Title: Simulations of cosmic structure formation with fuzzy dark matter - Simon May, Max Planck Institute of Astrophysics

Speakers:

Series: Cosmology & Gravitation

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Abstract: In the fuzzy dark matter model, dark matter consists of "axion-like" ultra-light scalar particles of mass around 10?²² eV. This candidate behaves similarly to cold dark matter on large scales, but exhibits different properties on smaller (galactic) scales due to macroscopic wave effects arising from the extremely light particles' large de Broglie wavelengths. It has both particle physics motivations and a rich astrophysical phenomenology, giving rise to notable differences in the structures on highly non-linear scales due to the manifestation of wave effects, which can impact a number of contentious small-scale tensions in the standard cosmological model, ?CDM. Some of the unique features include transient wave interference patterns and granules, the presence flat-density cores (solitons) at the centers of dark matter halos, and the formation of quantized vortices. I will present large numerical simulations of cosmic structure formation with this dark matter model - including the full non-linear wave dynamics - using a pseudo-spectral method to numerically solve the Schrödinger-Poisson equations, and the significant computational challenges associated with these equations. I will discuss several observables, such as the evolution of the matter power spectrum, the fuzzy dark matter halo mass function, dark matter halo density profiles, and the question of a fuzzy dark matter core-halo mass relation, using results obtained from these simulations, and contrast them with corresponding results for the cold dark matter model.

SIMULATIONS OF COSMIC STRUCTURE FORMATION WITH FUZZY DARK MATTER

Simon May
simon.may@mpa-garching.mpg.de

Max Planck Institute for Astrophysics

1st February 2022

MAX-PLANCK-GESELLSCHAFT

Outline

Introduction

- Fuzzy dark matter: Motivation and theoretical background
- Impact of fuzzy dark matter on structure formation
  - #1: Initial conditions
  - #2: Dynamics/structure
  - #3: Dynamics/objects

Numerical methods and challenges

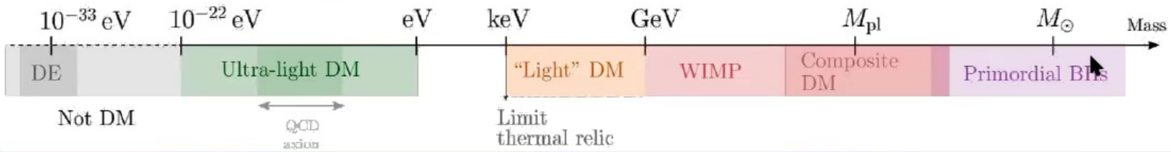
Cosmological full FDM wave simulations in “large” volumes

- Dark matter power spectrum
- First measurement of the halo mass function
- Dark matter halo profiles and cores vs. cusps
- The core–halo mass relation

Other results and constraints

Summary & outlook

What is "fuzzy dark matter"?



- F(C)DM, BECDM, ULDM, ELBDM, ψDM, quantum-wave DM, (ultra-light) axion(-like) DM (ULA, ALP)...
- New **extremely light scalar** particle ($m \approx 10^{-22}$ eV/c^2!)
  - → electron: 511 keV
- Non-thermal production mechanism (thus not ultra-hot)
- Aggregations of bosons can form a **Bose–Einstein condensate**
- Tiny mass
  - ⇒ large de Broglie wavelength ($\lambda \sim 1/m$)
  - ⇒ **macroscopic quantum (wave) effects** on kpc scales
  - Structures resist collapse below quantum wavelength (**Heisenberg uncertainty principle**/"quantum pressure")
- **Large scales: equivalent to cold dark matter**

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Motivation for fuzzy dark matter

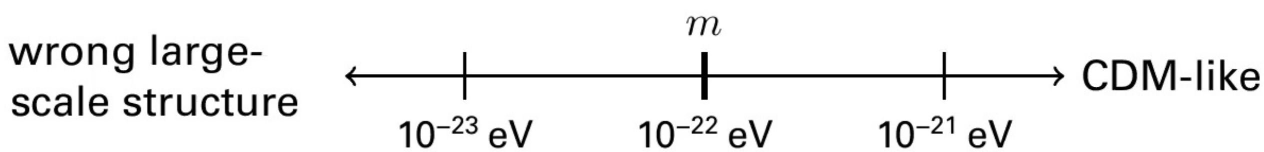
Particle physics perspective:

- Original concept – strong CP problem:
  Why doesn't QCD violate CP symmetry?
- Solved by Peccei–Quinn $\mathrm{U}(1)$ symmetry
  and (pseudo-)scalar field (*axion*!)
  Peccei and Quinn (1977)!
- Fuzzy dark matter is **not** the QCD axion, but axion-like particles are a common feature of early-universe theories ("axiverse")

Astrophysics perspective:

- Small-scale challenges (cusp–core, missing satellites, ...)

$\rightarrow$ Ultra-light scalars: WIMP alternative, could improve this



- **No sign of (WIMP) CDM**

The fuzzy dark matter equations: derivation

Schrödinger–Poisson system

- Add a scalar field to the Einstein–Hilbert action of general relativity

$$S = \frac{1}{\hbar c^2} \int \mathrm{d}^4x \sqrt{-g} \left(\frac{1}{2} g^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi) - \frac{1}{2} \frac{m^2 c^2}{\hbar^2} \phi^2 - \frac{\lambda}{\hbar^2 c^2} \phi^4 \right)$$

  - *Superfluid DM* without self-interaction ($\lambda = 0$ or $T \to 0$)
  - *QCD axion* case: originates from periodic potential like $V(\phi) \sim \Lambda^4 (1 - \cos(\phi / f_a))$ for $\phi \ll f_a$
- Rewrite

$$\phi = \sqrt{\frac{\hbar^3 c}{2m}} \frac{1}{2} \mathrm{Re}\left(\psi e^{-i \frac{mc^2}{\hbar} t} \right) = \sqrt{\frac{\hbar^3 c}{2m}} \left(\psi e^{-i \frac{mc^2}{\hbar} t} + \psi^* e^{i \frac{mc^2}{\hbar} t} \right)$$

  and take non-relativistic limit with perturbed FLRW metric

$$\mathrm{d}s^2 = \left(1 + \frac{2\Phi}{c^2} \right) c^2 \mathrm{d}t^2 - a(t)^2 \left(1 - \frac{2\Phi}{c^2} \right) \mathrm{d}\vec{x}^2$$

- Result: "Schrödinger" (Gross–Pitaevskii) equation

$$i\hbar \left(\partial_t \psi + \frac{3}{2} H \psi \right) = -\frac{\hbar^2}{2m} \nabla^2 \psi + m \Phi \psi$$

The fuzzy dark matter equations

Schrödinger–Poisson system

- Non-relativistic "Schrödinger equation":

$$i\hbar\left(\partial_t\psi + \frac{3}{2}H\psi\right) = -\frac{\hbar^2}{2m}\nabla^2\psi + m\Phi\psi$$

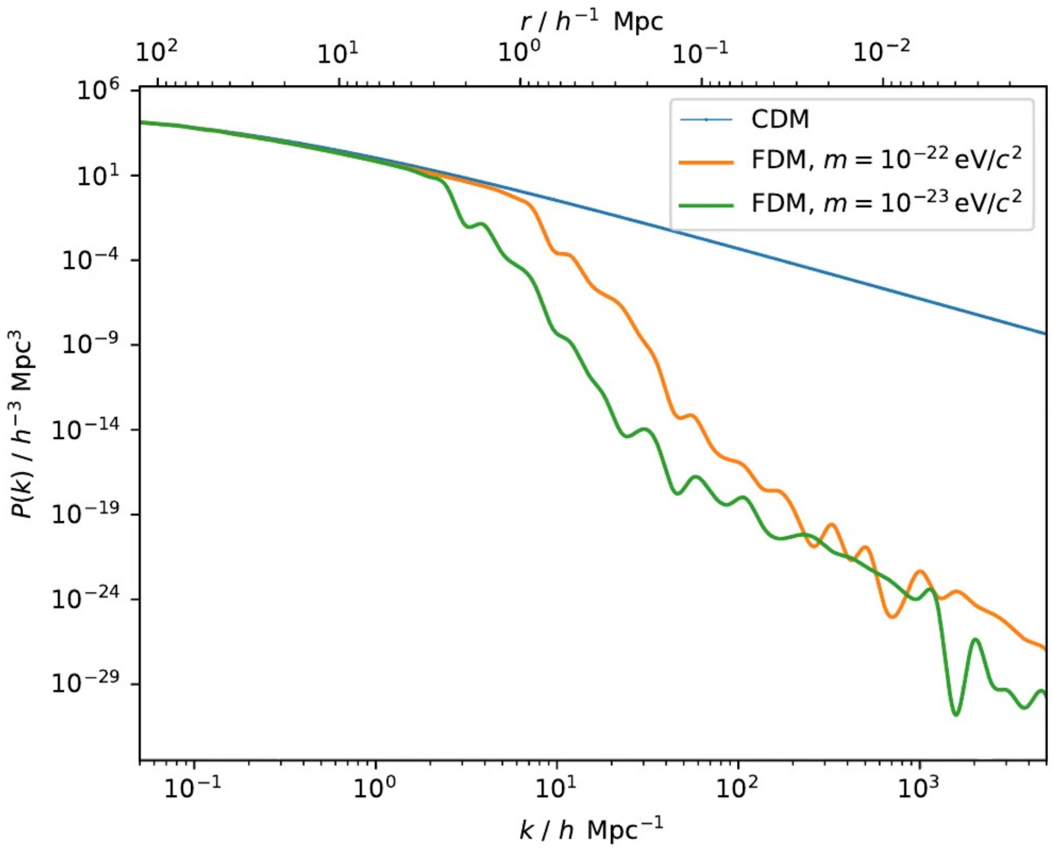
- Mean field approximation: interpretation as the **single macroscopic wave function of a Bose-Einstein condensate** with density $\rho = m|\psi|^2$
- **"FDM equations": nonlinear Schrödinger–Poisson** (or Gross–Pitaevskii–Poisson) **system of equations:**

$$i\hbar\partial_t\psi_\mathrm{c} = -\frac{\hbar^2}{2ma^2}\nabla_\mathrm{c}^2\psi_\mathrm{c} + \frac{m}{a}\Phi_\mathrm{c}\psi_\mathrm{c}$$

$$\nabla_\mathrm{c}^2\Phi_\mathrm{c} = 4\pi Gm(|\psi_\mathrm{c}|^2 - \langle|\psi_\mathrm{c}|^2\rangle)$$

Only a single scale, determined by $\hbar/m$ ($\rightarrow$ wavelength)

Impact of fuzzy dark matter – #1: Initial conditions

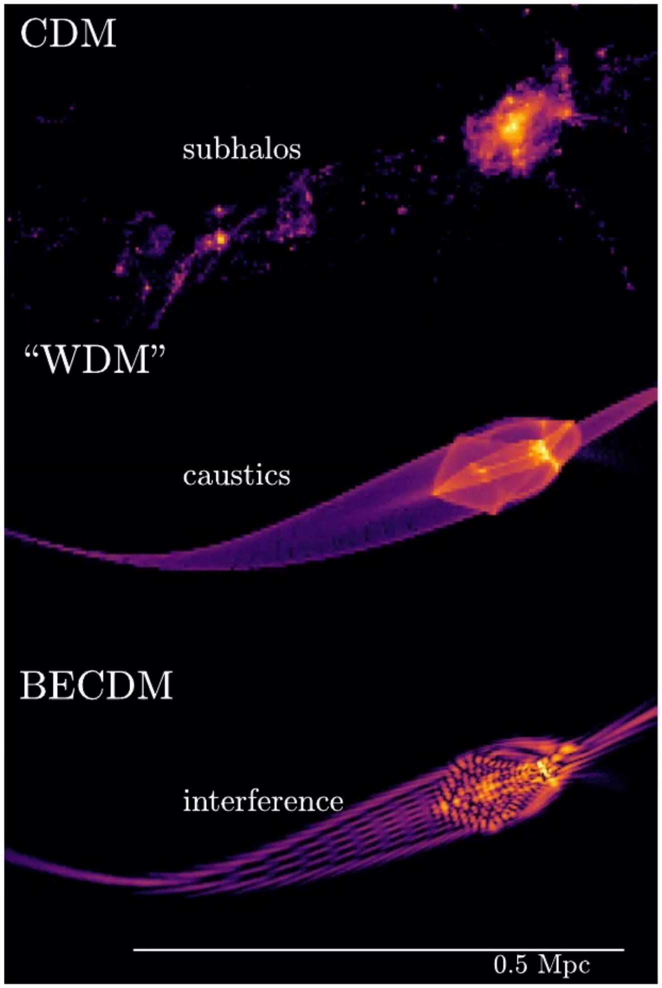


- Changes the primordial **matter power spectrum** compared to CDM
- $P(k)$ can be computed using `AxionCAMB`, Hlozek, Grin, Marsh, et al. (2015)
- Seeds of structure suppressed below quantum wavelength

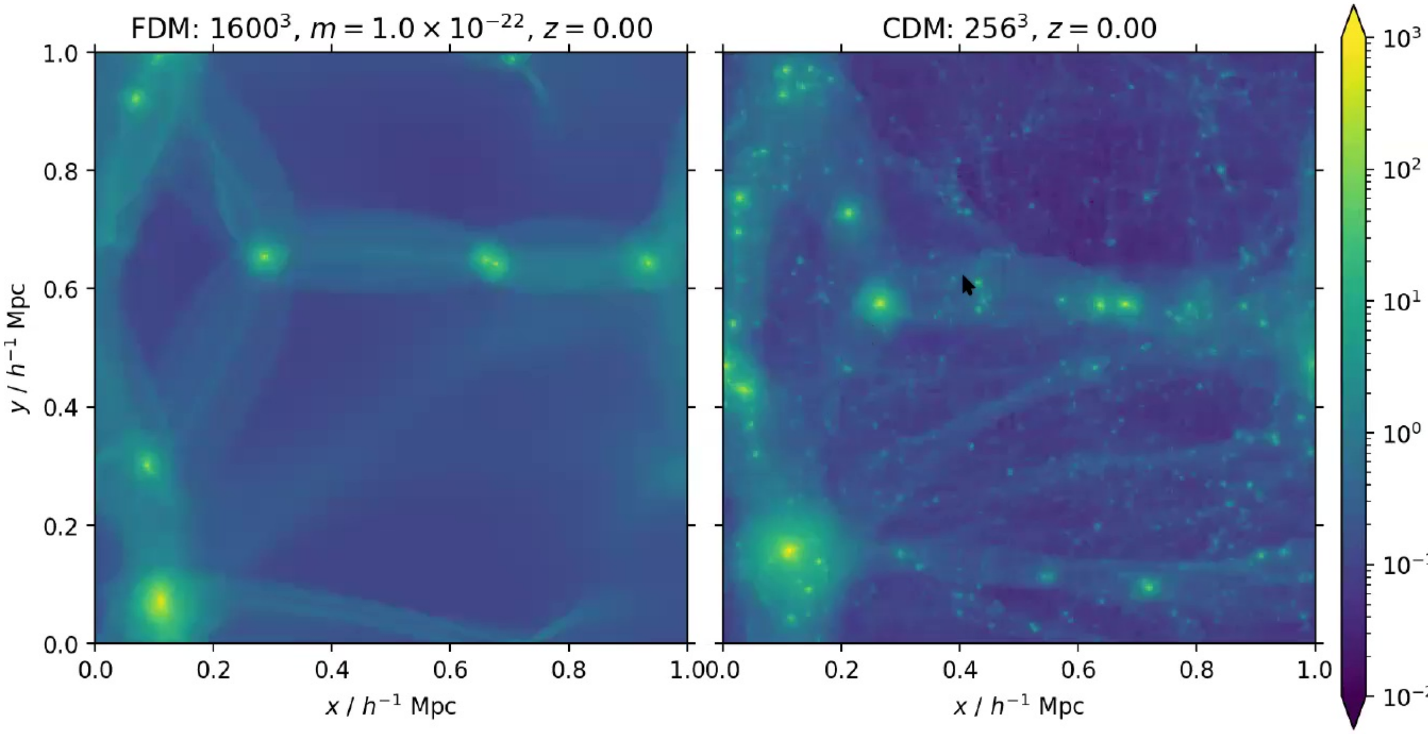
Impact of fuzzy dark matter – #2: Dynamics/structure

- Small-scale (sub-)structure suppressed compared to CDM
- Smooth (like WDM) instead of clumpy (like CDM)
- Quantum fluctuations, wave interference patterns

Mocz, Fialkov, Vogelsberger, et al. (2020)



Impact of fuzzy dark matter – #2: Dynamics/structure



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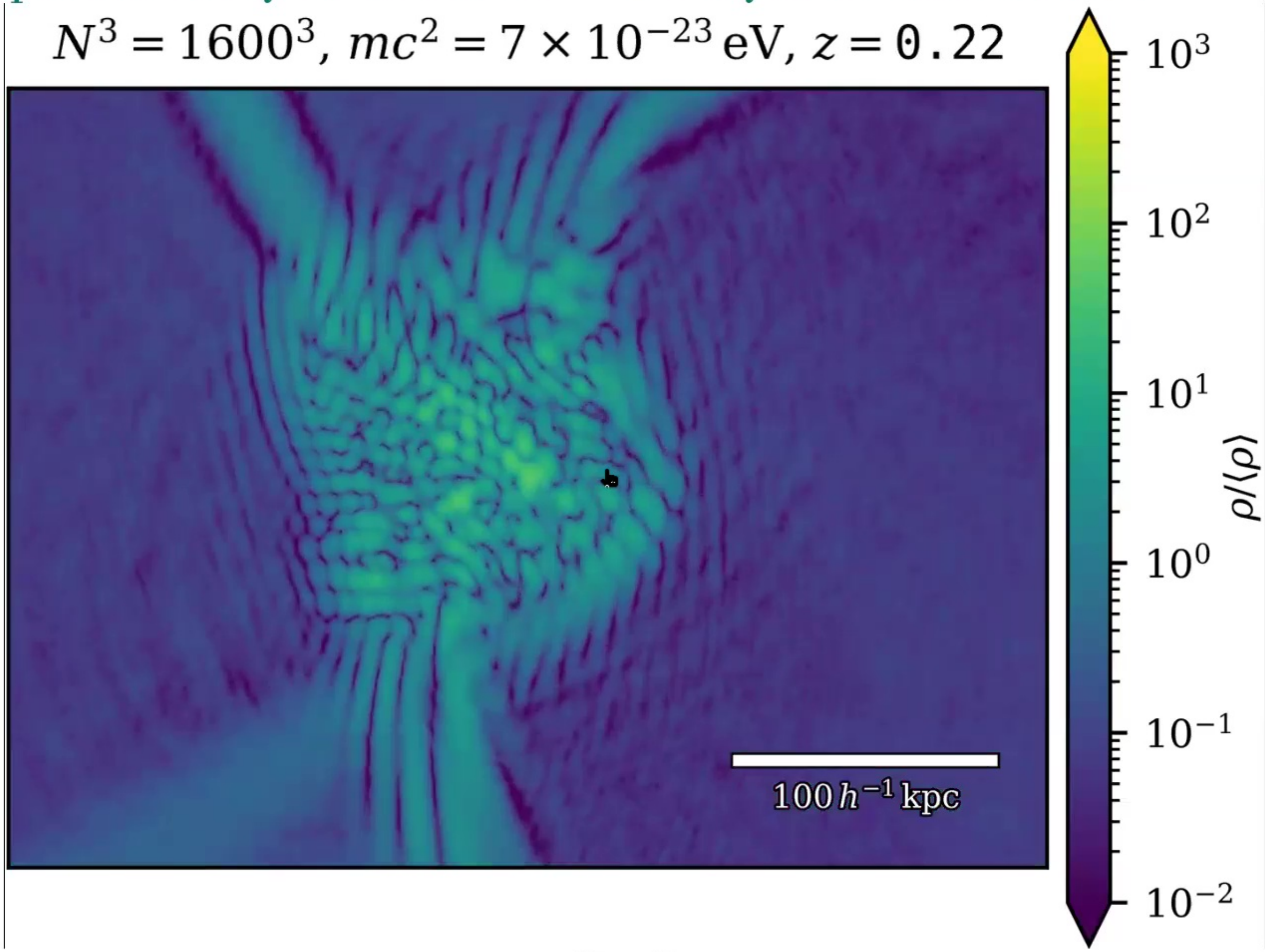
Impact of fuzzy dark matter – #2: Dynamics/structure



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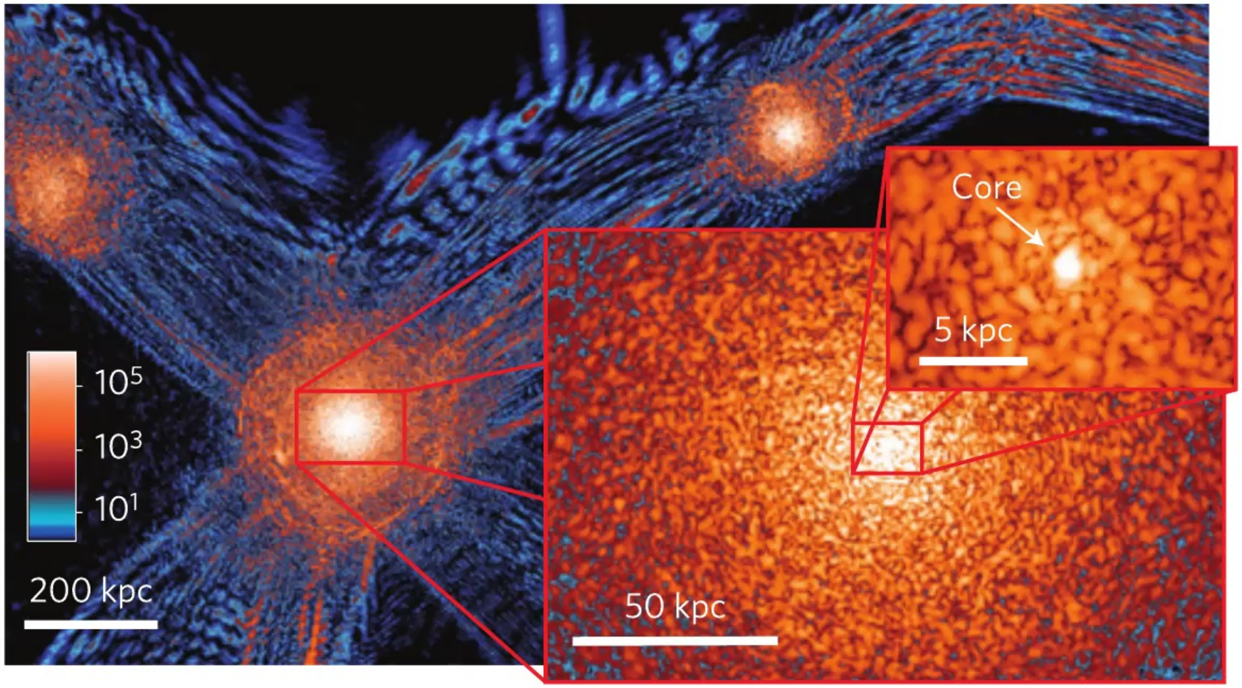
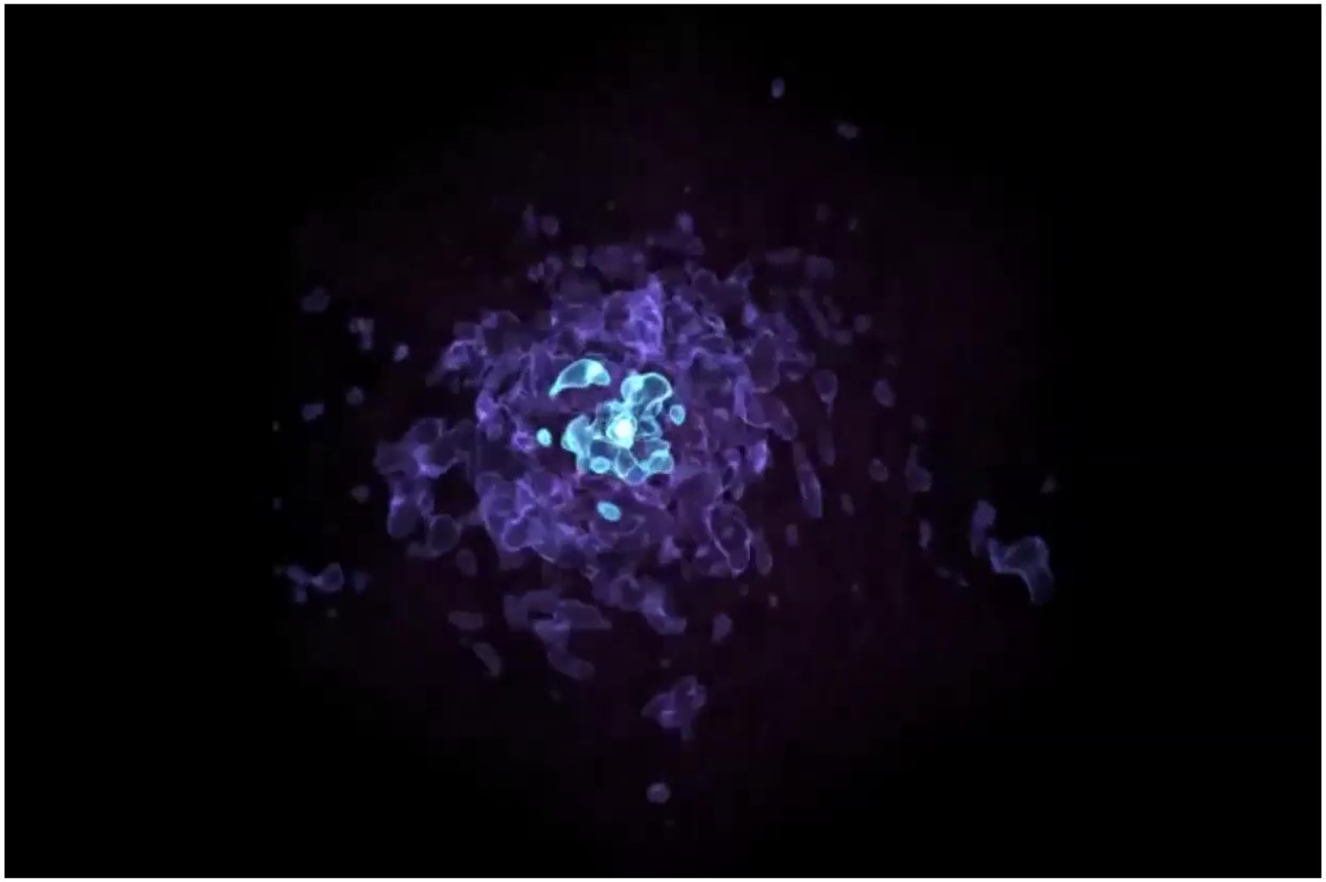
Impact of fuzzy dark matter – #3: Dynamics/objects (solitons)



Schive, Chiueh, and Broadhurst (2014)

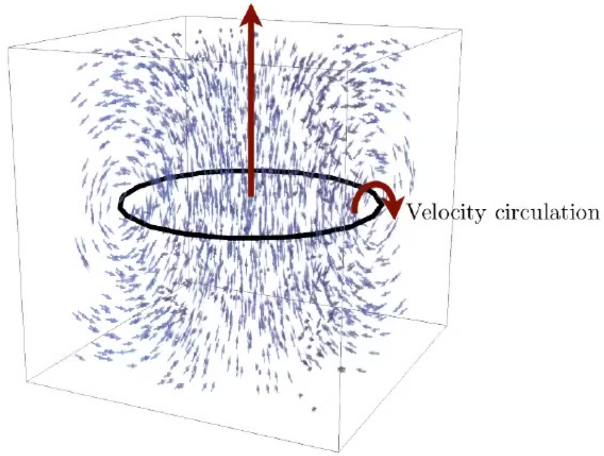
- Ground state of FDM equations: **soliton** (spherical "**core**")
- Soliton(-like) cores form at the center of all virialized halos
- Fluctuations around & within soliton cores ⇒ **dynamical heating** (e. g. of stars), potential **disruption** (e. g. of globular clusters)

Impact of fuzzy dark matter – #3: Dynamics/objects (solitons)

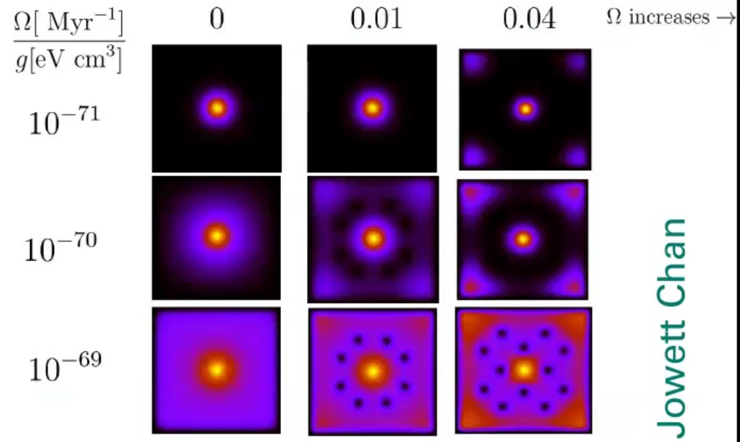


Mocz, Vogelsberger, Robles, et al. (2017)

Impact of fuzzy dark matter – #3: Dynamics/objects (vortices)



Hui, Joyce, Landry, et al. (2021)



Jowett Chan

- FDM fluid velocity is a gradient flow
- However, vorticity can form around interference regions ($\rho = 0$), where velocity is undefined

$$C = \oint \vec{v} \cdot \mathrm{d}\vec{\ell} = 2\pi n \frac{\hbar}{m} \quad (n \in \mathbb{Z}) \quad \to \text{vortex rings}$$

- With self-interactions: superfluid, forms vortices when rotating
- We know very little about these objects!

Numerical approaches to fuzzy dark matter simulations

I. Schrödinger–Poisson equations

$$i\hbar\partial_t\psi = -\frac{\hbar^2}{2ma^2}\nabla^2\psi + \frac{m}{a}\Phi\psi$$
$$\nabla^2\Phi = 4\pi G m(|\psi|^2 - \langle|\psi|^2\rangle)$$

II. Madelung formulation (fluid dynamics representation)

$$\partial_t\rho + \nabla\cdot\rho\vec{v} = 0$$
$$\partial_t\vec{v} + \frac{1}{a^2}(\vec{v}\cdot\nabla)\vec{v} = -\nabla\left(\frac{1}{a}\Phi - \underbrace{\frac{\hbar^2}{2m^2a^2}\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}}_{=Q}\right)$$
$$\nabla^2\Phi = 4\pi G(\rho - \langle\rho\rangle)$$

$$\psi = \sqrt{\frac{\rho}{m}}e^{i\alpha}$$
$$\rho = m|\psi|^2$$
$$\vec{v} = \frac{\hbar}{m}\nabla\alpha$$

- Phase is undefined for $\rho = 0$
  $\Rightarrow$ significant effects on overall evolution

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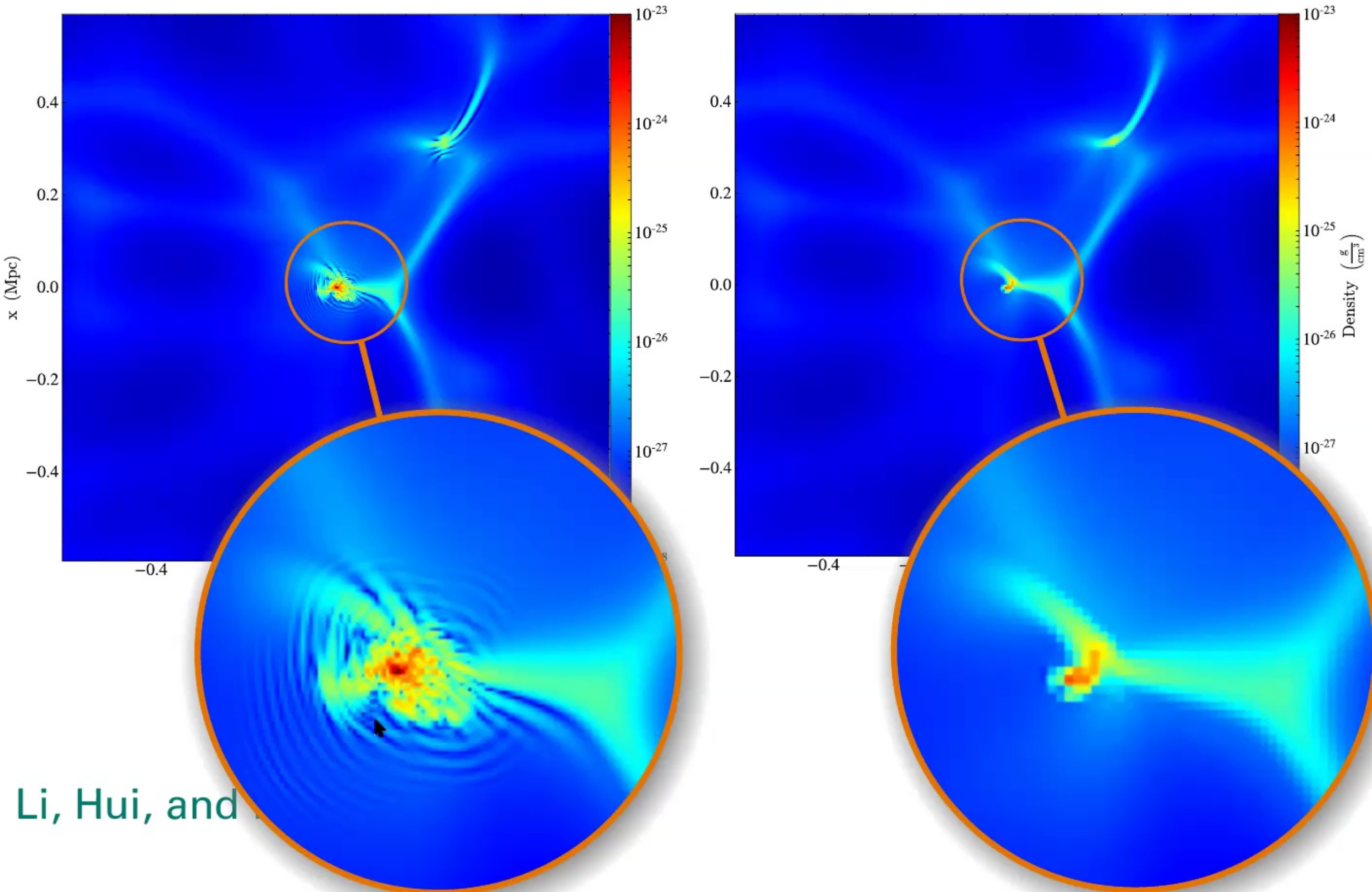
$$\psi = \sqrt{\frac{\rho}{m}}e^{i\alpha}$$
$$\rho = m|\psi|^2$$
$$\vec{v} = \frac{\hbar}{m}\nabla\alpha$$

"quantum potential"
"quantum pressure"

- Phase is undefined for $\rho = 0$
  $\Rightarrow$ significant effects on overall evolution

Schrödinger–Poisson vs. Madelung formulation



Li, Hui, and

Using pseudo-spectral methods to simulate fuzzy dark matter

$$i\hbar\partial_t\psi = -\frac{\hbar^2}{2ma^2}\nabla^2\psi + \frac{m}{a}\Phi\psi$$
$$\nabla^2\Phi = 4\pi G m(|\psi|^2 - \langle|\psi|^2\rangle)$$

- Symmetrized split-step Fourier method ("kick–drift–kick")
- Small time step Δt:

$$\psi(t+\Delta t,\vec{x}) = \mathcal{T} e^{-i\int_t^{t+\Delta t}\left(-\frac{\hbar}{2m}\frac{1}{a(t')^2}\nabla^2 + \frac{m}{\hbar}\frac{1}{a(t')}\Phi(t',\vec{x})\right)\mathrm{d}t'}\psi(t,\vec{x})$$
$$\approx e^{i\frac{\Delta t}{2}\left(\frac{\hbar}{m}\frac{1}{a(t)^2}\nabla^2 - \frac{m}{\hbar}\frac{1}{a(t)}\Phi(t+\Delta t,\vec{x}) - \frac{m}{\hbar}\frac{1}{a(t)}\Phi(t,\vec{x})\right)}\psi(t,\vec{x})$$
$$\approx \underbrace{e^{-i\frac{m}{\hbar}\frac{\Delta t}{2}\frac{1}{a(t)}\Phi(t+\Delta t,\vec{x})}}_{\text{“kick”}}\ \underbrace{e^{i\frac{\hbar}{m}\frac{1}{a(t)^2}\frac{\Delta t}{2}\nabla^2}}_{\text{“drift”}}\ \underbrace{e^{-i\frac{m}{\hbar}\frac{\Delta t}{2}\frac{1}{a(t)}\Phi(t,\vec{x})}}_{\text{“kick”}}\ \psi(t,\vec{x})$$

- Discretize ⇒ **algorithm** on a Cartesian mesh:

$$\psi \leftarrow e^{-i\frac{m}{\hbar}\frac{1}{a}\frac{\Delta t}{2}\Phi}\psi \qquad \text{kick}$$
$$\psi \leftarrow \mathrm{FFT}^{-1}\left(e^{-i\frac{\hbar}{m}\frac{1}{a^2}\frac{\Delta t}{2}k^2}\,\mathrm{FFT}(\psi)\right) \qquad \text{drift}$$
$$\Phi \leftarrow \mathrm{FFT}^{-1}\left(-\frac{1}{k^2}\mathrm{FFT}\big(4\pi G m(|\psi|^2 - \langle|\psi|^2\rangle)\big)\right) \qquad \text{update potential}$$
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Pseudo-spectral methods for fuzzy dark matter

Symmetrized split-step Fourier method ("kick–drift–kick")

$$i\hbar\partial_t\psi = -\frac{\hbar^2}{2ma^2}\nabla^2\psi + \frac{m}{a}\Phi\psi$$
$$\nabla^2\Phi = 4\pi Gm(|\psi|^2 - \langle|\psi|^2\rangle)$$

Algorithm:

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Choice of time step: $\Delta t < \min\left(\frac{4}{3\pi}\frac{m}{\hbar}a^2\Delta x^2\ ,\ 2\pi\frac{\hbar}{m}a\frac{1}{|\Phi_{\max}|}\right)$

- "Exact" solution
- Automatic conservation of mass
- Can adapt existing particle–mesh (PM) code
- Simple implementation

Pseudo-spectral methods for fuzzy dark matter

Symmetrized split-step Fourier method ("kick–drift–kick")

$$i\hbar\partial_t\psi = -\frac{\hbar^2}{2ma^2}\nabla^2\psi + \frac{m}{a}\Phi\psi$$
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- "Exact" solution
- Automatic conservation of mass
- Can adapt existing particle–mesh (PM) code
- Simple implementation
- **Challenges:** $\Delta t \propto \Delta x^2$ **and** strong resolution requirements!

Resolution requirements

The velocity criterion

- As seen in the Madelung formation, the fluid velocity is related to the gradient of the wave function's phase:

$$\psi = \sqrt{\frac{\rho}{m}} e^{i\alpha}$$

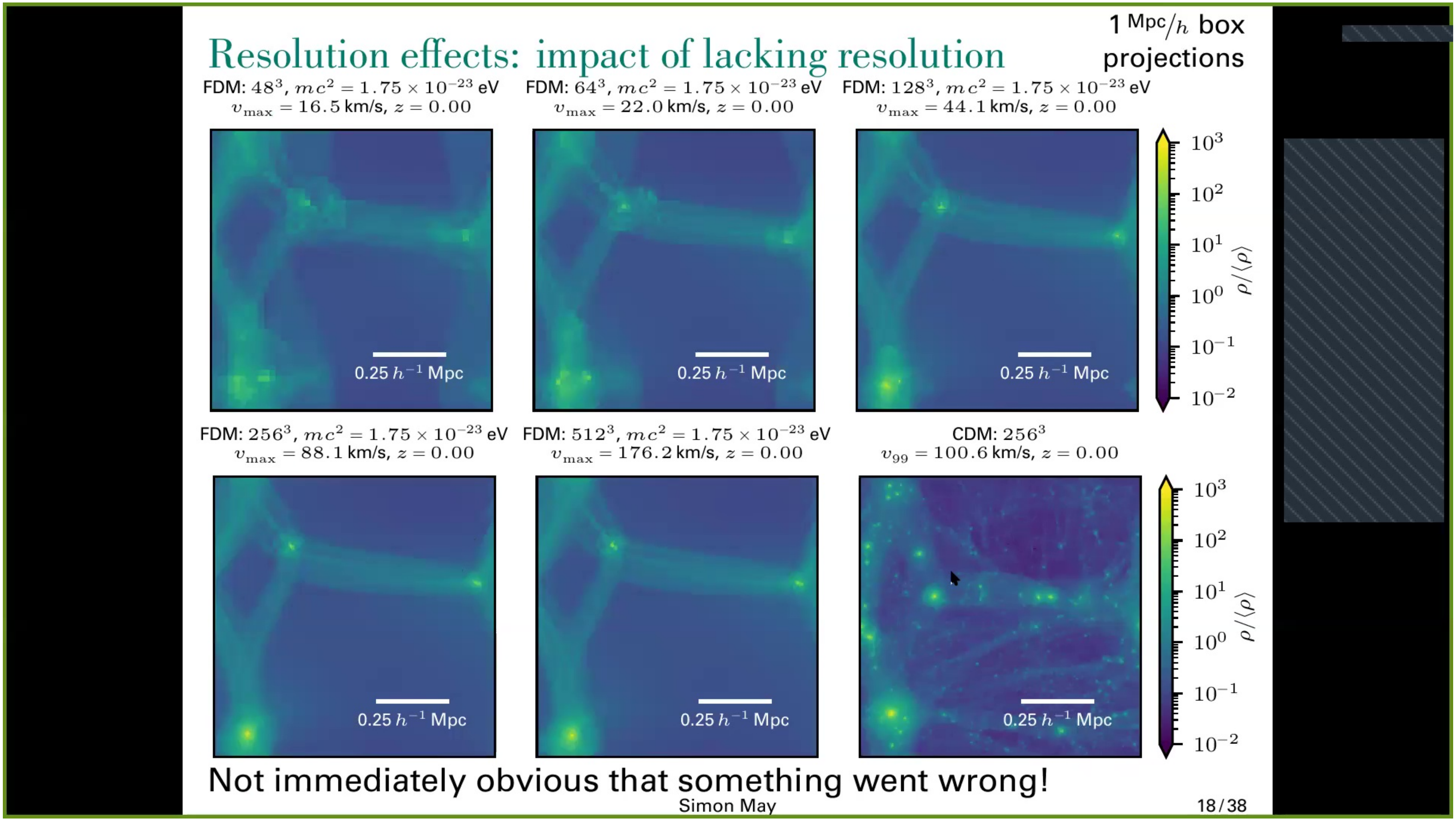
$$\rho = m|\psi|^2$$

$$\vec{v} = \frac{\hbar}{m} \nabla \alpha$$

- Grid discretization implies a **maximum velocity** which can be represented
  Mocz, Lancaster, Fialkov, et al. (2018),
  Mocz, Fialkov, Vogelsberger, et al. (2020)

$$v < \frac{\hbar}{m} \frac{\pi}{\Delta x}$$

$\Rightarrow$ Imposes a resolution limit Δx!


Resolution effects: impact of lacking resolution
1 Mpc/h box projections
FDM: 48³, mc² = 1.75 × 10⁻²³ eV
v_max = 16.5 km/s, z = 0.00
FDM: 64³, mc² = 1.75 × 10⁻²³ eV
v_max = 22.0 km/s, z = 0.00
FDM: 128³, mc² = 1.75 × 10⁻²³ eV
v_max = 44.1 km/s, z = 0.00
FDM: 256³, mc² = 1.75 × 10⁻²³ eV
v_max = 88.1 km/s, z = 0.00
FDM: 512³, mc² = 1.75 × 10⁻²³ eV
v_max = 176.2 km/s, z = 0.00
CDM: 256³
v_99 = 100.6 km/s, z = 0.00
0.25 h⁻¹ Mpc
ρ/⟨ρ⟩
Not immediately obvious that something went wrong!
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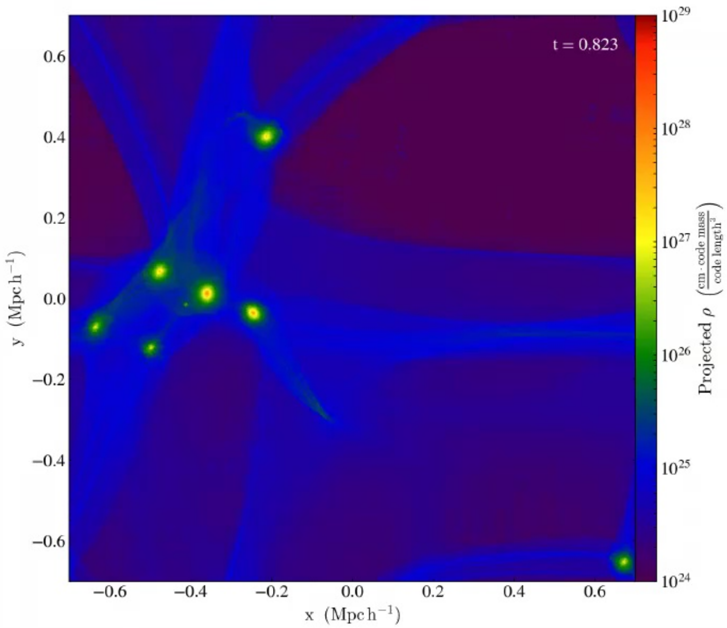
18/38

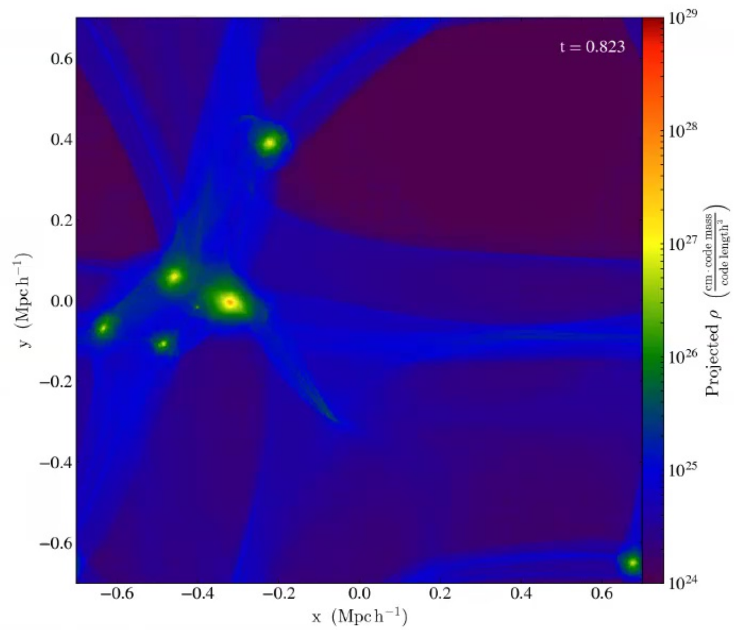
Resolution effects

Example: Dark matter halo formation

Lower resolution



Higher resolution



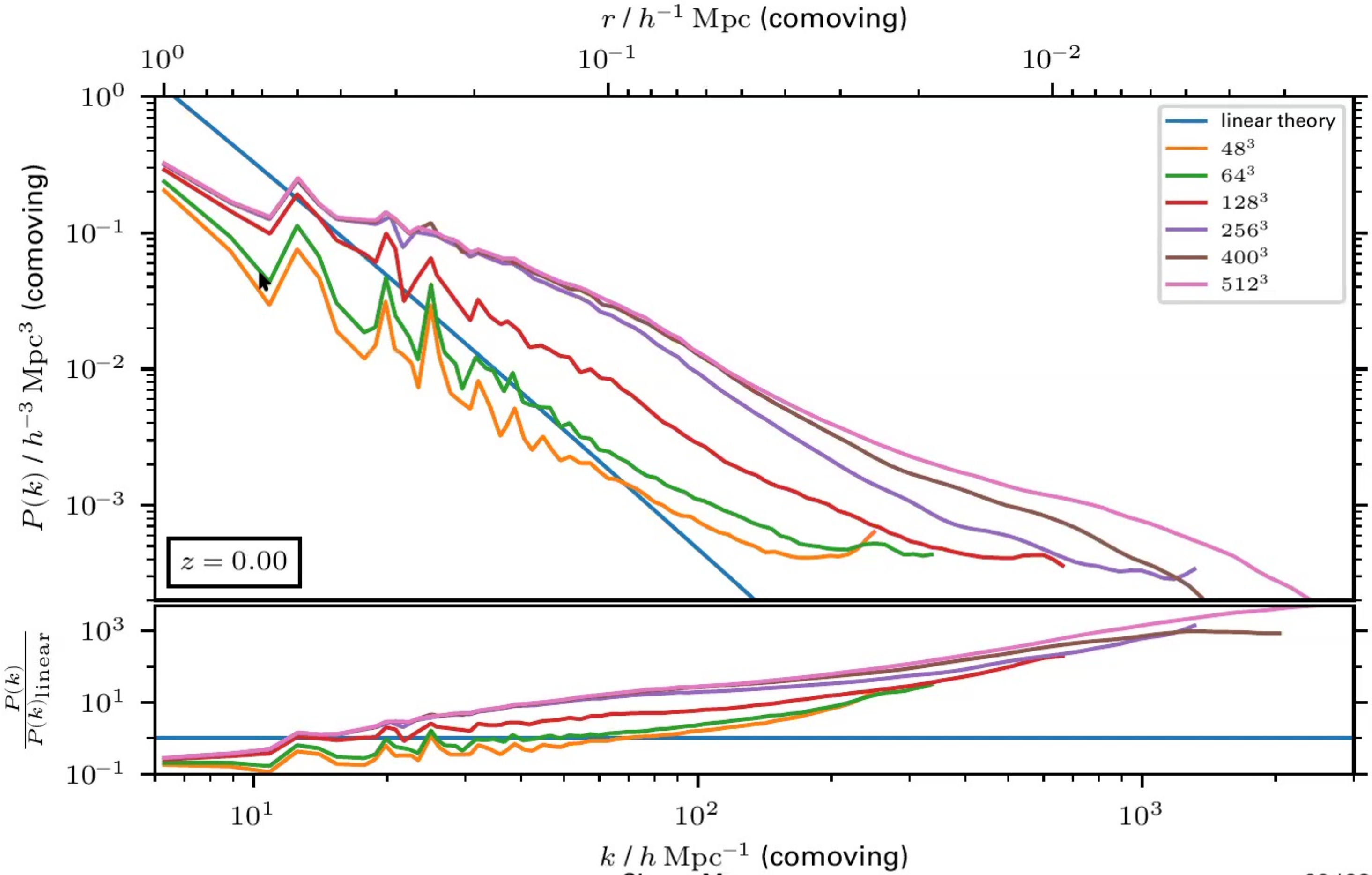
Evolution is delayed/halted for simulations with insufficient resolution

(images from Schive, FDM Workshop Göttingen 2020)

Resolution effects

FDM, $L = 1\,^{\mathrm{Mpc}}/_{\mathrm{h}}$, $m = 1.75\,^{\mathrm{eV}}/_{\mathrm{c}^2}$

Convergence of power spectra



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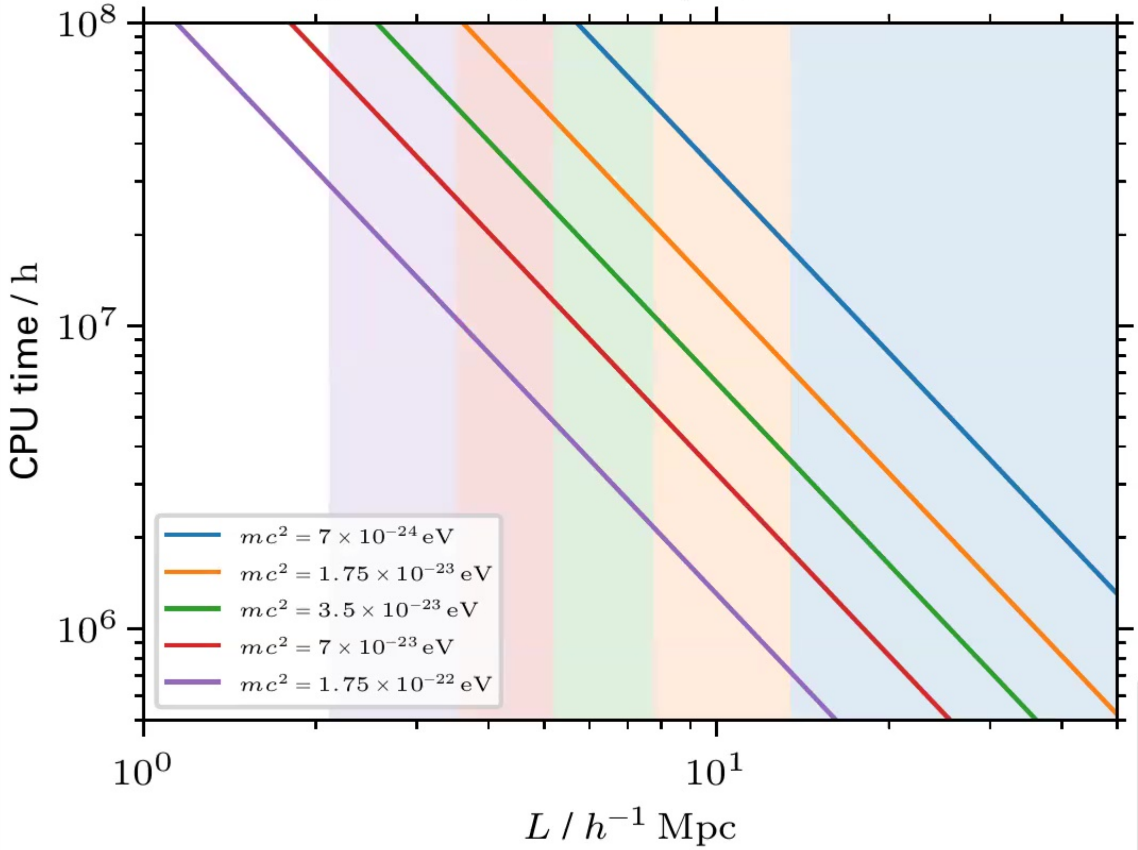
Computational cost of fuzzy dark matter simulations

Computational cost scales as $\propto m^{-1}$ with mass, $\propto L^{-2}$ with box size & $\propto N^5$ with grid size ($\Delta x = L/N$)!



Here: $N^3 = 8192^3$ grid (fixed N)

$$\Delta t < \frac{4}{3\pi}\frac{m}{\hbar}a^2\Delta x^2$$

$$v < \frac{\hbar}{m}\frac{\pi}{\Delta x}$$

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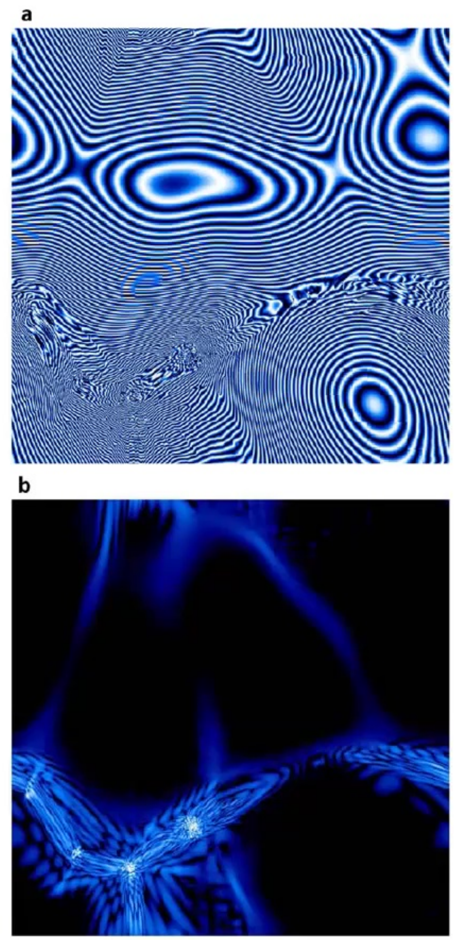
Why is it hard to simulate fuzzy dark matter?

Computational challenges

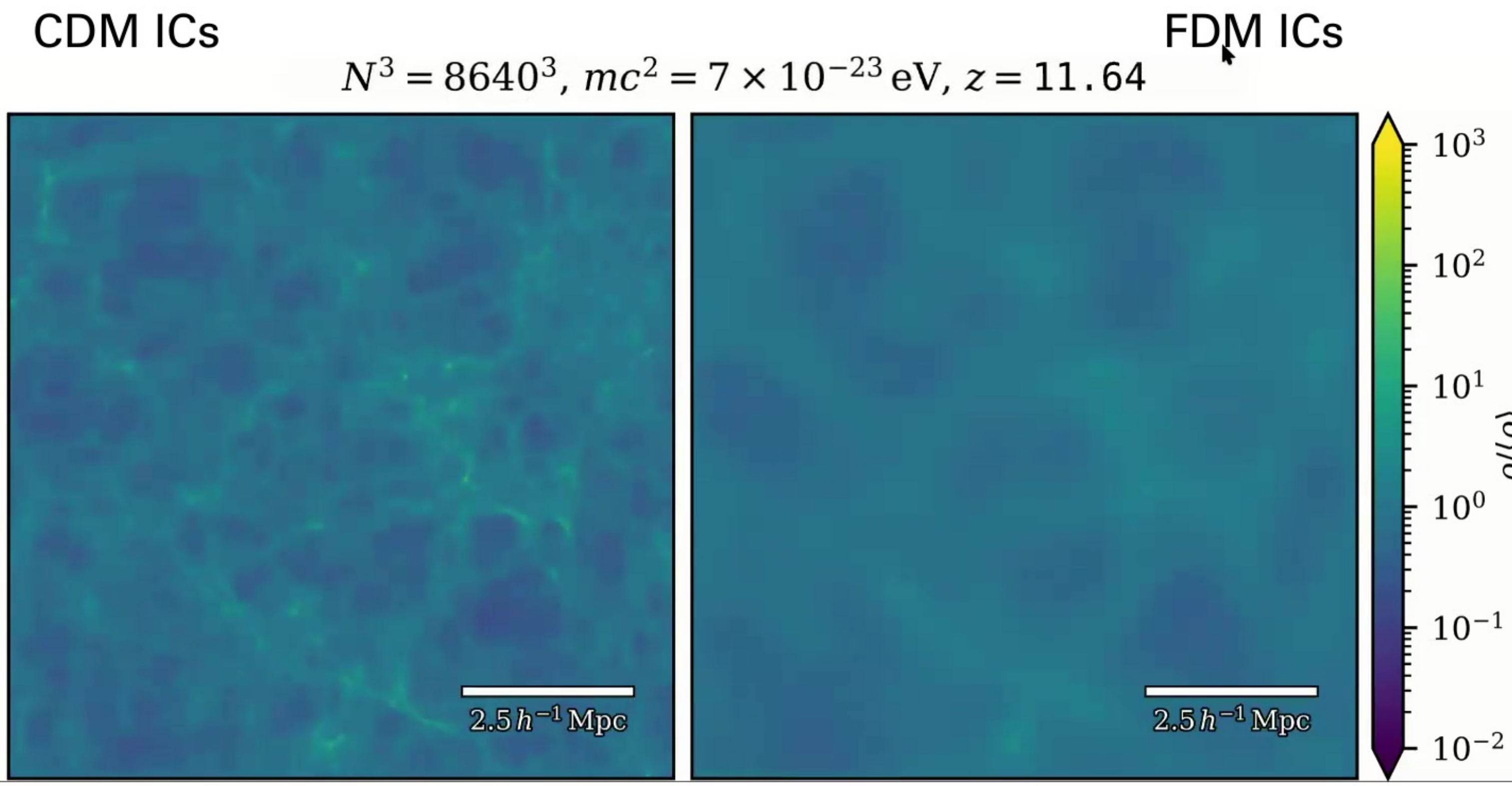
- **Large dynamic range**: **Both** large scales and small (kpc-scale) de Broglie wavelength must be resolved for correct evolution
- High velocities require **high resolution** even in low-density regions
  Velocity criterion: $v < \frac{\hbar}{m} \frac{\pi}{\Delta x}$
- **Time step criterion:** $\Delta t \propto \Delta x^2$ (seems to be approach-independent)
- (Tooling: Hydrodynamics codes are designed for N-body simulations)



Schive, Chiueh, and Broadhurst (2014)

Largest cosmological full FDM wave simulations to date

10 $^{\mathrm{Mpc}}/h$ boxes (slices through simulation volume)

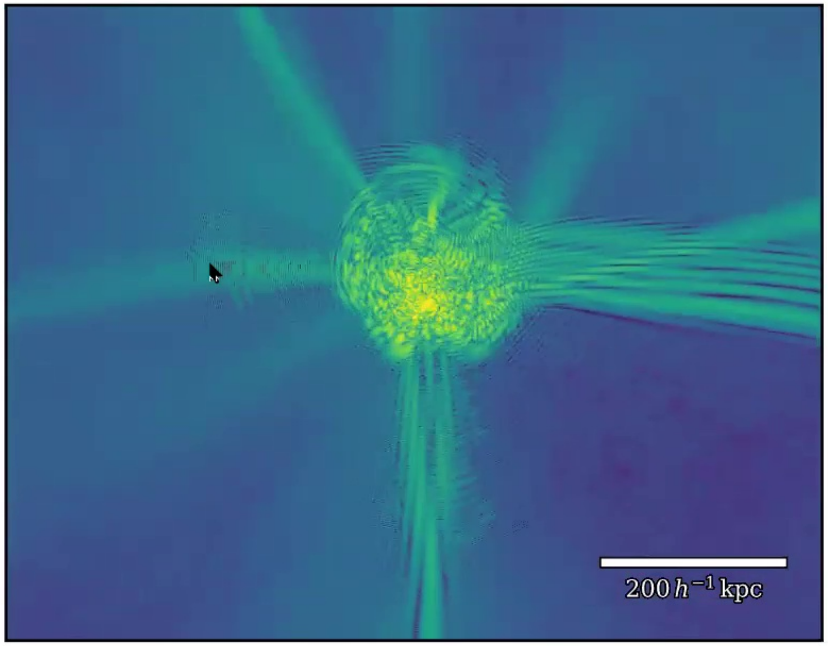


May and Springel (2021) & May et al., in prep.

Largest cosmological full FDM wave simulations to date

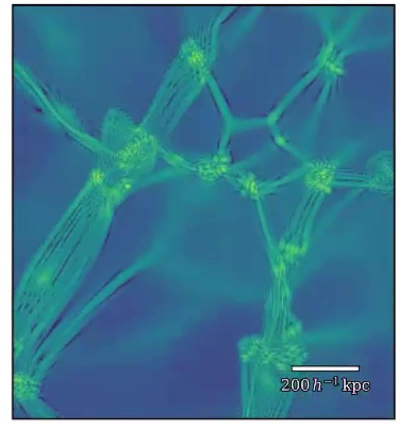
$10\,\mathrm{Mpc}/h$ box (slices through simulation volume)

$N^3 = 8640^3$, $mc^2 = 7 \times 10^{-23}\,\mathrm{eV}$, $z = 3.65$

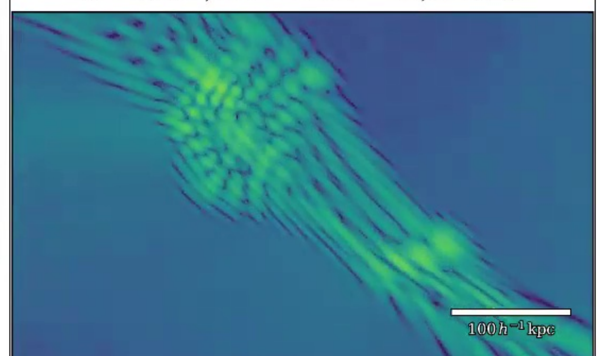


May and Springel (2021)

$N^3 = 8640^3$, $mc^2 = 7 \times 10^{-23}\,\mathrm{eV}$, $z = 3.60$



$N^3 = 8640^3$, $mc^2 = 7 \times 10^{-23}\,\mathrm{eV}$, $z = 3.60$



Largest cosmological full FDM wave simulations to date

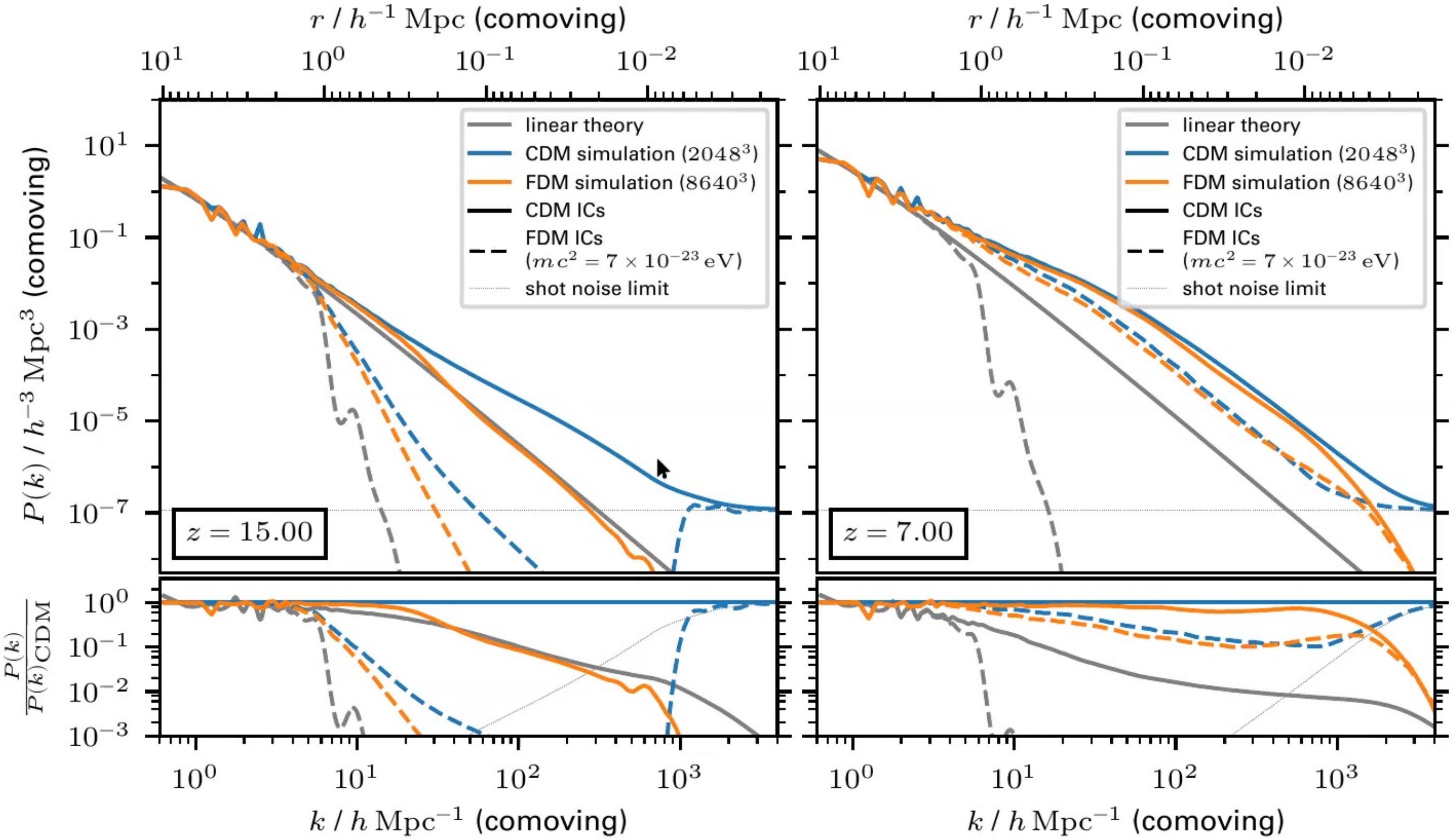
Computational scale

- Grid size: $N^3 = 8640^3 \approx 645$ billion
- Just storing the wave function takes 10 terabytes of memory
- Ran on 8640 parallel cores
- Total cost > 5 million CPU hours
- Several months until completion
- And still, could only reach this small volume (in LSS terms) of $(10\,\mathrm{Mpc}/\mathrm{h})^3$!

$\rightarrow$ Future: tackle the cost problem using hybrid methods?
($\rightarrow$ CDM limit of FDM)

Dark matter power spectrum

10 Mpc/h box simulations



May and Springel (2021) & May et al. (in prep.)

Dark matter power spectrum

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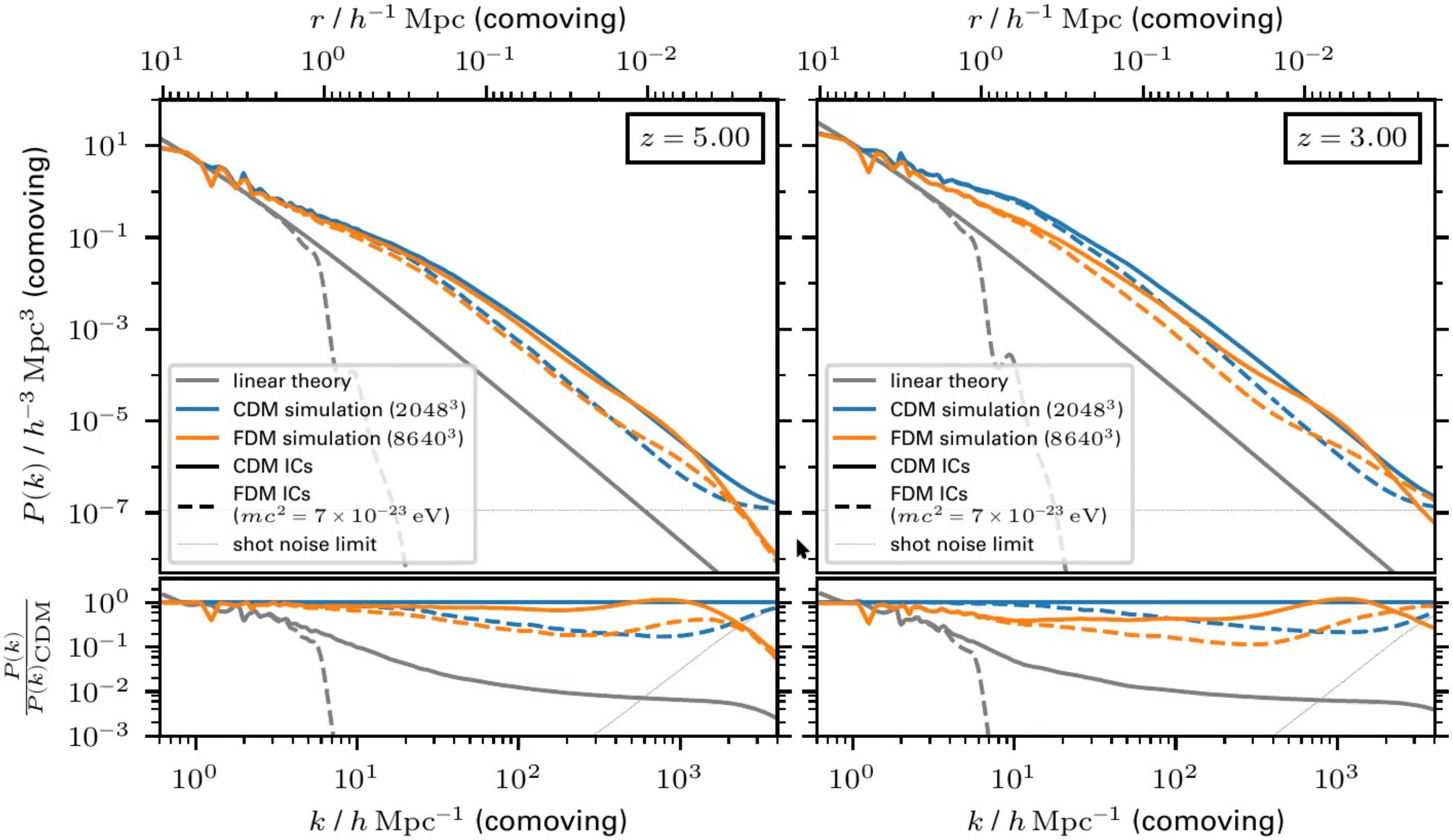
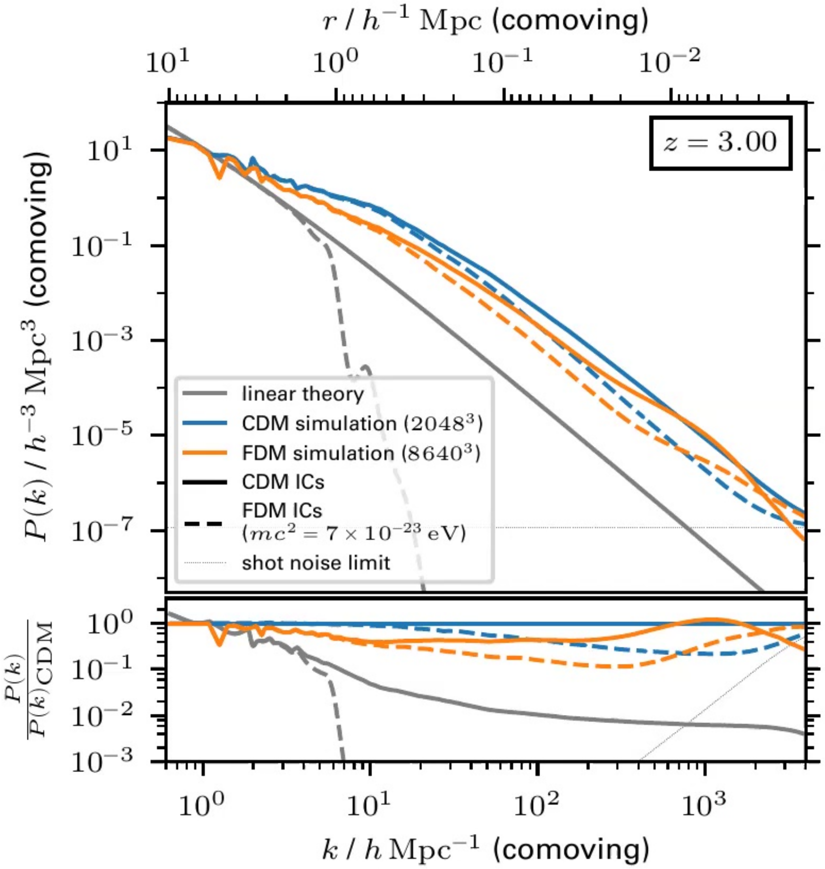


May and Springel (2021) & May et al. (in prep.)

Dark matter power spectrum

10 Mpc/h box simulations

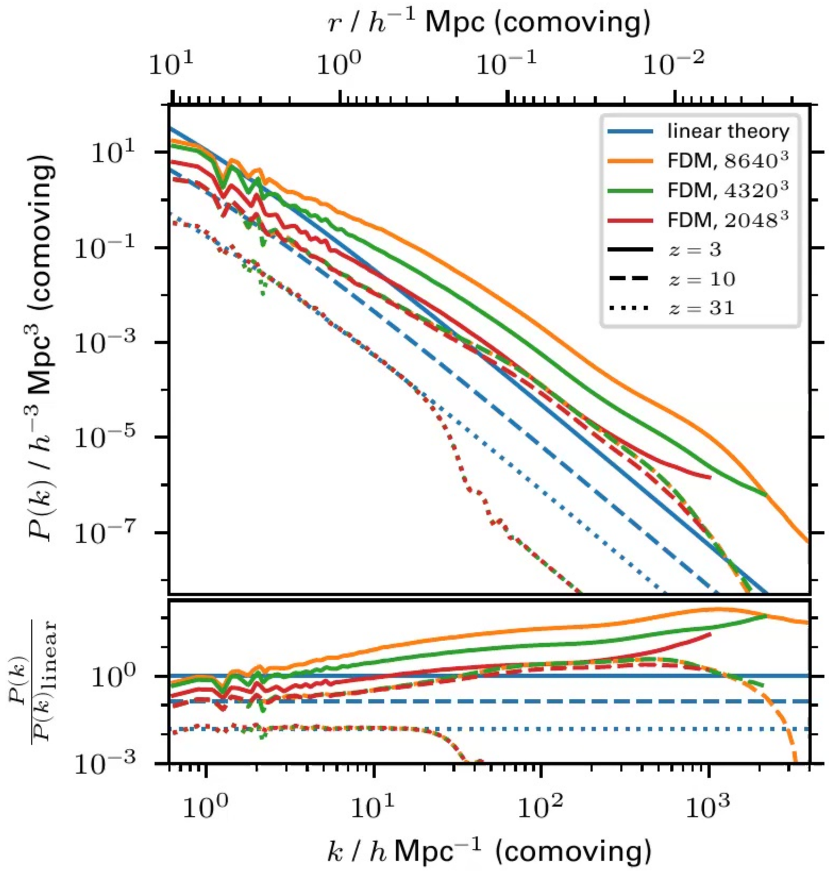


- Agreement on large scales
- Delayed structure formation
- Suppression on small scales
- Small-scale FDM power can even exceed CDM (→ interference patterns)

May and Springel (2021) & May et al. (in prep.)

Dark matter power spectrum

Redshift dependence of convergence & resolution requirements

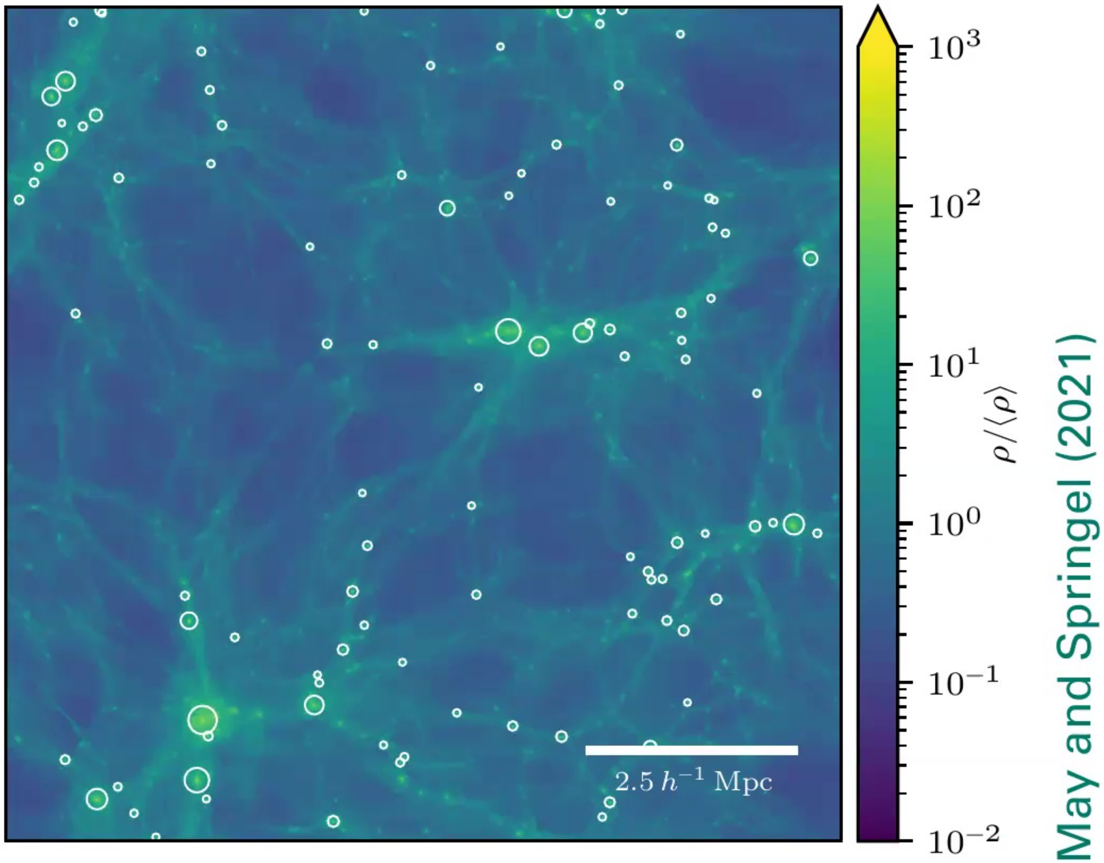


- Insufficient resolution causes structure to "freeze" below a certain redshift

→ Resolution criteria become more stringent as non-linear structure forms and velocities grow

May and Springel (2021)

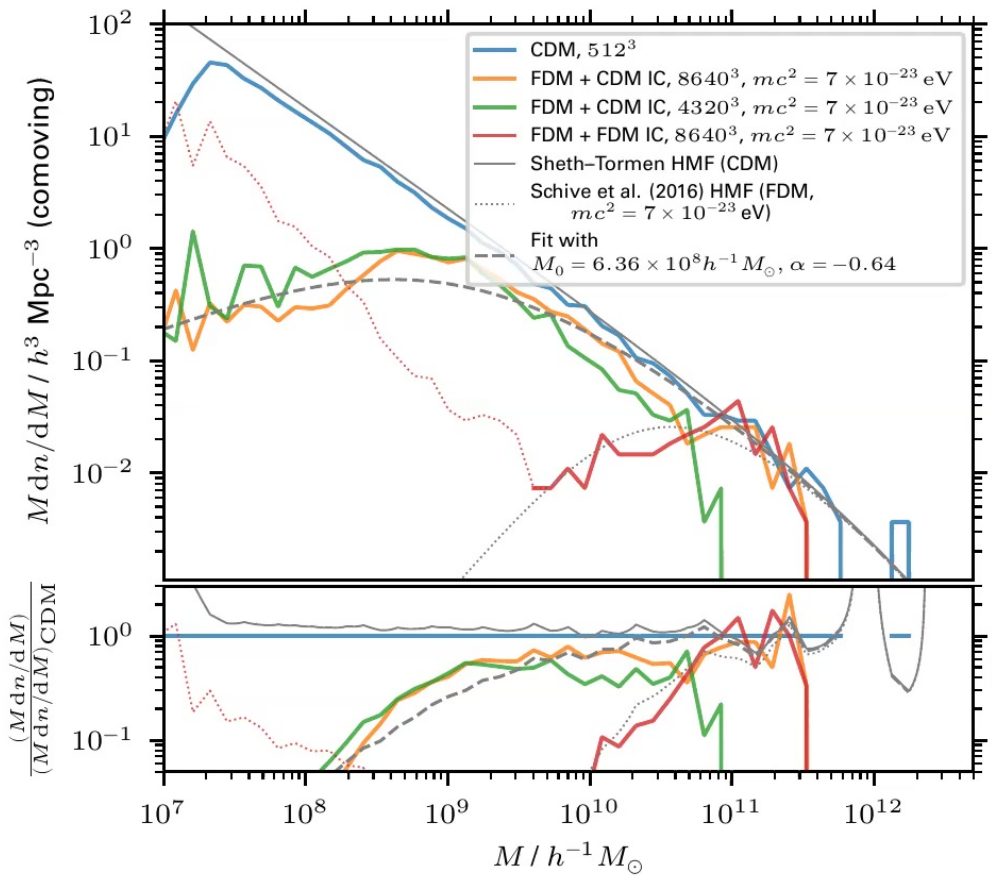
Finding dark matter halos on a grid



Developed modified MPI-parallel friends-of-friends halo finder to operate on a Cartesian grid in AREPO

Halo mass function

First measurements in full FDM wave simulations (10 Mpc/h box)



- Fewer low-mass objects
- Similar counts for the largest halos
- Halo finding problematic in FDM IC case!

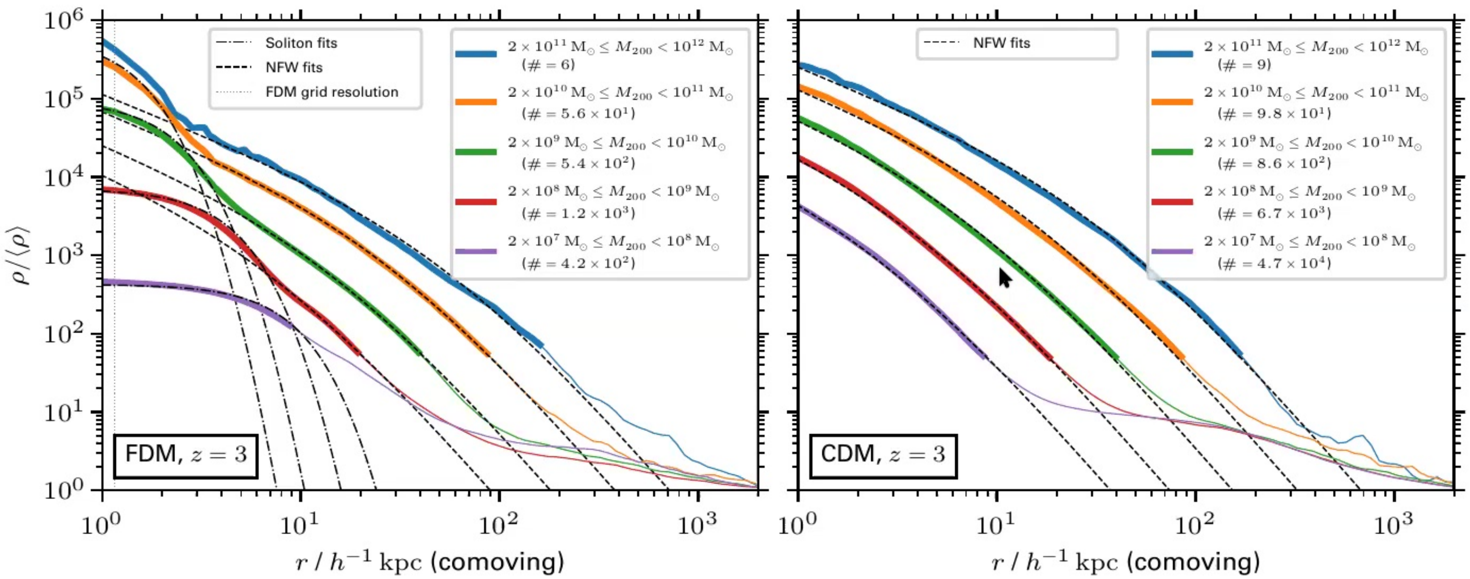
May and Springel (2021)
May et al., in prep.

Dark matter halo profiles

10 $^{\text{Mpc}}/h$ box simulations (CDM ICs)

$mc^2 = 7 \times 10^{-23}$ eV

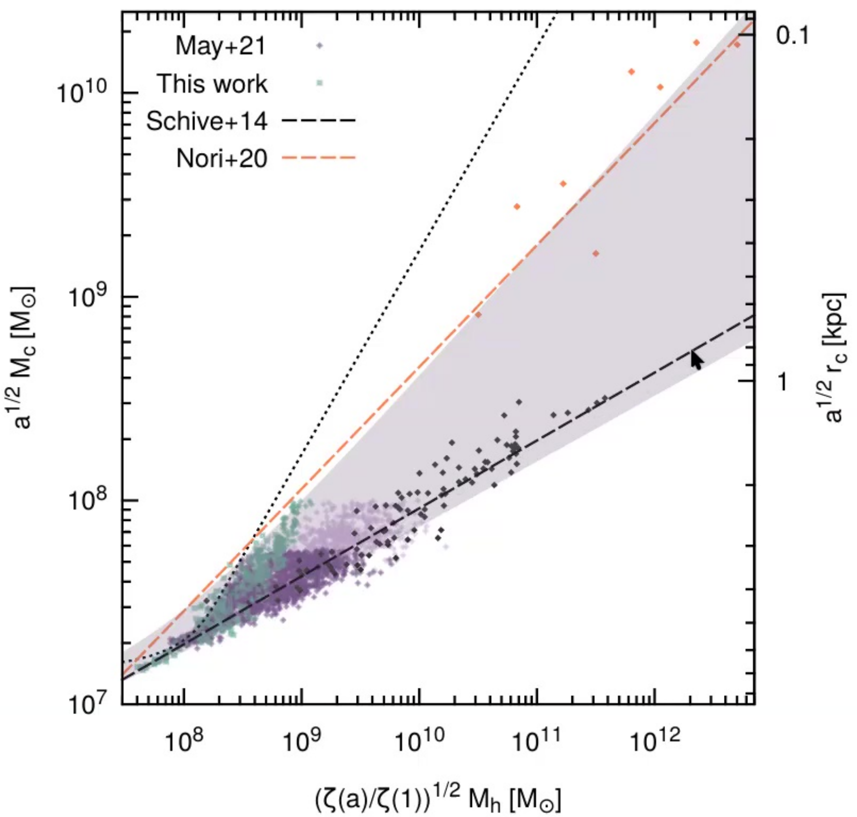
May and Springel (2021)



- Confirms: FDM halos form central cores with flat density profiles + outer NFW region
- But: adaptive resolution better suited to show inner halos

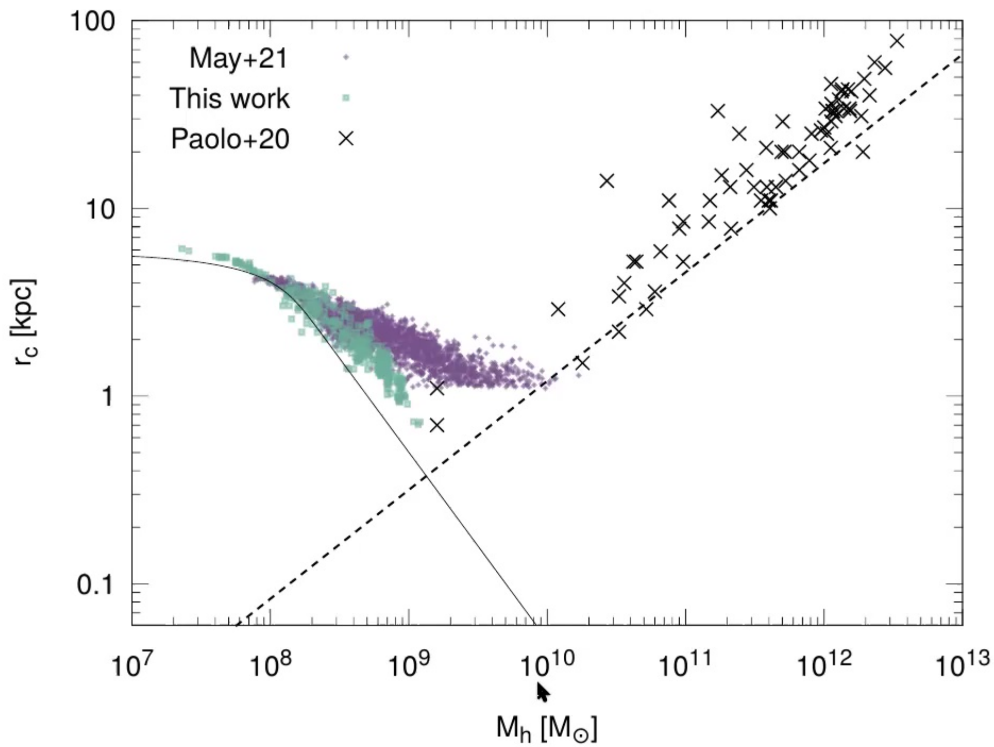
The core–halo mass relation

Chan, Ferreira, May, et al. (2021)



- ▶ Could the solitonic cores explain dwarf galaxy profiles?
- ▶ Demonstrated that there is no tight one–to–one relation between core & halo mass
- → Significant impact on dynamical modeling of dwarfs

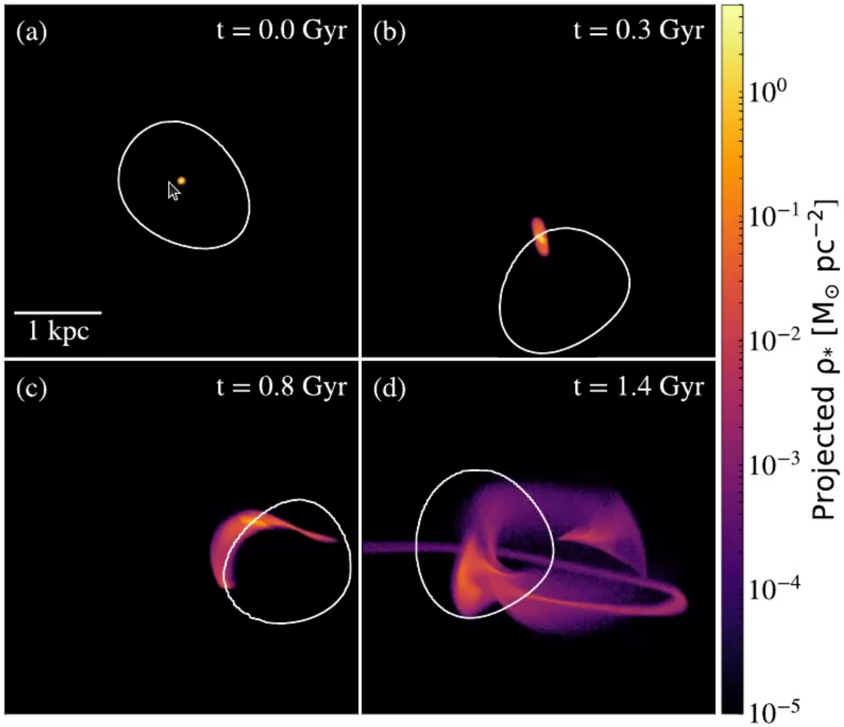
Core radius vs. halo mass



Chan, Ferreira, May, et al. (2021)

- Could the solitonic cores explain dwarf galaxy density profiles?
- Although there is no exact relationship between halo and core mass, unfortunately FDM generally goes in the wrong direction: Heavier FDM cores become more compact!

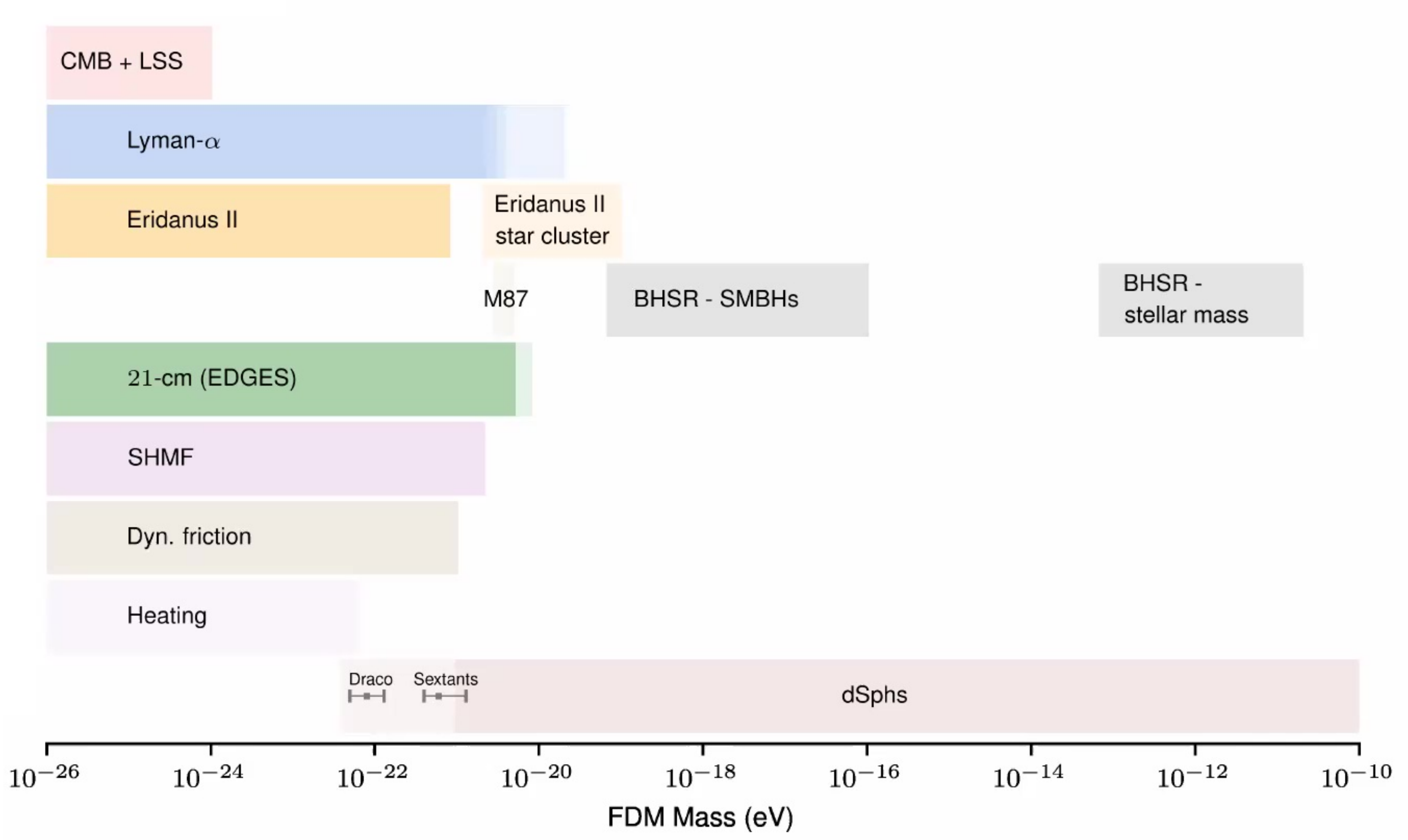
Small-scale constraints on fuzzy dark matter



Schive, Chiueh, and Broadhurst (2020)

- Disruption of stellar clusters
- Disruption of tidal stellar streams
- Dynamical heating of stars (e. g. in the Milky Way)
- Gravitational lensing

Bounds on the fuzzy dark matter particle mass



Ferreira (2021)

Summary

Features of FDM dynamics compared to CDM:

1. Modified **initial power spectrum**, small scales suppressed
2. **Suppression** of structure below de Broglie wavelength ($\approx$ kpc)
   $\rightarrow$ Heisenberg uncertainty principle
3. Formation of **halo cores** (solitons), **vortices**
4. Fluctuating $\approx$ kpc **granules** and **wave interference patterns**

Rich astrophysical phenomenology!

Main challenges for fuzzy dark matter simulations:

1. **Time integration** $\Delta t \sim \Delta x^2$
2. **Rapid oscillations** even in low-density regions
3. **Large dynamic range**: "large"-scale structure simulations must still resolve de Broglie wavelength, limited to $\lesssim 10\,\mathrm{Mpc}/h$ box
4. "New" field without decades of experience or refined codes/methods as for CDM

Outlook

Future plans:

- (Publish results from already completed simulations)
- Current WIP: Full FDM wave simulations of the Lyman-α forest with baryons (May et al., in prep./b)
  (Current bounds: $mc^2 \gtrsim 10^{-21}$ eV using simplified approximations/assumptions, e. g. Rogers and Peiris (2021))
- FDM simulations with baryons & galaxy formation
  1. Use adaptive resolution (e. g. AMR) to resolve smaller features
  2. Future of FDM simulations: hybrid CDM/FDM methods?
  3. Add existing well-developed galaxy formation machinery
- Investigate unique small-scale signatures, interplay of FDM with baryons
  - Stellar streams
  - Local universe, dwarf/satellite galaxies
  - Filament rotation caused by FDM vortices?
    Wang, Libeskind, Tempel, et al. (2021)
    Alexander, Capanelli, Ferreira, et al. (2021)