Title: Quantum gravity and statistical physics

Speakers: Jan de Boer

Series: Quantum Fields and Strings

Date: February 01, 2022 - 2:00 PM

URL: https://pirsa.org/22020048

Abstract: Several puzzles and issues in quantum gravity are related to the question of what type of high-energy information is actually available to low-energy observers. In this talk I will try to argue that the best description available to a low-energy observer is probably of a statistical nature. Such a statistical description naturally connects to e.g. the eigenstate thermalization hypothesis, and wormholes and the apparent lack of factorization. I will describe this statistical picture and the evidence we have for it, and what we could possibly learn from it. Partially based on work in progress.

Zoom Link: https://pitp.zoom.us/j/96646337719?pwd=UVRKN3Y4Q05xNXk1VExOenYwSFZvZz09

20:04 Tue 1 Feb

Note 1 Feb 2022
1 Feb 2022 at 08:41

QUANTUM GRAVITY & STATISTICAL PHYSICS

based on

2006.05499	Belin, JdB
2110.14649	Belin, JdB, Liska
2111.06373	Belin, JdB, Nayak, Sonner
2112.09143	Anous, Belin, JdB, Liska

SEMI CLASSICAL GRAVITY IS A VERY PARTICULAR THEORY. IT KNOWS ABOUT

* BLACK HOLE ENTROPY $\longleftrightarrow$ HIGH T PARTITION FUNCTION

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SEMI CLASSICAL GRAVITY IS A VERY PARTICULAR THEORY. IT KNOWS ABOUT

* BLACK HOLE ENTROPY $\longleftrightarrow$ HIGH T PARTITION FUNCTION

* THE PARTITION FUNCTION ON VARIOUS EUCLIDEAN MANIFOLDS (+ CORRELATORS)

* THE PAGE CURVE (REQUIRES ∞ CONTROL OVER EXTERNAL SYSTEM)

Jan de Boer

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* SPECTRAL CORRELATIONS

* LACK OF FACTORIZATION $\langle ZZ \rangle \neq \langle Z \rangle \langle Z \rangle$

NONE OF THESE RESOLVE EXACT UV INFORMATION

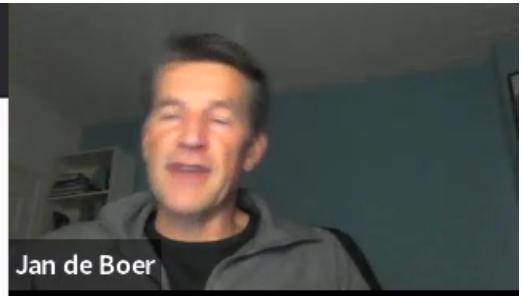
THEY PROVIDE COARSE GRAINED UV INFORMATION

EXAMPLE $Z_{EXACT} = \int \sum_i \delta(E-E_i) e^{-\beta E} dE$

$$\downarrow$$

$$Z_{LEEFT} = \int \rho(E) e^{-\beta E} dE$$

} CHAOTIC



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THEY PROVIDE COARSE GRAINED UV INFORMATION

EXAMPLE $Z_{EXACT} = \int \sum_i \delta(E-E_i) e^{-\beta E} dE$

$\downarrow$

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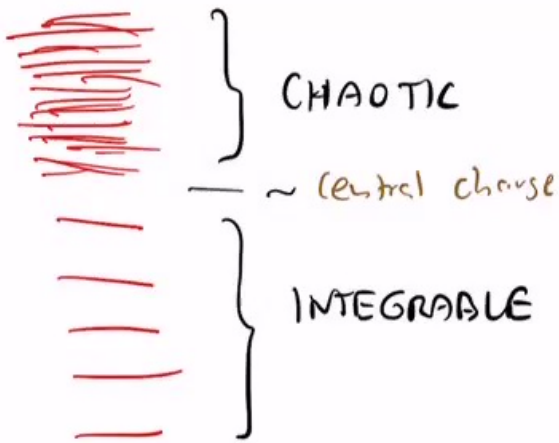

TYPICAL STRONGLY COUPLED CFT SPECTRUM

Main claim: semi-classical gravity is the theory of the statistics of the chaotic sector of the theory

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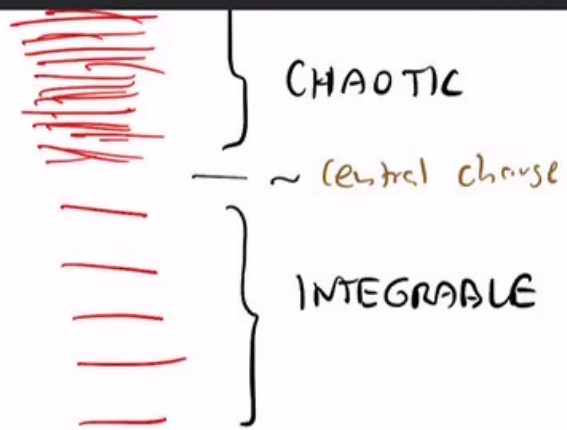

TYPICAL STRONGLY COUPLED CFT SPECTRUM

Main claim: semi classical gravity is the theory of the statistics of the chaotic sector of the theory. It can probe coarse grained higher moments of the relevant statistical distributions but not individual values. It cannot distinguish averaged from non-averaged theories as long as the averages yield the same moments of the statistical distribution.

details depend on what one means by "semi classical gravity"

Is this a "not in practice" or a "not even in principle" statement?

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Consider the operator ~~[illegible]~~ A

20:14 Tue 1 Feb

values. It cannot distinguish averaged from non-averaged theories as long as the averages yield the same moments of the statistical distribution.

Is this a “not in practice” or a “not even in principle” statement?

details depend on what one means by "semi classical gravity"

Consider for example an operator A

$$A = \frac{1}{N} \sum (\text{FIRST } N \text{ DIGITS OF } \pi)$$

pseudorandom!

$$\langle A \rangle \approx 4.5$$

$$\langle (A - 4.5)^2 \rangle \approx \frac{8.25}{N} \quad \left(\begin{array}{l} 8.24 \\ \text{for } N = 10^5 \end{array} \right)$$

like a wormhole!

agrees with result, we sampled

~~Average variance~~

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Consider for example an operator A

$$A = \frac{1}{N} \sum (\text{FIRST } N \text{ DIGITS OF } \pi)$$

pseudorandom!

$$\langle A \rangle \approx 4.5$$

$$\langle (A - 4.5)^2 \rangle \approx \frac{8.25}{N} \quad \left(\begin{array}{l} 8.24 \\ \text{for } N = 10^5 \end{array} \right)$$

like a wormhole!

agrees with result, we sampled random numbers. Average variance in digits of π = variance of a random digit

OPERATOR STATISTICS

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OPERATOR STATISTICS

OPE COEFFICIENTS $C_{ijk} = \langle \mathcal{O}_i(\infty)\, \mathcal{O}_j(1)\, \mathcal{O}_k(0) \rangle$

distinguish light (L) and heavy (H) operators

Gravity has access to the statistics of C_{LLH}, C_{LHH} and C_{HHH}, but not to their individual values

OPE randomness hypothesis:

$$C_{LLH} \sim f(E_H)\, R_H$$

(DelB, Belin)

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Gravity has access to the statistics of C_{LLH}, C_{LHH} and C_{HHH}, but not to their individual values

OPE randomness hypothesis:

$$C_{LLH} \sim f(E_H)\, R_H$$

(JdB, Belin)

$$C_{LHH'} \sim f(\bar{E}_H, \Delta E_H)\, R_{HH'}$$

$$C_{HH'H''} \sim f(\bar{E}_H, \Delta E_H)\, R_{HH'H''}$$

slowly varying function

pseudorandom
mean ~ 0
variance ~ 1

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Slowly varying function

pseudorandom
mean ~0
variance ~1

FAMOUS EXAMPLE: ETH

$$\langle i|O|j\rangle = f(\bar{E})\,\delta_{ij} + g(\bar{E},\Delta E)\,e^{-S(\bar{E})/2}\,R_{ij}$$

$$\bar{E} = (E_i + E_j)/2$$

$$\Delta E = E_i - E_j$$

ETH CORRECTLY REPRODUCES THERMAL ONE- AND TWO-POINT FUNCTIONS

How does one ~~check all this~~?

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FAMOUS EXAMPLE: ETH

$$\langle i|\mathcal{O}|j\rangle = f(\bar{E})\,\delta_{ij} + g(\bar{E},\Delta E)\,e^{-S(\bar{E})/2}\,R_{ij}$$

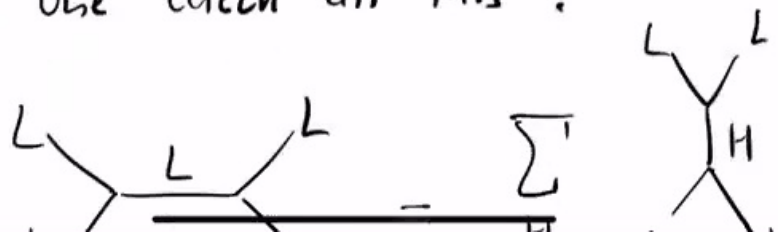
$$\bar{E} = (E_i + E_j)/2$$

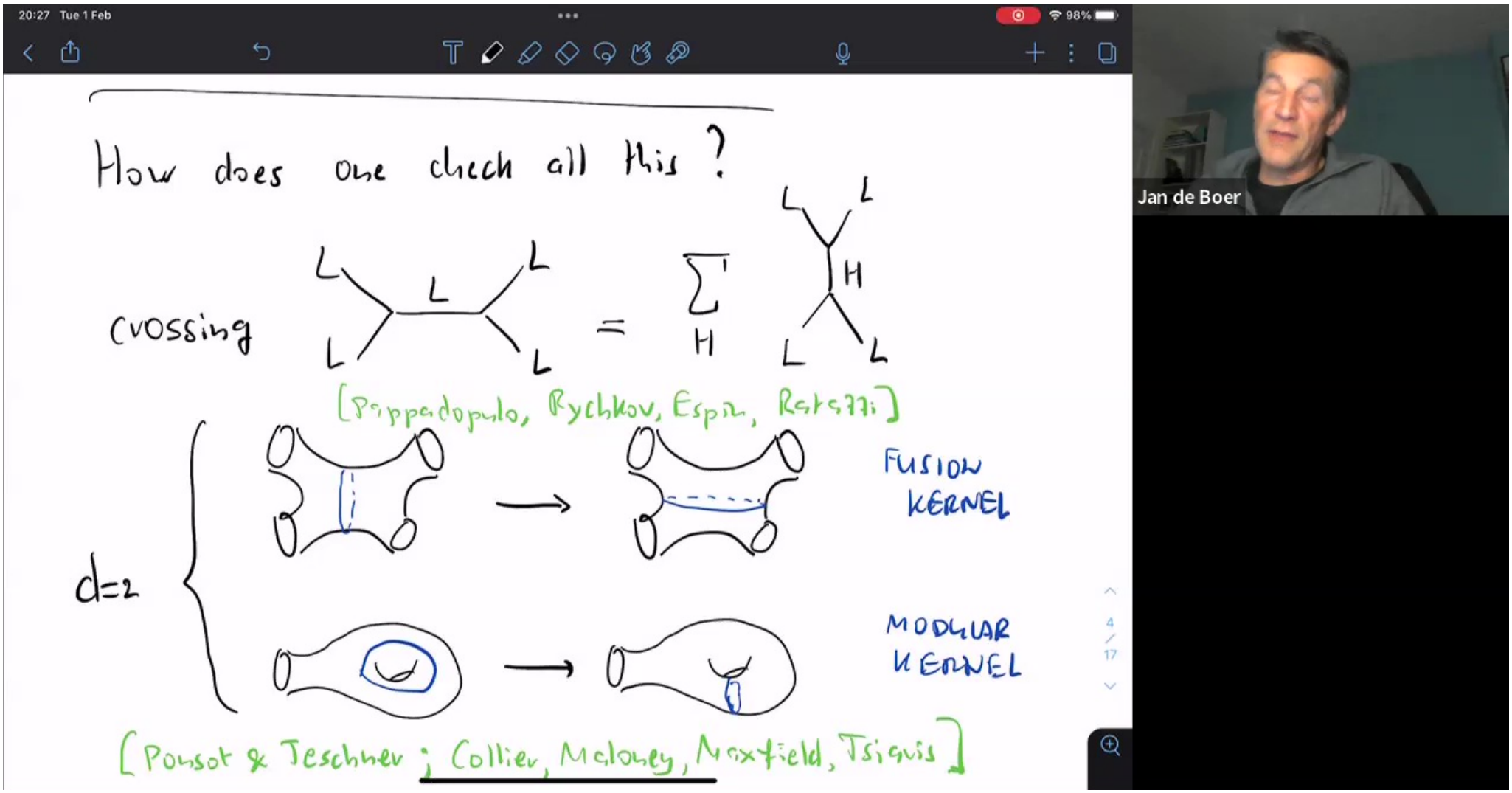
$$\Delta E = E_i - E_j$$

ETH CORRECTLY REPRODUCES THERMAL ONE- AND TWO-POINT FUNCTIONS

How does one check all this?

crossing




How does one check all this?
crossing
L
L
L
L
L
=
$\sum_H$
L
L
H
L
L
[Pappadopulo, Rychkov, Espin, Rattazzi]
d=2
FUSION KERNEL
MODULAR KERNEL
[Ponsot & Teschner ; Collier, Maloney, Maxfield, Tsiares]
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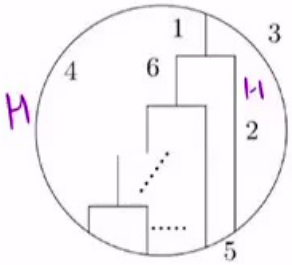
RESULTS :

$$C^{k}_{LHH} \sim e^{-(k-1)S}$$

Foini
Kurchan



$$C^{k}_{HHH} \sim e^{-\frac{5k-4}{4}S}$$

$$-\frac{9k-6}{8}S$$

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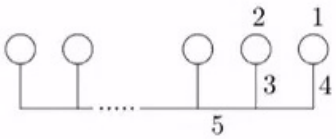
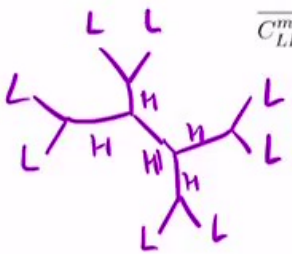


$$C^{k}_{HHH} \sim e^{-\frac{9k-6}{8}S}$$

$$\overline{C^3_{LLH}C_{HHH}}\Big|_{d\geq 2,\ \text{all op}} \sim \frac{\Delta_H^{6\Delta_L-3}}{\rho(\Delta_H)^3} \sim e^{-3S}$$

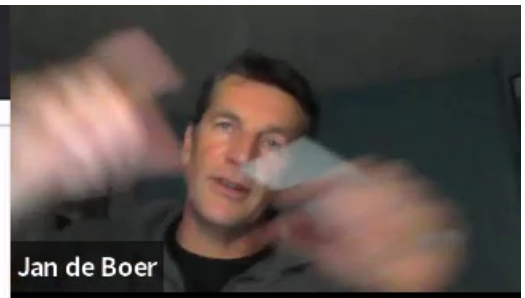
$$\overline{C^3_{LLH}C_{HHH}}\Big|_{d=2,\ \text{quasi-prim}} \sim \frac{\Delta_H^{6\Delta_L-6}}{\rho(\Delta_H)^3} \sim e^{-3S}$$

$$\overline{C^3_{LLH}C_{HHH}}\Big|_{d=2,\ \text{Vir-prim}} \sim \left(\frac{3\sqrt{3}}{16}\right)^{3\Delta_H} \frac{\Delta_H^{6\Delta_L-\frac{19+11c}{36}}}{\rho_{\text{vip}}(\Delta_H)^{\frac{9}{4}}} \sim e^{-\frac{9S}{4}}$$



$$\overline{C^{m+2}_{LLH}C^{m}_{HHH}C^{\eta}_{HHL}}\Big|_{d=2,\ \text{Vir-prim}} \sim C^{\eta}_{LLL}\frac{3^{\frac{9m}{2}\Delta_H}}{16^{(1+2m)\Delta_H}}\frac{\Delta_H^{\left(4+2m+\frac{\eta}{2}\right)\Delta_L-\frac{9(c+1)+2(5+c)m}{36}}}{\rho_{\text{vip}}(\Delta_H)^{\frac{1}{4}(2+4\eta+7m)}}$$

$$\sim e^{-S(2+4\eta+7m)/4}$$



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- GENERAL LESSON: CONNECTED HIGHER POINT FUNCTIONS ARE SUPPRESSED

- RESULTS APPLY IN ANY CFT, NOT NECESSARILY CHAOTIC

- QUALITATIVE BEHAVIOR CAN ALSO BE OBTAINED FROM (cf Pollack, Rozali, Sully, Wakeham)
  - MICROCANONICAL UNITARY AVERAGES
  - ALTLAND-SONNER σ-MODEL FOR CHAOS (Belin, JdB, Nayak, Sonner)

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$$Z[J_{abc}] = \exp\left[f_1 J_{abc} J^{abc} + f_2 J_{acb} J^{bcc} + \text{higher} \right]$$

SO THAT

$$\overline{CC\cdots C} = \frac{\delta}{\delta J} \cdots \frac{\delta}{\delta J} Z[J]\Big|_{J=0}$$

OPEN QUESTION:
- WHAT INDEX CONTRACTIONS APPEAR ?
- IS SINGLE SIDED INFORMATION SUFFICIENT ?

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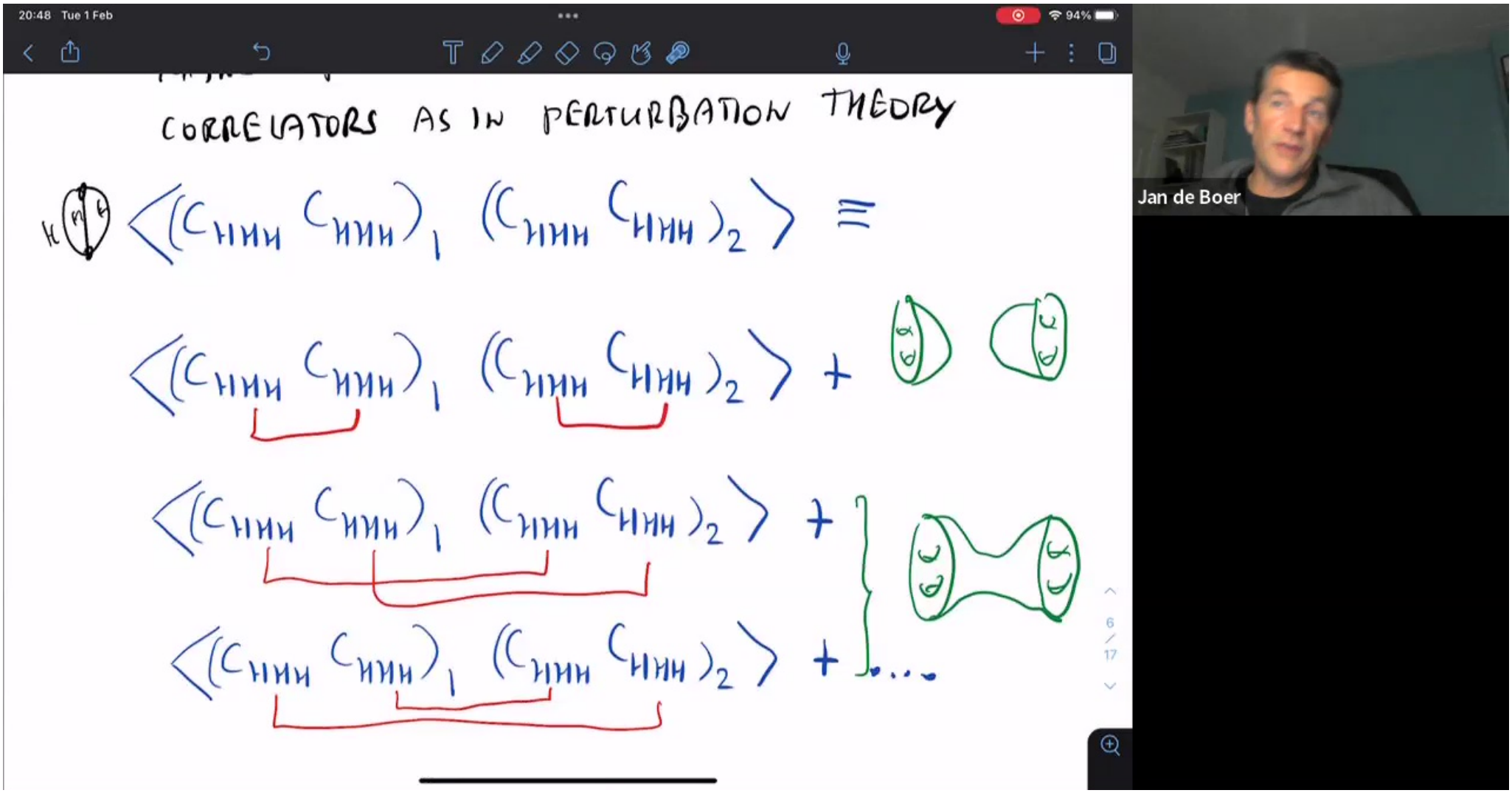
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CORRELATORS AS IN PERTURBATION THEORY
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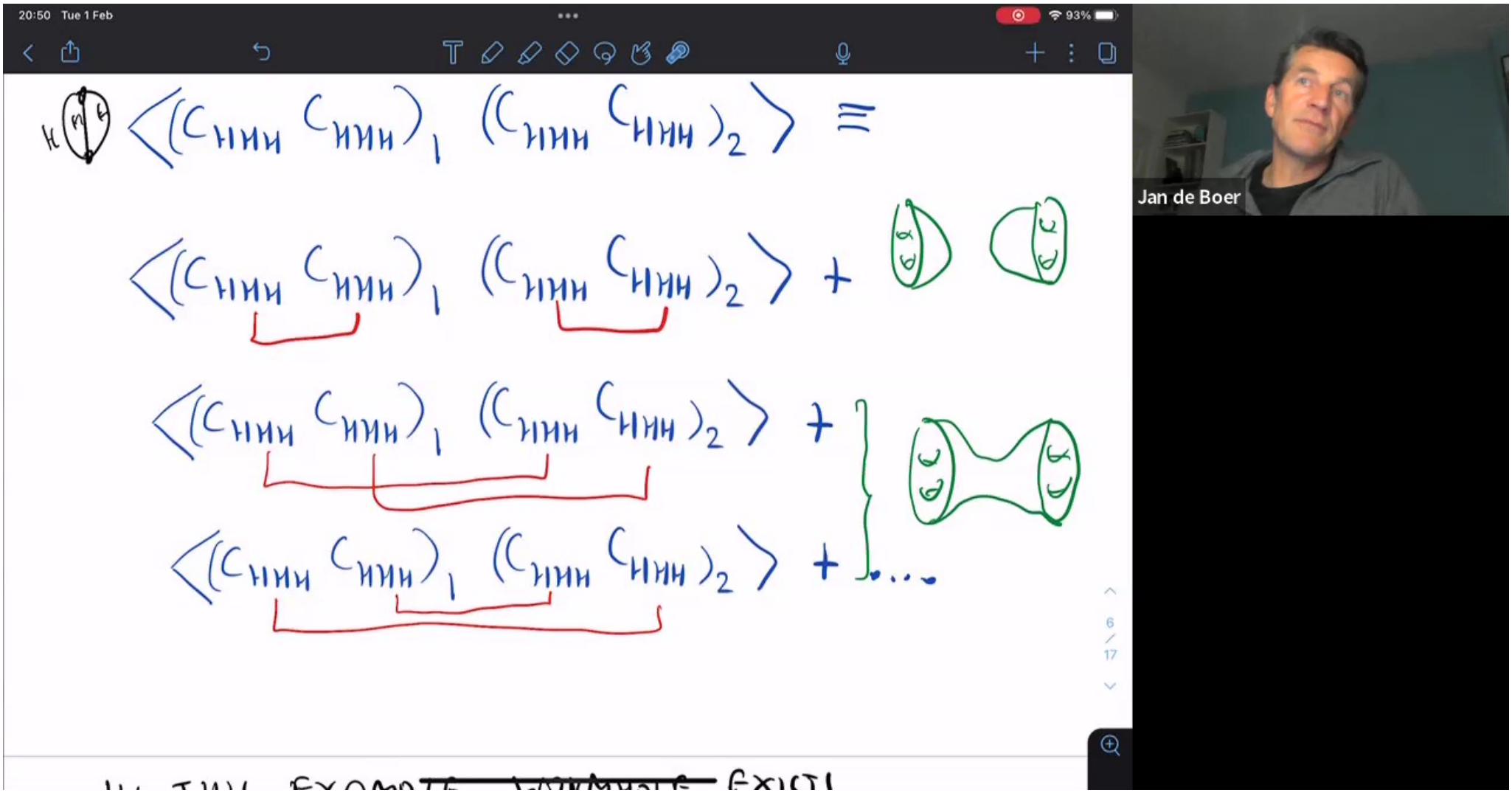
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(MAGDALENA – MAOZ) AND AGREES WITH ABOVE PREDICTION FROM "PETURBATION THEORY".

DISAGREEMENT COULD ALWAYS BE FIXED WITH ADDITIONAL QUARTIC VERTICES IN WHICH THE SAME INDEX APPEARS >2 TIMES

TWO WORMHOLE "PREDICTIONS"

(1) $\left\langle \left(\right)_1 \left(\right)_2 \right\rangle$ = disconnected

+

Gaussian, connected

+

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TWO WORMHOLE "PREDICTIONS"

(1) $\left\langle \left(\right)_1 \left(\right)_2 \right\rangle$ = disconnected

+

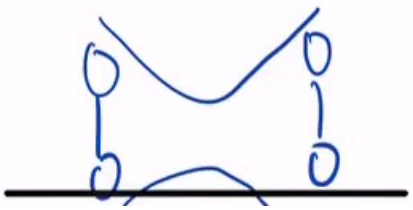
Gaussian, connected

+

4th order vertex, connected

dominates !

$\exists$? wormhole



$$Z = e^{\frac{25c - 360\Delta_x}{288}\frac{\pi^2}{\beta}}$$

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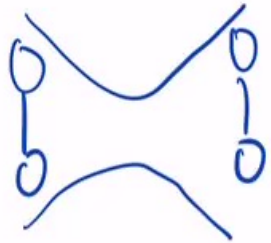
Gaussian, connected

+

4th order vertex, connected

dominates !

$\exists$? wormhole



$$Z = e^{\frac{25c - 360\Delta_x}{288}\frac{\pi^2}{\beta}}$$

$\gg$

$$Z = O(1)$$

(Maldacena–Maoz)

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92%

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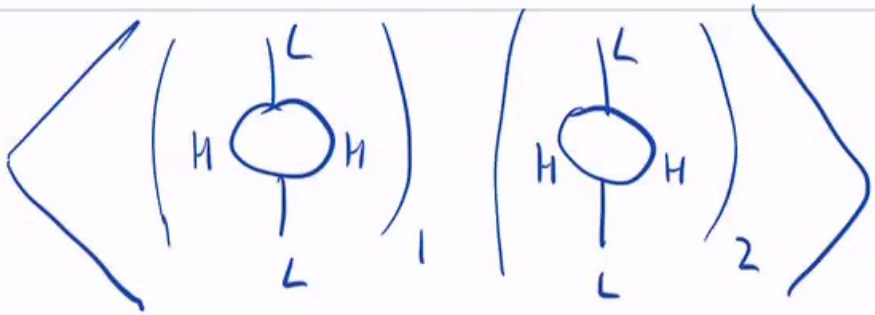
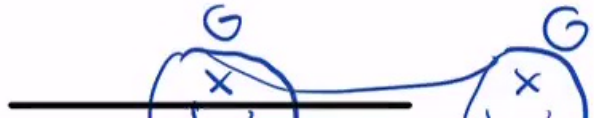
(2)



$$= \langle (C_{LHH} C_{LHH})_1 \, (C_{LHH} C_{LHH})_2 \rangle$$

HAS CONNECTED PIECES FROM WICK CONTRACTIONS.

?? WORMHOLE



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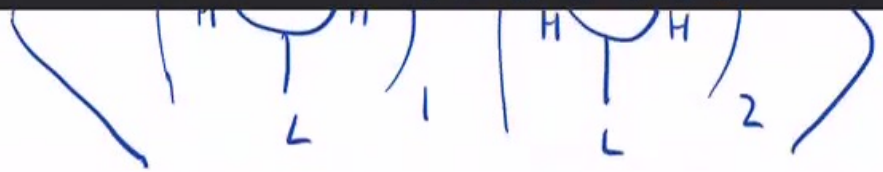
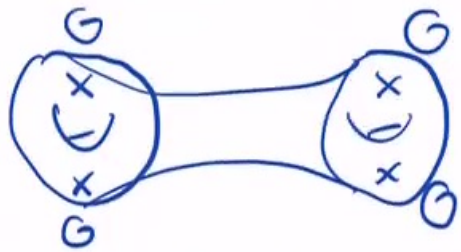


$$= \langle (C_{LHH} C_{LHH})_1 \, (C_{LHH} C_{LHH})_2 \rangle$$

HAS CONNECTED PIECES FROM WICK CONTRACTIONS.

$\exists$? WORMHOLE



STABILIZED BY MATTER?

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BESIDES OPERATOR STATISTICS THERE IS ALSO SPECTRAL STATISTICS (eg SPECTRAL FORM FACTOR)

UNCLEAR WHAT THE FORMALISM IS THAT CAPTURES EVERYTHING

ONE IDEA IS TO GENERALIZE STATISTICAL PHYSICS TO ALLOW FOR A PROBABILITY DISTRIBUTION ON THE SPACE OF DENSITY MATRICES

$$\left(\qquad -F[\rho] \right.$$

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CAPTURES EVERYTHING

ONE IDEA IS TO GENERALIZE STATISTICAL PHYSICS TO ALLOW FOR A PROBABILITY DISTRIBUTION ON THE SPACE OF DENSITY MATRICES

$$\rho \longrightarrow \int d\mu[\rho]\, e^{-F[\rho]}$$

THIS COULD BE CHOSEN TO BE SUCH THAT IT MAXIMIZES THE "UNCERTAINTY" GIVEN LOW ENERGY INPUT

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MATRICES

$$\rho \longrightarrow \int d\mu[\rho]\, e^{-F[\rho]}$$

THIS COULD BE CHOSEN TO BE SUCH THAT IT MAXIMIZES THE "UNCERTAINTY" GIVEN LOW ENERGY INPUT ...

- WHAT IS "UNCERTAINTY"?
- WHY THIS STRUCTURE? UNOBSERVABLE SUPERSELECTION SECTORS?

HERE IS A (TOO NAIVE) ATTEMPT:

(Araujo, Chapman, JdB)

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BOLTZMANN 2.0

- usual: maximize entropy give some expectation value of the energy

$$S(\rho) + \lambda\left(\mathrm{tr}(\rho H) - E\right) \xrightarrow[\text{vary } \rho]{} \rho_{ent} = \frac{e^{-\beta H}}{Z(\beta)} \qquad -\lambda \sim \beta$$

- proposal: do the same but now for a measure $\mu[\rho]$ on the space of density matrices

21:08 Tue 1 Feb

- proposal: do the same but now for a measure $\mu[\rho]$ on the space of density matrices

$$\int d\rho\, \mu[\rho] \left\{ -\log \mu[\rho] - \mathrm{Tr}(\rho \log \rho) + \lambda\left(\mathrm{Tr}(\rho H) - E\right) \right\} = 0$$

need to define measure

classical Shannon piece

quantum piece

$$\Rightarrow \quad \mu[\rho] = N e^{S[\rho] - \beta \mathrm{Tr}(\rho H)} = N' e^{-S(\rho | \rho_\beta)}$$

This measure is peaked at the thermal state ρ_β

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Write $\rho = \rho_\beta + \eta$ then up to order η^2

$$\mu(\rho) \sim N' e^{-\frac{1}{2} g_{ij} \eta_{ij} \eta_{ij}^*}$$

$$g_{ij} = Z(\beta)\beta \frac{E_i - E_j}{e^{-\beta E_j} - e^{-\beta E_i}}$$

$$\approx Z(\beta) e^{\beta \bar{E}} = e^{S(\bar{E})} \quad \text{for } E_i \sim E_j$$

① the size of a fluctuation in η is $e^{-S(\bar{E})/2}$ which is just like in ETH !

② We get approximate unitary invariance in an ~~energy band where~~ g_{ij} is

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$$\int dp\, \mu[p] \left\{ -\log \mu[p] - \mathrm{Tr}(\rho \log \rho) + \lambda\left(\mathrm{Tr}(\rho H) - E\right) \right\} = 0$$

need to define measure

classical Shannon piece

quantum piece

$$\Rightarrow \quad \mu[\rho] = N e^{S[\rho] - \beta \mathrm{Tr}(\rho H)} = N' e^{-S(\rho | \rho_\beta)}$$

This measure is peaked at the thermal state ρ_β but has support away from it

Write $\rho = \rho_\beta + \eta$ then up to order η^2

$$\mu[\rho] = N' e^{\frac{-1}{2} g_{ij} \eta_{ij} \eta^*_{ij}}$$

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CONCLUDING REMARKS

- CAN WE FIND NEW WORMHOLES?
- WHAT IS THE RIGHT STATISTICAL FRAMEWORK?
- CAN WE GET MORE INSIGHT FROM BOOTSTRAP APPROACHES?
- IS SINGLE SIDED INFORMATION ENOUGH?

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- CAN WE GET MORE INSIGHT FROM BOOTSTRAP APPROACHES?
- IS SINGLE SIDED INFORMATION ENOUGH?
- ANY DEEP LESSONS ABOUT CHAOTIC SYSTEMS IN NATURE?
- SEMI CLASSICAL GRAVITY IS "AVERAGING AGNOSTIC"