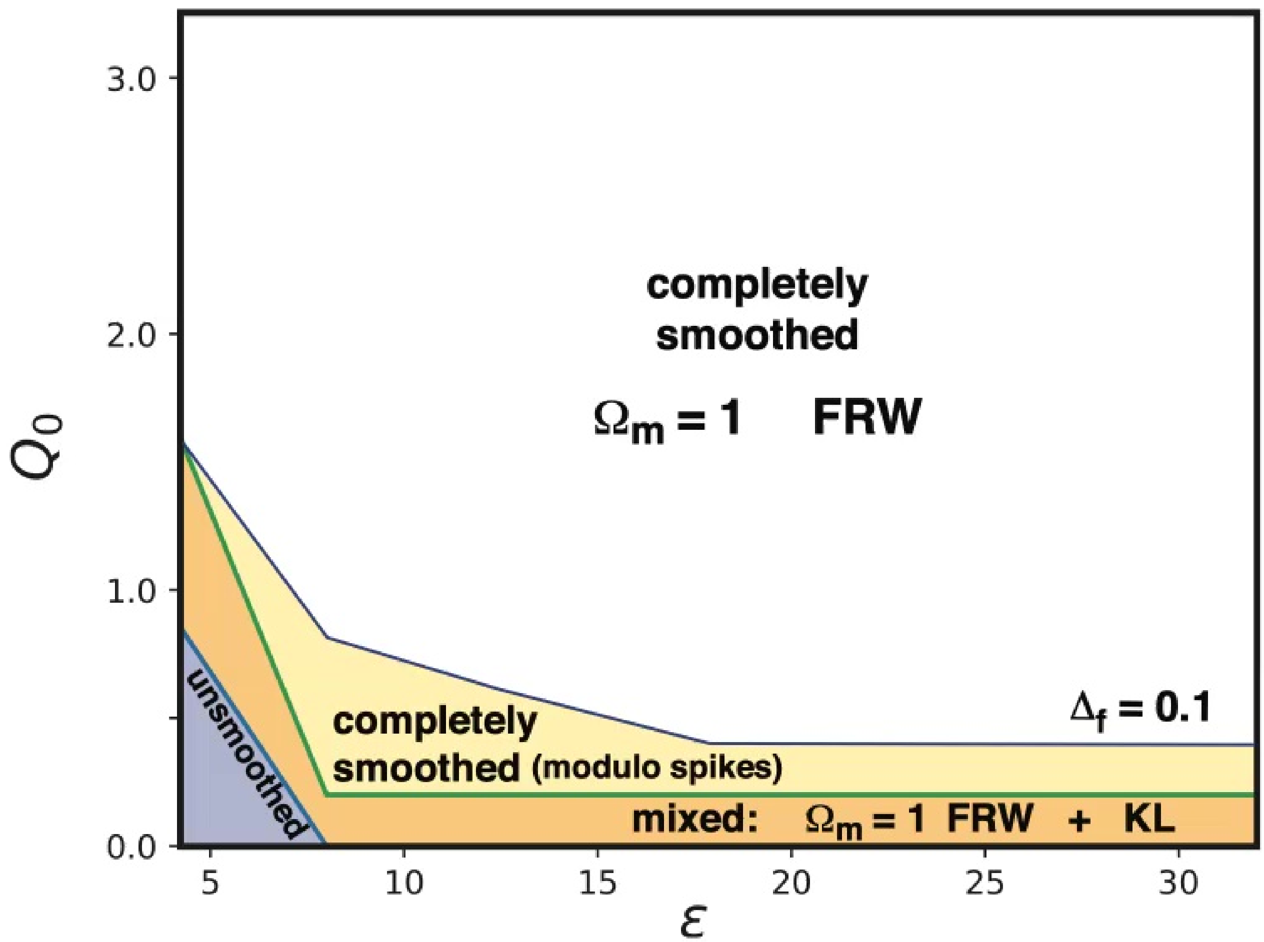
FRW fixed point has a large basin of attraction

arXiv: 2006.04999; 2006.01172



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For the Kasner-like fixed point,
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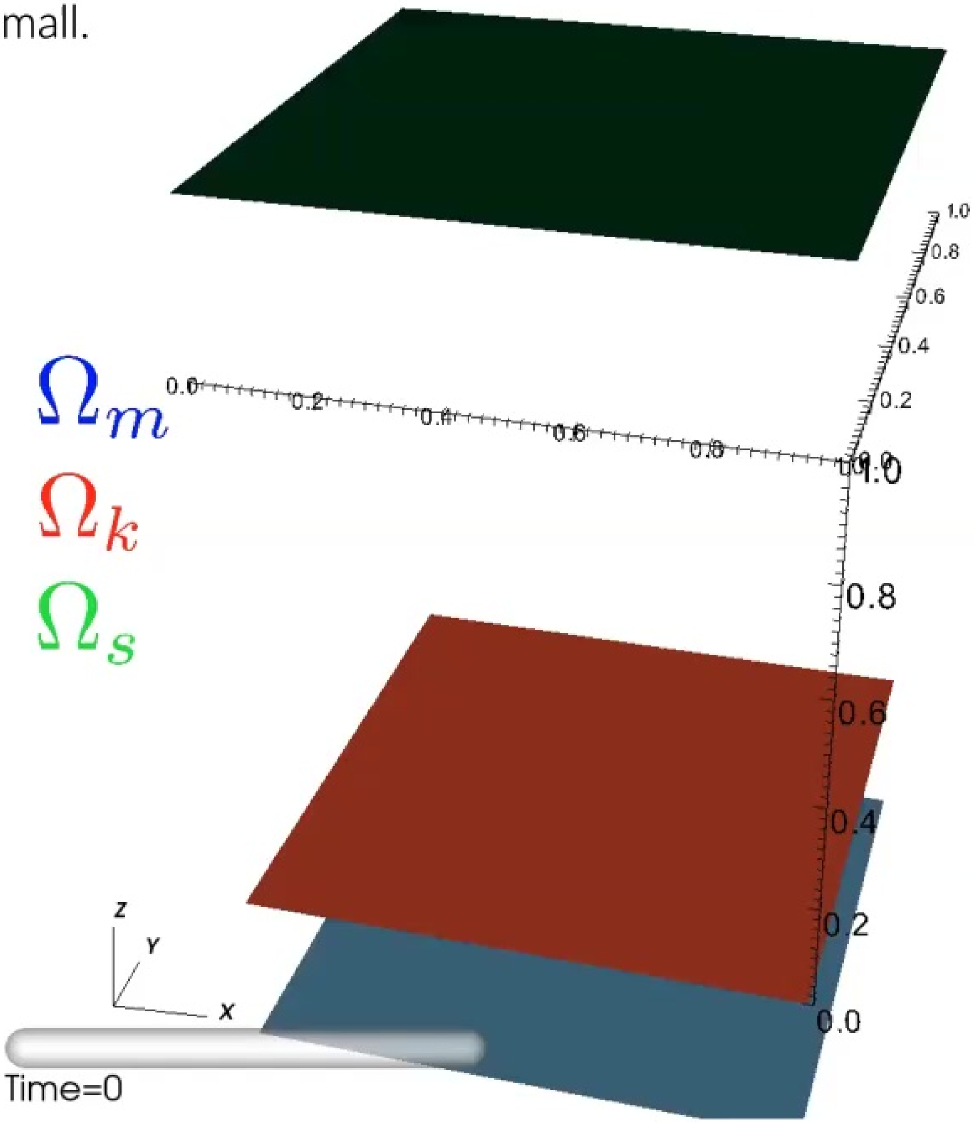
arXiv: 2104.12293



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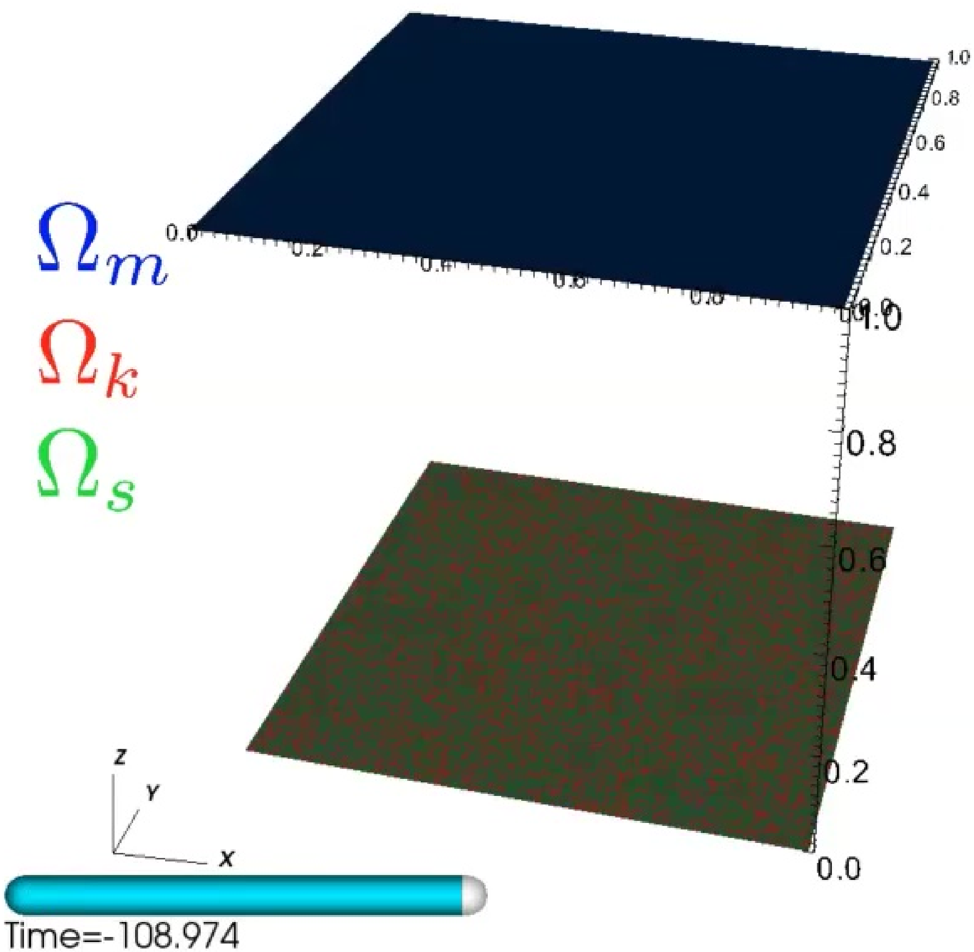

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corollary:

common descriptions of the early universe do not capture the true dynamics during smoothing slow contraction

Spin-Offs & Future Directions

Primordial Perturbations

arXiv: 2109.09768; 2102.03818; 2012.08249

Cyclic Cosmologies

arXiv: 1904.08022

Singularity Problem

arXiv: 1809.07010; 1710.05990

Dark Energy

arXiv: 2201.07704

Black Hole Physics

arXiv: 2108.07101

Key ideas of the analytic argument:

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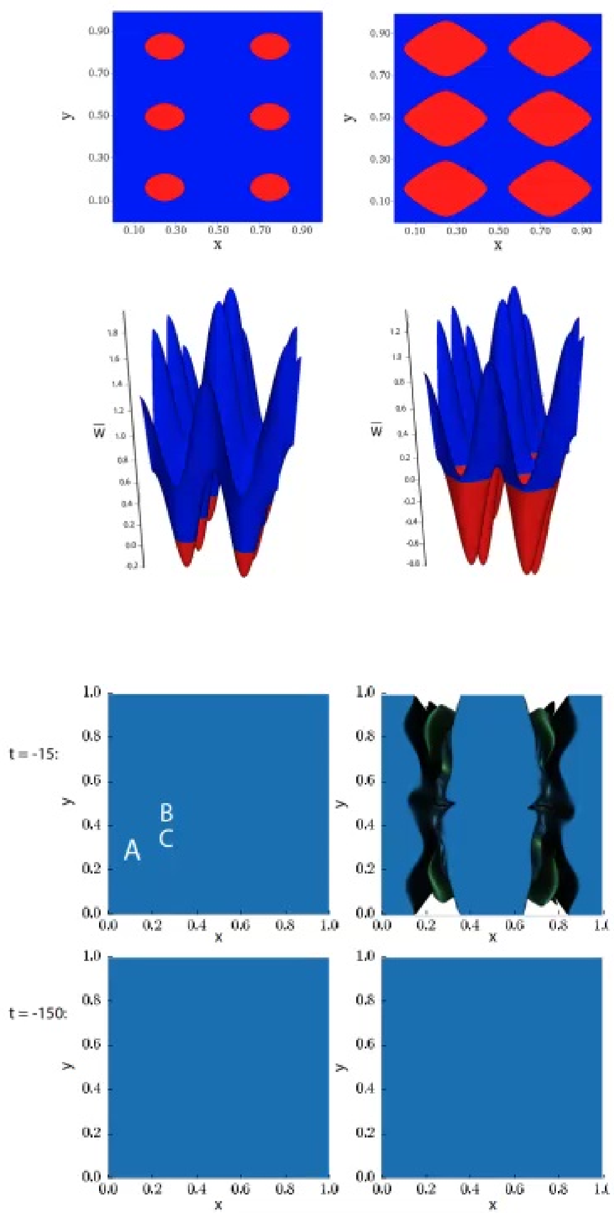

Ultralocality

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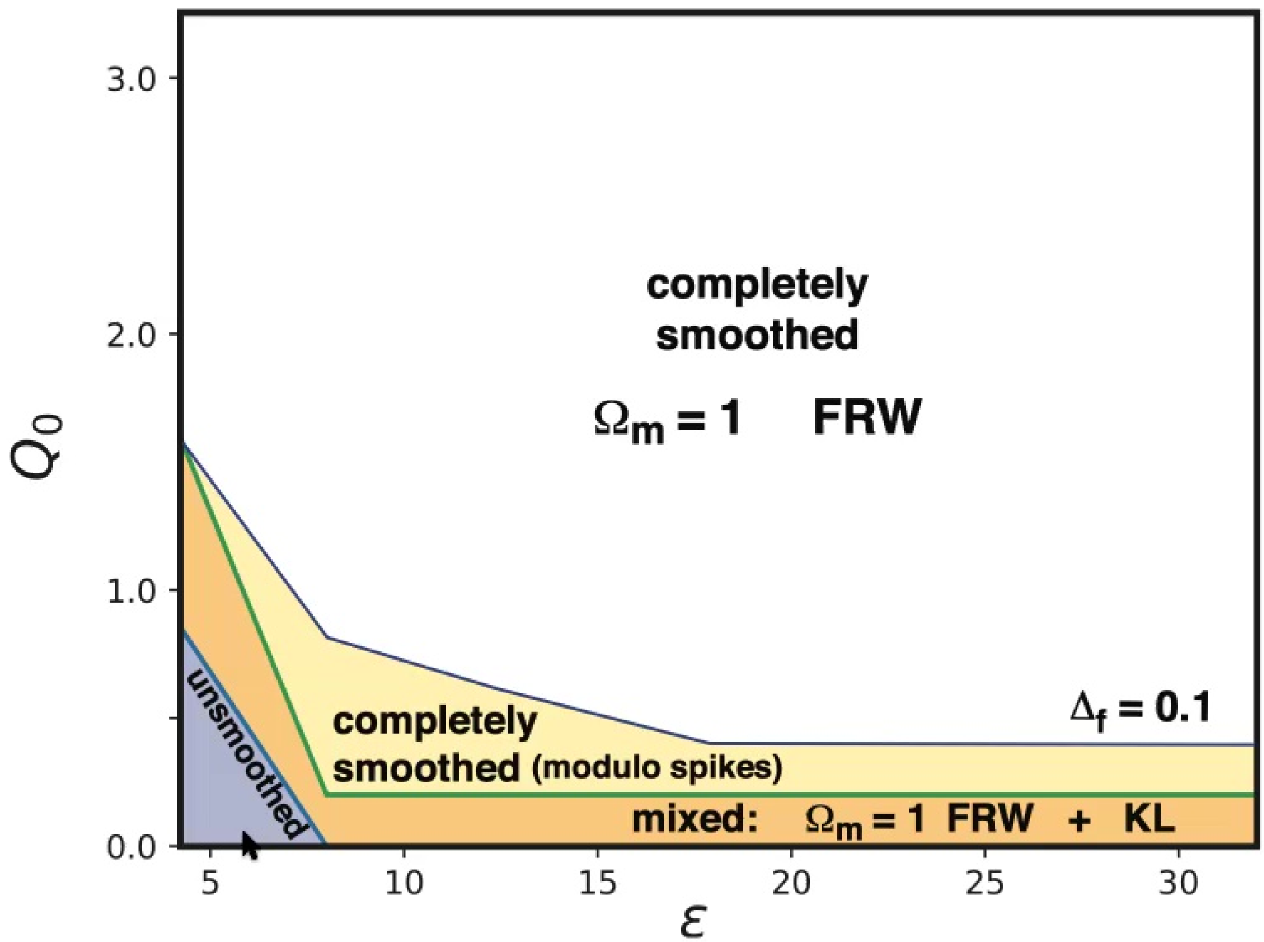

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Ultralocality

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