Title: Quantum Black Holes and Holographic Complexity

Speakers: Antonia Micol Frassino

Series: Quantum Gravity

Date: February 10, 2022 - 2:30 PM

URL: https://pirsa.org/22020056

Abstract: In this talk, I will consider quantum effects on some specific black hole solutions and take into account their gravitational backreaction. In particular, I will describe the holographic construction of the quantum BTZ black hole (quBTZ) from an exact four-dimensional bulk solution. I will present some of the thermodynamic properties of these black holes, focus on the generalized first law and analyze the different complexity proposals for the quBTZ. Our results indicate that Action Complexity fails to account for the additional quantum contributions and does not lead to the correct classical limit. On the other hand, the Volume Complexity admits a consistent quantum expansion and agrees with known limits.

QUANTUM BLACK HOLES:

THERMODYNAMICS
&
HOLOGRAPHIC COMPLEXITY

Antonia Micol Frassino
Barcelona University & ICC

In collaboration with:
R. Emparan,
M. Sasieta,
M. Tomašević
B. Way

Perimeter Institute
February 10, 2022

CLASSICAL VS QUANTUM BHS

- The derivation of the Hawking radiation assumes that the spacetime is *classical* (i.e., is described by GR)
- The quantum thermal radiation does, however, have energy, and therefore, will affect the spacetime geometry (back-reaction)
- This *back-reaction* is governed by the semiclassical Einstein equations

$$G_{\mu\nu}(g_{\alpha\beta}) = 8\pi G_N \langle T_{\mu\nu}(g_{\alpha\beta}) \rangle$$

QUANTUM BACKREACTION

$$G_{\mu\nu}(g_{\alpha\beta}) = 8\pi G_N \langle T_{\mu\nu}(g_{\alpha\beta}) \rangle$$

Classical Einstein tensor & metric $\propto$ Quantum matter renorm stress tensor (many fields)

Difficulties in solving these equations:

- Coupled system: metric+ <QFT>
- Very hard to solve simultaneously
- Perturbative backreaction: limited insight

Exact backreaction:

- 2D models: CGHS, JT+CFT
- **Holographic reformulation**

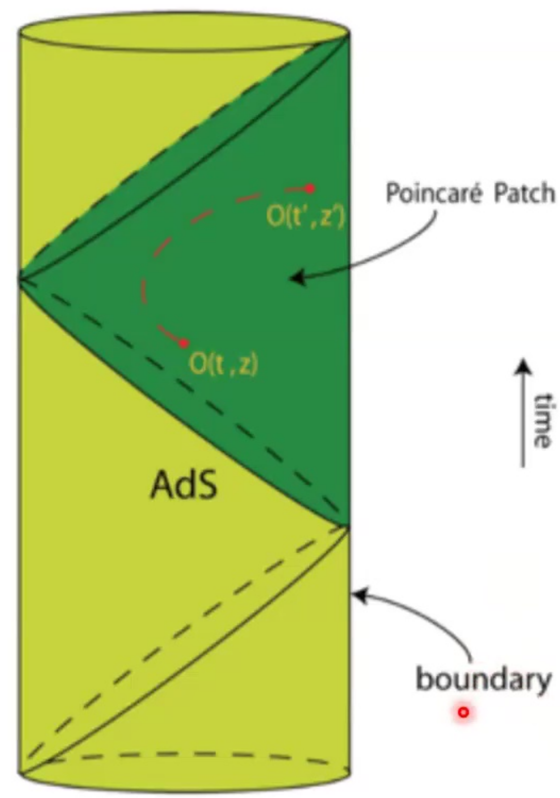
Emparan, Fabbri, Kalopper (2002)

HOLOGRAPHIC APPROACH

…an offspring of String Theory:
the AdS/CFT correspondence

AdS/CFT is a holographic equivalence between:

(quantum) gravity in asymptotically AdS spacetime –
the '*bulk*'
and
(conformal) field theories living on its '*boundary*'
(no gravity at all)



HOLOGRAPHIC APPROACH

$\langle T_{\mu\nu} \rangle$ CFT on boundary geometry of the dual classical bulk

➡ planar CFT ($N \to \infty$)

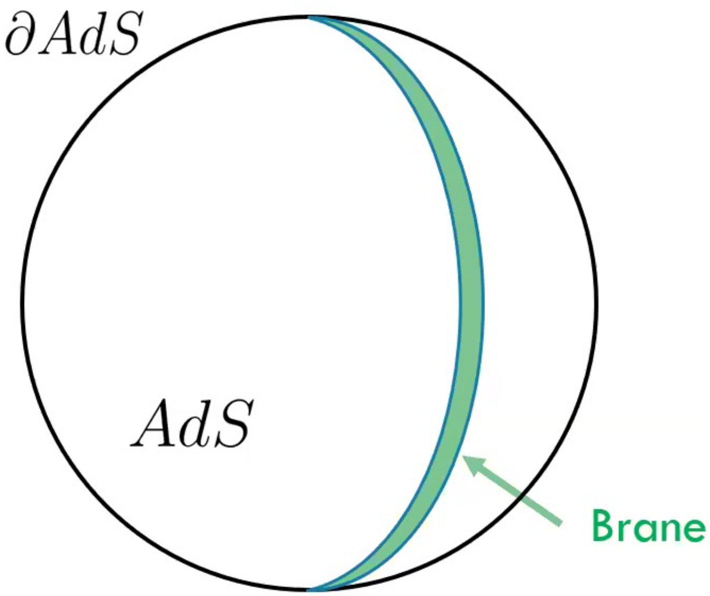
Conventional AdS/CFT has *fixed* boundary geometry

What we want is to make the boundary dynamical:

- Modify the boundary conditions: "setting the boundary free" — Compere, Marolf (2008); Andrade, Marof (2011)
- Move the boundary inwards: Braneworld holography — H. Verlinde, Gubser (1999)

BRANEWORLD HOLOGRAPHY

Basic idea of BRANEWORLD gravity is that you want to recover gravity localized on some lower dimensional surface of some higher dimensional bulk spacetime.



Cutting the bulk with a (Plank) brane

- Introduces a (D-1)-dimensional graviton massive mode localized on the brane
- The CFT is also cutoff in the UV

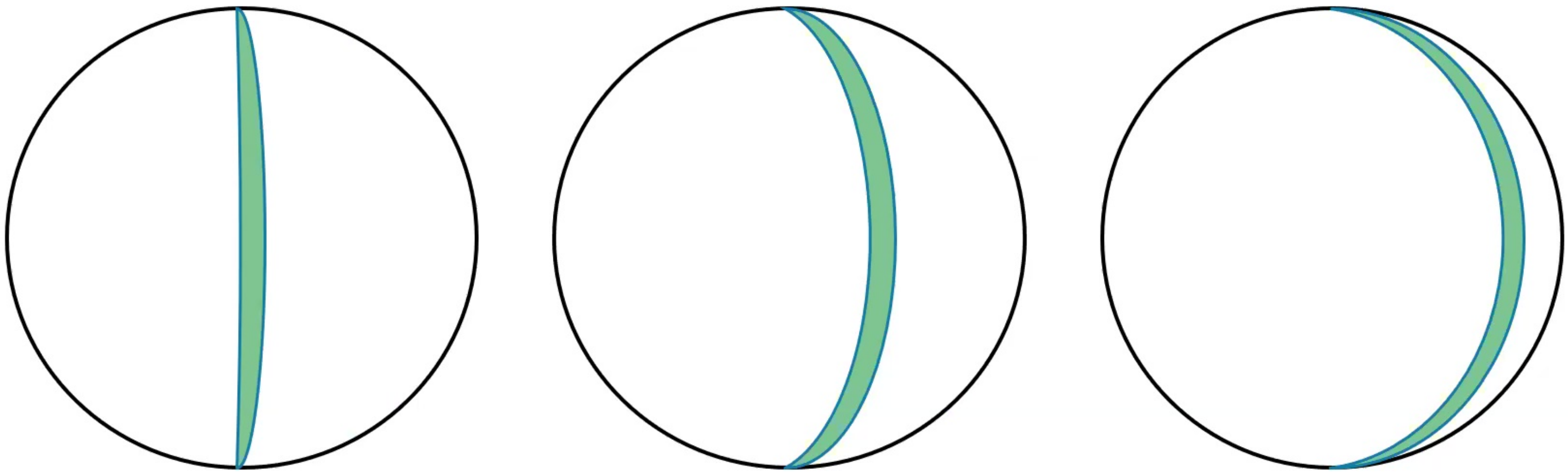
→ You get dynamics on the brane

Randall, Sundrum (1999)
Karch, Randall (2001)

BRANE LIMITS

Brane tension $\tau \propto \frac{1}{G_4 \ell}$

$\ell \to \infty$ | $0 < \ell < \infty$ | $\ell \to 0$



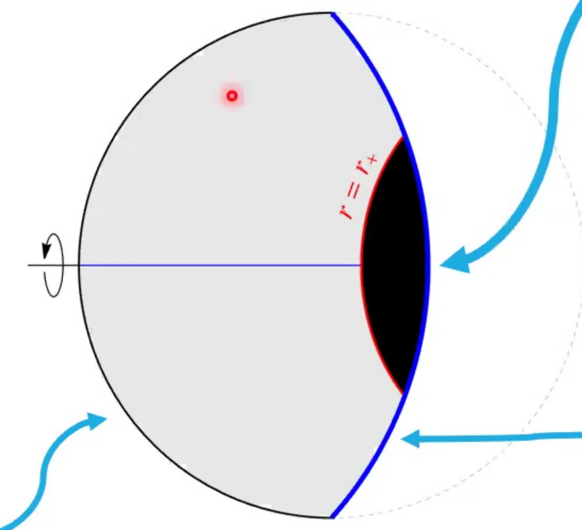
the graviton mass becomes as large as possible - gravity on the brane is completely 4D (only Z2 symmetry)

massive gravity theory on the brane

almost massless graviton

BLACK HOLE ON THE BRANE

The BH in the bulk is intersecting the brane



CFT3 on the brane

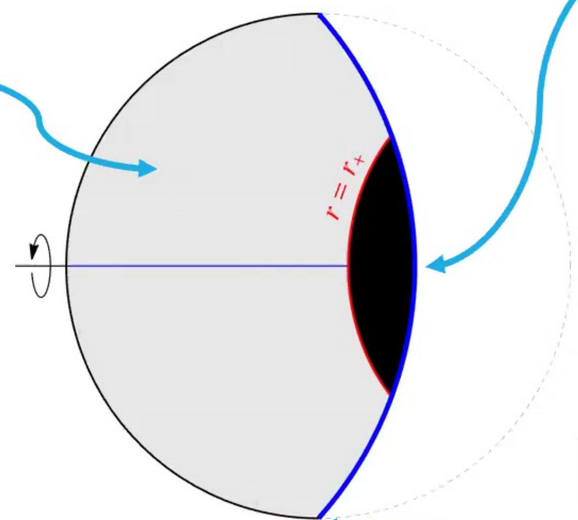
The 4D theory has an asymptotic boundary where the CFT3 lives

Emparan, Horowitz, Myers (1999)
Emparan, AMF, Way (2020)

The bulk spacetime is a solution of the vacuum EE with negative CC in 4D

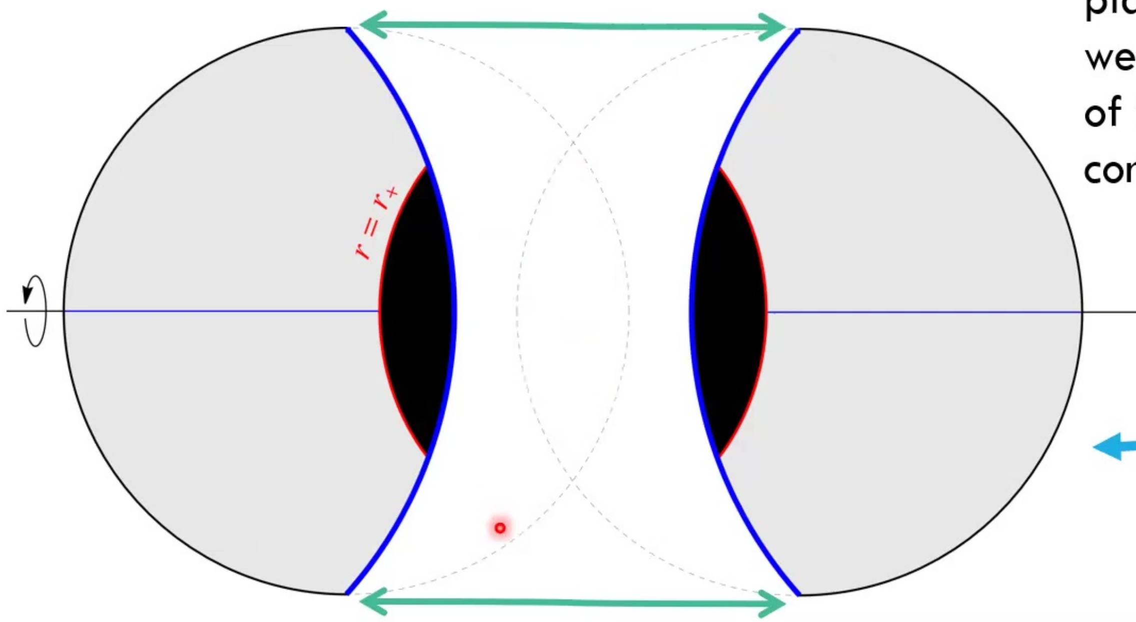
3D CC obtained from:
4D CC & brane tension

$$\frac{1}{\ell_4^2} = \frac{1}{\ell^2} + \frac{1}{\ell_3^2}$$



The BH in the bulk is intersecting the brane

Transparent boundary conditions for the CFT btw the boundary and the brane (we consider the dCFT)



When the brane is placed in the spacetime, we erase the other part of the spacetime and consider another copy

The gluing between the two parts of the spacetime is done using Israel gluing conditions along the brane

ADS4 C-METRIC

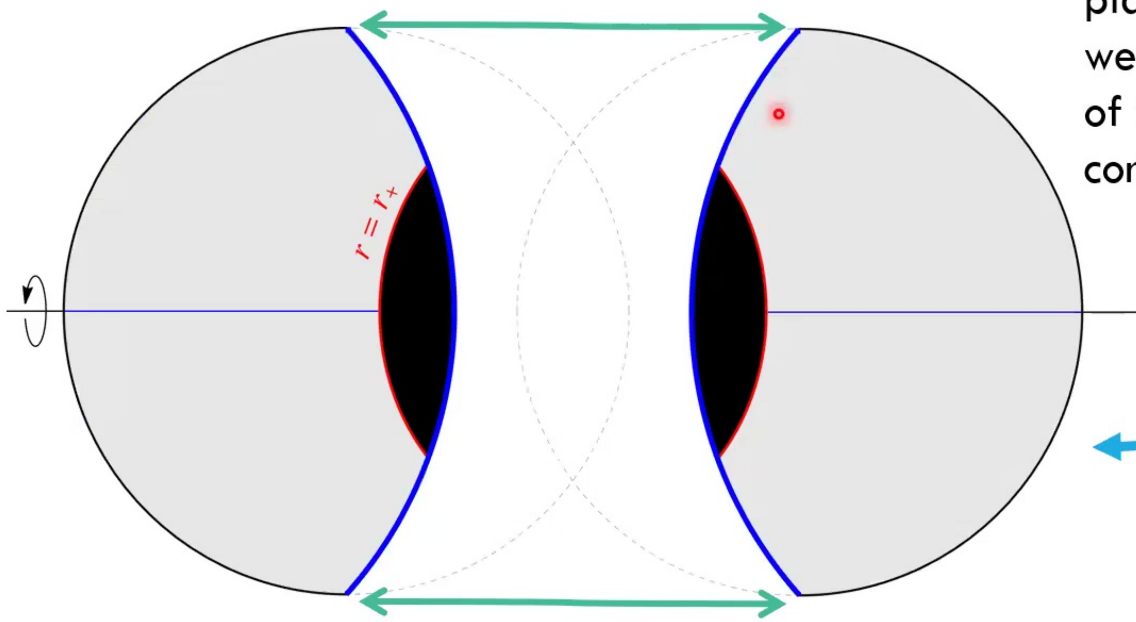
The classical metric of this state:

$$ds^2 = \frac{\ell^2}{(\ell + xr)^2}\left[-H(r)dt^2 + \frac{dr^2}{H(r)} + r^2\left(\frac{dx^2}{G(x)} + G(x)d\phi^2\right)\right]$$

$$H(r) = \frac{r^2}{\ell_3^2} + \kappa - \frac{\mu\ell}{r} \qquad G(x) = 1 - \kappa x^2 - \mu x^3$$

- ℓ_3 Brane curvature radius
- ℓ Brane position, tension^(-1)
- μ Quantum corrections on the brane

Note: the black hole is NOT in the center of AdS4 – it's accelerating



When the brane is placed in the spacetime, we erase the other part of the spacetime and consider another copy

The gluing between the two parts of the spacetime is done using Israel gluing conditions along the brane

ADS$_4$ C-METRIC

The classical metric of this state:

$$ds^2 = \frac{\ell^2}{(\ell + xr)^2}\left[-H(r)dt^2 + \frac{dr^2}{H(r)} + r^2\left(\frac{dx^2}{G(x)} + G(x)d\phi^2\right)\right]$$

$$H(r) = \frac{r^2}{\ell_3^2} + \kappa - \frac{\mu\ell}{r} \qquad G(x) = 1 - \kappa x^2 - \mu x^3$$

ℓ_3 Brane curvature radius

ℓ Brane position, tension^(-1)

μ Quantum corrections on the brane

Note: the black hole is NOT in the center of AdS$_4$ – it's accelerating

ADS4 C-METRIC

The classical metric of this state:

$$ds^2 = \frac{\ell^2}{(\ell + xr)^2}\left[-H(r)dt^2 + \frac{dr^2}{H(r)} + r^2\left(\frac{dx^2}{G(x)} + G(x)d\phi^2\right)\right]$$

$$H(r) = \frac{r^2}{\ell_3^2} + \kappa - \frac{\mu\ell}{r} \qquad G(x) = 1 - \kappa x^2 - \mu x^3$$

ℓ_3 Brane curvature radius

ℓ Brane position, tension^(-1)

$$ds^2 = \frac{\ell^2}{(\ell + xr)^2}\left[-H(r)dt^2 + \frac{dr^2}{H(r)} + r^2\left(\frac{dx^2}{G(x)} + G(x)d\phi^2\right)\right]$$

The rotating AdS C-metric:

- Rotation parameter a
- Similar but more complicated:
- Bulk structure similar to Kerr-AdS4 (inner and outer horizons, ring singularity)

Interesting features

Emparan, Tomašević (2020)
Emparan, AMF, Way (2020)

METRIC quBTZ

The 3D metric induced on the brane at x=0 (satisfies the Israel junction conditions) is:

$$ds^2 = \frac{\ell^2}{(\ell + xr)^2}\left[-H(r)dt^2 + \frac{dr^2}{H(r)} + r^2\left(\frac{dx^2}{G(x)} + G(x)d\phi^2\right)\right]$$

$$H(r) = \frac{r^2}{\ell_3^2} + \kappa - \frac{\mu\ell}{r} \qquad G(x) = 1 - \kappa x^2 - \mu x^3$$

METRIC quBTZ

Emparan, Horowitz, Myers (2000)
Emparan, Fabbri, Kaloper (2002)
Emparan, AMF, Way (2020)

3D metric induced on the brane at x=0

$$ds^2 = -\left(\frac{r^2}{\ell_3^2} + \kappa - \frac{\mu\ell}{r}\right) dt^2 + \frac{1}{\frac{r^2}{\ell_3^2} + \kappa - \frac{\mu\ell}{r}} dr^2 + r^2 d\phi^2$$

Classical limits $\mu = 0$

- $\kappa = -1$ BTZ
- $\kappa = +1$ Global or Conical AdS_3

$\mu \neq 0$ $\kappa = -1$ quBTZ, different properties of the horizon, has curvature singularity

METRIC quBTZ

Emparan, Horowitz, Myers (2000)
Emparan, Fabbri, Kaloper (2002)
Emparan, AMF, Way (2020)

3D metric induced on the brane at x=0

$$ds^2 = -\left(\frac{r^2}{\ell_3^2} + \kappa - \frac{\mu\ell}{r}\right) dt^2 + \frac{1}{\frac{r^2}{\ell_3^2} + \kappa - \frac{\mu\ell}{r}} dr^2 + r^2 d\phi^2$$

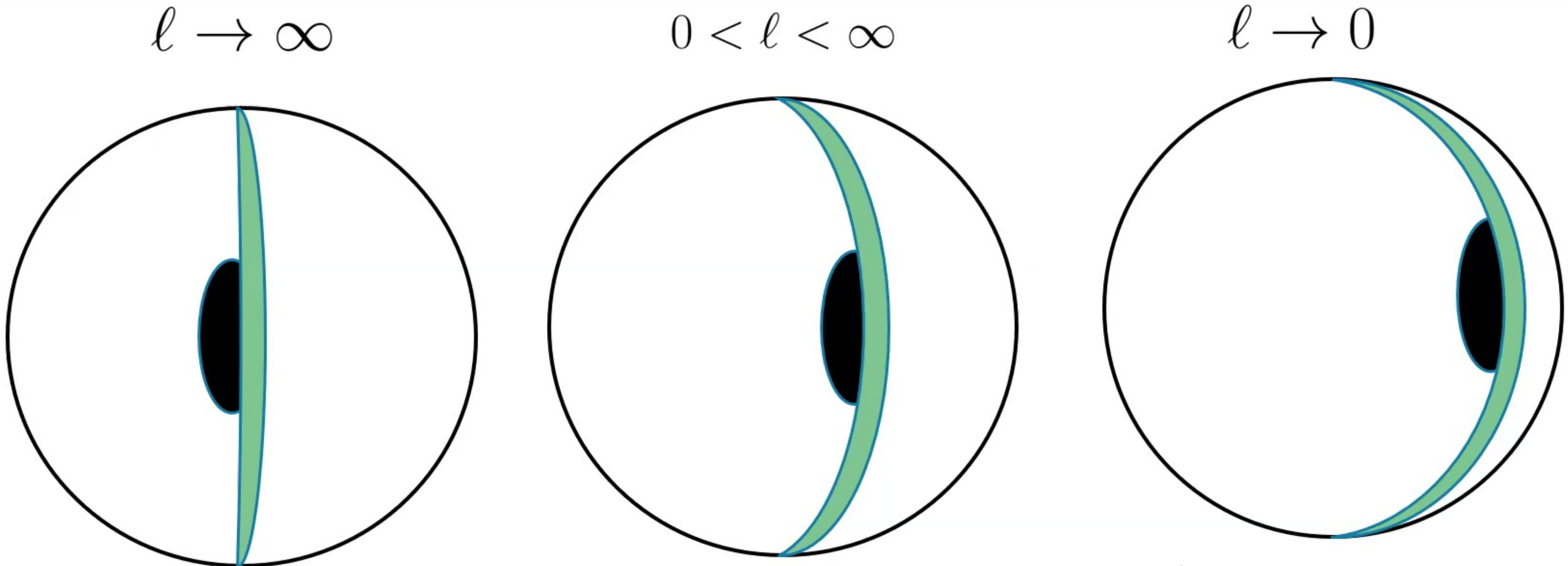
Classical limits $\mu = 0$

- $\kappa = -1$ BTZ
- $\kappa = +1$ Global or Conical AdS3

Interpretation: is as a solution of a theory of 3D gravity, with higher curvature terms, coupled to a large number of quantum conformal fields, namely, the holographic CFT dual to the 4D bulk.

GLOBAL ASPECTS OF THE BULK

Hubeny, Marolf, Rangamani (2009)



4D AdS BH (only Z2 symmetry)

BH+CFT

$\kappa = -1$ bd geometry has a BTZ

Double Wick rotation of the Schwarzschild-AdS_4

CFT STRESS TENSOR

$$\langle T^a{}_b \rangle = \frac{\ell}{16\pi G_3} \frac{F(M)}{r^3} \mathrm{diag}\{1, 1, -2\} + \ldots$$

Comparison with other calculations

Free conformal scalar in BTZ

- Method of images
- Transparent bc's at boundary of AdS_3
- Satisfy KMS, HH at horizon
- Same structure – different $F(M)$

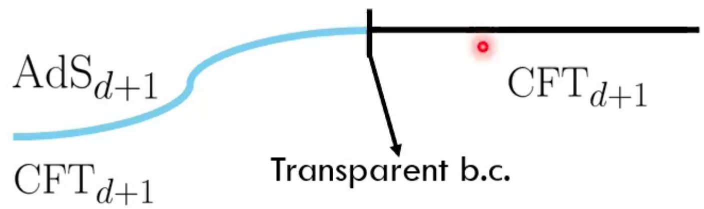
Holographic w/out backreaction:

- Exact BTZ at boundary AdS
- Very different method of construction
- but recovered as limit $\ell \to 0$ of ours

DOUBLY HOLOGRAPHIC INTERPRETATION

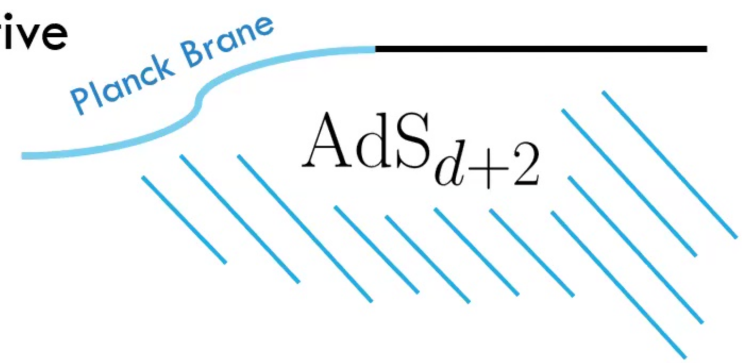
1. UV Perspective

2. Brane Perspective

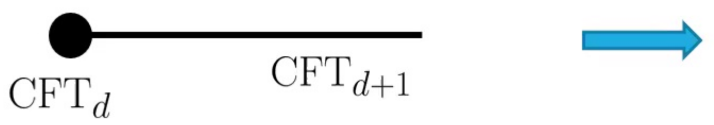


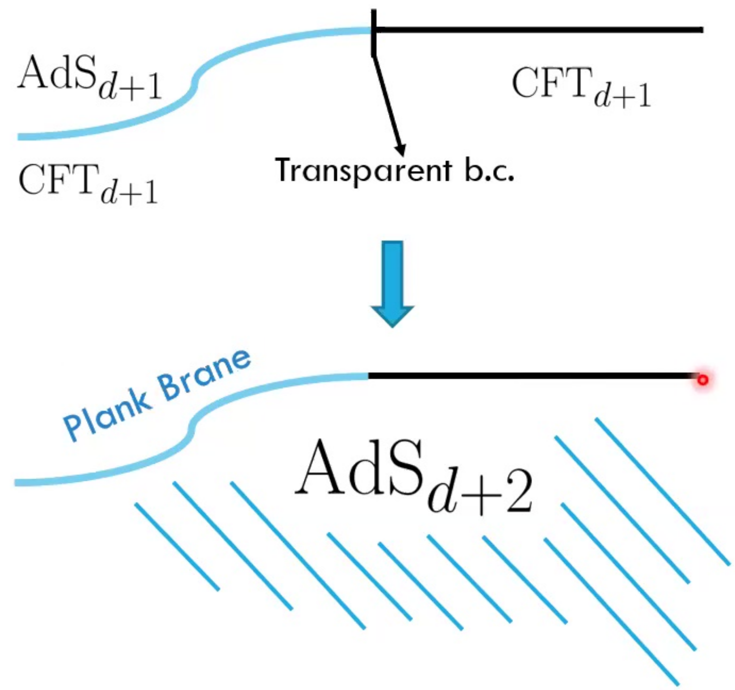
3. Doubly Holographic perspective

View the whole setup as a (d+2) AdS without any CFT



DOUBLY HOLOGRAPHIC INTERPRETATION





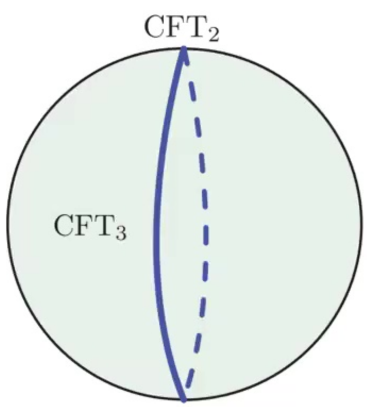
- Now this is just a dynamical (d+1) spacetime (a brane) in the (d+2) dimensional bulk.

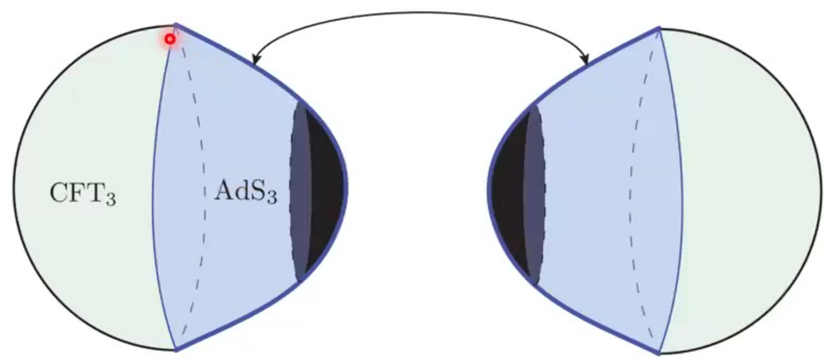
- The EFT of gravity in the (d+1) is just a localization of this (d+2) gravity on the brane (like in the RS setup)

OUR SETUP: BLACK HOLE ON THE BRANE

1. Start with a 3d CFT on a space that is topologically a sphere and then put a 2d defect on the equator

2. Replace the defect by a AdS_3 theory. Since the 2d CFT is in a thermal state, the AdS_3 contains a (semiclassical) BH

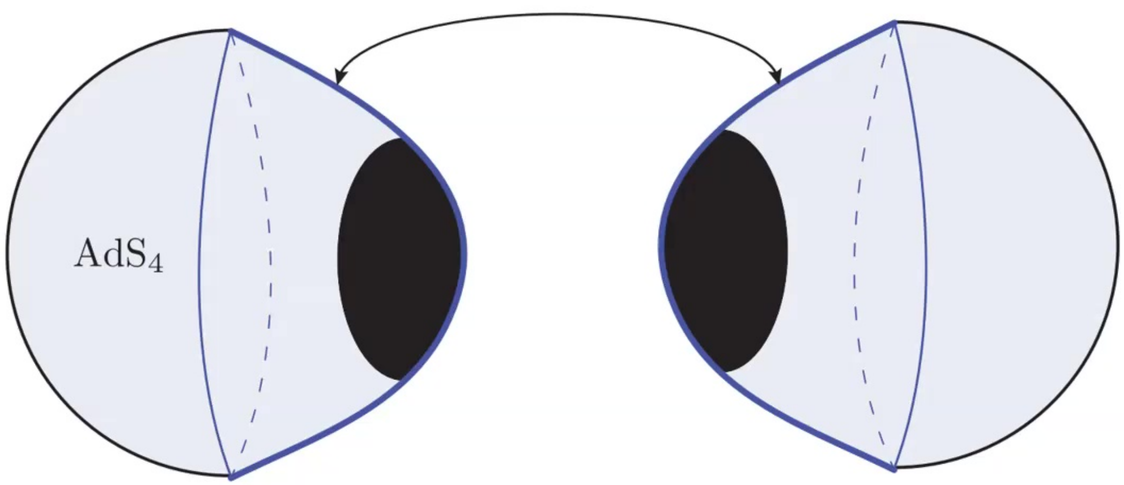




There are quantum backreaction effect of the CFT_3

OUR SETUP: BLACK HOLE ON THE BRANE

3. Put the whole system on a 4D bulk



We can see it from the induced effective action on the 3D brane

This is the idea of translating quantum effects in the brane perspective to classical quantities in to 4D perspective

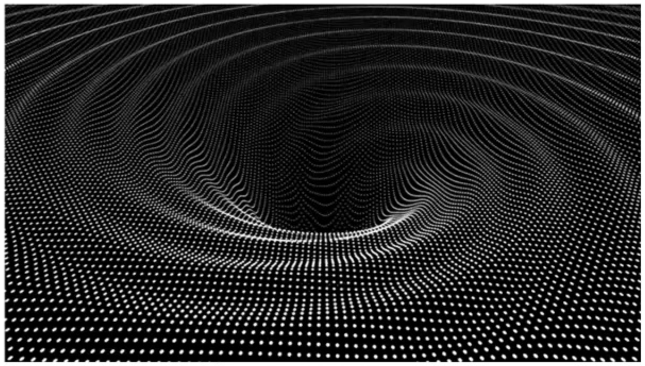
The precise limits we are taking to have this classical description are:

$$c_3 \to \infty \quad c_2 \to \infty \quad \text{and} \quad \frac{c_3}{c_2} \ll 1$$

WHAT CAN WE CALCULATE?

In holography, we can use the C-metric to compute quantum corrections using a classical set-up.

- I have shown you that this metric has a well-defined meaning as holographic setup.
- More can be explored even for the solution on the brane – even without the holographic interpretation.



EFFECTIVE ACTION

From the 4D classical gravitational action with a brane

$$I = \frac{1}{16\pi G_4} \int d^4x \sqrt{-g}(R - 2\ell_4) - \frac{1}{2\pi G_4 \ell} \int d^3y \sqrt{-h}$$

one can derive the effective 3D action for this theory:

$$I = \frac{1}{16\pi G_3} \int d^3x \sqrt{-h} \left(\frac{2}{\ell_3^2} + R + O(\ell^2) \right) + I_{CFT}$$

$$G_3 = G_4/2\ell_4$$

RESULT: GENERALIZED ENTROPY IN BW HOLOGRAPHY

Emparan, AMF, Way (2020)

$$S_{\text{gen}} = S_{\text{Wald}} + S_{\text{outside}}$$

Bulk BH entropy

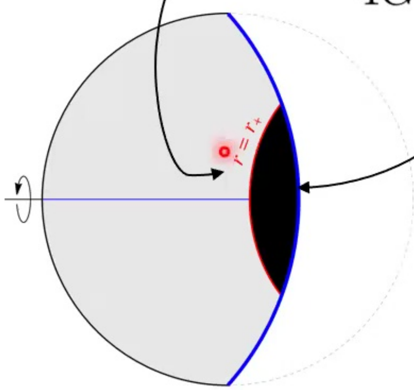
$$S_{gen} = \frac{A_{D+1}}{4G_{D+1}}$$

Brane BH entropy

$$S_{\text{Wald}} = \frac{A_D}{4G_D} + \dots$$

Entropy of the CFT that leaves in the presence of the BH
→ Obtained by subtraction:

$$S_{\text{ent}} = S_{\text{gen}} - S_{\text{Wald}}$$



Use this result in BH thermodynamics

QUANTUM ENTROPY: 1ST AND 2ND LAW

If the holographic interpretation of braneworld is consistent, then:

$$T dS_{\text{gen}} = dM - \Omega dJ$$

1st law: Not trivial

In D+1 dim bulk

On D dim brane w/ higher curvature gravity

$$\Delta S_{\text{gen}} \geqslant 0$$

2nd law: Trivial

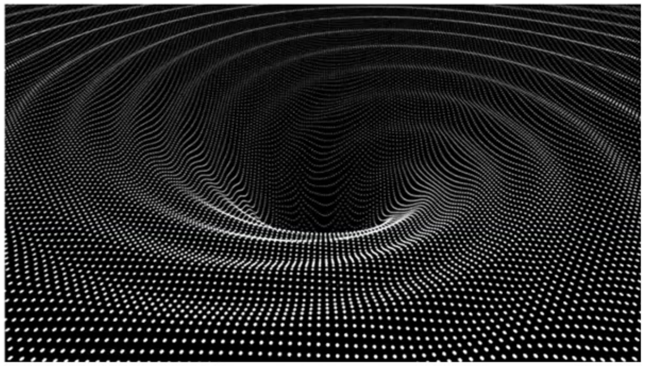
S_{Wald} alone will not satisfy these relations!

WHAT CAN WE CALCULATE?

Using the C-metric we can compute quantum corrections using a classical set-up.

- Generalized Entropy
- *Holographic complexity*

Even though, from the outside BHs appear to stay constant in size, the interior volume grows bigger and bigger all the time as space stretches toward the center point.



Puzzle: How is this growth encoded in the boundary?

Quantum computational complexity!

Susskind, 2015

HOLOGRAPHIC COMPLEXITY

The discussion so far implicitly assumes that the bulk is classical, which describes the CFT to leading order in the limit of large N, or more properly, large c.

→ Beyond this leading order in the large-c expansion, the defining properties of holographic complexity remain largely unexplored in $D \geq 3$.

Consider a next step in the generalizations of **CV** and **CA** that includes the leading finite-c effects in the CFT, corresponding to the leading quantum corrections in the bulk EFT.

HOLOGRAPHIC COMPLEXITY

- We can compute the CV complexity of this state by the volume in 4D or the CA by the action in 4D

$$C_V(|\mathrm{quBTZ}\rangle) = \frac{\mathrm{Vol}_4(\Sigma)}{G_4 L} \qquad C_A(|\mathrm{quBTZ}\rangle) = \frac{I_4}{\hbar}$$

Strong motivation from TN

Requirement for a more general functional

Belin, Myers, Ruan, Sárosi, Speranza (2021)

VOLUME COMPLEXITY

For CV, the tensor network rationale already hints at a generalization of the schematic form

$$\mathcal{C}_V\left(|\Psi\rangle\right) = \frac{\mathrm{Vol}(\Sigma)}{G\hbar\,L} + \frac{\delta\mathrm{Vol}(\Sigma) + \mathcal{V}(\Sigma)}{G\hbar\,L} + \mathcal{C}_V^{\mathrm{bulk}}\left(|\phi\rangle\right) + \ldots$$

Semiclassical backreaction

higher derivative corrections

complexity of the state of the quantum fields in the bulk

The expression resembles to the quantum-corrected holographic entanglement entropy

LATE TIME CV

The quantum-corrected VC formula reproduces the expected computation rate for a semiclassical black hole

$$\frac{\mathrm{d}\mathcal{C}_V}{\mathrm{d}\bar{t}}\bigg|_{t\gg\beta} = 2M\left(1 + \sqrt{2}\,\frac{\mu\ell}{\ell_3} + \dots\right)$$

- $$\frac{\mathrm{d}\mathcal{C}_V}{\mathrm{d}t}\bigg|_{t\gg\beta} \sim TS_{\text{gen}} > (TS)_{BTZ}$$

up to an O(1) coefficient that depends on the mass of the black hole.

LATE TIME CV

The quantum-corrected VC formula reproduces the expected computation rate for a semiclassical black hole

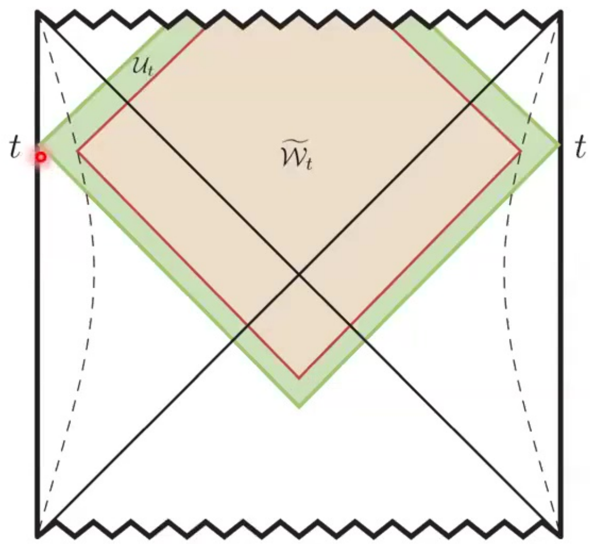
$$\left.\frac{\mathrm{d}\mathcal{C}_V}{\mathrm{d}\bar{t}}\right|_{t\gg\beta} = 2M\left(1 + \sqrt{2}\,\frac{\mu\ell}{\ell_3} + \ldots\right)$$

- $$\mathcal{C}_V\left(|\Psi\rangle\right) = \frac{\mathrm{Vol}(\Sigma)}{G\hbar\, L} + \frac{\delta\mathrm{Vol}(\Sigma) + \mathcal{V}(\Sigma)}{G\hbar\, L} + \mathcal{C}_V^{\mathrm{bulk}}\left(|\phi\rangle\right) + \ldots$$

There is no (renormalized) proper complexity of the bulk field at the leading order (b.c.)

ACTION COMPLEXITY

CA is also complicated, because the metric of the spacetime is not spherically symmetric (so the WdW patch will not be a diamond)



- Introduce a bulk IR-cutoff at $r = \infty$ which lies at finite proper distance from the brane.
- Then consider two different spacetime domains
- The WdW patch W lies in between the red causal diamond $\widetilde{W}$, and the green causal diamond U_t
- The $\widetilde{W}$ is anchored to a finite proper distance in the bulk

$$\widetilde{\mathcal{C}}_A(t) = \frac{I(\widetilde{\mathcal{W}}_t)}{\pi}$$

LATE TIME BEHAVIOR

The late time grows of CA is proportional to the mass of the quBTZ but, in the final results, is independent of the effective coupling on the brane

$$\frac{dC_A}{dt} = 8M f(G_3 M)$$

All the term in the calculation of the action on the WdW patch depend on the effective coupling (c_3/c_2) but then when we sum, there is a cancellation

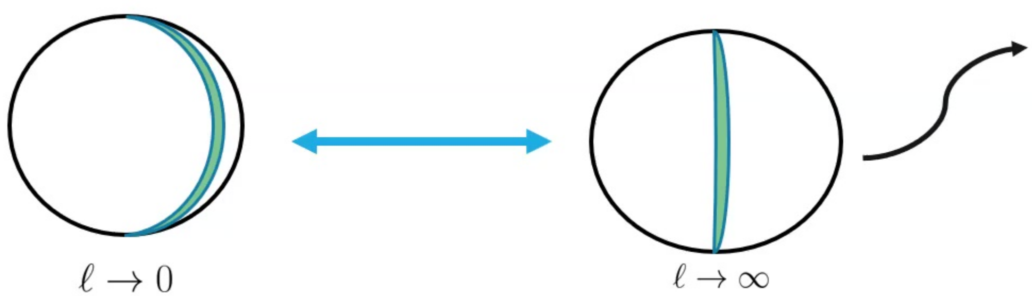
P. Braccia, A. L. Cotrone, and E. Tonni (2019)
S. Chapman, D. Ge, and G. Policastro (2018)

RESULTS FOR THE ACTION

- It doesn't reproduce the complexity rate of the BTZ:

$$\frac{dC_A}{dt} \neq (TS)_{BTZ} \text{ when } \frac{c_3}{c_2}\left(\text{or}\frac{\ell}{\ell_3}\right) \to 0$$

- And, since the same parameters control the inverse tension of the brane, it doesn't distinguish between different bulk geometry:



It gives the action growth for a 4D hyperbolic AdS4 Schw-BH

RESULTS FOR THE ACTION

- Discontinuity in the classical limit: $\frac{dC_A}{dt} \neq (TS)_{BTZ}$ when $\frac{c_3}{c_2}\left(\text{or}\,\frac{\ell}{\ell_3}\right) \to 0$

➢ The discontinuity of AC is (likely) coming from discontinuity in the character of the singularity

the WdW patch reaches the inner singularity, whose structure changes qualitatively due to quantum backreaction, and which gives rise to large quantum contributions to AC.

➢ Our result point to a more fundamental characteristic of AC: it is a magnitude that allows to probe the black hole interior in a more precise way than VC.

CONCLUSIONS

- Exact solution that allowed us to compute the generalize entropy including the von Neumann entropy of the CFT
- We verified the first law
- The solution allows to compute quantum corrections to Volume-Complexity and Action-Complexity
- The quantum-corrected VC formula correctly reproduces the expected computation rate for a semiclassical black hole TS_{gen}
- AC remains still puzzling because it does not give the correct classical limit
- Sensitivity of the AC to the singularity