Title: Possibility of causal loops without superluminal signalling -- a general framework

Speakers: Vilasini Venkatesh

Series: Quantum Foundations

Date: February 11, 2022 - 2:00 PM

URL: https://pirsa.org/22020051

Abstract: Causality is fundamental to science, but it appears in several different forms. One is relativistic causality, which is tied to a space-time structure and forbids signalling outside the future. On the other hand, causality can be defined operationally using causal models by considering the flow of information within a network of physical systems and interventions on them. From both a foundational and practical viewpoint, it is useful to establish the class of causal models that can coexist with relativistic principles such as no superluminal signalling, noting that causation and signalling are not equivalent. We develop such a general framework that allows these different notions of causality to be independently defined and for connections between them to be established. The framework first provides an operational way to model causation in the presence of cyclic, fine-tuned and non-classical causal influences. We then consider how a causal model can be embedded in a space-time structure and propose a mathematical condition (compatibility) for ensuring that the embedded causal model does not allow signalling outside the space-time future. We identify several distinct classes of causal loops that can arise in our framework, showing that compatibility with a space-time can rule out only some of them. We then demonstrate the mathematical possibility of causal loops embedded in Minkowski space-time that can be operationally detected through interventions, without leading to superluminal signalling. Our framework provides conditions for preventing superluminal signalling within arbitrary (possibly cyclic) causal models and also allows us to model causation in post-quantum theories admitting jamming correlations. Applying our framework to such scenarios, we show that post-quantumjamming can indeed lead to superluminal signalling contrary to previous claims. Finally, this work introduces a new causal modelling concept of ``higher-order affects relations" and several related technical results, which have applications for causal discovery in fined-tuned causal models.

Possibility of causal loops without superluminal signalling – a general framework

V. Vilasini [1,2] Roger Colbeck [2]

[1] Institute for Theoretical Physics, ETH Zurich, 8093 Zürich, Switzerland

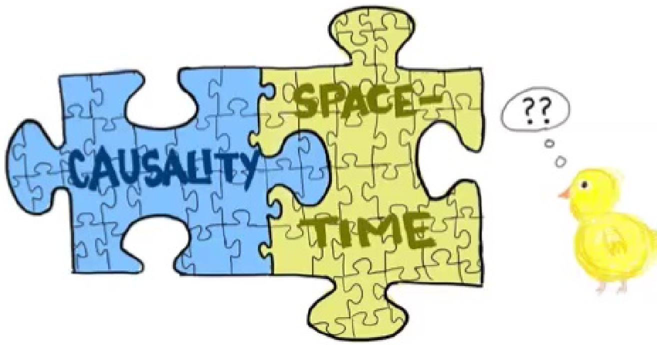
[2] Department of Mathematics, University of York, Heslington, York YO10 5DD

Perimeter Institute Quantum Foundations Seminar, February 2022

Based on

VV and RC 2021 arxiv:2109.12128 (Submitted to PRL+PRA)
VV's PhD thesis *arxiv:2102.02393*

Talk overview



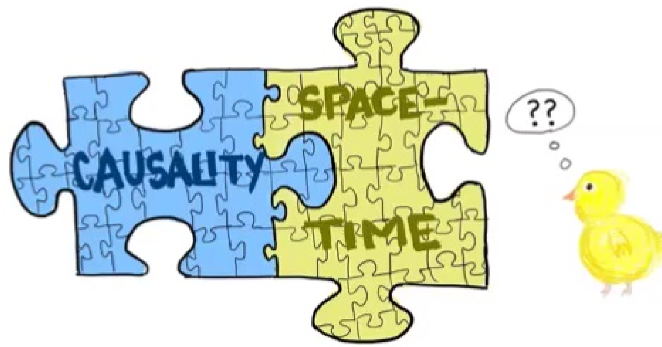
Motivation, post-quantum jamming

Framework: causality, space-time and how they fit together

Causal loops without superluminal signalling

Jamming allows for superluminal signalling

Talk overview



- Motivation, post-quantum jamming
- Framework: causality, space-time and how they fit together
- Causal loops without superluminal signalling
- Jamming allows for superluminal signalling
- Conclusions and open questions

The landscape of causality

NOTIONS OF CAUSATION

Relativistic notions

- "Cause" precedes "effect"
- No signalling outside the future
- No backward-in-time causal influences
- No closed timelike curves
- ...

Tied to space-time structure
"Causal structure" of Minkowski space-time is acyclic

Information-theoretic notions

- Causal models
- Compositional structure of circuits
- Conservation of probabilities
- Causal separability of process matrices
- ...

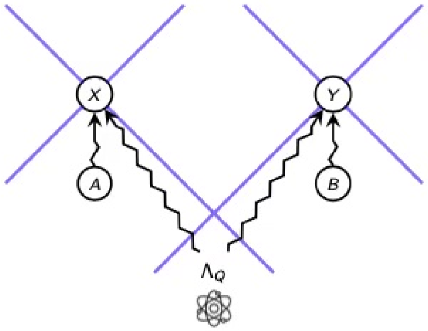
Independent of space-time structure
Allows for cyclic causation
Can have causation without signalling

Quantum theory challenges certain notions (e.g., classical causal models [1]) but is compatible with certain others (e.g., no superluminal signalling).

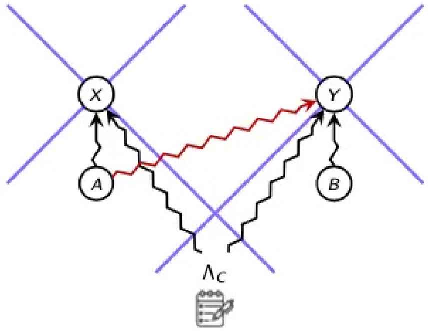
[1] Wood and Spekkens. NJP 2015

Causality, space-time and their compatibility

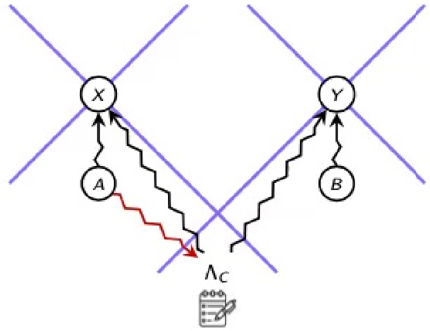
Several different approaches for describing quantum causation in response to Bell's theorem, e.g.,



(a) Local, not realist



(b) Non-local HV



(c) Retrocausality

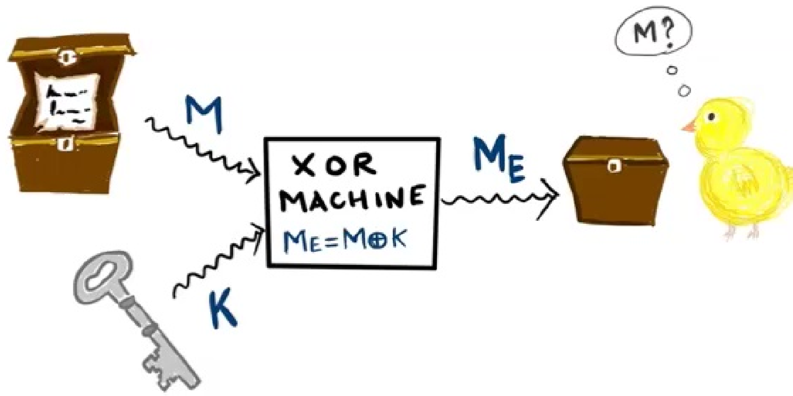
To consider all such possibilities, also beyond the Bell scenario, need general framework that:

1. Disentangles causality from space-time
2. Allows for causation w/o signalling, cyclic and non-classical causal influences
3. Models correlations and interventions
4. Specifies when a causal model can be embedded in space-time w/o superluminal signalling

Q: Is it possible to embed a cyclic causal structure in Minkowski space-time w/o superluminal signalling?

Applications of cyclic and fine-tuned causal models

Cryptography: Security of protocols such as the OTP rely on fine-tuned causal correlations. (*Fine-tuning* = *"causation without correlation"*)



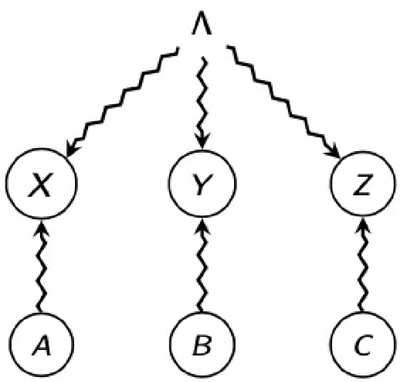
Post-quantum theories beyond GPTs: Post-quantum theories beyond standard no-signalling probabilistic theories, yet compatible with "relativistic causality" [1,2].

Causal discovery problem: "No fine-tuning" assumption is often made, understanding fine-tuned influences would help relax this.

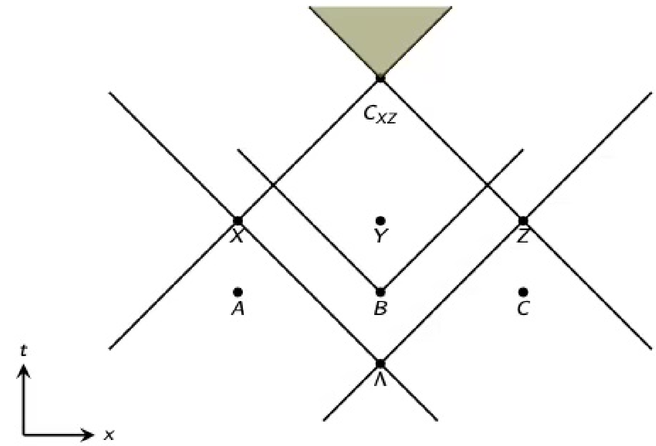
Cyclic quantum causal structures: Can model physical scenarios with feedback as well as exotic closed timelike curves, distinguished by space-time embedding.

[2] Grunhaus, Popescu, Rohrlich. PRA 1996.
[3] Horodecki, Ramanathan. Nat. Comms. 2019.

Jamming correlations in the tripartite Bell scenario



(a)



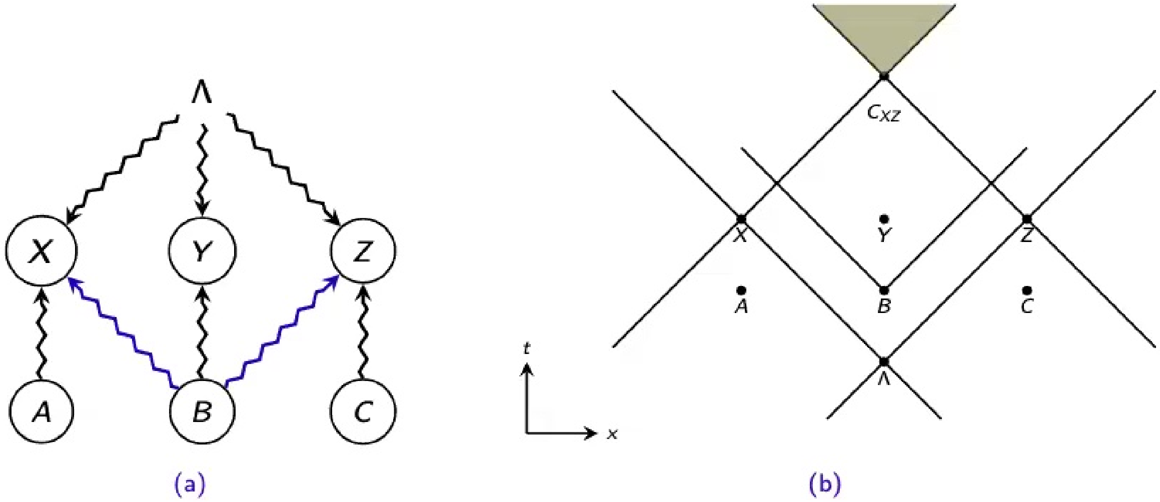
(b)

- **No signalling conditions:** Outcomes of any subset of parties cannot depend on the settings of complementary set of parties. In particular,

$$P(XZ|ABC) = P(XZ|AC), \quad P(X|ABC) = P(X|A), \quad P(Z|ABC) = P(Z|C).$$

- **Jamming [2,3]:** In space-time configuration (b), dropping the first constraint does not lead to superluminal signalling.

Tripartite Bell scenario



Jamming correlations

$$P(XZ|ABC) \neq P(XZ|AC), \quad P(X|ABC) = P(X|A), \quad P(Z|ABC) = P(Z|C).$$

Bob can "jam" correlations between space-like separated parties Alice and Charlie

FRAMEWORK:

CAUSALITY, SPACE-TIME AND HOW THEY FIT TOGETHER

Causality: causal structures and observed correlations

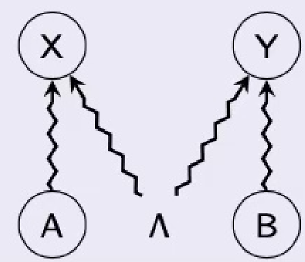
Causal structure

- Directed graph $\mathcal{G}$.
- Nodes can be observed (classical random variables) or unobserved (classical, quantum or GPT systems).
- Edges ⇝ denote causation

Pearl 2009. Henson, Lal, Pusey 2014.

Observed distribution

Joint probability distribution $P_{\mathcal{G}}(S_{obs})$ over all observed nodes S_{obs} of $\mathcal{G}$.
E.g., $P(ABXY)$ in $\mathcal{G}^{Bell}$



Graph separation (d-separation) in $\mathcal{G}$ implies conditional independence in $P_{\mathcal{G}}(S_{obs})$.
E.g., no signalling conditions in $\mathcal{G}^{Bell}$: $P(X|AB) = P(X|A), P(Y|AB) = P(Y|B)$.

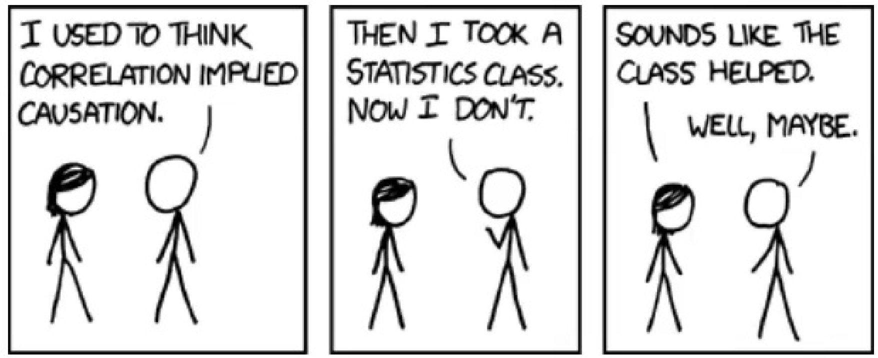
Compatibility of $P_{\mathcal{G}}(S_{obs})$ with $\mathcal{G}$

For all disjoint subsets X, Y, Z of observed nodes S_{obs},

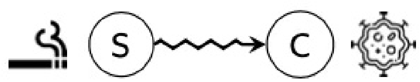
$$(X \perp^d Y|Z) \quad \Rightarrow \quad P_{\mathcal{G}}(XY|Z) = P_{\mathcal{G}}(X|Z)P_{\mathcal{G}}(Y|Z)$$

If converse holds, then **faithful**, otherwise **unfaithful/fine-tuned** causal model.

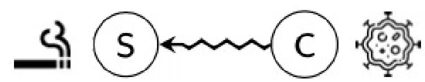
Interventions and affects relations



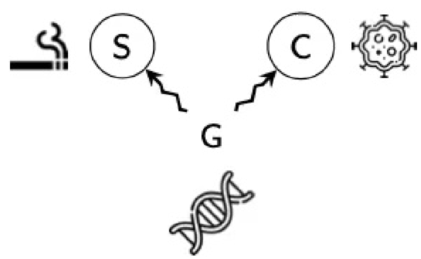
Correlation alone can't single out a causal explanation, need interventions!



(a)



(b)



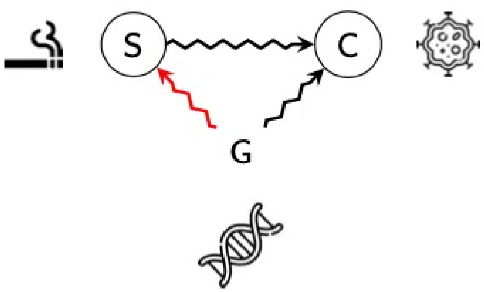
(c)

Interventions and affects relations

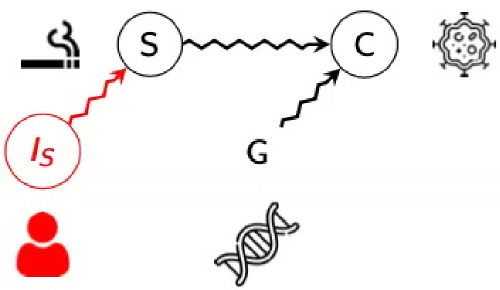
Interventions

An intervention on an observed node X in $\mathcal{G}$ corresponds to cutting off all incoming arrows to X, and independently forcing it to take some fixed value $X = x$, defines new causal structure $\mathcal{G}_{\text{do}(X)}$.

$\mathcal{G}$



$\mathcal{G}_{\text{do}(S)}$



Affects relation (captures the notion of signalling)

For any disjoint subsets $X, Y \subseteq S_{obs}$, we say that X *affects* Y if there exists a value x of X such that

$$P_{\mathcal{G}_{do(X)}}(Y|X = x) \neq P_{\mathcal{G}}(Y).$$

We derive rules for relating $P_{\mathcal{G}_{do(X)}}$ and $P_{\mathcal{G}}$ for these general class of causal models

Classification of causal arrows

X affects $Y \Rightarrow X$ is a cause of Y, but converse not true in fine-tuned models.

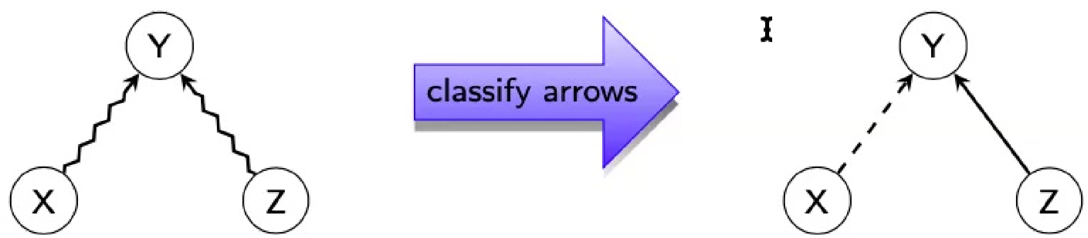
Motivates classification of causal arrows

Solid and dashed arrows

For any X, Y in S_{obs},
Causal arrow $X \rightsquigarrow Y$ with X affects Y is a solid arrow $X \longrightarrow Y$
Causal arrow $X \rightsquigarrow Y$ with X does not affect Y is a dashed arrow $X \dashrightarrow Y$

Important rule: whenever Y is correlated with a parentless node X, X affects Y.

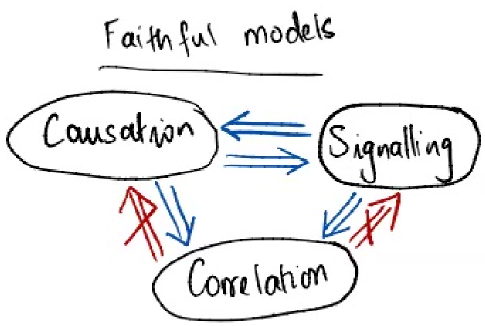
E.g.: $Y = X \oplus Z$ where X is non-uniform and Z is uniformly distributed, and all are classical bits.

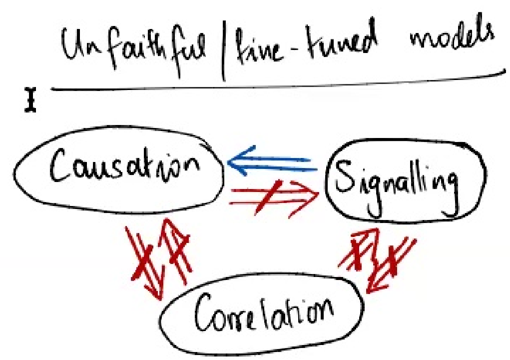


Quick recap

So far: causation without reference to space-time structure

- Directed graph who's topology imposes independence constraints on observed correlations.
- Beyond correlations: interventions and affects relations.
- Causation $\not\Rightarrow$ signalling: Classify causal arrows.





Signalling is characterised by affects relations, will be key for compatibility with space-time

Space-time structure

Model space-time by a partially ordered set $\mathcal{T}$, e.g. light-cones in Minkowski space-time.

To make operational statements about $\mathcal{T}$, we must embed physical systems (here, observed RVs)

Ordered random variable (ORV)

ORV $\mathcal{X}$ = Random variable X + location $O(X) \in \mathcal{T}$

Inclusive future of an ORV

$$\overline{\mathcal{F}}(\mathcal{X}) := \{\alpha \in \mathcal{T} : \alpha \geq O(X)\}.$$

Embedding a causal model in space-time

- Two types of order relations: pre-order $\rightsquigarrow$ of $\mathcal{G}$, the partial order $\prec$ of $\mathcal{T}$.
- How can $\mathcal{G}$ be "compatibly" embedded in $\mathcal{T}$? Depends on where ORVs can be accessed in $\mathcal{T}$.

Accessible region of an RV $X \in S_{obs}$

Set of space-time points $\mathcal{R}_X \subseteq \mathcal{T}$ at which it is possible to have a copy of X.

It follows that the AR of set of a RVs S_1 is given as $\mathcal{R}_{S_1} = \bigcap_{s_1 \in S_1} \mathcal{R}_{s_1}$.

Embedding = "random variable+location+accessible region"

An embedding $\mathscr{E}$ of a set of RVs S in a partial order $\mathcal{T}$ corresponds to an assignment of

- **locations** in $O(X) \in \mathcal{T}$ to each $X \in S$, to produce an ORV $\mathcal{X}$ and
- **accessible regions** (w.r.t. $\mathcal{T}$) to each $X \in S$.

Compatibility of causal model with space-time embedding (**compat**)

Captures the intuition of no signalling outside the future

In faithful models, X affects $Y \Rightarrow \overline{\mathcal{F}}(\mathcal{Y}) \subseteq \overline{\mathcal{F}}(\mathcal{X})$ is sufficient. But *not* in fine-tuned models!

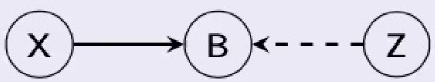
Example 1: "jamming"

B affects $\{X, Z\}$, B does not affect X, B does not affect Z

compat: $\overline{\mathcal{F}}(\mathcal{X}) \cap \overline{\mathcal{F}}(\mathcal{Z}) \subseteq \overline{\mathcal{F}}(\mathcal{B})$

Example 3: another collider

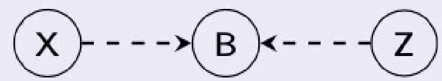
$\{X, Z\}$ affects B, X affects B, Z does not affect B



compat: $\overline{\mathcal{F}}(\mathcal{B}) \subseteq \overline{\mathcal{F}}(\mathcal{X}) \cap \overline{\mathcal{F}}(\mathcal{Z})$

Example 2: collider

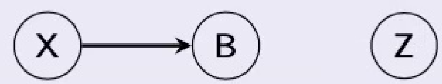
$\{X, Z\}$ affects B, X does not affect B, Z does not affect B



compat: $\overline{\mathcal{F}}(\mathcal{B}) \subseteq \overline{\mathcal{F}}(\mathcal{X}) \cap \overline{\mathcal{F}}(\mathcal{Z})$

Example 4: not a collider

$\{X, Z\}$ affects B, X affects B, Z does not affect B



compat: $\overline{\mathcal{F}}(\mathcal{B}) \subseteq \overline{\mathcal{F}}(\mathcal{X})$

Compatibility of causal model with space-time embedding (**compat**)

Captures the intuition of no signalling outside the future

In faithful models, X affects $Y \Rightarrow \overline{\mathcal{F}}(\mathcal{Y}) \subseteq \overline{\mathcal{F}}(\mathcal{X})$ is sufficient. But *not* in fine-tuned models!

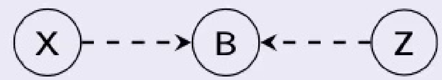
Example 1: "jamming"

B affects $\{X, Z\}$, B does not affect X, B does not affect Z

compat: $\overline{\mathcal{F}}(\mathcal{X}) \cap \overline{\mathcal{F}}(\mathcal{Z}) \subseteq \overline{\mathcal{F}}(\mathcal{B})$

Example 2: collider

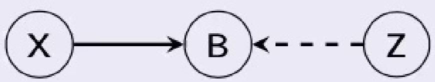
$\{X, Z\}$ affects B, X does not affect B, Z does not affect B



compat: $\overline{\mathcal{F}}(\mathcal{B}) \subseteq \overline{\mathcal{F}}(\mathcal{X}) \cap \overline{\mathcal{F}}(\mathcal{Z})$

Example 3: another collider

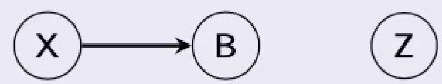
$\{X, Z\}$ affects B, X affects B, Z does not affect B



compat: $\overline{\mathcal{F}}(\mathcal{B}) \subseteq \overline{\mathcal{F}}(\mathcal{X}) \cap \overline{\mathcal{F}}(\mathcal{Z})$

Example 4: not a collider

$\{X, Z\}$ affects B, X affects B, Z does not affect B



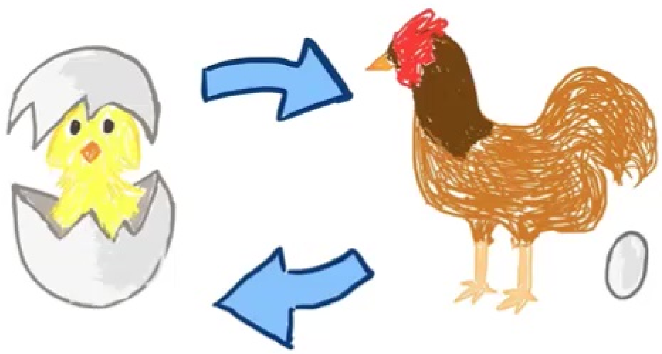
compat: $\overline{\mathcal{F}}(\mathcal{B}) \subseteq \overline{\mathcal{F}}(\mathcal{X})$

To distinguish between Eg 3 and 4, we introduce new concept of higher order affects relations

Causal loops

A causal loop is a directed cycle in $\mathcal{G}$

Recall: We had to classify causal arrows (solid vs dashed) based on affects relations. Must classify causal loops as well (affects vs hidden loops).



- **Affects causal loop (ACL1):** X affects Y and Y affects X.
- **Affects causal loop Type 2 (ACL2):** $\{X, Z\}$ affects Y and Y affects $\{X, Z\}$.
- **Hidden causal loop (HCL):** Directed cycles in $\mathcal{G}$ that are not affects loops.

ACL1, ACL2, and generalisations operationally certify the cyclicity of underlying causal structure

Causal loops and compatible space-time embeddings

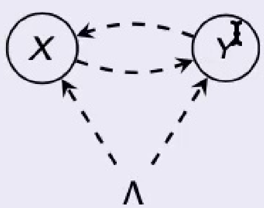
An affects causal loop of Type 1

$X = Y$, X affects Y, Y affects X.

No non-trivial and compatible space-time embedding!

A hidden causal loop

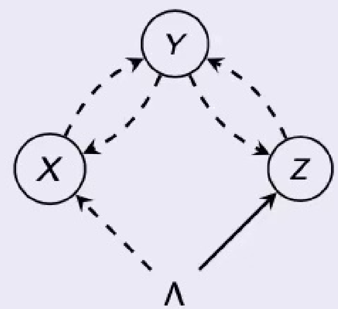
Λ unobserved



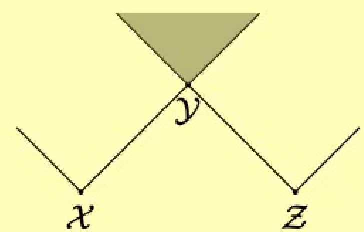
Compatible with any space-time embedding

An affects causal loop of Type 2

$Y = X \oplus Z$, Y affects $\{X, Z\}$ and $\{X, Z\}$ affects Y.



Exists non-trivial and compatible space-time embedding: $\overline{\mathcal{F}}(\mathcal{Y}) = \overline{\mathcal{F}}(\mathcal{X}) \cap \overline{\mathcal{F}}(\mathcal{Z})$

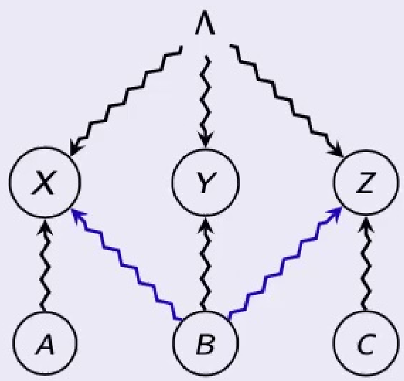


JAMMING AND SUPERLUMINAL SIGNALLING

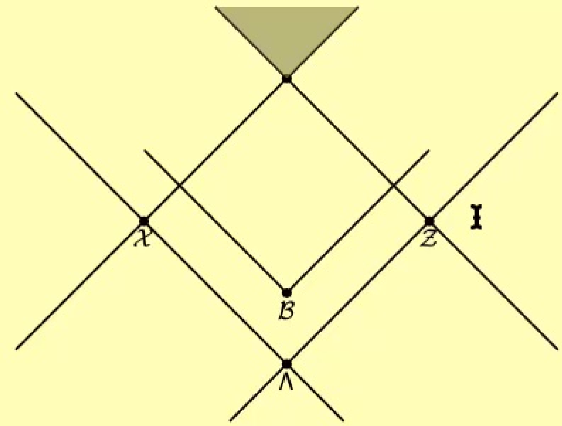
Jamming can lead to superluminal signalling

Jamming causal structure ($\mathcal{G}^{jam}$)

B affects $\{X, Z\}$, B neither affects X nor Z



Space-time embedding ($\mathscr{E}^{jam}$)



[1] Grunhaus, Popescu, Rohrlich, PRA 1996.
[2] Horodecki, Ramanathan, Nat. Comm. 2019.

Summary and conclusions

- **General framework** for analysing causality and its compatibility with space-time.
- **Causal loops w/o superluminal signalling** possibility of causal loops that can be operationally detected yet do not allow signalling outside space-time future
- **Post-quantum jamming theories** provides a causal modelling framework for these.
- **Jamming can lead to superluminal signalling** shows missing assumptions in previous results, new features of such post-quantum theories.
- **Applications for causal discovery** New technical results reg. interventions in fine-tuned causal models

Take home message: Important to separate

- Causality from space-time
- Causation from correlation
- Different notions of causation

Outlook

- Theories with causal loops w/o superluminal signalling, allowed correlations?
- Causality and consistency in the presence of causal loops, time-symmetry vs cyclic causation.
- Further properties of jamming theories: tensor product structure, local tomography?
- More general space-time embeddings (in preparation with Renato Renner)
  - Embedding quantum systems in space-time
  - Classical/quantum uncertainties in space-time location
  - Superpositions of space-time trajectories

Outlook

- Theories with causal loops w/o superluminal signalling, allowed correlations?
- Causality and consistency in the presence of causal loops, time-symmetry vs cyclic causation.
- Further properties of jamming theories: tensor product structure, local tomography?
- More general space-time embeddings (in preparation with Renato Renner)
  - Embedding quantum systems in space-time
  - Classical/quantum uncertainties in space-time location
  - Superpositions of space-time trajectories