Title: Comments on Euclidean wormholes and holography

Speakers: Panagiotis Betzios

Series: Quantum Fields and Strings

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Abstract: Euclidean wormholes are exotic types of gravitational solutions that we still don't understand completely. In the first part of the talk, I will analyze asymptotically AdS wormhole solutions from a gravitational point of view. By studying correlation functions of local and non-local operators, the universal properties that any putative holographic dual should exhibit, become manifest. In the second part, I will describe some concrete field theoretic models (both effective and microscopic) that share these properties.

Euclidean Wormholes and Interacting Systems

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arXiv:1903.05658 and arXiv:2110.14655
+ in progress

Perimeter Institute

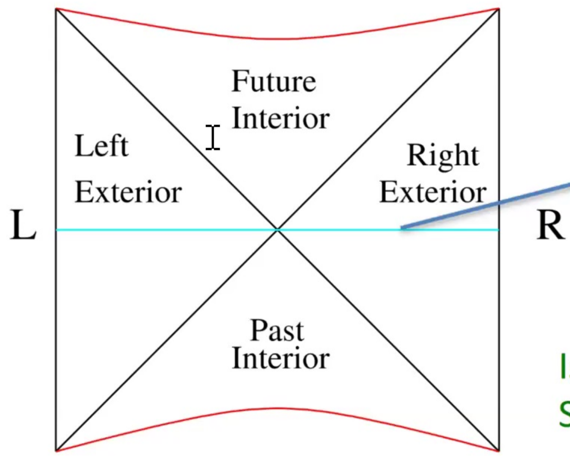
15 February 2022

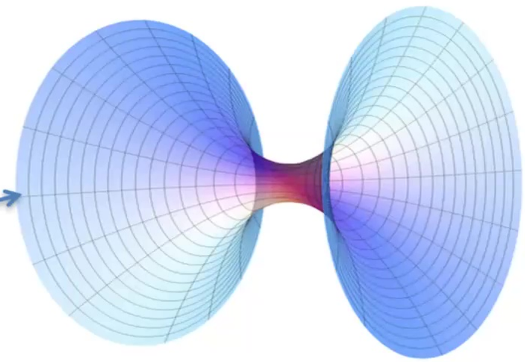
Why to study Euclidean Wormholes

Wormholes are interesting exotic solutions of GR + matter

- Motivation
  - Understand Holography in the presence of multiple boundaries
  - Universal properties of the dual QFT description?
  - Is there a microscopic model that can describe such geometries?
  - Can we understand string theory on target space wormhole backgrounds?
- Clear distinction
  - Lorentzian vs Euclidean
  - Macroscopic vs. Microscopic "gas of wormholes" (α-parameters)
  - Different characteristic scales
    $L_P \ll L_W \sim L_{AdS}$ vs. $L_P \leq L_W \ll L_{AdS}$
- Plan of the talk
  - Review
  - 1st Part: Bulk gravitational perspective
  - 2nd Part: Field theory models
  - Summary and future directions

Lorentzian wormholes or "ER = EPR"





Israel Maldacena
Susskind

- Wormhole = Einstein Podolski Rosen pair of two black holes in a particular entangled state of two non-interacting QFT's:

$$|\Psi\rangle = \sum_n e^{-\beta E_n/2} |E_n\rangle_L^{CPT} \times |E_n\rangle_R$$

- Large amounts of entanglement can give rise

to a geometric connection!

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Shapes

Lorentzian Wormholes & Holographic comments

- Einstein - Rosen Bridge: Connects the two sides of the eternal black hole
- Holography $\Rightarrow$ interpret such a geometry as describing a pair of decoupled but entangled QFT's
- We cannot communicate a message between the two sides
- The two boundaries are always separated by a horizon, once the null energy condition is assumed for the gravitational theory
- Upon analytically continuing to Euclidean signature the space smoothly caps off disconnecting the different asymptotic boundaries
- Entanglement cannot keep the Euclidean space connected!
- Traversable Wormholes: Lorentzian signature solutions for which the null energy condition is violated $\Rightarrow$ Signals can pass through the wormhole
- Local interactions that couple the two boundaries $\int d^d x \mathcal{O}_L(x) \mathcal{O}_R(x)$

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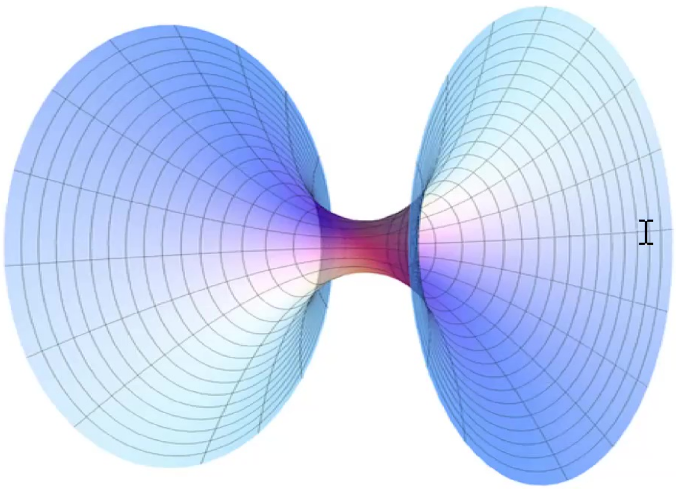
Euclidean Wormholes - "Entanglement is not enough!"



- Euclidean "spacetime" Wormholes: Not! slices of Lorentzian manifolds
- Analogous to instantons or bounces
- To have such solutions, one needs locally negative Euclidean "Energy" to support the throat from collapsing
- Such energy can be provided by axions or "magnetic" fluxes etc...

Holographic comments [PB - Kiritsis - Papadoulaki (2019)]

- Euclidean geometric connection $\Rightarrow$ Entanglement is irrelevant
- Global symmetries for the boundary theories? $\leftrightarrow$ A common Bulk "Gauss Law constraint"
- Naively: different QFTs on $\partial\mathcal{M} = \cup_i \partial\mathcal{M}_i \Rightarrow$ Cross-correlations factorise
- Common Bulk dictates otherwise $\Rightarrow$ Some form of interaction?

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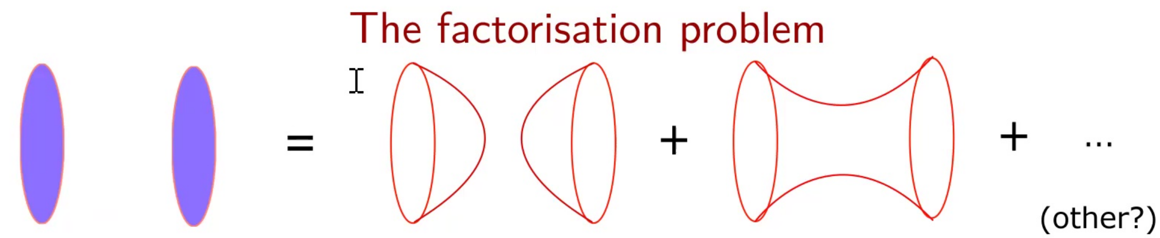
The factorisation problem

[Maldacena - Maoz ...]

The problem that $Z(J_1, J_2) \neq Z_1(J_1)Z_2(J_2)$ is :



Several ideas:

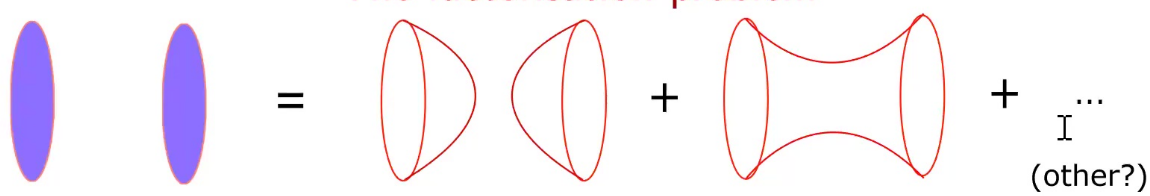
- The bulk QGR path integral corresponds to an average over QFT's: $\langle Z(J_1)Z(J_2)\rangle$ ⇒ average over what and how?
- Unitarity? , higher d canonical example of $\mathcal{N} = 4$ SYM / IIB ?
- An approximate statistical averaging (ETH - Chaos): ⇒ "Statistical wormholes" (above the BH threshold)
- After summing over bulk topologies and other non-perturbative states the correlators factorise ? ⇒ Need non-perturbative info

The factorisation problem

[Maldacena - Maoz ...]

The problem that $Z(J_1, J_2) \neq Z_1(J_1)Z_2(J_2)$ is :

The factorisation problem



Straightforward resolution:

- Interactions between QFT's $\Rightarrow$ subtle properties...
  "How come a pair of boundaries and not a single one?"
- Could the partition function be of the form (S some "sector")

$$Z(J_1, J_2) = \sum_S e^{w(S)} Z_S^{(QFT1)}(J_1) Z_S^{(QFT2)}(J_2)$$

  in a unitary setting?
- Cross correlators $\Rightarrow$ averages of lower point correlators

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Part I : Bulk perspective

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Types of solutions

[PB - Kiritsis - Papadoulaki] - 2019

We studied various Euclidean solutions with two asympotic AdS boundaries ("bottom up")

- Solutions of Einstein - Maxwell - Dilaton in two dimensions, where the throat of the wormhole is supported by the gauge field flux or Dilaton
- Solutions of Einstein - Dilaton theory in various dimensions with hyperbolic slicings, a running dilaton and a constant potential
- Solutions of Einstein - Yang - Mills with spherical slices in four dimensions, supported by the flux of the non abelian gauge field

- There are many more such solutions
- A recent paper where also stability has been studied is [Marolf - Santos], where they showed that a subset of such solutions are perturbatively stable $\Rightarrow$ one needs to take these solutions seriously...
- Hard to find a "clean" string theory embedding in $10/11d$...

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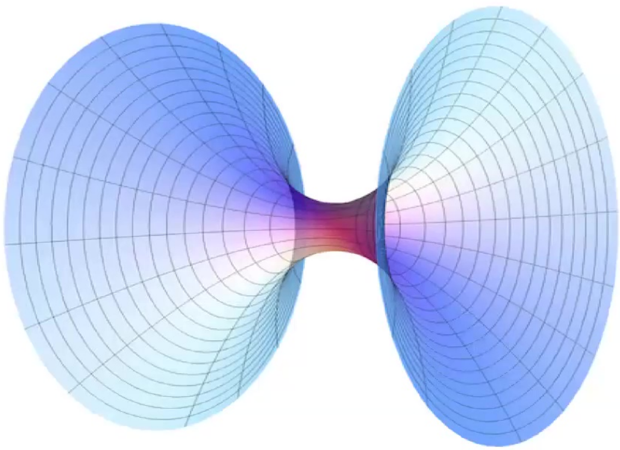
Local observables: Two boundary correlators



- To unravel the physics of Euclidean wormholes in holography we should study correlation functions
- To study correlators for local boundary operators $\mathcal{O}_1$, $\mathcal{O}_2$ $\Rightarrow$ Study the (2nd order) bulk fluctuation equation for the dual field ϕ

- We have two boundaries, where we can insert operators or sources
- The extra freedom provides for two types of correlation functions, one on a single boundary which we label by $\langle \mathcal{O}_1 \mathcal{O}_1 \rangle$ or $\langle \mathcal{O}_2 \mathcal{O}_2 \rangle$, and one cross-correlator across the two boundaries $\langle \mathcal{O}_1 \mathcal{O}_2 \rangle$

- $EAdS_2$ we can compute the correlators analytically, other cases: numerical results

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Prescription for Scalar Correlators

- For a probe scalar

$$S[\phi] = \frac{1}{2}\int_{\mathcal{M}} d^d x du \sqrt{g}\, \phi \left(-\Box + m^2\right)\phi - \frac{1}{2}\int_{\partial\mathcal{M}} d^d x \sqrt{\gamma}\, \phi\, \vec{n}\cdot\partial\phi$$

- The solution has two asymptotic falloffs $\sim u^{\Delta^i_\pm}$ near each of the boundaries, the boundary to Bulk (btB) propagators satisfy

$$(-\Box + m^2)K^i(u,x;x') = 0$$

$$K^i(u,x;x')|_{\partial\mathcal{M}_i} = \epsilon_i^{\Delta^i_-}\delta_{\epsilon_i}(x-x'), \quad K^i(u,x;x')|_{\partial\mathcal{M}_j} = 0\,, \quad \text{for}\, i \neq j$$

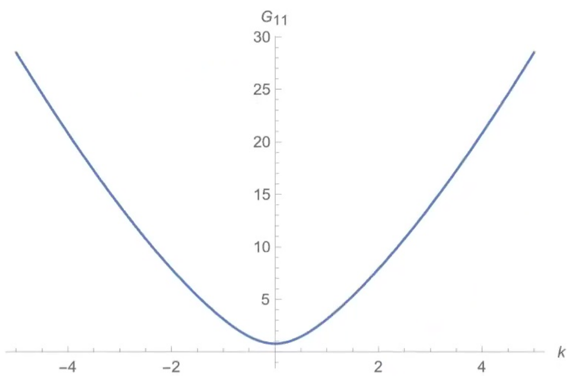
$$\phi(u,x) = \sum_i \int d^d x' K^i(u,x;x')\phi^s_i(x')$$

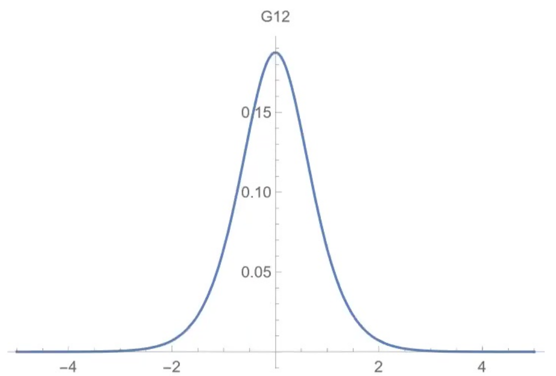
- The last formula gives the reconstruction of a bulk field from given sources ϕ^s_i at the boundaries
- Plugging it in the on-shell action we find

$$\langle\mathcal{O}_1\mathcal{O}_1\rangle = \left[u^{-d}u\partial_u f^1_k(u)\right]^{u=\epsilon}_{\partial\mathcal{M}_1}\,, \quad \langle\mathcal{O}_1\mathcal{O}_2\rangle = \left[u^{-d}u\partial_u f^2_k(u)\right]^{u=\epsilon}_{\partial\mathcal{M}_1}$$

with $f^i_k(u) = \epsilon_i^{\Delta^i_-}\frac{\sqrt{\gamma_i(\epsilon_i)}}{\sqrt{\gamma_i(u)}}\frac{K^i(k,u)}{K^i(k,\epsilon_i)}$

Scalar Correlators: Universal properties





- The $\langle \mathcal{O}_1 \mathcal{O}_1 \rangle$ and $\langle \mathcal{O}_2 \mathcal{O}_2 \rangle$ have a similar behaviour in the UV as when there is only one boundary (power law divergence)
- In the IR they saturate to a constant positive value
- The cross correlator $\langle \mathcal{O}_1 \mathcal{O}_2 \rangle$ goes to zero in the UV and has a finite maximum in the IR
- In position space $(EAdS_2)$ they behave as $\sim 1/\sinh^{2\Delta_+}(\Delta\tau)$ and $\sim 1/\cosh^{2\Delta_+}(\Delta\tau)$ respectively $\Rightarrow$ No short distance singularity for the cross-correlator
- The qualitative behavior of the correlators is the same for all the types of solutions $\Rightarrow$ Universality

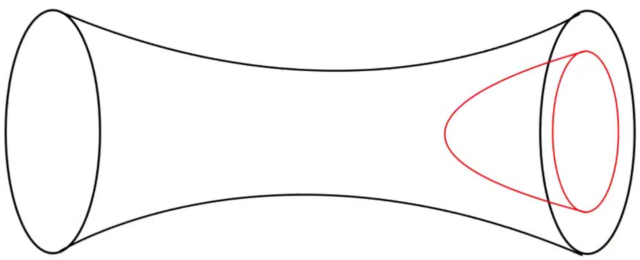
Shapes

Non-local observables: Wilson Loops

- Interesting to study non-local observables such as correlators of Wilson loops

$$W(C) = \mathrm{tr}\left(\mathcal{P}\exp\, i\oint_C A_\mu dx^\mu\right)$$

- In holography: Find the string worldsheet ending on the corresponding loop on a boundary and minimize its area

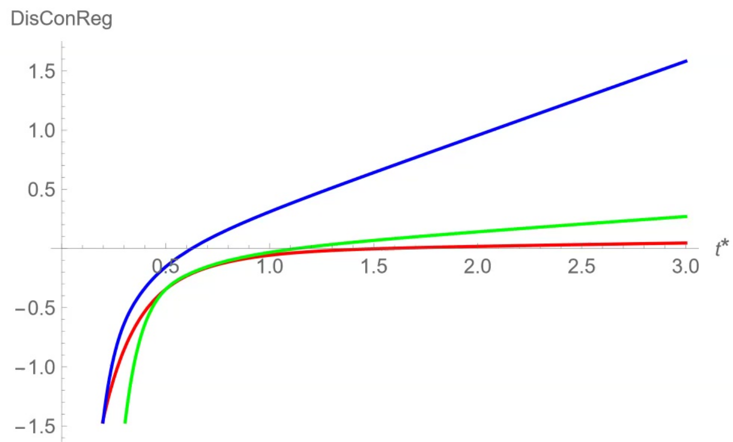


- The disk slices are three spheres and the loop is a circle sitting on the S^3

- Simplest observable: expectation value of a Wilson loop $\langle W(C)\rangle$ on one of the two boundaries
- In the limit of a large loop we can probe the IR properties of the boundary dual
- Large loops on the boundary penetrate further in the bulk
- The dual bulk string minimal surface, does not pass through the throat

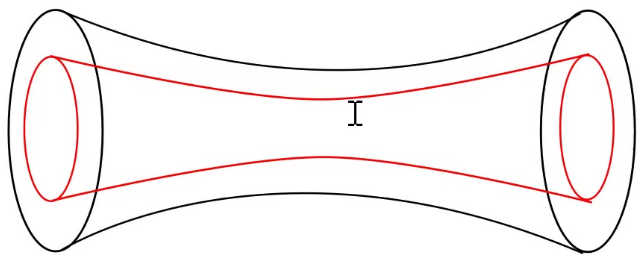
One circular Wilson loop: properties

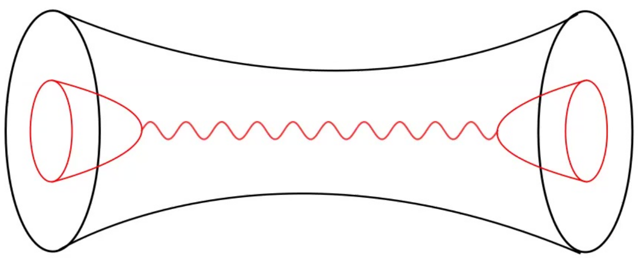
- Plot of the regularised action S^{reg} as a function of t^*
- $0 < t^* < \pi$ governs the boundary loop size (S^3 - Euler)
- $L_{throat}/L_{AdS} = 0.6, 0.7, 0.9$ from red to blue



- For a large loop the action scales approximately linearly with t^*
- This scaling is an Area law
- One boundary: indicative of IR confining behaviour only in the infinite volume limit of the S^3 [Witten]
- Here two asymptotic boundaries

Wilson Loop correlators





- Study loop - loop correlators $\langle W(C_1)W(C_2)\rangle$, with the two loops residing on different boundaries
- In the regime of large Wilson loops, the leading contribution originates from a single surface connecting the two loops having a cylinder topology $S^1 \times R$
- In the disconnected case the loops can interact only via exchange of perturbative bulk modes
- As we shrink the boundary loops, we find that the leading configuration of lowest action is the one for two disconnected loops
- Large loops $\Rightarrow$ Strong IR cross-coupling! ("cross-confining")

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Part II: QFT models

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Universal properties of a putative dual

- Traversable (Lorentzian) Wormholes: $\Rightarrow$ Couple the two boundary CFTs by a double trace deformation `[Gao - Jafferis - Wall ...]`

$$S_{int} = \int dt d^d x \, h(t,x) O_1(t,x) O_2(-t,x)$$

- This is a local interaction term
- Euclidean case: cross interactions are softer at shorter distances, they increase and become strong in the IR ("cross-confinement")
- The two theories interact "mildly" and the mixed correlators do not have short distance singularities
- It is not possible to have this behaviour by introducing a local term
- Initial Bottom up proposal: Take two Euclidean theories S_1 and S_2 and local operators $O_1(x) \in S_1$ and $O_2(x) \in S_2$. Introduce effective (non-local) cross-interactions

$$S = S_1 + S_2 + \lambda \int d^d x \, d^d y \, O_1(x) O_2(y) \, f(x-y)$$

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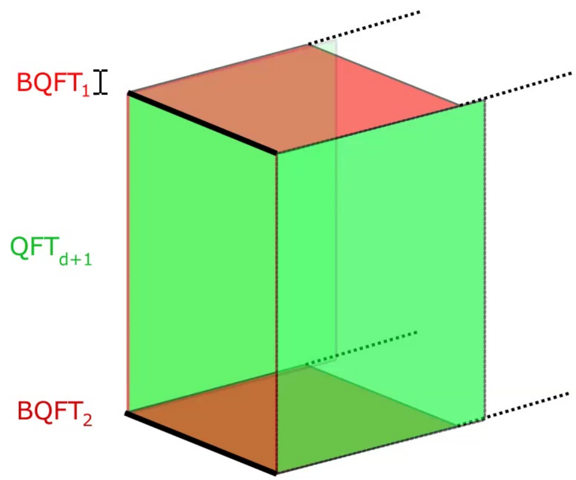
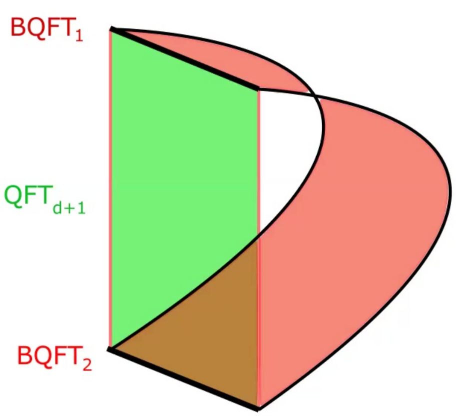
"Sandwich" construction

[van Raamsdonk], [PB - Kiritsis - Papadoulaki]





- A class of models resolving the issues of the previous proposal is the following:
- Two d-dim (holographic) BQFT's coupled through a $d+1$-dim intermediate (messenger) theory $\Rightarrow$ normally a $d+2$-dim gravitational theory
- Consider a system for which $c_{d+1} \ll c_d$
- The system should also flow to a gapped/confining theory in the IR
- The hope: The dual bulk gravity can localise on $d+1$-dim EOW branes that bend and connect in the IR
- We study the limit where the messenger theory is (quasi) topological

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Microscopic "Sandwich" model $(2d - 1d)$

- Take a $2d$ gYM (τ, z) and couple it to two $1d$ $U(N)$ matrix quantum mechanics theories at the endpoints of an interval I $(z = \pm L)$.
- The action is ($\Phi(B) = B^2$ (2d YM).)

$$S_{gYM} = \frac{1}{g_{YM}^2}\int_\Sigma \mathrm{tr}\, BF + \frac{\theta}{g_{YM}^2}\int_\Sigma \mathrm{tr}\, B\, d\mu - \frac{1}{2g_{YM}^2}\int_\Sigma \mathrm{tr}\,\Phi(B)\, d\mu$$

where $F = dA + A \wedge A$ and $A_\tau(\tau, z = \pm L) = A_\tau^{1,2}(\tau)$ is the restriction of the 2d gauge field on the two boundaries

$$S_{MQM} = \int d\tau\, \mathrm{tr}\left(\frac{1}{2}(D_\tau M_{1,2})^2 - V(M_{1,2})\right)\,,\; D_\tau M_{1,2} = \partial_\tau M_{1,2} + i[A_\tau^{1,2}, M_{1,2}]$$

- For τ non-compact there is a non-perturbative constraint

$$\frac{1}{2g_{YM}^2 L} J_{YM}^\tau = \frac{1}{2g_{YM}^2 L}[W^{-1}, \partial^\tau W] = J_{MQM_1}^\tau - J_{MQM_2}^\tau\,, \quad W = \mathcal{P}\exp\left(\int_{-L}^{L} dz A_z\right)$$

with W a Wilson line extending across the boundaries

- $J_{MQM_{1,2}}^\tau = \delta S_{MQM_{1,2}}/\delta A_\tau^{1,2}$ are the two MQM charges/currents

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"Entangling" the representations

- Each MQM Hamiltonian is $(M = U^\dagger \Lambda U,\ J = U^\dagger K U)$

$$\hat{H}^R_{MQM} = \left[-\frac{1}{2} \sum_i \left(\frac{\partial^2}{\partial \lambda_i^2} + V(\lambda_i) \right) + \frac{1}{2} \sum_{i<j} \frac{K^R_{ij} K^R_{ij}}{(\lambda_i - \lambda_j)^2} \right]$$

acting on wave-functions $\Psi_R(\lambda) = \prod_{i<j}(\lambda_i - \lambda_j)\tilde{\Psi}_R(\lambda)$ transforming in the $U(N)$ representation R

- The representations of $MQM_{1,2}$ are "entangled" by the constraint
- Place the system on $I \times S^1$
- The 2dYM partition function on the cylinder is

$$Z_{YM}(U_1, U_2) = \sum_R \chi_R(U_1)\chi_R(U_2) e^{-L \frac{g^2_{YM}}{N} C^{(2)}_R + i\theta C^{(1)}_R}$$

and depends on the two asymptotic holonomies $U_{1,2} = \exp \oint d\tau A^{1,2}_\tau$

- R a $U(N)$ representation, $C^{1,2}_R$ its Casimirs and $\chi_R(U)$ are $U(N)$ characters/wavefunctions

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"Entangling" the representations

- This should be coupled to the two MQM partition functions $Z_{1,2}^{MQM}(U_{1,2})$ and then integrated over $U_{1,2}$
- The complete partition function on the cylinder is

$$Z(\beta_1,\,\beta_2) = \sum_R e^{-L\frac{g_{YM}^2}{N}C_R^{(2)}+i\theta C_R^{(1)}}\, Z_R^{MQM_1}(\beta_1) Z_R^{MQM_2}(\beta_2)\,,$$

$$Z_R^{MQM}(\beta) = \mathrm{tr}_{\mathcal{H}_R}\, e^{-\beta \hat{H}_R^{MQM}}$$

with β the S^1 size and R a $U(N)$ representation and $C_R^{1,2}$ its Casimirs

- The MQM partition functions carry common representations R $\Rightarrow$ What we called "the sectors"

  $\sum_R \Rightarrow$ is a form of "averaging", consistent with unitarity (reflection positivity) for a single quantum mechanical system

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Liberating messenger and MQM gauge groups

- Couple the MQMs $M_{ij}^{1,2}$ with the 2d YM by introducing bifundamental fields $\psi_{\alpha,i}^{1,2}$ on the boundaries that couple to the 2d YM gauge field $A_{\alpha\beta}$
- Latin indices $i = 1, ..., N_{1,2}$ associated to the $U(N_{1,2})$ MQM groups and Greek indices $\alpha = 1, ..., n$ associated to the $U(n)$ 2d YM group
- The partition function takes a similar form as a sum over representations
- We now have additional parameters to tune: "ratio of ranks" $n/N_{1,2}$
- Aside: The analysis is most easily performed using Hall-Littlewood/Kostka polynomials
- For $n \ll N_{1,2} \to \infty$ we generally have a very small deformation of two decoupled MQM systems (the dual geometries are almost factorised but possibly connected via "quantum microscopic wormholes")
- For $n \sim N_{1,2} \to \infty$ the leading saddle is very different from that of two independent MQMs

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Cross-Correlators

- The n-point cross-correlator takes the general form

$$\langle O_{i_1}(\tau_{i_1})\dots\tilde{O}_{i_2}(\tau_{i_2})\dots\rangle = \sum_R \langle O_{i_1}(\tau_{i_1})\dots\rangle_1^R \langle\tilde{O}_{i_2}(\tau_{i_2})\dots\rangle_2^R \, e^{-L\frac{g_{YM}^2}{n}C_R^{(2)}+i\theta|R|}$$

- where i_1 refers to MQM theory 1 and i_2 to MQM theory 2
- This correlator generically only depends separately on the differences $\tau_{i_1}-\tau_{j_1}$ and $\tau_{i_2}-\tau_{j_2}$ and not on time differences that mix the $1,2$ sub-indices or normal with tilde operators
- Cross-communication arises at a leading saddle point level when we take both $N_{1,2}, n \to \infty$
- When $N_{1,2}\to\infty$, but n is finite the leading saddle is factorised and there is a subleading cross-communication from very long thin partitions with $O(N_{1,2})$ boxes
- The two point cross correlators of simple traces of the matrices $M_{1,2}(\tau)$ are simply constants (translational invariance on the circle)

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Dual geometry

- The singlet sector of one gauged MQM (taking a double scaling limit) is dual to $2d$ linear dilaton background of the $c = 1$-Liouville string
  $\Rightarrow$ One asymptotic region of space
- Non trivial reps with few boxes in their Young diagrams are related to long strings - Large reps deform the background geometry (possibly creating black holes)
  [Gaiotto, Maldacena, Kazakov-Kostov-Kutasov, P.B.-Papadoulaki ...]
- We took the large representation limit (continuous Tableaux) and studied the saddle point equations in order to find the corresponding geometry,
  $\rightarrow$ technically difficult, hard to read-off the dual geometry
  (bulk reconstruction for large reps ?)
- However, we were able to prove the existence of different saddles some of which we expect to correspond to disconnected and others to connected geometries
- Interesting to study phase transitions between them

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Higher Dimensional Examples

- Consider a sandwich spacetime, with the topology $\mathcal{M} = \Sigma \times I$
- The two holographic QFT's on the interval ends $\Sigma_{1,2}$ are coupled via a (quasi) - topological QFT in $\mathcal{M}$

2D dimensional example

- Two $2d$ BCFT's coupled through a Chern-Simons theory living in $3d$

3D dimensional example

- A natural choice of a messenger theory in $d = 4$, is a BF-type action in analogy to the simple 2d gYM theory

- Generalise also to higher dimensions

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$2d$ - $3d$ model

- Consider a $3d$ CS theory on $\mathcal{M} = \Sigma \times I$. In radial quantisation ($A_r = 0$, $\mu, \nu = 1, 2$) one finds

$$S_{CS} = \frac{k_{CS}}{4\pi} \int dr \int_\Sigma d^2x \epsilon^{\mu\nu} \, \mathrm{tr}\, (A_\mu \partial_r A_\nu) \,, \quad \frac{\delta S_{CS}}{\delta A_r} = \epsilon^{\mu\nu} F_{\mu\nu} = 0$$

- First order action (various choices of conjugate pairs). In holomorphic quantisation

$$[A_z^a(z) \,, \, A_{\bar{w}}^b(w)] = \frac{2\pi}{k} \delta^{ab} \delta^{(2)}(z - w)$$

- Take $\Sigma = T^2$ and decompose

$$A_{\bar{z}} = \partial_{\bar{z}} \chi + i\pi\tau_2^{-1} \alpha$$

with χ a complex function and α a complex group valued vector (in the abelian case simply a $U(1)$ compact complex function)

- In the abelian $U(1)$ case the wavefunctions are

$$\Psi_m(\chi, \alpha) = \frac{e^{\frac{k\pi}{2} \alpha \tau_2^{-1} \alpha}}{\eta(\tau)} \, e^{\frac{k}{2\pi} \int_\Sigma \partial_{\bar{z}} \chi \partial_z \chi} \, \vartheta \begin{bmatrix} \frac{m}{k} \\ 0 \end{bmatrix} (k\alpha, k\tau) \,, \quad m \in [0, k-1]$$

- In the non-abelian case they are Weyl-Kac characters $\chi_{\mu,k}(\alpha, \tau)$ (in analogy with $\chi_R(U)$ of the $2d$ BF model)

- We should now couple the 3d CS theory to two BCFT's at $\Sigma_{1,2}$
- For consistency depending on the choice of polarisation one needs to add appropriate boundary terms
- We then define a CS transition amplitude with wavefunctions the characters on $\Sigma_{1,2}$
- The result for $U(1)$ is ($i = 1, 2$ stands for the two boundaries)

$$\mathcal{Z} = \sum_{m=0}^{k-1} Z_m^{(1)}(J_1) Z_m^{(2)}(J_2)$$

$$Z_m^{(i)}(J_i) = \int \mathcal{D}\left(\chi_i, \alpha_i\right) Z_{BCFT}^{(i)}\left(\chi_i, \alpha_i; J_i\right) \Psi_m^{(i)}(\chi_i, \alpha_i)$$

with $Z_{BCFT}^{(i)}\left(\chi_i, \alpha_i; J_i\right)$ being each BCFT partition function in the presence of external sources

- In the non-abelian case the simple discrete sum is again replaced by a sum over reps

Models of non self-interacting "messengers"

- Another class of tripartite models with soft cross correlators: "Non self interacting massive messengers"
- A simple model of this type ($S_{1,2}$ are separated by an interval $|y_1 - y_2| = L$)

$$S_{1,2} = -\frac{1}{2}\int d^dx \ \mathrm{tr}\, \phi_{1,2}(x)\left(\Box_d - m^2\right)\phi_{1,2}(x) \ + \ S^{int.}_{1,2}$$

$$S_{mess.} = -\frac{1}{2}\int d^dx dy \ \mathrm{tr}\, \Phi(x,y)\left(\Box_{d+1} - M^2\right)\Phi(x,y)$$

$$S_{int} = g\int d^dx \ \mathrm{tr}\, \phi_1(x)\Phi(x,y_1) + g\int d^dx \ \mathrm{tr}\, \phi_2(x)\Phi(x,y_2)$$

- Integrate out $\Phi(x,y)$. The source functional is

$$Z[J_1(x_1), J_2(x_2)] = \int \mathcal{D}\phi_1\mathcal{D}\phi_2\, e^{-S_1 - S_2 - S_{12} - \int_1 d^dx_1\, \mathrm{tr}\, \phi_1(x_1)J_1(x_1) - \int_2 d^dx_2\, \mathrm{tr}\, \phi_2(x_2)J_2(}$$

with

$$S_{12} = g^2\int \frac{d^dp}{(2\pi)^d}\,\phi_1(p)\Sigma_{12}(p)\phi_2(-p)\,, \quad \Sigma_{12}(p) = \frac{\sinh^{-1}(L\sqrt{p^2+M^2})}{2\sqrt{p^2+M^2}}$$

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Models of non self-interacting "messengers"

- Using the Hubbard-Stratonovich trick

$$Z[J_1(x_1), J_2(x_2)] = \int \mathcal{D}\zeta_1 \mathcal{D}\zeta_2 \; Z_1[J_1+i\zeta_1] Z_2[J_2+i\zeta_2] \, e^{-\frac{1}{g^2}\int \frac{d^d p}{(2\pi)^d} \; \mathrm{tr}\, \zeta_1(p)\, \Sigma_{12}^{-1}(p)\, \zeta_2(-p)}$$

- All the cross correlators are sufficiently soft since $\Sigma_{12}(p) \sim \exp(-Lp)$ in the UV
- One can imagine that the two theories are strong (self)-interacting holographic theories like $\mathcal{N} = 4$ SYM ($\phi_{1,2} \to X^I_{1,2}\,,\;\; I = 1, ...6$)
- It is not clear though if the combined system has a weakly coupled gravitational dual (higher spins?)

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Reflection positivity?

- One can go to the diagonal basis for the propagators and consider the option that the messenger either propagates on an infinite space or on the interval ($s = p^2$)

$$S = \int \frac{d^d p}{(2\pi)^d} \left[\phi_+(p) D_+(p) \phi_+(-p) + \phi_-(p) D_-(p) \phi_-(-p)\right] + \ldots, \quad \phi_\pm = \frac{\phi_1 \pm \phi_2}{\sqrt{2}}$$

$$D_\pm(s) = s + m^2 + \frac{g^2 \left(1 \pm e^{-L\sqrt{s+M^2}}\right)}{2\sqrt{s+M^2}},$$

$$D_+^I(s) = s + m^2 + g^2 \left(\frac{\coth(L\sqrt{s+M^2}/2)}{2\sqrt{s+M^2}}\right),$$

$$D_-^I(s) = s + m^2 + g^2 \left(\frac{\tanh(L\sqrt{s+M^2}/2)}{2\sqrt{s+M^2}}\right)$$

- There is a regime of parameters for which $D_\pm^{-1}(x)$ and $D_\pm^{-1\,I}(x)$ are well defined and reflection positive

Properties of analytic continuation

- Lorentzian Continuation: Two choices
- Analytic Continuation of one of the boundary directions ($s = \vec{p}^2 - \omega^2$):
- Well defined in the case of $D^I(s)$. For $D(s)$ existence of transcendental branch-cut ($s = -M^2$) with no obvious interpretation
- In the case of analytic continuation along the messenger direction y, the boundary theories $\phi_{1,2}$ remain Euclidean
- Only the case of $D(x)$ is well defined (reflection positive Euclidean propagator). In the case of $D^I(x)$ the propagators are not reflection positive
- It would be interesting to analyse how these results change in a model at strong coupling

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Summary and Future

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Summary and Future Directions

Summary and Future Directions

Summary

- Variety of Euclidean wormhole solutions
- Computed correlators of both local and non-local observables
- Found common properties: Cross-correlators do not factorize and they are soft in the UV, while strong in the IR
- Dual description in terms of a system of interacting QFT's
- The field theoretic correlators are compatible with geometric dual computations
- A microscopic model in terms of two matrix quantum mechanics or BCFTs coupled through a topological messenger theory
- Concrete generalisations in higher d
- These models are unitary and require no additional averaging (exept the sum over representations) - no large deviation from the usual holographic prescription

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General structure of the Hilbert space

- For Lorentzian wormholes (B田): $\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$ and $|\Psi\rangle_{TFD}$
- Proposed models for Euclidean wormholes: due to the presence of gauge constraints the Hilbert space does not factorise
- Consider the enlarged space $\mathcal{H} = \sum_{R_1} \mathcal{H}^1_{R_1} \otimes \sum_{R_2} \mathcal{H}^2_{R_2}$ where $|R_1\rangle \otimes |R_2\rangle$ a product state for two independent reps
- The constraint reduces this into $\mathcal{H} = \sum_R \mathcal{H}^1_R \otimes \mathcal{H}^2_R$
- We now correlate ("entangle") representations and not energies as in the TFD...

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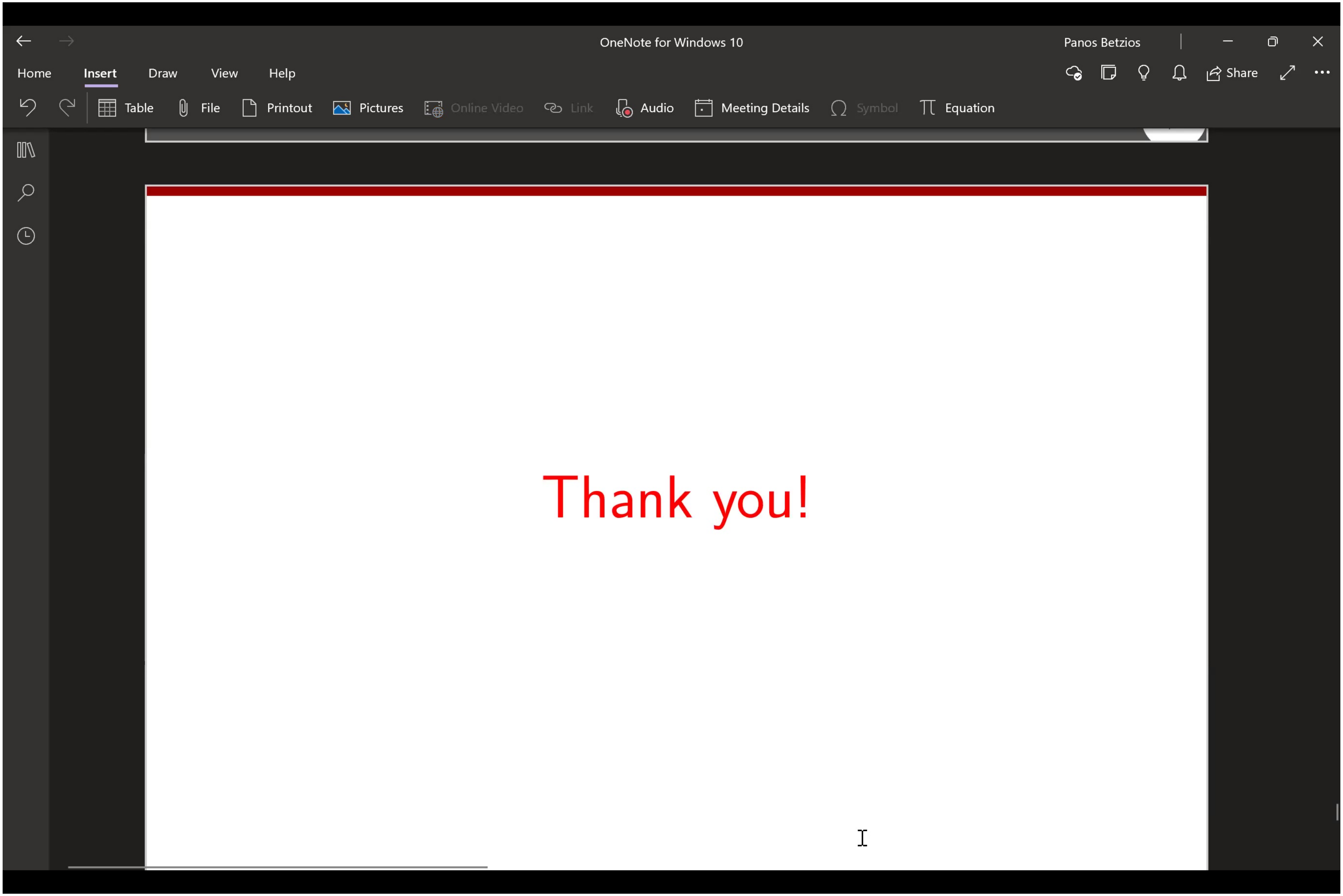
Future Directions

- Case of multiple boundaries
- Find a method to reconstruct the geometry from the microscopic QFT duals (large rep limit)
- Study Euclidean wormhole (target) backgrounds from the Liouville worldsheet perspective
- The non-singlet sectors are also relevant for black hole physics and involve similar sums over reps [Maldacena, PB - Papadoulaki]
- Understand better the analytic continuations of these field theoretic setups and their holographic duals (cosmologies, bang/crunch universe / traversable wormholes)
- Top down constructions embeddable in critical string theory (ex: Gaiotto Witten type of systems [van Raamsdonk, Bachas - Lavdas])
- Microscopic wormhole gas and α - parameters?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