Title: Quantum gravity from quantum matter

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Series: Quantum Gravity

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Abstract: We present a model of quantum gravity in which dimension, topology and geometry of spacetime are collective dynamical variables that describe the pattern of entanglement of underlying quantum matter. As spacetimes with arbitrary dimensions can emerge, the gauge symmetry is generalized to a group that includes diffeomorphisms in general dimensions. The gauge symmetry obeys a first-class constraint operator algebra, and is reduced to a generalized hypersurface deformation algebra in states that exhibit classical spacetimes. In the semi-classical limit, we find a saddle-point solution that describes a series of (3+1)-dimensional de Sitter-like spacetimes with the Lorentzian signature bridged by Euclidean spaces in between.

Quantum Gravity from Quantum Matter

Sung-Sik Lee

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arXiv:1912.12291
arXiv:2012.14416

Dynamical geometry, topology and dimension

- In the classical general relativity, geometry is dynamical, but topology and dimension of a spacetime manifold are fixed
  - In quantum gravity, topology and dimension may not remain well defined in the presence of strong quantum fluctuations
- In the AdS/CFT correspondence, gravitational degrees of freedom are collective variables of underlying quantum matter [Maldacena] [Gubser, Klebanov, Polyakov] [Witten]
  - It is plausible that topology and dimension of bulk spacetime are also dynamical in non-perturbative formulations of the bulk theory
- This talk : a model of quantum gravity in which geometry, topology, dimension are dynamical

[earlier works with similar goals :
Quantum graphity, Konopka, Markopoulou, Smolin (06);
Geometry from entanglement, Cao, Carroll, Michalakis(17), ..]

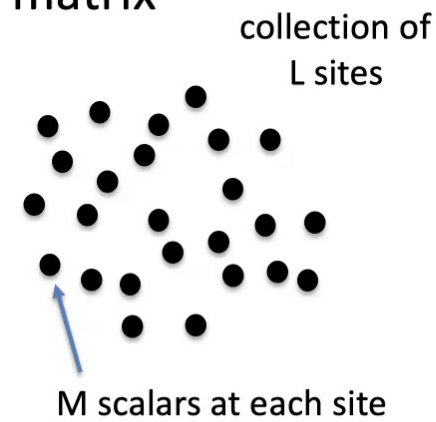
Kinematic Hilbert space

- Fundamental degree of freedom : M × L real matrix

$$\Phi^A{}_i \qquad A = 1, 2, .., M, \quad i = 1, 2, .., L$$

  – row index (A) : flavor
  – column index (i) : site

(M≫L>>1)



- Hilbert space is spanned by $\{\ |\Phi\rangle\ \}$

$$\hat{\Phi}^A{}_i|\Phi\rangle = \Phi^A{}_i|\Phi\rangle \qquad |\Phi\rangle = \otimes_{i,A}|\Phi_i^A\rangle$$

- Inner product : $\langle\Phi'|\Phi\rangle = \prod_{i,A}\delta\left(\Phi'^A_i - \Phi^A{}_i\right)$

Frame

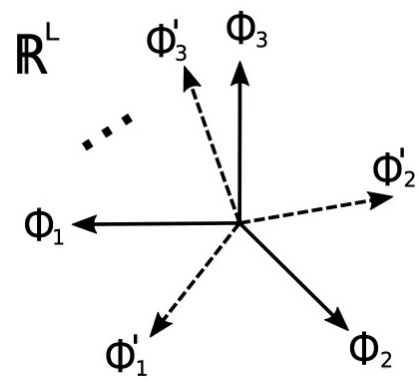
- A decomposition of the total Hilbert space as a direct product of local Hilbert spaces

$$\mathbb{H} = \otimes_i \mathbb{H}_i$$

$$\mathbb{H}_i = \{\otimes_A |\Phi_i^A\rangle\}$$

- Frame can be rotated

$$\Phi'^{A}{}_{I} = \Phi^{A}{}_{i}\, g^{i}_{I} \qquad g \in SL(L, \mathbb{R})$$

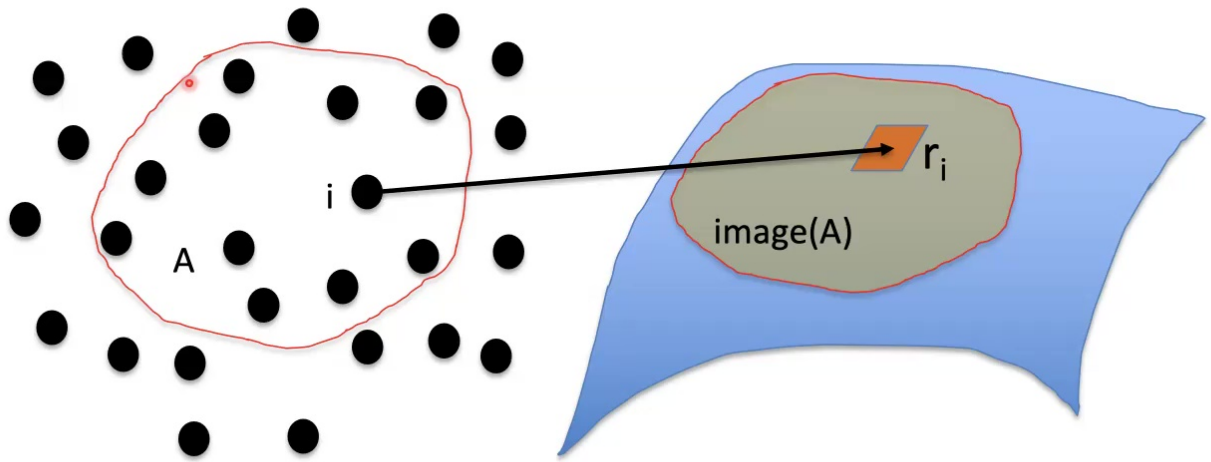


- Total Hilbert space admits alternative partitioning

$$\mathbb{H} = \otimes_I \mathbb{H}'_I$$

- Quantum information stored in one site in a frame is spread over multiple sites in another frame

Local structure



- A state is defined to have a local structure in a frame if
  - there exists a mapping from sites into a Riemannian manifold
  - the state viewed as a state defined on the Riemannian manifold is short-range entangled : the entanglement entropy of any sub-region A is proportional to the volume of ∂(image(A))

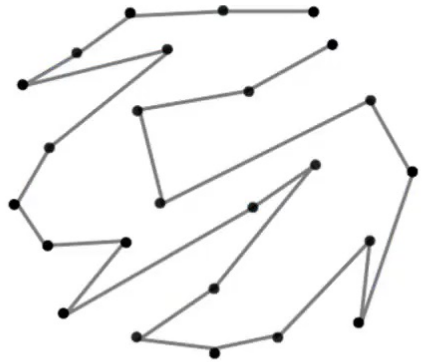
Examples of states with/without local structures

$$|t\rangle = \int d\Phi \; e^{itr[t\Phi^T\Phi]} |\Phi\rangle$$

$$tr[t\Phi^T\Phi] = t^{ij}\Phi^A_i\Phi^A_j$$

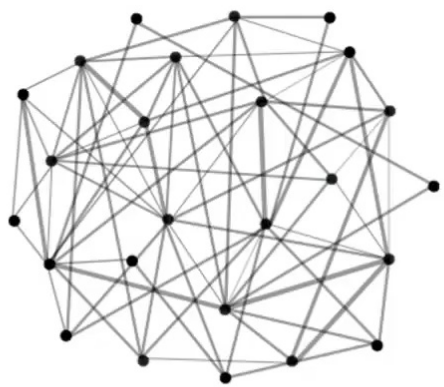
t^{ij} describes entanglement bonds

$$I_{ij} = 2M\left(-\ln\frac{|t^{ij}|^2}{4Imt^{ii}Imt^{jj}}+1\right)\frac{|t^{ij}|^2}{4Imt^{ii}Imt^{jj}} + \ldots$$



1D local structure

No local structure



2D local structure

Generalized spatial diffeomorphism

- In GR, the momentum constraint generates spatial diffeomorphism of a fixed dimension [Arnowitt, Deser, Misner]

$$\left\{P\left[\vec{\xi}_1\right], P\left[\vec{\xi}_2\right]\right\}_{PB} = P\left[\mathscr{L}_{\vec{\xi}_1}\vec{\xi}_2\right] \qquad \vec{\xi} \quad : \quad \text{shift}$$

- In the present theory, the dimension and topology of manifold are all dynamical

- Spatial diffeomorphism should be generalized to include
  - diffeomorphism in any dimension and topology

- SL(L,R) frame rotation

L × M M× L

$$\hat{G}_y = \frac{1}{2}\mathrm{tr}\left(\hat{\Pi}\hat{\Phi}y\right) + h.c.$$

$$\Pi^i{}_A = -i\frac{\partial}{\partial\Phi^A{}_i}$$

y : L × L traceless matrix (shift tensor)

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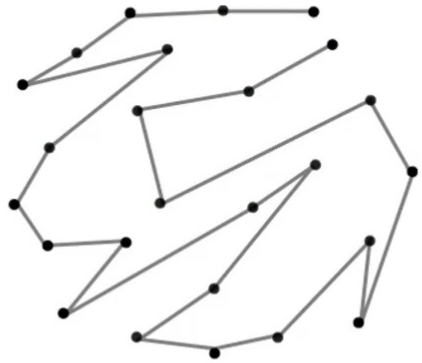
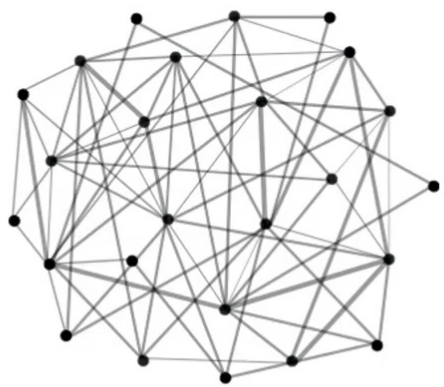


1D local structure

No local structure



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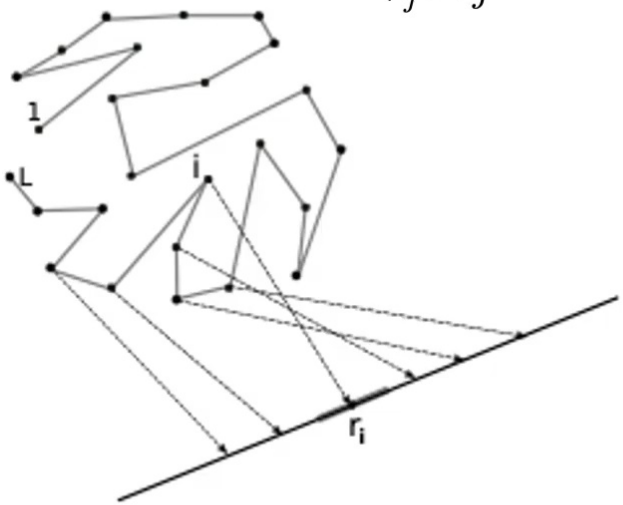
Example

$$|t\rangle = \int d\Phi \, e^{it^{ij}\Phi^A_i \Phi^A_j} |\Phi\rangle$$

$$t^{ij} = i\left(\delta_{ij} + \epsilon\delta_{|i-j|,1}\right)$$

$$r_j = j$$



$$\hat{G}_y = \mathrm{tr}\left\{\hat{G}y\right\}$$ with

$$y^j_i = \frac{\xi_i}{2}\left(\delta_{j,i+1} - \delta_{j,i-1}\right)$$

generates active diffeomorphism
with shift vector $\xi(r_i) = \xi_i$
in the continuum limit

$$e^{-i\hat{G}_{\epsilon y}} \, \hat{\Phi}^A{}_i \, e^{i\hat{G}_{\epsilon y}} = \hat{\Phi}^A(r_i) - \epsilon \, \xi^\mu(r_i)\partial_\mu \hat{\Phi}^A(r_i) + O(\partial^2)$$

Hamiltonian constraint

- In GR, Hamiltonian density transforms as a scalar density under spatial diffeomorphism

$$\left\{P\left[\vec{\xi}\right], H\left[\theta\right]\right\}_{PB} = H\left[\mathscr{L}_{\vec{\xi}}\theta\right] \qquad \theta \quad : \quad \text{lapse}$$

- More importantly, the Poisson bracket between Hamiltonians should be proportional to the momentum constraint

$$\left\{H\left[\theta_1\right], H\left[\theta_2\right]\right\}_{PB} = P\left[\vec{\xi}_{\theta_1,\theta_2}\right] \qquad \xi^{\mu}_{\theta_1,\theta_2} = -\boxed{\mathcal{S} g^{\mu\nu}}\left(\theta_1\nabla_\nu\theta_2 - \theta_2\nabla_\nu\theta_1\right)$$

signature ($\mathcal{S}$, +, +, ..)

Hamiltonian constraint

- A Hamiltonian that satisfies $[H, H] \sim G$ is

$$\hat{H}_v = \mathrm{tr}\left\{\left(-\overset{L\times M}{\hat{\Pi}}\hat{\Pi}^T + \frac{\tilde{\alpha}}{M^2}\hat{\Pi}\hat{\Pi}^T\hat{\Phi}^T\hat{\Phi}\hat{\Pi}\hat{\Pi}^T\right)v\right\}$$

lapse tensor
(symmetric matrix)

Physical meaning

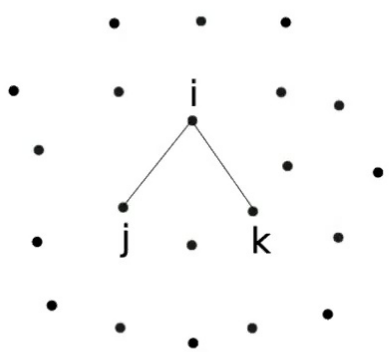
In the frame in which the lapse is diagonal,

diagonal element of v

$$\hat{H}_v = \sum_i S_i \left[-\hat{\Pi}'^i_A \hat{\Pi}'^i_A + \frac{\tilde{\alpha}}{M^2} \sum_{j,k} \hat{\Pi}'^i{}_A \hat{\Pi}'^j{}_A \hat{\Phi}'^B{}_j \hat{\Phi}'^B{}_k \hat{\Pi}'^k{}_C \hat{\Pi}'^i{}_C \right]$$

ultra-local kinetic term

Relatively local hopping term



$$\left\langle \hat{\Pi}'^i{}_A \hat{\Pi}'^j{}_A \right\rangle \hat{\Phi}'^B{}_j \hat{\Phi}'^B{}_k \left\langle \hat{\Pi}'^k{}_C \hat{\Pi}'^i{}_C \right\rangle$$

$$\frac{1}{M} \frac{\langle t | (\hat{\Pi}^j{}_A \hat{\Pi}^i{}_A) | t \rangle}{\langle t | t \rangle} = -2i(t^{-1} - t^{*-1})^{-1}_{ij}$$

the strength of hopping between sites j and k is determined by the amount of entanglement formed between the sites through third sites

$$|t\rangle = \int d\Phi \, e^{it^{ij}\Phi^A_i \Phi^A_j} |\Phi\rangle$$

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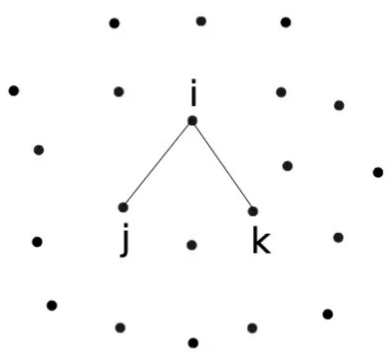

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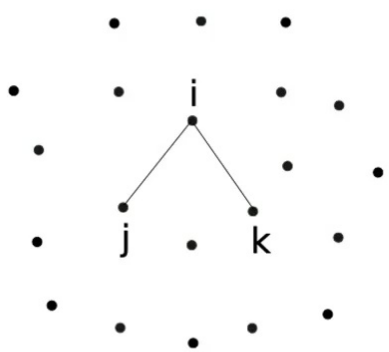

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Constraint Algebra

$$\left[\hat{G}^{i}{}_{j}, \hat{G}^{k}{}_{l}\right] = A^{ikn}_{jlm}\ \hat{G}^{m}{}_{n}$$

$$\left[\hat{G}^{i}{}_{j}, \hat{H}^{kl}\right] = B^{ikl}_{jmn}\ \hat{H}^{mn}$$

sub-leading

$$\left[\hat{H}^{ij}, \hat{H}^{kl}\right] = \hat{C}^{ijkln}_{m}\ \hat{G}^{m}{}_{n} + \frac{1}{M}\hat{D}^{ijkl}_{mn}\hat{H}^{mn}$$

$$\begin{aligned}
A^{ikn}_{jlm} &= \delta^k_j\delta^i_m\delta^n_l - \delta^i_l\delta^k_m\delta^n_j, \\
B^{ikl}_{jmn} &= \delta^k_j\delta^{il}_{mn} + \delta^l_j\delta^{ki}_{mn}, \\
\hat{C}^{ijkln}_{m} &= -4\tilde{\alpha}\left[U^{n[j}U^{i][l}\delta^{k]}_m - U^{n[l}U^{k][j}\delta^{i]}_m\right] \\
&\quad +4\tilde{\alpha}^2\Big[U^{n[j}U^{i]m'}Q_{m'n'}U^{n'[l}\delta^{k]}_m + U^{n[j}U^{i][l}U^{k]m'}Q_{m'n'}\delta^{n'}_m \\
&\qquad -U^{n[l}U^{k]m'}Q_{m'n'}U^{n'[j}\delta^{i]}_m - U^{n[l}U^{k][j}U^{i]m'}Q_{m'n'}\delta^{n'}_m\Big]
\end{aligned}$$

$$U = (\Pi\Pi^T)$$
$$Q = (\Phi^T\Phi)$$

The constraints obey a **first-class quantum algebra**

Constraint algebra in the continuum

$$\left[\hat{G}^i{}_j, \hat{G}^k{}_l\right] = A^{ikn}_{jlm}\, \hat{G}^m{}_n$$
$$\left[\hat{G}^i{}_j, \hat{H}^{kl}\right] = B^{ikl}_{jmn}\, \hat{H}^{mn}$$
$$\left[\hat{H}^{ij}, \hat{H}^{kl}\right] = \hat{C}^{ijkln}_m\, \hat{G}^m{}_n + \frac{1}{M}\hat{D}^{ijkl}_{mn}\hat{H}^{mn}$$

$$\left\{P\left[\vec{\xi}_1\right], P\left[\vec{\xi}_2\right]\right\}_{PB} = P\left[\mathcal{L}_{\vec{\xi}_1}\vec{\xi}_2\right] + O(\partial^2)$$
$$\left\{P\left[\vec{\xi}\right], H\left[\theta\right]\right\}_{PB} = H\left[\mathcal{L}_{\vec{\xi}}\,\theta\right] + O(\partial^2)$$
$$\left\{H\left[\theta_1\right], H\left[\theta_2\right]\right\}_{PB} = P\left[\vec{\xi}_{\theta_1,\theta_2}\right] + O(\partial^2)$$
+(Weyl and higher spin symmetry generators)

$$\mathscr{P}_\mu(r_i) = V_i^{-1}\left.\frac{\partial \mathscr{G}^i{}_j}{\partial r^\mu_j}\right|_{j=i} \qquad \xi^\mu_y(r_i) = \sum_j y^j{}_i(r^\mu_j - r^\mu_i)$$
$$\mathscr{H}(r_i) = V_i^{-1}\mathscr{H}^{ii} \qquad \theta_v(r_i) = v_{ii}$$

(V_i : coordinate volume assigned to site i)

$$-\mathcal{S}\; g^{\mu\nu}(r_m) = \frac{1}{2}\sum_{i,k,n} C^{iikkn}_m\,(r^\mu_n - r^\mu_m)\,(r^\nu_k - r^\nu_i)$$

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Dynamics in a constrained system

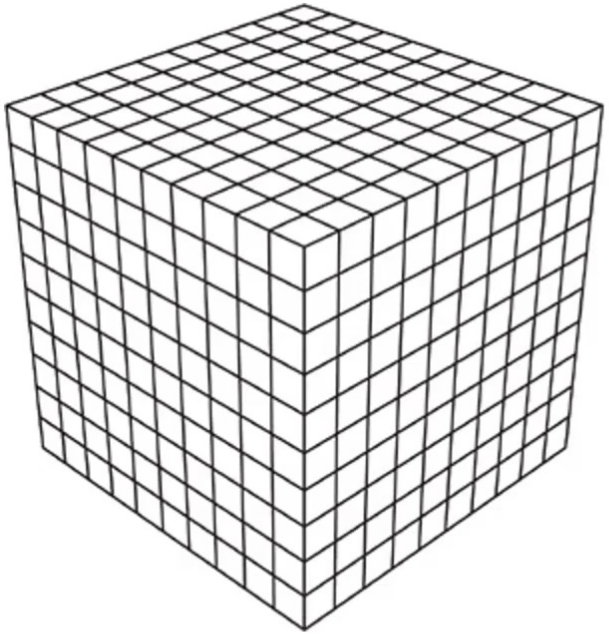
- Gauge invariant states that satisfy $\hat{G}_y|0\rangle = \hat{H}_v|0\rangle = 0$ for all v and y have infinite norm
  - Gauge group is non-compact, and wavefunctions of gauge invariant states are extended in the space of ϕ
  - A gauge invariant states captures a whole spacetime history not just a state at a moment of time

- Dynamical information is encoded in the correlation among kinematic variables along the gauge orbit

$$|\chi(\tau)\rangle = \mathcal{T} e^{-i\int_0^\tau d\tau' \left(\hat{H}_{v(\tau')} + \hat{G}_{y(\tau')}\right)} |\chi\rangle$$

Dynamics

An example : initial state with a classical 3d local structure



- Three torus with nearest neighbor entanglement bonds in three-dimensional local structure
- In the large N,M limit, the fluctuations of the collective variables are small and the semi-classical approximation can be used

$$|t\rangle \sim \int d\Phi \, e^{it^{ij}\Phi^A_i\Phi^A_j} |\Phi\rangle$$

Classical evolution

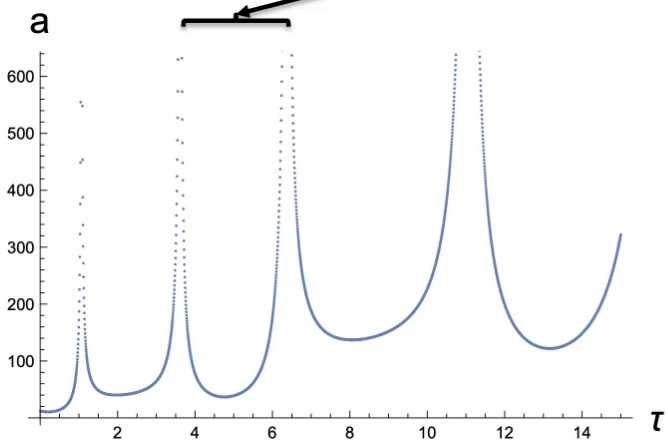
numerical solution for $L=10^6$

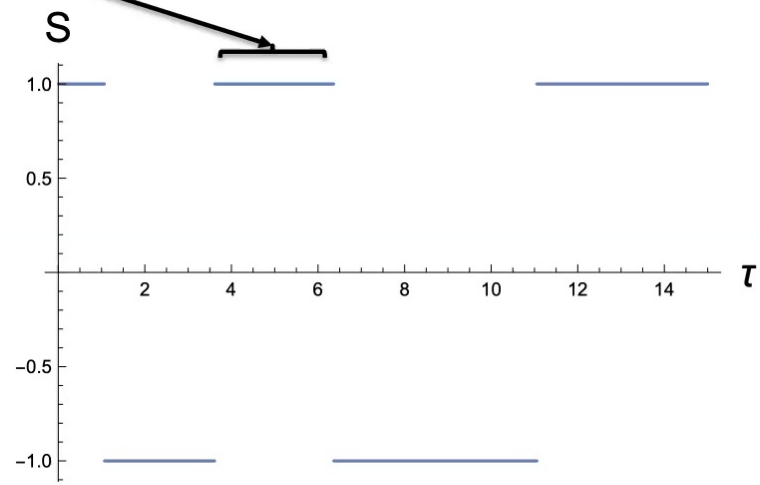
$g_{\mu\nu} = a\delta_{\mu\nu}$

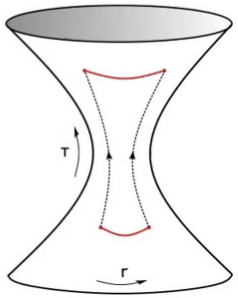
Scale factor of space

Euclidean spacetime

Signature of time







One epoch of de Sitter-like spacetime with Lorentzian signature

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$$\mathscr{P}_\mu(r_i) = V_i^{-1}\left.\frac{\partial \mathscr{G}^i{}_j}{\partial r^\mu_j}\right|_{j=i} \qquad \xi^\mu_y(r_i) = \sum_j y^j_i (r^\mu_j - r^\mu_i)$$

$$\mathscr{H}(r_i) = V_i^{-1}\mathscr{H}^{ii} \qquad \theta_v(r_i) = v_{ii}$$

(V_i : coordinate volume assigned to site i)

$$-\mathcal{S}\, g^{\mu\nu}(r_m) = \frac{1}{2}\sum_{i,k,n} \boxed{C^{iikkn}_m}\,(r^\mu_n - r^\mu_m)\,(r^\nu_k - r^\nu_i)$$

Classical evolution

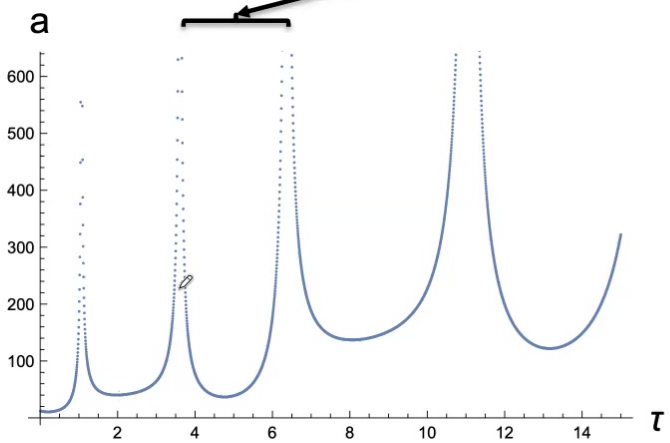
numerical solution for $L=10^6$

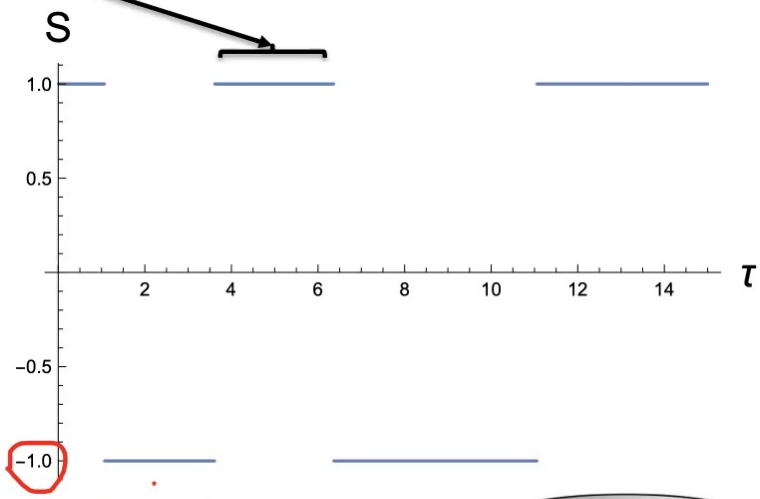
$g_{\mu\nu} = a\delta_{\mu\nu}$

Scale factor of space

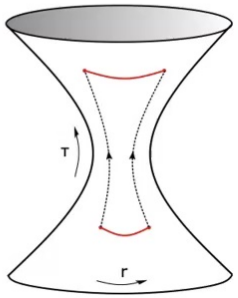
Euclidean spacetime

Signature of time

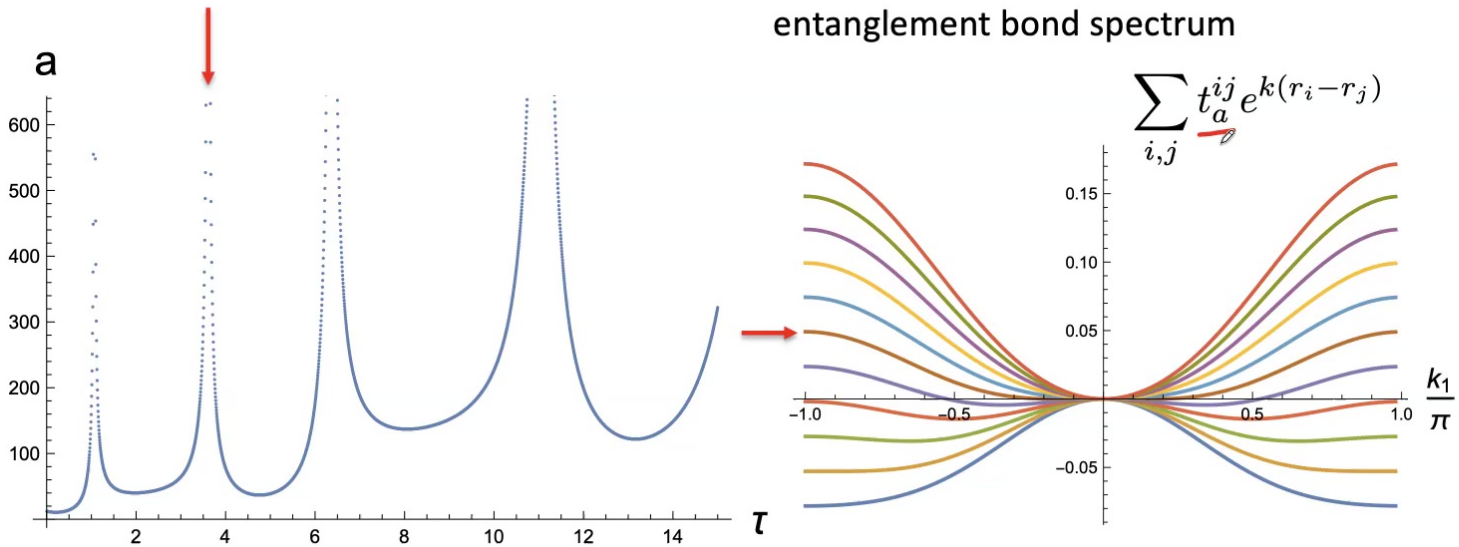




One epoch of de Sitter-like spacetime with Lorentzian signature

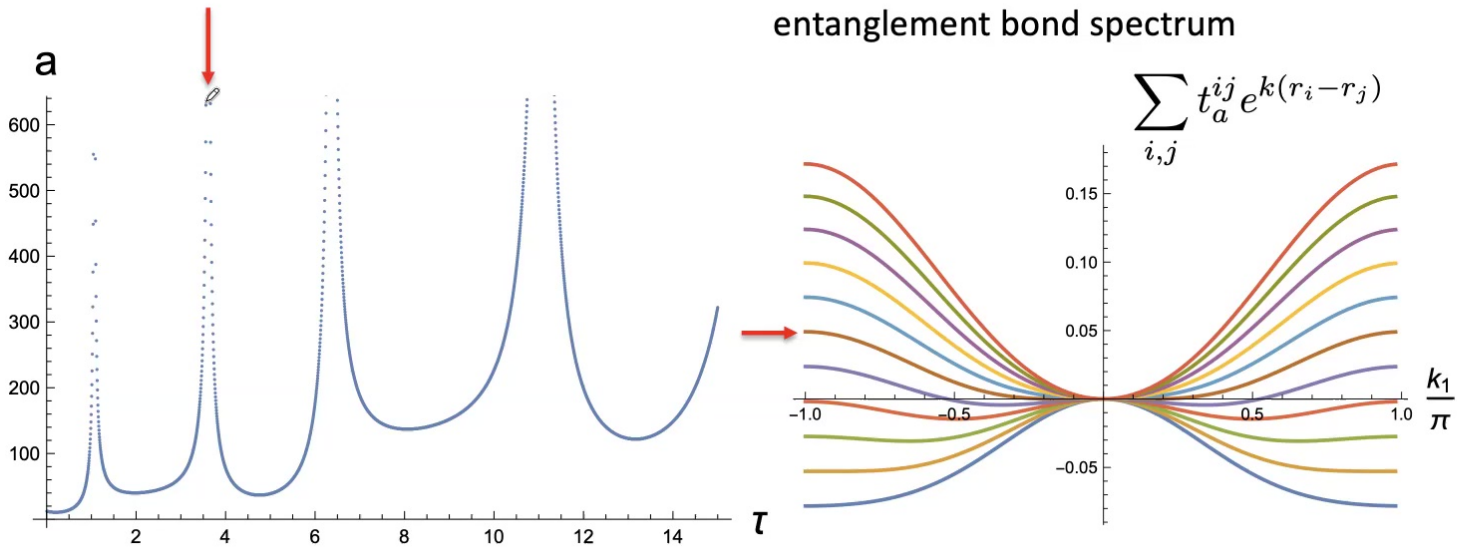


Dynamical Lifshitz transition



- The metric is proportional to the second moment of entanglement bond
- At the critical points, the the entanglement bond spectrum becomes locally flat with zero second moment

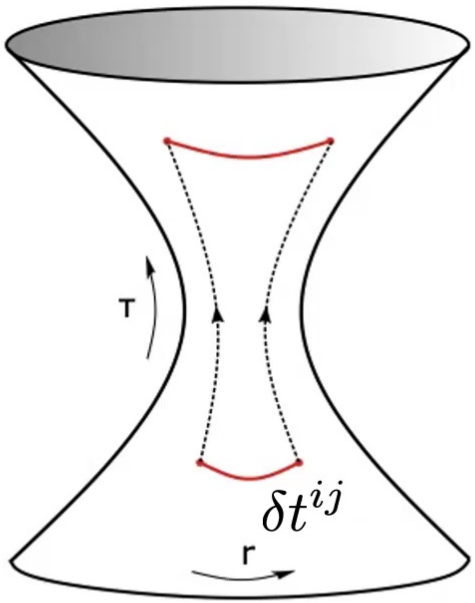
‘ER ⊊ EPR’



- The geometry alone does not fully specify the pattern of entanglement
  - There exist non-geometric entanglement that is not captured by the metric

[cf. ER=EPR, Susskind, Maldacena]

Low-energy effective theory



- Bi-local fields propagate (obeying local dynamics) in the background spacetime formed by the saddle-point configuration
- The theory that describes the excitations is local

Remarks and Open Questions

- Under the enlarged gauge symmetry, the notion of local site is not fixed
  - A site in one frame is made of a linear `superposition' of multiple sites in another frame
  - The spacetime perceived by a set of observers local in one frame is different from the spacetime perceived by another set of observers local in a different frame
- For GR to emerge as an effective theory, the higher order gauge symmetry needs to be spontaneously broken
- Physical spectrum to be understood